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APPLIED THERMODYNAMICS

Onkar Singh



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APPLIED THERMODYNAMICS

(THIRD EDITON)

Onkar Singh

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Preface to the Third Edition

I feel extremely encouraged at the good response to this textbook. Looking upon the feed back received from its readers third edition of the book is being presented here.

In this edition number of solved and unsolved problems have been added in some of the chapters and a few new topics have also been added.

I wish to express my sincere thanks to Professors and students for their valuable suggestions and recommending the book to their students and friends.

I strongly feel that the book would prove to be further useful to students. I would be obliged for the errors, omissions and suggestions brought to my notice for improvement of the book in its next edition.

Onkar Singh

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Preface to the First Edition

During teaching of the course of engineering thermodynamics and applied thermodynamics I have felt that the students at the undergraduate level of engineering and technology face difficulty in understanding the concepts of engineering thermodynamics and their applications in the course of applied thermodynamics. Also, the students face great difficulty in referring to the number of text-books for different topics. The present book is an effort in the direction of presenting the concepts of engineering thermodynamics and their applications in clear, concise and systematic manner at one place. Presentation is made in very simple and easily understandable language and well supported with wide ranging illustrations and numerical problems.

The subject matter in this book covers the syllabus of the basic and advanced course on engineering thermodynamics/thermal engineering being taught in different institutions and universities across the country. There are total 18 chapters in this book. The initial seven chapters cover the basic course on engineering thermodynamics and remaining chapters cover the advanced course in thermal engineering. These deal with “Fundamental concepts and definitions”, “Zeroth law and thermodynamics”, “First law of thermodynamics”, “Second law of thermodynamics”, “Entropy”, “Availability and general thermodynamic relations”, “Thermodynamic properties of pure substances”, “Fuels and combustion”, “Boilers and boiler calculations”, “Vapour power cycles”, “Gas power cycles”, “Steam engines”, “Nozzles”, “Steam turbines, Steam condenser”, “Reciprocating and rotary compressors”, “Introduction to internal combustion engines” and “Introduction to refrigeration and air conditioning”. Each chapter has been provided with sufficient number of typical numerical problems of solved and unsolved type. The book is written in SI system of units and the various tables such as steam tables, refrigeration tables, Mollier chart, psychrometry chart etc. are also provided at the end of the book for quick reference. I hope that the students and teachers referring to this book will find it useful.

I am highly indebted to my family members for their continuous encouragement and cooperation during the preparation of manuscript. I would like to place on record my gratitude and apologies to my wife Parvin and kids Sneha and Prateek who patiently endured certain neglect and hardships due to my preoccupation with the preparation of this manuscript.

I am thankful to AICTE, New Delhi for the financial support provided to me in the Young Teacher Career Award.

I am also thankful to Mr. L.N. Mishra and other staff members of New Age International for their cooperation throughout the preparation of the textbook. At the end I thank to all those who supported directly or indirectly in the preparation of this book.

I shall be extremely grateful to all the readers of text book for their constructive criticism, indicating any errors and omissions etc. for improving its quality and form.

Onkar Singh

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Fundamental Concepts and Definitions

1.1 INTRODUCTION AND DEFINITION OF THERMODYNAMICS

Thermodynamics is a branch of science which deals with energy. Engineering thermodynamics is modified name of this science when applied to design and analysis of various energy conversion systems. Thermodynamics has basically a few fundamental laws and principles applied to a wide range of problems. Thermodynamics is core to engineering and allows understanding of the mechanism of energy conversion. It is really very difficult to identify any area where there is no interaction in terms of energy and matter. It is a science having its relevance in every walk of life. Thermodynamics can be classified as 'Classical thermodynamics' and 'Statistical thermodynamics'. Here in engineering systems analysis the classical thermodynamics is employed.

“Thermodynamics is the branch of physical science that deals with the various phenomena of energy and related properties of matter, especially of the laws of transformations of heat into other forms of energy and vice-versa.”

Internal combustion engines employed in automobiles are a good example of the energy conversion equipments where fuel is being burnt inside the piston cylinder arrangement and chemical energy liberated by the fuel is used for getting the shaft work from crankshaft. Thermodynamics lets one know the answer for the questions as, what shall be the amount of work available from engine?, what shall be the efficiency of engine?, etc.

For analysing any system there are basically two approaches available in engineering thermodynamics. Approach of thermodynamic analysis means how the analyser considers the system. Macroscopic approach is the one in which complete system as a whole is considered and studied without caring for what is there constituting the system at microscopic level.

Contrary to this the microscopic approach is one having fragmented the system under consideration upto microscopic level and analysing the different constituent subsystems/microsystems. In this approach study is made at the microscopic level. For studying the system the microlevel studies are put together to see the influences on overall system. Thus, the statistical techniques are used for integrating the studies made at microscopic level. This is how the studies are taken up in statistical thermodynamics. In general it can be said that, Macroscopic approach analysis = Σ (Microscopic approach analysis).

1.2 DIMENSIONS AND UNITS

“Dimension” refers to certain fundamental physical concepts that are involved in the process of nature and are more or less directly evident to our physical senses, thus dimension is used for characterizing

any physical quantity. Dimensions can be broadly classified as “primary dimensions” and “secondary or derived dimensions”. “Basic dimensions such as mass ‘M’, length ‘L’, time ‘t’ and temperature ‘T’ are called primary dimensions, while quantities which are described using primary dimensions are called secondary dimensions such as for energy, velocity, force, volume, etc”.

“Units” are the magnitudes assigned to the dimensions. Units assigned to “primary dimensions” are called “basic units” whereas units assigned to “secondary dimensions” are called “derived units”. Various systems of units have prevailed in the past such as FPS (Foot-Pound-Second), CGS (Centimetre-Gram-Second), MKS (Metre-Kilogram-Second) etc. but at present SI system (System-International) of units has been accepted world wide. Here in the present text also SI system of units has been used. Following table gives the basic and derived units in SI system of units.

Table 1.1 *SI system of units*

<i>Quantity</i>	<i>Unit</i>	<i>Symbol</i>
Basic Units		
Length (L)	Metre	m
Mass (M)	Kilogram	kg
Time (t)	Second	s
Temperature (T)	Kelvin	K
Plane angle	Radian	rad
Solid angle	Steradian	sr
Luminous intensity	Candela	cd
Molecular substance	Mole	mol.
Electric Current	Ampere	A
Derived Units		
Force (F)	Newton	N {kg.m/s ² }
Energy (E)	Joule	J {N.m = kg. m ² /s ² }
Power	Watt	W {J/s = kg. m ² /s ³ }
Pressure	Pascal	Pa {N/m ² = kg/(ms ²)}

Equivalence amongst the various systems of unit for basic units is shown in table 1.2.

Table 1. 2 *Various systems of units*

Unit - (Symbol)

<i>Quantity</i>	<i>SI</i>	<i>MKS</i>	<i>CGS</i>	<i>FPS</i>
Length	Metre (m)	Metre (m)	Centimetre (cm)	Foot (ft)
Mass	Kilogram (kg)	Kilogram (kg)	Gram (gm)	Pound (lb)
Time	Second (s)	Second (s)	Second (s)	Second (s)
Temperature	Kelvin (K)	Centigrade (°C)	Centigrade (°C)	Fahrenheit (°F)

The various prefixes used with SI units are given as under :

Prefix	Factor	Symbol	Prefix	Factor	Symbol
deca	10	da	deci	10 ⁻¹	d
hecto	10 ²	h	centi	10 ⁻²	c
kilo	10 ³	k	milli	10 ⁻³	m
mega	10 ⁶	M	micro	10 ⁻⁶	μ
giga	10 ⁹	G	nano	10 ⁻⁹	n
tera	10 ¹²	T	pico	10 ⁻¹²	p
peta	10 ¹⁵	P	femto	10 ⁻¹⁵	f
exa	10 ¹⁸	E	atto	10 ⁻¹⁸	a

The conversion table for one unit into the other is given in table 1.3.

Table 1.3 Unit conversion table

1 ft = 0.3048 m	1 ft ² = 0.09290 m ²
1 in = 0.0254 m	1 in ² = 6.45 cm ²
1 lb = 453.6 gm	1 lb = 0.4536 kg
1 lbf = 4.45 N	1 kgf = 9.81 N
1 lbf/in ² = 6.989 kN/m ² = 0.0689 bar = 703 kgf/m ²	
1 bar = 10 ⁵ N/m ² = 14.5038 lbf/in ² = 0.9869 atm	
= 1.0197 kgf/cm ²	
1 ft. lbf = 1.356 Joules	
1 Btu = 778.16 ft. lbf = 1.055 kJ	
1Btu/lb = 2.326 kJ/kg	
1 ft ³ /lb = 0.0624 m ³ /kg,	1 Cal = 4.18 J

1.3 CONCEPT OF CONTINUUM

In Macroscopic approach of thermodynamics the substance is considered to be continuous whereas every matter actually comprises of myriads of molecules with intermolecular spacing amongst them. For analyzing a substance in aggregate it shall be desired to use laws of motion for individual molecules and study at molecular level be put together statistically to get the influence upon aggregate. In statistical thermodynamics this microscopic approach is followed, although it is often too cumbersome for practical calculations.

In engineering thermodynamics where focus lies upon the gross behaviour of the system and substance in it, the statistical approach is to be kept aside and classical thermodynamics approach be followed. In classical thermodynamics, for analysis the atomic structure of substance is considered to be continuous. For facilitating the analysis this concept of continuum is used in which the substance is treated free from any kind of discontinuity. As this is an assumed state of continuum in substance so the order of analysis or scale of analysis becomes very important. Thus, in case the scale of analysis is large enough and the discontinuities are of the order of intermolecular spacing or mean free path then due to relative order of discontinuity being negligible it may be treated continuous.

In the situations when scale of analysis is too small such that even the intermolecular spacing or mean free path are not negligible i.e. the mean free path is of comparable size with smallest significant dimension in analysis then it can not be considered continuous and the microscopic approach for analysis should be followed. For example, whenever one deals with highly rarefied gases such as in rocket flight at very high altitudes or electron tubes, the concept of continuum of classical thermodynamics

should be dropped and statistical thermodynamics using microscopic approach should be followed. Thus, in general it can be said that the assumption of continuum is well suited for macroscopic approach where discontinuity at molecular level can be easily ignored as the scale of analysis is quite large. The concept of continuum is thus a convenient fiction which remains valid for most of engineering problems where only macroscopic or phenomenological informations are desired.

For example, let us see density at a point as a property of continuum. Let us take some mass of fluid Δm in some volume ΔV enveloping a point 'P' in the continuous fluid. Average mass density of fluid within volume ΔV shall be the ratio $(\Delta m/\Delta V)$. Now let us shrink the volume ΔV enveloping the point to volume $\Delta V'$. It could be seen that upon reducing the volume, $\Delta V'$ may be so small as to contain relatively few molecules which may also keep on moving in and out of the considered very small volume, thus average density keeps on fluctuating with time. For such a situation the definite value of density can not be given.

Therefore, we may consider some limiting volume ΔV_{limit} such that the fluid around the point may be treated continuous and the average density at the point may be given by the ratio $\left(\frac{\Delta m}{\Delta V_{\text{limit}}} \right)$.

Thus, it shows how the concept of continuum although fictitious is used for defining density at a point as given below,

$$\text{Average density at the point} = \lim_{\Delta V \rightarrow \Delta V_{\text{limit}}} \left(\frac{\Delta m}{\Delta V} \right)$$

1.4 SYSTEMS, SURROUNDINGS AND UNIVERSE

In thermodynamics the 'system' is defined as the quantity of matter or region in space upon which the attention is concentrated for the sake of analysis. These systems are also referred to as thermodynamic systems. For the study these systems are to be clearly defined using a real or hypothetical boundary. Every thing outside this real/hypothetical boundary is termed as the 'surroundings'. Thus, the surroundings may be defined as every thing surrounding the system. System and surroundings when put together result in universe.

$$\text{Universe} = \text{System} + \text{Surroundings}$$

The system is also some times defined as the control system and the boundary defined for separating it from surroundings is called control boundary, the volume enclosed within the boundary is control volume and the space enclosed within the boundary is called control space.

Based on the energy and mass interactions of the systems with surroundings/other systems across the boundary the system can be further classified as the open, close, and isolated system. The 'open system' is one in which the energy and mass interactions take place at the system boundary, for example automobile engine etc.

'Closed system' is the system having only energy interactions at its boundary, for example, boiling water in a closed pan etc. The mass interactions in such system are absent. 'Isolated system' refers to the system which neither has mass interaction nor energy interaction across system boundary, for example Thermos Flask etc. Thus, the isolated system does not interact with the surroundings/systems in any way.

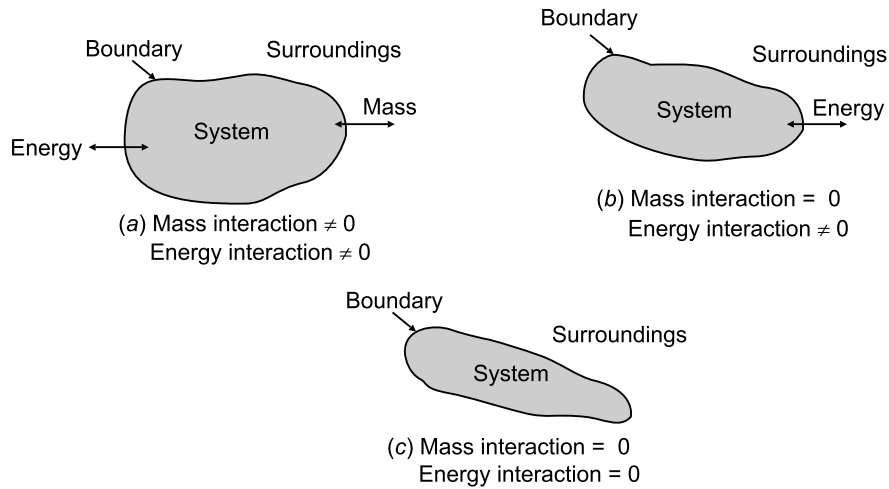


Fig. 1.1 (a) Open system (b) Closed system (c) Isolated system

1.5 PROPERTIES AND STATE

For defining any system certain parameters are needed. ‘Properties’ are those observable characteristics of the system which can be used for defining it. Thermodynamic properties are observable characteristics of the thermodynamic system. Pressure, temperature, volume, viscosity, modulus of elasticity etc. are the examples of property. These properties are some times observable directly and some times indirectly. Properties can be further classified as the ‘intensive property’ and ‘extensive property’. The *intensive properties* are those properties which have same value for any part of the system or the properties that are independent of the mass of system are called intensive properties, e.g. pressure, temperature etc. *Extensive properties* on the other hand are those which depend upon the mass of system and do not maintain the same value for any path of the system. e.g. mass, volume, energy, enthalpy etc. These extensive properties when estimated on the unit mass basis result in intensive property which is also known as *specific property*, e.g. specific heat, specific volume, specific enthalpy etc.

‘State’ of a system indicates the specific condition of the system. To know the characteristics of the system quantitatively refers to knowing the state of system. Thus, when the properties of system are quantitatively defined then it refers to the ‘state’. For completely specifying the state of a system number of properties may be required which depends upon the complexity of the system. Thermodynamic state in the same way refers to the quantitative definition of the thermodynamic properties of a thermodynamic system e.g. for defining a gas inside the cylinder one may have to define the state using pressure and temperature as 12 bar, 298 K. When the thermodynamic properties defining a state undergo a change in their values it is said to be the ‘change of state’.

1.6 THERMODYNAMIC PATH, PROCESS AND CYCLE

Thermodynamic system undergoes changes due to the energy and mass interactions. Thermodynamic state of the system changes due to these interactions. The mode in which the change of state of a system takes place is termed as the *process* such as constant pressure process, constant volume process etc. Let us take gas contained in a cylinder and being heated up. The heating of gas in the cylinder shall result in change in state of gas as its pressure, temperature etc. shall increase. However, the mode in

which this change of state in gas takes place during heating shall be constant volume mode and hence the process shall be called constant volume heating process.

The *path* refers to the series of state changes through which the system passes during a process. Thus, path refers to the locii of various intermediate states passed through by a system during a process.

Cycle refers to a typical sequence of processes in such a fashion that the initial and final states are identical. Thus, a cycle is the one in which the processes occur one after the other so as to finally bring the system at the same state. Thermodynamic path in a cycle is in closed loop form. After the occurrence of a cyclic process system shall show no sign of the processes having occurred.

Mathematically, it can be said that the cyclic integral of any property in a cycle is zero, i.e.,

$$\oint dp = 0, \text{ where } p \text{ is any thermodynamic property.}$$

Thermodynamic processes, path and cycle are shown on p - v diagram in Fig. 1.2

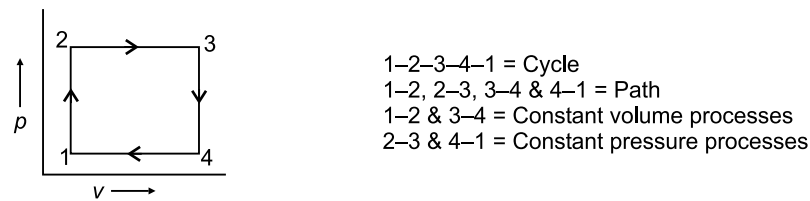


Fig. 1.2 Thermodynamic process, path and cycle.

1.7 THERMODYNAMIC EQUILIBRIUM

Equilibrium of a system refers to the situation in which its “state” does not undergo any change in itself with passage of time without the aid of any external agent. Equilibrium state of a system can be examined by observing whether the change in state of the system occurs or not. If no change in state of system occurs then the system can be said in equilibrium. Thermodynamic equilibrium is a situation in which thermodynamic system does not undergo any change in its state. Let us consider a steel glass full of hot milk kept in open atmosphere. It is quite obvious that the heat from milk shall be continuously transferred to atmosphere till the temperature of milk, glass and atmosphere are not alike. During the transfer of heat from milk the temperature of milk could be seen to decrease continually. Temperature attains some final value and does not change any more. This is the equilibrium state at which the properties stop showing any change in themselves.

Generally, thermodynamic equilibrium of a system may be ensured by ensuring the mechanical, thermal, chemical and electrical equilibriums of the system. ‘Mechanical equilibrium’ of the system can be well understood from the principles of applied mechanics which say that the net force and moment shall be zero in case of such equilibrium. Thus, in the state of mechanical equilibrium the system does not have any tendency to change mechanical state as it is the state at which the applied forces and developed stresses are fully balanced.

‘Thermal equilibrium’ is that equilibrium which can be stated to be achieved if there is absence of any heat interactions. Thus, if the temperature states of the system do not change then thermal equilibrium is said to be attained. Equality of temperature of the two systems interacting with each other shall ensure thermal equilibrium.

‘Chemical equilibrium’ is the one which can be realized if the chemical potential of the systems interacting are same. The equality of forward rate of chemical reaction and backward rate of chemical reaction can be taken as criterion for ensuring the chemical equilibrium. Similar to this, in case the electrical potential of the systems interacting are same, the ‘electrical equilibrium’ is said to be attained.

Thus, a system can be said to be in thermodynamic equilibrium if it is in mechanical, thermal, chemical and electrical equilibrium.

1.8 REVERSIBILITY AND IRREVERSIBILITY

Thermodynamic processes may have the change of state occurring in two ways. One is the change of state occurring so that if the system is to restore its original state, it can be had by reversing the factors responsible for occurrence of the process. Other change of state may occur such that the above restoration of original state is not possible. Thermodynamic system that is capable of restoring its original state by reversing the factors responsible for occurrence of the process is called reversible system and the thermodynamic process involved is called *reversible process*. Thus, upon reversal of a process there shall be no trace of the process being occurred, i.e. state changes during the forward direction of occurrence of a process are exactly similar to the states passed through by the system during the reversed direction of the process. It is quite obvious that the such reversibility can be realised only if the system maintains its thermodynamic equilibrium throughout the occurrence of process.

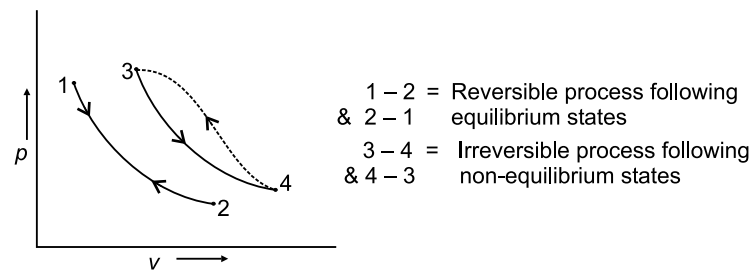


Fig. 1.3 Reversible and Irreversible processes

The *irreversibility* is the characteristics of the system which forbids system from retracing the same path upon reversal of the factors causing the state change. Thus, irreversible systems are those which do not maintain equilibrium during the occurrence of a process. Various factors responsible for the nonattainment of equilibrium are generally the reasons responsible for irreversibility. Presence of friction, dissipative effects etc. have been identified as a few of the prominent reasons for irreversibility. The reversible and irreversible processes are shown on p - v diagram in Fig. 1.3 by '1–2 and 2–1' and '3–4 and 4–3' respectively.

1.9 QUASI-STATIC PROCESS

Thermodynamic equilibrium of a system is very difficult to be realised during the occurrence of a thermodynamic process. It may be understood that this kind of equilibrium is rather practically impossible. In case such equilibrium could not be attained then the thermodynamic analysis cannot be done, as the exact analysis of a system not in equilibrium is impossible. 'Quasi-static' consideration is one of the ways to consider the real system as if it is behaving in thermodynamic equilibrium and thus permitting the thermodynamic study. Actually, system does not attain thermodynamic equilibrium only certain assumptions make it akin to a system in equilibrium, for the sake of study and analysis.

Quasi-static literally refers to "almost static" and the infinite slowness of the occurrence of a process is considered as the basic premise for attaining near equilibrium in the system. Here it is considered that the change in state of a system occurs at infinitely slow pace, thus consuming very large time for completion of the process. During the dead slow rate of state change the magnitude of change in a state

shall also be infinitely small. This infinitely small change in state when repeatedly undertaken one after the other consecutively, results in overall state change. Quasi-static process is considered to remain in thermodynamic equilibrium just because of infinitesimal state changes taking place during the occurrence of the process. Quasi static process can be understood from the following example.

Let us consider the heating of gas in a container with certain mass ' W ' kept on the top of lid (lid is such that it does not permit leakage across its interface with vessel wall) of the vessel as shown in Fig. 1.4. After certain amount of heat being added to the gas it is found that the lid gets raised up. Thermodynamic state change is shown in figure. The "change in state" is significant. During the change of state since the states could not be considered to be in equilibrium, hence for unsteady state of system, thermodynamic analysis could not be extended. Let us now assume that the total mass comprises of infinitesimal small masses of ' w ' such that all ' w ' masses put together become equal to W . Now let us start heat addition to vessel and as soon as the lifting of lid is observed put first fraction mass ' w ' over the lid so as to counter the lifting and estimate the state change. During this process it is found that the state change is negligible. Let us further add heat to the vessel and again put the second fraction mass ' w ' as soon as the lift is felt so as to counter it. Again the state change is seen to be negligible. Continue with the above process and at the end it shall be seen that all fraction masses ' w ' have been put over the lid, thus

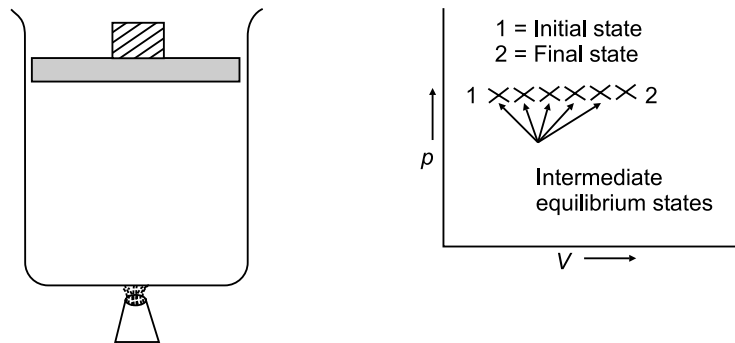


Fig. 1.4 Quasi static process

amounting to mass ' W ' kept over the lid of vessel and the state change occurred is exactly similar to the one which occurred when the mass kept over the lid was ' W '. In this way the equilibrium nature of system can be maintained and the thermodynamic analysis can be carried out. p - v representation for the series of infinitesimal state changes occurring between states 1 and 2 is shown in Fig. 1.4.

1.10 SOME THERMODYNAMIC PROPERTIES

Pressure, temperature, density, volume etc. are some of the thermodynamic properties frequently used. Pressure is defined as the force per unit area. Mathematically, it can be given by the ratio of force applied on a area (say F) divided by the area (say A) as ;

$$p = F/A, (\text{N/m}^2).$$

In general during analysis one comes across the following four types of pressure,

- | | |
|--------------------------|------------------------|
| (i) Atmospheric pressure | (ii) Absolute pressure |
| (iii) Gauge pressure | (iv) Vacuum pressure |

Atmospheric pressure is defined as the pressure exerted by the atmosphere. It is said to be equal to 760 mm of mercury column at 0°C for mercury density of $0.0135951 \text{ kg/cm}^3$, gravitational acceleration of 9.80665 m/s^2 and has magnitude of 1.013 bar ($= 1.013 \times 10^5 \text{ N/m}^2$). The instrument used for

measuring this pressure is called *barometer*. Italian scientist Torricelli was first to construct the barometer to measure the pressure. In his honour the pressure exerted by one millimeter column of mercury under atmospheric conditions is known as 'Torr' ($1 \text{ atm} = 760 \text{ Torr}$).

Absolute pressure of gas refers to the actual pressure of the gas. Let us consider a U-tube manometer as shown in Fig. 1.5. It shows the manometer with its one limb connected to bulb containing the gas while other limb is open to atmosphere. Fig. 1.5a describes a special case in which the pressure of the gas is more than the atmospheric pressure and it is the reason for the rise in level of mercury in the open limb. The difference in the pressure of fluid and atmosphere which is measurable by the rise of mercury column ($= h.d.g.$ where h is the rise in mercury column, d is the density of mercury, g is the gravitational acceleration) is known as the *Gauge pressure*. Mathematically, it can be shown that,

$$\text{Absolute pressure} = \text{Atmospheric pressure} + \text{Gauge pressure}$$

Figure 1.5b shows another typical case in which the pressure of gas is less than the atmospheric pressure and that is why the mercury column is depressed in the open limb. The amount by which the pressure of gas is less than the atmospheric pressure is called *Vacuum pressure*. Thus, the vacuum pressure is the negative gauge pressure. Mathematically it can be shown by,

$$\text{Absolute pressure} = \text{Atmospheric pressure} - \text{Vacuum pressure}$$

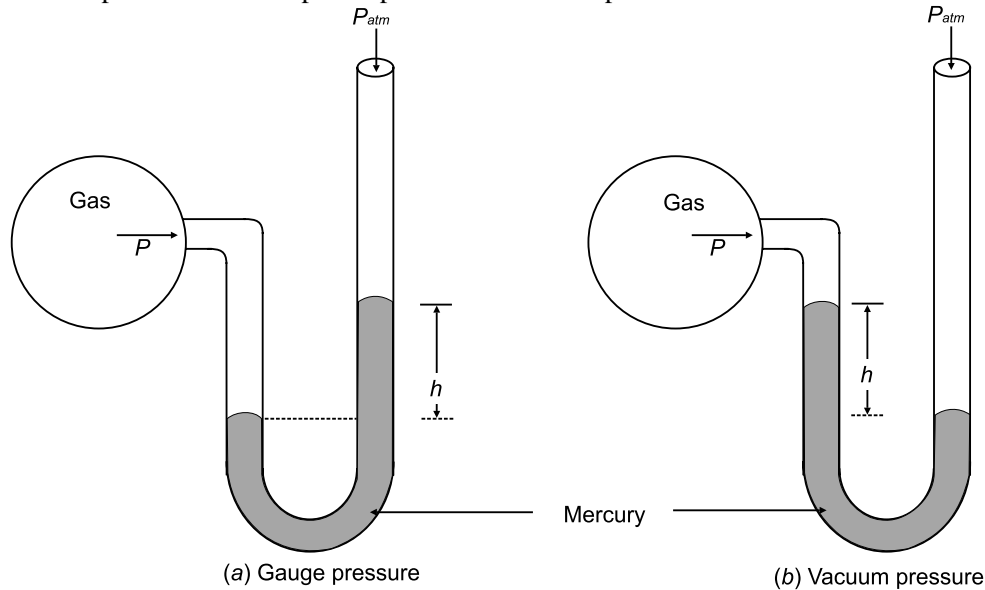


Fig. 1.5 U-tube manometer

The bar chart shown in Fig. 1.6 further clarifies the interrelationship amongst the different pressures.

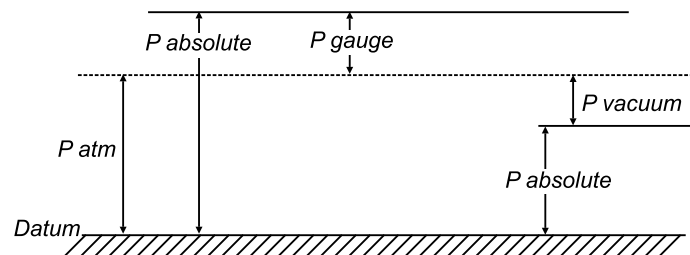


Fig. 1.6 Different pressures

Pressure could also be measured by a Bourdan tube. Bourdan tube has a flattened cross section (oval) closed at one end. Other end of tube connects to the region whose pressure is to be measured. Gas whose pressure is being measured acts on inside of tube surface, thus causing it to change its section from oval section to circular section. Pressure exerted by gas works against tube stresses and air pressure. This change in cross-section from elliptical to circular causes straightening of tube and thus deflecting free end of tube through some distance 'd' as shown in figure 1.7. This deflection in free end of tube measures the pressure difference between gas pressure and atmospheric pressure. Generally this free end of tube is connected to an indicating hand sweeping over a graduated dial showing the gauge pressure directly.

Temperature is another thermodynamic property which is normally used in Kelvin scale in engineering thermodynamic analysis. It is dealt in detail in subsequent chapter.

Density which refers to the mass per unit volume is the ratio of mass and volume occupied. Its units are kg/m^3 .

$$\text{Density} = (\text{Mass/Volume})$$

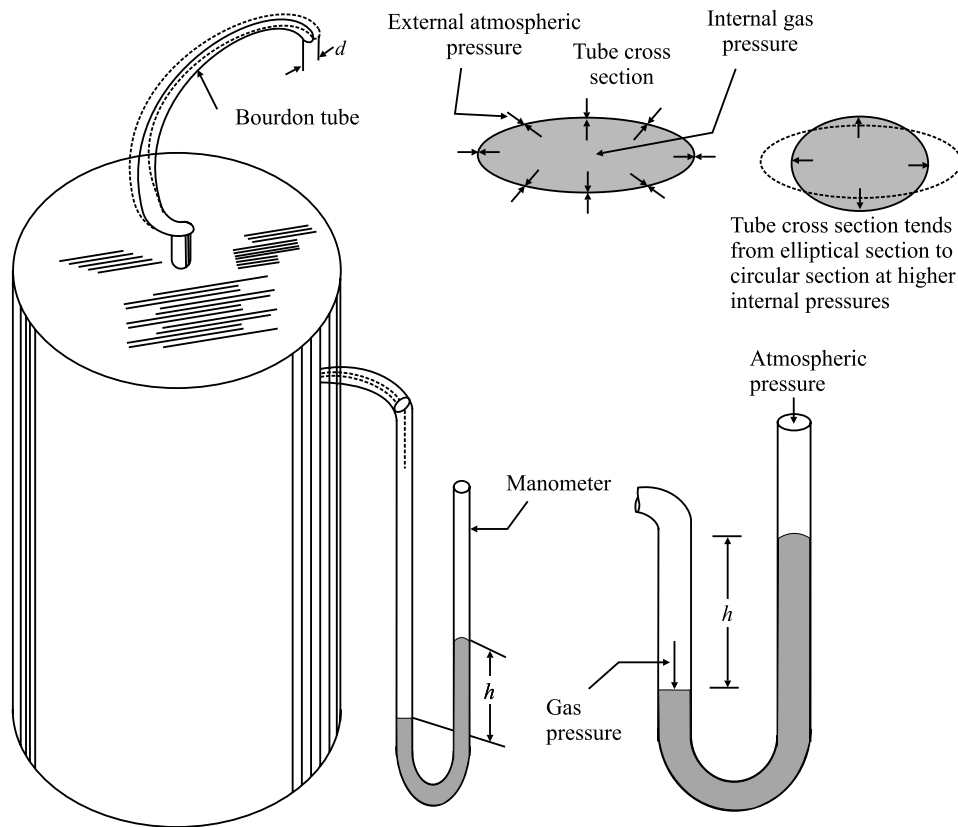


Fig. 1.7 Bourdan tube for pressure measurement

The *specific volume* is the volume per unit mass of the substance. It is defined by ratio of the volume occupied and the mass of substance. Its units are m^3/kg .

$$\text{Specific volume} = (\text{Volume/Mass})$$

Density or specific volume conform to the definitive specification of a thermodynamic property and are capable of getting associated with other properties such as temperature, pressure and internal

energy. Also, the volume occupied by a material is a measure of distance between molecules and thus indicates their molecular energy.

Weight is actually the force due to gravity acting on any substance. Thus, it is the product of mass and gravitational acceleration. Its units are Newtons.

$$\text{Weight} = (\text{mass} \times \text{gravitational acceleration})$$

Specific weight of a substance is the ratio of weight of substance and volume of substance.

$$\begin{aligned}\text{Specific weight} &= (\text{Weight/Volume}) \\ &= (\text{density} \times \text{gravitational acceleration})\end{aligned}$$

Specific gravity is defined as the ratio of the density of any substance and standard density of some reference substance. For solids and liquids the water density at some specified temperature say 0°C or 4°C is taken as standard density.

1.11 ENERGY AND ITS FORMS

“Energy is usually defined as the ability to do mechanical work”. It is indeed quite difficult to precisely define the “energy”. We feel energy at every moment and can sense it very oftenly.

Another broader definition of energy says that “energy refers to the capacity for producing effects.”

Total energy at any moment may be the algebraic summation of the different forms of energy. Conversion of energy from one to other is also possible. In thermodynamics we are primarily interested in studying the change in total energy of a system. Thus, for analysis relative value of energy is considered instead of absolute value.

Energy can be classified in the following general categories;

- (a) *Energy in transition*: It refers to the energy that is in process of transition between substances or regions because of some driving potential, such as difference or gradient of force, or of temperature, or of electrical potential etc. For example heat, work etc.
- (b) *Energy stored in particular mass*: It refers to the potential and kinetic energy associated with masses that are elevated or moving with respect to the earth.

Apart from above broad classification the energy can also be categorised into various forms.

- (i) *Macroscopic energy*: It refers to the energy possessed by a system considered at macroscopic level such as kinetic energy, potential energy etc.
- (ii) *Microscopic energy*: It refers to the energy defined at molecular level. Summation of energy at molecular level or microscopic energy results in internal energy.

Some of the popular forms of energy are described below :

Potential energy: This type of energy is based on relative position of bodies in a system, i.e. elevation in a gravitational field.

Potential energy for a mass m at elevation z is given as :

$$\text{P.E.} = m.g.z$$

Here g is the gravitational acceleration and elevation is measured from some reference point.

Kinetic energy: It is based on the relative movement of bodies. For a mass m moving with certain velocity c it could be mathematically expressed as;

$$\text{K.E.} = (1/2) m.c^2$$

Internal energy: Internal energy of a system is the energy associated with the molecular structure at molecular level.

Let us study fall of a ‘weight’ from certain height on the floor. Upon hitting the floor, ‘weight’ comes to dead stop and its potential energy and kinetic energy both reduce to zero. Question arises, where does the vanishing potential energy and kinetic energy go upon ‘weight’ touching the floor. If we touch the point of contact between ‘weight’ and floor, it is observed that both these points are slightly hotter than before impact. Thus, it is obvious that the energy has changed its form from potential and kinetic to internal energy and causes rise in temperature of ‘weight’ and floor at the points of contact.

Internal energy is sum of energy associated with molecules which may have translational, vibrational and rotational motions etc. and respective energies causing these motions.

Internal energy may be thus comprising of sensible energy, latent energy, chemical energy, nuclear energy etc. ‘*Sensible energy*’ refers to the kinetic energy associated with molecules. ‘*Latent energy*’ refers to the energy associated with phase of a substance.

‘*Chemical energy*’ refers to the energy associated with molecular bonds. ‘*Nuclear energy*’ refers to the energy associated with the bonds within nucleus of atom itself.

Total energy of a system may be given as summation of different forms of energy at a moment. Mathematically;

$$\text{T.E (Total energy)} = \text{K.E} + \text{P.E} + \text{I.E}$$

where
 K.E = Kinetic energy
 P.E = Potential energy
 I.E = Internal energy

Some different forms of energy interaction and associated work interactions with block diagram are given in table 1.4.

Table 1.4 Some forms of energy and the associated work interactions

S. No.	Macroscopic form of energy	Governing equation	Energy interaction	Work interaction	Block diagram
1.	Kinetic energy (translation)	$F = m \cdot \frac{dV}{dt}$	$\Delta E = \frac{1}{2} m \cdot (V_2^2 - V_1^2)$	$= - F \cdot dx$	
2.	Kinetic energy (rotational)	$T = J \cdot \frac{d\omega}{dt}$	$\Delta E = \frac{1}{2} J \cdot (\omega_2^2 - \omega_1^2)$	$= - T \cdot d\theta$	
3.	Spring stored energy (translational)	$F = kx$	$\Delta E = \frac{1}{2} k \cdot (x_2^2 - x_1^2)$	$= - F \cdot dx$	
4.	Spring stored energy (rotational)	$T = K \cdot \theta$	$\Delta E = \frac{1}{2} K \cdot (\theta_2^2 - \theta_1^2)$	$= - T \cdot d\theta$	
5.	Gravitational energy	$F = mg$	$\Delta E = mg \cdot (Z_2 - Z_1)$	$= - F \cdot dz$	
6.	Electrical energy (capacitance)	$u = \frac{q}{c}$	$\Delta E = \frac{1}{2} \frac{q^2}{c} = \frac{1}{2} cu^2$	$= - u \cdot dq$	
7.	Electrical energy (inductance)	$\phi = L \cdot i$	$\Delta E = \frac{1}{2} Li^2 = \frac{1}{2} \frac{\phi^2}{L}$	$= - i \cdot d\phi$	

1.12. HEAT AND WORK

When two systems at different temperatures are brought into contact there are observable changes in some of their properties and changes continue till the two don't attain the same temperature if contact is prolonged. Thus, there is some kind of energy interaction at the boundary which causes change in temperatures. This form of energy interaction is called heat. *Thus 'heat' may be termed as the energy interaction at the system boundary which occurs due to temperature difference only.* Heat is observable in transit at the interface i.e. boundary, it can not be contained in a system. In general the heat transfer to the system is assigned with positive (+) sign while the heat transfer from the system is assigned with negative (–) sign. Its units are Calories.

In thermodynamics the work can be defined as follows:

“Work shall be done by the system if the total effect outside the system is equivalent to the raising of weight and this work shall be positive work”.

In above definition the work has been defined as positive work and says that there need not be actual raising of weight but the effect of the system behaviour must be reducible to the raising of a weight and nothing else. Its units are N. m or Joule. Heat and work are two transient forms of energy.

Let us look at a piston cylinder mechanism (closed system), where high pressure air is filled inside the cylinder fitted with a piston exerting force against some resistance. As the piston moves a distance say ' l ', the work would be done. It can be reduced to the raising of weight by replacing this resisting system by a frictionless pulley and lever such that a weight W is raised, Fig. 1.8.

For example, if an electrical battery and resistance is considered as a system, then this system shall do work when electric current flows in an external resistance as this resistance could be replaced by an ideal frictionless motor driving a frictionless pulley and raising a weight.

Here, also in reference to work it is obvious that the work is the entity which is available at the boundary of system, thus work can not be stored rather it is energy interaction in transit at the boundary. From the thermodynamic definition of work the sign convention established as positive work shall be the one which is done by the system while the negative work shall be the one that is done upon the system.

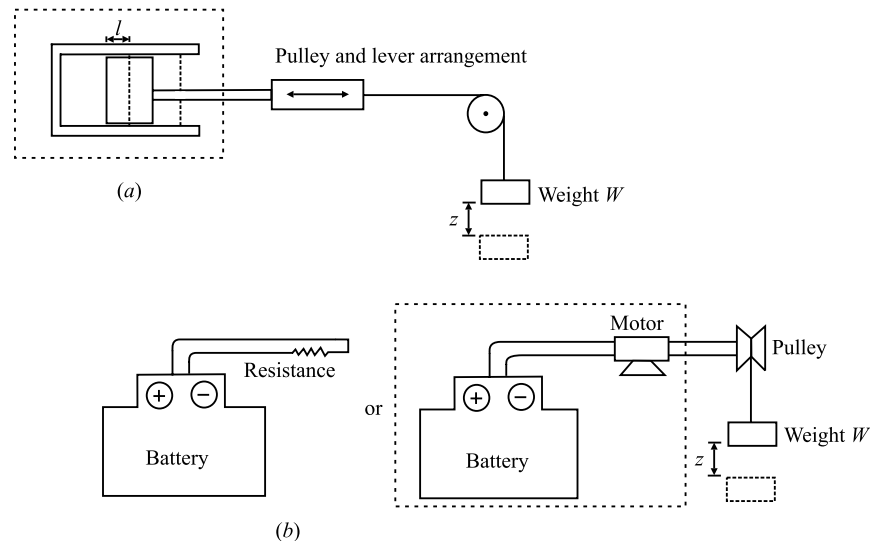


Fig. 1.8 Thermodynamic work

1.13 GAS LAWS

Thermodynamic analysis relies largely upon the gas laws, which are known as Boyle's law (1662) and Charles's law (1787). Boyle's law says that if temperature of a gas is held constant then its molar volume is inversely proportional to the pressure. Mathematically it can be related as $p\bar{v} = \text{constant}$. Here p is the pressure and \bar{v} is the molar volume of gas, i.e. volume per mole.

Charles's law says that for the pressure of gas held constant the volume of gas is directly proportional to the temperature of gas. Mathematically it can be given as $\bar{v}/T = \text{constant}$, where T is the temperature of the gas. It also says that if the molar volume of gas is held constant, the pressure of gas is directly proportional to temperature, i.e. $p/T = \text{constant}$. Figure 1.9 shows the graphical representation.

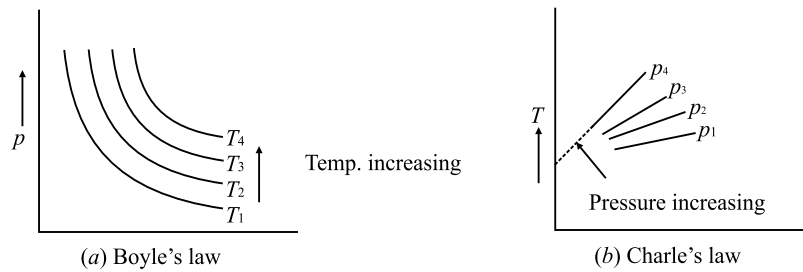


Fig 1.9 Graphical representations of gas laws at constant temperature and at constant pressure

Boyle's and Charles's law when combined together result in,

$$p\bar{v}/T = \text{constant}$$

or $p\bar{v} = \bar{R}T$, where \bar{R} is the universal gas constant.

1.14 IDEAL GAS

Engineering thermodynamics deals with different systems having gaseous working fluids. Some gases behave as ideal gas and some as non-ideal gas. Based on the experimental methods various equations of state of gases have been developed.

For perfect gas the ideal gas equation shows that

$p\bar{v} = \bar{R}T$, where \bar{R} is the universal gas constant and can be related as $R = \bar{R}/M$, here R is the characteristic gas constant and M is the molar mass or molecular weight of the substance, \bar{v} is volume per mole. Universal gas constant has value given as 8.31441 kJ/k mol.K. or $pV = mRT$, where m is mass of the substance, V is the volume of substance,

$$\text{i.e. } V = n \cdot \bar{v}$$

$$m = n \cdot M, \text{ where 'n' is no. of moles.}$$

Gas constant is also related to specific heats at constant pressure and volume as follows,

$$R = c_p - c_v$$

Upon plotting the variables P , V , T for an ideal gas on three mutually perpendicular axes, the three dimensional entity generated is called P - V - T surface and can be used for studying the thermodynamic properties of ideal gas. Figure 1.10 shows the typical P - V - T surface for an ideal gas.

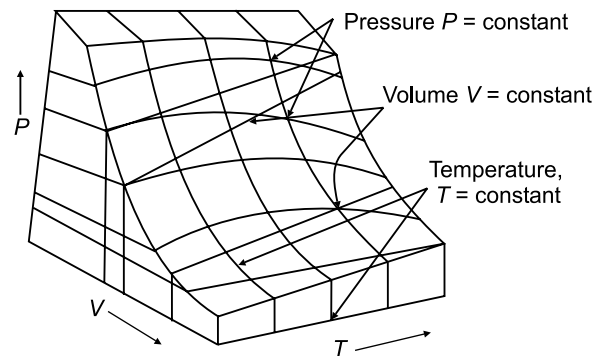


Fig. 1.10 P-V-T surface for ideal gas

For certain gases the molecular weight and gas constant are given in table 1.5.

Table 1.5

Gas	Molecular weight, kg/kmol	Gas constant, kJ/kg.K
Air	28.97	0.287
Carbon dioxide	44.01	0.189
Hydrogen	2.016	4.124
Helium	4.004	2.077
Nitrogen	28.01	0.297
Oxygen	32.00	0.260
Steam	18.02	0.461

1.15 DALTON'S LAW, AMAGAT'S LAW AND PROPERTY OF MIXTURE OF GASES

Dalton's law of partial pressures states that the "total pressure of a mixture of gases is equal to the sum of partial pressures of constituent gases." Partial pressure of each constituent can be defined as the pressure exerted by the gas if it alone occupied the volume at the same temperature.

Thus, for any mixture of gases having 'j' gases in it, the mathematical statement of Dalton's law says,

$$p = p_1 + p_2 + p_3 + \dots + p_j$$

if $V = V_1 = V_2 = V_3 = \dots = V_j$

and $T = T_1 = T_2 = T_3 = \dots = T_j$

Dalton's law can be applied to both mixture of real gases and ideal gases.

$$(a) \begin{array}{|c|} \hline m_1, p_1 \\ \hline V, T \\ \hline \end{array} + \begin{array}{|c|} \hline m_2, p_2 \\ \hline V, T \\ \hline \end{array} \rightarrow \begin{array}{|c|} \hline m, p \\ \hline V, T \\ \hline \end{array} \quad (b) \begin{array}{|c|} \hline m_1, V_1 \\ \hline p, T \\ \hline \end{array} + \begin{array}{|c|} \hline m_2, V_2 \\ \hline p, T \\ \hline \end{array} \rightarrow \begin{array}{|c|} \hline m, V \\ \hline p, T \\ \hline \end{array}$$

constituent gases Mixture

Fig. 1.11 (a) Dalton's law of partial pressures, (b) Amagat's law

Let us take mixture of any three, perfect gases, say, 1, 2, 3 in a container having volume 'V' and temperature T.

Equation of state for these gases shall be,

$$p_1 V = m_1 R_1 T ; p_2 V = m_2 \cdot R_2 \cdot T, p_3 V = m_3 \cdot R_3 \cdot T$$

The partial pressures of three gases shall be,

$$p_1 = \frac{m_1 R_1 T}{V}, p_2 = \frac{m_2 \cdot R_2 \cdot T}{V}, p_3 = \frac{m_3 \cdot R_3 \cdot T}{V}$$

From Dalton's law;

$$p = p_1 + p_2 + p_3 = (m_1 R_1 + m_2 R_2 + m_3 R_3) \cdot \frac{T}{V}$$

or, it can be given in general form as,

$$pV = T \cdot \sum_{i=1}^j m_i \cdot R_i$$

where i refers to constituent gases

Amagat's law of additive volumes states that volume of a gas mixture is equal to the sum of volumes each gas would occupy at the mixture pressure and temperature.

$$V = V_1 + V_2 + V_3 \dots\dots\dots + V_j$$

$$p = p_1 = p_2 = p_3 \dots\dots\dots p_j$$

$$T = T_1 = T_2 = T_3 = \dots\dots\dots T_j$$

Mass balance upon mixture yields $m = m_1 + m_2 + m_3$

or

$$m = \sum_{i=1}^j m_i$$

From above the gas constant for the mixture can be given as;

$$R = \frac{m_1 R_1 + m_2 R_2 + m_3 R_3}{(m_1 + m_2 + m_3)}$$

or, in general form,

$$R = \frac{\sum_{i=1}^j m_i \cdot R_i}{\sum_{i=1}^j m_i}$$

Mole fraction x_i of a constituent gas can be defined as the ratio of number of moles of that constituent to the total number of moles of all constituents.

Thus mole fractions of three gases, if number of moles of three gases are n_1 , n_2 and n_3 ;

$$x_1 = \frac{n_1}{n_1 + n_2 + n_3}$$

$$x_2 = \frac{n_2}{n_1 + n_2 + n_3}$$

$$x_3 = \frac{n_3}{n_1 + n_2 + n_3}$$

or, in general $x_i = \frac{n_i}{\sum n_i}$

Total no. of moles,

$$n = n_1 + n_2 + n_3 \text{ or, } n = \sum_{i=1}^j n_i$$

Sum of mole fractions of all constituent equals to 1,

$$\sum x_i = \frac{\sum n_i}{n} = 1$$

Number of moles of any constituent gas,

$$n_i = n \cdot x_i$$

For M_i being the molecular weight of a constituent gas, the mass m_i of that constituent shall be

$$m_i = n_i \cdot M_i$$

or,

$$m_i = n \cdot x_i \cdot M_i$$

and the total mass m , shall be

$$m = \sum m_i = n \cdot \sum x_i \cdot M_i$$

Molecular weight of mixture shall be:

$$M = \frac{m}{n} = \sum x_i \cdot M_i$$

1.16 REAL GAS

When a gas is found to disobey the perfect gas law, i.e. the equation of state for ideal gas, then it is called 'real gas'. Real gas behaviour can also be shown by a perfect gas at the changed thermodynamic states such as high pressure etc.

Deviation of real gas from ideal gas necessitates the suitable equation of state which can be used for interrelating the thermodynamic properties P , V , and T .

From the kinetic theory of gases it is obvious that the ideal gas equation of state suits the gas behaviour when intermolecular attraction and volume occupied by the molecules themselves is negligibly small in reference to gas volume. At high pressures intermolecular forces and volume of molecules both increase and so the gas behaviour deviates from ideal gas to real gas.

A number of mathematical equations of state suggested by Van der-Waals, Berthelot, Dieterici, Redlich-Kwong, Beattie-Bridgeman and Martin-Hou etc. are available for analysing the real gas behaviour.

Dalton's law and Amagat's law can also be used for real gases with reasonable accuracy in conjunction with modified equations of state.

As the ideal gas equation does not conform to the real gas behaviour in certain ranges of pressures and temperatures, so the perfect gas equation should be modified using compressibility factor for the gas at given pressure and temperature.

Such modified form of equations shall be;

$$P \bar{v} = Z \cdot \bar{R} \cdot T$$

Here Z is the compressibility factor, a function of pressure and temperature.

Thus, compressibility factor is like a correction factor introduced in ideal equation of state for suiting the real gas behaviour. Compressibility factor is an indication of deviation of a gas from ideal gas behaviour and can be mathematically given as;

$$Z = f(P, T)$$

or
$$Z = \frac{v_{\text{actual}}}{v_{\text{ideal}}}$$

Here, $v_{\text{ideal}} = \frac{RT}{P}$ i.e. $Z = 1$ for ideal gases while Z can be greater than or less than unity.

Individual graphical representations are available for getting the compressibility factor as shown in Fig 1.12. Compressibility factor charts are available for different substances. Compressibility factors for various substances can also be shown on a generalized compressibility chart using reduced properties. Reduced properties are non-dimensional properties given as ratio of existing property to critical property of substance. Such as reduced pressure is ratio of pressure of gas to critical pressure of gas. Similarly, reduced temperature can be given by ratio of temperature of gas to critical temperature of gas.

