Tutorial Sheet 2

Question 1: Filtering (30 pts)

For this question, we'll design filters to perform various operations:

(a) (10 points) Specify the entries of a 3x3 **cross-correlation** kernel that computes this operation: $g(x,y) = [f(x,y+1) - f(x,y-1)] / 2$. Assume x increases left-to-right, y increases bottom-to-top.

What kind of operation does this perform?

(b) (5 points) Specify the **convolution** kernel that performs the same operation: $g(x,y) = [f(x,y+1) - f(x,y-1)] / 2$.

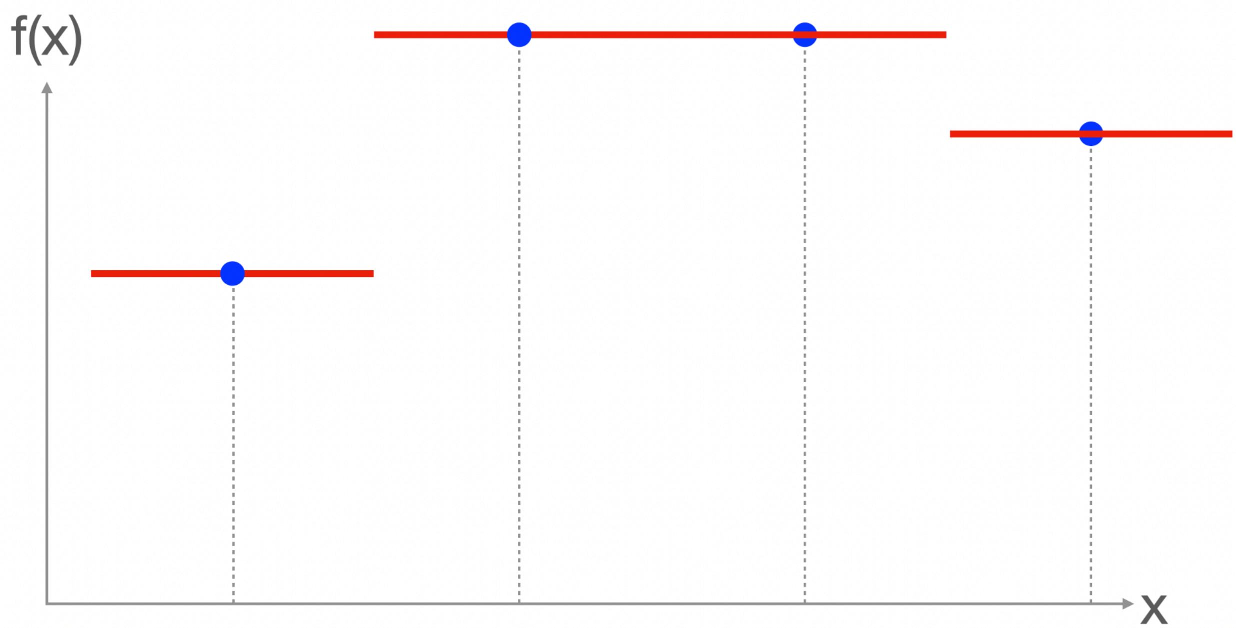
(c) (5 points) Specify the entries of a 3x3 **cross-correlation** kernel that translates the image one pixel to the right. I.e., after the filter is applied, each pixel should move one pixel to the right.

(d) (5 points) Specify the entries of a *single* 3x3 **cross-correlation** kernel that performs a 3x3 mean operation and multiplies each pixel value by 3.

(e) (5 points) Specify the 1D **cross-correlation** kernel that performs the interpolation operation shown below. The blue points are input points, and the red line segments are the result of performing a 1D cross-correlation with your filter kernel – the filter draws a line segment from ½ unit to the left to ½ unit to the right of each input point. Define your kernel h(u) mathematically. For example, here's the definition of the hat "connect the dots" filter that we discussed in class/cartoon:

$h(u) = u+1$, for $u \in [-1,0)$
$\quad 1-u$, for $u \in [0,1]$
$\quad 0$, otherwise

Give your kernel here:
h(u) =



Question 2: Shading (26 pts)

Suppose **N** is the unit surface normal, **V** the unit vector pointing towards the viewer, and **L** the unit vector pointing toward the light source.

(a) (5 points) A retro-reflective surface reflects most of its light back towards the light source. Give an equation for the intensity of a *diffuse* retro-reflector. I.e., your equation should be brightest when **V** = **L**, and fall off gradually as these two vectors diverge. It should not go below 0. Your equation should have a form that's similar to the equations for diffuse or specular reflection from class.

I(**N**,**V**,**L**) =

(b) (5 points) Give an equation for the intensity of a surface that's brightest on the silhouette – points where the surface normal is perpendicular to the viewer, and falls off gradually away from the silhouette. It should not go below 0.

I(**N**,**V**,**L**) =

(c) (5 points) Suppose you have a surface that's given by this equation from class

$$k_d max\{0, \mathbf{N}\cdot\mathbf{L}\} + k_s max\{0, (\mathbf{H}\cdot\mathbf{N})^\alpha\}$$

Assume k_d, k_s, and α are scalar values. What values of **V** and **L** would produce the brightest possible spot at a point with normal **N**?

(d) (6 points) Given the formula in part (c), specify the values of k_d, k_s, and α that you would set to achieve the following:

(i) A surface that reflects light equally in all directions (above the surface).

(ii) A surface that reflects light in only one direction

(e) (5 points) Now suppose you're modeling three channel RGB images, and $\mathbf{k_d}$ and $\mathbf{k_s}$ are now 3D vectors:

$$\mathbf{k_d} max\{0, \mathbf{N}\cdot\mathbf{L}\} + \mathbf{k_s} max\{0, (\mathbf{H}\cdot\mathbf{N})^\alpha\}$$

How would you set $\mathbf{k_d}$ and $\mathbf{k_s}$ to create a diffuse blue surface with a white highlight?

Question 3: Projection (30 pts)

In this question you'll compute vanishing points through a series of steps.
Points in homogeneous coordinates are equivalent up to scale. I.e., this point is

$$s\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Represents the same scene point as

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

(a) (5 points) Consider the point along the Z-axis distance *t* units from the origin:

$$\begin{bmatrix} 0 \\ 0 \\ t \\ 1 \end{bmatrix}$$

Since homogeneous vectors are equivalent up to scale, divide this vector by t, and take the limit as t->∞. What 4D homogeneous vector do you get? (your answer should *not* contain ∞)

(b) (5 points) Your answer to part (a) is a point at infinity. The projection of this point into the image is a *vanishing point*. What is the 2D projection of this point in the image? You should give your answer as a 2D vector (x,y), *not* in homogeneous coordinates. Assume the projection matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Is this vanishing point at a finite or infinite position in the image?

(c) (10 points) Consider the line

$$\begin{bmatrix} X + tA \\ Y + tB \\ Z + tC \\ 1 \end{bmatrix}$$

What is the vanishing point for this line? Assume the same projection matrix as part (b). Express your answer in 2D (not homogeneous coordinates), of the form (x, y). Show your work.

(d) (10 points) An orthographic projection of a point

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

is

$$\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Give the 3x4 projection matrix corresponding to orthographic projection. How is an orthographic projection different from perspective projection?