Reduced pressure,
$$p_R = \frac{P}{p_c}$$

Reduced temperature,
$$T_R = \frac{T}{T_c}$$

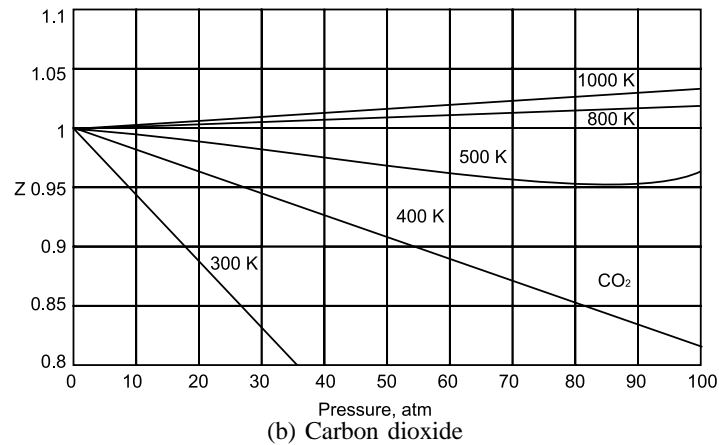
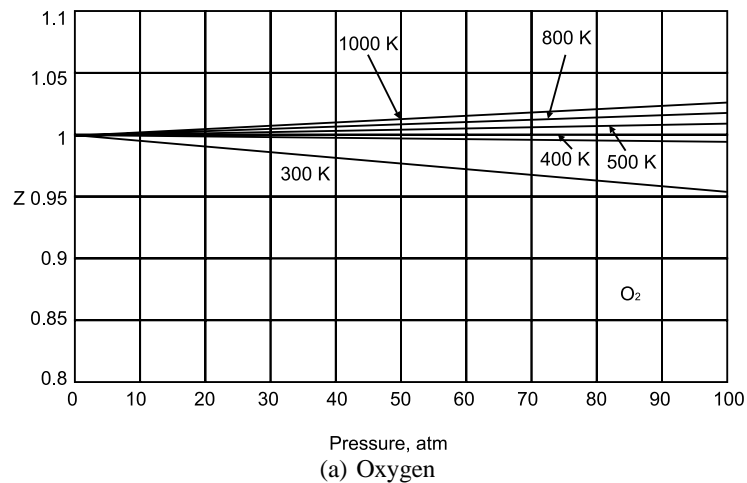


Fig. 1.12 Compressibility factors, Z

where P_c and T_c denote critical pressure and critical temperature respectively. These reduced pressure and reduced temperature are used for getting the generalized compressibility chart of the form, $Z = f(p_R, T_R)$ where Z for all gases is approximately same. This similar behaviour of compressibility correction factor for different gases in respect to reduced pressures and temperatures is called "principle of corresponding states." Fig. 1.13 shows a generalized compressibility chart. In generalized compressibility chart a set of curves is fitted through a set of experimentally determined Z values plotted against reduced

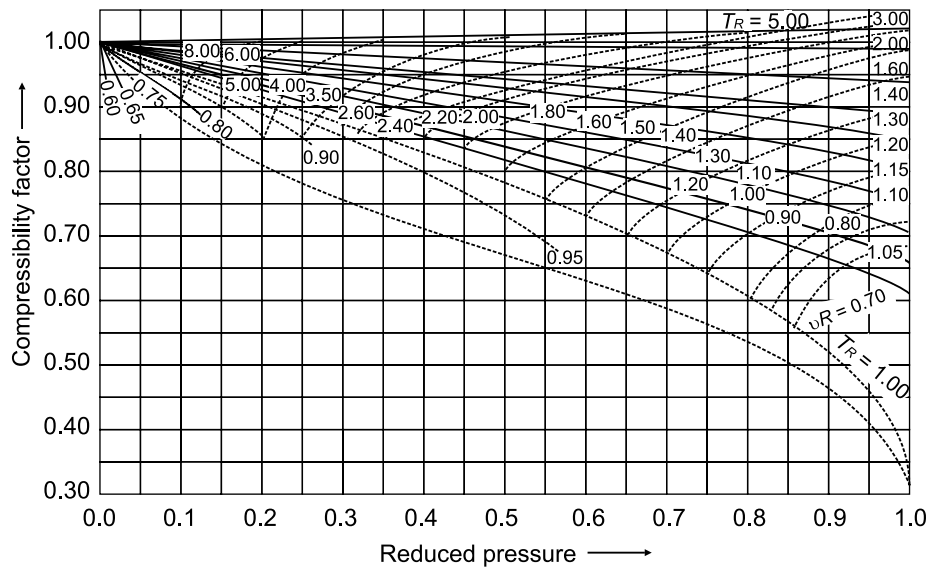


Fig. 1.13 (a) Generalized compressibility chart, $p_R \leq 1.0$

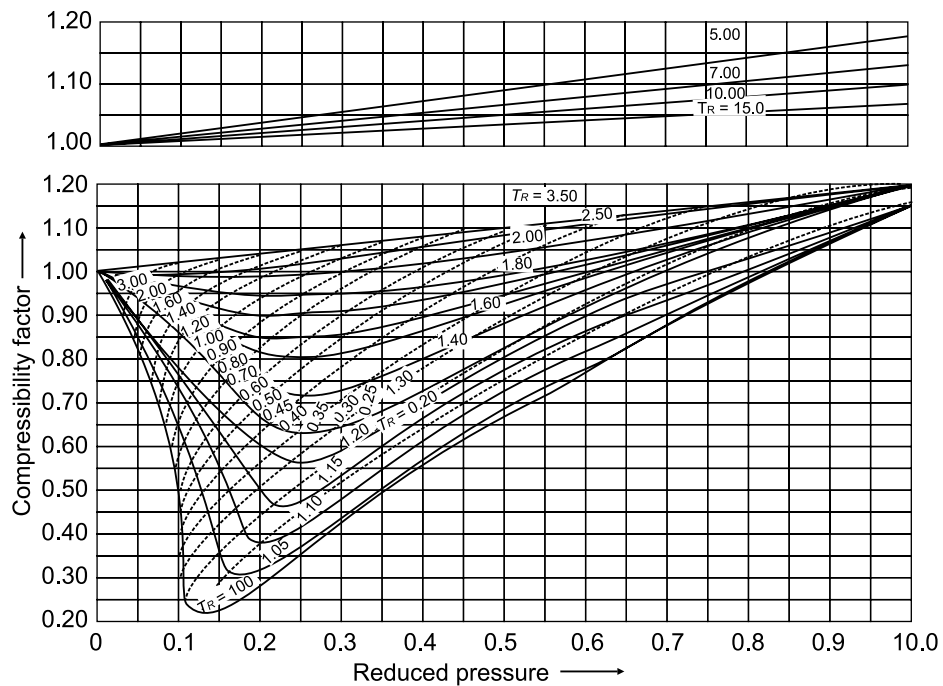


Fig. 1.13 (b) Generalized compressibility chart, $p_R \leq 10.0$

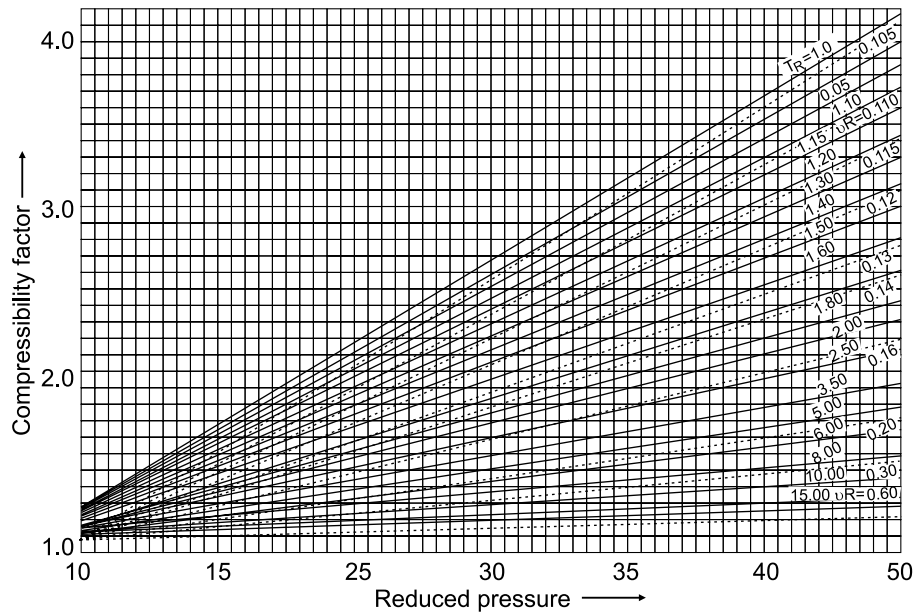


Fig. 1.13 (c) Generalized compressibility chart, $10 \leq p_R \leq 40$

pressure and reduced temperatures for several gases. On the generalized compressibility chart it could be seen that at “very small pressures the gases behave as an ideal gas irrespective of its temperature” and also at “very high temperatures the gases behave as ideal gas irrespective of its pressure”.

1.17 VANDER' WAALS AND OTHER EQUATIONS OF STATE FOR REAL GAS

Vander' Waals suggested the equation of state for real gas in 1873. The equation has been obtained applying the laws of mechanics at molecular level and introducing certain constants in the equation of state for ideal gas. This equation agrees with real gas behaviour in large range of gas pressures and temperatures.

Vander' Waals equation of state for real gas is as follows,

$$\left(p + \frac{a}{\bar{v}^2}\right)(\bar{v} - b) = \bar{R}T$$

where ‘a’ is the constant to take care of the mutual attraction amongst the molecules and thus $\left(\frac{a}{\bar{v}^2}\right)$ accounts for cohesion forces.

Table 1.6 Vander' Waals constant

Gas	Constant a , $N.m^4/(kg.mol)^2$	Constant b , $m^3/kg.mol$
Helium	34176.2×10^2	2.28×10^{-2}
Hydrogen	251.05×10^2	2.62×10^{-2}
Oxygen	1392.5×10^2	3.14×10^{-2}
Air	1355.22×10^2	3.62×10^{-2}
Carbon dioxide	3628.50×10^2	3.14×10^{-2}

Constant ‘ b ’ accounts for the volumes of molecules which are neglected in perfect gas equation, thus it denotes “co-volume”, Mathematically,

$$a = \frac{27\bar{R}^2 T_c^2}{64p_c}, \quad b = (\bar{R} \cdot T_c) / (8p_c)$$

Here, p_c , T_c are critical point pressures and temperatures having values as given in appendix.

Thus these constants ‘ a ’ & ‘ b ’ are determined from behaviour of substance at the critical point.

In general it is not possible to have a single equation of state which conforms to the real gas behaviour at all pressures and temperatures.

A few more equations of state for real gas as suggested by various researchers are as follows.

Redlich-Kwong equation of state for real gas,

$$p = \frac{\bar{R}T}{(\bar{v} - b)} - \frac{a}{\bar{v} \cdot (\bar{v} + b) \cdot \sqrt{T}}$$

where

$$a = 0.4278 \left(\frac{\bar{R}^2 \cdot T_c^{2.5}}{p_c} \right) \text{ and } b = 0.08664 \left(\frac{\bar{R} \cdot T_c}{p_c} \right)$$

Berthelot equation of state for real gas,

$$p = \frac{\bar{R}T}{(\bar{v} - b)} - \frac{a}{T \cdot \bar{v}^2},$$

where

$$a = \left(\frac{27 \cdot \bar{R}^2 \cdot T_c^3}{64 \cdot p_c} \right) \text{ and } b = \left(\frac{\bar{R} \cdot T_c}{8 p_c} \right)$$

Here a and b refer to the constants as suggested in respective equations.

Beattie-Bridgeman equation of state given in 1928, for real gas has five constants determined experimentally. It is,

$$p = \frac{\bar{R} \cdot T}{(\bar{v})^2} \left(1 - \frac{C}{\bar{v} \cdot T^3} \right) (\bar{v} + B) - \frac{A}{(\bar{v})^2}$$

where,

$$A = A_0 \left(1 - \frac{a}{\bar{v}} \right) \text{ and } B = B_0 \left(1 - \frac{b}{\bar{v}} \right)$$

Constants used in Beattie – Bridgeman equation are given in Table 1.7 when p is in $k p_a$, \bar{v} is in $m^3/k \text{ mol}$, T is in K , and $\bar{R} = 8.314 k p_a m^3/k \text{ mol} \cdot K$.

Table 1.7. Beattie -Bridgeman constants

Gas	A_0	a	B_0	b	c
Helium	2.1886	0.05984	0.01400	0.0	40
Hydrogen	20.0117	−0.00506	0.02096	−0.04359	504
Oxygen	151.0857	0.02562	0.04624	0.004208	4.80×10^4
Air	131.8441	0.01931	0.04611	−0.001101	4.34×10^4
Carbon dioxide	507.2836	0.07132	0.10476	0.07235	6.60×10^5

Virial equations of state propose a form of equation which can be suitably modified and used for real gases.

These equations of state are in the form,

$$\frac{p\bar{v}}{RT} = A_0 + A_1 \cdot p + A_2 \cdot p^2 + A_3 \cdot p^3 + \dots$$

or

$$\frac{p\bar{v}}{RT} = B_0 + \frac{B_1}{\bar{v}} + \frac{B_2}{\bar{v}^2} + \frac{B_3}{\bar{v}^3} + \dots$$

Where $A_0, A_1, A_2, A_3, \dots$ and $B_0, B_1, B_2, B_3, \dots$

are called the "virial coefficients" and depend upon temperature alone. Virial equations of state can be suitably modified based on experimental P, v, T data for the real gas analysis. Virial constants can be calculated if the suitable model for describing the forces of interaction between the molecules of gas under consideration is known.

EXAMPLES

1. Find out the pressure difference shown by the manometer deflection of 30 cm of Mercury. Take local acceleration of gravity as 9.78 m/s^2 and density of mercury at room temperature as $13,550 \text{ kg/m}^3$.

Solution:

From the basic principles of fluid statics,

$$\begin{aligned} \text{Pressure difference} &= \rho \cdot gh \\ &= 13550 \times 30 \times 10^{-2} \times 9.78 \\ &= \mathbf{39755.70 \text{ Pa} \quad \text{Ans.}} \end{aligned}$$

2. An evacuated cylindrical vessel of 30 cm diameter is closed with a circular lid. Estimate the effort required for lifting the lid, if the atmospheric pressure is 76 cm of mercury column (Take $g = 9.78 \text{ m/s}^2$)

Solution:

Effort required for lifting the lid shall be equal to the force acting upon the lid. Thus, effort required = Pressure \times Area

$$\begin{aligned} &= (76 \times 10^{-2} \times 13550 \times 9.78) \times (3.14 \times (30 \times 10^{-2})^2/4) \\ &= \mathbf{7115.48 \text{ N} \quad \text{Ans.}} \end{aligned}$$

3. Calculate the actual pressure of air in the tank if the pressure of compressed air measured by manometer is 30 cm of mercury and atmospheric pressure is 101 kPa. (Take $g = 9.78 \text{ m/s}^2$)

Solution:

Pressure measured by manometer on the tank is gauge pressure, which shall be

$$\begin{aligned} &= \rho \cdot g \cdot h \\ &= (13550 \times 9.78 \times 30 \times 10^{-2}) \\ &= 39755.70 \text{ Pa} \\ &= 39.76 \text{ kPa} \end{aligned}$$

Actual pressure of air = Gauge pressure + atmospheric pressure

$$\begin{aligned} &= 39.76 + 101 \\ &= \mathbf{140.76 \text{ kPa} \quad \text{Ans.}} \end{aligned}$$

4. Determine gauge pressure at a depth of 1 m in a tank filled with oil of specific gravity 0.8. Take density of water as 1000 kg/m^3 and $g = 9.81 \text{ m/s}^2$.

Solution:

Density of oil = Specific gravity \times Density of water

$$\rho_{\text{oil}} = 0.8 \times 1000$$

$$\rho_{\text{oil}} = 800 \text{ kg/m}^3$$

$$\begin{aligned} \text{Gauge pressure} &= (\rho_{\text{oil}} \times g \times h) \\ &= 800 \times 9.81 \times 1 \\ &= 7848 \text{ N/m}^2 \end{aligned}$$

or

$$\text{Gauge pressure} = \mathbf{7.848 \text{ kPa. Ans.}}$$

5. Calculate the gas pressure using a mercury manometer with one limb open to atmosphere as shown in Fig. 1.14. Barometer reading is 76 cm and density of mercury is $13.6 \times 10^3 \text{ kg/m}^3$. Take $g = 9.81 \text{ m/s}^2$.

Solution:

Figure shows that the difference of height in mercury columns is 40 cm.

In reference to level AB the pressure exerted by gas, p_{gas} can be written as sum of atmospheric pressure and pressure due to mercury column at AB

$$\begin{aligned} p_{\text{gas}} &= (\rho_{\text{mercury}} \times 9.81 \times 40 \times 10^{-2}) + \text{Atmospheric pressure} \\ &= (13.6 \times 10^3 \times 9.81 \times 40 \times 10^{-2}) + (13.6 \times 10^3 \times 9.81 \times 76 \times 10^{-2}) \\ &= 154762.56 \text{ N/m}^2 \end{aligned}$$

or

$$P_{\text{gas}} = \mathbf{154.76 \text{ kPa Ans.}}$$

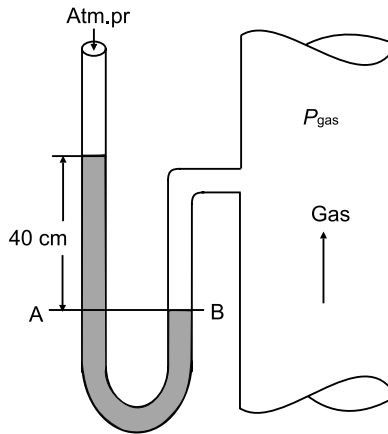


Fig. 1.14

6. 1 kg of water falls from an altitude of 1000 m above ground level. What will be change in the temperature of water at the foot of fall, if there are no losses during the fall. Take specific heat of water as 1 kcal/kg·K

Solution:

Initially when water is at 1000 m, it shall have potential energy in it. This potential energy shall get transformed upon its fall and change the temperature of water.

By law of energy conservation

Potential energy = Heat required for heating water

$$= \frac{1 \times 9.81 \times 1000}{4.18} = 1 \times 1 \times 10^3 \times \Delta T$$

or

$$\Delta T = 2.35^\circ\text{C}$$

$$\text{Change in temperature of water} = \mathbf{2.35^\circ\text{C Ans.}}$$

7. A spring balance is used for measurement of weight. At standard gravitational acceleration it gives weight of an object as 100 N. Determine the spring balance reading for the same object when measured at a location having gravitational acceleration as 8.5 m/s^2 .

Solution:

$$\begin{aligned}\text{At standard gravitational acceleration, mass of object} &= \frac{100}{9.81} \\ &= 10.194 \text{ kg} \\ \text{Spring balance reading} &= \text{Gravitational force in mass} \\ &= 10.194 \times 8.5 \\ &= 86.649 \text{ N} \\ &= \mathbf{86.65 \text{ N} \quad \text{Ans.}}\end{aligned}$$

8. An incompressible gas in the cylinder of 15 cm diameter is used to support a piston, as shown. Manometer indicates a difference of 12 cm of Hg column for the gas in cylinder. Estimate the mass of piston that can be supported by the gas. Take density of mercury as $13.6 \times 10^3 \text{ kg/m}^3$.

Solution:

Piston shall be supported by the gas inside, therefore, let mass of piston be ' m ' kg.

Weight of piston = Upward thrust by gas

$$m \cdot g = p \times \pi \times \frac{d^2}{4}$$

$$m \times 9.81 = (12 \times 10^{-2} \times 13.6 \times 10^3 \times 9.81) \times \frac{\pi}{4} \times (15 \times 10^{-2})^2$$

$$m = 28.84 \text{ kg}$$

Mass of piston = **28.84 kg. Ans.**

9. Determine pressure of steam flowing through a steam pipe when the U-tube manometer connected to it indicates as shown in figure 1.16. During pressure measurement some steam gets condensed in manometer tube and occupies a column of height 2 cm (AB) while mercury gets raised by 10 cm (CD) in open limb. Consider barometer reading as 76 cm of Hg, density of mercury and water as $13.6 \times 10^3 \text{ kg/m}^3$ and 1000 kg/m^3 respectively.

Solution:

Let us make pressure balance at plane BC.

$$P_{\text{steam}} + P_{\text{water, AB}} = P_{\text{atm}} + P_{\text{Hg, CD}}$$

$$P_{\text{steam}} = P_{\text{atm}} + P_{\text{Hg, CD}} - P_{\text{water, AB}}$$

$$P_{\text{atm}} = (13.6 \times 10^3 \times 76 \times 10^{-2} \times 9.81)$$

$$P_{\text{atm}} = 101396.16 \text{ N/m}^2$$

$$P_{\text{water, AB}} = (1000 \times 2 \times 10^{-2} \times 9.81)$$

$$P_{\text{water, AB}} = 196.2 \text{ N/m}^2$$

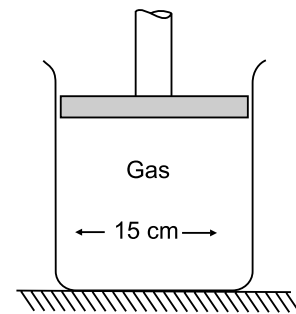


Fig. 1.15

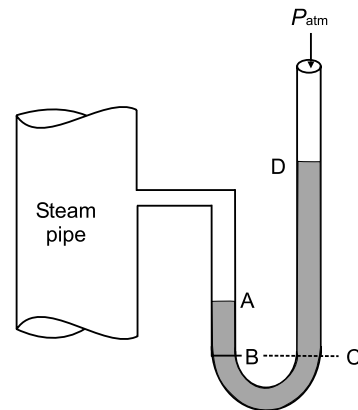


Fig. 1.16

$$P_{\text{Hg, CD}} = (13.6 \times 10^3 \times 10 \times 10^{-2} \times 9.81)$$

$$P_{\text{Hg, CD}} = 13341.6 \text{ N/m}^2$$

Substituting for getting steam pressure,

$$P_{\text{steam}} = 101396.16 + 13341.6 - 196.2$$

$$P_{\text{steam}} = 114541.56 \text{ N/m}^2$$

or

$$P_{\text{steam}} = \mathbf{114.54 \text{ kPa} \text{ Ans.}}$$

10. A vessel has two compartments 'A' and 'B' as shown with pressure gauges mounted on each compartment. Pressure gauges of A and B read 400 kPa and 150 kPa respectively. Determine the absolute pressures existing in each compartment if the local barometer reads 720 mm Hg.

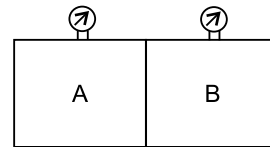


Fig. 1.17

Solution:

Atmospheric pressure from barometer

$$= (9810) \times (13.6) \times (0.720)$$

$$= 96060 \text{ Pa}$$

$$= 96.06 \text{ kPa}$$

Absolute pressure in compartment A,

$$P_{\text{abs, A}} = P_{\text{gauge, A}} + P_{\text{atm}}$$

$$= 400 + 96.06 = 496.06 \text{ kPa}$$

Absolute pressure in compartment B,

$$P_{\text{abs, B}} = P_{\text{gauge, B}} + P_{\text{atm}}$$

$$= 150 + 96.06 = 246.06 \text{ kPa}$$

Absolute pressure in compartments A & B

$$= \mathbf{496.06 \text{ kPa} \text{ \& } 246.06 \text{ kPa Ans.}}$$

11. Determine the air pressure in a tank having multifluid manometer connected to it, with the tube open to atmosphere as shown in figure. Tank is at an altitude where atmospheric pressure is 90 kPa. Take densities of water, oil and mercury as 1000 kg/m^3 , 850 kg/m^3 and 13600 kg/m^3 respectively.

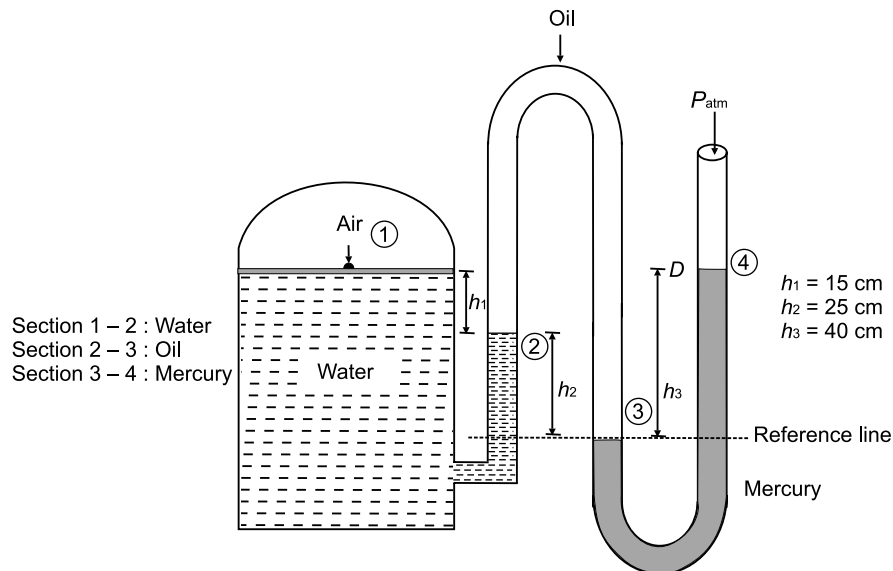


Fig. 1.18

Solution:

It is obvious that the lengths of different fluids in U-tube are due to the air pressure and the pressure of air in tank can be obtained by equalizing pressures at some reference line. Starting from point (1) the pressure can be given as under for equilibrium,

$$p_1 + \rho_{\text{water}} \cdot g \cdot h_1 + \rho_{\text{oil}} \cdot g \cdot h_2 = p_{\text{atm}} + \rho_{\text{mercury}} \cdot g \cdot h_3$$

Given: $\rho_{\text{water}} = 1000 \text{ kg/m}^3$, $\rho_{\text{oil}} = 850 \text{ kg/m}^3$, $\rho_{\text{mercury}} = 13600 \text{ kg/m}^3$

$$h_1 = 0.15 \text{ m},$$

$$h_2 = 0.25 \text{ m},$$

$$h_3 = 0.40 \text{ m},$$

$$p_{\text{atm}} = 90 \text{ kPa}$$

Substituting we get $p_1 = 139.81 \text{ kPa}$

Air pressure = **139.81 kPa** Ans.

12. Estimate the kinetic energy associated with space object revolving around earth with a relative velocity of 750 m/s and subjected to gravitational force of 4000 N. Gravitational acceleration may be taken as 8 m/s².

Solution:

$$\text{Mass of object} = \frac{\text{Gravitational force}}{\text{Gravitational acceleration}}$$

$$= \frac{4000}{8}$$

$$= 500 \text{ kg}$$

$$\text{Kinetic energy} = \frac{1}{2} \times 500 \times (750)^2$$

$$= 140625000 \text{ J}$$

Kinetic energy = **$1.4 \times 10^8 \text{ J}$** Ans.

13. Determine the molecular weight of a gas if its specific heats at constant pressure and volume are $c_p = 2.286 \text{ kJ/kg K}$ and $c_v = 1.768 \text{ kJ/kg K}$.

Solution:

Gas constant shall be,

$$R = c_p - c_v$$

$$= 0.518 \text{ kJ/kg.K}$$

Molecular weight of gas

$$= \frac{\bar{R}}{R} = \frac{\text{Universal gas constant}}{\text{Characteristic gas constant}}$$

$$= \frac{8.3143}{0.518}$$

= **16.05 kg/k mol** Ans.

14. A perfect gas at pressure of 750 kPa and 600 K is expanded to 2 bar pressure. Determine final temperature of gas if initial and final volume of gas are 0.2 m³ and 0.5 m³ respectively.

Solution:

Initial states = 750×10^3 Pa, 600 K, 0.2 m^3

Final states = 2 bar or 2×10^5 Pa, 0.5 m^3 .

Using perfect gas equation,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\frac{750 \times 10^3 \times 0.2}{600} = \frac{2 \times 10^5 \times 0.5}{T_2}$$

$$T_2 = 400 \text{ K}$$

Final temperature = **400 K or 127° C** Ans.

15. A vessel of 5 m^3 capacity contains air at 100 kPa and temperature of 300K. Some air is removed from vessel so as to reduce pressure and temperature to 50 kPa and 7°C respectively. Find the amount of air removed and volume of this mass of air at initial states of air. Take $R = 287 \text{ J/kg.K}$ for air.

Solution:

Initial states : 100×10^3 Pa, 300 K, 5 m^3

Final states : 50×10^3 Pa, 280 K, 5 m^3

Let initial and final mass of air be m_1 and m_2 . From perfect gas equation of air,

$$m_1 = \frac{p_1 V_1}{RT_1}; \quad m_2 = \frac{p_2 V_2}{RT_2}$$

$$m_1 = \frac{100 \times 10^3 \times 5}{287 \times 300}; \quad m_2 = \frac{50 \times 10^3 \times 5}{287 \times 280}$$

Mass of removed, $(m_1 - m_2)$

$$= \left(\frac{100 \times 10^3 \times 5}{287 \times 300} \right) - \left(\frac{50 \times 10^3 \times 5}{287 \times 280} \right)$$

$$m_1 - m_2 = 2.696 \text{ kg}$$

Volume of this mass of air at initial states i.e 100 kPa and 300 K;

$$V = \frac{(m_1 - m_2) \cdot RT_1}{p_1}$$

$$= \frac{2.696 \times 287 \times 300}{100 \times 10^3}$$

$$\text{Volume} = 2.32 \text{ m}^3$$

Mass of air removed = **2.696 kg**

Volume of air at initial states = **2.32 m³** Ans.

16. A cylindrical vessel of 1 m diameter and 4 m length has hydrogen gas at pressure of 100 kPa and 27°C. Determine the amount of heat to be supplied so as to increase gas pressure to 125 kPa. For hydrogen take $C_p = 14.307 \text{ kJ/kg.K}$, $C_v = 10.183 \text{ kJ/kg K}$.

Solution:

Assuming hydrogen to be perfect gas let initial and final states be denoted by subscript 1 and 2.

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}, \text{ Here } V_2 = V_1$$

$$T_2 = \frac{p_2 V_2 \cdot T_1}{p_1 V_1} = \frac{p_2 T_1}{p_1} = \frac{125 \times 10^3 \times 300}{100 \times 10^3}$$

$$T_2 = 375 \text{ K}$$

As it is constant volume heating so, heat supplied,

$$Q = m \cdot C_v (T_2 - T_1)$$

From perfect gas characteristics, $R = C_p - C_v$

$$R = 4.124 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Mass of hydrogen, } m = \frac{p_1 V_1}{RT_1} = \frac{100 \times 10^3 \times \pi \times (0.5)^2 \times 4}{4.124 \times 10^3 \times 300}$$

$$m = 0.254 \text{ kg}$$

$$\text{Heat added, } Q = 0.254 \times 10.183 \times (375 - 300)$$

$$\text{Heat to be supplied} = \mathbf{193.99 \text{ kJ} \quad \text{Ans.}}$$

17. Two cylindrical vessels of 2 m^3 each are inter connected through a pipe with valve in-between. Initially valve is closed and one vessel has 20 kg air while 4 kg of air is there in second vessel. Assuming the system to be at 27°C temperature initially and perfectly insulated, determine final pressure in vessels after the valve is opened to attain equilibrium.

Solution:

When the valve is opened then the two vessels shall be connected through pipe and transfer of air shall take place in order to attain equilibrium state. After attainment of equilibrium total mass of air shall be 24 kg.

$$\text{Final total volume} = 2 \times 2 = 4 \text{ m}^3$$

Using perfect gas equation.

$$pV = mRT$$

$$p = \frac{mRT}{V}$$

$$\text{For air, } R = 287 \text{ J/kg K}$$

$$\begin{aligned} \text{Substituting values, } p &= \frac{24 \times 287 \times 300}{4} \\ &= 516600 \text{ N/m}^2 \end{aligned}$$

$$\text{Final pressure} = \mathbf{516.6 \text{ kPa} \quad \text{Ans.}}$$

18. Determine the pressure of 5 kg carbon dioxide contained in a vessel of 2 m^3 capacity at 27°C , considering it as

(i) perfect gas

(ii) real gas.

Solution:

Given : Volume, $V = 2 \text{ m}^3$, Universal gas constt. = $8.314 \text{ kJ/kg} \cdot \text{K}$

Temperature, $T = 27^\circ\text{C}$

$$= (273 + 27) \text{ K}$$

$$T = 300 \text{ K}$$

Mass, $m = 5 \text{ kg}$

Let pressure exerted be ' p '.

(i) Considering it as perfect gas,

$$pV = mR_{\text{CO}_2}T$$

$$R_{\text{CO}_2} = \frac{\text{Universal gas constt.}}{\text{Molecular weight of CO}_2}$$

$$R_{\text{CO}_2} = \frac{8.314 \times 10^3}{44.01}$$

$$R_{\text{CO}_2} = 188.9 \text{ J/kg} \cdot \text{K}$$

Substituting in perfect gas equation,

$$p = \frac{5 \times 188.9 \times 300}{2} = 141675 \text{ N/m}^2$$

$$\text{Pressure} = \mathbf{1.417 \times 10^5 \text{ N/m}^2} \quad \text{Ans.}$$

(ii) Considering it as real gas let us use Vander-Waals equation;

$$\left(p + \frac{a}{\bar{v}^2} \right) (\bar{v} - b) = \bar{R}T$$

where ' \bar{v} ' is molar specific volume and constants ' a ' and ' b ' can be seen from Table 1.6.

$$\bar{R} = 8.314 \times 10^3$$

$$\text{Molar specific volume, } \bar{v} = \frac{2 \times 44.01}{5}$$

$$\bar{v} = 17.604 \text{ m}^3/\text{kg} \cdot \text{mol}$$

Vander-Waals Constant,

$$a = 3628.5 \times 10^2 \text{ N} \cdot \text{m}^4/(\text{kg} \cdot \text{mol})^2$$

$$b = 3.14 \times 10^{-2} \text{ m}^3/\text{kg} \cdot \text{mol}$$

Substituting values in Vander Waals equation,

$$\left(p + \frac{3628.5 \times 10^2}{(17.604)^2} \right) (17.604 - 3.14 \times 10^{-2}) = (8.314 \times 10^3 \times 300)$$

$$p + 1170.86 = 141936.879$$

$$p = 140766.019 \text{ N/m}^2$$

$$\text{Pressure} = 1.408 \times 10^5 \text{ N/m}^2$$

For CO_2 as perfect gas = $1.417 \times 10^5 \text{ N/m}^2$

For CO_2 as real gas = $\mathbf{1.408 \times 10^5 \text{ N/m}^2} \quad \text{Ans.}$

(using Vander-Waals equation)

19. Determine the specific volume of steam at 17672 kPa and 712 K considering it as (a) perfect gas, (b) considering compressibility effects. Take critical pressure = 22.09 MPa, critical temperature = 647.3 K, $R_{\text{steam}} = 0.4615 \text{ kJ/kg}\cdot\text{K}$.

Solution:

(a) Considering steam as perfect gas,

$$\begin{aligned}\text{Sp. volume} &= \frac{R_{\text{steam}} \cdot T}{p} \\ &= \frac{0.4615 \times 712}{17672}\end{aligned}$$

Specific volume = **0.0186 m³/kg** Ans.

(b) Considering compressibility effects, the specific volume can be given by product of compressibility factor 'Z' and "specific volume when perfect gas".

$$\begin{aligned}\text{Reduced pressure} &= \frac{p}{\text{Critical pressure}} \\ &= \frac{17672}{22.09 \times 10^3}\end{aligned}$$

Reduced pressure = 0.8

$$\begin{aligned}\text{Reduced temperature} &= \frac{T}{\text{Critical temperature}} \\ &= \frac{712}{647.3}\end{aligned}$$

Reduced temperature = 1.1

From generalized compressibility chart compressibility factor 'Z' can be seen for reduced pressure and reduced temperatures of 0.8 and 1.1. We get,

$$Z = 0.785$$

$$\begin{aligned}\text{Actual specific volume} &= 0.785 \times 0.0186 \\ &= \mathbf{0.0146 \text{ m}^3/\text{kg}} \quad \text{Ans.}\end{aligned}$$

20. A spherical balloon of 5 m diameter is filled with Hydrogen at 27°C and atmospheric pressure of 1.013 bar. It is supposed to lift some load if the surrounding air is at 17°C. Estimate the maximum load that can be lifted.

Solution:

Balloon filled with H_2 shall be capable of lifting some load due to buoyant forces.

$$\text{Volume of balloon} = \frac{4}{3} \cdot \pi \cdot \left(\frac{5}{2}\right)^3 = 65.45 \text{ m}^3$$

Mass of H_2 in balloon can be estimated considering it as perfect gas.

$$\text{Gas constant for } H_2 = \frac{8.314 \times 10^3}{2} = 4.157 \times 10^3 \text{ J/kg} \cdot \text{K}$$

$$\begin{aligned}\text{Mass of } H_2 \text{ in balloon} &= \frac{P_{\text{balloon}} \cdot V_{\text{balloon}}}{R_{H_2} \cdot T_{H_2}} \\ &= \frac{1.013 \times 10^5 \times 65.45}{4.157 \times 10^3 \times 300}\end{aligned}$$

$$m_{H_2} = 5.316 \text{ kg}$$

$$\begin{aligned}\text{Volume of air displaced} &= \text{Volume of balloon} \\ &= 65.45 \text{ m}^3\end{aligned}$$

$$\text{Mass of air displaced} = \frac{1.013 \times 10^5 \times 65.45}{R_{\text{air}} \times (17 + 273)}$$

$$R_{\text{air}} = 0.287 \text{ kJ/kg} \cdot \text{K}$$

$$m_{\text{air}} = \frac{1.013 \times 10^5 \times 65.45}{0.287 \times 10^3 \times 290}$$

$$m_{\text{air}} = 79.66 \text{ kg}$$

$$\begin{aligned}\text{Load lifting capacity due to buoyant force} &= m_{\text{air}} - m_{H_2} \\ &= 79.66 - 5.316 \\ &= \mathbf{74.344 \text{ kg} \quad \text{Ans.}}\end{aligned}$$

21. A pump draws air from large air vessel of 20 m^3 at the rate of $0.25 \text{ m}^3/\text{min}$. If air is initially at atmospheric pressure and temperature inside receiver remains constant then determine time required to reduce the receiver pressure to $\frac{1}{4}$ th of its original value.

Solution:

Let volume of receiver be V , m^3 and volume sucking rate of pump be $v \text{ m}^3/\text{min}$, then theoretically problem can be modelled using perfect gas equation. Here p is pressure in receiver and T is temperature in vessel.

$$pV = mRT$$

Here pressure ' p ' and temperature ' T ' shall change with respect to time t . Differentiating perfect gas equation with respect to time.

$$V \cdot \frac{dp}{dt} = RT \frac{dm}{dt}$$

Here $\left(\frac{dm}{dt}\right)$ is mass extraction rate from receiver in kg/min . This mass rate can be given using perfect gas equation when volume flow rate (m^3/min) is given as v . So.

$$\frac{dm}{dt} = -\frac{pv}{RT} \quad (\text{-ve as mass gets reduced with time})$$

Substituting,

$$V \cdot \frac{dp}{dt} = -RT \cdot \frac{pv}{RT}$$

$$V \cdot \frac{dp}{dt} = -pv$$

$$\int_0^t dt = - \frac{V}{v} \int_0^t \frac{dp}{p}$$

$$t = -\frac{V}{v} \ln \left(\frac{p_2}{p_1} \right)$$

Here final pressure, $p_2 = \frac{p_1}{4}$, $V = 20 \text{ m}^3$, $v = 0.25 \text{ m}^3/\text{min}$

So time,
$$t = \frac{V}{v} \ln \left(\frac{1}{4} \right)$$

$$\begin{aligned} \text{time} &= \frac{20}{0.25} \ln(4) = 110.9 \text{ minutes} \\ &= \mathbf{110.9 \text{ minutes} \quad \text{Ans.}} \end{aligned}$$

22. In 5 kg mixture of gases at 1.013 bar and 300 K the various constituent gases are as follows, 80% N_2 , 18% O_2 , 2% CO_2 .

Determine the specific heat at constant pressure, gas constant for the constituents and mixture and also molar mass of mixture taking $\gamma = 1.4$ for N_2 and O_2 and $\gamma = 1.3$ for CO_2 .

Universal gas constant = 8314 J/kg · K

Solution:

Gas constants for constituent gases shall be,

$$R_{N_2} = \frac{8314}{\text{mol. wt. of } N_2} = \frac{8314}{28} = \mathbf{296.9 \text{ J/kg} \cdot \text{K}}$$

$$R_{O_2} = \frac{8314}{\text{mol. wt. of } O_2} = \frac{8314}{32} = \mathbf{259.8 \text{ J/kg} \cdot \text{K}}$$

$$R_{CO_2} = \frac{8314}{\text{mol. wt. of } CO_2} = \frac{8314}{44} = \mathbf{188.9 \text{ J/kg} \cdot \text{K}}$$

Gas constant for mixture, R_{mixture}

$$\begin{aligned} &= \left(\frac{m_{N_2}}{M} \right) R_{N_2} + \left(\frac{m_{O_2}}{M} \right) R_{O_2} + \left(\frac{m_{CO_2}}{M} \right) R_{CO_2} \\ &= (0.80 \times 296.9) + (0.18 \times 259.8) + (0.02 \times 188.9) \end{aligned}$$

$$R_{\text{mixture}} = \mathbf{288.06 \text{ J/kg} \cdot \text{K}}$$

Specific heat at constant pressure for constituent gases.

$$C_{p, N_2} = \left(\frac{\gamma}{\gamma - 1} \right) R_{N_2} = \left(\frac{1.4}{0.4} \right) \times 296.9 = \mathbf{1.039 \text{ kJ/kg} \cdot \text{K}}$$

$$C_{p, O_2} = \left(\frac{\gamma}{\gamma - 1} \right) \cdot R_{O_2} = \left(\frac{1.4}{0.4} \right) \times 259.8 = \mathbf{0.909 \text{ kJ/kg} \cdot \text{K}}$$

$$C_{p, CO_2} = \left(\frac{\gamma}{\gamma - 1} \right) \cdot R_{CO_2} = \left(\frac{1.3}{0.3} \right) \times 188.9 = \mathbf{0.819 \text{ kJ/kg} \cdot \text{K}}$$

$$C_{p, \text{mixture}} = \left(\frac{m_{N_2}}{M} \right) \cdot C_{p, N_2} + \left(\frac{m_{O_2}}{M} \right) \cdot C_{p, O_2} + \left(\frac{m_{CO_2}}{M} \right) \cdot C_{p, CO_2}$$

$$C_{p, \text{mixture}} = (0.80 \times 1.039) + (0.18 \times 0.909) + (0.02 \times 0.819) \\ = \mathbf{1.0276 \text{ kJ/kg} \cdot \text{K}}$$

$$\text{Molar mass of mixture} = \sum x_i \cdot M_i = \frac{\text{Total mass of mixture}}{\text{Total no. of moles}}$$

$$x_i = \frac{n_i}{\sum n_i}, \text{ here } M_i = \text{mol. wt. of } i\text{th constituent.}$$

No. of moles of constituent gases,

$$n_{N_2} = \frac{m_{N_2}}{\text{Mol. wt. } N_2} = \frac{0.8 \times 5}{28} = 0.143$$

$$n_{O_2} = \frac{m_{O_2}}{\text{Mol. wt. } O_2} = \frac{0.18 \times 5}{32} = 0.028$$

$$n_{CO_2} = \frac{m_{CO_2}}{\text{Mol. wt. } CO_2} = \frac{0.02 \times 5}{44} = 0.0023$$

$$\begin{aligned} \text{Total mole numbers in mixture} &= n_{N_2} + n_{O_2} + n_{CO_2} \\ &= (0.143 + 0.028 + 0.0023) \\ \sum n_i &= 0.1733 \end{aligned}$$

Mole fractions of constituent gases,

$$x_{N_2} = \frac{n_{N_2}}{\sum n_i} = \frac{0.143}{0.1733} = 0.825$$

$$x_{O_2} = \frac{n_{O_2}}{\sum n_i} = \frac{0.028}{0.1733} = 0.162$$

$$x_{CO_2} = \frac{n_{CO_2}}{\sum n_i} = \frac{0.0023}{0.1733} = 0.0133$$

$$\begin{aligned} \text{Molecular wt. of mixture} &= \text{Molar mass of mixture} = \sum x_i \cdot M_i \\ &= (0.825 \times 28) + (0.162 \times 32) + (0.0133 \times 44) \\ &= \mathbf{28.87 \text{ kg/kmol} \quad \text{Ans.}} \end{aligned}$$

23. A gas mixture comprises of 18% O_2 , 75% N_2 and 7% CO_2 by volume at 0.5 MPa and 107°C. For 5 kg mass of mixture carry out gravimetric analysis and determine the partial pressure of gases in mixture.

Solution:

$$\text{Mole fraction of constituents} \Rightarrow x_i = \frac{n_i}{n} = \frac{V_i}{V}$$

where ' n_i ' and ' V_i ' are no. of moles and volume fraction of constituent while ' n ' and ' V ' are total no. of moles and total volume of mixture.

$$x_{O_2} = \frac{0.18}{1} = 0.18$$

$$x_{N_2} = \frac{0.75}{1} = 0.75$$

$$x_{CO_2} = \frac{0.07}{1} = 0.07$$

$$\begin{aligned} \text{Molecular weight of mixture} &= \text{Molar mass} \\ &= (0.18 \times 32) + (0.75 \times 28) + (0.07 \times 44) \\ &= 29.84 \end{aligned}$$

Gravimetric analysis refers to the mass fraction analysis.

Mass fraction of constituents

$$= \frac{m_i}{m} = \frac{(V_i / V) \times \text{Mol. wt. of constituent}}{\text{Mol. wt. of mixture}}$$

$$\left. \begin{aligned} \text{Mole fraction of } O_2 &= \frac{0.18 \times 32}{29.84} = \mathbf{0.193} \\ \text{Mole fraction of } N_2 &= \frac{0.75 \times 28}{29.84} = \mathbf{0.704} \\ \text{Mole fraction of } CO_2 &= \frac{0.07 \times 44}{29.84} = \mathbf{0.104} \end{aligned} \right\} \text{Ans.}$$

Partial pressures of constituents = Volume fraction \times Pressure of mixture

$$\left. \begin{aligned} \text{Partial pressure of } O_2 &= 0.18 \times 0.5 = \mathbf{0.09 \text{ MPa}} \\ \text{Partial pressure of } N_2 &= 0.75 \times 0.5 = \mathbf{0.375 \text{ MPa}} \\ \text{Partial pressure of } CO_2 &= 0.07 \times 0.5 = \mathbf{0.35 \text{ MPa}} \end{aligned} \right\} \text{Ans.}$$

24. A steel insulated tank of 6 m³ volume is equally divided into two chambers using a partition. The two portions of tank contain N_2 gas at 800 kPa and 480 K and CO_2 gas at 400 kPa and 390 K. Determine the equilibrium temperature and pressure of mixture after removing the partition. Use $\gamma = 1.4$ for N_2 , $\gamma = 1.3$ for CO_2 .

Solution:

Since tank is insulated so adiabatic mixing can be considered. Let us consider N_2 and CO_2 to behave as perfect gas.

No. of moles of N_2

$$n_{N_2} = \frac{p_{N_2} \cdot V_{N_2}}{\bar{R} \cdot T_{N_2}} = \frac{800 \times 10^3 \times 3}{8314 \times 480} = 0.601$$

No of moles of CO_2

$$n_{CO_2} = \frac{p_{CO_2} \cdot V_{CO_2}}{\bar{R} \cdot T_{CO_2}} = \frac{400 \times 10^3 \times 3}{8314 \times 390} = 0.370$$

Total no. of moles of mixture,

$$\begin{aligned} n &= n_{N_2} + n_{CO_2} \\ &= 0.601 + 0.370 \\ &= 0.971 \end{aligned}$$

Specific heat for N_2 at constant volume,

$$C_{v, N_2} = \frac{R_{N_2}}{(\gamma_{N_2} - 1)} = \frac{(8314 / 28)}{(1.4 - 1)}$$

$$C_{v, N_2} = 742.32 \text{ J/kg} \cdot \text{K}$$

Specific heat for CO_2 at constant volume,

$$C_{v, CO_2} = \frac{R_{CO_2}}{(\gamma_{CO_2} - 1)} = \frac{(8314 / 44)}{(1.3 - 1)}$$

$$C_{v, CO_2} = 629.85 \text{ J/kg} \cdot \text{K}$$

Mass of $N_2 = n_{N_2} \times \text{Mol. wt. of } N_2 = 0.601 \times 28 = 16.828 \text{ kg}$

Mass of $CO_2 = n_{CO_2} \times \text{Mol. wt. of } CO_2 = 0.370 \times 44 = 16.28 \text{ kg.}$

Let us consider the equilibrium temperature of mixture after adiabatic mixing at T . Applying energy conservation principle :

$$\begin{aligned} m_{N_2} \cdot C_{v, N_2} \cdot (T - T_{N_2}) + m_{CO_2} \cdot C_{v, CO_2} \cdot (T - T_{CO_2}) &= 0 \\ \{16.828 \times 742.32 (T - 480)\} + \{16.28 \times 629.85 (T - 390)\} &= 0 \\ 22745.7 \cdot T &= 9995088.881 \end{aligned}$$

Equilibrium temperature, **$T = 439.4 \text{ K}$ Ans.**

Equilibrium pressure of mixture, $T_{\text{mixture}} = 439.4 \text{ K}$, $V_{\text{mixture}} = 6 \text{ m}^3$

$$p_{\text{mixture}} = \frac{n \cdot \bar{R} \cdot T_{\text{mixture}}}{V_{\text{mixture}}} = \frac{0.971 \times 8314 \times 439.4}{6}$$

Equilibrium pressure = **591.205 kPa Ans.**

25. 2 kg of Hydrogen and 3 kg of Helium are mixed together in an insulated container at atmospheric pressure and 100 K temperature. Determine the specific heat of final mixture if specific heat at constant pressure is 11.23 kJ/kg . K and 5.193 kJ/kg . K for H_2 and He respectively.

Solution:

Two gases are non reacting, therefore, specific heat of final mixture can be obtained by following

for adiabatic mixing.

$$C_{p, \text{mixture}} = \frac{c_{p, H_2} \cdot m_{H_2} + c_{p, He} \cdot m_{He}}{(m_{H_2} + m_{He})}$$

Substituting values,

$$= \frac{(2 \times 11.23) + (3 \times 5.193)}{(3 + 2)}$$

$$C_{p, \text{mixture}} = \mathbf{7.608 \text{ kJ/kg} \cdot \text{K} \text{ Ans.}}$$

26. A mixture of 18 kg hydrogen, 10 kg nitrogen and 2 kg of carbon dioxide is put into a vessel at atmospheric conditions. Determine the capacity of vessel and the pressure in vessel if it is heated upto twice of initial temperature. Take ambient temperature as 27°C.

Solution:

Gas constant for mixture can be obtained as;

$$R_{\text{mixture}} = \frac{(m_{H_2} \cdot R_{H_2} + m_{N_2} \cdot R_{N_2} + m_{CO_2} \cdot R_{CO_2})}{(m_{H_2} + m_{N_2} + m_{CO_2})}$$

$$R_{H_2} = \frac{8.314}{2} \text{ kJ/kg} \cdot \text{K}$$

$$R_{N_2} = \frac{8.314}{28} \text{ kJ/kg} \cdot \text{K}$$

$$R_{CO_2} = \frac{8.314}{44} \text{ kJ/kg} \cdot \text{K}$$

$$R_{H_2} = 4.15 \text{ kJ/kg} \cdot \text{K}$$

$$R_{N_2} = 0.297 \text{ kJ/kg} \cdot \text{K}$$

$$R_{CO_2} = 0.189 \text{ kJ/kg} \cdot \text{K}$$

$$R_{\text{mixture}} = \frac{(18 \times 4.15 + 10 \times 0.297 + 2 \times 0.189)}{30}$$

$$R_{\text{mixture}} = 2.606 \text{ kJ/kg} \cdot \text{K}$$

Considering mixture to be perfect gas;

$$\text{Capacity of vessel } V_{\text{mixture}} = \frac{m_{\text{mixture}} \cdot R_{\text{mixture}} \cdot T}{p}$$

Here, $p = 101.325 \text{ kPa}$

$$V_{\text{mixture}} = \frac{30 \times 2.606 \times 300.15}{101.325}$$

$$\text{Capacity of vessel} = \mathbf{231.58 \text{ m}^3 \text{ Ans.}}$$

For constant volume heating, final pressure shall be,

$$p_{\text{final}} = p_{\text{initial}} \times \frac{T_{\text{final}}}{T_{\text{initial}}}$$

$$p_{\text{final}} = 101.325 \times 2 = \mathbf{202.65 \text{ kPa} \text{ Ans.}}$$

27. Determine the ratio of exit to inlet diameter of a duct having heating element in the mid of duct. Atmospheric air enters the duct at 27°C and gets heated up to 500 K before leaving the duct. The kinetic and potential energy changes during passage of air through duct are negligible.

Solution:

Said air heating process is the case of constant pressure process. Let inlet state be '1' and exit state '2'.

Therefore, by Charle's law volume and temperature can be related as;

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$\text{or} \quad \frac{\left(\frac{\pi}{4} \times d_2^2\right) \times \text{Velocity at 2}}{\left(\frac{\pi}{4} \times d_1^2\right) \times \text{Velocity at 1}} = \frac{T_2}{T_1}$$

$$\text{Since} \quad \Delta \text{K.E} = 0, \text{ so } \frac{d_2^2}{d_1^2} = \frac{T_2}{T_1}$$

$$\text{or} \quad \frac{d_2}{d_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\text{Exit to inlet diameter ratio} = \sqrt{\frac{500}{300.15}} = 1.29 = \mathbf{1.29 \text{ Ans.}}$$

28. A vessel of 2 m³ volume contains hydrogen at atmospheric pressure and 27°C temperature. An evacuating pump is connected to vessel and the evacuation process is continued till its pressure becomes 70 cm of Hg vacuum.

Estimate the mass of hydrogen pumped out. Also determine the final pressure in vessel if cooling is carried up to 10°C. Take atmospheric pressure as 76 cm of Hg and universal gas constant as 8.314 kJ/kg. K

Solution:

$$\text{For hydrogen, gas constant, } R = \frac{8.314}{2}$$

$$R = 4.157 \text{ kJ/kg} \cdot \text{K}$$

Say initial and final states are given by '1' and '2'.

Mass of hydrogen pumped out shall be difference of initial and final mass inside vessel.

Final pressure of hydrogen = Atm. pr. – Vacuum pr.

$$\begin{aligned}
 &= 76 - 70 \\
 &= 6 \text{ cm of Hg.} \\
 \text{Therefore, pressure difference} &= 76 - 6 \\
 &= 70 \text{ cm of Hg.} \\
 &= \frac{70}{76} \times 101.325 \text{ kPa} \\
 &= 93.33 \text{ kPa}
 \end{aligned}$$

$$\begin{aligned}
 \text{Mass pumped out} &= \frac{p_1 V_1}{RT_1} - \frac{p_2 V_2}{RT_2}; \text{ here } V_1 = V_2 = V \text{ and } T_1 = T_2 = T. \\
 &= \frac{V}{RT} (p_1 - p_2) \\
 &= \frac{2 \times 93.33 \times 10^3}{4.157 \times 300.15 \times 10^3} \\
 &= \mathbf{0.15 \text{ kg.} \quad \text{Ans.}}
 \end{aligned}$$

During cooling upto 10°C, the process may be considered as constant volume process. Say the state before and after cooling are denoted by suffix 2 and 3.

$$\begin{aligned}
 \text{Therefore,} \quad p_3 &= \frac{T_3}{T_2} \cdot p_2 \\
 &= \frac{283.15}{300.15} \times \frac{6 \times 101.325}{76} \\
 \text{Final pressure after cooling} &= \mathbf{7.546 \text{ kPa.} \quad \text{Ans.}}
 \end{aligned}$$

EXERCISE

- 1.1 Define thermodynamics and discuss different approaches to study of thermodynamics.
- 1.2 Write short notes on the following:
Thermodynamic properties, state, path, process, closed system, isolated system, open system, extensive and intensive properties.
- 1.3 What is meant by quasi-static process? Also discuss its physical significance.
- 1.4 Describe thermodynamic equilibrium of a system.
- 1.5 State thermodynamic definition of work. Also differentiate between heat and work.
- 1.6 What is energy? What are different forms of it?
- 1.7 Explain the concept of continuum.
- 1.8 Define perfect gas.
- 1.9 Differentiate between characteristic gas constant and universal gas constant.
- 1.10 State the Dalton's law of partial pressures and assumptions for it.
- 1.11 What is meant by real gas? Why ideal equation of state cannot be used for it?
- 1.12 Write equations of state for real gas.
- 1.13 Define compressibility factor.

- 1.14** Write Boyle's law and Charle's law.
- 1.15** Determine the absolute pressure of gas in a tank if the pressure gauge mounted on tank reads 120 kPa pressure. [221.3 kPa]
- 1.16** What shall be the volume of a fluid having its specific gravity as 0.0006 and mass as 10 kg? [16.67 m³]
- 1.17** Determine the pressure of compressed air in an air vessel, if the manometer mounted on it shows a pressure of 3 m of mercury. Assume density of mercury to be $13.6 \times 10^3 \text{ kg/m}^3$ and atmospheric pressure as 101 kPa. [501.25 kPa]
- 1.18** Calculate the kinetic energy of a satellite revolving around the earth with a speed of 1 km/s. Assume acceleration due to gravity as 9.91 m/s^2 and gravitational force of 5 kN. [254.8 MJ]
- 1.19** If the gauge pressure of oil in a tube is 6.275 kPa and oil's specific gravity is 0.8, then determine depth of oil inside tube. [80 cm]
- 1.20** Determine the work required for displacing a block by 50 m and a force of 5 kN. [250 kJ]
- 1.21** Determine the barometer reading in millimetres of Hg if the vacuum measured on a condenser is 74.5 cm of Hg and absolute pressure is 2.262 kPa. [760 mm]
- 1.22** Determine the absolute pressures for the following:
(i) Gauge pressure of 1.4 MPa
(ii) Vacuum pressure of 94.7 kPa
Take barometric pressure as 77.2 cm of Hg and density of mercury as $13.6 \times 10^3 \text{ kg/m}^3$. [1.5 MPa, 8.3 kPa]
- 1.23** Determine the pressure acting upon surface of a vessel at 200 m deep from surface of sea. Take barometric pressure as 101 kPa and specific gravity of sea water as 1.025. [2.11 MPa]
- 1.24** A vacuum gauge gives pressure in a vessel as 0.1 bar, vacuum. Find absolute pressure within vessel in bars. Take atmospheric pressure as 76 cm of mercury column, $g = 9.8 \text{ m/s}^2$, density of mercury = 13.6 g/cm^3 . [0.91 bar]
- 1.25** Determine the work done upon a spring having spring constant of 50 kN/m. Spring is stretched to 0.1 m from its unstretched length of 0.05 m. [0.0625 kJ]
- 1.26** Determine the mass of oxygen contained in a tank of 0.042 m^3 at 298 K and $1.5 \times 10^7 \text{ Pa}$ considering it as perfect gas. Also determine the mass using compressibility charts. [8.25, 8.84]
- 1.27** What will be specific volume of water vapour at 1 MPa and 523 K, if it behaves as ideal gas? Also determine the same considering generalized compressibility chart. [0.241 m³/kg, 0.234 m³/kg]
- 1.28** Calculate the pressure of CO₂ gas at 27°C and 0.004 m³/kg treating it as ideal gas. Also determine the pressure using Van der Waals equation of state. [14.17 MPa, 6.9 MPa]
- 1.29** Determine molecular weight and gas constant for a mixture of gases having 65% N₂, 35% CO₂ by mole. [33.6 kg/k mol, 0.247 kJ/kg . K]
- 1.30** Considering air as a mixture of 78% N₂, 22% O₂ by volume determine gas constant, molecular weight, C_p and C_v for air at 25°C. [0.2879 kJ/kg . K, 28.88 kg/K mol, 1.0106 kJ/kg . K, 0.722 kJ/kg . K]
- 1.31** What minimum volume of tank shall be required to store 8 kmol and 4 kmol of O₂ and CO₂ respectively at 0.2 MPa, 27°C ? [149.7 m³]
- 1.32** Two tanks A and B containing O₂ and CO₂ have volumes of 2 m³ and 4 m³ respectively. Tank A is at 0.6 MPa, 37°C and tank B is at 0.1 MPa and 17°C. Two tanks are connected through some pipe so as to allow for adiabatic mixing of two gases. Determine final pressure and temperature of mixture. [0.266 MPa, 30.6°C]
- 1.33** Determine the molecular weight and gas constant for some gas having $C_p = 1.968 \text{ kJ/kg . K}$, $C_v = 1.507 \text{ kJ/kg . K}$. [18.04 kg/kmol, 0.461 kJ/kg . K]

Zeroth Law of Thermodynamics

2.1 INTRODUCTION

Thermodynamics is the branch of science which deals with the energy interactions. In order to find whether energy interactions are taking place or not some measurable mathematical parameters are needed. These parameters are called thermodynamic properties. Out of number of thermodynamic properties discussed earlier the 'temperature' is one property.

One is well familiar with the qualitative statement of the state of a system such as cold, hot, too cold, too hot etc. based on the day to day experience. The degree of hotness or coldness is relative to the state of observer. For example, let us take an iron bar. Obviously the bar shall have initial temperature equal to the room temperature. Now let us heat this metal bar. Observations at the molecular level show that upon heating the molecular activity inside the bar gets increased. This may be attributed to the more agitated state of molecules as energy is given to them in the form of heating of the bar. From the physiological sensations it can be felt that this has resulted in increase in the degree of hotness of the bar. This qualitative indication of the relative hotness can be exactly defined by using thermodynamic property known as *temperature*. If this hot bar is brought in contact with another bar at room temperature one can feel that after some time the two bars which were initially at high and low temperatures attain the same temperature which is lying between the two temperatures. It is indicative of the fact that there has been exchange of some entity between two bars resulting in the attainment of final equilibrium temperature. This state of attainment of common equilibrium temperature is also termed as the state of thermal equilibrium. Thus, the temperature becomes a potential indicator of the energy interactions in the systems.

A look at the history shows that for quantitative estimation of temperature a German instrument maker Mr. Gabriel Daniel Fahrenheit (1686-1736) came up with idea of instrument like thermometer and developed mercury in glass thermometer. Here he observed that height of mercury column used to change as the bulb of thermometer was brought in the environments having different degrees of hotness. In the year 1742, a Swedish astronomer Mr. Anders Celsius described a scale for temperature measurement. This scale later on became very popular and is known as *Centigrade Scale*. For calibration of these measuring instruments some reference states of different substances were used initially and the relative state of temperature of the substance could be quantified. Later on with the passage of time things were standardised and internationally acceptable temperature scales and instruments were developed.

2.2 PRINCIPLE OF TEMPERATURE MEASUREMENT AND ZEROTH LAW OF THERMODYNAMICS

After the identification of 'Temperature' as a thermodynamic property for quantification of the energy interactions the big question was its estimation. Based on the relative degree of coldness/hotness concept it was concluded that the absolute value of temperature is difficult to be described. Hence it was mooted

to make temperature estimations in reference to certain widely acceptable known thermal states of the substances. Temperature is thus the intensive parameter and requires reference states. These acceptable known thermal states are such as the boiling point of water commonly called steam point, freezing point of water commonly called ice point etc. These easily reproducible and universally acceptable states of the substance are known as *reference states* and the temperature values assigned to them are called *reference temperatures*. Since these reference points and reference temperatures maintain their constant value, therefore these are also called *fixed points* and *fixed temperatures* respectively. A list of these fixed points is given in Table 2.1.

Table 2.1 Some fixed points used for International Practical Temperature Scale

Sl. No.	Reference State	Temperature °C
1.	Ice point	0
2.	Steam point	100
3.	Triple point of water	0.010
4.	Triple point of hydrogen	-259.34
5.	Triple point of oxygen	-218.79
6.	Oxygen point (normal boiling point)	-182.96
7.	Silver point (normal freezing point)	961.93
8.	Gold point (normal freezing point)	1064.43
9.	Zinc point (normal freezing point)	419.58
10.	Neon point (normal boiling point)	-246.05
11.	Sulphur point (normal boiling point)	444.60

The methodology adopted was to first develop a temperature measurement system which could show some change in its characteristics (property) due to heat interactions taking place with it. Such systems are called *thermometers*, the characteristics of property which shows change in its value is termed *thermometric property* and the substance which shows change in its thermometric property is called *thermometric substance*. Science that deals with the temperature and its measurement is called *thermometry*. For example in case of *clinical thermometer* the *mercury* in glass is the *thermometric substance* and since there is change in length of mercury column due to the heat interactions taking place between the thermometer and the body whose temperature is to be measured, therefore the *length* is the *thermometric property*. Thus, the underlying principle of temperature measurement is to bring the thermometer in thermal equilibrium with the body whose temperature is to be measured, i.e. when there is no heat interaction or the state when two (thermometer and body) attain same temperature. In this process it is to be noted that thermometer is already calibrated using some standard reference points by bringing thermometer in thermal equilibrium with reference states of the substance.

Zeroth law of thermodynamics states that if the bodies A and B are in thermal equilibrium with a third body C separately then the two bodies A and B shall also be in thermal equilibrium with each other. This is the principle of temperature measurement. Block diagram shown in Fig. 2.1a and 2.1b show the zeroth law of thermodynamics and its application for temperature measurement respectively.

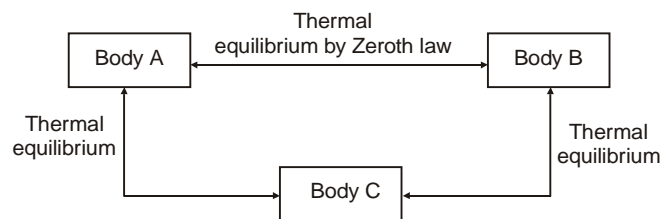


Fig. 2.1a Zeroth law of thermodynamics

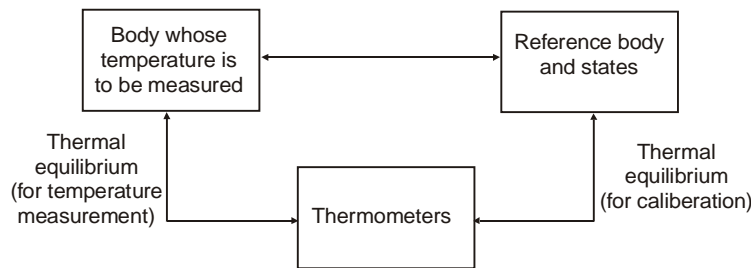


Fig. 2.1b Application of Zeroth law for temperature measurement

2.3 TEMPERATURE SCALES

Number of temperature measuring scales came up from time to time. The text ahead gives a brief idea of the different temperature scales used in thermometry. Different temperature scales have different names based on the names of persons who originated them and have different numerical values assigned to the reference states.

(a) Celsius Scale or Centigrade Scale

Anders Celsius gave this Celsius or Centigrade scale using ice point of 0°C as the lower fixed point and steam point of 100°C as upper fixed point for developing the scale. It is denoted by letter *C*. Ice point refers to the temperature at which freezing of water takes place at standard atmospheric pressure. Steam point refers to the temperature of water at which its vaporization takes place at standard atmospheric pressure. The interval between the two fixed points was equally divided into 100 equal parts and each part represented 1°C or 1 degree celsius.

(b) Fahrenheit Scale

Fahrenheit gave another temperature scale known as Fahrenheit scale and has the lower fixed point as 32 F and the upper fixed point as 212 F. The interval between these two is equally divided into 180 part. It is denoted by letter *F*. Each part represents 1 F.

(c) Rankine Scale

Rankine scale was developed by William John MacQuorn Rankine, a Scottish engineer. It is denoted by letter *R*. It is related to Fahrenheit scale as given below.

$$T_R = T_F + 459.67$$

(d) Kelvin Scale

Kelvin scale proposed by Lord Kelvin is very commonly used in thermodynamic analysis. It also defines the absolute zero temperature. Zero degree Kelvin or absolute zero temperature is taken as -273.15°C . It is denoted by letter *K*.

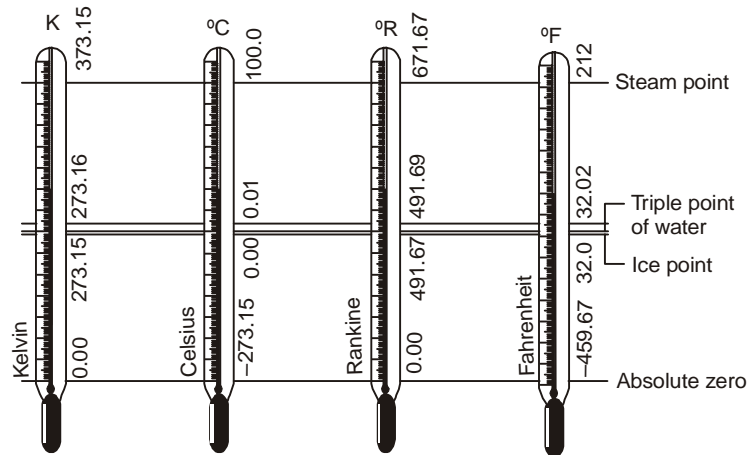


Fig. 2.2 Different temperature scales

Detailed discussion on Kelvin scale has been done in chapter 4 along with absolute thermodynamic temperature scale. Mathematically, it is related to the different temperature scales as follows,

$$\frac{T_C}{100} = \frac{T_F - 32}{180} = \frac{T_K - 273.15}{100} = \frac{T_R - 491.67}{180}$$

$$\frac{T_K}{100} = \frac{T_R}{180}$$

2.4 TEMPERATURE MEASUREMENT

For measurement of temperature number of thermometers are available using different thermometric properties of the thermometric substances. Length, volume, pressure, resistance, e.m.f. etc. are the commonly used thermometric properties for thermometers. Different thermometers developed using these thermometric properties are given below.

(a) Liquid Thermometer

Liquid thermometers are those thermometers that employ liquids as the thermometric substance and the change in volume of liquid with heat interaction is the characteristics used for temperature measurement. Commonly used liquids in such thermometers are Mercury and Alcohol. Fig. 2.3 shows the mercury in glass thermometer. In this the change in volume of the mercury results in the rise or fall in the level of mercury column in the glass tube. Out of the two liquids mercury is preferred over alcohol as it has low specific heat and hence absorbs little heat from body. Mercury is comparatively a good conductor of heat. Mercury can be seen in a fine capillary tube conveniently. Mercury does not wet the wall of the tube. Mercury has a uniform coefficient of expansion over a wide range of temperature and remains liquid over a large range as its freezing and boiling points are -39°C and 357°C respectively.

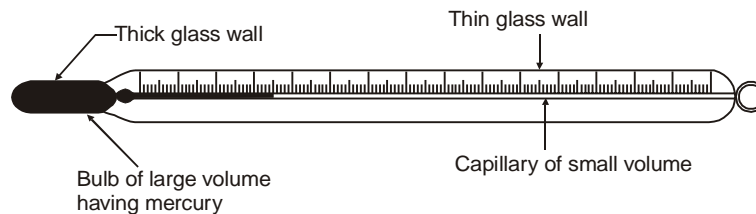


Fig. 2.3 Mercury in glass thermometer

(b) Gas Thermometers

Thermometers using gaseous thermometric substance are called gas thermometers. Gas thermometers are advantageous over the liquid thermometers as the coefficient of expansion of gases is more compared to liquids therefore these are more sensitive. Also thermal capacity of a gas is low compared to liquid so even a small change can also be recorded accurately. Gas thermometers are not suitable for routine work as they are large, cumbersome and can be used only in certain fixed conditions. These are used mainly for calibration and standardization purpose. Main types of gas thermometers are discussed ahead.

(i) **Constant volume gas thermometer** : Fig. 2.4 shows a typical constant volume gas thermometer having a glass bulb 'B' connected to glass tube. Other end of glass tube is connected to mercury reservoir through a rubber tube. There is a fixed marking 'M' over the glass tube. Difference in levels of mercury in reservoir with reference to mark 'M' is seen on the scale. Bulb 'B' is generally filled with 1/7th of its volume by mercury so as to compensate for expansion of bulb 'B'. This is done so as to keep volume of air in bulb upto the fixed mark 'M'.

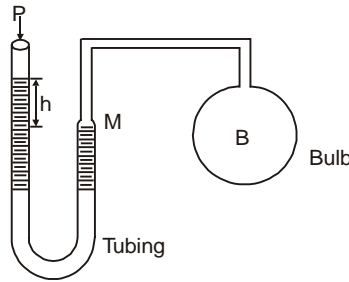


Fig. 2.4 Constant volume gas thermometer

Initially the bulb 'B' is kept in melting ice and reservoir level is suitably adjusted so that mercury level is at mark 'M'. Corresponding to this difference in level of reservoir and mark 'M' of h_i height, the ice point pressure shall be,

$$P_i = P + (h_i \cdot \rho \cdot g) = P_0$$

Bulb is kept at the boiling water (steam point) and again the reservoir is adjusted so as to keep mercury at the fixed mark. For difference in mercury levels between mark 'M' and reservoir level being h_s the pressure corresponding to steam point shall be

$$P_s = P + (h_s \cdot \rho \cdot g) = P_{100}$$

Now for the bulb 'B' kept in the bath whose temperature is to be measured, again the reservoir is to be adjusted so as to keep mercury level at mark 'M'. At this state if the difference in mercury levels is h_t , then the pressure shall be,

$$P_t = P + (h_t \cdot \rho \cdot g)$$

For a fixed volume, the pressure variation with respect to temperature can be given as,

$$P = P_0 (1 + \alpha \cdot t)$$

Similarly making appropriate substitutions one can give the temperature t as follows,

$$t = \frac{(h_t - h_i) \times 100}{(h_s - h_i)}$$

(ii) **Constant pressure gas thermometer** : These thermometers are based on the principle that, pressure remaining constant the volume of a given mass of gas is directly proportional to its absolute temperature. Fig. 2.5 shows a constant pressure gas thermometer having a silica bulb 'B' connected to

the reservoir 'R' containing mercury through a *connecting tube 'A'*, *compensating bulb 'C'* having a compensating tube with volume equal to the connecting tube. Manometer tube contains sulphuric acid.

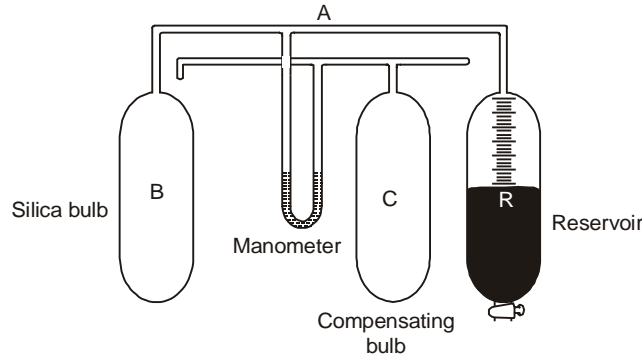


Fig. 2.5 Constant pressure gas thermometer

Initially the reservoir is filled with mercury upto zero marking and the stop cock is closed. The bulbs 'B' 'R' and 'C' are immersed in melting ice. Tubes are sealed when the pressure on the two sides as shown by manometer is the same i.e. the pressure in silica bulb 'B' and compensating bulb 'C' are same. When the pressure on two sides of the manometer containing sulphuric acid is same the acid level in two limbs shall be same. This way the pressure of gas and air can be maintained same. Now let us assume the silica bulb 'B' to have definite number of molecules of air. Also, the compensating bulb and compensating tube contain the same number of molecules of air. If the silica bulb is immersed in the environment whose temperature is to be measured and compensating bulb being kept in melting ice. Both connecting tube and compensating tubes are at the room temperature and the air in silica bulb attains temperature equal to the temperature to be measured.

(c) Electrical resistance thermometer

Electrical resistance thermometer first developed by Siemen in 1871, also known as 'Platinum Resistance Thermometer' works on the principle of change in resistance of the thermometric substance (platinum) with temperature. Thus resistance is the thermometric property used in these thermometers. It consists of a pure platinum wire wound in a double spiral on a mica plate. Two ends of the platinum wire are connected to the copper leads (for low temperatures) or platinum leads (for high temperatures). Principle of Wheatstone bridge is employed in these thermometers, as shown in Fig. 2.6. It has a set of compensating leads having exactly similar resistance as leads used. Platinum wire and the compensating leads are enclosed in a sealed glazed porcelain tube having binding terminals at the top. The resistance of wire can be mathematically related as

$$R_t = R_0 \cdot (1 + a \cdot t + b \cdot t^2)$$

where a and b are the constants having their values depending upon the nature of material used.

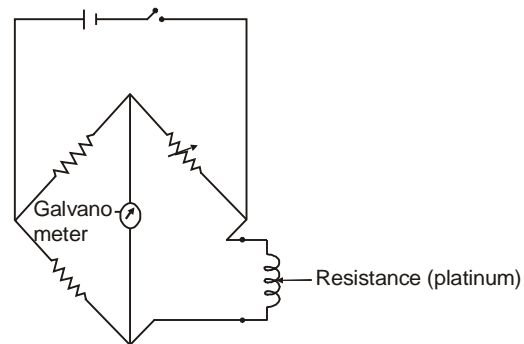


Fig. 2.6 Electrical resistance thermometer, (principle of wheatstone bridge)

Using fixed points of ice point and steam point the temperature can be mathematically obtained by substituting the different parameters in the following,

$$t = \frac{(R_t - R_i) \times 100}{(R_s - R_i)},$$

where R_i & R_s are resistance values for ice and steam points

(d) Thermoelectric Thermometer

Thermo electric thermometer works on the principle of Seebeck effect. *Seebeck effect* says that a current flows or e.m.f. is produced in a circuit of two dissimilar metals having one junction as hot while other as cold junction. Current produced in this way is called thermo electric current while the e.m.f. produced is called thermo e.m.f. Measurement of temperature is being done by knowing the e.m.f. produced which is the thermometric property here.

In such type of thermometer a sensitive galvanometer is connected with thermocouple as shown in Fig. 2.7. One junction is kept at ice point and other in oil bath having any temperature. Upon heating the oil bath it is seen that the thermal e.m.f. is produced by Seebeck effect. Temperature of the oil bath is measured by some calibrated thermometer of any other type. Further the temperature of oil bath is changed to known temperatures and for different temperatures the e.m.f. is noted and a graph is plotted between the temperature of bath and e.m.f.

Now for using this thermocouple the cold junction shall still be maintained at the ice point while the hot junction may be kept in contact with the bath whose temperature is to be measured. To get the temperature depending upon the e.m.f. available the calibrated graph is used and corresponding temperature noted from there.

In these thermometers the potentiometers may also be used as shown in Fig. 2.8. Here also the one junction is maintained at ice point while the other junction is put at the temperature to be measured. The potentiometer wire is directly calibrated to measure temperature.

Here the length of the potentiometer wire at which the balance point is obtained is used for getting temperature.

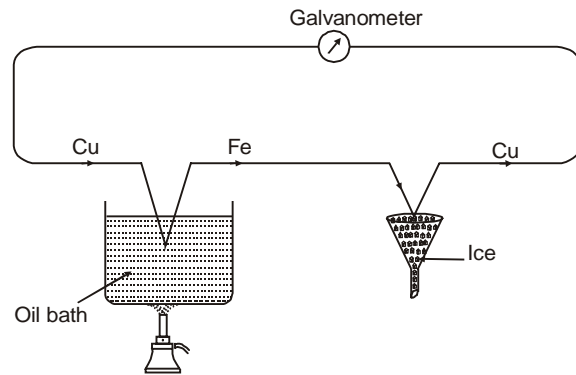


Fig. 2.7 Thermoelectric thermometer using a galvanometer

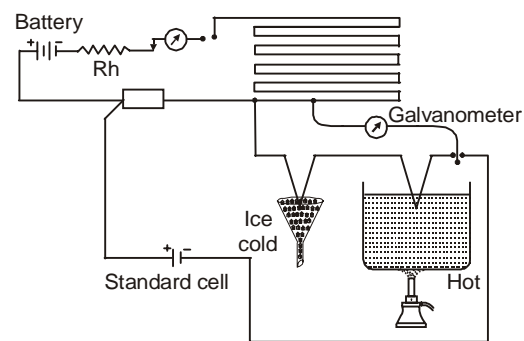


Fig. 2.8 Thermoelectric thermometer using a potentiometer

EXAMPLES

1. Determine the human body temperature in degree celsius ($^{\circ}\text{C}$) if the temperature in Fahrenheit is 98.6°F .

Solution:

Degree Celsius and Fahrenheit are related as below,

$$T(^{\circ}\text{C}) = \frac{T(^{\circ}\text{F}) - 32}{1.8}$$

Substituting values.

$$T(^{\circ}\text{C}) = \frac{98.6 - 32}{1.8} = 37^{\circ}\text{C}$$

Temperature in degree celsius shall be **37°C. Ans.**

2. A temperature scale is being developed using the following relation.

$$t = a \cdot \ln(p) + \left(\frac{b}{2}\right)$$

where 'p' is thermometric property and 'a' and 'b' are constants. Determine celsius temperature corresponding to thermometric property of 6.5, if ice point and steam point give thermometric property value of 3 and 8.

Solution:

For Ice point; $t = 0^{\circ}\text{C}$ and $p = 3$

For Steam point; $t = 100^{\circ}\text{C}$ and $p = 8$

Using thermometric relation,

$$0 = a \ln(3) + \left(\frac{b}{2}\right)$$

$$100 = a \ln(8) + \frac{b}{2}$$

Solving the above two equations, we get

$$a = 101.95$$

$$b = 224$$

Thus,
$$t = 101.95 \cdot \ln(p) + \left(\frac{224}{2}\right)$$

$$t = 101.95 \ln(p) + 112$$

For $p = 6.5$, $t = \mathbf{302.83^{\circ}\text{C}}$ **Ans.**

3. In a thermoelectric thermometer for $t^{\circ}\text{C}$ temperature, the emf is given as;

$$E = 0.003 \cdot t - 5 \times 10^{-7} \cdot t^2 + 0.5 \times 10^{-3}, \text{ volts}$$

Thermometer is having reference junction at ice point and is calibrated at ice point and steam points. What temperature shall be shown by the thermometer for a substance at 30°C ?

Solution:

At ice point; $t = 0^{\circ}\text{C}$, $E_0 = 0.5 \times 10^{-3}$, volts

At steam point, $t = 100^{\circ}\text{C}$, $E_{100} = 0.0265$, volts

When $t = 30^{\circ}\text{C}$

$$E_{30} = 9.14 \times 10^{-3} \text{ volts}$$

Thus temperature shown by this thermometer;

$$t = \left(\frac{E_{30} - E_0}{E_{100} - E_0}\right) \times (T_{100} - T_0)$$

$$= \left(\frac{9.14 \times 10^{-3} - 0.5 \times 10^{-3}}{0.0265 - 0.5 \times 10^{-3}} \right) \times 100$$

$$= 33.23^\circ\text{C Ans.}$$

4. Estimate the % variation in temperature reading from a thermocouple having its test junction in gas and other reference junction at ice point. The temperature of gas using gas thermometer is found 50°C . Thermocouple is calibrated with emf varying linearly between ice point and steam point. When thermocouple's test junction is kept in gas at $t^\circ\text{C}$ and reference junction at ice point, the e.m.f. produced in millivolts is,

$$e = 0.18 \cdot t - 5.2 \times 10^{-4} \times t^2, \text{ millivolts.}$$

Solution:

As ice point and steam points are two reference points, so

at ice point having $t = 0^\circ\text{C}$, e.m.f. = 0

at steam point having $t = 100^\circ\text{C}$, e.m.f. = 12.8 mV

at gas temperature of 50°C , e.m.f. = 7.7 mV

Since e.m.f. variation is linear so, temperature at e.m.f. of 7.7 mV;

$$= \frac{(100-0) \times 7.7}{(12.8-0)}$$

$$= 60.16^\circ\text{C}$$

Temperature of gas using thermocouple = 60.16°C

% variation in temperature reading with respect to gas thermometer reading of 50°C .

$$= \frac{60.16 - 50}{50} \times 100$$

$$= 20.32\% \text{ Ans.}$$

5. In an unknown temperature scale freezing point of water is 0°X and boiling point of water is 1000°X . Obtain a conversion relation between degrees X and degree celsius. Also determine the absolute zero in degree X.

Solution:

Let the conversion relation be $X = aC + b$

where C is temperature in degree celsius, a & b are constants and X is temperature in $^\circ\text{X}$.

At freezing point, temperature = 0°C , 0°X

$$\text{or, } 0 = a \cdot 0 + b$$

$$\Rightarrow b = 0$$

At boiling point, temperature = 100°C , 1000°X

$$1000 = a \cdot 100 + b$$

$$\Rightarrow a = 10$$

Conversion relation

$$\boxed{X = 10 \cdot C} \text{ Ans.}$$

Absolute zero temperature in $^\circ\text{C} = -273.15^\circ\text{C}$

Absolute zero temperature in $^\circ\text{X} = -2731.5^\circ\text{X}$

$$\boxed{-2731.5^\circ\text{X}} \text{ Ans.}$$

EXERCISE

- 2.1 State Zeroth law of thermodynamics.
- 2.2 Explain, how the Zeroth law of thermodynamics can be used for temperature measurement.
- 2.3 Write short notes on the following:
Thermometry, thermometric substance, thermometric property, Constant volume gas thermometer.
- 2.4 Sketch and explain the working of constant pressure thermometer.
- 2.5 Write equivalence amongst different temperature scales. Also write brief note on each of them.
- 2.6 Obtain triple point of water in Fahrenheit, Rankine and Kelvin scale.
- 2.7 Heating of a body causes its temperature to change by 30°F . Find out the increase in temperature in $^{\circ}\text{R}$ and $^{\circ}\text{C}$.
- 2.8 Temperature of an object changes by 10°C . What is the change in temperature in $^{\circ}\text{R}$, $^{\circ}\text{F}$.
(-18°R , -18°F)
- 2.9 Prove that the difference between the two temperatures in Celsius scale is same as that in Kelvin scale.
- 2.10 On some temperature scale 0°C is equivalent to 100°B and 100°C is equivalent to 300°B . Determine the temperature in $^{\circ}\text{C}$ corresponding to 200°B .
(50°C)
- 2.11 During temperature measurement of a body it is seen that the same numerical reading is obtained in Celsius and Fahrenheit scales. What is the temperature in degree Rankine?
(419.67°R)
- 2.12 Write a generic computer program for conversion of temperature in $^{\circ}\text{C}$, K, $^{\circ}\text{F}$ and $^{\circ}\text{R}$ into one another.

First Law of Thermodynamics

3.1 INTRODUCTION

Let us take water in a container and heat it from the bottom. What will happen? Container and the water inside shall start getting heated up. This heating is being sensed by either touching it or by measuring its initial and final temperatures. What has caused it to happen so?

Let us take bicycle wheel and paddle it very fast, Chained wheel starts rotating very fast. Now let us apply the brake. The wheel gets stopped. What has made it to stop? Also, if we touch the brake shoe and observe its temperature it shall show that the brake shoe has got heated up. Why has it happened so?

Answer for the above question lies in the energy interactions.

The heating up of the container and water has been caused by the heat being added through the burner flame. Heat available in the flame is being transferred to the container resulting into temperature rise of the container and water.

The fast rotation of the bicycle wheel by paddling has been due to work done in the form of paddling and causing wheel to rotate. Subsequent application of the brake has brought wheel to rest as the driving potential with the wheel is gradually dissipated due to the friction between the brake shoe and wheel. Thus the energy of wheel gets transferred to brake shoe bringing it to rest and heating up of the brake shoe (observed by the rise in temperature of brake shoe).

Thus, it is obvious that there is some entity which is responsible for the above phenomenon. This entity is called the energy which is available in different forms, some times causing the wheel rotation, heating up of water etc. Similar to the cases discussed above one may look at other real life systems and understand phenomenon occurring in them. It can be concluded that it is the energy interaction in some form which is responsible for occurrence of such phenomenon. These energy interactions only permit the transformation of energy from one form to other while making the creation and destruction of energy impossible. Hence, it is true that “energy can neither be produced nor destroyed, it can only change it’s form”. The prevailing law of “energy conservation” also states the same thing.

Here in this chapter we shall look into the first law of thermodynamics, heat, work and its definition, flow and non-flow work, their estimation, steady and unsteady flow processes, their analysis and limitations of the first law of thermodynamics.

3.2 THERMODYNAMIC PROCESSES AND CALCULATION OF WORK

Thermodynamic processes can be precisely categorized as cyclic process and non-cyclic process. The cyclic process is the one in which the initial and final states are identical i.e. system returns to its initial states after occurrence of process. The non cyclic process is the one in which the initial and final states are different i.e. the occurrence of process is accompanied by the state change. Thermodynamic work and its explanation has already been given in Article 1.12.

Let us consider a system consisting of a tank filled with water and fitted with a stirrer at room temperature, Fig. 3.1. Work can be transferred to the system by the stirrer and the temperature of water shall rise. When stirring stops, the system shall cool down till it reaches to the room temperature. Thus, the process is cyclic as the initial and final states are identical.

Let us now take a cylinder having piston and gas filled inside. If the gas is made to expand due to heating, the piston shall undergo displacement and say the piston displacement is dx . If the force exerted by gas on face of piston is F and the cross section area of piston is A , then the displacement work done may be given by :

$$dW = F \cdot dx$$

For the gas pressure being p , the force may be given by $F = p \cdot A$. Substituting for F ,

$$dW = p \cdot A \cdot dx$$

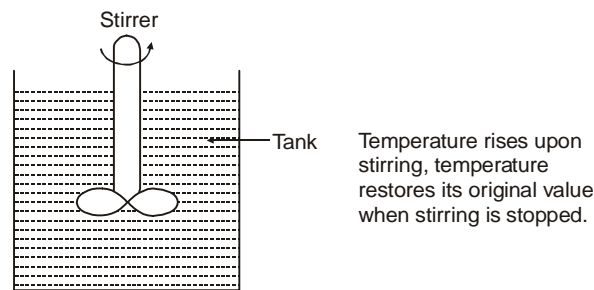


Fig. 3.1 Cyclic process

or, $dW = p \cdot dV$, where dV is the elemental change in volume or the volumetric displacement.

If the total displacement of piston is given by L then the total work can be had by integrating the above dW with respect to x for displacement L , or with respect to volume for volume change.

$$W = \int p \cdot dV = \int p \cdot A \cdot dx$$

Now, let us examine whether the work estimated above is in conformity to thermodynamic definition of work or not. If the piston displacement is transferred to a suitable link then the weight can be raised, thus it satisfies thermodynamic definition of work. What about the nature of process? cyclic or non cyclic. It is obvious that the initial and final states are not identical therefore, it is a non-cyclic process.

Thus, the work W as defined above refers to thermodynamic work for a non-cyclic process.

Thermodynamic processes can be further classified based on the thermodynamic constraints under which they occur. Different types of thermodynamic processes are as detailed below.

(i) Constant pressure process or isobaric process: It refers to the thermodynamic process in which there is no change in pressure during the process. Such type of processes are also known as isobaric processes. To understand let us take a cylindrical vessel having gas in it. It has a piston above it. Piston is free to reciprocate in the cylinder. Under normal situation piston shall be subjected to atmospheric pressure. Now, let heat be added to cylinder from bottom of cylinder. Due to heat addition, presuming energy transfer taking place reversibly and system always remaining in equilibrium, the gas shall try to expand. Expansion of gas results in raising up of the piston and it attains a new state say 2. Process is shown on p - V diagram in Fig. 3.2.

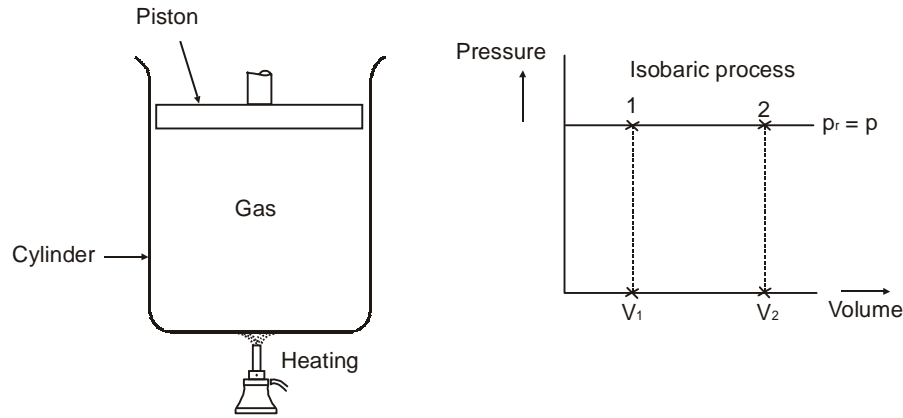


Fig. 3.2 Isobaric process

The work involved in the raising of piston shall be given by,

$$W_{1-2} = \int_{V_1}^{V_2} P \cdot dV = P \cdot (V_2 - V_1)$$

Mathematically from the first law of thermodynamics, it can be given that,

$$dQ = dU + dW$$

$$\int_1^2 dQ = \int_1^2 dU + \int_1^2 dW$$

$$\begin{aligned} Q_{1-2} &= m c_v (T_2 - T_1) + P(V_2 - V_1) \\ &= m c_v (T_2 - T_1) + mR(T_2 - T_1) \end{aligned}$$

Substituting for c_v , i.e. $c_v = \frac{R}{(\gamma - 1)}$

$$Q_{1-2} = mR (T_2 - T_1) \left\{ \frac{1}{(\gamma - 1)} + 1 \right\}$$

(ii) Constant volume process or isochoric process: When a fluid undergoes a thermodynamic process in a fixed enclosed space such that the process occurs at constant volume, then the process is called constant volume process or isochoric process. Let us consider heating of a gas in fixed enclosure at constant volume. On p - V diagram this process is represented by a vertical line as shown in Fig. 3.3. Area under the process line is zero which indicates that there is rise in pressure but there is no work done as there is no change in volume. Work involved shall be,

$$W_{1-2} = \int_{V_1=V}^{V_2=V} P \cdot dV = 0$$

From first law of thermodynamics,

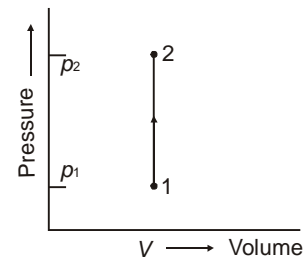


Fig. 3.3 Isochoric process

$$dQ = dU + dW$$

$$\int_1^2 dQ = \int_1^2 dU + \int_1^2 dW = \int_1^2 dU + 0$$

or

$$Q_{1-2} = U_2 - U_1 = mc_v (T_2 - T_1)$$

Thus, it indicates that the effect of heat addition in constant volume process is to increase the temperature and consequently the internal energy of system.

(iii) Constant temperature process or isothermal process: Thermodynamic process in which the temperature remains constant is called constant temperature or isothermal process. In this case the gas or vapour may be heated at constant temperature and there shall be no change in internal energy. The work done will be equal to the amount of heat supplied, as shown ahead. For a perfect gas during isothermal process;

$$p_1 V_1 = p_2 V_2 = \text{Constant, or, } P = \frac{P_1 V_1}{V}$$

so work involved,
$$W_{1-2} = \int_{V_1}^{V_2} P \cdot dV$$

$$W_{1-2} = \int_{V_1}^{V_2} \frac{P_1 V_1}{V} dV = P_1 V_1 \ln \left(\frac{V_2}{V_1} \right)$$

$$W_{1-2} = P_1 V_1 \ln r$$

where r = ratio of final and initial volumes.

By first law of thermodynamics

$$\int_1^2 dQ = \int_1^2 dW + \int_1^2 dU$$

$$Q_{1-2} = W_{1-2} + (U_2 - U_1) = W_{1-2} + 0$$

as

$$U_2 - U_1 = mc_v (T_2 - T_1), \text{ and } T_1 = T_2$$

(iv) Adiabatic process: An adiabatic process is the thermodynamic process in which there is no heat interaction during the process, i.e. during the process, $Q = 0$. In these processes the work interaction is there at the expense of internal energy. If we talk of adiabatic expansion then it shall mean that work is done at the cost of its own internal energy. The adiabatic process follows the law $PV^\gamma = \text{constant}$ where γ is called adiabatic index and is given by the ratio of two specific heats. Thus, it is obvious that adiabatic expansion shall be accompanied by the fall in temperature while temperature will rise during adiabatic compression. The adiabatic expansion process is shown on Fig. 3.5. Work done during expansion shall be,

$$W_{1-2} = \int_{V_1}^{V_2} P \cdot dV, \text{ where } PV^\gamma = \text{constant, therefore solving after substitution. Work shall be,}$$

$$W_{1-2} = \frac{P_1 V_1 - P_2 V_2}{(\gamma - 1)}$$

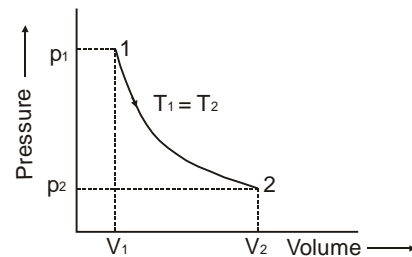


Fig. 3.4 Isothermal process

From first law of thermodynamics $\int_1^2 dQ = \int_1^2 dU + \int_1^2 dW$

$$Q_{1-2} = (U_2 - U_1) + \frac{p_1 V_1 - p_2 V_2}{(\gamma - 1)}$$

$$0 = mc_v (T_2 - T_1) + \frac{p_1 V_1 - p_2 V_2}{(\gamma - 1)}$$

$$\Rightarrow W_{1-2} = mc_v (T_1 - T_2)$$

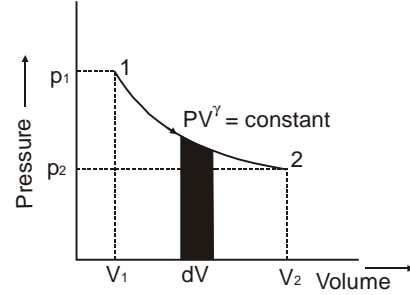


Fig. 3.5 Adiabatic expansion

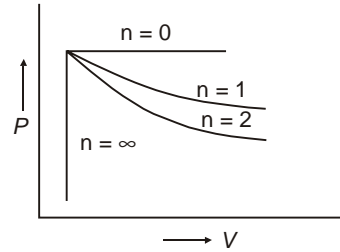
(v) Polytropic process: Polytropic process is the most commonly used process in practice. In this, the thermodynamic process is said to be governed by the law $PV^n = \text{constant}$ where n is the index which can vary from $-\infty$ to $+\infty$. Figure 3.6 shows some typical cases in which the value of n is varied and the type of process indicated for different values of n . Thus the various thermodynamics processes discussed above are special cases of polytropic process. Work interaction in case of polytropic process can be given as,

$$W_{1-2} = \int_{V_1}^{V_2} p \cdot dV$$

where $P_1 V_1^n = P_2 V_2^n = \text{constant}$

Solving the above, we get

$$W_{1-2} = \frac{p_1 V_1 - p_2 V_2}{(n - 1)}$$



$n = 0$, isobaric process
 $n = 1$, isothermal process
 $n = 2$, polytropic process
 $n = \infty$, isochoric process

Fig. 3.6 Polytropic process

From first law of thermodynamics,

$$\int_1^2 dQ = \int_1^2 dU + \int_1^2 dW$$

or

$$Q_{1-2} = (U_2 - U_1) + \frac{p_1 V_1 - p_2 V_2}{n - 1}$$

or

$$Q_{1-2} = mc_v (T_2 - T_1) + \frac{mR(T_1 - T_2)}{(n - 1)}$$

also

$$c_v = \frac{R}{\gamma - 1} \text{ or } R = c_v (\gamma - 1)$$

$$Q_{1-2} = mc_v \left\{ (T_2 - T_1) + \frac{(\gamma - 1)}{(n - 1)} \cdot (T_1 - T_2) \right\}$$

Thus heat transfer during a polytropic process for a perfect gas;

$$Q_{1-2} = mc_v (T_2 - T_1) \times \left(\frac{\gamma - n}{1 - n} \right)$$

or also, substituting for c_v

$$Q_{1-2} = \left(\frac{\gamma - n}{\gamma - 1} \right) \times W$$

(vi) Hyperbolic process: Hyperbolic process is the one in which product of pressure and volume remains constant during the process. The curve for such an expansion process is a rectangular hyperbola and hence this is known as hyperbolic expansion.

For a perfect gas $\frac{PV}{T} = \text{Constant}$, if T is also constant then it means that for a perfect gas the hyperbolic process shall also be isothermal process. Figure 3.7 shows hyperbolic expansion process between 1 and 2. Work done during process shall be

$$W_{1-2} = \int_1^2 p \cdot dV \text{ and } p_1 V_1 = p_2 V_2 = \text{constant}$$

$$W_{1-2} = \int_{V_1}^{V_2} \frac{p_1 V_1}{V} dV = p_1 V_1 \ln \frac{V_2}{V_1}$$

or, $W_{1-2} = p_1 V_1 \ln r$, where $r = \frac{V_2}{V_1}$, ratio of final and initial volumes from first law of thermodynamics,

$$\int_1^2 dQ = \int_1^2 dU + \int_1^2 dW = (U_2 - U_1) + p_1 V_1 \ln \left(\frac{V_2}{V_1} \right)$$

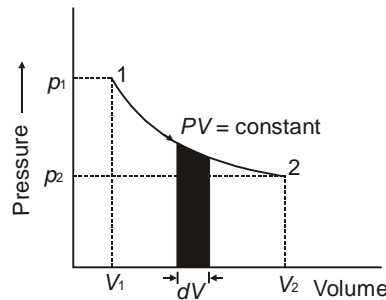


Fig. 3.7 Hyperbolic expansion

(vii) Free Expansion: Free expansion, as the name implies refers to the unrestrained expansion of a gas. Let us take an insulated tank having two compartments separated by a partition, say A and B. Let us assume that compartment A is filled with gas while B is having vacuum. If now the partition is removed and gas allowed to occupy the whole volume of tank, then the gas expands to fill the complete volume space. New pressure of gas will be lesser as compared to initial pressure of gas occupying the compartment A. A close look at the expansion process shows that the expansion due to removal of partition is unresisted expansion due to gas expanding in vacuum. This is also known as free expansion. The reverse of free expansion is impossible and so it is an irreversible process.

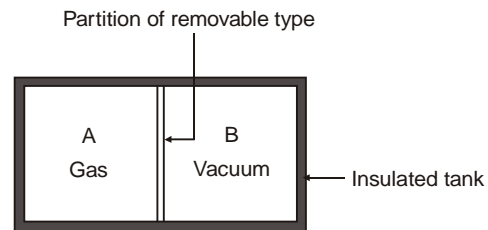


Fig. 3.8 Free expansion

During free expansion no work shall be done by the gas or on the gas due to no boundary displacement in the system.

$$W_{\text{free expansion}} = 0$$

Also in the above there shall be no heat interaction as tank is insulated. From first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

$$0 = \Delta U + 0$$

or, $U_{A+B} = U_A$, i.e. initial and final internal energies are same, which means for a perfect gas initial and final temperatures of gas are same.

Table 3.1 Thermodynamic processes

Sl. No.	Process	Governing equations	Heat interaction	Displacement work or non flow work during state change from 1 to 2 $W = \int_1^2 p.dV$
1.	Isobaric process	$p = \text{constant}$ $\frac{T_2}{T_1} = \frac{v_2}{v_1}$ index $n = 0$	$q = c_p \times (T_2 - T_1)$	$W = p(V_2 - V_1)$
2.	Isochoric process	$V = \text{constant}$ $\frac{T_1}{T_2} = \frac{p_1}{p_2}$ index, $n = \infty$	$q = c_v \times (T_2 - T_1)$	$W = 0$
3.	Isothermal process	$T = \text{constant}$ $p_1 V_1 = p_2 V_2$ index, $n = 1$	$q = p_1 V_1 \times \ln \left(\frac{V_2}{V_1} \right)$	$W = P_1 V_1 \ln \frac{V_2}{V_1}$
4.	Adiabatic process	$p_1 V_1^\gamma = p_2 V_2^\gamma$ $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$ $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$ index, $n = \gamma$	$q = 0$	$W = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$

5.	Polytropic process	$p_1 V_1^n = p_2 V_2^n$ $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{n-1}$ $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}}$ $C_n = c_v \left(\frac{\gamma - n}{1 - n} \right)$ where C_n is specific heat for polytropic process.	$q = c_v \left(\frac{\gamma - n}{1 - n} \right) \times (T_2 - T_1)$ or, $q = \left(\frac{\gamma - n}{\gamma - 1} \right) \times \text{work}$	$W = \frac{p_1 V_1 - p_2 V_2}{n - 1}$
6.	Hyperbolic process	$pV = \text{constant}$ but not necessarily $T = \text{constant}$	$q = c_v (T_2 - T_1) + RT_1 \ln \left(\frac{V_2}{V_1} \right)$	$W = RT_1 \ln \left(\frac{V_2}{V_1} \right)$
7.	Free expansion in adiabatic conditions	Unresisted expansion	$q = 0$	$W = 0$

3.3 NON-FLOW WORK AND FLOW WORK

Work interaction taking place in a system can be classified as flow work or non-flow work based on the nature of process.

Two basic types of processes are

- (i) Flow process
- (ii) Non flow process

3.3.1 Flow Process

Flow process is the one in which fluid enters the system and leaves it after work interaction, which means that such processes occur in the systems having open boundary permitting mass interaction across the system boundary.

Figure 3.9 shows the flow process occurring in a system. Flow processes can be further classified into steady flow and non-steady flow processes. Examples of engineering systems having steady flow processes are flow through nozzles, turbines, compressors etc. and the examples of nonsteady flow processes are the filling or emptying of any vessel. Flow process shown indicates various energy and mass interactions taking place across the system boundary.

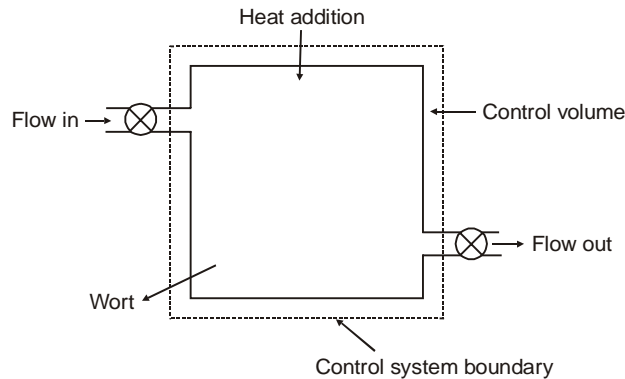


Fig. 3.9 Flow process

As the mass interaction takes place here so for every mass fraction crossing the boundary, work is done upon or by the system. Thus, a portion of work is always required to push the fluid mass into or out of the system. This amount of work is called flow work, or, “work required for causing flow of fluid to or from the system is called flow work”.

Here in the control volume shown say, some mass of fluid element is to be pushed into the control volume.

Fluid mass can be injected into the control volume with certain force, say F . The force required for pushing (F) owing to the pressure P of fluid element may be quantified as; $F = P \cdot A$, where A is cross-section area of the passage. For injecting entire mass of fluid, the force F must act through a distance L . Thus, work done in injecting the fluid element across the boundary will be,

$$W = F \cdot L = P \cdot A \cdot L, \text{ (kJ)}$$

or

$$w = P \cdot v, \text{ (kJ/kg)}$$

This work is the flow work. Thus, flow work per unit mass can be given as the product of pressure and specific volume. It is also referred to as flow energy or convected energy or transport energy.

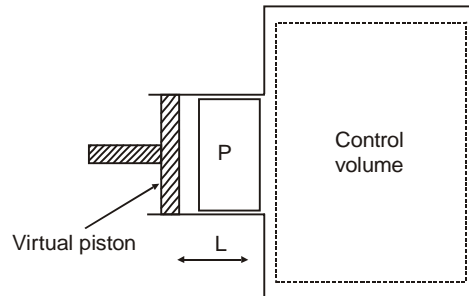


Fig. 3.10 Flow work

3.3.2 Non-Flow Processes

Non-flow process is the one in which there is no mass interaction across the system boundaries during the occurrence of the process.

Figure 3.11 shows block diagram of a piston-cylinder arrangement in which a constant mass of fluid inside the cylinder is shown to undergo compression. Thus, during compression the type of process shall be non-flow process and the work interaction shall be non-flow work.

Say, the force exerted by piston is F , and cross-section area of piston being A , the elemental work done in compressing along the length dL shall be

$$dW = F \cdot dL$$

If pressure of fluid is P then $F = P \cdot A$.

so $dW = P \cdot A \cdot dL$

The total work done in piston displacement, from 1 to 2 shall be,

$$\int_1^2 dW = \int_1^2 P \cdot A \cdot dL$$

$$\int_1^2 dW = \int_1^2 P \cdot dV$$

or
$$W_{1-2} = \int_1^2 P \cdot dV$$

Thus, this is called the non-flow work or displacement work.

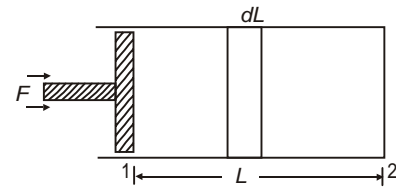


Fig. 3.11 Non-flow process

3.4 FIRST LAW OF THERMODYNAMICS

Benjamin Thompson (Count Rumford) 1753-1814 discovered the equivalence of work and heat in the course of manufacturing canon (1797) by boring solid metal submerged in the water. He was intrigued by the water boiling because of mechanical work of boring, as no heat had been added to the water. In his words, “is it possible that such a quantity of heat as would have caused five pounds of ice cold water to boil could have been furnished by so inconsiderable a quantity of metallic dust merely in consequence of a change in its capacity for heat?” Other experiments later discovered more evidence until some fifty years after the above experiment.

Let us take a bicycle, tyre pump and use it for inflating the bicycle tyre. It is observed that the pump becomes hotter during use. This phenomenon of heating of pump is obviously not from heat transfer but because of the work done. Although the heating of pump could also be realized by heat transfer. It indicates that some effects can be caused equivalently by heat or work and that there exist some relationship between heat and work.

James Prescott Joule (1818-1889) an English scientist and one time student assistant to John Dalton (1766-1844) with assistance from Lord Kelvin showed conclusively that mechanical work and heat are equivalent.

For example, let us take a closed system which permits work interaction and heat interaction both, as in case of stirring in a container, fig. 3.12. As a result of stirring it is seen that the temperature of water gets raised up. This rise in temperature can be accounted by quantifying the amount of heat supplied for raising this temperature. Thus, it is obvious that for any closed system undergoing a cycle

$\oint W = J \cdot \oint Q$, where J is Joule’s constant.

i.e., the net heat interaction is proportional to the work interaction. Also the constant is known as “Joule’s mechanical equivalent of heat”. Joule’s constant is described as;

$$\frac{W}{Q} = J = 4.18 \frac{\text{Joules}}{\text{Calories}}$$

Thus, J is a numerical conversion factor which could be unity if the heat is also given in joules.

For any cyclic process in the closed system the relationship between heat and work shall be, (if the consistent units are used)

$$\oint \delta q = \oint \delta w.$$

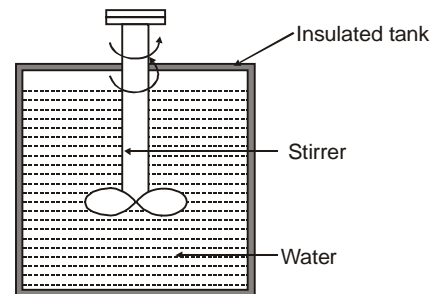


Fig. 3.12 Closed system

Thus first law of thermodynamics states that “in a closed system undergoing a cyclic process, the net work done is proportional to the net heat taken from the surroundings” or “for any cycle of a closed system the net heat transfer equals the net work”.

First law of thermodynamics can't be proved but it is supported by a large number of experiments and no exceptions have been observed. It is therefore termed as the law of nature.

Mathematical expression for the first law of thermodynamics can be rearranged and it shall be,

$$\oint (\delta q - \delta W) = 0$$

which shows that the quantity $(\delta q - \delta W)$ is a thermodynamic property.

For non-cyclic process: Let us now take up a system undergoing a non-cyclic process where transfer of heat and work take place and there is some change in the state of system i.e. initial and final states are different.

Figure 3.13 shows the non-cyclic process occurring between states 1 and 2. The change in state is accomplished by the energy interactions. If we assume the system to have the heat interaction ΔQ and work interaction ΔW , then from the basic principles it can be said that :

Energy lost = Energy gained

as the energy can neither be created nor destroyed.

Therefore, between states 1-2 one can write energy balance as,

$$Q_{1-2} - W_{1-2} = U_{1-2}$$

where Q_{1-2} , W_{1-2} and U_{1-2} are the heat, work and stored energy values. *This stored energy is called as internal energy for a system having negligible electrical, magnetic, solid distortion and surface tension effects.*

General expression based on above can be given as follows :

$$\Delta Q - \Delta W = \Delta U$$

$$\text{or} \quad \int_1^2 dQ - \int_1^2 dW = \int_1^2 dU$$

or, for elemental interactions; $dQ - dW = dU$

$$dQ = dU + dW$$

Thus, the first law of thermodynamics for non-cyclic processes can be given by

$$\int dQ = \int dU + \int dW$$

Above equations make it obvious that the internal energy change in the closed system during any non-cyclic process is obtained by subtracting the net amount of work done by the system from the net amount of heat added to the system i.e. $\Delta U = Q - W$.

Actually, there is no absolute value of internal energy of any system. Therefore its value may be taken to be zero for any particular state of the system and absolute value in reference to arbitrarily assumed state may be easily defined.

Mathematically, it can be shown that the internal energy is a thermodynamic property, as explained ahead. Let us consider the non-cyclic process following paths A, B and C in the directions as shown in Fig. 3.14.

As the processes A & B and A & C constitute a thermodynamic cycle starting and finishing at state 1, the first law of thermodynamics for cyclic process can be employed,

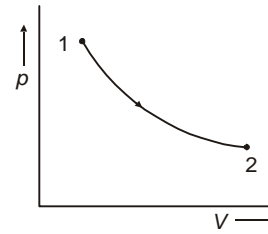


Fig. 3.13 Non cyclic process

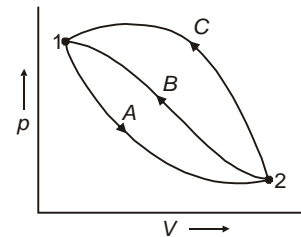


Fig. 3.14 Two different thermodynamic cycles.

$$\oint (\delta Q - \delta W) = 0$$

For the cycle following path 1-A-2-B-1, the first law of thermodynamics says,

$$\oint_{1-A-2-B-1} (\delta Q - \delta W) = 0$$

$$\text{or} \quad \int_{1,A}^2 (\delta Q - \delta W) + \int_{2,B}^1 (\delta Q - \delta W) = 0$$

which can be rewritten as,

$$\int_{1,A}^2 (\delta Q - \delta W) = - \int_{2,B}^1 (\delta Q - \delta W)$$

$$\text{or} \quad \int_{1,A}^2 (\delta Q - \delta W) = \int_{1,B}^2 (\delta Q - \delta W) \quad (i)$$

Also, for the cycle following path 1-A-2-C-1, the first law of thermodynamics can be applied as,

$$\oint_{1-A-2-C-1} (\delta Q - \delta W) = 0$$

$$\text{or} \quad \int_{1,A}^2 (\delta Q - \delta W) + \int_{2,C}^1 (\delta Q - \delta W) = 0$$

$$\text{or} \quad \int_{1,A}^2 (\delta Q - \delta W) = - \int_{2,C}^1 (\delta Q - \delta W)$$

$$\text{or} \quad \int_{1,A}^2 (\delta Q - \delta W) = \int_{1,C}^2 (\delta Q - \delta W) \quad (ii)$$

From equations (i) & (ii) it is obvious that

$$\int_{1,A}^2 (\delta Q - \delta W) = \int_{1,B}^2 (\delta Q - \delta W) = \int_{1,C}^2 (\delta Q - \delta W)$$

which shows that $(\delta Q - \delta W)$ is some property as it is independent of the path being followed.

Also, it can be rewritten as,

$$\int_{1,A}^2 \delta U = \int_{1,B}^2 \delta U = \int_{1,C}^2 \delta U$$

$$\text{or} \quad \Delta U_{1-2, A} = \Delta U_{1-2, B} = \Delta U_{1-2, C}$$

which means the change in internal energy is independent of the path followed and therefore *internal energy is a thermodynamic property*.

3.5 INTERNAL ENERGY AND ENTHALPY

Let us take a mass at certain elevation in earth's gravitational field and make it move with certain velocity. Energy considerations say that the mass shall have the potential energy ($P.E = mgz$) and kinetic energy ($K.E = (1/2) \cdot mC^2$) stored in it. Similarly, several other forms of energy such as due to magnetic, electrical, solid distortion and surface tension effects can be estimated as the contributory components of stored energy.

Difference of heat and work interactions yield the stored energy as given below; $E = Q - W$.

If the energy at macroscopic level as discussed above could be separated from the total stored energy E , then the amount of energy left shall be called internal energy.

Mathematically,

Internal energy, $U = (\text{Stored energy}) - (\text{Kinetic energy}) - (\text{Potential energy}) - (\text{Magnetic energy}) - (\text{Electrical energy}) - (\text{Surface tension energy}) - (\text{Solid distortion energy})$.

Therefore, stored energy is summation of internal energy, potential energy, kinetic energy, magnetic, electrical, surface tension, solid distortion etc. types of energy.

For the situation when magnetic, electric, surface tension, solid distortion effects are negligible, the stored energy shall be;

$$E = U + KE + PE$$

or,

$$E = U + \frac{mC^2}{2} + mgz$$

or, on unit mass basis;

$$e = u + \frac{C^2}{2} + gz$$

and the change in stored energy relative to some reference state shall be given as,

$$\Delta E = \Delta U + \Delta KE + \Delta PE.$$

Enthalpy (H) of a substance at any point is quantification of energy content in it, which could be given by summation of internal energy and flow energy. Enthalpy is very useful thermodynamic property for the analysis of engineering systems.

Mathematically, it is given as,

$$H = U + PV$$

On unit mass basis, the specific enthalpy could be given as,

$$h = u + pV$$

A look at expression of enthalpy shows that as we can't have absolute value of internal energy, the absolute value of enthalpy can not be obtained. Therefore only change in enthalpy of substance is considered. For certain frequently used substances such as steam, the enthalpy values of steam are available in tabulated form in Steam Tables at different thermodynamic states.

From the definition of enthalpy;

$$h = u + pV$$

or

$$dh = du + p \cdot dV + V \cdot dp.$$

For a constant pressure process, $dp = 0$.

$$dh = du + p dV$$

or,

$$dh = dq_{p = \text{constt}} \quad (\text{From first law of thermodynamics})$$

3.6 SPECIFIC HEATS AND THEIR RELATION WITH INTERNAL ENERGY AND ENTHALPY

Specific heats of the substance refer to the amount of heat interaction required for causing unit change in temperature of the unit mass of substance. This unit change in temperature may be realized under constant volume and constant pressure conditions separately.

Therefore, the above heat value obtained with heat interaction occurring under constant volume conditions is called specific heat at constant volume, denoted as c_v . Whereas the above heat value obtained with heat interaction occurring under constant pressure conditions is called specific heat at constant pressure, denoted as c_p .

Mathematically, the heat interaction causing ΔT temperature change in m mass of substance can be given as,

For isochoric conditions;

$$Q_v = m \cdot c_v \cdot \Delta T$$

and for isobaric conditions

$$Q_p = m \cdot c_p \cdot \Delta T$$

or

$$c_v = \frac{Q_v}{m \cdot \Delta T}$$

or,

$$c_p = \frac{Q_p}{m \cdot \Delta T}$$

For getting the specific heat values, substituting $m = 1$, $\Delta T = 1$,

$$\boxed{c_v = Q_v} \text{ and } \boxed{c_p = Q_p}$$

The specific heat at constant volume can also be given as the partial derivative of internal energy with respect to temperature at constant volume.

Thus

$$c_v = \left(\frac{\partial u}{\partial T} \right)_v$$

or,

$$c_v = \left(\frac{du}{dT} \right)_v$$

Also from first law of thermodynamics, on unit mass basis

$$dq = du + p dv$$

at constant volume, $dv = 0$

$$dq = du$$

or

$$dq = c_v \cdot dT = du$$

$$\boxed{c_v = \frac{dq}{dT}}, \text{ for } v = \text{constant}$$

Specific heat at constant pressure can be given as the partial derivative of enthalpy with respect to temperature at constant pressure.

Mathematically:

$$c_p = \left(\frac{\partial h}{\partial T} \right)_p$$

or

$$c_p = \left(\frac{dh}{dT} \right)_p$$

From definition of enthalpy, at unit mass basis.

$$h = u + p v$$

or

$$dh = du + p dv + v dp$$

at constant pressure, $dp = 0$

$$dh = du + p dv$$

substituting from first law of thermodynamics $dq = du + p dv$

$$dh = dq$$

or

$$dq = c_p \cdot dT = dh$$

$$\boxed{c_p = \frac{dq}{dT}}, \text{ for } p = \text{constant}$$

Let us try to establish relationship between c_p and c_v .

From enthalpy definition, at unit mass basis

$$h = u + p v$$

or

$$h = u + RT \text{ \{for ideal gas\}}$$

Taking partial derivative,

$$dh = du + R dT$$

Also we know for an ideal gas, c_p

$$dh = c_p \cdot dT; du = c_v \cdot dT$$

Substituting dh and du

$$c_p \cdot dT = c_v \cdot dT + R \cdot dT$$

or

$$c_p = c_v + R$$

or

$$\boxed{c_p - c_v = R}$$

Difference of specific heats at constant pressure and volume is equal to the gas constant for an ideal gas.

Also the ratio of specific heats at constant pressure and volume could be given as γ ,

$$\frac{c_p}{c_v} = \gamma$$

Combining above two relations of c_p and c_v we get,

$$c_p = \frac{\gamma \cdot R}{(\gamma - 1)} \text{ and } c_v = \frac{R}{(\gamma - 1)}$$

3.7 FIRST LAW OF THERMODYNAMICS APPLIED TO OPEN SYSTEMS

Let us consider an open system as shown in Fig. 3.15 having inlet at section 1-1 and outlet at section 2-2. The cross-section area, pressure, specific volume, mass flow rate, energy at section 1-1 and 2-2 are

Section 1-1 = A_1, p_1, v_1, m_1, e_1

Section 2-2 = A_2, p_2, v_2, m_2, e_2

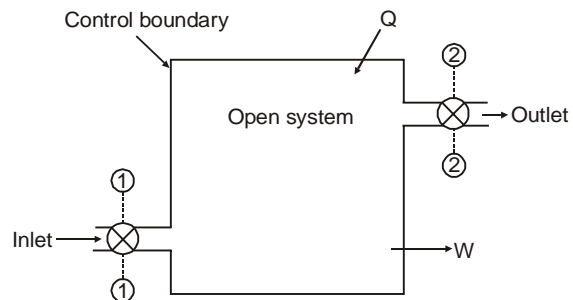


Fig. 3.15

Open system is also having heat and work interactions Q , W as shown in figure above.

Applying the energy balance at the two sections, it can be given as,

$$\begin{aligned} \text{Energy added to the system} + \text{Stored energy of the fluid at inlet} \\ = \text{Stored energy of the fluid at outlet} \end{aligned}$$

Quantifying the various energies;

Energy of fluid at inlet shall comprise of stored energy and flow energy as given here.

$$= m_1(e_1 + p_1 v_1)$$

Similarly, energy of fluid at outlet shall comprise of stored energy and flow energy,

$$= \text{Stored energy} + \text{Flow energy}$$

$$= m_2(e_2 + p_2 v_2)$$

The energy added to the system shall be the net energy interaction due to heat and work interactions.

$$= Q - W$$

Writing energy balance, mathematically;

$$Q - W + m_1(e_1 + p_1 v_1) = m_2(e_2 + p_2 v_2)$$

$$\text{or} \quad Q + m_1(e_1 + p_1 v_1) = W + m_2(e_2 + p_2 v_2)$$

If the mass flow rates at inlet and exit are same, then

$$Q + m(e_1 + p_1 v_1) = W + m(e_2 + p_2 v_2)$$

On unit mass basis

$$q + e_1 + p_1 v_1 = w + e_2 + p_2 v_2$$

Thus,

$$\text{Heat} + (\text{Stored energy} + \text{Flow energy})_1 = \text{Work} + (\text{Stored energy} + \text{Flow energy})_2$$

Stored energy at inlet and outlet can be mathematically given as,

$$e_1 = u_1 + \frac{C_1^2}{2} + gz_1$$

and

$$e_2 = u_2 + \frac{C_2^2}{2} + gz_2$$

where C_1 and C_2 are velocities at inlet and exit, u_1 and u_2 are internal energy at inlet and outlet, z_1 and z_2 are elevations of inlet and exit.

3.8 STEADY FLOW SYSTEMS AND THEIR ANALYSIS

Steady flow refers to the flow in which its properties at any point remain constant with respect to time. Steady system is the system whose properties are independent of time, i.e. any property at a point in system shall not change with time.

Let us take an open system having steady flow. Figure 3.16 shows steady flow system having inlet at section 1-1, outlet at section 2-2, heat addition Q and work done by the system W .

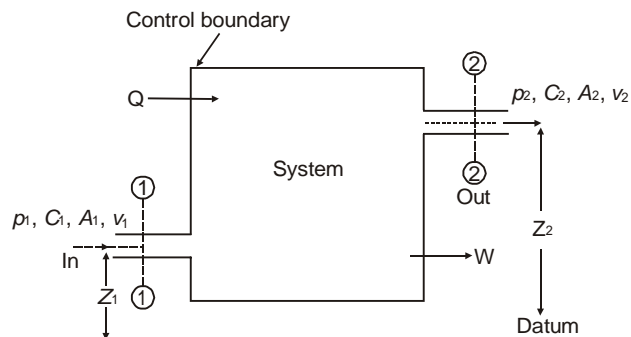


Fig. 3.16 Steady flow system

	At section 1-1	At section 2-2
Pressure, (N/m ²)	p_1	p_2
Sp volume, (m ³ /kg)	v_1	v_2
Velocity, (m/s)	C_1	C_2
Elevation, (m)	z_1	z_2
Cross-section area, (m ²)	A_1	A_2
Mass flow rate, (kg/s)	m_1	m_2
Internal energy, (J/kg)	u_1	u_2

As described in earlier article the energy balance when applied to open system results in

$$Q + m_1(e_1 + p_1 v_1) = W + m_2(e_2 + p_2 v_2)$$

Substituting for e_1 and e_2

$$Q + m_1 \left(u_1 + \frac{C_1^2}{2} + gz_1 + p_1 v_1 \right) = W + m_2 \left(u_2 + \frac{C_2^2}{2} + gz_2 + p_2 v_2 \right)$$

and from definition of enthalpy,

$$\begin{aligned} h_1 &= u_1 + p_1 v_1 \\ h_2 &= u_2 + p_2 v_2 \end{aligned}$$

therefore,

$$Q + m_1 \left(h_1 + \frac{C_1^2}{2} + gz_1 \right) = W + m_2 \left(h_2 + \frac{C_2^2}{2} + gz_2 \right)$$

Above equation is known as steady flow energy equation (S.F.E.E.). If the mass flow rates at inlet and exit are same, i.e. $m_1 = m_2 = m$

then,

$$Q + m \left(h_1 + \frac{C_1^2}{2} + gz_1 \right) = W + m \left(h_2 + \frac{C_2^2}{2} + gz_2 \right)$$

or, on unit mass basis the S.F.E.E. shall be;

$$q + h_1 + \frac{C_1^2}{2} + gz_1 = w + h_2 + \frac{C_2^2}{2} + gz_2$$

where

$$q = \frac{Q}{m}, w = \frac{W}{m}$$

The steady flow energy equation can be used as a tool for carrying out thermodynamic analysis of engineering system with suitable modifications.

Special Case : Such as for any system of perfectly insulated type, $Q = 0$

The steady flow energy equation gets modified to;

$$h_1 + \frac{C_1^2}{2} + gz_1 = w + h_2 + \frac{C_2^2}{2} + gz_2$$

Application of Continuity equation results in,

$$m_1 = m_2$$

$$\text{or, } \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2}$$

For any system having more than one inlets, outlets and energy interactions the example is shown below.

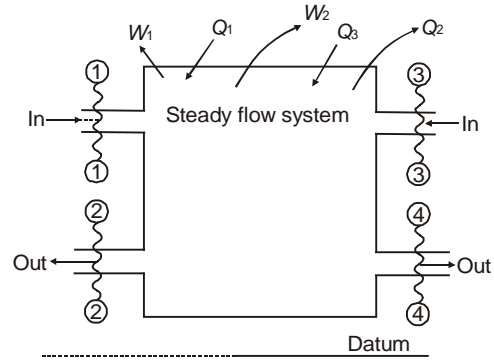


Fig. 3.17 Example of steady flow system

Salient properties at different sections are tabulated as under

	Section 1-1	Section 2-2	Section 3-3	Section 4-4
Pressure, (N/m ²)	p_1	p_2	p_3	p_4
Sp. volume, (m ³ /kg)	v_1	v_2	v_3	v_4
Mass flow rate, (kg/s)	m_1	m_2	m_3	m_4
Internal energy, (J/kg)	u_1	u_2	u_3	u_4
Velocity, (m/s)	C_1	C_2	C_3	C_4
Elevation, (m)	z_1	z_2	z_3	z_4
Cross-section area, (m ²)	A_1	A_2	A_3	A_4

Net heat added, $Q = Q_1 - Q_2 + Q_3$

Net work done, $W = W_1 + W_2$

Applying steady flow energy equation on the system as shown in Fig 3.17;

$$Q + m_1 \left(u_1 + \frac{C_1^2}{2} + gz_1 + p_1 v_1 \right) + m_3 \left(u_3 + \frac{C_3^2}{2} + gz_3 + p_3 v_3 \right)$$

$$= W + m_2 \left(u_2 + \frac{C_2^2}{2} + gz_2 + p_2 v_2 \right) + m_4 \left(u_4 + \frac{C_4^2}{2} + gz_4 + p_4 v_4 \right)$$

Substituting enthalpy values, h_1, h_2, h_3, h_4 and for Q and W ;

$$Q_1 - Q_2 + Q_3 + m_1 \left(h_1 + \frac{C_1^2}{2} + gz_1 \right) + m_3 \left(h_3 + \frac{C_3^2}{2} + gz_3 \right)$$

$$= (W_1 + W_2) + m_2 \left(h_2 + \frac{C_2^2}{2} + gz_2 \right) + m_4 \left(h_4 + \frac{C_4^2}{2} + gz_4 \right)$$

Case 1

If the inlet and exit velocities are negligible, then

$$KE_1 = KE_2 = KE_3 = KE_4 = 0$$

and S.F.E.E. is modified to

$$\begin{aligned} Q_1 - Q_2 + Q_3 + m_1(h_1 + gz_1) + m_3(h_3 + gz_3) \\ = W_1 + W_2 + m_2(h_2 + gz_2) + m_4(h_4 + gz_4) \end{aligned}$$

Case 2

If there is no change in elevation and mass flow rates at all inlets and outlets are same, then, $m_1 = m_2 = m_3 = m_4 = m$

$$Q_1 - Q_2 + Q_3 + m \cdot h_1 + m h_3 = W_1 + W_2 + m h_2 + m h_4$$

or, on unit mass basis

$$q_1 - q_2 + q_3 + h_1 + h_3 = w_1 + w_2 + h_2 + h_4$$

3.9 FIRST LAW APPLIED TO ENGINEERING SYSTEMS

Here the first law of thermodynamics applied to different engineering systems is discussed. It is assumed in general that the processes are of steady flow type and so the steady flow energy equation can be directly used with modifications in it.

(a) *Turbine*: It is the device in which the high temperature and high pressure fluid is expanded to low temperature and pressure resulting in generation of positive work at turbine shaft. Thus, turbine is a work producing device.

Turbines using gas as working fluid are called gas turbine where as turbines using steam are called steam turbines.

Expansion in turbine is assumed to be of adiabatic type so that the maximum amount of work is produced.

Assuming change in kinetic energy, potential energy to be negligible, the steady flow energy equation can be modified and written between 1 and 2 as,

$$\begin{aligned} 0 + mh_1 &= W_T + mh_2 \\ W_T &= m(h_1 - h_2) \end{aligned}$$

i.e., $Q = 0$ and total energy interaction is available in the form of work

$$\text{Turbine work} = m(h_1 - h_2) = m c_p(T_1 - T_2)$$

Here m is mass flow rate and T_1, T_2 are temperatures at inlet and outlet.

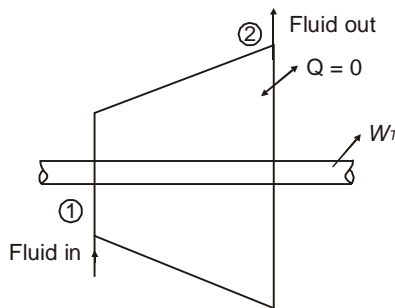
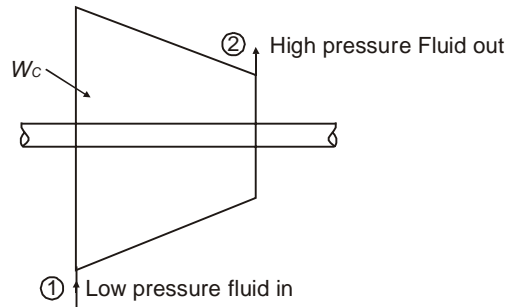


Fig. 3.18 Turbine

(b) *Compressor*: Compressor is a work absorbing device used for increasing the pressure of a fluid. Pressure of a fluid is increased by doing work upon it, which is accompanied by increase in temperature depending on the gas properties.

**Fig. 3.19 Compressor**

For compression of a gas adiabatic process is used as in this there is no heat loss and so minimum work requirement.

Let us assume change in kinetic energy and potential energy to be negligible between 1 & 2 and also flow to be of steady type.

Applying steady flow energy equation in modified form:

$$Q = 0$$

$$\Delta KE = 0$$

$$\Delta PE = 0$$

$$W_c = (-ve) \text{ work for compression}$$

$$mh_1 = -W_c + mh_2$$

or

$$W_c = m(h_2 - h_1)$$

$$\text{Adiabatic compression work} = m(h_2 - h_1) = mc_p (T_2 - T_1)$$

Here T_1 , T_2 are temperatures at inlet and outlet and m is mass flow rate.

(c) **Pump:** A pump is used for pumping liquid or suction of liquid. In case of pump the following assumptions can be made for using S.F.E.E.

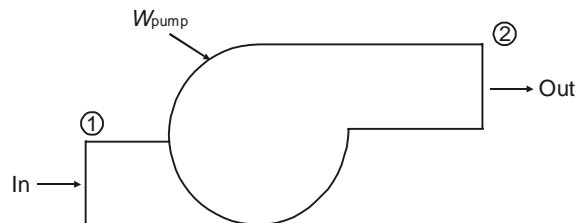
(i) Heat transfer is zero, $Q = 0$

(ii) Change in internal energy is zero, $\Delta U = 0$

Therefore

$$m \left(p_1 v_1 + \frac{C_1^2}{2} + gz_1 \right) = m \left(p_2 v_2 + \frac{C_2^2}{2} + gz_2 \right) - W_{\text{pump}}$$

$$\text{or, } W_{\text{pump}} = m \left\{ (p_2 v_2 - p_1 v_1) + \frac{C_2^2 - C_1^2}{2} + g(z_2 - z_1) \right\}$$

**Fig. 3.20 Pump**

(d) **Boiler:** Boiler is the engineering device used for steam generation at constant pressure. Heat is supplied externally to the boiler for steam generation depending upon state of steam desired.

Boiler may be assumed similar to a closed vessel having no work interaction, no change in kinetic energy, no change in potential energy.

i.e. $W = 0, \Delta KE = 0, \Delta PE = 0.$

Applying steady flow energy equation

$$Q_{\text{boiler}} + m(h_1) = m(h_2)$$

or $Q_{\text{boiler}} = m(h_2 - h_1) = m c_p (T_2 - T_1)$

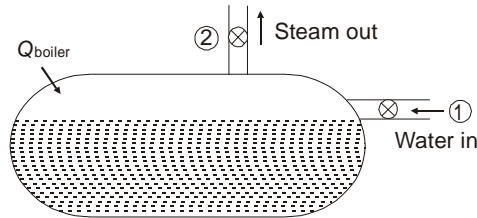


Fig. 3.21 Boiler

(e) **Condenser:** Condenser is the device used for condensing vapour into liquid at constant pressure. It is a type of heat exchanger in which another cool fluid is used for condensing the vapours into liquid. Heat exchange between the hot fluid and cold fluid takes place indirectly as cold fluid passes through the tubes and hot vapours are outside tubes in the shell.

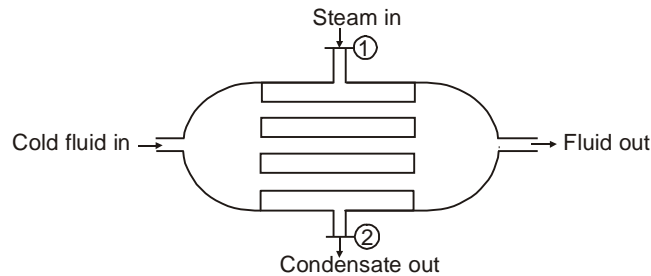


Fig. 3.22 Condenser

Steady flow energy equation can be applied with the following assumptions :

- (i) No work interaction, $W = 0$
- (ii) No change in kinetic energy, $\Delta KE = 0$
- (iii) No change in potential energy, $\Delta PE = 0$

Heat lost by steam,

$$Q = m (h_1 - h_2)$$

(f) **Nozzle:** Nozzle is the engineering device in which expansion of fluid takes place and pressure drops simultaneously. Thus in nozzle the velocity of fluid increases from inlet to exit. In case of subsonic flow the nozzle has converging cross-section area in the duct where as in supersonic flow the nozzle has diverging cross-section area in the duct.

Let us take a converging cross-section area duct as shown in Fig. 3.23.

Flow through the nozzle may be analysed with following assumptions:

- (i) No heat interaction, i.e. $Q = 0$, during passage through duct.
- (ii) No work interaction, i.e. $W = 0$, during passage through duct.

(iii) No change in elevation from 1 to 2, i.e. $\Delta PE = 0$.

Applying S.F.E.E on nozzle,

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

or,
$$\frac{C_2^2 - C_1^2}{2} = h_1 - h_2$$

$$C_2 = \sqrt{C_1^2 + 2(h_1 - h_2)}$$

or,
$$C_2 = \sqrt{C_1^2 + 2c_p(T_1 - T_2)}$$

In case, the velocity at inlet to nozzle is very small, then C_1 may be neglected and velocity at nozzle exit shall be:

$$C_2 = \sqrt{2c_p(T_1 - T_2)}$$

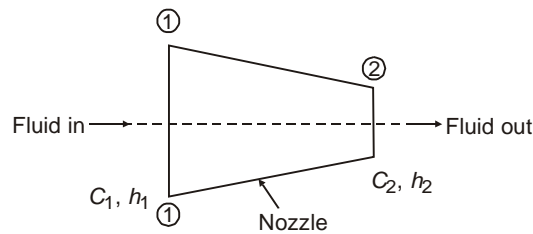


Fig. 3.23 Nozzle

(g) *Throttling*: Throttling refers to passage of a fluid through some restricted opening under isenthalpic conditions. Thus in the figure shown below the fluid passes through a restriction from section 1 to 2 and undergoes drop in its pressure and increase in volume, but during this passage enthalpy remains constant, such that $h_1 = h_2$.

Based on above throttling process the device called “throttle valve” has been developed in which pressure drop is realized without involving any work and heat interaction, change in kinetic energy and potential energy. Temperature may drop or increase during the throttling process and shall depend upon the Joule-Thomson coefficient, a property based on characteristic of substance.

$$\text{Joule-Thomson coefficient } \mu = \left(\frac{\partial T}{\partial p} \right)_{h=\text{const.}}$$

and if $\mu = 0$, Temperature remains constant

$\mu > 0$, Temperature decreases.

$\mu < 0$ Temperature increases.

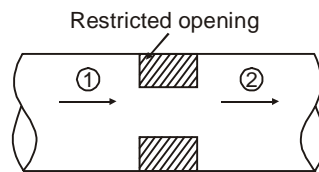


Fig. 3.24 Throttling process

(h) *Combustion chamber*: Combustion chambers are commonly used in gas turbine installations, in which fuel is injected at high pressure into a chamber having high pressure, high temperature air in it and ignited for heat release at constant pressure.

Mass balance yields:

$$m_2 = m_1 + m_f$$

where m_1 , m_2 , m_f are mass flow rates at 1, 2, and f shown in figure.

S.F.E.E. may be applied with assumptions of $\Delta KE = 0$,

$$\Delta PE = 0,$$

$$W = 0$$

Here, $Q = m_f \times \text{Calorific value of fuel}$

or

$$Q = m_f \times CV$$

$$Q + m_1 h_1 + m_f h_f = m_2 h_2$$

substituting for Q

$$m_f CV + m_1 h_1 + m_f h_f = m_2 h_2$$

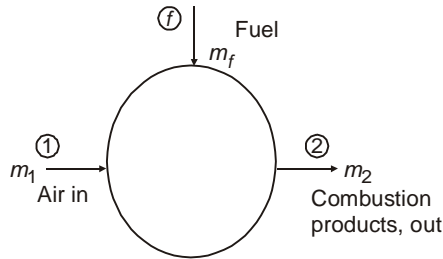


Fig. 3.25 Combustion chamber

(i) *Adiabatic mixing*: Adiabatic mixing refers to mixing of two or more streams of same or different fluids under adiabatic conditions.

Let us consider two streams of same fluid with mass flow rates m_1 and m_2 to get mixed together adiabatically.

Assumptions for applying S.F.E.E shall be;

(i) No heat interaction, $Q = 0$

(ii) No work interaction, $W = 0$

(iii) No change in kinetic energy, $\Delta KE = 0$

(iv) No change in potential energy, $\Delta PE = 0$.

Thus,

$$m_1 \cdot h_1 + m_2 \cdot h_2 = m_3 \cdot h_3$$

or

$$m_1 \cdot c_p \cdot T_1 + m_2 \cdot c_p \cdot T_2 = m_3 \cdot c_p \cdot T_3$$

or

$$T_3 = \frac{m_1 T_1 + m_2 T_2}{m_3}$$

By mass balance,

$$m_1 + m_2 = m_3$$

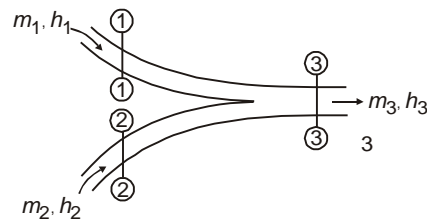


Fig. 3.26 Adiabatic mixing

3.10 UNSTEADY FLOW SYSTEMS AND THEIR ANALYSIS

In earlier discussions, for a steady flow system, it has been assumed that the properties do not change with time.

However, there exist a number of systems such as filling up of a bottle or emptying of a vessel etc. in which properties change continuously as the process proceeds. Such systems can not be analysed with the steady state assumptions. Unsteady flow processes are also known as transient flow processes or variable flow processes.

Let us take example of filling up of the bottle.

The bottle is filled up gradually, therefore it is case of an unsteady system. By conservation of mass, the unsteady process over a period of time ' dt ' can be expressed as following in generic form.

(Mass entering the control volume in time dt)

– (Mass leaving the control volume in time dt)

= Net change in mass in control volume in time dt .

If the mass flow rate at inlet and exit are given as m_i , m_e then

$$\frac{dm_i}{dt} - \frac{dm_e}{dt} = \frac{dm_{cv}}{dt}$$

and also,

$$\sum m_i - \sum m_e = (m_{\text{final}} - m_{\text{initial}})_{cv}$$

By the conservation of energy principle applied on control volume for time ' t ', energy balance yields;

Net energy interaction across the boundary in time dt

+ Energy entering into control volume in time dt

– Energy leaving out of control volume in time dt

= Change in energy in control volume in time dt

Mathematically, it can be given as:

$$(Q - W) + \sum E_i - \sum E_e = \Delta E_{cv}$$

where

$$E_i = \int_0^t m_i \left(h_i + \frac{C_i^2}{2} + gz_i \right) \cdot dt$$

$$E_e = \int_0^t m_e \left(h_e + \frac{C_e^2}{2} + gz_e \right) \cdot dt$$

Thus, the above mass balance and energy balance can be used for analysing the unsteady flow systems with suitable assumptions.

It may be assumed that the control volume state is uniform and fluid properties are uniform and steady at inlet and exit.

Simplified form of energy balance written above can be given as;

$$Q - W + \sum m_i \left(h_i + \frac{C_i^2}{2} + gz_i \right) - \sum m_e \left(h_e + \frac{C_e^2}{2} + gz_e \right)$$

$$= (m_{\text{final}} \cdot u_{\text{final}} - m_{\text{initial}} \cdot u_{\text{initial}})_{cv}$$

If the changes in kinetic energy and potential energy are negligible, then energy balance gets modified as;

$$Q - W + \sum m_i \cdot h_i - \sum m_e \cdot h_e = (m_{\text{final}} \cdot u_{\text{final}} - m_{\text{initial}} \cdot u_{\text{initial}})_{cv}$$

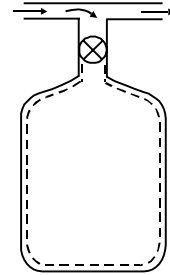


Fig. 3.27 Filling of the bottle

Case 1: Let us now use the energy and mass balance to the unsteady flow process of filling up a bottle as shown in Figure 3.27. Bottle is initially empty and connected to a pipe line through valve for being filled.

Let us denote initial state of system by subscript 1 and final state by 2.

Initially as bottle is empty, so $m_1 = 0$

From mass balance

$$\sum m_i - \sum m_e = (m_2 - 0)_{cv}$$

Here there is no exit from the bottle so $m_e = 0$

hence,

$$\sum m_i = m_2$$

or,

$$m_i = m_2$$

Mass entered into bottle = Final mass inside the bottle

Applying the energy balance assuming change in kinetic and potential energy to be negligible, treating bottle filling process to be occurring in insulated environment, and no work interaction, we get

$$Q \approx 0, W \approx 0, \Delta KE \approx 0, \Delta PE \approx 0,$$

Initial internal energy in bottle = 0

Mass leaving = 0

$$0 = - \sum m_i \cdot h_i + (m_2 \cdot u_2)_{cv}$$

or

$$m_i \cdot h_i = m_2 u_2$$

also

$$h_i = u_2 \text{ as } m_i = m_2$$

Enthalpy of fluid entering bottle = Final internal energy of fluid in bottle.

If fluid is ideal gas, then $c_p \cdot T_i = c_v \cdot T_2$

or

$$T_2 = \gamma \cdot T_i$$

where

$$\frac{c_p}{c_v} = \gamma$$

Case 2: Let us now take a case of emptying of bottle. Arrangement is shown in Fig. 3.28.

Initially bottle has mass m_1 and finally as a result of emptying, say mass left is m_2 after some time.

Applying mass balance, (as mass entering is zero),

$$\text{or, } 0 - \sum m_e = (m_2 - m_1)_{cv}$$

or

$$\sum m_e = (m_1 - m_2)_{cv}$$

or

$$m_e = (m_1 - m_2)_{cv}$$

Total mass leaving the bottle = (Mass reduced in bottle)

Applying energy balance, with the assumptions given below;

(i) No heat interaction i.e. $Q = 0$

(ii) No work interaction i.e. $W = 0$

(iii) No change in kinetic energy i.e. $\Delta KE = 0$

(iv) No change in potential energy i.e. $\Delta PE = 0$

$$- \sum m_e \cdot h_e = (m_2 u_2 - m_1 u_1)_{cv}$$

$$\text{or, } (- m_e \cdot h_e) = (m_2 u_2 - m_1 u_1)_{cv}$$

Substituting for ' m_e ' we get $(m_2 - m_1)_{cv} \cdot h_e = (m_2 u_2 - m_1 u_1)_{cv}$

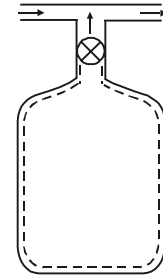


Fig. 3.28 Emptying of bottle

In case of complete emptying, $m_2 = 0$

and so, $h_e = u_1$

3.11 LIMITATIONS OF FIRST LAW OF THERMODYNAMICS

First law of thermodynamics based on law of energy conservation has proved to be a powerful tool for thermodynamic analysis. But over the period of time when it was applied to some real systems, it was observed that theoretically first law stands valid for the processes which are not realizable practically. It was then thought that there exist certain flaws in first law of thermodynamics and it should be used with certain limitations.

Say for example let us take a bicycle wheel and paddle it to rotate. Now apply brake to it. As a result of braking wheel comes to rest upon coming in contact with brake shoe. Stopping of wheel is accompanied by heating of brake shoe. Examining the situation from 1st law of thermodynamics point of view it is quite satisfying that rotational energy in wheel has been transformed into heat energy with shoe, thus causing rise in its temperature:

Now, if we wish to introduce the same quantity of heat into brake shoe and wish to restore wheel motion then it is not possible simply, whereas theoretically first law permits the conversion from heat to work (rotation of wheel in this case) as well.

Therefore, it is obvious that 1st law of thermodynamics has certain limitations as given below:

- (i) First law of thermodynamics does not differentiate between heat and work and assures full convertibility of one into other whereas full conversion of work into heat is possible but the vice-versa is not possible.
- (ii) First law of thermodynamics does not explain the direction of a process. Such as theoretically it shall permit even heat transfer from low temperature body to high temperature body which is not practically feasible. Spontaneity of the process is not taken care of by the first law of thermodynamics.

Perpetual motion machine of the first kind (PMM-I) is a hypothetical device conceived, based on violation of First law of thermodynamics. Let us think of a system which can create energy as shown below.

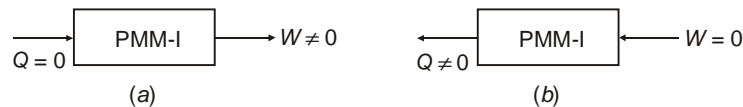


Fig. 3.29 PMM-I, based on violation of 1st law of thermodynamics

Here a device which is continuously producing work without any other form of energy supplied to it has been shown in (a), which is not feasible.

Similarly a device which is continuously emitting heat without any other form of energy supplied to it has been shown in (b), which is again not feasible.

Above two imaginary machines are called Perpetual Motion Machines of 1st kind.

EXAMPLES

1. Figure shows a system comprising of gas in cylinder at pressure of 689 kPa.

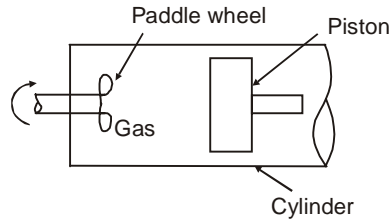


Fig. 3.30

Fluid expands from a volume of 0.04 m^3 to 0.045 m^3 while pressure remains constant. Paddle wheel in the system does a work of 4.88 kJ on the system. Determine (a) work done by system on the piston (b) the net amount of work done on or by the system.

Solution:

- (a) It is a closed system. If the pressure on face of piston is uniform, then the work done on piston can be obtained as,

$$W = p \int_1^2 dV$$

$$= 689 \times 10^3 (0.045 - 0.04)$$

Work done on piston, $W = 3445 \text{ J}$ or 3.445 kJ

Work done on piston = **3.445 kJ Ans.**

- (b) Paddle work done on the system = -4.88 kJ

$$\text{Net work of system} \Rightarrow W_{\text{net}} = W_{\text{piston}} + W_{\text{paddle}}$$

$$= 3445 - 4880$$

$$W_{\text{net}} = -1435 \text{ J}$$

Work done on system = 1435 J or 1.435 kJ .

Work done on system = **1.435 kJ Ans.**

2. A gas at 65 kPa, 200°C is heated in a closed, rigid vessel till it reaches to 400°C . Determine the amount of heat required for 0.5 kg of this gas if internal energy at 200°C and 400°C are 26.6 kJ/kg and 37.8 kJ/kg respectively.

Solution:

Given

$$m = 0.5 \text{ kg}$$

$$u_1 = 26.6 \text{ kJ/kg}$$

$$u_2 = 37.8 \text{ kJ/kg}$$

As the vessel is rigid therefore work done shall be zero.

$$W = 0$$

From first law of thermodynamics;

$$Q = U_2 - U_1 + W = m(u_2 - u_1) + 0$$

$$Q = 0.5 (37.8 - 26.6)$$

$$Q = 5.6 \text{ kJ}$$

Heat required = **5.6 kJ Ans.**

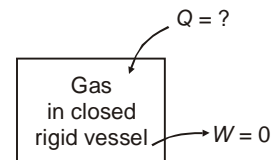


Fig. 3.31

3. Carbon dioxide passing through a heat exchanger at a rate of 50 kg/hr is to be cooled down from 800°C to 50°C. Determine the rate of heat removal assuming flow of gas to be of steady and constant pressure type. Take $c_p = 1.08 \text{ kJ/kg K}$.

Solution:

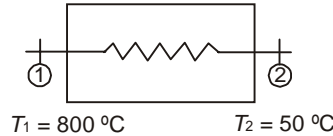


Fig. 3.32

Given, $m = 50 \text{ kg/hr}$

Writing down the steady flow energy equation.

$$q + h_1 + \frac{C_1^2}{2} + gz_1 = h_2 + \frac{C_2^2}{2} + gz_2 + w$$

Here let us assume changes in kinetic and potential energy to be negligible. During flow the work interaction shall also be zero.

Hence

$$\begin{aligned} q &= h_2 - h_1 \\ Q &= m (h_2 - h_1) \\ &= m \cdot c_p \cdot (T_2 - T_1) \\ &= 50 \times 1.08 \times (750) \\ &= 40500 \text{ kJ/hr} \end{aligned}$$

or

Heat should be removed at the rate of **40500 kJ/hr** **Ans.**

4. A completely evacuated cylinder of 0.78 m³ volume is filled by opening its valve to atmosphere and air rushing into it. Determine the work done by the air and by surroundings on system.

Solution : Total work done by the air at atmospheric pressure of 101.325 kPa,

$$\begin{aligned} W &= \int_{\text{cylinder}} p \cdot dv + \int_{\text{air}} p \cdot dv \\ &= 0 + p \cdot \Delta v, \text{ it is } -\text{ve work as air boundary shall contract} \end{aligned}$$

$$\begin{aligned} \text{Work done by air} &= -101.325 \times 0.78 = -79.03 \text{ kJ} \\ \text{Work done by surroundings on system} &= +79.03 \text{ kJ} \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Work done by air} \\ \text{Work done by surroundings on system} \end{aligned}} \right\} \text{Ans.}$$

5. A system comprising of a gas of 5 kg mass undergoes expansion process from 1 MPa and 0.5 m³ to 0.5 MPa. Expansion process is governed by, $p \cdot v^{1.3} = \text{constant}$. The internal energy of gas is given by, $u = 1.8 pv + 85, \text{ kJ/kg}$. Here 'u' is specific internal energy, 'p' is pressure in kPa, 'v' is specific volume in m³/kg. Determine heat and work interaction and change in internal energy.

Solution:

Given mass of gas, $m = 5 \text{ kg}$, $pv^{1.3} = \text{constant}$

Assuming expansion to be quasi-static, the work may be given as,

$$\begin{aligned} W &= m \int p \cdot dv \\ &= \frac{p_2 V_2 - p_1 V_1}{(1-n)} \end{aligned}$$

From internal energy relation, change in specific internal energy,

$$\Delta u = u_2 - u_1 = 1.8 (p_2 v_2 - p_1 v_1), \text{ kJ/kg}$$

Total change, $\Delta U = 1.8 \times m \times (p_2 v_2 - p_1 v_1), \text{ kJ}$

$$\Delta U = 1.8 \times (p_2 V_2 - p_1 V_1)$$

Between states 1 and 2,

$$p_1 V_1^{1.3} = p_2 V_2^{1.3}$$

or

$$p_1 V_1^{1.3} = p_2 V_2^{1.3}$$

$$\Rightarrow V_2 = (0.5) \cdot \left(\frac{1}{0.5} \right)^{1/1.3}$$

$$V_2 = 0.852 \text{ m}^3$$

Total change in internal energy, $\Delta U = -133.2 \text{ kJ}$

Work,
$$W = \frac{(0.5 \times 0.852 - 1 \times 0.5) \times 10^3}{(1 - 1.3)}$$

$$W = 246.67 \text{ kJ}$$

From first law,

$$\begin{aligned} \Delta Q &= \Delta U + W \\ &= -133.2 + 246.7 \end{aligned}$$

$$\Delta Q = 113.5 \text{ kJ}$$

$$\left. \begin{array}{l} \text{Heat interaction} = \mathbf{113.5 \text{ kJ}} \\ \text{Work interaction} = \mathbf{246.7 \text{ kJ}} \\ \text{Change in internal energy} = \mathbf{-133.2 \text{ kJ}} \end{array} \right\} \text{Ans.}$$

6. A gas contained in a cylinder is compressed from 1 MPa and 0.05 m³ to 2 MPa. Compression is governed by $pV^{1.4} = \text{constant}$. Internal energy of gas is given by;

$$U = 7.5 pV - 425, \text{ kJ.}$$

where p is pressure in kPa and V is volume in m³.

Determine heat, work and change in internal energy assuming compression process to be quasi-static.

Also find out work interaction, if the 180 kJ of heat is transferred to system between same states. Also explain, why is it different from above.

Solution:

Final state, volume
$$V_2 = \left(\frac{p_1}{p_2} \right)^{1/1.4} \cdot V_1$$

$$= \left(\frac{1}{2} \right)^{1/1.4} \cdot 0.05$$

$$V_2 = 0.03 \text{ m}^3$$

Change in internal energy,

$$\begin{aligned} \Delta U &= U_2 - U_1 = (7.5 p_2 V_2 - 7.5 p_1 V_1) \\ &= 7.5 \times 10^3 (2 \times 0.03 - 1 \times 0.05) \end{aligned}$$

$$\Delta U = 75 \text{ kJ}$$

For quasi-static process,

$$\begin{aligned}
 \text{Work, } W &= \int_1^2 p \cdot dV \\
 &= \frac{p_2 V_2 - p_1 V_1}{1 - n} \\
 &= \frac{(2 \times 0.03 - 1 \times 0.05) \times 10^3}{(1 - 1.4)} \\
 W &= 25 \text{ kJ, (-ve)}
 \end{aligned}$$

From first law of thermodynamics,

$$\begin{aligned}
 \text{Heat interaction } \Delta Q &= \Delta U + W \\
 &= 75 + (-25) \\
 &= 50 \text{ kJ}
 \end{aligned}$$

$$\left. \begin{aligned}
 \text{Heat} &= \mathbf{50 \text{ kJ}} \\
 \text{Work} &= \mathbf{25 \text{ kJ (-ve)}} \\
 \text{Internal energy change} &= \mathbf{75 \text{ kJ}}
 \end{aligned} \right\} \text{ Ans.}$$

If 180 kJ heat transfer takes place, then from 1st law,

$$\Delta Q = \Delta U + W$$

Since end states remain same, therefore ΔU , i.e. change in internal energy remains unaltered.

$$180 = 75 + W$$

or

$$W = \mathbf{105 \text{ kJ}}$$

This work is different from previous work because the process is not quasi-static in this case.

Ans.

7. Determine the heat transfer and its direction for a system in which a perfect gas having molecular weight of 16 is compressed from 101.3 kPa, 20°C to a pressure of 600 kPa following the law $pV^{1.3} = \text{constant}$. Take specific heat at constant pressure of gas as 1.7 kJ/kg.K.

Solution:

$$\text{Characteristic gas constant, } R = \frac{\text{Universal gas constant}}{\text{Molecular weight}}$$

$$= \frac{8.3143 \times 10^3}{16}, \text{ J/kg.K}$$

$$= 519.64, \text{ J/kg.K}$$

or

$$= 0.51964, \text{ kJ/kg.K}$$

$$R = 0.520, \text{ kJ/kg.K}$$

$$C_v = C_p - R$$

$$= 1.7 - 0.520$$

$$C_v = 1.18, \text{ kJ/kg.K}$$

or

$$\gamma = \frac{C_p}{C_v} = \frac{1.7}{1.18} = 1.44$$

For polytropic process,

$$V_2 = \left(\frac{p_1}{p_2} \right)^{1/1.3} \cdot V_1$$

or

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{1.3-1}{1.3}}$$

$$T_2 = 293 \cdot \left(\frac{600}{101.3} \right)^{0.231}$$

$$T_2 = 441.9 \text{ K}$$

Work,

$$W = \frac{R(T_1 - T_2)}{(1.3 - 1)}$$

$$W = 258.1 \text{ kJ/kg}$$

For polytropic process

Heat,

$$Q = \left(\frac{\gamma - n}{\gamma - 1} \right) \cdot W = \left(\frac{1.44 - 1.3}{1.44 - 1} \right) \times 258.1$$

$$= \mathbf{82.12, \text{ kJ/kg (+ve) Ans.}}$$

8. In a nozzle air at 627°C and twice atmospheric pressure enters with negligible velocity and leaves at a temperature of 27°C. Determine velocity of air at exit, assuming no heat loss and nozzle being horizontal. Take $C_p = 1.005 \text{ kJ/kg.K}$ for air.

Solution: Applying steady flow energy equation with inlet and exit states as 1, 2 with no heat and work interaction and no change in potential energy.

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

Given that, $C_1 \approx 0$, negligible inlet velocity

$$C_2 = \sqrt{2(h_1 - h_2)}$$

Exit velocity, $C_2 = \sqrt{2 \cdot C_p \cdot (T_1 - T_2)}$

Given, $T_1 = 900 \text{ K}$, $T_2 = 300 \text{ K}$

or $C_2 = \sqrt{2 \times 1.005 \times 10^3 (900 - 300)}$

$$C_2 = 1098.2 \text{ m/s}$$

Exit velocity = **1098.2 m/s. Ans.**

9. An air compressor requires shaft work of 200 kJ/kg of air and the compression of air causes increase in enthalpy of air by 100 kJ/kg of air. Cooling water required for cooling the compressor picks up heat of 90 kJ/kg of air. Determine the heat transferred from compressor to atmosphere.

Solution:

Work interaction, $W = -200 \text{ kJ/kg}$ of air

Increase in enthalpy of air = 100 kJ/kg of air

Total heat interaction, $Q = \text{Heat transferred to water} + \text{Heat transferred to atmosphere.}$

Writing steady flow energy equation on compressor, for unit mass of air entering at 1 and leaving at 2.

$$h_1 + \frac{C_1^2}{2} + gZ_1 + Q = h_2 + \frac{C_2^2}{2} + gZ_2 + W$$

Assuming no change in potential energy and kinetic energy;

$$\Delta KE = \Delta PE = 0$$

$$Q = (h_2 - h_1) + W$$

$$= 100 + (-200)$$

$$Q = -100 \text{ kJ/kg of air}$$

Also

$$Q = -100 = \text{Heat transferred to water} + \text{Heat transferred to atmosphere}$$

or

$$-100 = (-90) + \text{Heat transferred to atmosphere}$$

$$\text{Heat transferred to atmosphere} = -10 \text{ kJ/kg of air Ans.}$$

10. In a cinema hall with seating capacity of 500 persons the comfort conditions are created by circulating hot water through pipes in winter season. Hot water enters the pipe with enthalpy of 80 kcal/kg and leaves the pipe with enthalpy of 45 kcal/kg. The difference in elevation of inlet pipe and exit pipe is 10 m with exit pipe being higher than inlet pipe. Heat requirement per person is 50 kcal/hr. Estimate the quantity of water circulated per minute, neglecting changes in velocity.

Solution:

Above problem can be solved using steady flow energy equations upon hot water-flow,

$$Q + m_1 \left(h_1 + \frac{C_1^2}{2} + gz_1 \right) = W + m_2 \left(h_2 + \frac{C_2^2}{2} + gz_2 \right)$$

$$\text{Here total heat to be supplied} = 500 \times 50$$

$$\text{Heat lost by water (-ve), } Q = -25000 \text{ kcal/hr.}$$

There shall be no work interaction and change in kinetic energy, so, steady flow energy equation shall be,

$$Q + m_1(h_1 + gz_1) = m_2(h_2 + gz_2)$$

$$\text{Here } m_1 = m_2 = m, h_1 = 80 \text{ kcal/kg, } h_2 = 45 \text{ kcal/kg}$$

$$Q + m(h_1 - h_2) = m(gz_2 - gz_1)$$

$$-(25000 \times 10^3 \times 4.18) + m(80 - 45) \times 10^3 \times 4.18 = m \times (9.81 \times 10)$$

$$m = 714.76 \text{ kg/hr}$$

or, Water circulation rate = **11.91 kg/min. Ans.**

11. Figure 3.33 shows steam injector for lifting water from a depth of 2 m from axis of injector. Determine the rate at which steam should be supplied for pumping unit mass of water. Enthalpy of steam entering and water entering injector are 720 kcal/kg and 24.6 kcal/kg respectively.

Enthalpy of steam and water mixture leaving injector is 100 kcal/kg. Velocity of steam entering injector is 50 m/s while mixture leaves with 25 m/s. Heat loss from injector to surroundings is 12 kcal/kg of steam sent through injector.

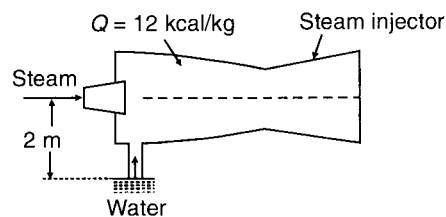


Fig. 3.33

Solution:

Let mass of steam to be supplied per kg of water lifted be ' m ' kg. Applying law of energy conservation upon steam injector, for unit mass of water lifted.

Energy with steam entering + Energy with water entering = Energy with mixture leaving + Heat loss to surroundings.

$$\begin{aligned}
 & m \left[\frac{(50)^2}{2} + (720 \times 10^2 \times 4.18) \right] + 1 \times [(24.6 \times 10^3 \times 4.18) + (9.81 \times 2)] \\
 & \text{K.E.} \qquad \qquad \text{Enthalpy} \qquad \qquad \text{Enthalpy} \qquad \qquad \text{P.E.} \\
 & = (1 + m) \left[(100 \times 10^3 \times 4.18) + \frac{(25)^2}{2} \right] + [m \times 12 \times 10^3 \times 4.18] \\
 & \qquad \qquad \qquad \text{Enthalpy} \qquad \qquad \text{K.E.} \qquad \qquad \text{Heat loss}
 \end{aligned}$$

$$m [3010850] + [102847.62] = (1 + m) \cdot (418312.5) + m[50160]$$

Upon solving, $m = 0.124$ kg steam/kg of water

Steam supply rate = **0.124 kg/s** per kg of water. **Ans.**

12. An inelastic flexible balloon is inflated from initial empty state to a volume of 0.4 m^3 with H_2 available from hydrogen cylinder. For atmospheric pressure of 1.0313 bar determine the amount of work done by balloon upon atmosphere and work done by atmosphere.

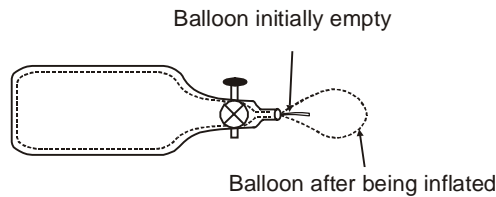
Solution:

Fig. 3.34

Here let us assume that the pressure is always equal to atmospheric pressure as balloon is flexible, inelastic and unstressed and no work is done for stretching balloon during its filling. Figure 3.34 shows the boundary of system before and after filling balloon by firm line and dotted line respectively.

$$\text{Displacement work, } W = \int_{\text{cylinder}} p \cdot dV + \int_{\text{balloon}} p \cdot dV$$

$$\int p \cdot dV = 0 \text{ as cylinder shall be rigid.}$$

$$\begin{aligned}
 &= 0 + p \cdot \Delta V \\
 &= 0 + 1.013 \times 10^5 \times 0.4 \\
 &= 40.52 \text{ kJ}
 \end{aligned}$$

$$\left. \begin{aligned}
 &\text{Work done by system upon atmosphere} = \mathbf{40.52 \text{ kJ}} \\
 &\text{Work done by atmosphere} = \mathbf{-40.52 \text{ kJ}}
 \end{aligned} \right\} \text{Ans.}$$

13. In a steam power plant 5 kW of heat is supplied in boiler and turbine produces 25% of heat added while 75% of heat added is rejected in condenser. Feed water pump consumes 0.2% of this heat added

for pumping condensate to boiler. Determine the capacity of generator which could be used with this plant.

Solution:

Given,

$$Q_{\text{add}} = 5000 \text{ J/s}$$

$$W_T = 0.25 \times 5000 = 1250 \text{ J/s}$$

$$Q_{\text{rejected}} = 0.75 \times 5000 = 3750 \text{ J/s}$$

$$W_p = (-) 0.002 \times 5000 = 10 \text{ J/s}$$

$$\begin{aligned} \text{Capacity of generator} &= W_T - W_p \\ &= 1250 - 10 \\ &= 1240 \text{ J/s or } 1240 \text{ W} \\ &= \mathbf{1.24 \text{ kW Ans.}} \end{aligned}$$

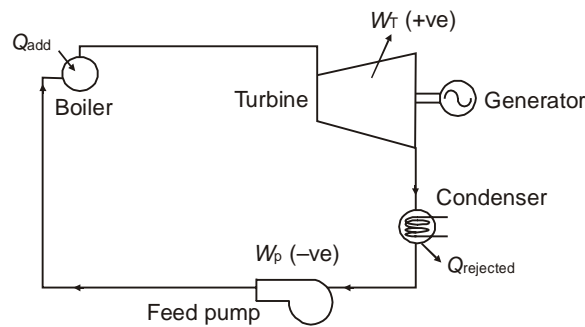


Fig. 3.35

14. In a gas turbine installation air is heated inside heat exchanger upto 750°C from ambient temperature of 27°C . Hot air then enters into gas turbine with the velocity of 50 m/s and leaves at 600°C . Air leaving turbine enters a nozzle at 60 m/s velocity and leaves nozzle at temperature of 500°C . For unit mass flow rate of air determine the following assuming adiabatic expansion in turbine and nozzle,

- heat transfer to air in heat exchanger
- power output from turbine
- velocity at exit of nozzle.

Take c_p for air as $1.005 \text{ kJ/kg}^\circ\text{K}$.

Solution:

In heat exchanger upon applying S.F.E.E. with assumptions of no change in kinetic energy, no work interaction, no change in potential energy, for unit mass flow rate of air,

$$h_1 + Q_{1-2} = h_2$$

$$Q_{1-2} = h_2 - h_1$$

$$Q_{1-2} = C_p \cdot (T_2 - T_1)$$

Heat transfer to air in heat exchanger $Q_{1-2} = \mathbf{726.62 \text{ kJ Ans.}}$

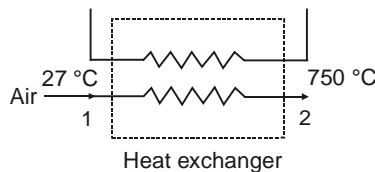


Fig. 3.36

In gas turbine let us use S.F.E.E., assuming no change in potential energy, for unit mass flow rate of air

$$\begin{aligned}
 h_2 + \frac{C_2^2}{2} &= h_3 + \frac{C_3^2}{2} + W_T \\
 W_T &= (h_2 - h_3) + \left(\frac{C_2^2 - C_3^2}{2} \right) \\
 &= C_p(T_2 - T_3) + \left(\frac{C_2^2 - C_3^2}{2} \right) \\
 &= 1.005 (750 - 600) + \left(\frac{50^2 - 60^2}{2} \right) \times 10^{-3}
 \end{aligned}$$

Power output from turbine = **150.2 kJ/s Ans.**

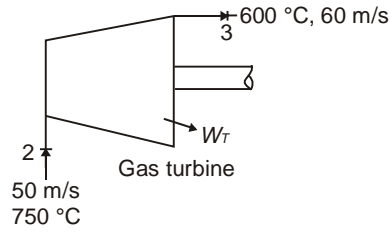


Fig. 3.37

Applying S.F.E.E. upon nozzle assuming no change in potential energy, no work and heat interactions, for unit mass flow rate,

$$\begin{aligned}
 h_3 + \frac{C_3^2}{2} &= h_4 + \frac{C_4^2}{2} \\
 \frac{C_4^2}{2} &= (h_3 - h_4) + \frac{C_3^2}{2} \\
 &= C_p(T_3 - T_4) + \frac{C_3^2}{2} \\
 &= 1.005 (600 - 500) + \left(\frac{60^2}{2} \right) \times 10^{-3} \\
 \frac{C_4^2}{2} &= 102.3
 \end{aligned}$$

$$C_4 = 14.3 \text{ m/s}$$

Velocity at exit of nozzle = **14.3 m/s Ans.**

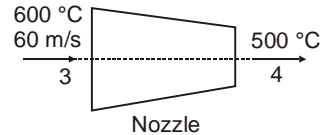


Fig. 3.38

15. One mol of air at 0.5 MPa and 400 K, initially undergoes following processes, sequentially

(a) heating at constant pressure till the volume gets doubled.

(b) expansion at constant temperature till the volume is six times of initial volume.

Determine the work done by air.

Solution:

For constant pressure heating, say state changes from 1 to 2

$$W_a = \int_1^2 p_1 dV$$

$$W_a = p_1 (V_2 - V_1)$$

It is given that

$$V_2 = 2V_1$$

so

$$W_a = p_1 V_1$$

$$W_a = RT_1$$

For subsequent expansion at constant temperature say state changes from 2 to 3.

Also given that $\frac{V_3}{V_1} = 6$, so $\frac{V_3}{V_2} = 3$

Work,

$$W_b = \int_2^3 p dV$$

$$= \int_2^3 \frac{RT}{V} dV = RT_2 \ln \frac{V_3}{V_2}$$

$$W_b = RT_2 \ln (3)$$

Temperature at 2 can be given by perfect gas considerations as,

$$\frac{T_2}{T_1} = \frac{V_2}{V_1}$$

or

$$T_2 = 2 T_1$$

Total work done by air,

$$W = W_a + W_b$$

$$= RT_1 + RT_2 \ln (3)$$

$$= RT_1 + 2RT_1 \ln (3)$$

$$= RT_1 (1 + 2 \ln 3) = 8.314 \times 400 (1 + 2 \ln 3)$$

$$\text{Work done} = \mathbf{10632.69 \text{ kJ Ans.}}$$

16. Determine the work done by gas for the arrangement shown in Fig. 3.39. Here spring exerts a force upon piston which is proportional to its deformation from equilibrium position. Spring gets deflected due to heating of gas till its volume becomes thrice of original volume. Initial states are 0.5 MPa and 0.5 m^3 while final gas pressure becomes 1 MPa. Atmospheric pressure may be taken as $1.013 \times 10^5 \text{ Pa}$.

Solution:

Let stiffness of spring be k and it undergoes a deflection by ' x ' along x -axis. Force balance at any equilibrium position of piston shall be,

$$p \cdot A = p_{\text{atm}} \cdot A + kx,$$

here x shall be linear displacement of piston due to expansion of gas. Let volume of gas change from V_0

to some value V . Then, $x = \frac{V - V_0}{A}$, V_0 is volume of gas when spring is at its natural length.

$$p \cdot A = p_{\text{atm}} \cdot A + k \left(\frac{V - V_0}{A} \right)$$

or
$$(p - p_{\text{atm}}) = \frac{k(V - V_0)}{A^2}$$

Work done by gas between initial and final states, $W = \int_i^f p \cdot dV$

$$\begin{aligned} W &= p_{\text{atm}} (V_f - V_i) + \frac{k}{A^2} \left\{ \frac{V^2}{2} - V_0 \cdot V \right\}_i^f \\ &= p_{\text{atm}} (V_f - V_i) + \frac{k}{A^2} \left\{ \frac{V_f^2 - V_i^2}{2} - V_0 V_f + V_0 \cdot V_i \right\} \\ &= (V_f - V_i) p_{\text{atm}} + \frac{k}{A^2} \left\{ \left(\frac{V_f^2 - V_i^2 - 2V_0 V_f + 2V_0 \cdot V_i}{2} \right) \right\} \\ &= (V_f - V_i) p_{\text{atm}} + \left\{ \frac{k}{2A^2} ((V_f - V_0) + (V_i - V_0))(V_f - V_i) \right\} \\ &= (V_f - V_i) \left\{ p_{\text{atm}} + \frac{k}{2A^2} ((V_f - V_0) + (V_i - V_0)) \right\} \end{aligned}$$

from above force balance, $(p_f - p_{\text{atm}}) = \frac{k}{A^2} (V_f - V_0)$

$$(p_i - p_{\text{atm}}) = \frac{k}{A^2} (V_i - V_0)$$

or
$$W = (V_f - V_i) \left\{ p_{\text{atm}} + \left(\frac{p_f - p_{\text{atm}}}{2} \right) + \left(\frac{p_i - p_{\text{atm}}}{2} \right) \right\}$$

$$= (V_f - V_i) \left\{ \frac{p_i + p_f}{2} \right\}, \text{ substituting pressure and volume values, } V_f = 3V_i$$

$W = 0.75 \times 10^6 \text{ J Ans.}$

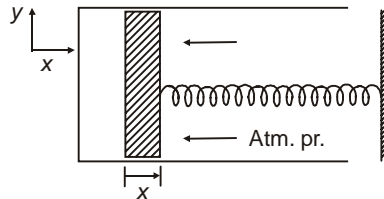


Fig. 3.39

17. A closed insulated container has frictionless and smooth moving insulated partition as shown in Fig. 3.40 such that it equally divides total 1 m^3 of volume, when both the gases are at initial pressure of 0.5 MPa and ambient temperature of 27°C . Subsequently the nitrogen is heated using electrical heating element such that volume of N_2 becomes $3/4$ of total volume of container. Determine, (i) final pressure of hydrogen, (ii) Work done by partition, (iii) Work done by N_2 and H_2 (iv) Heat added to N_2 by electric heater.

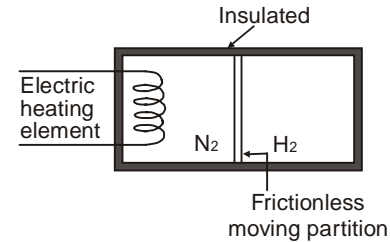


Fig. 3.40

Take $C_{p, \text{N}_2} = 1.039 \text{ kJ/kg} \cdot \text{K}$, $C_{p, \text{H}_2} = 14.307 \text{ kJ/kg} \cdot \text{K}$,

$R_{\text{N}_2} = 0.2968 \text{ kJ/kg} \cdot \text{K}$, $R_{\text{H}_2} = 4.1240 \text{ kJ/kg} \cdot \text{K}$

Solution:

With the heating of N_2 it will get expanded while H_2 gets compressed simultaneously. Compression of H_2 in insulated chamber may be considered of adiabatic type.

Adiabatic Index of compression for H_2 can be obtained as,

$$C_{p, \text{H}_2} = R_{\text{H}_2} \left(\frac{\gamma_{\text{H}_2}}{\gamma_{\text{H}_2} - 1} \right)$$

$$14.307 = 4.124 \left(\frac{\gamma_{\text{H}_2}}{\gamma_{\text{H}_2} - 1} \right)$$

$$\gamma_{\text{H}_2} = 1.405$$

Adiabatic Index of expansion for N_2 , $C_{p, \text{N}_2} = R_{\text{N}_2} \left(\frac{\gamma_{\text{N}_2}}{\gamma_{\text{N}_2} - 1} \right)$

$$1.039 = 0.2968 \left(\frac{\gamma_{\text{N}_2}}{\gamma_{\text{N}_2} - 1} \right)$$

$$\gamma_{\text{N}_2} = 1.399$$

(i) For hydrogen, $p_1 V_1^\gamma = p_2 V_2^\gamma$

Here $\gamma = \gamma_{\text{H}_2} = 1.405$, $V_1 = 0.5 \text{ m}^3$

$p_1 = 0.5 \times 10^6 \text{ Pa}$, $V_2 = 0.25 \text{ m}^3$

$$\text{Final pressure of } \text{H}_2 = 0.5 \times 10^6 \left(\frac{0.5}{0.25} \right)^{1.405}$$

$$= \mathbf{1.324 \text{ MPa Ans.}}$$

(ii) Since partition remains in equilibrium throughout hence no work is done by partition. It is a case similar to free expansion.

Partition work = 0. Ans.

(iii) Work done upon H_2 ,

$$W_{\text{H}_2} = \frac{p_1 V_1 - p_2 V_2}{(\gamma_{\text{H}_2} - 1)}$$

Here $p_1 = 0.5 \times 10^6$ Pa, $p_2 = 1.324 \times 10^6$ Pa, $V_1 = 0.5$ m³, $V_2 = 0.25$ m³.

$$\begin{aligned}\text{Work done by hydrogen, } W_{H_2} &= \frac{(-)0.081 \times 10^6}{0.405} \\ &= (-) 2 \times 10^5 \text{ J Ans.}\end{aligned}$$

Work done by N₂ = Work done upon H₂

Work done by nitrogen = + 2 × 10⁵ J Ans.

(iv) Heat added to N₂ can be obtained using first law of thermodynamics as

$$Q_{N_2} = \Delta U_{N_2} + W_{N_2} \Rightarrow Q_{N_2} = mc_v(T_2 - T_1) + W_{N_2}$$

Final temperature of N₂ can be obtained considering it as perfect gas.

$$\text{Therefore, } T_2 = \frac{p_2 V_2 T_1}{p_1 V_1}$$

p_2 = Final pressure of N₂ which will be equal to that of H₂ as the partition is free and frictionless.

$$p_2 = 1.324 \times 10^6 \text{ Pa.}$$

$$\begin{aligned}T_2 = \text{Final temperature of N}_2 &= \frac{1.324 \times 10^6 \times 0.75 \times 300}{0.5 \times 10^6 \times 0.5} \\ &= 1191.6 \text{ K}\end{aligned}$$

$$\text{mass of N}_2, m = \frac{p_1 V_1}{RT_1} = \frac{0.5 \times 10^6 \times 0.5}{0.2968 \times 10^3 \times 300} = 2.8 \text{ kg.}$$

Specific heat at constant volume, $C_v = C_p - R \Rightarrow C_{v, N_2} = 0.7422$ kJ/kg . K.

$$\begin{aligned}\text{Heat added to N}_2, Q_{N_2} &= \{2.8 \times (1191.6 - 300) \times 0.7422 \times 10^3\} + 2 \times 10^5 \\ &= 2052.9 \text{ kJ Ans.}\end{aligned}$$

18. A cylinder of 2 m³ has air at 0.5 MPa and temperature of 375°K. Air is released in atmosphere through a valve on cylinder so as to run a frictionless turbine. Find the amount of work available from turbine assuming no heat loss and complete kinetic energy being used for running turbine. Take $C_{p, \text{air}} = 1.003$ kJ/kg . K, $C_{v, \text{air}} = 0.716$ kJ/kg . K, $R_{\text{air}} = 0.287$ kJ/kg . K.

Solution:

Let initial states and final states of air inside cylinder be given by m_1, p_1, V_1, T_1 , and m_2, p_2, V_2, T_2 respectively. It is a case of emptying of cylinder.

$$\text{Initial mass of air, } m_1 = \frac{p_1 V_1}{P_{\text{air}} \cdot T_1} = 9.29 \text{ kg.}$$

For adiabatic expansion during release of air through valve from 0.5 MPa to atmospheric pressure.

$$\begin{aligned}T_2 &= T_1 \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \\ &= 375 \left(\frac{1.013 \times 10^5}{0.5 \times 10^6} \right)^{\frac{1.4-1}{1.4}}\end{aligned}$$

$$T_2 = 237.65 \text{ K}$$

Final mass of air left in tank, $m_2 = \frac{p_2 V_2}{RT_2}$

$$m_2 = \frac{1.013 \times 10^5 \times 2}{(0.287 \times 10^3 \times 237.65)} = 2.97 \text{ kg}$$

Writing down energy equation for unsteady flow system

$$(m_1 - m_2) \left(h_2 + \frac{C^2}{2} \right) = m_1 u_1 - m_2 u_2$$

$$(m_1 - m_2) \frac{C^2}{2} = (m_1 u_1 - m_2 u_2) - (m_1 - m_2) h_2$$

Kinetic energy available for running turbine

$$= (m_1 C_v T_1 - m_2 C_v T_2) - (m_1 - m_2) \cdot C_p \cdot T_2$$

$$= (9.29 \times 0.716 \times 10^3 \times 375) - (2.97 \times 0.716 \times 10^3 \times 237.65) - \{(9.29 - 2.97) \times 1.003 \times 10^3 \times 237.65\}$$

$$= 482.54 \times 10^3 \text{ J}$$

Amount of work available = **482.54 kJ Ans.**

19. A rigid and insulated tank of 1 m^3 volume is divided by partition into two equal volume chambers having air at 0.5 MPa , 27°C and 1 MPa , 500 K . Determine final pressure and temperature if the partition is removed.

Solution:

Using perfect gas equation for the two chambers having initial states as 1 and 2 and final state as 3.

$$p_1 = 0.5 \times 10^6 \text{ Pa}, V_1 = 0.5 \text{ m}^3, T_1 = 300 \text{ K}$$

$$n_1 = \frac{p_1 V_1}{RT_1}$$

$$= \frac{0.5 \times 10^6 \times 0.5}{8314 \times 300}$$

$$n_1 = 0.1002$$

and $n_2 = \frac{p_2 V_2}{RT_2}$ where $p_2 = 1 \times 10^6 \text{ Pa}$, $V_2 = 0.5 \text{ m}^3$, $T_2 = 500 \text{ K}$

$$= \frac{1 \times 10^6 \times 0.5}{8314 \times 500}$$

$$n_2 = 0.1203$$

For tank being insulated and rigid we can assume, $\Delta U = 0$, $W = 0$, $Q = 0$, so writing ΔU ,

$$\Delta U = n_1 C_v (T_3 - T_1) + n_2 C_v (T_3 - T_2) = 0$$

or $T_3 = 409.11 \text{ K}$

Using perfect gas equation for final mixture,

$$p_3 = \frac{(n_1 + n_2) \bar{R} T_3}{(V_1 + V_2)} = 0.75 \text{ MPa}$$

Final pressure and temperature = **0.75 MPa, 409.11 K Ans.**

20. An evacuated bottle of 0.5 m^3 volume is slowly filled from atmospheric air at 1.0135 bars until the pressure inside the bottle also becomes 1.0135 bar. Due to heat transfer, the temperature of air inside the bottle after filling is equal to the atmospheric air temperature. Determine the amount of heat transfer.

[U.P.S.C., 1994]

Solution:

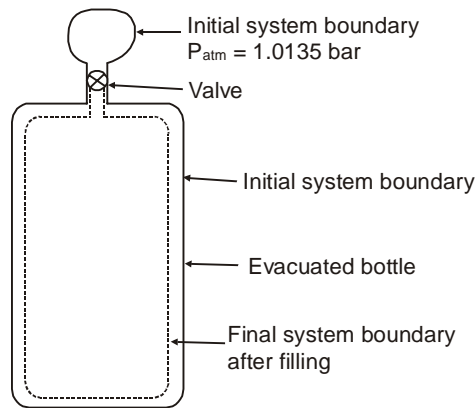


Fig. 3.41

Displacement work; $W = 1.0135 \times 10^5 \times (0 - 0.5)$

$$W = -0.50675 \times 10^5 \text{ Nm}$$

Heat transfer, $Q = 0.50675 \times 10^5 \text{ Nm}$

Heat transfer = **$0.50675 \times 10^5 \text{ Nm}$** **Ans.**

21. A compressed air bottle of 0.3 m^3 volume contains air at 35 bar, 40°C . This air is used to drive a turbogenerator supplying power to a device which consumes 5 W. Calculate the time for which the device can be operated if the actual output of the turbogenerator is 60% of the maximum theoretical

output. The ambient pressure to which the tank pressure has fallen is 1 bar. For air, $\frac{C_p}{C_v} = 1.4$.

[U.P.S.C. 1993]

Solution:

Here turbogenerator is fed with compressed air from a compressed air bottle. Pressure inside bottle gradually decreases from 35 bar to 1 bar. Expansion from 35 bar to 1 bar occurs isentropically. Thus, for the initial and final states of pressure, volume, temperature and mass inside bottle being given as P_1 , V_1 , T_1 & m_1 and P_2 , V_2 , T_2 & m_2 respectively. It is transient flow process similar to emptying of the bottle.

$$\left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}, \text{ Given: } P_1 = 35 \text{ bar, } T_1 = 40^\circ\text{C or } 313 \text{ K}$$

$$V_1 = 0.3 \text{ m}^3; V_2 = 0.3 \text{ m}^3$$

$$P_2 = 1 \text{ bar.}$$

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = 113.22 \text{ K}$$

By perfect gas law, initial mass in bottle, $m_1 = \frac{P_1 V_1}{RT_1} = \frac{35 \times 10^2 \times 0.3}{0.287 \times 313}$

$$m_1 = 11.68 \text{ kg}$$

Final mass in bottle, $m_2 = \frac{P_2 V_2}{RT_2} = \frac{1 \times 10^2 \times 0.3}{0.287 \times 113.22}$

$$m_2 = 0.923 \text{ kg}$$

Energy available for running turbo generator or work;

$$W + (m_1 - m_2) h_2 = m_1 u_1 - m_2 u_2$$

$$W = (m_1 u_1 - m_2 u_2) - (m_1 - m_2) h_2$$

$$= (m_1 c_v T_1 - m_2 c_v T_2) - (m_1 - m_2) \cdot c_p \cdot T_2$$

Taking $c_v = 0.718 \text{ kJ/kg} \cdot \text{K}$ and $c_p = 1.005 \text{ kJ/kg} \cdot \text{K}$

$$W = \{(11.68 \times 0.718 \times 313) - (0.923 \times 0.718 \times 113.22)\} - \{(11.68 - 0.923) \times 1.005 \times 113.22\}$$

$$W = 1325.86 \text{ kJ}$$

This is the maximum work that can be had from the emptying of compressed air bottle between given pressure limits.

Turbogenerator's actual output = 5 kJ/s

$$\text{Input to turbogenerator} = \frac{5}{0.6} = 8.33 \text{ kJ/s.}$$

Time duration for which turbogenerator can be run;

$$\Delta t = \frac{1325.86}{8.33}$$

$$\Delta t = 159.17 \text{ sec.}$$

Duration \approx **160 seconds** Ans.

22. 3 kg of air at 1.5 bar pressure and 77°C temperature at state 1 is compressed polytropically to state 2 at pressure 7.5 bar, index of compression being 1.2. It is then cooled at constant temperature to its original state 1. Find the net work done and heat transferred. [U.P.S.C. 1992]

Solution:

Different states as described in the problem are denoted as 1, 2 and 3 and shown on p - V diagram.

Process 1-2 is polytropic process with index 1.2

So,
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}}$$

or,
$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}}$$

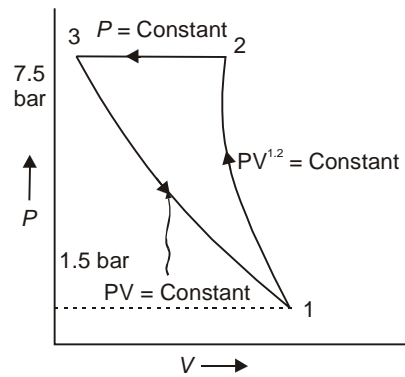


Fig. 3.42

$$= 350 \cdot \left(\frac{7.5}{1.5} \right)^{\frac{1.2-1}{1.2}}$$

$$T_2 = 457.68 \text{ K}$$

At state 1,

$$P_1 V_1 = mRT_1$$

$$\frac{1.5 \times 10^5}{10^3} \cdot V_1 = 3 \times 0.287 \times 350$$

or,

$$V_1 = 2.009 \approx 2.01 \text{ m}^3$$

For process 1-2,

$$V_2^{1.2} = \frac{P_1 V_1^{1.2}}{P_2}, \quad V_2 = \left\{ \frac{1.5 \times (2.01)^{1.2} \times 10^5}{7.5 \times 10^5} \right\}^{\frac{1}{1.2}}$$

or,

$$V_2 = 0.526 \text{ m}^3$$

Process 2-3 is constant pressure process, so $\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}$ gets modified as,

$$\Rightarrow V_3 = \frac{V_2 \cdot T_3}{T_2}$$

Here process 3-1 is isothermal process, so $T_1 = T_3$

$$\text{or, } V_3 = \frac{0.526 \times 350}{457.68}$$

$$\text{or, } V_3 = 0.402 \text{ m}^3$$

During process 1-2 the compression work;

$$\begin{aligned} W_{1-2} &= \frac{m \cdot R \cdot (T_1 - T_2)}{1 - n} \\ &= \frac{3 \times 0.287 (457.68 - 350)}{(1 - 1.2)} \\ W_{1-2} &= -463.56 \text{ kJ} \end{aligned}$$

Work during process 2-3,

$$\begin{aligned} W_{2-3} &= P_2 (V_3 - V_2) \\ &= 7.5 \times 10^5 (0.402 - 0.526) \\ &= -93 \text{ kJ} \end{aligned}$$

Work during process 3-1,

$$W_{3-1} = P_3 V_3 \ln \left(\frac{V_1}{V_3} \right) = 7.5 \times 10^5 \times 0.402 \times \ln \left(\frac{2.01}{0.402} \right)$$

$$W_{3-1} = 485.25 \text{ kJ}$$

Net work,

$$\begin{aligned} W_{\text{net}} &= W_{1-2} + W_{2-3} + W_{3-1} \\ &= -463.56 - 93 + 485.25 \end{aligned}$$

$$\boxed{\text{Network} = -71.31 \text{ kJ}} \text{ Ans.}$$

–ve work shows work done upon the system. Since it is the cycle, so

$$W_{\text{net}} = Q_{\text{net}}$$

$$\oint dW = \oint dQ = -71.31 \text{ kJ}$$

$$\boxed{\text{Heat transferred from system} = 71.31 \text{ kJ}} \text{ Ans.}$$

23. A compressed air bottle of volume 0.15 m^3 contains air at 40 bar and 27°C . It is used to drive a turbine which exhausts to atmosphere at 1 bar. If the pressure in the bottle is allowed to fall to 2 bar, determine the amount of work that could be delivered by the turbine. [U.P.S.C. 1998]

Solution:

$$c_p = 1.005 \text{ kJ/kg} \cdot \text{K}, c_v = 0.718 \text{ kJ/kg} \cdot \text{K}, \gamma = 1.4$$

$$\text{Initial mass of air in bottle} \Rightarrow m_1 = \frac{p_1 V_1}{RT_1} = \frac{40 \times 10^2 \times 0.15}{0.287 \times 300}$$

$$m_1 = 6.97 \text{ kg}$$

$$\text{Final mass of air in bottle} \Rightarrow m_2 = \frac{p_2 V_2}{RT_2}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}, m_2 = \frac{2 \times 10^2 \times 0.15}{0.287 \times 127.36}$$

$$= \left(\frac{2}{40} \right)^{\frac{1.4-1}{1.4}}, m_2 = 0.821 \text{ kg.}$$

$$T_2 = 127.36 \text{ K}$$

Energy available for running of turbine due to emptying of bottle,

$$\begin{aligned} &= (m_1 c_v T_1 - m_2 c_v T_2) - (m_1 - m_2) \cdot c_p \cdot T_2 \\ &= \{(6.97 \times 0.718 \times 300) - (0.821 \times 0.718 \times 127.36)\} \\ &\quad - \{(6.97 - 0.821) \times 1.005 \times 127.35\} \\ &= 639.27 \text{ kJ.} \end{aligned}$$

$$\boxed{\text{Work available from turbine} = 639.27 \text{ kJ}} \text{ Ans.}$$

EXERCISE

- 3.1 Define the first law of thermodynamics. Also give supporting mathematical expression for it.
- 3.2 How the first law of thermodynamics is applied to a closed system undergoing a non-cyclic process?
- 3.3 Show that internal energy is a property.
- 3.4 Explain the following :
- Free expansion
 - Polytropic process
 - Hyperbolic process
- Also obtain expressions for work in each case.
- 3.5 Show that for a polytropic process.

$$Q = \left(\frac{\gamma - n}{\gamma - 1} \right) W$$

where Q and W are heat and work interactions and n is polytropic index.

- 3.6 Derive the steady flow energy equation.
- 3.7 Explain a unsteady flow process.
- 3.8 Show that for a given quantity of air supplied with a definite amount of heat at constant volume, the rise in pressure shall be directly proportional to initial absolute pressure and inversely proportional to initial absolute temperature.
- 3.9 How much work is done when 0.566 m^3 of air initially at a pressure of 1.0335 bar and temperature of 7°C undergoes an increase in pressure upto 4.13 bar in a closed vessel? [0]
- 3.10 An ideal gas and a steel block are initially having same volumes at same temperature and pressure. Pressure on both is increased isothermally to five times of its initial value. Show with the help of P - V diagram, whether the quantities of work shall be same in two processes or different. If different then which one is greater. Assume processes to be quasi-static.
- 3.11 An inventor has developed an engine getting 1055 MJ from fuel and rejecting 26.375 MJ in exhaust and delivering 25 kWh of mechanical work. Is this engine possible? [No]
- 3.12 For an ideal gas the pressure is increased isothermally to ' n ' times its initial value. How high would the gas be raised if the same amount of work were done in lifting it? Assume process to be quasi-static.
- 3.13 A system's state changes from a to b as shown on P - V diagram

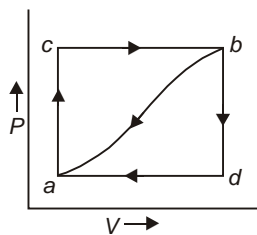


Fig. 3.43

Along path ' acb ' 84.4 kJ of heat flows into the system and system does 31.65 kJ of work. Determine heat flow into the system along path ' adb ' if work done is 10.55 kJ. When system returns from ' b ' to ' a ' following the curved path then work done on system is 21.1 kJ. How much heat is absorbed or rejected? If internal energy at ' a ' and ' d ' are 0 and 42.2 kJ, find the heat absorbed in processes ' ad ' and ' db '. [63.3 kJ, -73.85 kJ, 52.75 kJ, 10.55 kJ]

- 3.14 A tank contains 2.26 m^3 of air at a pressure of 24.12 bar. If air is cooled until its pressure and temperature becomes 13.78 bar and 21.1°C respectively. Determine the decrease of internal energy. [-5857.36 kJ]

- 3.15** Water in a rigid, insulating tank is set in rotation and left. Water comes to rest after some time due to viscous forces. Considering the tank and water to constitute the system answer the following.
- (i) Is any work done during the process of water coming to rest?
 - (ii) Is there a flow of heat?
 - (iii) Is there any change in internal energy (U)?
 - (iv) Is there any change in total energy (E)? [No, No, Yes, No]
- 3.16** Fuel-air mixture in a rigid insulated tank is burnt by a spark inside causing increase in both temperature and pressure. Considering the heat energy added by spark to be negligible, answer the following :
- (i) Is there a flow of heat into the system?
 - (ii) Is there any work done by the system?
 - (iii) Is there any change in internal energy (U) of system?
 - (iv) Is there any change in total energy (E) of system? [No, No, No, No]
- 3.17** Calculate the work if in a closed system the pressure changes as per relation $p = 300 \cdot V + 1000$ and volume changes from 6 to 4 m³. Here pressure ' p ' is in Pa and volume ' V ' is in m³. [-5000J]
- 3.18** Hydrogen from cylinder is used for inflating a balloon to a volume of 35m³ slowly. Determine the work done by hydrogen if the atmospheric pressure is 101.325 kPa. [3.55 MJ]
- 3.19** Show that the work done by an ideal gas is mRT_1 , if gas is heated from initial temperature T_1 to twice of initial temperature at constant volume and subsequently cooled isobarically to initial state.
- 3.20** Derive expression for work done by the gas in following system. Piston-cylinder device shown has a gas initially at pressure and volume given by P_1, V_1 . Initially the spring does not exert any force on piston. Upon heating the gas, its volume gets doubled and pressure becomes P_2 .

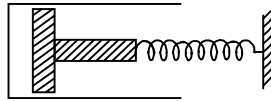


Fig. 3.44 Piston-cylinder arrangement

- 3.21** An air compressor with pressure ratio of 5, compresses air to $\frac{1}{4}$ th of the initial volume. For inlet temperature to be 27°C determine temperature at exit and increase in internal energy per kg of air. [101.83°C, 53.7 kJ/kg]
- 3.22** In a compressor the air enters at 27°C and 1 atm and leaves at 227°C and 1 MPa. Determine the work done per unit mass of air assuming velocities at entry and exit to be negligible. Also determine the additional work required, if velocities are 10 m/s and 50 m/s at inlet and exit respectively. [200.9 kJ/kg, 202.1 kJ/kg]
- 3.23** Turbojet engine flies with velocity of 270 m/s at the altitude where ambient temperature is -15°C. Gas temperature at nozzle exit is 873 K and fuel air ratio is 0.019. Corresponding enthalpy values for air and gas at inlet and exit are 260 kJ/kg and 912 kJ/kg respectively. Combustion efficiency is 95% and calorific value of fuel is 44.5 MJ/kg. For the heat losses from engine amounting to 21 kJ/kg of air determine the velocity of gas jet at exit. [613.27 m/s]
- 3.24** Oxygen at 3MPa and 300°C flowing through a pipe line is tapped out to fill an empty insulated rigid tank. Filling continues till the pressure equilibrium is not attained. What shall be the temperature of the oxygen inside the tank? If $\gamma = 1.39$. [662.5°C]
- 3.25** Determine work done by fluid in the thermodynamic cycle comprising of following processes :
- (a) Unit mass of fluid at 20 atm and 0.04 m³ is expanded by the law $PV^{1.5} = \text{constant}$, till volume gets doubled.
 - (b) Fluid is cooled isobarically to its original volume.
 - (c) Heat is added to fluid till its pressure reaches to its original pressure, isochorically. [18.8 kJ]

- 3.26** An air vessel has capacity of 10 m^3 and has air at 10 atm and 27°C . Some leakage in the vessel causes air pressure to drop sharply to 5 atm till leak is repaired. Assuming process to be of reversible adiabatic type determine the mass of air leaked. [45.95 kg]
- 3.27** Atmospheric air leaks into a cylinder having vacuum. Determine the final temperature in cylinder when inside pressure equals to atmospheric pressure, assuming no heat transferred to or from air in cylinder. [144.3°C]
- 3.28** Determine the power available from a steam turbine with following details;
Steam flow rate = 1 kg/s
Velocity at inlet and exit = 100 m/s and 150 m/s
Enthalpy at inlet and exit = 2900 kJ/kg, 1600 kJ/kg
Change in potential energy may be assumed negligible. [1293.75 kW]
- 3.29** Determine the heat transfer in emptying of a rigid tank of 1 m^3 volume containing air at 3 bar and 27°C initially. Air is allowed to escape slowly by opening a valve until the pressure in tank drops to 1 bar pressure. Consider escape of air in tank to follow polytropic process with index $n = 1.2$ [76.86 kJ]
- 3.30** A pump is used for pumping water from lake at height of 100 m consuming power of 60 kW. Inlet pipe and exit pipe diameters are 150 mm and 180 mm respectively. The atmospheric temperature is 293 K. Determine the temperature of water at exit of pipe. Take specific heat of water as 4.18 kJ/kg.K [293.05K]
- 3.31** Air at 8 bar, 100°C flows in a duct of 15 cm diameter at rate of 150 kg/min. It is then throttled by a valve upto 4 bar pressure. Determine the velocity of air after throttling and also show that enthalpy remains constant before and after throttling. [37.8 m/s]
- 3.32** Determine the power required by a compressor designed to compress atmospheric air (at 1 bar, 20°C) to 10 bar pressure. Air enters compressor through inlet area of 90 cm^2 with velocity of 50 m/s and leaves with velocity of 120 m/s from exit area of 5 cm^2 . Consider heat losses to environment to be 10% of power input to compressor. [50.4 kW]

Second Law of Thermodynamics

4.1 INTRODUCTION

Earlier discussions in article 3.11 throw some light on the limitations of first law of thermodynamics. A few situations have been explained where first law of thermodynamics fails to mathematically explain non-occurrence of certain processes, direction of process etc. Therefore, need was felt to have some more law of thermodynamics to handle such complex situations. Second law came up as embodiment of real happenings while retaining the basic nature of first law of thermodynamics. Feasibility of process, direction of process and grades of energy such as low and high are the potential answers provided by IInd law. Second law of thermodynamics is capable of indicating the maximum possible efficiencies of heat engines, coefficient of performance of heat pumps and refrigerators, defining a temperature scale independent of physical properties etc.

4.2 HEAT RESERVOIR

Heat reservoir is the system having very large heat capacity i.e. it is a body capable of absorbing or rejecting finite amount of energy without any appreciable change in its' temperature. Thus in general it may be considered as a system in which any amount of energy may be dumped or extracted out and there shall be no change in its temperature. Such as *atmosphere* to which large amount of heat can be rejected without measurable change in its temperature. Large river, sea etc. can also be considered as reservoir, as dumping of heat to it shall not cause appreciable change in temperature.

Heat reservoirs can be of two types depending upon nature of heat interaction i.e. heat rejection or heat absorption from it. Heat reservoir which rejects heat from it is called *source*. While the heat reservoir which absorbs heat is called *sink*. Some times these heat reservoirs may also be called Thermal Energy Reservoirs (TER).

4.3 HEAT ENGINE

Heat engine is a device used for converting heat into work as it has been seen from nature that conversion from work to heat may take place easily but the vice-versa is not simple to be realized. Heat and work have been categorized as two forms of energy of low grade and high grade type. Conversion of high grade of energy to low grade of energy may be complete (100%), and can occur directly whereas complete conversion of low grade of energy into high grade of energy is not possible. For converting low grade of energy (heat) into high grade of energy (work) some device called heat engine is required.

Thus, heat engine may be precisely defined as “a device operating in cycle between high temperature source and low temperature sink and producing work”. Heat engine receives heat from source, transforms

some portion of heat into work and rejects balance heat to sink. All the processes occurring in heat engine constitute cycle.

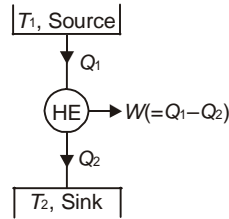


Fig. 4.1 Heat engine

Block diagram representation of a heat engine is shown above. A practical arrangement used in gas turbine plant is also shown for understanding the physical significance of heat engine.

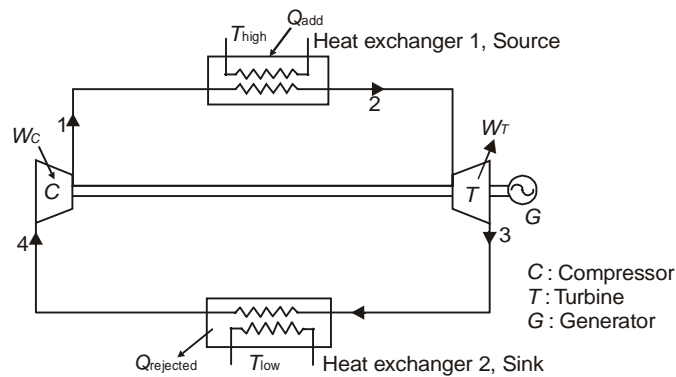


Fig. 4.2 Closed cycle gas turbine power plant

Gas turbine installation shows that heat is added to working fluid from 1–2 in a ‘heat exchanger 1’ and may be treated as heat supply by source. Working fluid is expanded in turbine from 2–3 and produces positive work. After expansion fluid goes to the ‘heat exchanger 2’ where it rejects heat from it like heat rejection in sink. Fluid at state 4 is sent to compressor for being compressed to state 1. Work required for compression is quite small as compared to positive work available in turbine and is supplied by turbine itself.

Therefore, heat engine model for it shall be as follows,

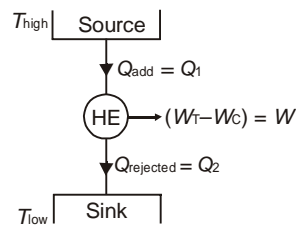


Fig. 4.3 Heat engine representation for gas turbine plant

Efficiency of heat engine can be given by the ratio of net work and heat supplied.

$$\eta_{\text{heat engine}} = \frac{\text{Net work}}{\text{Heat supplied}} = \frac{W}{Q_1}$$

For gas turbine plant shown

$$W = W_T - W_C$$

and

$$Q_1 = Q_{\text{add}}$$

Also since it is operating in cycle, so;

$$W_T - W_C = Q_{\text{add}} - Q_{\text{rejected}}$$

therefore, efficiency of heat engine can be given as;

$$\begin{aligned}\eta_{\text{heat engine}} &= \frac{W_T - W_C}{Q_{\text{add}}} \\ &= \frac{Q_{\text{add}} - Q_{\text{rejected}}}{Q_{\text{add}}} \\ \eta_{\text{heat engine}} &= 1 - \frac{Q_{\text{rejected}}}{Q_{\text{add}}}\end{aligned}$$

4.4 HEAT PUMP AND REFRIGERATOR

Heat pump refers to a device used for extracting heat from a low temperature surroundings and sending it to high temperature body, while operating in a cycle. In other words heat pump maintains a body or system at temperature higher than temperature of surroundings, while operating in cycle. Block diagram representation for a heat pump is given below:

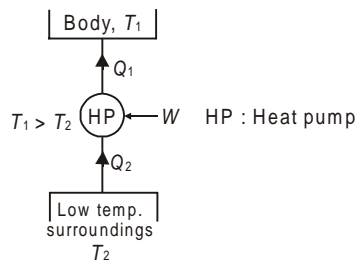


Fig. 4.4 Heat pump

As heat pump transfers heat from low temperature to high temperature, which is non spontaneous process, so external work is required for realizing such heat transfer. Heat pump shown picks up heat Q_2 at temperature T_2 and rejects heat Q_1 for maintaining high temperature body at temperature T_1 .

For causing this heat transfer heat pump is supplied with work W as shown.

As heat pump is not a work producing machine and also its objective is to maintain a body at higher temperature, so its performance can't be defined using efficiency as in case of heat engine. Performance of heat pump is quantified through a parameter called coefficient of performance (C.O.P). *Coefficient of performance* is defined by the ratio of desired effect and net work done for getting the desired effect.

$$\text{C.O.P.} = \frac{\text{Desired effect}}{\text{Net work done}}$$

For heat pump :

$$\text{Net work} = W$$

$$\text{Desired effect} = \text{heat transferred } Q_1 \text{ to high temperature body at temperature, } T_1.$$

$$(\text{COP})_{\text{HP}} = \frac{Q_1}{W}$$

also

$$W = Q_1 - Q_2$$

so

$$(\text{COP})_{\text{HP}} = \frac{Q_1}{Q_1 - Q_2}$$

Refrigerator is a device similar to heat pump but with reverse objective. It maintains a body at temperature lower than that of surroundings while operating in a cycle. Block diagram representation of refrigerator is shown in Fig 4.5.

Refrigerator also performs a non spontaneous process of extracting heat from low temperature body for maintaining it cool, therefore external work W is to be done for realizing it.

Block diagram shows how refrigerator extracts heat Q_2 for maintaining body at low temperature T_2 at the expense of work W and rejects heat to high temperature surroundings.

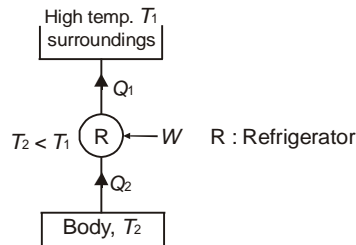


Fig. 4.5 Refrigerator

Performance of refrigerator is also quantified by coefficient of performance, which could be defined as:

$$(\text{COP})_{\text{refrigerator}} = \frac{\text{Desired effect}}{\text{Net work}} = \frac{Q_2}{W}$$

Here

$$W = Q_1 - Q_2$$

or

$$(\text{COP})_{\text{refrigerator}} = \frac{Q_2}{Q_1 - Q_2}$$

COP values of heat pump and refrigerator can be interrelated as:

$$(\text{COP})_{\text{HP}} = (\text{COP})_{\text{refrigerator}} + 1$$

4.5 STATEMENTS FOR IIND LAW OF THERMODYNAMICS

Rudolph Julius Emmanuel Clausius, a German physicist presented a first general statement of second law of thermodynamics in 1850 after studying the work of Sadi Carnot. It was termed as Clausius statement of second law. Lord Kelvin and Max Planck also came up with another statement of second law which was termed as Kelvin-Planck statement for second law of thermodynamics. Thus, there are two statements of second law of thermodynamics, (although they are equivalent as explained ahead).

Clausius statement of second law of thermodynamics: “It is impossible to have a device that while operating in a cycle produces no effect other than transfer of heat from a body at low temperature to a body at higher temperature.”

Above statement clearly indicates that if a non spontaneous process such as transferring heat from low temperature body to high temperature body is to be realized then some other effects such as

external work requirement is bound to be there. As already seen in case of refrigerator the external work is required for extracting heat from low temperature body and rejecting it to high temperature body.

Kelvin-Planck statement of second law of thermodynamics: “It is impossible for a device operating in a cycle to produce net work while exchanging heat with bodies at single fixed temperature”.

It says that in order to get net work from a device operating in cycle (i.e. heat engine) it must have heat interaction at two different temperatures or with body/reservoirs at different temperatures (i.e. source and sink).

Thus, above two statements are referring to feasible operation of heat pump/refrigerator and heat engine respectively. Devices based on violation of IInd law of thermodynamics are called Perpetual motion machines of 2nd kind (PMM-II). Fig 4.6 shows such PMM-II.

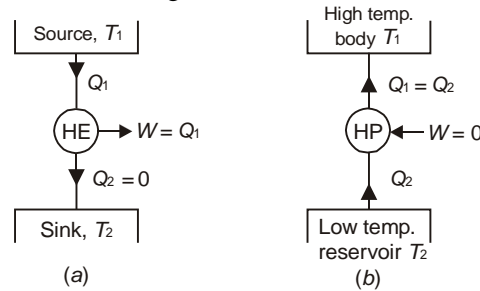


Fig. 4.6 Perpetual Motion Machine of IInd kind

PMM-II shown in Fig. 4.6a, refers to a heat engine which produces work while interacting with only one reservoir. PMM-II shown in Fig. 4.6b, refers to the heat pump which transfers heat from low temperature to high temperature body without spending work.

4.6 EQUIVALENCE OF KELVIN-PLANCK AND CLAUSIUS STATEMENTS OF IIND LAW OF THERMODYNAMICS

Kelvin-Planck and Clausius statements of IInd law of thermodynamics are actually two different interpretations of the same basic fact. Here the equivalence of two statements has been shown. For establishing equivalence following statements may be proved.

- (a) System based on violation of Kelvin-Planck statement leads to violation of Clausius statement.
- (b) System based on violation of Clausius statement leads to violation of Kelvin-Planck statement.

The explanation for equivalence based on above two is explained ahead.

- (a) Let us assume a heat engine producing net work while exchanging heat with only one reservoir at temperature T_1 , thus based on violation of Kelvin Planck statement. Let us also have a perfect heat pump operating between two reservoirs at temperatures T_1 and T_2 . Work requirement of heat pump may be met from the work available from heat engine. Layout shown ahead explains the proposed arrangement.

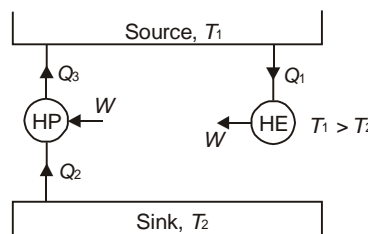


Fig. 4.7 System based on violation of Kelvin Planck statement

If heat pump takes input work from output of heat engine then,

$$Q_3 = Q_2 + W$$

and

$$W = Q_1$$

or

$$Q_3 = Q_1 + Q_2$$

Combination of heat engine and heat pump shall thus result in an equivalent system working as heat pump transferring heat from low temperature T_2 to high temperature T_1 without expense of any external work. This heat pump is based on violation of Clausius statement and therefore not possible.

Hence, it shows that violation of Kelvin Planck statement leads to violation of Clausius statement.

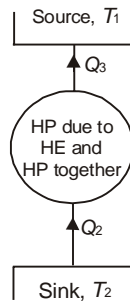


Fig. 4.8 Equivalent system

- (b) Let us assume a heat pump which operating in cycle transfers heat from low temperature reservoir to high temperature reservoir without expense of any work, thus based on violation of Clausius statement.

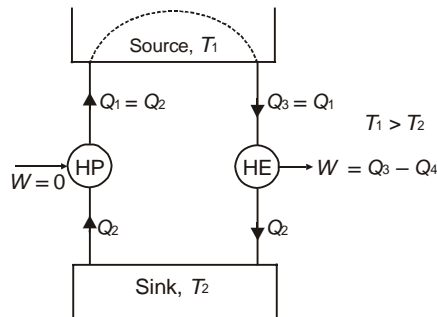


Fig. 4.9 System based on violation of Clausius statement

Heat pump transfers heat Q_1 to high temperature reservoir while extracting heat Q_2 from low temperature reservoir. Mathematically, as no work is done on pump, so

$$Q_2 = Q_1$$

Let us also have a heat engine between same temperature limits of T_1 and T_2 and produce net work W . Heat engine receives heat Q_3 from source which may be taken equal to Q_1 . Let us now devise for heat rejected from heat pump be given directly to heat engine. In such a situation the combination of heat pump and heat engine results in equivalent heat engine which produces work ' W ' while exchanging heat with only one reservoir at temperature T_2 . Arrangement is shown by dotted lines. This type of equivalent system is producing work as a result of only one heat interaction and thus violation of Kelvin Planck statement.

Thus, it shows that violation of Clausius statement also causes violation of Kelvin Planck statement.

Hence from (a) and (b) proved above it is obvious that the Clausius and Kelvin-Planck statements are equivalent. Conceptually the two statements explain the basic fact that,

- (i) net work can't be produced without having heat interactions taking place at two different temperatures.
- (ii) non spontaneous process such as heat flow from low temperature body to high temperature body is not possible without spending work.

4.7 REVERSIBLE AND IRREVERSIBLE PROCESSES

Reversible processes as described in chapter 1 refer to “the thermodynamic processes occurring in the manner that states passed through are always in thermodynamic equilibrium and no dissipative effects are present.” Any reversible process occurring between states 1–2 upon reversal, while occurring from 2–1 shall not leave any mark of process ever occurred as states traced back are exactly similar to those in forward direction. Reversible processes are thus very difficult to be realized and also called ideal processes. All thermodynamic processes are attempted to reach close to the reversible process in order to give best performance.

Thermodynamic process which does not fulfil conditions of a reversible process are termed irreversible processes. Irreversibilities are the reasons causing process to be irreversible. Generally, the irreversibilities can be termed as internal irreversibility and external irreversibility. Internal irreversibility is there because of internal factors whereas external irreversibility is caused by external factors at the system-surrounding interface. Generic types of irreversibilities are due to;

- (i) Friction,
 - (ii) Electrical resistance,
 - (iii) Inelastic solid deformations,
 - (iv) Free expansion
 - (v) Heat transfer through a finite temperature difference,
 - (vi) Non equilibrium during the process, etc.
- (i) **Friction:** Friction is invariably present in real systems. It causes irreversibility in the process as work done does not show equivalent rise in kinetic or potential energy of the system. Fraction of energy wasted due to frictional effects leads to deviation from reversible states.
 - (ii) **Electrical resistance:** Electrical resistance in the system also leads to presence of dissipation effects and thus irreversibilities. Due to electric resistance dissipation of electrical work into internal energy or heat takes place. The reverse transformation from heat or internal energy to electrical work is not possible, therefore leads to irreversibility.
 - (iii) **Inelastic solid deformation:** Deformation of solids, when of inelastic type is also irreversible and thus causes irreversibility in the process. If deformation occurs within elastic limits then it does not lead to irreversibility as it is of reversible type.
 - (iv) **Free expansion:** Free expansion as discussed earlier in chapter 3, refers to the expansion of unresisted type such as expansion in vacuum. During this unresisted expansion the work interaction is zero and without expense of any work it is not possible to restore initial states. Thus, free expansion is irreversible.
 - (v) **Heat transfer through a finite temperature difference:** Heat transfer occurs only when there exist temperature difference between bodies undergoing heat transfer. During heat transfer if heat addition is carried out in finite number of steps then after every step the new

state shall be a non-equilibrium state. In order to have equilibrium states in between, the heat transfer process may be carried out in infinite number of steps. Thus, infinitesimal heat transfer every time causes infinitesimal temperature variation. These infinitesimal state changes shall require infinite time and process shall be of quasi-static type, therefore reversible. Heat transfer through a finite temperature difference which practically occurs is accompanied by irreversible state changes and thus makes processes irreversible.

- (vi) **Non equilibrium during the process:** Irreversibilities are introduced due to lack of thermodynamic equilibrium during the process. Non equilibrium may be due to mechanical inequilibrium, chemical inequilibrium, thermal inequilibrium, electrical inequilibrium etc. and irreversibility are called mechanical irreversibility, chemical irreversibility, thermal irreversibility, electrical irreversibility respectively. Factors discussed above are also causing non equilibrium during the process and therefore make process irreversible.

Comparative study of reversible and irreversible processes shows the following major differences.

Difference between reversible and irreversible processes

<i>Reversible process</i>	<i>Irreversible process</i>
<p>(i) Reversible process can not be realized in practice</p> <p>(ii) The process can be carried out in the reverse direction following the same path as followed in forward direction</p> <p>(iii) A reversible process leaves no trace of occurrence of process upon the system and surroundings after its' reversal.</p> <p>(iv) Such processes can occur in either directions without violating second law of thermodynamics.</p> <p>(v) A system undergoing reversible processes has maximum efficiency. So the system with reversible processes are considered as reference systems or bench marks.</p> <p>(vi) Reversible process occurs at infinitesimal rate i.e. quasi-static process.</p> <p>(vii) System remains throughout in thermodynamic equilibrium during occurrence of such process.</p> <p>(viii) Examples; Frictionless motion, controlled expansion and compression, Elastic deformations, Electric circuit with no resistance, Electrolysis, Polarization and magnetisation process etc.</p>	<p>(i) All practical processes occurring are irreversible processes</p> <p>(ii) Process, when carried out in reverse direction follows the path different from that in forward direction.</p> <p>(iii) The evidences of process having occurred are evident even after reversal of irreversible process.</p> <p>(iv) Occurrence of irreversible processes in either direction is not possible, as in one direction it shall be accompanied with the violation of second law of thermodynamics.</p> <p>(v) System having irreversible processes do not have maximum efficiency as it is accompanied by the wastage of energy.</p> <p>(vi) Irreversible processes occur at finite rate.</p> <p>(vii) System does not remain in thermodynamic equilibrium during occurrence of irreversible processes.</p> <p>(viii) Examples; Viscous fluid flow, inelastic deformation and hysteresis effect, Free expansion, Electric circuit with resistance, Mixing of dissimilar gases, Throttling process etc.</p>

4.8 CARNOT CYCLE AND CARNOT ENGINE

Nicholas Leonard Sadi Carnot, an engineer in French army originated use of cycle (Carnot) in thermodynamic analysis in 1824 and these concepts provided basics upon which second law of thermodynamics was stated by Clausius and others.

Carnot cycle is a reversible thermodynamic cycle comprising of four reversible processes.

Thermodynamic processes constituting Carnot cycle are;

- (i) Reversible isothermal heat addition process, (1–2, Q_{add})
- (ii) Reversible adiabatic expansion process (2–3, W_{expn} +ve)
- (iii) Reversible isothermal heat release process (3–4, Q_{rejected})
- (iv) Reversible adiabatic compression process (4–1, W_{compr} –ve)

Carnot cycle is shown on P – V diagram between states 1, 2, 3 4, and 1. A reciprocating piston-cylinder assembly is also shown below P – V diagram.

Process 1 –2 is isothermal heat addition process of reversible type in which heat is transferred to system isothermally. In the piston cylinder arrangement heat Q_{add} can be transferred to gas from a constant temperature source T_1 through a cylinder head of conductor type.

First law of thermodynamics applied on 1–2 yields;

$$Q_{\text{add}} = U_2 - U_1 + W_{1-2}$$

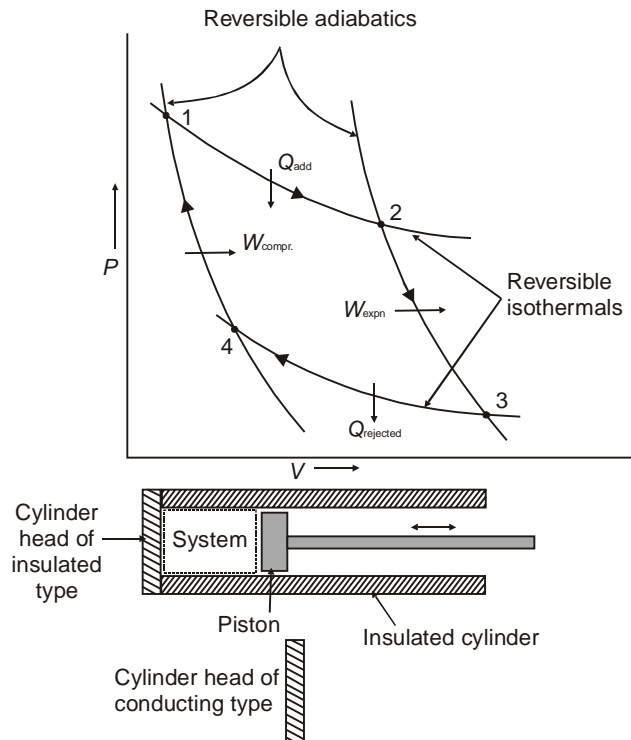


Fig. 4.10 Carnot cycle

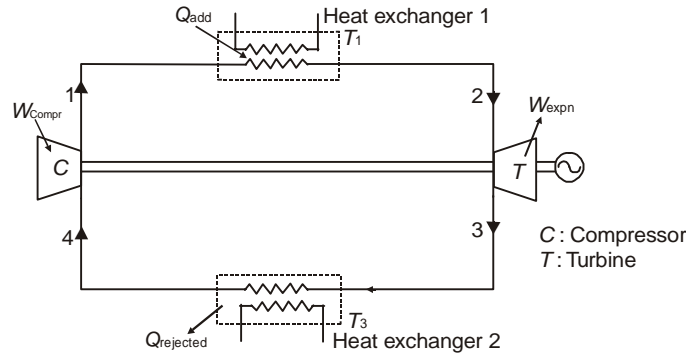


Fig. 4.11 Gas turbine plant: Carnot heat engine

For the perfect gas as working fluid in isothermal process no change in internal energy occurs, therefore $U_2 = U_1$

and

$$Q_{\text{add}} = W_{1-2}$$

Process 2–3 is reversible adiabatic expansion process which may be had inside cylinder with cylinder head being replaced by insulating type cylinder head so that complete arrangement is insulated and adiabatic expansion carried out.

During adiabatic expansion say work W_{expn} is available,

$$Q_{2-3} = 0$$

From first law of thermodynamics;

$$0 = (U_3 - U_2) + W_{\text{expn}}$$

or

$$W_{\text{expn}} = (U_2 - U_3)$$

Process 3–4 is reversible isothermal heat rejection for which cylinder head of insulating type may be replaced by conducting type as in 1–2 and heat (Q_{rejected}) be extracted out isothermally.

From first law of thermodynamics applied on process 3–4,

$$-Q_{\text{rejected}} = (U_4 - U_3) + (-W_{3-4})$$

for perfect gas internal energy shall remain constant during isothermal process. Thus, $U_3 = U_4$

$$-Q_{\text{rejected}} = -W_{3-4}$$

or

$$Q_{\text{rejected}} = W_{3-4}$$

Process 4–1 is the reversible adiabatic compression process with work requirement for compression. In the piston cylinder arrangement cylinder head of conducting type as used in 3–4 is replaced by insulating type, so that the whole arrangement becomes insulated and adiabatic compression may be realized,

From first law applied on process 4–1

For adiabatic process;

$$Q_{4-1} = 0$$

$$\Rightarrow 0 = (U_1 - U_4) + (-W_{\text{compr}})$$

or

$$W_{\text{compr}} = (U_1 - U_4)$$

Efficiency of reversible heat engine can be given as;

$$\eta_{\text{rev, HE}} = \frac{\text{Net work}}{\text{Heat supplied}}$$

$$\text{Here, Net work} = W_{\text{expn}} - W_{\text{compr}}$$

and heat is supplied only during process 1–2, therefore

$$\text{heat supplied} = Q_{\text{add}}$$

Substituting in the expression for efficiency.

$$\eta_{\text{rev, HE}} = \frac{W_{\text{expn}} - W_{\text{compr}}}{Q_{\text{add}}}$$

Also for a cycle

$$\sum_{\text{cycle}} W = \sum_{\text{cycle}} Q$$

$$W_{\text{net}} = Q_{\text{add}} - Q_{\text{rejected}}$$

so

Hence

$$\eta_{\text{rev, HE}} = 1 - \frac{Q_{\text{rejected}}}{Q_{\text{add}}}$$

As the heat addition takes place at high temperature, while heat rejection takes place at low temperature, so writing these heat interactions as Q_{high} , Q_{low} we get,

$$\eta_{\text{rev, HE}} = 1 - \frac{Q_{\text{low}}}{Q_{\text{high}}}$$

$$\eta_{\text{Carnot}} = 1 - \frac{Q_{\text{low}}}{Q_{\text{high}}}$$

Piston-cylinder arrangement shown and discussed for realizing Carnot cycle is not practically feasible as;

- (i) Frequent change of cylinder head i.e. of insulating type and diathermic type for adiabatic and isothermal processes is very difficult.
- (ii) Isothermal heat addition and isothermal-heat rejection are practically very difficult to be realized
- (iii) Reversible adiabatic expansion and compression are not possible.
- (iv) Even if near reversible isothermal heat addition and rejection is to be achieved then time duration for heat interaction should be very large i.e. infinitesimal heat interaction occurring at dead slow speed. Near reversible adiabatic processes can be achieved by making them to occur fast. In a piston-cylinder reciprocating engine arrangement such speed fluctuation in a single cycle is not possible.

Carnot heat engine arrangement is also shown with turbine, compressor and heat exchangers for adiabatic and isothermal processes. Fluid is compressed in compressor adiabatically, heated in heat exchanger at temperature T_1 , expanded in turbine adiabatically, cooled in heat exchanger at temperature T_3 and sent to compressor for compression. Here also following practical difficulties are confronted;

- (i) Reversible isothermal heat addition and rejection are not possible.
- (ii) Reversible adiabatic expansion and compression are not possible.

Carnot cycle can also operate reversibly as all processes constituting it are of reversible type. Reversed Carnot cycle is shown below;

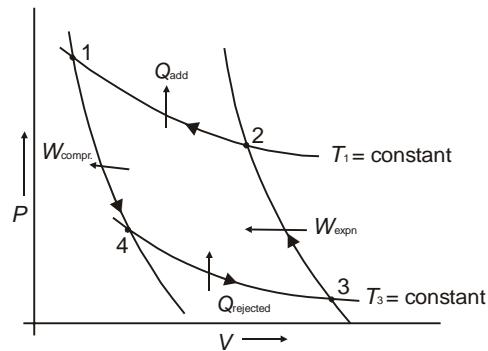


Fig. 4.12 Reversed Carnot cycle

Heat engine cycle in reversed form as shown above is used as ideal cycle for refrigeration and called “Carnot refrigeration cycle”.

4.9 CARNOT THEOREM AND ITS COROLLARIES

Carnot theorem states that “any engine cannot have efficiency more than that of reversible engine operating between same temperature limits.”

Different corollaries of Carnot theorem are,

- (i) Efficiency of all reversible engines operating between same temperature limits is same.
- (ii) Efficiency of a reversible engine does not depend on the working fluid in the cycle.

Using Clausius and Kelvin Planck statements, the Carnot theorem can be proved easily. Let us take two heat engines HE_I and HE_{II} operating between same temperature limits T_1 , T_2 of source and sink as shown in Fig. 4.13a.

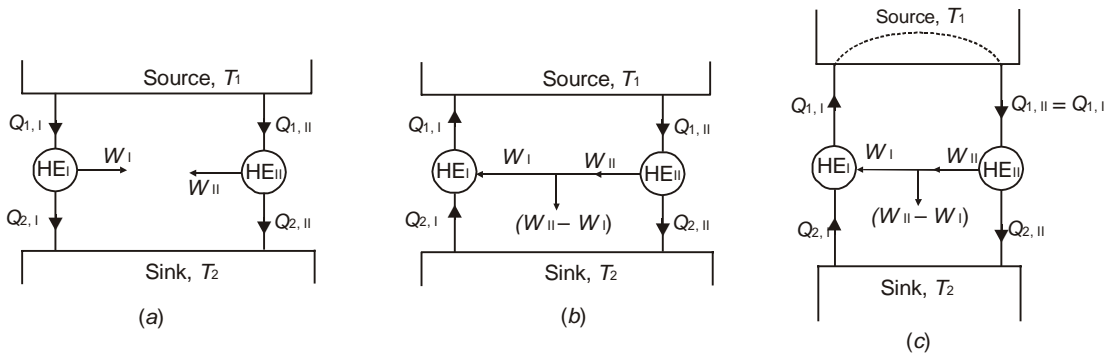


Fig. 4.13 Proof of Carnots theorem

Arrangement shown has heat engine, HE_I getting $Q_{1,I}$ from source, rejecting $Q_{2,I}$ and producing work W_I . Heat engine, HE_{II} receives $Q_{1,II}$, rejects $Q_{2,II}$ and produces work W_{II} .

$$W_I = Q_{1,I} - Q_{2,I}$$

$$W_{II} = Q_{1,II} - Q_{2,II}$$

Efficiency of engines HE_I , HE_{II}

$$\eta_{HE_I} = \frac{W_I}{Q_{1,I}}$$

$$\eta_{HE_{II}} = \frac{W_{II}}{Q_{I,II}}$$

Now let us assume that engine HE_I is reversible engine while HE_{II} is any engine.

As per Carnot's theorem efficiency of HE_I (reversible engine) is always more than that of HE_{II} . Let us start with violation of above statement, i.e., efficiency of HE_{II} is more than that of HE_I

$$\eta_{HE_I} < \eta_{HE_{II}}$$

or

$$\frac{W_I}{Q_{I,I}} < \frac{W_{II}}{Q_{I,II}}$$

Let us take the heat addition to each engine to be same i.e.

$$Q_{I,I} = Q_{I,II}$$

Hence

$$W_I < W_{II}$$

Also we have assumed that engine HE_I is of reversible type, so let us operate it in reversed manner, as shown in Figure 4.13b.

Let us also assume that the work requirement of reversed heat engine, HE_I be fed by work output W_{II} of the heat engine HE_{II} . Since W_{II} is more than W_I , a net work ($W_{II} - W_I$) shall also be available as output work after driving HE_I . Also since $Q_{I,I}$ and $Q_{I,II}$ are assumed to be same, the heat rejected by reversed HE_I may be supplied to heat engine, HE_{II} as shown in figure 4.13c by dotted lines. Thus, it results into an equivalent heat engine which produces net work ($W_{II} - W_I$) while heat interaction takes place with only one reservoir at temperature, T_2 . This is a violation of Kelvin Planck statement, so the assumption made in beginning that efficiency of reversible engine is less than that of other engine, is not correct.

Hence, it is established that out of all heat engines operating within same temperature limits, the reversible engine has highest efficiency.

Similarly for showing the correctness of corollaries of Carnot theorem the heat engines and their combinations be considered like above and proved using Kelvin-Planck and Clausius statements.

4.10 THERMODYNAMIC TEMPERATURE SCALE

After the Carnot's theorem and its corollary were stated and verified, it was thought to have a thermodynamic temperature scale, independent of thermometric substance and principles of thermometry. Such a temperature scale can be developed with the help of reversible heat engine concept and is called thermodynamic temperature scale. Defining thermodynamic temperature scale refers to the assigning of numerical values to different temperatures using reversible heat engines.

From the previous discussions on heat engines it is obvious that the efficiency of a reversible heat engine depends on the temperatures of reservoir with which heat interaction takes place.

Mathematically, it can be easily given by any unknown function 'f';

$$\eta_{rev, HE} = f(T_{high}, T_{low})$$

where T_{high} and T_{low} are the two temperatures of high temperature source and low temperature sink.

or

$$\eta_{rev, HE} = 1 - \frac{Q_{low}}{Q_{high}} = f(T_{high}, T_{low})$$

Unknown function 'f' may be substituted by another unknown function, say ϕ

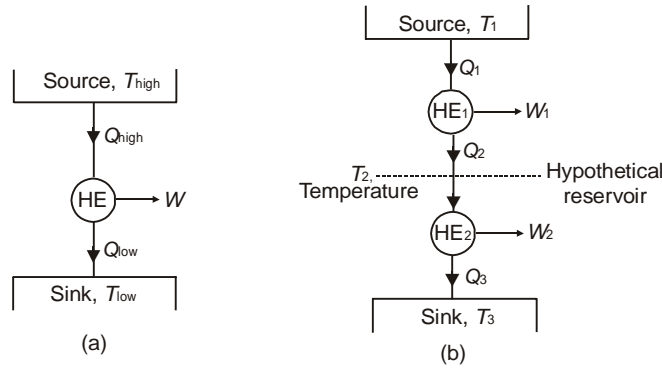


Fig. 4.14 Reversible heat engine and its combinations

$$\frac{Q_{\text{high}}}{Q_{\text{low}}} = \phi(T_{\text{high}}, T_{\text{low}})$$

Thus, some functional relationship as defined by ‘ ϕ ’ is established between heat interactions and temperatures.

Let us now have more than one reversible heat engines operating in series as shown in Figure 4.14b, between source and sink having T_1 and T_3 temperatures. In between an imaginary reservoir at temperature T_2 may be considered.

From the above for two reversible heat engines;

$$\frac{Q_1}{Q_2} = \phi(T_1, T_2)$$

and

$$\frac{Q_2}{Q_3} = \phi(T_2, T_3)$$

Combination of two heat engines may be given as shown here,

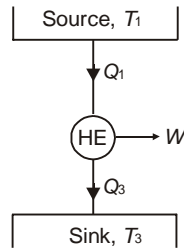


Fig. 4.15 Equivalent heat engine for two reversible heat engines operating in series.

$$\frac{Q_1}{Q_3} = \frac{Q_1/Q_2}{Q_2/Q_3}$$

or

$$\frac{Q_1}{Q_3} = \phi(T_1, T_3)$$

$$\frac{Q_1}{Q_3} = \frac{Q_1}{Q_2} \cdot \frac{Q_2}{Q_3}$$

or

$$\phi(T_1, T_3) = \phi(T_1, T_2) \cdot \phi(T_2, T_3)$$

Above functional relation is possible only if it is given by another function ψ as follows.

$$\phi(T_1, T_2) = \frac{\psi(T_1)}{\psi(T_2)}$$

$$\phi(T_2, T_3) = \frac{\psi(T_2)}{\psi(T_3)}$$

$$\phi(T_1, T_3) = \frac{\psi(T_1)}{\psi(T_3)}$$

Thus,

$$\frac{Q_1}{Q_2} = \frac{\psi(T_1)}{\psi(T_2)}$$

$$\frac{Q_2}{Q_3} = \frac{\psi(T_2)}{\psi(T_3)}$$

$$\frac{Q_1}{Q_3} = \frac{\psi(T_1)}{\psi(T_3)}$$

Lord Kelvin based upon his observations proposed that the function $\psi(T)$ can be arbitrarily chosen based on Kelvin scale or absolute thermodynamic temperature scale as;

$$\psi(T) = \text{Temperature } T \text{ in Kelvin Scale}$$

Therefore,

$$\frac{Q_1}{Q_2} = \frac{\psi(T_1)}{\psi(T_2)} = \frac{T_1}{T_2}$$

$$\frac{Q_2}{Q_3} = \frac{\psi(T_2)}{\psi(T_3)} = \frac{T_2}{T_3}$$

$$\frac{Q_1}{Q_3} = \frac{\psi(T_1)}{\psi(T_3)} = \frac{T_1}{T_3}$$

where T_1, T_2, T_3 are temperatures in *absolute thermodynamic scale*.

Here heat absorbed and heat rejected is directly proportional to temperatures of reservoirs supplying and accepting heats to heat engine. For a Carnot heat engine or reversible heat engine operating between reservoirs at temperature T and triple point of water, T_t ;

$$\frac{Q}{Q_t} = \frac{T}{T_t} = \frac{T}{273.16}$$

or

$$T = 273.16 \cdot \frac{Q}{Q_t}$$

Here for a known Q and Q_t values the temperature T can be defined. Thus, heat interaction acts as thermometric property in thermodynamic temperature scale, which is independent of thermometric substance. It may be noted that negative temperatures cannot exist on thermodynamic temperature scale.

Let us now have a large number of reversible heat engines (Carnot engines) operating in series as shown in Figure 4.16.

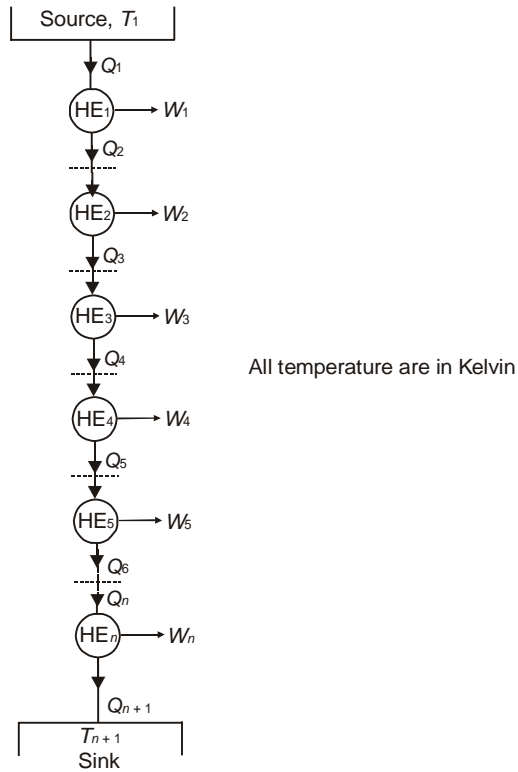


Fig. 4.16 Series of reversible heat engines

From thermodynamic temperature scale for different engines,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q_2}{Q_3} = \frac{T_2}{T_3}$$

$$\frac{Q_3}{Q_4} = \frac{T_3}{T_4}$$

For n th engine

$$\frac{Q_n}{Q_{n+1}} = \frac{T_n}{T_{n+1}}$$

Here work output from each engine shall continuously diminish the heat supplied to subsequent heat engine. Let us assume work outputs from 'n' engines to be same; i.e.

$$W_1 = W_2 = W_3 = W_4 = \dots = W_n$$

$$\text{or } (Q_1 - Q_2) = (Q_2 - Q_3) = (Q_3 - Q_4) = \dots = (Q_n - Q_{n+1})$$

$$\text{or } (T_1 - T_2) = (T_2 - T_3) = (T_3 - T_4) = \dots = (T_n - T_{n+1})$$

It is obvious that for a large number of heat engines the heat rejected by n th engine shall be negligible i.e. for very large value of n , $Q_{n+1} \rightarrow 0$

or for $\lim n \rightarrow \infty$, $\lim Q_{n+1} \rightarrow 0$

Thus, from thermodynamic temperature scale when heat rejection approaches zero, the temperature of heat rejection also tends to zero as a limiting case. But in such a situation when heat rejection is zero, the heat engine takes form of a perpetual motion machine of 2nd kind, where work is produced with only heat supplied to it. Thus, it leads to violation of Kelvin-Planck statement. Hence it is not possible.

Also it can be said that "it is impossible to attain absolute zero temperature in finite number of operations." There exists absolute zero temperature on thermodynamic temperature scale, but cannot be attained without violation of second law of thermodynamics. This fact is popularly explained by *third law of thermodynamics*.

Carnot cycle efficiency can now be precisely defined as function of source and sink temperatures.

$$\eta_{\text{carnot}} = 1 - \frac{Q_{\text{low}}}{Q_{\text{high}}}$$

$$\eta_{\text{carnot}} = 1 - \frac{T_{\text{low}}}{T_{\text{high}}}$$

Thus, it is seen that Carnot cycle efficiency depends only upon lower and higher temperatures. Carnot cycle efficiency is high for small values of sink temperature (T_{low}) and larger values of source temperature (T_{high}).

Therefore for maximum efficiency, Carnot cycle must operate between maximum possible source and minimum possible sink temperatures.

EXAMPLES

1. Using IInd law of thermodynamics show that the following are irreversible

- (i) Free expansion.
- (ii) Heat transfer through finite temperature difference.

Solution:

- (i) Let us consider a perfectly insulated tank having two compartments divided by thin wall. Compartment I has gas while II has vacuum. When wall is punctured then gas in I expands till pressure in I and II gets equalised. Let us assume that free expansion is reversible i.e. the gas in II returns into I and original states are restored.

When gas is allowed to expand, say it produces work W from a device D due to expansion. This work W is available due to change in internal energy of gas. Internal energy of gas can be restored by adding equivalent heat Q to it from a source as shown. This whole arrangement if consolidated can be

treated as a device which is producing work while exchanging heat with single body. Thus, it is violation of IInd law of thermodynamics, therefore the assumption that free expansion is reversible is incorrect. Free expansion is irreversible.

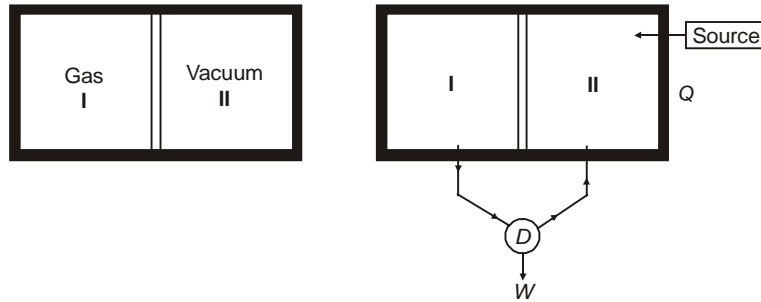


Fig. 4.17 Free expansion

- (ii) For showing that the heat transfer through finite temperature difference is irreversible, let us start with the fact that such heat transfer is reversible. Let us take a heat source (T_1) and sink (T_2) and assume that a heat Q_{1-2} flows from T_1 to T_2 .

Let us have a heat engine operating between T_1 and T_2 as shown and producing work W . Let us reverse heat transfer process from T_2 to T_1 i.e. Q_{2-1} , as assumed. Let us assume $Q_2 = Q_{2-1}$.

This assumption paves the way for eliminating sink. Let us now remove sink and directly supply Q_2 as Q_{2-1} ($= Q_2$). This results in formation of a heat engine which produces work while exchanging heat with single reservoir, the violation of IInd law of thermodynamics. (Kelvin Planck statement).

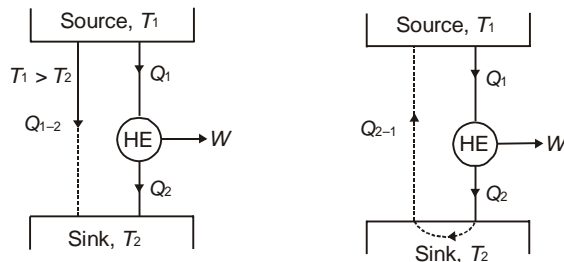


Fig. 4.18 Heat transfer through a finite temperature difference

Hence, assumption that heat transfer through finite temperature is reversible, stands incorrect. Therefore, heat transfer through finite temperature difference is irreversible.

2. Determine the heat to be supplied to a Carnot engine operating between 400°C and 15°C and producing 200 kJ of work.

Solution :

To find out

$$Q_1 = ?$$

In Carnot engine from thermodynamic temperature scale;

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

and work

$$W = Q_1 - Q_2$$

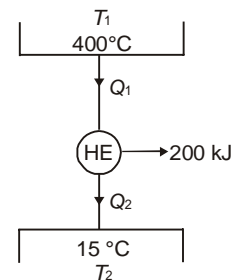


Fig. 4.19

$$\text{Thus } \frac{Q_1}{Q_2} = \frac{673}{288} \quad (1)$$

$$\text{and } Q_1 - Q_2 = 200 \text{ kJ} \quad (2)$$

From equations 1 and 2, upon solving

$$Q_1 = 349.6 \text{ kJ}$$

$$\text{and } Q_2 = 149.6 \text{ kJ}$$

Heat to be supplied = **349.6 kJ Ans.**

3. A refrigerator operates on reversed Carnot cycle. Determine the power required to drive refrigerator between temperatures of 42°C and 4°C if heat at the rate of 2 kJ/s is extracted from the low temperature region.

Solution:

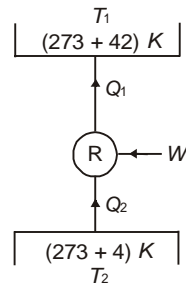


Fig. 4.20

To find out,

$$W = ?$$

Given :

$$T_1 = 315 \text{ K}, T_2 = 277 \text{ K}$$

and

$$Q_2 = 2 \text{ kJ/s}$$

From thermodynamic temperature scale;

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\text{or } \frac{Q_1}{2} = \frac{315}{277}$$

$$\text{or } Q_1 = 2.274 \text{ kJ/s}$$

$$\begin{aligned} \text{Power/Work input required} &= Q_1 - Q_2 \\ &= 2.274 - 2 \end{aligned}$$

$$\text{Power required} = 0.274 \text{ kJ/s}$$

Power required for driving refrigerator = **0.274 kW Ans.**

4. A reversible heat engine operates between two reservoirs at 827°C and 27°C . Engine drives a Carnot refrigerator maintaining -13°C and rejecting heat to reservoir at 27°C . Heat input to the engine is 2000 kJ and the net work available is 300 kJ . How much heat is transferred to refrigerant and total heat rejected to reservoir at 27°C ?

Solution:

Block diagram based on the arrangement stated;

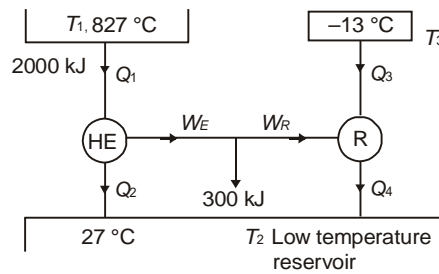


Fig. 4.21

We can write, for heat engine,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q_1}{Q_2} = \frac{1100}{300}$$

Substituting $Q_1 = 2000$ kJ, we get $Q_2 = 545.45$ kJ

Also

$$W_E = Q_1 - Q_2 = 1454.55 \text{ kJ}$$

For refrigerator,

$$\frac{Q_3}{Q_4} = \frac{260}{300} \quad (1)$$

Also,

$$W_R = Q_4 - Q_3 \quad (2)$$

and

$$W_E - W_R = 300$$

or

$$W_R = 1154.55 \text{ kJ}$$

Equations (1) & (2) result in,

$$Q_4 - Q_3 = 1154.55 \quad (3)$$

From equations (1) & (3),

$$Q_3 = 7504.58 \text{ kJ}$$

$$Q_4 = 8659.13 \text{ kJ}$$

Total heat transferred to low temperature reservoir

$$= Q_2 + Q_4 = 9204.68 \text{ kJ}$$

$$\left. \begin{array}{l} \text{Heat transferred to refrigerant} = 7504.58 \text{ kJ} \\ \text{Total heat transferred to low temperature reservoir} = 9204.68 \text{ kJ} \end{array} \right\} \text{Ans.}$$

5. In a winter season when outside temperature is -1°C , the inside of house is to be maintained at 25°C . Estimate the minimum power required to run the heat pump of maintaining the temperature. Assume heating load as 125 MJ/h .

Solution:

$$COP_{HP} = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{1}{\left(1 - \frac{Q_2}{Q_1}\right)}$$

Also we know

Thus
$$\frac{Q_1}{Q_2} = \frac{298.15}{272.15}$$

Also
$$COP_{HP} = 11.47$$

therefore
$$COP_{HP} = \frac{Q_1}{W}, \text{ Substituting } Q_1$$

or,
$$W = 10.89 \text{ MJ/h}$$

Minimum power required = **3.02 kW Ans.**

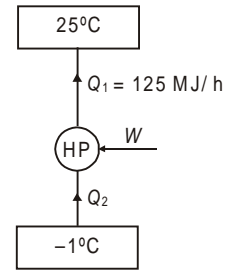


Fig. 4.22

6. A cold storage plant of 40 tonnes of refrigeration capacity runs with its performance just $\frac{1}{4}$ th of its Carnot COP. Inside temperature is -15°C and atmospheric temperature is 35°C . Determine the power required to run the plant. [Take : One ton of refrigeration as 3.52 kW]

Solution:

Cold storage plant can be considered as a refrigerator operating between given temperatures limits.

Capacity of plant = Heat to be extracted = 140.8 kW

$$\begin{aligned} \text{Carnot COP of plant} &= \frac{1}{\left(\frac{308}{258.15}\right) - 1} \\ &= 5.18 \end{aligned}$$

Actual
$$COP = \frac{5.18}{4} = 1.295$$

Also actual
$$COP = \frac{Q_2}{W}, \text{ hence } W = 108.73 \text{ kW.}$$

Power required = **108.73 kW Ans.**

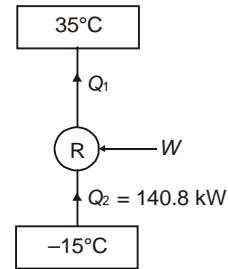


Fig. 4.23

7. What would be maximum efficiency of engine that can be had between the temperatures of 1150°C and 27°C ?

Solution:

Highest efficiency is that of Carnot engine, so let us find the Carnot cycle efficiency for given temperature limits.

$$\begin{aligned} \eta &= 1 - \left(\frac{273 + 27}{273 + 1150} \right) \\ \eta &= \mathbf{0.7891 \text{ or } 78.91\% \text{ Ans.}} \end{aligned}$$

8. A domestic refrigerator maintains temperature of -8°C when the atmospheric air temperature is 27°C . Assuming the leakage of 7.5 kJ/min from outside to refrigerator determine power required to run this refrigerator. Consider refrigerator as Carnot refrigerator.

Solution:

Here heat to be removed continuously from refrigerated space shall be 7.5 kJ/min or 0.125 kJ/s.

For refrigerator, C.O.P. shall be,

$$\frac{0.125}{W} = \frac{265}{(300 - 265)}$$

or

$$W = 0.0165 \text{ kJ/s.}$$

Power required = **0.0165 kW Ans.**

9. Three reversible engines of Carnot type are operating in series as shown between the limiting temperatures of 1100 K and 300 K. Determine the intermediate temperatures if the work output from engines is in proportion of 3 : 2 : 1.

Solution:Here, $W_1 : W_2 : W_3 = 3 : 2 : 1$ Efficiency of engine, HE_1 ,

$$\frac{W_1}{Q_1} = \left(1 - \frac{T_2}{1100}\right) \Rightarrow Q_1 = \frac{1100.W_1}{(1100 - T_2)}$$

for HE_2 engine,

$$\frac{W_2}{Q_2} = \left(1 - \frac{T_3}{T_2}\right)$$

for HE_3 engine,

$$\frac{W_3}{Q_3} = \left(1 - \frac{300}{T_3}\right)$$

From energy balance on engine, HE_1

$$Q_1 = W_1 + Q_2 \Rightarrow Q_2 = Q_1 - W_1$$

Above gives,

$$Q_1 = \left\{ \frac{1100 W_1}{(1100 - T_2)} - W_1 \right\} = W_1 \left\{ \frac{T_2}{1100 - T_2} \right\}$$

Substituting Q_2 in efficiency of HE_2

$$\frac{W_2}{W_1 \left(\frac{T_2}{1100 - T_2} \right)} = \left(1 - \frac{T_3}{T_2}\right)$$

or

$$\frac{W_2}{W_1} = \left(\frac{T_2}{1100 - T_2} \right) \left(\frac{T_2 - T_3}{T_2} \right) = \left(\frac{T_2 - T_3}{1100 - T_2} \right)$$

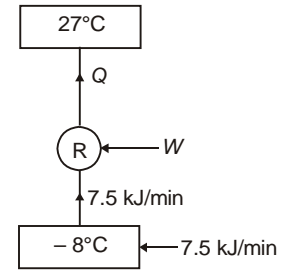
or

$$\left\{ \frac{2}{3} = \left(\frac{T_2 - T_3}{1100 - T_2} \right) \right\}$$

or

$$2200 - 2T_2 = 3T_2 - 3T_3$$

$$\boxed{5T_2 - 3T_3 = 2200}$$

**Fig. 4.23**

Energy balance on engine HE_2 gives,

$$Q_2 = W_2 + Q_3$$

Substituting in efficiency of HE_2 ,

$$\frac{W_2}{(W_2 + Q_3)} = \left(\frac{T_2 - T_3}{T_2} \right)$$

or

$$W_2 \cdot T_2 = (W_2 + Q_3) (T_2 - T_3)$$

or

$$Q_3 = \frac{W_2 T_3}{(T_2 - T_3)}$$

Substituting Q_3 in efficiency of HE_3 ,

$$\frac{W_3}{\left(\frac{W_2 T_3}{T_2 - T_3} \right)} = \frac{T_3 - 300}{T_3}$$

$$\frac{W_3}{W_2} = \left(\frac{T_3}{T_2 - T_3} \right) \left(\frac{T_3 - 300}{T_3} \right)$$

$$\frac{1}{2} = \frac{T_3 - 300}{T_2 - T_3}$$

$$3T_3 - T_2 = 600$$

Solving, equations of T_2 and T_3 , $T_3 = 433.33$ K

$$T_2 = 700 \text{ K}$$

Intermediate temperatures: **700 K** and **433.33 K** Ans.

10. A Carnot engine getting heat at 800 K is used to drive a Carnot refrigerator maintaining 280 K temperature. Both engine and refrigerator reject heat at some temperature, T , when heat given to engine is equal to heat absorbed by refrigerator. Determine efficiency of engine and C.O.P. of refrigerator.

Solution:

Efficiency of engine,

$$\frac{W}{Q_1} = \left(\frac{800 - T}{800} \right)$$

For refrigerator, COP

$$\frac{Q_3}{W} = \frac{280}{(T - 280)}$$

It is given that
so, from engine

$$Q_1 = Q_3 = Q$$

$$\frac{W}{Q} = \left(\frac{800 - T}{800} \right)$$

From refrigerator,

$$\frac{Q}{W} = \frac{280}{T - 280}$$

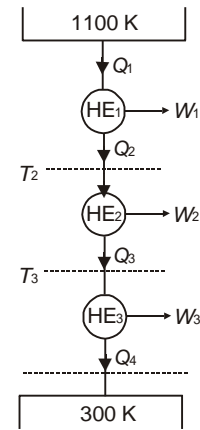


Fig. 4.25

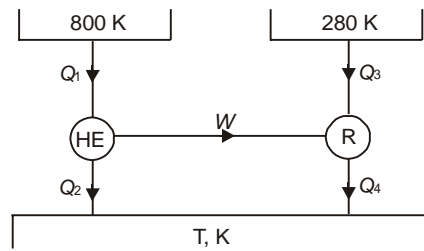


Fig. 4.26

From above two $\left(\frac{Q}{W}\right)$ may be equated,

$$\frac{T - 280}{280} = \frac{800 - T}{800}$$

Temperature, $T = 414.8 \text{ K}$

$$\text{Efficiency of engine} = \left(\frac{800 - 414.8}{800}\right) = \mathbf{0.4815 \text{ Ans.}}$$

$$\text{C.O.P. of refrigerator} = \left(\frac{280}{414.8 - 280}\right) = \mathbf{2.077 \text{ Ans.}}$$

II. 0.5 kg of air executes a Carnot power cycle having a thermal efficiency of 50%. The heat transfer to the air during isothermal expansion is 40 kJ. At the beginning of the isothermal expansion the pressure is 7 bar and the volume is 0.12 m^3 . Determine the maximum and minimum temperatures for the cycle in Kelvin, the volume at the end of isothermal expansion in m^3 , and the work and heat transfer for each of the four processes in kJ. For air $c_p = 1.008 \text{ kJ/kg} \cdot \text{K}$, $c_v = 0.721 \text{ kJ/kg} \cdot \text{K}$. [U.P.S.C. 1993]

Solution:

Given :

$$\eta_{\text{Carnot}} = 0.5, m = 0.5 \text{ kg}$$

$$P_2 = 7 \text{ bar}, V_2 = 0.12 \text{ m}^3$$

Let thermodynamic properties be denoted with respect to salient states;

$$\text{Carnot efficiency } \eta_{\text{Carnot}} = 1 - \frac{T_1}{T_2}$$

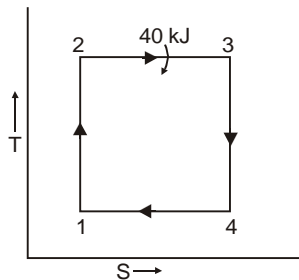


Fig. 4.27

or, $\frac{T_1}{T_2} = 0.5$

or, $T_2 = 2T_1$

Corresponding to state 2, $P_2 V_2 = mRT_2$
 $7 \times 10^5 \times 0.12 = 0.5 \times 287 \times T_2$
 $T_2 = 585.36 \text{ K}$

Heat transferred during process 2-3 (isothermal expansion), $Q_{23} = 40 \text{ kJ}$

$$Q_{23} = W_{23} = P_2 V_2 \ln \left(\frac{V_3}{V_2} \right)$$

$$40 = mRT_2 \ln \left(\frac{V_3}{V_2} \right) = 0.5 \times 0.287 \times 585.36 \ln \left(\frac{V_3}{0.12} \right)$$

$$V_3 = 0.1932 \text{ m}^3$$

Temperature at state 1, $T_1 = \frac{T_2}{2}$

$$T_1 = 292.68 \text{ K}$$

During process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$$\gamma = \frac{c_p}{c_v} = \frac{1.008}{0.721}, \quad \gamma = 1.398$$

Thus,

$$P_1 = 0.613 \text{ bar}$$

$$P_1 V_1 = mRT_1$$

$$0.613 \times 10^5 \times V_1 = 0.5 \times 287 \times 292.68$$

$$V_1 = 6.85 \times 10^{-4} \text{ m}^3$$

Heat transferred during process 4-1 (isothermal compression) shall be equal to the heat transferred during process 2-3 (isothermal expansion).

For isentropic process, $dQ = 0$

$$dW = dU$$

During process 1-2, isentropic process, $W_{12} = -mc_v (T_2 - T_1)$

$$Q_{12} = 0, W_{12} = -0.5 \times 0.721 (585.36 - 292.68)$$

$$W_{12} = -105.51 \text{ kJ, (-ve work)}$$

During process 3-4, isentropic process, $W_{34} = -mc_v (T_4 - T_3)$

$$Q_{34} = 0, W_{34} = +0.5 \times 0.721 \times (585.36 - 292.68)$$

$$W_{34} = +105.51 \text{ kJ (+ve work)}$$

Ans.

Process	Heat transfer	Work interaction
1-2	0	-105.51, kJ
2-3	40 kJ	40 kJ
3-4	0	+105.51, kJ
4-1	-40 kJ	-40 kJ

Maximum temperature of cycle = 585.36 K
 Minimum temperature of cycle = 292.68 K
 Volume at the end of isothermal expansion = 0.1932 m³

12. A reversible engine as shown in figure during a cycle of operation draws 5 mJ from the 400 K reservoir and does 840 kJ of work. Find the amount and direction of heat interaction with other reservoirs. [U.P.S.C. 1999]

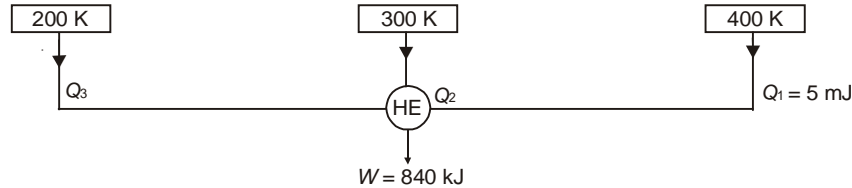


Fig. 4.28

Solution:

Let us assume that heat engine rejects Q_2 and Q_3 heat to reservoir at 300 K and 200 K respectively. Let us assume that there are two heat engines operating between 400 K and 300 K temperature reservoirs and between 400 K and 200 K temperature reservoirs. Let each heat engine receive Q'_1 and Q''_1 from reservoir at 400 K as shown below:

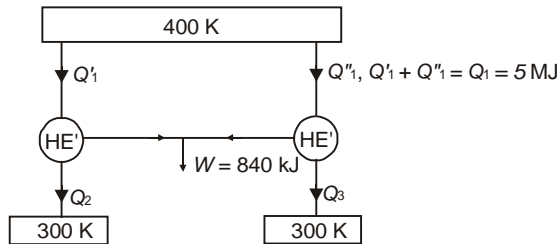


Fig. 4.29 Assumed arrangement

Thus,

$$Q'_1 + Q''_1 = Q_1 = 5 \times 10^3 \text{ kJ}$$

also,

$$\frac{Q'_1}{Q_2} = \frac{400}{300}, \text{ or, } Q'_1 = \frac{4}{3} Q_2$$

and

$$\frac{Q''_1}{Q_3} = \frac{400}{200} \text{ or, } Q''_1 = 2Q_3$$

Substituting Q'_1 and Q''_1

$$\frac{4}{3} Q_2 + 2Q_3 = 5000$$

Also from total work output, $Q'_1 + Q''_1 - Q_2 - Q_3 = W$

$$5000 - Q_2 - Q_3 = 840$$

$$Q_2 + Q_3 = 4160$$

$$Q_3 = 4160 - Q_2$$

Substituting Q_3 , $\frac{4}{3} Q_2 + 2(4160 - Q_2) = 5000$

$$\frac{4}{3} Q_2 - 2 Q_2 = 5000 - 8320$$

$$\frac{-2Q_2}{3} = -3320$$

$$Q_2 = 4980 \text{ kJ}$$

and

$$Q_3 = -820 \text{ kJ}$$

Negative sign with Q_3 shows that the assumed direction of heat Q_3 is not correct and actually Q_3 heat will flow from reservoir to engine. Actual sign of heat transfers and magnitudes are as under:

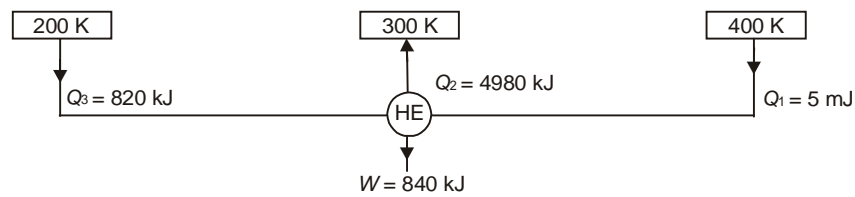


Fig 4.30

$Q_2 = 4980 \text{ kJ}$, from heat engine
 $Q_3 = 820 \text{ kJ}$, to heat engine

Ans.

13. A heat pump working on a reversed Carnot cycle takes in energy from a reservoir maintained at 3°C and delivers it to another reservoir where temperature is 77°C . The heat pump drives power for its operation from a reversible engine operating within the higher and lower temperature limits of 1077°C and 77°C . For 100 kJ/s of energy supplied to the reservoir at 77°C , estimate the energy taken from the reservoir at 1077°C . [U.P.S.C. 1994]

Solution:

Arrangement for heat pump and heat engine operating together is shown here. Engine and pump both reject heat to the reservoir at 77°C (350 K).

For heat engine.

$$\eta_E = 1 - \frac{350}{1350} = \frac{W}{Q_1}$$

$$0.7407 = \frac{Q_1 - Q_2}{Q_1}$$

$$0.7407 = 1 - \frac{Q_2}{Q_1}$$

$$Q_2 = 0.2593 Q_1$$

For heat pump

$$COP_{HP} = \frac{Q_4}{Q_4 - Q_3}$$

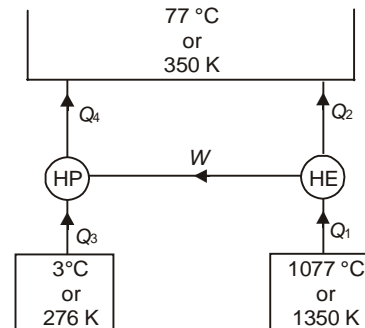


Fig. 4.31

$$\text{COP}_{\text{HP}} = \frac{350}{350 - 276} = \frac{Q_4}{Q_4 - Q_3}$$

\Rightarrow

$$Q_4 = 1.27 Q_3$$

Work output from engine = Work input to pump

$$Q_1 - Q_2 = Q_4 - Q_3 \Rightarrow Q_1 - 0.2593 Q_1 = Q_4 - \frac{Q_4}{1.27}$$

Also it is given that $Q_2 + Q_4 = 100$

Substituting Q_2 and Q_4 as function of Q_1 in following expression,

$$Q_2 + Q_4 = 100$$

$$0.2593 Q_1 + \frac{Q_1}{0.287} = 100$$

$$Q_1 = 26.71 \text{ kJ}$$

Energy taken by engine from reservoir at 1077°C
= **26.71 kJ**

Ans.

14. A reversible engine is used for only driving a reversible refrigerator. Engine is supplied 2000 kJ/s heat from a source at 1500 K and rejects some energy to a low temperature sink. Refrigerator is desired to maintain the temperature of 15°C while rejecting heat to the same low temperature sink. Determine the temperature of sink if total 3000 kJ/s heat is received by the sink.

Solution:

Let temperature of sink be T_{sink} K.

Given: $Q_{\text{sink, HE}} + Q_{\text{sink, R}} = 3000 \text{ kJ/s}$

Since complete work output from engine is used to run refrigerator so,

$$\begin{aligned} 2000 - Q_{\text{sink, HE}} &= Q_{\text{sink, R}} - Q_R \\ Q_R &= 3000 - 2000 = 1000 \text{ kJ/s} \end{aligned}$$

Also for engine,

$$\frac{2000}{1500} = \frac{Q_{\text{sink, HE}}}{T_{\text{sink}}} \Rightarrow Q_{\text{sink, HE}} = \frac{4}{3} T_{\text{sink}}$$

For refrigerator,

$$\frac{Q_R}{288} = \frac{Q_{\text{sink, R}}}{T_{\text{sink}}} \Rightarrow Q_{\text{sink, R}} = \frac{1000 T_{\text{sink}}}{288}$$

Substituting $Q_{\text{sink, HE}}$ and $Q_{\text{sink, R}}$ values.

$$\frac{4}{3} T_{\text{sink}} + \frac{1000 T_{\text{sink}}}{288} = 3000$$

\Rightarrow

$$T_{\text{sink}} = 624.28 \text{ K}$$

Temperature of sink = **351.28°C Ans.**

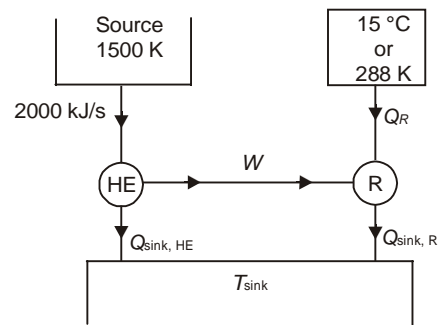


Fig. 4.32

15. A reversible heat engine runs between 500°C and 200°C temperature reservoirs. This heat engine is used to drive an auxiliary and a reversible heat pump which runs between reservoir at 200°C and the body at 450°C. The auxiliary consumes one third of the engine output and remaining is consumed for driving heat pump. Determine the heat rejected to the body at 450°C as fraction of heat supplied by reservoir at 500°C.

Solution:

Let the output of heat engine be W . So $\frac{W}{3}$ is consumed for driving auxiliary and remaining $\frac{2W}{3}$ is consumed for driving heat pump for heat engine,

$$\eta = \frac{W}{Q_1} = 1 - \frac{473}{773}$$

$$\frac{W}{Q_1} = 0.3881$$

$$\text{COP of heat pump} = \frac{723}{723 - 473} = \frac{Q_3}{2W/3}$$

$$\Rightarrow 2.892 = \frac{3Q_3}{2W}$$

Substituting W ,

$$\frac{Q_3}{Q_1} = 0.7482$$

Ratio of heat rejected to body at 450°C to the heat supplied by the reservoir = **0.7482** Ans.

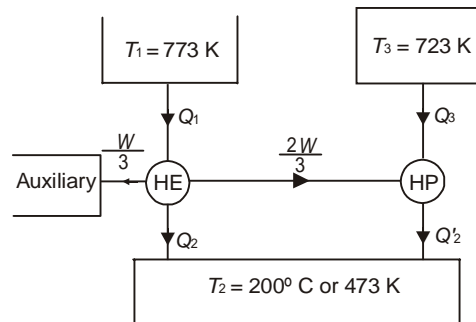


Fig. 4.33

16. A reversible heat engine operates between a hot reservoir at T_1 and a radiating surface at T_2 . Heat radiated from the surface is proportional to the surface area and temperature of surface raised to power 4. Determine the condition for minimum surface area for a given work output.

Solution:

Heat rejected = Heat radiated from surface at $T_2 = K \cdot A \cdot T_2^4$, where A is surface area and K is proportionality constant.

$$\frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$$

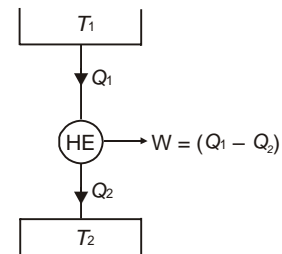


Fig. 4.34

$$\frac{K \cdot A \cdot T_2^4}{W} = \frac{T_2}{T_1 - T_2}$$

$$\Rightarrow A = \frac{W}{T_2^3 (T_1 - T_2) \cdot K}$$

In order to have minimum surface area the denominator in above expression of A should be maximum i.e. $T_2^3 (T_1 - T_2)$ should be maximum. Differentiating with respect to T_2 .

$$\frac{d}{dT_2} \{ (T_2^3 (T_1 - T_2)) \} = 0$$

$$3T_1 \cdot T_2^2 - 4T_2^3 = 0$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{3}{4} \Rightarrow T_2 = T_1 \cdot \frac{3}{4}$$

Taking second differential

$$\frac{d^2}{dT_2^2} \{ T_2^3 \cdot (T_1 - T_2) \} = 6T_1 \cdot T_2 - 12T_2^2$$

Upon substitution it is -ive so $\left(\frac{T_2}{T_1} = \frac{3}{4} \right)$ is the condition for $\{ T_2^3 (T_1 - T_2) \}$ to be maximum and so the minimum surface area

$$\boxed{\frac{T_2}{T_1} = \frac{3}{4}} \text{ Ans.}$$

17. A cold body is to be maintained at low temperature T_2 when the temperature of surrounding is T_3 . A source is available at high temperature T_1 . Obtain the expression for minimum theoretical ratio of heat supplied from source to heat absorbed from cold body.

Solution:

Let us consider a refrigerator for maintaining cold body and also a reversible heat engine for driving refrigerator to operate together, Fig 4.35.

To obtain; $\frac{Q_1}{Q_3}$

$$\text{For heat engine, } \frac{W}{Q_1} = \frac{T_1 - T_3}{T_1}$$

$$\text{For refrigerator, } \frac{Q_3}{W} = \frac{T_2}{T_3 - T_2}$$

Combining the above two:

$$\frac{Q_1}{Q_3} = \frac{T_1 \times (T_3 - T_2)}{T_2 \times (T_1 - T_3)}$$

$$\boxed{\text{Ratio of heat supplied from source to heat absorbed from cold body} = \frac{T_1 \cdot (T_3 - T_2)}{T_2 \cdot (T_1 - T_3)}} \text{ Ans.}$$

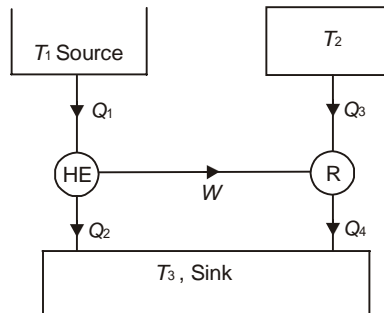


Fig. 4.35

18. A heat pump is run by a reversible heat engine operating between reservoirs at 800°C and 50°C . The heat pump working on Carnot cycle picks up 15 kW heat from reservoir at 10°C and delivers it to a reservoir at 50°C . The reversible engine also runs a machine that needs 25 kW . Determine the heat received from highest temperature reservoir and heat rejected to reservoir at 50°C .

Solution:

Schematic arrangement for the problem is given in figure.

For heat engine,

$$\eta_{HE} = \frac{W_{HE}}{Q_1} = 1 - \frac{323}{1173}$$

\Rightarrow

$$\frac{W_{HE}}{Q_1} = 0.7246$$

For heat pump,

$$W_{HP} = Q_4 - Q_3 = Q_4 - 15$$

$$COP = \frac{T_4}{T_4 - T_3} = \frac{Q_4}{Q_4 - Q_3}$$

\Rightarrow

$$\frac{323}{(323 - 283)} = \frac{Q_4}{Q_4 - 15}$$

\Rightarrow

$$Q_4 = 17.12\text{ kW}$$

\Rightarrow

$$W_{HP} = 17.12 - 15 = 2.12\text{ kW}$$

Since,

$$W_{HE} = W_{HP} + 25$$

\Rightarrow

$$W_{HE} = 27.12\text{ kW}$$

$$\eta_{HE} = 0.7246 = \frac{W_{HE}}{Q_1}$$

\Rightarrow

$$Q_1 = 37.427\text{ kW}$$

\Rightarrow

$$Q_2 = Q_1 - W_{HE}$$

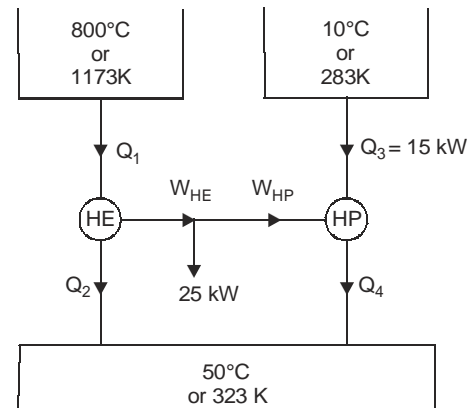


Fig. 4.36

$$= 37.427 - 27.12$$

$$Q_2 = 10.307 \text{ kW}$$

Hence heat rejected to reservoir at 50°C

$$= Q_2 + Q_4$$

$$\Rightarrow = 10.307 + 17.12$$

$$= \mathbf{27.427 \text{ kW Ans.}}$$

Heat received from highest temperature reservoir = 37.427 kW Ans.

19. Two insulated tanks are connected through a pipe with closed valve in between. Initially one tank having volume of 1.8 m^3 has argon gas at 12 bar, 40°C and other tank having volume of 3.6 m^3 is completely empty. Subsequently valve is opened and the argon pressure gets equalized in two tanks. Determine, (a) the final pressure & temperature (b) the change of enthalpy and (c) the work done considering argon as perfect gas and gas constant as 0.208 kJ/kg. K

Solution:

$$\text{Total volume, } V = V_1 + V_2 = 5.4 \text{ m}^3$$

$$\text{By perfect gas law, } p_1 V_1 = mRT_1$$

$$12 \times 10^2 \times 1.8 = m \times 0.208 \times 313$$

$$\Rightarrow m = 33.18 \text{ kg}$$

By gas law for initial and final state,

$$p_1 V_1 = p_{\text{final}} \times V_{\text{final}}$$

$$12 \times 10^2 \times 1.8 = p_{\text{final}} \times 5.4$$

Final pressure $\Rightarrow p_{\text{final}} = 400 \text{ kPa}$ or 4 bar **Ans.**

Here since it is insulated system and it has no heat transfer so, there will be no change in internal energy, hence there will be no change in temperature. Also by 1st law of thermodynamics, since there is no heat transfer due to system being insulated and no work due to frictionless expansion;

Final temperature = 313 K .

$$dq = du + dw \Rightarrow du = 0$$

$$\text{i.e. } T_{\text{initial}} = T_{\text{final}}$$

$$\text{Change in enthalpy} = 0 \quad \mathbf{Ans.}$$

$$\text{Work done} = 0 \quad \mathbf{Ans.}$$

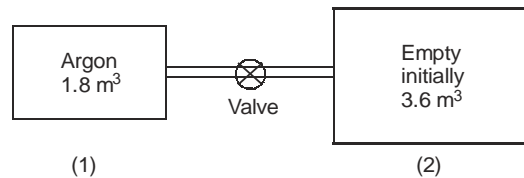


Fig. 4.37

EXERCISE

- 4.1** State the Kelvin Planck and Clausius statements of 2nd law of thermodynamics.
- 4.2** Show the equivalence of two statements of 2nd law of thermodynamics.
- 4.3** Write short notes on the following:
Heat reservoir, Heat engine, Heat pump and refrigerator.
- 4.4** Explain the reversible and irreversible processes.
- 4.5** Describe Carnot cycle and obtain expression for its efficiency as applied to a heat engine.

- 4.6 Why Carnot cycle is a theoretical cycle? Explain.
- 4.7 Show that coefficient of performance of heat pump and refrigerator can be related as;

$$\text{COP}_{\text{Ref}} = \text{COP}_{\text{HP}} - 1$$
- 4.8 State Carnot theorem. Also prove it.
- 4.9 Show that the efficiencies of all reversible heat engines operating between same temperature limits are same.
- 4.10 Show that efficiency of an irreversible engine is always less than the efficiency of reversible engine operating between same temperature limits.
- 4.11 Assume an engine to operate on Carnot cycle with complete reversibility except that 10% of work is required to overcome friction. For the efficiency of reversible cycle being 30%, what shall be the efficiency of assumed engine.
 For same magnitude of energy required to overcome friction, if machine operated as heat pump, then what shall be ratio between refrigerating effect and work required. [27%, 2.12]
- 4.12 A Carnot engine operating between certain temperature limits has an efficiency of 30%. Determine the ratio of refrigerating effect and work required for operating the cycle as a heat pump between the same temperature limits. [2.33]
- 4.13 An inventor claims to have developed an engine that takes in 1055 mJ at a temperature of 400K and rejects 42.2 MJ at a temperature of 200 K while delivering 15kWh of mechanical work. Check whether engine is feasible or not. [Engine satisfies 1st law but violates 2nd law]
- 4.14 Determine which of the following is the most effective way to increase Carnot engine efficiency
 (i) To increase T_2 while keeping T_1 fixed.
 (ii) To decrease T_1 while keeping T_2 fixed. [If T_1 is decreased]
- 4.15 A refrigerator has COP one half as great as that of a Carnot refrigerator operating between reservoirs at temperatures of 200 K and 400 K, and absorbs 633 KJ from low temperature reservoir. How much heat is rejected to the high temperature reservoir? [1899 kJ]
- 4.16 Derive a relationship between COP of a Carnot refrigerator and the efficiency of same refrigerator when operated as an engine.
 Is a Carnot engine having very high efficiency suited as refrigerator?
- 4.17 Calculate COP of Carnot refrigerator and Carnot heat pump, if the efficiency of the Carnot engine between same temperature limits is 0.17. [5, 6]
- 4.18 For the reversible heat engines operating in series, as shown in figure 4.36. Show the following, if work output is twice that of second.

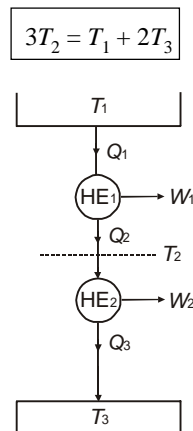


Fig. 4.36

- 4.19 A domestic refrigerator is intended to freeze water at 0°C while water is available at 20°C. COP of refrigerator is 2.5 and power input to run it is 0.4 kW. Determine capacity of refrigerator if it takes 14 minutes to freeze. Take specific heat of water as 4.2 kJ/kg. °C. [10 kg]

- 4.20** A cold storage plant of 49.64 hp power rating removes 7.4 MJ/min and discharges heat to atmospheric air at 30°C. Determine the temperature maintained inside the cold storage. [-40°C]
- 4.21** A house is to be maintained at 21°C from inside during winter season and at 26°C during summer. Heat leakage through the walls, windows and roof is about 3×10^3 kJ/hr per degree temperature difference between the interior of house and environment temperature. A reversible heat pump is proposed for realizing the desired heating/cooling. What minimum power shall be required to run the heat pump in reversed cycle if outside temperature during summer is 36°C? Also find the lowest environment temperature during winter for which the inside of house can be maintained at 21°C. [0.279 kW, 11°C]
- 4.22** Estimate the minimum power requirement of a heat pump for maintaining a commercial premises at 22°C when environment temperature is -5°C. The heat load on pump is 1×10^7 kJ/day.
- 4.23** A reversible engine having 50% thermal efficiency operates between a reservoir at 1527°C and a reservoir at some temperature T . Determine temperature T in K .
- 4.24** A reversible heat engine cycle gives output of 10 kW when 10 kJ of heat per cycle is supplied from a source at 1227°C. Heat is rejected to cooling water at 27°C. Estimate the minimum theoretical number of cycles required per minute. [75]
- 4.25** Some heat engine A and a reversible heat engine B operate between same two heat reservoirs. Engine A has thermal efficiency equal to two-third of that of reversible engine B . Using second law of thermodynamics show that engine A shall be irreversible engine.
- 4.26** Show that the COP of a refrigeration cycle operating between two reservoirs shall be, $\text{COP}_{\text{ref}} = \left(\frac{1}{\eta_{\text{max}}} - 1 \right)$, if η_{max} refers to thermal efficiency of a reversible engine operating between same temperature limits.
- 4.27** A heat pump is used for maintaining a building at 20°C. Heat loss through roofs and walls is at the rate of 6×10^4 kJ/h. An electric motor of 1 kW rating is used for driving heat pump. On some day when environment temperature is 0°C, would it be possible for pump to maintain building at desired temperature? [No]
- 4.28** Three heat engines working on carnot cycle produce work output in proportion of 5 : 4 : 3 when operating in series between two reservoirs at 727°C and 27°C. Determine the temperature of intermediate reservoirs. [435.34°C, 202°C]
- 4.29** Determine the power required for running a heat pump which has to maintain temperature of 20°C when atmospheric temperature is -10°C. The heat losses through the walls of room are 650 W per unit temperature difference of inside room and atmosphere. [2 kW]
- 4.30** A heat pump is run between reservoirs with temperatures of 7°C and 77°C. Heat pump is run by a reversible heat engine which takes heat from reservoir at 1097°C and rejects heat to reservoir at 77°C. Determine the heat supplied by reservoir at 1097°C if the total heat supplied to reservoir at 77°C is 100 kW. [25.14 kW]
- 4.31** A refrigerator is used to maintain temperature of 243K when ambient temperature is 303K. A heat engine working between high temperature reservoir of 200°C and ambient temperature is used to run this refrigerator. Considering all processes to be reversible, determine the ratio of heat transferred from high temperature reservoir to heat transferred from refrigerated space. [0.69]

Entropy

5.1 INTRODUCTION

Till now the detailed explanation of Zeroth law, first law and second law of thermodynamics have been made. Also we have seen that the first law of thermodynamics defined a very useful property called internal energy. For overcoming the limitations of first law, the second law of thermodynamics had been stated. Now we need some mathematical parameter for being used as decision maker in respect of feasibility of process, irreversibility, nature of process etc. Here in this chapter a mathematical function called 'entropy' has been explained. 'Entropy' is the outcome of second law and is a thermodynamic property. Entropy is defined in the form of calculus operation, hence no exact physical description of it can be given. However, it has immense significance in thermodynamic process analysis.

5.2 CLAUSIUS INEQUALITY

Let us take any reversible process 1–2 as shown on P – V diagram. Let us also have a reversible adiabatic process 1–1' followed by reversible isothermal process 1'–2' and a reversible adiabatic process 2'–2, as approximation to the original process 1 – 2 such that area under 1 – 2 equals to that under 1–1'–2'–2. By first law of thermodynamics for process shown by 1–2.

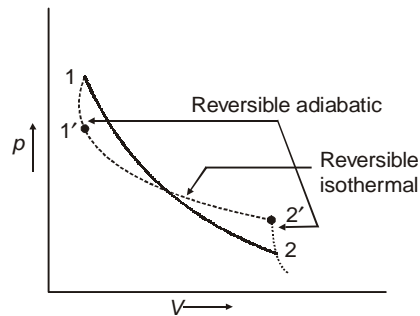


Fig. 5.1 Reversible adiabatic, isothermal and reversible process

$$Q_{1-2} = (U_2 - U_1) + W_{1-2}$$

First law on 1–1'–2'–2 processes; (Heat and work are path functions and internal energy is point function)

$$Q_{1-1'-2'-2} = (U_2 - U_1) + W_{1-1'-2'-2}$$

As already assumed that

$$W_{1-2} = W_{1-1'-2'-2}$$

so

$$Q_{1-1'-2'-2} = Q_{1-2}$$

In the path $1 - 1' - 2' - 2$ during adiabatic processes $1 - 1'$ & $2 - 2'$ there is no heat interaction so the total heat interaction in $1 - 2$ is getting occurred during isothermal process $1' - 2'$ only.

Hence, it is always possible to replace any reversible process by a series of reversible adiabatic, reversible isothermal and reversible adiabatic processes between the same end states provided the heat interaction and work involved remains same.

If the number of reversible adiabatic and reversible isothermal processes is quite large then the series of such processes shall reach close to the original reversible process.

Let us undertake this kind of substitution for the processes in a reversible cycle

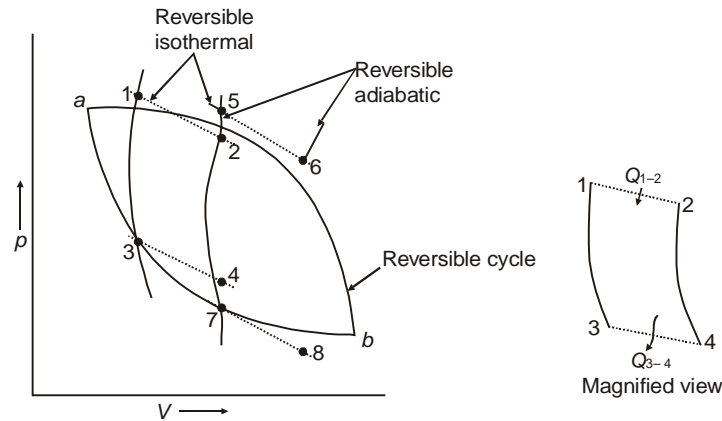


Fig. 5.2 A reversible cycle replaced by reversible adiabatics and reversible isotherms

Figure 5.2 shows replacement of original processes in cycle $a - b - a$ by adiabatic and isotherms. This shall result in a number of Carnot cycles appearing together in place of original cycle. Two Carnot cycles thus formed are shown by $1 - 2 - 4 - 3$ and $5 - 6 - 8 - 7$. Magnified view of first Carnot cycle is also shown separately where heat supplied at high temperature is Q_{1-2} and heat rejected at low temperature is Q_{3-4} .

From thermodynamic temperature scale;

For Carnot cycle $1 - 2 - 4 - 3$,

$$\frac{Q_{1-2}}{Q_{3-4}} = \frac{T_{1-2}}{T_{3-4}}$$

or

$$\frac{Q_{1-2}}{T_{1-2}} = \frac{Q_{3-4}}{T_{3-4}}$$

For Carnot cycle $5 - 6 - 8 - 7$,

$$\frac{Q_{5-6}}{T_{5-6}} = \frac{Q_{7-8}}{T_{7-8}}$$

Now taking sign conventions for heat added and rejected;

$$\frac{Q_{1-2}}{T_{1-2}} + \frac{Q_{3-4}}{T_{3-4}} = 0$$

and

$$\frac{Q_{5-6}}{T_{5-6}} + \frac{Q_{7-8}}{T_{7-8}} = 0$$

Hence, if there are 'n' number of Carnot cycles replacing the original reversible cycle, then

$$\left(\frac{Q_{1-2}}{T_{1-2}} + \frac{Q_{3-4}}{T_{3-4}} \right) + \left(\frac{Q_{5-6}}{T_{5-6}} + \frac{Q_{7-8}}{T_{7-8}} \right) + \dots = 0$$

or, it can be given as summation of the ratio of heat interaction (Q) to the temperature (T) at which it occurs, being equal to zero.

$$\sum \frac{Q}{T} = 0$$

If number of Carnot cycles is very large, then the zig-zag path formed due to replacing adiabatics and isotherms shall reach very close to original cycle.

In such situation the cyclic integral of $\frac{Q}{T}$ may be given in place of above.

or
$$\oint \left(\frac{dQ}{T} \right)_{\text{rev}} = 0$$

Here it indicates that $\left(\frac{dQ}{T} \right)$ is some thermodynamic property. Above expression developed for a reversible heat engine cycle also remains valid for internally reversible engines. In case of internally reversible engines T shall be temperature of working fluid at the time of heat interaction.

$$\oint \left(\frac{dQ}{T} \right)_{\text{int, rev}} = 0$$

Let us now try to find out what happens to $\oint \frac{dQ}{T}$ when we have an irreversible engine cycle. Let there be a reversible and irreversible heat engine operating between same temperature limits, such that heat added to them is same.

From Carnot's theorem for both reversible and irreversible heat engine cycles,

$$\eta_{\text{rev}} > \eta_{\text{irrev}}$$

or
$$\left(1 - \frac{Q_{\text{rejected}}}{Q_{\text{add}}} \right)_{\text{rev}} > \left(1 - \frac{Q_{\text{rejected}}}{Q_{\text{add}}} \right)_{\text{irrev}}$$

or
$$\left(\frac{Q_{\text{rejected}}}{Q_{\text{add}}} \right)_{\text{irrev}} > \left(\frac{Q_{\text{rejected}}}{Q_{\text{add}}} \right)_{\text{rev}}$$

For same heat added, i.e. $Q_{\text{add, rev}} = Q_{\text{add, irrev}} = Q_{\text{add}}$
 $Q_{\text{rejected, irrev}} > Q_{\text{rejected, rev}}$

or
$$\frac{Q_{\text{rejected, irrev}}}{Q_{\text{rejected, rev}}} > 1$$

For absolute thermodynamic temperature scale,

$$\left(\frac{Q_{\text{add}}}{Q_{\text{rejected}}} \right)_{\text{rev}} = \frac{T_{\text{add}}}{T_{\text{rejected}}}, \text{ upon substitution we get,}$$

$$\frac{Q_{\text{rejected, irrev}}}{Q_{\text{add}}} > \frac{T_{\text{rejected}}}{T_{\text{add}}}$$

or

$$\frac{Q_{\text{rejected, irrev}}}{T_{\text{rejected}}} > \frac{Q_{\text{add}}}{T_{\text{add}}}$$

Upon substituting sign convention, we get

$$\frac{Q_{\text{add}}}{T_{\text{add}}} + \frac{Q_{\text{rejected, irrev}}}{T_{\text{rejected}}} < 0$$

or

$$\frac{Q_{\text{add, irrev}}}{T_{\text{add}}} + \frac{Q_{\text{rejected, irrev}}}{T_{\text{rejected}}} < 0$$

If it is given in the form of cyclic integral.

or

$$\oint \left(\frac{dQ}{T} \right)_{\text{irrev}} < 0$$

Now combining for reversible and irreversible paths it can be given as;

$$\oint \left(\frac{dQ}{T} \right) \leq 0$$

This is called *Clausius inequality*.

here,

$$\oint \left(\frac{dQ}{T} \right) = 0 \text{ for reversible cycle}$$

$$\oint \left(\frac{dQ}{T} \right) < 0, \text{ for irreversible cycle}$$

$$\oint \left(\frac{dQ}{T} \right) > 0, \text{ for impossible cycle}$$

5.3 ENTROPY—A PROPERTY OF SYSTEM

From Clausius inequality mathematically it is shown that for a reversible cycle.

$$\oint \left(\frac{dQ}{T} \right)_{\text{rev}} = 0$$

Let us take a reversible cycle comprising of two processes *A* and *B* as shown and apply Clausius inequality.

$$\int_{a \text{ path } A}^b \frac{dQ}{T} + \int_{b \text{ path } B}^a \frac{dQ}{T} = 0$$

$$\int_{a \text{ path } A}^b \frac{dQ}{T} = - \int_{b \text{ path } B}^a \frac{dQ}{T}$$

or

$$\int_{a \text{ path } A}^b \frac{dQ}{T} = \int_{a \text{ path } A}^b \frac{dQ}{T}$$

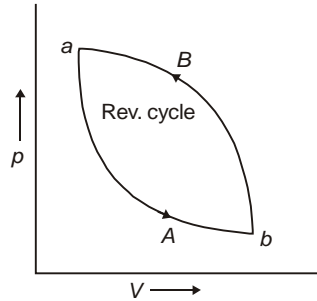


Fig. 5.3 Reversible cycle

Hence, it shows that $\left(\frac{dQ}{T}\right)$ is some property and does not depend upon path followed. This thermodynamic property is called “entropy”. Entropy is generally denoted by ‘S’ or ‘ ϕ ’.

Thus, the energy interactions in the form of heat are accompanied by entropy changes.

Writing it as function of entropy change.

$$\int_{a \text{ path } A}^b \frac{dQ}{T} = \int_{a \text{ path } A}^b \frac{dQ}{T} = S_b - S_a$$

or

$$\int \left(\frac{dQ}{T}\right)_{\text{rev}} = \int dS$$

Since entropy is point function and depends only upon end states therefore entropy change for any process following reversible or irreversible path shall be same.

$$(S_b - S_a)_{\text{rev, path}} = (S_b - S_a)_{\text{irrev, path}}$$

$$\boxed{\int_a^b \frac{dQ_{\text{rev}}}{T} = \Delta S_{\text{rev, path}} = \Delta S_{\text{irrev, path}}}$$

Entropy is an extensive property and has units J/K. Specific entropy may be given on unit mass basis;

$$s = \frac{S}{m} \text{ (J/kg K)}$$

Entropy, as obvious from definition is defined for change in entropy value, therefore absolute value of entropy cannot be defined. Entropy is always given as change, i.e. relative values can be expressed.

Let us now have two thermodynamic cycles $a - b - a$ following paths $(a - R - b, b - R - a)$ and $(a - R - b, b - I - a)$.

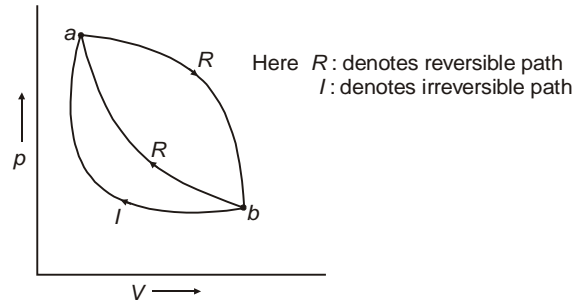


Fig. 5.4 Reversible and irreversible cycle

We have from Clausius inequality.

For reversible cycle $a - R - b - R - a$

$$\oint \left(\frac{dQ}{T} \right)_{\text{rev}} = 0$$

$$\int_{a,R}^b \frac{dQ}{T} + \int_{b,R}^a \frac{dQ}{T} = 0$$

For irreversible cycle $a - R - b - I - a$

$$\oint \left(\frac{dQ}{T} \right) < 0$$

or
$$\int_{a,R}^b \frac{dQ}{T} + \int_{b,I}^a \frac{dQ}{T} < 0$$

or
$$\int_{a,R}^b \frac{dQ}{T} < - \int_{b,I}^a \frac{dQ}{T}$$

Also from definition of entropy.

$$\int \left(\frac{dQ}{T} \right)_{\text{rev}} = \int dS$$

so from above

$$\int_{a,R}^b \frac{dQ}{T} = \int_a^b dS$$

or
$$\left(\frac{dQ}{T} \right)_{\text{rev}} = dS$$

For reversible cycle.

$$\int_{a,R}^b \frac{dQ}{T} = - \int_{b,R}^a \frac{dQ}{T}$$

Substituting it in expression for irreversible cycle.

$$- \int_{b,R}^a \frac{dQ}{T} < - \int_{b,I}^a \frac{dQ}{T}$$

also from definition of entropy.

$$- \int_b^a dS < - \int_{b,I}^a \frac{dQ}{T}$$

or

$$\int_{b,I}^a \frac{dQ}{T} < \int_b^a dS$$

or, in general it can be given as,

$$\left(\frac{dQ}{T} \right)_{\text{irrev}} < dS$$

Combining the above two we get inequality as following,

$$\boxed{dS \geq \frac{dQ}{T}}$$

where

$$dS = \frac{dQ}{T} \text{ for reversible process}$$

$$dS > \frac{dQ}{T} \text{ for irreversible process}$$

Mathematical formulation for entropy ($dQ_{\text{rev}} = T \cdot dS$) can be used for getting property diagrams between “temperature and entropy” ($T - S$), “enthalpy and entropy” ($h - S$).

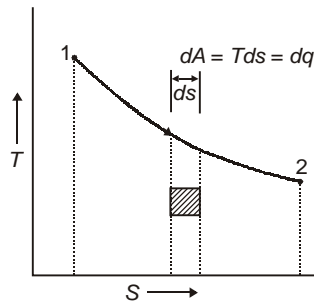


Fig. 5.5 T - S diagram

Area under process curve on T - S diagram gives heat transferred, for internally reversible process

$$dQ_{\text{int, rev}} = T \cdot dS$$

$$\boxed{Q_{\text{int, rev}} = \int_1^2 T \cdot dS}$$

5.4 PRINCIPLE OF ENTROPY INCREASE

By second law, entropy principle has been obtained as,

$$dS \geq \frac{dQ}{T}$$

For an isolated system:

$$dQ = 0, \text{ therefore } dS_{\text{isolated}} \geq 0$$

for a reversible process $dS_{\text{isolated}} = 0$

$$\text{i.e. } S_{\text{isolated}} = \text{constant}$$

for an irreversible process $dS_{\text{isolated}} > 0$

which means the entropy of an isolated system always increases, if it has irreversible processes,

$$\text{In general form } dS_{\text{isolated}} \geq 0$$

It may be concluded here that the entropy of an isolated system always increases or remains constant in case of irreversible and reversible processes respectively. This is known as “Principle of entropy increase” or “entropy principle”.

Universe which comprises of system and surroundings may also be treated as isolated system and from entropy principle;

$$dS_{\text{Universe}} \geq 0$$

Which means that entropy of universe either increases continuously or remains constant depending upon whether processes occurring in it are of “irreversible” or “reversible” type respectively.

Since, Universe = System + Surrounding.

therefore

$$dS_{\text{system}} + dS_{\text{surrounding}} \geq 0$$

or

$$\Delta S_{\text{system}} + \Delta S_{\text{surrounding}} \geq 0$$

Since most of the processes occurring generally are of irreversible type in universe, so it can be said that in general entropy of universe keeps on increasing and shall be maximum after attaining the state of equilibrium, which is very difficult to attain.

In the above expression, system and surroundings are treated as two parts of universe (an isolated system). So the total entropy change during a process can be given by sum of “entropy change in system” and “entropy change in surroundings”. This total entropy change is also called “entropy generation” or “entropy production”. Entropy generation will be zero in a reversible process.

Therefore

$$\Delta S_{\text{total}} = S_{\text{gen}} = \Delta S_{\text{system}} + \Delta S_{\text{surrounding}}$$

For closed systems

In case of closed systems there is no mass interaction but heat and work interactions are there. Entropy change is related to heat interactions occurring in system and surroundings.

Total entropy change.

or

Entropy generated

$$\Delta S_{\text{total}} = S_{\text{gen}} = \Delta S_{\text{system}} + \Delta S_{\text{surrounding}}$$

For system changing its state from 1 to 2 i.e. initial and final state.

$$\Delta S_{\text{system}} = S_2 - S_1$$

For surroundings, entropy change depends upon heat interactions.

$$\Delta S_{\text{surrounding}} = \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}$$

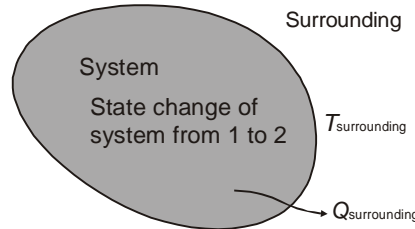


Fig. 5.6 Closed system

or

$$-\Delta S_{\text{total}} = S_{\text{gen}} = (S_2 - S_1) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}$$

or

$$S_2 - S_1 = m(s_2 - s_1)$$

where m is mass in system and s_1 and s_2 are specific entropy values at initial and final state, then

$$S_{\text{gen}} = m(s_2 - s_1) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}$$

For open systems

In case of open systems the mass interactions also take place along with energy interactions. Here mass flow into and out of system shall also cause some entropy change, so a control volume as shown in figure is to be considered. Entropy entering and leaving at section $i-i$ and $o-o$ are considered. Mass flow carries both energy and entropy into or out of control volume. Entropy transfer with mass flow is called “entropy transport”. It is absent in closed systems.

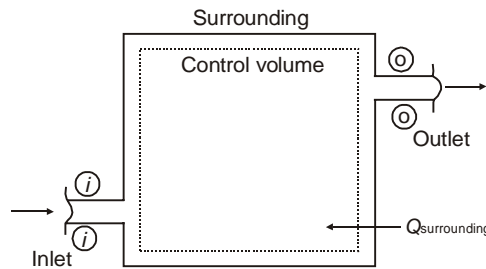


Fig. 5.7 Open system

If control volume undergoes state change from 1 to 2, then entropy change in control volume shall be $(S_2 - S_1)$ while entropy entering and leaving out may be given as S_i and S_o respectively.

By principle of entropy increase, total entropy change shall be,

$$\Delta S_{\text{total}} = S_{\text{gen}} = (S_2 - S_1) + (S_o - S_i) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}$$

Entropy entering and leaving out may be given as sum of entropy of all mass flows into and out of system in case of *uniform flow process*.

Therefore

$$\begin{aligned} S_i &= \sum m_i \cdot s_i \\ S_o &= \sum m_o \cdot s_o \end{aligned}$$

where m_i and m_o are mass flows into and out of system, and s_i and s_o are specific entropy associated with mass entering and leaving.

Substituting,

$$\Delta S_{\text{total}} = S_{\text{gen}} = (S_2 - S_1) + \left(\sum m_o \cdot s_o - \sum m_i \cdot s_i \right) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}$$

In case of steady flow process since properties do not change with respect to time during any process, therefore within control volume there shall be no change in entropy.

i.e.

$$S_1 = S_2$$

Total entropy change or entropy generation for this case shall be;

$$\Delta S_{\text{total}} = S_{\text{gen}} = (S_o - S_i) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}$$

$$\Delta S_{\text{total}} = S_{\text{gen}} = \left(\sum m_o \cdot s_o - \sum m_i \cdot s_i \right) + \frac{Q_{\text{surrounding}}}{T_{\text{surrounding}}}$$

In all the cases discussed above $\Delta S_{\text{total}} \geq 0$ or $S_{\text{gen}} \geq 0$.

Entropy generated can be taken as criterion to indicate feasibility of process as follows;

- * If S_{gen} or $\Delta S_{\text{total}} = 0$ then process is reversible.
- * If S_{gen} or $\Delta S_{\text{total}} > 0$ then process is irreversible.
- * If S_{gen} or $\Delta S_{\text{total}} < 0$ then process is impossible.

One thing is very important about entropy generated that S_{gen} is not a thermodynamic property and its value depends on the path followed whereas entropy change of system ($S_2 - S_1$) is a point function and so thermodynamic property. It is because of the fact that entropy change is cumulative effect of entropy transfer/change in system and surroundings.

5.5 ENTROPY CHANGE DURING DIFFERENT THERMODYNAMIC PROCESSES

Isothermal process

Let us find out entropy change for *isothermal heat addition* process. As isothermal process can be considered internally reversible, therefore entropy change shall be;

$$\Delta S_{a-b} = \int_a^b \left(\frac{dQ}{T} \right)$$

or

$$\Delta S_{a-b} = \frac{1}{T} \int_a^b dQ$$

or

$$\Delta S_{a-b} = \frac{Q_{a-b}}{T}$$

where Q_{a-b} is total heat interaction during state change $a - b$ at temperature T .

Isentropic process

It is the process during which change in entropy is zero and entropy remains constant during process.

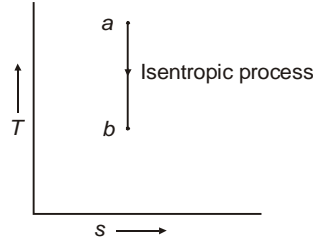


Fig. 5.8 *Isentropic process*

It indicates that when $\Delta S_{a-b} = 0$.

then

$$Q_{a-b} = 0$$

which means there is no heat interaction during such process and this is adiabatic process.

Hence, it can be said that "a reversible isentropic process shall be adiabatic, whereas if isentropic process is adiabatic then it may or may not be reversible".

Thus, adiabatic process may or may not be reversible. It means in reversible adiabatic process all states shall be in equilibrium and no dissipative effects are present along with no heat interaction whereas in adiabatic process there is no heat interaction but process may be irreversible.

Finally, it can be concluded that an adiabatic process may or may not be isentropic whereas a reversible adiabatic process is always isentropic.

An adiabatic process of non isentropic type is shown below where irreversibility prevails, say due to internal friction.

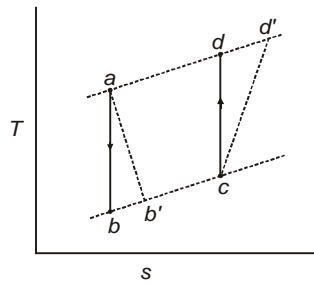


Fig. 5.9 *Isentropic and non-isentropic processes*

Here $a - b$ is reversible adiabatic expansion of isentropic type.

Non-isentropic or adiabatic expansion is shown by $a - b'$.

Isentropic expansion efficiency may be defined as ratio of actual work to ideal work available during expansion.

$$\eta_{\text{isen, expn}} = \frac{\text{Actual work in expansion}}{\text{Ideal work in expansion}}$$

$$\eta_{\text{isen, expn}} = \frac{h_a - h_{b'}}{h_a - h_b}$$

Similarly, isentropic and non-isentropic compression process are shown as $c - d$ and $c - d'$ respectively.

Isentropic compression efficiency can be defined on same lines as,

$$\eta_{\text{isen, compr}} = \frac{\text{Ideal work in expansion}}{\text{Actual work in expansion}}$$

$$\eta_{\text{isen, compr}} = \frac{h_d - h_c}{h_{d'} - h_c}$$

For ideal gases

Combination of first and second law yields;

$$Tds = du + pdv$$

also we know $du = c_v \cdot dT$ and for perfect gas $p = \frac{RT}{v}$

substituting for du and R

$$T \cdot ds = c_v \cdot dT + \frac{RT}{v} \cdot dv$$

or
$$ds = c_v \cdot \frac{dT}{T} + \frac{Rdv}{v}$$

or
$$s_2 - s_1 = \int_1^2 c_v \cdot \frac{dT}{T} + \int_1^2 \frac{Rdv}{v}$$

If c_v is function of temperature then,

$$s_2 - s_1 = \int_1^2 c_v(T) \cdot \frac{dT}{T} + R \cdot \ln \frac{v_2}{v_1}$$

If specific heat is constant then,

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \cdot \ln \frac{v_2}{v_1}$$

Also combination of Ist and IInd law yields following using; $h = u + pv$,

or $dh = du + pdv + vdp$

$$T \cdot ds = dh - v dp$$

substituting $dh = c_p \cdot dT$, and $v = \frac{RT}{p}$

$$T \cdot ds = c_p \cdot dT - v \cdot dp$$

or
$$ds = c_p \cdot \frac{dT}{T} - \frac{Rdp}{p}$$

entropy change

$$s_2 - s_1 = \int_1^2 \frac{c_p dT}{T} - \int_1^2 \frac{Rdp}{p}$$

If specific heat is function of temperature then

$$s_2 - s_1 = \int_1^2 \frac{c_p(T) dT}{T} - R \ln \frac{p_2}{p_1}$$

If specific heat is constant, then

$$s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

Above expressions given in enclosed box may be suitably used for getting the change in entropy.

Polytropic Process

Entropy change in a polytropic process having governing equation as $p v^n = \text{constant}$, can be obtained as below,

For polytropic process between 1 and 2, $p_1 v_1^n = p_2 v_2^n$

or
$$\left(\frac{p_1}{p_2} \right) = \left(\frac{v_2}{v_1} \right)^n$$

Also, from gas laws,

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1} \times \frac{T_1}{T_2} \right)$$

Above two pressure ratios give,

$$\frac{v_2}{v_1} = \left(\frac{T_1}{T_2} \right)^{\left(\frac{1}{n-1} \right)}$$

Substituting $\left(\frac{v_2}{v_1} \right)$ in the entropy change relation derived earlier.

$$s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$= c_v \ln \left(\frac{T_2}{T_1} \right) + R \ln \left(\frac{T_1}{T_2} \right)^{\frac{1}{n-1}}$$

$$s_2 - s_1 = c_v \ln \left(\frac{T_2}{T_1} \right) + \left(\frac{R}{n-1} \right) \ln \frac{T_1}{T_2}$$

For perfect gas

$$R = c_p - c_v$$

$$R = \gamma \cdot c_v - c_v$$

$$R = c_v (\gamma - 1)$$

Substituting R in entropy change

$$\begin{aligned} s_2 - s_1 &= c_v \ln \left(\frac{T_2}{T_1} \right) + \frac{c_v(\gamma-1)}{(n-1)} \ln \left(\frac{T_1}{T_2} \right) \\ &= c_v \ln \frac{T_2}{T_1} \left\{ 1 - \left(\frac{\gamma-1}{n-1} \right) \right\} \\ s_2 - s_1 &= c_v \ln \frac{T_2}{T_1} \left(\frac{n-\gamma}{n-1} \right) \end{aligned}$$

Entropy change in polytropic process.

$$s_2 - s_1 = \left(c_v \cdot \ln \frac{T_2}{T_1} \right) \left(\frac{n-\gamma}{n-1} \right)$$

5.6 ENTROPY AND ITS RELEVANCE

Entropy has been introduced as a property based on the concept of IInd law of thermodynamics and derived from the thermodynamics involved in heat engines. A large number of definitions are available for entropy. To understand entropy let us take some gas in a closed vessel and heat it. Upon heating of gas the motion of gas molecules inside the vessel gets increased. State of molecular motion inside vessel depends upon the quantum of heat supplied. If we measure new kinetic energy of gas molecules, it is found to be larger than that initially. Also, the rate of intermolecular collision and randomness in molecular motion gets increased. In nutshell it could be said that heating has caused increase in energy level of gas molecules and thus resulting in increased disorderness inside the vessel. Higher is the energy with molecules, higher shall be the degree of disorderness. Entropy is closely defined using the degree of disorderness. It is said that greater is the molecular disorderness in the system greater shall be entropy. Mathematically, it can be supported by greater entropy value due to large heat supplied (dQ/T).

Thus “entropy can be defined as a parameter for quantifying the degree of molecular disorderness in the system”. “Entropy is a measure of driving potential available for occurrence of a process”.

Entropy is also an indicator of the direction of occurrence of any thermodynamic process. Mathematically, it has been seen from second law of thermodynamics that entropy of an isolated system always increases. Therefore, a process shall always occur in such a direction in which either entropy does not change or increases. In general almost all real processes are of irreversible type so entropy tends to increase. As entropy cannot be defined absolutely so the change in entropy should always have a positive or zero value.

5.7 THERMODYNAMIC PROPERTY RELATIONSHIP

Different thermodynamic properties such as P , V , T , U , H , S etc. can be related to one another using the combination of mathematical forms of first law, second law of thermodynamics and definitions of properties. Here specific values of properties are related.

For a non-flow process in closed system.

or

$$\begin{aligned} dq &= du + dw \\ dq &= du + p \cdot dv \end{aligned}$$

Also, for a reversible process from definition of entropy, by second law we can write

$$dq = Tds$$

Combining above two,

$$Tds = du + p \cdot dv$$

From definition of enthalpy, specific enthalpy

$$h = u + pv$$

or

$$dh = du + p \cdot dv + v \cdot dp$$

substituting from above

$$dh = T \cdot ds + v \cdot dp$$

Above relations may be used for getting the variation of one property with the other, such as for constant pressure process, $dh = T \cdot ds$

or
$$\left(\frac{dh}{ds} \right)_{p=\text{constt.}} = T$$

which means slope of constant pressure line on enthalpy – entropy diagram ($h-s$) is given by temperature.

Also from above two relations

$$T \cdot ds = c_v \cdot dT + p \cdot dv \quad \{ \text{as } du = c_v \cdot dT \}$$

Substituting for dh and rearranging, $dh = T \cdot ds + v \cdot dp$

$$\{ \text{as } dh = c_p \cdot dT \}$$

or

$$Tds = c_p \cdot dT - v \cdot dp$$

For a constant pressure process above yields

$$\left(\frac{dT}{ds} \right)_{p=\text{constt.}} = \frac{T}{c_p}$$

It gives the slope of constant pressure line on $T-s$ diagram.

Similarly, for a constant volume process,

$$\left(\frac{dT}{ds} \right)_{v=\text{const.}} = \frac{T}{c_v}$$

It gives the slope of constant volume line on $T-s$ diagram.

It can be concluded from the above mathematical explanations for slope that *slope of constant volume line is more than the slope of constant pressure line as $c_p > c_v$.*

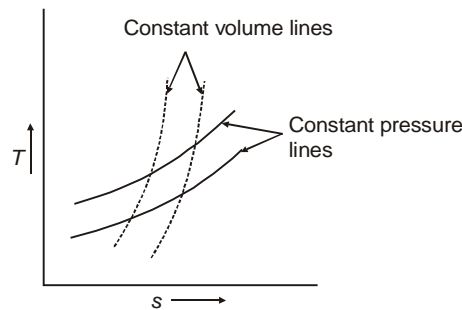


Fig. 5.10 $T-s$ diagram showing isobaric and isochoric process.

5.8 THIRD LAW OF THERMODYNAMICS

'Third law of thermodynamics', an independent principle uncovered by 'Nernst' and formulated by 'Planck', states that the "Entropy of a pure substance approaches zero at absolute zero temperature." This fact can also be corroborated by the definition of entropy which says it is a measure of molecular disorderness. At absolute zero temperature substance molecules get frozen and do not have any activity, therefore it may be assigned zero entropy value at crystalline state. Although the attainment of absolute zero temperature is impossible practically, however theoretically it can be used for defining absolute entropy value with respect to zero entropy at absolute zero temperature. Second law of thermodynamics also shows that absolute zero temperature can't be achieved, as proved earlier in article 4.10. Third law of thermodynamics is of high theoretical significance for the sake of absolute property definitions and has found great utility in thermodynamics.

EXAMPLES

1. Calculate the change in entropy of air, if it is throttled from 5 bar, 27°C to 2 bar adiabatically.

Solution:

Here $p_1 = 5$ bar, $T_1 = 300$ K.

$$p_2 = 2 \text{ bar, } c_{p \text{ air}} = 1.004 \text{ kJ/kg.K}$$

$$R = 0.287 \text{ kJ/kg.K}$$

Entropy change may be given as;

$$s_2 - s_1 = \left\{ c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1} \right\}$$

for throttling process $h_1 = h_2$

$$\text{i.e. } c_p T_1 = c_p T_2$$

or

$$T_1 = T_2$$

Hence,

$$\begin{aligned} \text{Change in entropy} &= 1.004 \ln(1) - 0.287 \ln\left(\frac{2}{5}\right) \\ &= 0.263 \text{ kJ/kg.K} \end{aligned}$$

Change in entropy = **0.263 kJ/kg.K Ans.**

2. Find the change in entropy of steam generated at 400°C from 5 kg of water at 27°C and atmospheric pressure. Take specific heat of water to be 4.2 kJ/kg.K, heat of vaporization at 100°C as 2260 kJ/kg and specific heat for steam given by; $c_p = R(3.5 + 1.2T + 0.14T^2)$, J/kg.K

Solution:

Total entropy change = Entropy change during water temperature rise (ΔS_1).

+ Entropy change during water to steam change (ΔS_2)

+ Entropy change during steam temperature rise (ΔS_3)

$$\Delta S_1 = \frac{Q_1}{T_1} \text{ where } Q_1 = m c_p \cdot \Delta T$$

Heat added for increasing water temperature from 27°C to 100°C.

$$\begin{aligned} &= 5 \times 4.2 \times (100 - 27) \\ &= 1533 \text{ kJ} \end{aligned}$$

therefore,
$$\Delta S_1 = \frac{1533}{300} = 5.11 \text{ kJ/K}$$

Entropy change during phase transformation;

$$\Delta S_2 = \frac{Q_2}{T_2}$$

Here $Q_2 = \text{Heat of vaporization} = 5 \times 2260 = 11300 \text{ kJ}$

$$\text{Entropy change, } \Delta S_2 = \frac{11300}{373.15} = 30.28 \text{ kJ/K.}$$

Entropy change during steam temperature rise;

$$\Delta S_3 = \int_{373.15}^{673.15} \frac{dQ}{T}$$

Here $dQ = mc_p \cdot dT$; for steam $R = \frac{8.314}{18} = 0.462 \text{ kJ/kg.K}$

Therefore, c_p for steam $= 0.462 (3.5 + 1.2 \cdot T + 0.14T^2) \times 10^{-3}$
 $= (1.617 + 0.5544 T + 0.065 T^2) \times 10^{-3}$

or
$$\Delta S_3 = \int_{373.15}^{673.15} 5 \times 10^{-3} \times \left(\frac{1.617}{T} + 0.5544 + 0.065 T \right) dT$$

$$= 51843.49 \times 10^{-3} \text{ kJ/K}$$

$$\Delta S_3 = 51.84 \text{ kJ/K}$$

 Total entropy change $= 5.11 + 30.28 + 51.84$
 $= \mathbf{87.23 \text{ kJ/K Ans.}}$

3. Oxygen is compressed reversibly and isothermally from 125 kPa and 27°C to a final pressure of 375 kPa. Determine change in entropy of gas?

Solution:

Gas constant for oxygen:

$$R = \frac{8.314}{32} = 0.259 \text{ kJ/kg.K}$$

For reversible process the change in entropy may be given as;

$$\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

Substituting values of initial & final states

$$\Delta s = -R \ln \left(\frac{375}{125} \right) = -0.285 \text{ kJ/kg.K}$$

Entropy change $= - \mathbf{0.285 \text{ kJ/kg. K Ans.}}$

4. Determine the change in entropy of universe if a copper block of 1 kg at 150°C is placed in a sea water at 25°C. Take heat capacity of copper as 0.393 kJ/kg K.

Solution:

Entropy change in universe

$$\Delta S_{\text{universe}} = \Delta S_{\text{block}} + \Delta S_{\text{water}}$$

where $\Delta S_{\text{block}} = mC \ln \frac{T_2}{T_1}$

Here hot block is put into sea water, so block shall cool down upto sea water at 25°C as sea may be treated as sink.

Therefore, $T_1 = 150^\circ\text{C}$ or 423.15 K
and $T_2 = 25^\circ\text{C}$ or 298.15 K

$$\begin{aligned}\Delta S_{\text{block}} &= 1 \times 0.393 \times \ln \left(\frac{298.15}{423.15} \right) \\ &= -0.1376 \text{ kJ/K}\end{aligned}$$

$$\begin{aligned}\text{Heat lost by block} &= \text{Heat gained by water} \\ &= -1 \times 0.393 \times (423.15 - 298.15) \\ &= -49.125 \text{ kJ}\end{aligned}$$

Therefore, $\Delta S_{\text{water}} = \frac{49.125}{298.15} = 0.165 \text{ kJ/k}$

Thus, $\Delta S_{\text{universe}} = -0.1376 + 0.165$
 $= 0.0274 \text{ kJ/k}$ or 27.4 J/K

Entropy change of universe = **27.4 J/K Ans.**

5. Determine change in entropy of universe if a copper block of 1 kg at 27°C is dropped from a height of 200 m in the sea water at 27°C. (Heat capacity for copper = 0.393 kJ/kg.K)

Solution:

$$\Delta S_{\text{universe}} = \Delta S_{\text{block}} + \Delta S_{\text{sea water}}$$

Since block and sea water both are at the same temperature so,

$$\Delta S_{\text{universe}} = \Delta S_{\text{sea water}}$$

Conservation of energy equation yields;

$$Q - W = \Delta U + \Delta PE + \Delta KE$$

Since in this case, $W = 0$, $\Delta KE = 0$, $\Delta U = 0$

$$Q = \Delta PE$$

$$\begin{aligned}\text{Change in potential energy} &= \Delta PE = mgh = 1 \times 9.81 \times 200 \\ &= 1962 \text{ J} \\ Q &= 1962 \text{ J}\end{aligned}$$

$$\Delta S_{\text{universe}} = \Delta S_{\text{sea water}} = \frac{1962}{300} = 6.54 \text{ J/kg K}$$

Entropy change of universe = **6.54 J/kg.K Ans.**

6. Determine entropy change of universe, if two copper blocks of 1 kg & 0.5 kg at 150°C and 0°C are joined together. Specific heats for copper at 150°C and 0°C are 0.393 kJ/kg K and 0.381 kJ/kg K respectively.

Solution:

Here, $\Delta S_{\text{universe}} = \Delta S_{\text{block 1}} + \Delta S_{\text{block 2}}$

Two blocks at different temperatures shall first attain equilibrium temperature. Let equilibrium temperature be T_f .

Then from energy conservation.

$$1 \times 0.393 \times (423.15 - T_f) = 0.5 \times 0.381 \times (T_f - 273.15)$$

$$T_f = 374.19 \text{ K}$$

Hence, entropy change in block 1, due to temperature changing from 423.15 K to 374.19 K.

$$\Delta S_1 = 1 \times 0.393 \times \ln \left(\frac{374.19}{423.15} \right) = -0.0483 \text{ kJ/K}$$

Entropy change in block 2

$$\Delta S_2 = 0.5 \times 0.381 \times \ln \left(\frac{374.19}{273.15} \right) = 0.0599 \text{ kJ/K}$$

$$\begin{aligned} \text{Entropy change of universe} &= 0.0599 - 0.0483 \\ &= 0.0116 \text{ kJ/K} \end{aligned}$$

Entropy change of universe = **0.0116 kJ/K** Ans.

7. A cool body at temperature T_1 is brought in contact with high temperature reservoir at temperature T_2 . Body comes in equilibrium with reservoir at constant pressure. Considering heat capacity of body as C , show that entropy change of universe can be given as;

$$C \left[\left(\frac{T_1 - T_2}{T_2} \right) - \ln \frac{T_1}{T_2} \right]$$

Solution:

Since body is brought in contact with reservoir at temperature T_2 , the body shall come in equilibrium when it attains temperature equal to that of reservoir, but there shall be no change in temperature of the reservoir.

$$\text{Entropy change of universe } \Delta S_{\text{universe}} = \Delta S_{\text{body}} + \Delta S_{\text{reservoir}}$$

$$\Delta S_{\text{body}} = C \ln \frac{T_2}{T_1}$$

$$\Delta S_{\text{reservoir}} = \frac{-C(T_2 - T_1)}{T_2}$$

$$\begin{aligned} \text{as, heat gained by body} &= \text{Heat lost by reservoir} \\ &= C(T_2 - T_1) \end{aligned}$$

$$\text{Thus, } \Delta S_{\text{universe}} = C \ln \frac{T_2}{T_1} - \frac{C(T_2 - T_1)}{T_2}$$

or, rearranging the terms,

$$\Delta S_{\text{universe}} = \frac{C(T_1 - T_2)}{T_2} - C \ln \left(\frac{T_1}{T_2} \right)$$

Hence proved.

8. Determine the rate of power loss due to irreversibility in a heat engine operating between temperatures of 1800 K and 300 K. Engine delivers 2 MW of power when heat is added at the rate of 5 MW.

Solution:

For irreversible operation of engine

$$\begin{aligned}\text{Rate of entropy generation} &= \frac{Q_1}{T_1} + \frac{Q_2}{T_2} \\ &= \frac{-5}{1800} + \frac{Q_2}{T_2}\end{aligned}$$

Also,

$$W = Q_1 - Q_2 = 5 \times 10^6 - Q_2$$

given

$$W = 2 \text{ MW} = 2 \times 10^6 \text{ W}$$

so

$$Q_2 = 3 \times 10^6 \text{ W}$$

Therefore,

$$\text{entropy generated} = \left(\frac{-5}{1800} + \frac{3}{300} \right) \times 10^6$$

$$\Delta S_{\text{gen}} = 7222.22 \text{ W/K}$$

Work lost

$$= T_2 \times \Delta S_{\text{gen}}$$

$$= 300 \times 7222.22$$

$$= 2.16 \times 10^6 \text{ W}$$

or

$$= 2.16 \text{ MW.}$$

$$\text{Work lost} = \mathbf{2.16 \text{ MW Ans.}}$$

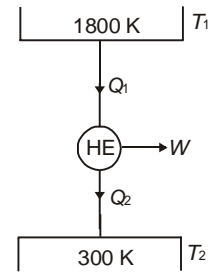


Fig. 5.11

9. A system at 500 K and a heat reservoir at 300 K are available for designing a work producing device. Estimate the maximum work that can be produced by the device if heat capacity of system is given as;

$$C = 0.05 T^2 + 0.10T + 0.085, \text{ J/K}$$

Solution:

System and reservoir can be treated as source and sink. Device thought of can be a Carnot engine operating between these two limits. Maximum heat available from system shall be the heat rejected till it's temperature drops from 500 K to 300 K.

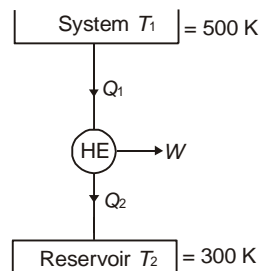


Fig. 5.12

Therefore,

$$\text{Maximum heat} \quad Q_1 = \int_{T_1}^{T_2} C \cdot dT$$

$$Q_1 = \int_{500}^{300} (0.05 T^2 + 0.10T + 0.085) dT$$

$$Q_1 = 1641.35 \times 10^3 \text{ J}$$

Entropy change of system, $\Delta S_{\text{system}} = \int_{500}^{300} C \frac{dT}{T}$

$$= -4020.043 \text{ J/K}$$

$$\Delta S_{\text{reservoir}} = \frac{Q_2}{T_2} = \frac{Q_1 - W}{T_2}$$

$$= \left(\frac{1641.35 \times 10^3 - W}{300} \right)$$

Also, we know from entropy principle

$$\Delta S_{\text{universe}} \geq 0$$

and $\Delta S_{\text{universe}} = \Delta S_{\text{system}} + \Delta S_{\text{reservoir}}$

Thus, upon substituting

$$(\Delta S_{\text{system}} + \Delta S_{\text{reservoir}}) \geq 0$$

$$\left\{ -4020.043 + \left(\frac{1641.35 \times 10^3 - W}{300} \right) \right\} \geq 0$$

or $1451.123 - \frac{W}{300} \geq 0$

$$1451.123 \geq \frac{W}{300}$$

$$W \leq 435337.10$$

or $W \leq 435.34 \text{ kJ}$

Hence Maximum work = **435.34 kJ**

For the given arrangement, device can produce maximum **435.34 kJ** of work. **Ans.**

10. Determine the change in enthalpy and entropy if air undergoes reversible adiabatic expansion from 3MPa, 0.05 m³ to 0.3 m³.

Solution:

For reversible adiabatic process, governing equation for expansion,

$$PV^{1.4} = \text{Constt.}$$

Also, for such process entropy change = 0.

Initial state : 3MPa, 0.05 m³

Final state : 0.3 m³

$$\text{Using } \frac{p_2}{p_1} = \left(\frac{V_1}{V_2} \right)^{1.4} \text{ or } V = \left(\frac{p_1 V_1^{1.4}}{p} \right)^{\frac{1}{1.4}}$$

we get

$$p_2 = 0.244 \text{ MPa}$$

From first law, second law and definition of enthalpy;

$$dH = T \cdot dS + V dp$$

or, for adiabatic process of reversible type, $dS = 0$.

$$dH = V \cdot dp$$

$$\int_1^2 dH = \int_1^2 V \cdot dp$$

Substituting V , and actual states

$$H_2 - H_1 = \int_{3000}^{244} \left(\frac{3000 \times 0.05^{1.4}}{p} \right)^{\frac{1}{1.4}} dp$$

or

$$\Delta H = 268.8 \text{ kJ}$$

Enthalpy change = **268.8 kJ.**

Entropy change = **0 Ans.**

11. During a free expansion 2 kg air expands from 1 m³ to 10 m³ volume in an insulated vessel.

Determine entropy change of (a) the air (b) the surroundings (c) the universe.

Solution:

During free expansion temperature remains same and it is an irreversible process. For getting change in entropy let us approximate this expansion process as a reversible isothermal expansion.

(a) Change in entropy of air

$$\begin{aligned} \Delta S_{\text{air}} &= m \cdot R \ln \frac{V_2}{V_1} \\ &= 2 \times 287 \ln \left(\frac{10}{1} \right) \end{aligned}$$

$$\begin{aligned} \Delta S_{\text{air}} &= 1321.68 \text{ J/K} \\ &= \mathbf{1321.68 \text{ J/K Ans.}} \end{aligned}$$

(b) During free expansion on heat is gained or lost to surroundings so,

$$\Delta S_{\text{surroundings}} = 0$$

Entropy change of surroundings,

$$= \mathbf{0 \text{ Ans.}}$$

(c) Entropy change of universe

$$\begin{aligned} \Delta S_{\text{universe}} &= \Delta S_{\text{air}} + \Delta S_{\text{surroundings}} \\ &= 1321.68 \text{ J/K} \\ &= \mathbf{1321.68 \text{ J/K Ans.}} \end{aligned}$$

12. Determine the change in entropy of 0.5 kg of air compressed polytropically from $1.013 \times 10^5 \text{ Pa}$ to 0.8 MPa and 800 K following index 1.2. Take $C_v = 0.71 \text{ kJ/kg} \cdot \text{K}$.

Solution:

Let initial and final states be denoted by 1 and 2.

For polytropic process pressure and temperature can be related as,

$$\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = \frac{T_2}{T_1}$$

or
$$T_2 = 800 \times \left(\frac{0.8 \times 10^6}{1.013 \times 10^5} \right)^{\frac{1.2-1}{1.2}}$$

Temperature after compression = 1128.94 K

Substituting in entropy change expression for polytropic process,

$$\begin{aligned} (s_2 - s_1) &= C_v \left(\frac{n-\gamma}{n-1} \right) \cdot \ln \frac{T_2}{T_1} \\ &= 0.71 \times 10^3 \left(\frac{1.2-1.4}{1.2-1} \right) \ln \left(\frac{1128.94}{800} \right) \\ &= -244.54, \text{ kJ/kg} \cdot \text{K} \end{aligned}$$

$$\begin{aligned} \text{Total entropy change} &= m (s_2 - s_1) \\ &= 0.5 \times 244.54 \end{aligned}$$

$$\Delta S = 122.27 \text{ J/K Ans.}$$

13. A heat engine is working between the starting temperature limits of T_1 and T_2 of two bodies. Working fluid flows at rate ' m ' kg/s and has specific heat at constant pressure as C_p . Determine the maximum obtainable work from engine.

Solution:

In earlier discussions we have seen that in order to have highest output from engine, it should operate in reversible cycle and satisfy following relation,

$$\oint \frac{dQ}{T} = 0$$

Let us assume that the two bodies shall attain final temperature of T_f and engine shall then get stopped. so,

$$\int_{T_1}^{T_f} mC_p \frac{dT}{T} + \int_{T_2}^{T_f} mC_p \frac{dT}{T} = 0$$

or
$$mC_p \left\{ \ln \left(\frac{T_f}{T_1} \right) + \ln \left(\frac{T_f}{T_2} \right) \right\} = 0$$

or
$$mC_p \cdot \ln \left(\frac{T_f^2}{T_1 \cdot T_2} \right) = 0$$

Here, $mC_p \neq 0$, so, $\ln \left(\frac{T_f^2}{T_1 \cdot T_2} \right) = 0$

or
$$T_f = \sqrt{T_1 \cdot T_2}$$

$$\begin{aligned} \text{Maximum work} &= Q_{\text{supplied}} - Q_{\text{rejected}} \\ &= mC_p(T_1 - T_f) - mC_p(T_f - T_2) \\ &= mC_p\{T_1 - 2T_f + T_2\} \end{aligned}$$

$$= mC_p \{T_1 - 2 \cdot \sqrt{T_1 \cdot T_2} + T_2\}$$

$$\text{Maximum work} = mC_p \{ \sqrt{T_1} - \sqrt{T_2} \}^2 \text{ Ans.}$$

14. A heat engine operates between source at 600 K and sink at 300 K. Heat supplied by source is 500 kcal/s. Evaluate feasibility of engine and nature of cycle for the following conditions. (i) Heat rejected being 200 kcal/s, (ii) Heat rejected being 400 kcal/s (iii) Heat rejected being 250 kcal/s.

Solution:

Clausius inequality can be used for cyclic process as given below; consider '1' for source and '2' for sink.

$$\oint \frac{dQ}{T} = \frac{Q_1}{T_1} - \frac{Q_2}{T_2}$$

(i) For $Q_2 = 200$ kcal/s

$$\oint \frac{dQ}{T} = \frac{500}{600} - \frac{200}{300} = 0.1667$$

As $\oint \frac{dQ}{T} > 0$, therefore under these conditions engine is *not possible*. **Ans.**

(ii) For $Q_2 = 400$ kcal/s

$$\oint \frac{dQ}{T} = \frac{500}{600} - \frac{400}{300} = -0.5$$

Here $\oint \frac{dQ}{T} < 0$, so engine is *feasible* and cycle is *irreversible*. **Ans.**

(iii) For $Q_2 = 250$ kcal/s

$$\oint \frac{dQ}{T} = \frac{500}{600} - \frac{250}{300} = 0$$

Here, $\oint \frac{dQ}{T} = 0$, so engine is *feasible* and cycle is *reversible*. **Ans.**

15. Along a horizontal and insulated duct the pressure and temperatures measured at two points are 0.5 MPa, 400 K and 0.3 MPa, 350 K. For air flowing through duct determine the direction of flow.

Solution:

Let the two points be given as states 1 and 2, so,

$$p_1 = 0.5 \text{ MPa}, T_1 = 400 \text{ K}$$

$$p_2 = 0.3 \text{ MPa}, T_2 = 350 \text{ K}$$

Let us assume flow to be from 1 to 2

$$\text{So entropy change} \quad \Delta s_{1-2} = s_1 - s_2 = C_p \ln \left(\frac{T_1}{T_2} \right) - R \ln \left(\frac{p_1}{p_2} \right)$$

For air,

$$R = 0.287 \text{ kJ/kg} \cdot \text{K}$$

$$C_p = 1.004 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Hence} \quad s_1 - s_2 = 1.004 \ln \left(\frac{400}{350} \right) - 0.287 \ln \left(\frac{0.5}{0.3} \right)$$

$$= -0.01254 \text{ kJ/kg} \cdot \text{K}$$

or $s_1 - s_2 = 0.01254 \text{ kJ/kg} \cdot \text{K}$

It means $s_2 > s_1$ hence the assumption that flow is from 1 to 2 is correct as from second law of thermodynamics the entropy increases in a process i.e. $s_2 \geq s_1$.

Hence flow occurs from 1 to 2 i.e. from **0.5 MPa, 400 K** to **0.3 MPa & 350 K** **Ans.**

16. An ideal gas is heated from temperature T_1 to T_2 by keeping its volume constant. The gas is expanded back to its initial temperature according to the law $p v^n = \text{constant}$. If the entropy change in the two processes are equal, find the value of 'n' in terms of adiabatic index γ . [U.P.S.C. 1997]

Solution:

During constant volume process change in entropy $\Delta S_{12} = mc_v \cdot \ln \frac{T_2}{T_1}$

Change in entropy during polytropic process, $\Delta S_{23} = mc_v \left(\frac{\gamma - n}{n - 1} \right) \ln \frac{T_2}{T_1}$

Since the entropy change is same, so

$$\Delta S_{12} = \Delta S_{23}$$

$$mc_v \ln \frac{T_2}{T_1} = mc_v \left(\frac{\gamma - n}{n - 1} \right) \ln \frac{T_2}{T_1}$$

or

$$\boxed{n = \frac{\gamma + 1}{2}} \text{ Ans.}$$

17. A closed system executed a reversible cycle 1–2–3–4–5–6–1 consisting of six processes. During processes 1–2 and 3–4 the system receives 1000 kJ and 800 kJ of heat, respectively at constant temperatures of 500 K and 400 K, respectively. Processes 2–3 and 4–5 are adiabatic expansions in which the steam temperature is reduced from 500 K to 400 K and from 400 K to 300 K respectively. During process 5–6 the system rejects heat at a temperature of 300 K. Process 6–1 is an adiabatic compression process. Determine the work done by the system during the cycle and thermal efficiency of the cycle.

[U.P.S.C. 1995]

Solution:

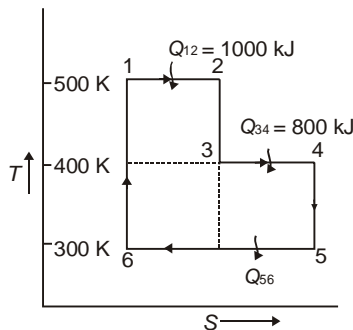


Fig. 5.13

$$\text{Heat added} = Q_{12} + Q_{34}$$

$$\text{Total heat added} = 1800 \text{ kJ}$$

For heat addition process 1–2,

$$Q_{12} = T_1 \cdot (S_2 - S_1)$$

$$1000 = 500 \cdot (S_2 - S_1)$$

or,

$$S_2 - S_1 = 2$$

For heat addition process 3–4, $Q_{34} = T_3 \cdot (S_4 - S_3)$

$$800 = 400 \cdot (S_4 - S_3)$$

or,

$$S_4 - S_3 = 2$$

Heat rejected in process 5–6

$$Q_{56} = T_5 \cdot (S_5 - S_6)$$

$$= T_5 \cdot \{(S_2 - S_1) + (S_4 - S_3)\}$$

$$Q_{56} = 300 \cdot \{2 + 2\} = 1200 \text{ kJ}$$

Net work done = Net heat

$$= (Q_{12} + Q_{34}) - Q_{56}$$

$$W_{\text{net}} = 1800 - 1200$$

$$W_{\text{net}} = 600 \text{ kJ}$$

$$\text{Thermal efficiency of cycle} = \frac{W_{\text{net}}}{\text{Heat added}}$$

$$= \frac{600}{1800}$$

$$= 0.3333 \text{ or } 33.33\%$$

<p>Work done = 600 kJ</p> <p>Thermal efficiency = 33.33% Ans.</p>

18. A reversible heat engine has heat interaction from three reservoirs at 600 K, 700 K and 800 K. The engine rejects 10 kJ/s to the sink at 320 K after doing 20 kW of work. The heat supplied by reservoir at 800 K is 70% of the heat supplied by reservoir at 700 K then determine the exact amount of heat interaction with each high temperature reservoir.

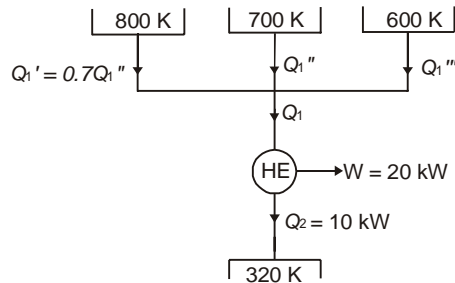


Fig. 5.14

Solution:

Let heat supplied by reservoir at 800 K, 700 K and 600 K be Q'_1 , Q''_1 , Q'''_1 .

Here,

$$Q_1 - Q_2 = W$$

\Rightarrow

$$Q_1 = 30 \text{ kJ/s}$$

Also given that,

$$Q'_1 = 0.7 Q''_1$$

$$Q'''_1 = Q_1 - (0.7 Q''_1 + Q''_1)$$

$$Q'''_1 = Q_1 - 1.7 Q''_1$$

For reversible engine

$$\Rightarrow \frac{Q'_1}{800} + \frac{Q''_1}{700} + \frac{Q'''_1}{600} - \frac{Q_2}{320} = 0$$

$$\Rightarrow \frac{0.7Q'_1}{800} + \frac{Q''_1}{700} + \frac{(Q_1 - 1.7Q''_1)}{600} - \frac{10}{320} = 0$$

$$\Rightarrow \begin{aligned} Q''_1 &= 88.48 \text{ kJ/s} \\ Q'_1 &= 61.94 \text{ kJ/s} \\ Q'''_1 &= -120.42 \text{ kJ/s} \end{aligned}$$

Heat supplied by reservoir at 800 K = **61.94 kJ/s**

Heat supplied by reservoir at 700 K = **88.48 kJ/s**

Heat supplied to reservoir at 600 K = **120.42 kJ/s Ans.**

19. A rigid insulated tank is divided into two chambers of equal volume of 0.04 m^3 by a frictionless, massless thin piston, initially held at position with a locking pin. One chamber is filled with air at 10 bar & 25°C and other chamber is completely evacuated. Subsequently pin is removed and air comes into equilibrium. Determine whether the process is reversible or irreversible. Consider, $R = 0.287 \text{ kJ/kg.K}$ and $c_v = 0.71 \text{ kJ/kg.K}$.

Solution:

Let us assume process to be adiabatic and so the heat interaction would not be there. Also in view of this expansion being frictionless expansion there would be no work done, i.e., $W = 0$, $Q = 0$.

Let initial and final states be indicated by subscripts 1 and 2.

Using first law of thermodynamics;

$$dQ = dW + dU$$

$$\Rightarrow 0 = 0 + mc_v(T_2 - T_1)$$

$$\Rightarrow T_1 = T_2 = 298 \text{ K}$$

Volume changes are, $V_2 = 2V_1 = 0.08 \text{ m}^3$

Using gas laws, $p_1 V_1 = p_2 V_2 \Rightarrow p_2 = 0.5 p_1 = 5 \text{ bar}$

$$\text{Initial Mass of air, } m_1 = \frac{p_1 V_1}{RT_1} \Rightarrow m_1 = \frac{10 \times 10^2 \times 0.04}{0.287 \times 298} = 0.4677 \text{ kg}$$

$$\text{Change of entropy, } (S_2 - S_1) = mR \ln \left(\frac{V_2}{V_1} \right) + mc_v \ln \left(\frac{T_2}{T_1} \right)$$

$$\begin{aligned} (S_2 - S_1) &= 0.4677 \{0.287 \times \ln 2 + 0.71 \ln 1\} \\ &= 0.09304 \text{ kJ/K} \end{aligned}$$

From reversibility/irreversibility considerations the entropy change should be compared with

$$\int_1^2 \left(\frac{dQ}{T} \right)_{rev}$$

In this case; $\int_1^2 \left(\frac{dQ}{T} \right)_{rev} = 0$, while entropy change = 0.09304 kJ/K.

Here $(S_2 - S_1) > \int_1^2 \left(\frac{dQ}{T} \right)_{rev}$, which means the **process is irreversible.** **Ans.**

20. Two tanks A and B are connected through a pipe with valve in between. Initially valve is closed and tanks A and B contain 0.6 kg of air at 90°C, 1 bar and 1 kg of air at 45°C, 2 bar respectively. Subsequently valve is opened and air is allowed to mix until equilibrium. Considering the complete system to be insulated determine the final temperature, final pressure and entropy change.

Solution:

In this case due to perfectly insulated system, $Q = 0$, Also $W = 0$

Let the final state be given by subscript f' and initial states of tank be given by subscripts 'A' and 'B'. $p_A = 1$ bar, $T_A = 363$ K, $m_A = 0.6$ kg; $T_B = 318$ K, $m_B = 1$ kg, $p_B = 2$ bar

$$\Delta Q = \Delta W + \Delta U$$

$$0 = 0 + \{(m_A + m_B) \cdot C_v \cdot T_f - (m_A \cdot C_v \cdot T_A) - (m_B \cdot C_v \cdot T_B)\}$$

$$T_f = \frac{(m_A \cdot C_v \cdot T_A + m_B \cdot C_v \cdot T_B)}{(m_A + m_B) \cdot C_v} = \frac{(0.6 \times 363 + 1 \times 318)}{(0.6 + 1)}$$

$$T_f = 334.88 \text{ K, Final temperature} = \mathbf{334.88 \text{ K}} \quad \mathbf{Ans.}$$

Using gas law for combined system after attainment of equilibrium,

$$p_f = \frac{(m_A + m_B) \cdot R T_f}{(V_A + V_B)}$$

$$V_A = \frac{m_A R T_A}{p_A}; V_B = \frac{m_B R T_B}{p_B}$$

$$V_A = 0.625 \text{ m}^3; V_B = 0.456 \text{ m}^3$$

$$\Rightarrow p_f = \frac{(1 + 0.6) \times 0.287 \times 334.88}{(0.625 + 0.456)} = 142.25 \text{ kPa}$$

$$\text{Final pressure} = \mathbf{142.25 \text{ kPa}} \quad \mathbf{Ans.}$$

Entropy change;

$$\Delta S = \{(m_A + m_B) \cdot s_f - (m_A \cdot s_A + m_B \cdot s_B)\}$$

$$\Delta S = \{m_A (s_f - s_A) + m_B (s_f - s_B)\}$$

$$= \left\{ m_A \left(C_p \ln \frac{T_f}{T_A} - R \ln \frac{p_f}{p_A} \right) + m_B \left(C_p \ln \frac{T_f}{T_B} - R \ln \frac{p_f}{p_B} \right) \right\}$$

Considering $C_p = 1.005 \text{ kJ/kg.K}$

$$\Delta S = \left\{ 0.6 \left(1.005 \ln \frac{334.88}{363} - 0.287 \ln \frac{142.25}{100} \right) + 1 \left(1.005 \ln \frac{334.88}{318} - 0.287 \ln \frac{142.25}{200} \right) \right\}$$

$$\Delta S = \{ -0.1093 + 0.14977 \}$$

$$= 0.04047 \text{ kJ/K}$$

Entropy produced = **0.04047 kJ/K** Ans.

21. Three tanks of equal volume of 4m^3 each are connected to each other through tubes of negligible volume and valves in between. Tank A contains air at 6 bar, 90°C , tank B has air at 3 bar, 200°C and tank C contains nitrogen at 12 bar, 50°C . Considering adiabatic mixing determine (i) the entropy change when valve between tank A and B is opened until equilibrium, (ii) the entropy change when valves between tank C; tank A and tank B are opened until equilibrium. Consider $R_{\text{Air}} = 0.287 \text{ kJ/kg.K}$, $g_{\text{Air}} = 1.4$, $R_{\text{Nitrogen}} = 0.297 \text{ kJ/kg.K}$ and $g_{\text{Nitrogen}} = 1.4$.

Solution:

Let states in tanks A, B & C be denoted by subscripts A, B & C respectively.

(i) When tank A and B are connected;

$$C_{v,\text{Air}} = \frac{R_{\text{Air}}}{(r_{\text{Air}} - 1)} = 0.718 \text{ kJ/kg.K}$$

After adiabatic mixing let the states be denoted by subscript 'D'.

Internal energy before mixing = Internal energy after mixing

$$m_A \cdot C_{v,\text{Air}} \cdot T_A + m_B \cdot C_{v,\text{Air}} \cdot T_B = (m_A + m_B) \cdot C_{v,\text{Air}} \cdot T_D$$

$$\Rightarrow T_D = \frac{(m_A \cdot T_A + m_B \cdot T_B)}{(m_A + m_B)}$$

$$\text{Using gas laws, } m_A = \frac{p_A \cdot V_A}{R_{\text{Air}} T_A} = \frac{6 \times 10^2 \times 4}{0.287 \times 363} = 23.04 \text{ kg}$$

$$m_B = \frac{p_B \cdot V_B}{R_{\text{Air}} T_B} = \frac{3 \times 10^2 \times 4}{0.287 \times 573} = 7.29 \text{ kg}$$

$$\text{Final temperature } \Rightarrow T_D = \left\{ \frac{(23.04 \times 363) + (7.29 \times 573)}{(23.04 + 7.29)} \right\} = 413.47 \text{ K}$$

$$\text{Final pressure, } p_D = \frac{R_{\text{Air}} T_D \cdot m_D}{V_D} = \frac{R_{\text{Air}} T_D \cdot (m_A + m_B)}{(V_A + V_B)}$$

$$p_D = \frac{0.287 \times 413.47 \times (23.04 + 7.29)}{(4 + 4)} = 449.89 \text{ kPa}$$

$$p_D = 4.4989 \text{ bar}$$

Entropy change, $\Delta_{\Delta S} = (S_D - S_A) + (S_D - S_B)$

$$\Delta S = \left\{ (m_A \cdot C_{p,Air} \ln \frac{T_D}{T_A} - m_A \cdot R_{Air} \ln \frac{p_D}{p_A}) + \right. \\ \left. (m_B \cdot C_{p,Air} \ln \frac{T_D}{T_B} - m_B \cdot R_{Air} \ln \frac{p_D}{p_B}) \right\} \\ \Delta S = \left\{ \left(23.04 \times 1.005 \ln \left(\frac{413.47}{363} \right) \right) - \right. \\ \left(23.04 \times 0.287 \ln \left(\frac{449.89}{600} \right) \right) + \left(7.29 \times 1.005 \ln \left(\frac{413.47}{573} \right) \right) \\ \left. - \left(7.29 \times 0.287 \ln \left(\frac{449.89}{300} \right) \right) \right\} \\ = \{ 3.014 + 1.904 + (-2.391) - 0.848 \}$$

Entropy change, $\Delta S = \mathbf{1.679 \text{ kJ/K}}$ **Ans.**

(ii) After the three tanks A, B, and C are interconnected then the equilibrium will be attained amongst three. Equilibrium between A & B will result in state D as estimated in part (i) above. Thus it may be considered as the mixing of state D and nitrogen in tank C. Let the final state attained be 'F'.

After adiabatic mixing the final gas properties (as a result of mixing of air, state D and nitrogen, state C) may be estimated as under

$$m_c = \frac{p_c \cdot V_c}{R_{Nitrogen} \cdot T_c} = \frac{12 \times 10^2 \times 4}{0.297 \times 323} \\ m_c = 50.04 \text{ kg} ; m_D = m_A + m_B = 30.33 \text{ kg} \\ C_{v, Nitrogen} = \frac{R_{Nitrogen}}{\gamma_{Nitrogen} - 1} = \frac{0.297}{(1.4 - 1)} = 0.7425 \text{ kJ/kgK} \\ C_{p, Nitrogen} = \gamma_{Nitrogen} \cdot C_{v, Nitrogen} = 1.4 \times 0.7425 \\ = 1.0395 \text{ kJ/kg.K} \\ m_F = (m_D + m_c) = (23.04 + 7.29 + 50.04) = 80.37 \text{ kg} \\ C_{v,F} = \{ (m_D \cdot C_{v, Air} + m_c \cdot C_{v, Nitrogen}) / (m_D + m_c) \} \\ = 0.733 \text{ kJ/kg.K} \\ R_F = \frac{m_D \cdot R_{Air} + m_c \cdot R_{Nitrogen}}{(m_D + m_c)}$$

$$= \left\{ \frac{(30.33 \times 0.287 + 50.04 \times 0.297)}{80.37} \right\}$$

$$R_F = 0.293 \text{ kJ/kg.K}$$

By first law of thermodynamics,

$$\Delta Q = \Delta W + \Delta U; \quad \text{here } \Delta Q = 0, \Delta W = 0$$

Internal energy before mixing = Internal energy after mixing

$$m_D \cdot C_{v,Air} \cdot T_D + m_C \cdot C_{v,Nitrogen} \cdot T_C = (m_D + m_C) C_{v_F} \cdot T_F$$

$$\Rightarrow (30.33 \times 0.718 \times 413.47) + (50.04 \times 0.7425 \times 323) = 80.37 \times 0.733 \times T_F$$

$$\Rightarrow T_F = 356.55 \text{ K; Final temperature after mixing} = 356.55 \text{ K}$$

Final pressure after mixing;

$$p_F = \frac{m_F \cdot R_F \cdot T_F}{V_F} = \frac{(80.37 \times 0.293 \times 356.55)}{(4 + 4 + 4)}$$

$$p_F = \mathbf{699.68 \text{ kPa; Final pressure.}}$$

Entropy change after mixing, $\Delta S = (S_F - S_D) + (S_F - S_C)$

$$\begin{aligned} \Delta S &= m_D \left\{ C_{p,Air} \ln \frac{T_F}{T_D} - R_{Air} \ln \frac{p_F}{p_D} \right\} + m_C \left\{ C_{p,Nitrogen} \ln \frac{T_F}{T_C} - R_{Nitrogen} \ln \frac{p_F}{p_C} \right\} \\ &= 30.33 \left\{ 1.005 \ln \frac{356.55}{413.47} - 0.287 \ln \frac{699.68}{449.89} \right\} + 50.04 \left\{ 1.0395 \ln \frac{356.55}{323} - 0.297 \ln \frac{699.68}{1200} \right\} \\ &= -8.359 + 13.158 \end{aligned}$$

$$\Delta S = \mathbf{4.799 \text{ kJ/kg.K} \quad \text{Ans.}}$$

EXERCISE

- 5.1 Discuss the significance of Clausius inequality.
- 5.2 Define the 'entropy'. Also explain how it is a measure of irreversibility?
- 5.3 Explain the difference between isentropic process and adiabatic process.
- 5.4 How does the second law of thermodynamics overcome limitations of first law of thermodynamics?
- 5.5 Show that entropy of universe is increasing.
- 5.6 Is the adiabatic mixing of fluids irreversible? If yes, explain.
- 5.7 Why does entropy generally increase? Explain.
- 5.8 Explain the entropy principle and apply it to a closed system.
- 5.9 How the feasibility of any process can be ensured?
- 5.10 Give the third law of thermodynamics.
- 5.11 Explain why the slope of constant volume line is more than the slope of constant pressure line on T-S diagram.
- 5.12 Explain, whether the arrangement shown below for a reversible engine is feasible. If no then why? Give the correct arrangement.

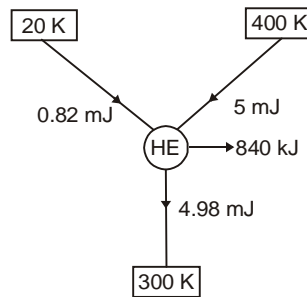


Fig. 5.15

- 5.13** Using second law of thermodynamics check the following and also indicate nature of cycle.
- Heat engine receiving 1000 kJ of heat from a reservoir at 500 K and rejecting 700 kJ heat to a sink at 27°C.
 - Heat engine receiving 1000 kJ of heat from a reservoir at 500 K and rejecting 600 kJ of heat to a sink at 27°C.
- (i) Possible, irreversible cycle
(ii) Possible, reversible cycle
- 5.14** Determine the change in entropy of air during it's heating in a perfectly insulated rigid tank having 5 kg of air at 2 atm. Air is heated from 40°C to 80°C temperature.
- 5.15** Calculate change in entropy of air during the process in which a heat engine rejects 1500 kJ of heat to atmosphere at 27°C during its operation. [5 kJ/K]
- 5.16** Determine the final temperature and total entropy change during a process in which metal piece of 5 kg at 200°C falls into an insulated tank containing 125 kg of water at 20°C. Specific heat of metal = 0.9 kJ/kg.K, Specific heat of water = 4.184 kJ/kg.K. [21.53°C, 0.592 kJ/K]
- 5.17** Show that for air undergoing isentropic expansion process;
- $$ds = c_p \frac{dv}{v} + c_v \frac{dp}{p}$$
- 5.18** Determine the change in entropy of air, if it is heated in a rigid tank from 27°C to 150°C at low pressure. [246.8 J/kg.K]
- 5.19** An electrical resistance of 100 ohm is maintained at constant temperature of 27°C by a continuously flowing cooling water. What is the change in entropy of the resistor in a time interval of one minute ? [0]
- 5.20** A water tank of steel is kept exposed to sun. Tank has capacity of 10 m³ and is full of water. Mass of steel tank is 50 kg and during bright sun temperature of water is 35°C and by the evening water cools down to 30°C. Estimate the entropy change during this process. Take specific heat for steel as 0.45 kJ/kg.K and water as 4.18 kJ/kg.K. [5.63 kJ/K]
- 5.21** Heat engine operating on Carnot cycle has a isothermal heat addition process in which 1 MJ heat is supplied from a source at 427°C. Determine change in entropy of (i) working fluid, (ii) source, (iii) total entropy change in process. [1.43 kJ/K, -1.43 kJ/K, 0]
- 5.22** A system operating in thermodynamic cycle receives Q_1 heat at T_1 temperature and rejects Q_2 at temperature T_2 . Assuming no other heat transfer show that the net work developed per cycle is given as,

$$W_{\text{cycle}} = Q_1 \frac{Q_1}{T_1} + \left(1 - \frac{T_2}{T_1}\right) - T_2 \cdot S_{\text{gen}}$$

where S_{gen} is amount of entropy produced per cycle due to irreversibilities in the system.

- 5.23** A rigid tank contains 5 kg of ammonia at 0.2 MPa and 298 K. Ammonia is then cooled until its pressure drops to 80 kPa. Determine the difference in entropy of ammonia between initial and final state.
[−14.8 kJ/K]
- 5.24** Determine the change in entropy in each of the processes of a thermodynamic cycle having following processes;
 (i) Constant pressure cooling from 1 to 2, $P_1 = 0.5$ MPa, $V_1 = 0.01$ m³
 (ii) Isothermal heating from 2 to 3, $P_3 = 0.1$ MPa, $T_3 = 25^\circ\text{C}$, $V_3 = 0.01$ m³
 (iii) Constant volume heating from 3 to 1.
 Take $C_p = 1$ kJ/kg · K for perfect gas as fluid.
 [−0.0188 kJ/kg · K, 0.00654 kJ/kg · K, 0.0134 kJ/kg · K]
- 5.25** Conceptualize some toys that may approach close to perpetual motion machines. Discuss them in detail.
- 5.26** Heat is added to air at 600 kPa, 110°C to raise its temperature to 650°C isochorically. This 0.4 kg air is subsequently expanded polytropically up to initial temperature following index of 1.32 and finally compressed isothermally up to original volume. Determine the change in entropy in each process and pressure at the end of each process. Also show processes on p-V and T-s diagram, Assume $C_v = 0.718$ kJ/kg.K, $R = 0.287$ kJ/kg.K
 [0.2526 kJ/K, 0.0628 kJ/K, 0.3155 kJ/K
 1445 kPa, 38.45 kPa]
- 5.27** Air expands reversibly in a piston-cylinder arrangement isothermally at temperature of 260°C while its volume becomes twice of original. Subsequently heat is rejected isobarically till volume is similar to original. Considering mass of air as 1 kg and process to be reversible determine net heat interaction and total change in entropy. Also show processes on T-s diagram.
 [− 161.8 kJ/kg, − 0.497 kJ/kg.K]
- 5.28** Ethane gas at 690 kPa, 260°C is expanded isentropically up to pressure of 105 kPa, 380K. Considering initial volume of ethane as 0.06 m³ determine the work done if it behaves like perfect gas.
 Also determine the change in entropy and heat transfer if the same ethane at 105 kPa, 380K is compressed up to 690 kPa following $p.V^{1.4} = \text{constant}$.
 [0.8608 kJ/K, 43.57 kJ]
- 5.29** Determine the net change in entropy and net flow of heat from or to the air which is initially at 105 kPa, 15°C. This 0.02 m³ air is heated isochorically till pressure becomes 420 kPa and then cooled isobarically back up to original temperature.
 [− 0.011 kJ/K, − 6.3 kJ]
- 5.30** Air initially at 103 kPa, 15°C is heated through reversible isobaric process till it attains temperature of 300°C and is subsequently cooled following reversible isochoric process up to 15°C temperature. Determine the net heat interaction and net entropy change.
 [101.9 kJ, 0.246 kJ/K]
- 5.31** Calculate the entropy change when 0.05 kg of carbon dioxide is compressed from 1 bar, 15°C to 830 kPa pressure and 0.004 m³ volume. Take $C_p = 0.88$ kJ/kg.K. This final state may be attained following isobaric and isothermal process.
 [0.0113 kJ/K]
- 5.32** Two insulated tanks containing 1 kg air at 200 kPa, 50°C and 0.5 kg air at 100 kPa, 80°C are connected through pipe with valve. Valve is opened to allow mixing till the equilibrium. Calculate the amount of entropy produced.
 [0.03175 kJ/K]

Thermodynamic Properties of Pure Substance

6.1 INTRODUCTION

Engineering systems have an inherent requirement of some substance to act as working fluid i.e. transport agent for energy and mass interactions. Number of working fluids are available and are being used in different systems suiting to the system requirements. Steam is also one of such working fluids used exhaustively because of its favourable properties. In thermal power plants steam is being extensively used. Water has capability to retain its chemical composition in all of its' phases i.e. steam and ice, and also it is almost freely available as gift of nature.

Pure substance refers to the “*substance with chemical homogeneity and constant chemical composition.*” H_2O is a pure substance as it meets both the above requirements. Any substance, which undergoes a chemical reaction, cannot be pure substance.

6.2 PROPERTIES AND IMPORTANT DEFINITIONS

Pure substance as defined earlier is used for operating various systems, such as steam is used for power generation in steam power plants. Hence, for thermodynamic analysis thermodynamic properties are required. Pressure and temperature are the properties that can be varied independently over wide range in a particular phase. Therefore, the behaviour of properties of pure substance have to be studied and mathematical formulations be made for their estimation.

Various dependent properties discussed ahead shall be enthalpy, internal energy, specific volume, entropy etc.

Some of terms used in discussion ahead are given as under.

- (a) *Sensible heating*: It refers to the heating of substance in single phase. It causes rise in temperature of substance. In case of cooling in above conditions it shall be called sensible cooling.
- (b) *Latent heating*: It is the heating of substance for causing its phase change without any change in its temperature. If heat is extracted for causing phase change without any change in its temperature it will be called latent cooling.
- (c) *Normal boiling point*: It is the temperature at which vapour pressure equals to atmospheric pressure and at this temperature phase change from liquid to gas begins.
- (d) *Melting point*: It is the temperature at which phase change from solid to liquid takes place upon supplying latent heat.
- (e) *Saturation states*: Saturation state of a substance refers to the state at which its phase transformation takes place without any change in pressure and temperature. These can be saturated solid state, saturated liquid state and saturated vapour state. For example saturated

vapour state refers to the state of water at which its phase changes to steam without varying pressure and temperature.

- (f) *Saturation pressure*: It is the pressure at which substance changes its phase for any given temperature. Such as at any given temperature water shall get converted into steam at a definite pressure only, this pressure is called saturation pressure corresponding to given temperature. For water at 100°C the saturation pressure is 1 atm pressure.
- (g) *Saturation temperature*: It refers to the temperature at which substance changes its phase for any given pressure. For water at 1 atm pressure the saturation temperature is 100°C.
- (h) *Triple point*: Triple point of a substance refers to the state at which substance can coexist in solid, liquid and gaseous phase in equilibrium. For water it is 0.01°C i.e. at this temperature ice, water and steam can coexist in equilibrium. Table 6.1 given below gives triple point data for number of substances.

Table 6.1 Triple point and critical point

Substance	Triple point		Critical point	
	Pressure, kPa	Temperature, °C	Pressure, MPa	Temperature, °C
Water	0.611	0.01	22.12	374.15
Helium	5.1	– 271	0.23	– 268
Hydrogen	7.0	– 259	1.3	– 213
Oxygen	0.15	– 219	5.0	– 119
Nitrogen	12.5	– 210	3.4	– 147
Ammonia	6.1	– 78	11.3	132
Carbondioxide	517	– 57	7.39	31
Mercury	1.65×10^{-7}	– 39	18.2	899

- (i) *Critical states*: “Critical state refers to that state of substance at which liquid and vapour coexist in equilibrium.” In case of water at 22.12 MPa, and 374.15°C the water and vapour coexist in equilibrium, thus it is the highest pressure and temperature at which distinguishable water and vapour exist together. Data for critical state of many substances is given in the table 6.1. Specific volume at critical point for water is 0.00317 m³/kg.
- (j) *Dryness fraction*: It is the mass fraction of vapour in a mixture of liquid and vapour at any point in liquid-vapour mixture region. It is generally denoted by ‘x’. It is also called quality of steam.
- (k) *Compressed liquid or subcooled liquid*: Liquid at temperature less than saturation temperature corresponding to a given pressure is called compressed liquid or subcooled liquid. Degree of subcooling is given by the temperature difference between liquid temperature and saturation temperature of liquid at given pressure.
Degree of subcooling = Saturation temperature at given pressure – Temperature of liquid.
- (l) *Superheated steam*: Steam having temperature more than the saturation temperature corresponding to given pressure is called superheated steam. Amount of superheating is quantified by degree of superheating. Degree of superheating is given by difference between temperature of steam and saturation temperature at given pressure.
Degree of superheating = Temperature of steam – Saturation temperature at given pressure.

6.3 PHASE TRANSFORMATION PROCESS

Let us study phase transformation from ice to steam by taking ice at -20°C in an open vessel i.e. at atmospheric pressure, and heat it from bottom. Salient states passed through the phase change are as given under. Melting point of ice is 0°C and boiling point of water is 100°C for water at 1 atmospheric pressure.

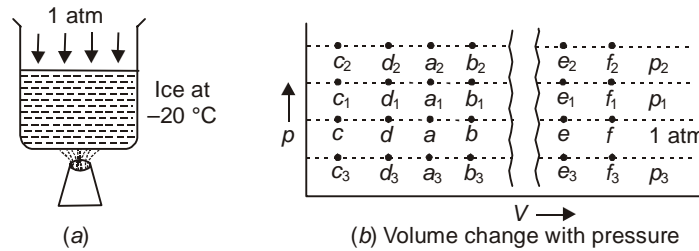


Fig. 6.1 Phase transformation process

Say initial state is given by 'a' at -20°C and 1 atmospheric pressure.

- (i) Upon heating the ice its temperature increases from -20°C to 0°C while being in solid phase. Temperature increase is accompanied by increase in volume and new state 'b' is attained. This heating is sensible heating as heating causes increase in temperature in same phase.
- (ii) After ice reaches to 0°C , the melting point, it is ready for phase transformation into water. Further heat addition to it causes melting and now water at 0°C is available. This heating is called latent heating and heat added is called latent heat. New state attained is 'c' and volume gets reduced due to typical characteristic of water. As defined earlier state 'b' is called *saturated solid state* as phase can change here without any change in pressure and temperature. State 'c' is called *saturated liquid state* with respect to solidification.
- (iii) Further heating of water at 0°C shall cause increase in its temperature upto 100°C . This heat addition is accompanied by increase in volume and state changes from 'c' to 'd' as shown on p - V diagram. Here typical behaviour of water from 0 to 4°C is neglected. This heating is sensible heating in liquid phase. State 'd' is called *saturated liquid state* with respect to vaporization. Thus, there are two saturated liquid states 'c' and 'd' depending upon direction of transformation.
- (iv) Water at 100°C and 1 atmosphere is ready for getting vaporized with supply of latent heat of vaporization. Upon adding heat to it the phase transformation begins and complete liquid gradually gets transformed into steam at state 'e'. This phase change is accompanied by large increase in volume. Heating in this zone is called latent heating. State 'e' is called *saturated vapour state* or *saturated steam state*.
- (v) Steam at 100°C upon heating becomes hotter and its temperature rises. Say, the heating causes temperature rise upto 200°C . This increase in temperature is also accompanied by increase in volume up to state 'f' as shown on p - V diagram. This heating is sensible heating in gaseous phase.

Similar phase transformations can be realized at different pressures and such salient states be identified and marked. Joining all saturated solid states at different pressures results in a locii, which is called "saturated solid line." Similarly, joining all saturated liquid states with respect to solidification and saturated liquid states with respect to vaporization results in two 'saturated liquid lines'. Locii of all saturated vapour states at different pressure generates 'saturated vapour line'. The lines thus obtained are shown in Fig. 6.2 detailing p - V diagram for water. Point at which "saturated liquid line" with respect

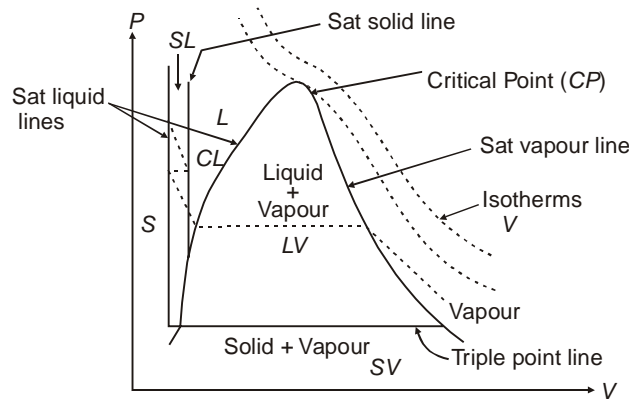


Fig. 6.2 P - V diagram for water

to vaporization meets with “saturated vapour line” is called “Critical point”. Critical point is also sometimes referred to as “Critical state”.

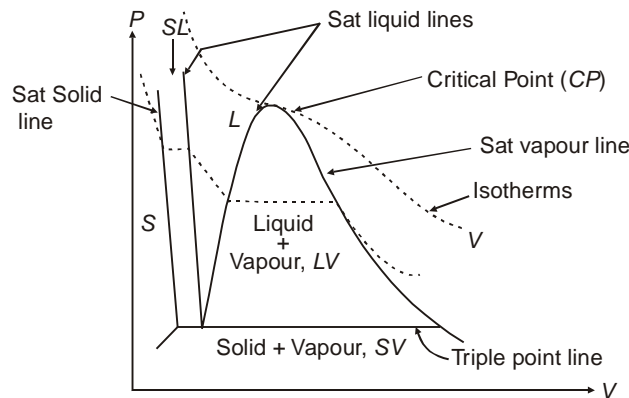


Fig. 6.3 P - V diagram for Carbon dioxide

On P - V diagram region marked S shows the solid region, mark SL shows solid-liquid mixture region, mark LV shows liquid-vapour mixture region, mark CL shows compressed liquid region. Triple point line is also shown and it indicates coexistence of solid, liquid and gas in equilibrium. Region marked SV and lying below triple point line is sublimation region where solid gets transformed directly into vapour upon supply of latent heat of sublimation. P - V diagram for substance which has different characteristics of contraction upon freezing is also shown here, Fig. 6.3. Carbon dioxide is such substance. Difference in p - V diagrams for two different substances (water and CO_2) may be understood from here.

6.4 GRAPHICAL REPRESENTATION OF PRESSURE, VOLUME AND TEMPERATURE

Graphical representations based on variation of thermodynamic properties can be obtained from the study of actual phase transformation process. In earlier discussion the variation of pressure and volume

during phase transformation from ice to steam has been shown and explained. On these lines the standard graphical representations in terms of p - V , T - V , p - T and p - V - T for water can be obtained. p - V diagram as obtained has already been discussed in article 6.2.

(i) **T - V diagram:** It gives variation of temperature with volume. Let us look at different steps involved in phase transformation and how are the temperature variations.

$a - b$: Temperature rises from -20°C to 0°C with volume increase, (ice)

$b - c$: Temperature remains constant at 0°C due to phase change and volume decreases, (ice to water)

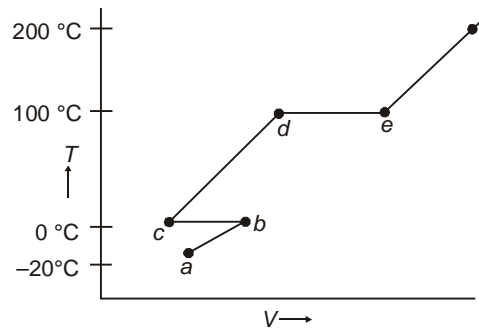


Fig. 6.4 Temperature-Volume variation at 1 atm pressure

$c - d$: Temperature increases from 0°C to 100°C and volume increases, (water).

$d - e$: Temperature remains constant at 100°C , phase changes from liquid to vapour and volume increases, (water to steam).

$e - f$: Temperature increases from 100°C to 200°C and volume increases, (steam).

Similar to above, the T - V variations at different pressures can be obtained and identical states i.e. saturation states be joined to get respective saturation lines as shown in Fig. 6.5 ahead.

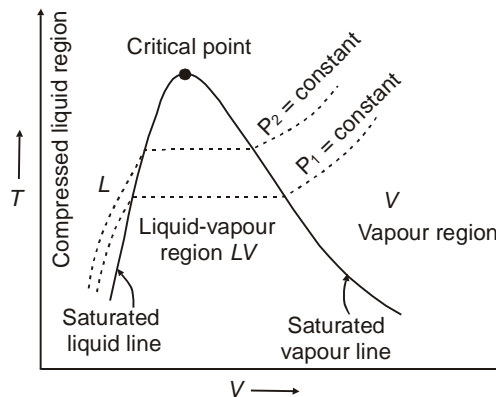


Fig. 6.5 T - V diagram for water

(ii) **P - T diagram:** It is the property diagram having pressure on Y-axis and temperature on X-axis.

This can also be obtained by identifying and marking salient states during phase transformation and subsequently generating locii of identical states at different pressures.

For the phase change discussed in article 6.3, pressure and temperature variation shall be as described ahead. This phase change occurs at constant pressure of 1 atm.

- $a - b$: Temperature rises from -20°C to 0°C , phase is solid.
 $b - c$: Temperature does not rise, phase changes from ice to water (solid to liquid)
 $c - d$: Temperature rises from 0°C to 100°C , phase is liquid
 $d - e$: Temperature does not rise, phase changes from liquid to gas (water to steam)
 $e - f$: Temperature rises from 100°C to 200°C , phase is gas (steam).

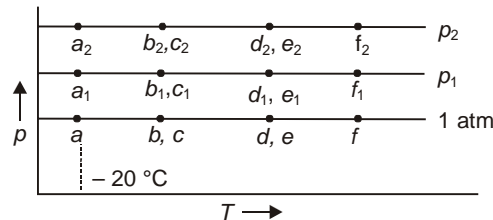


Fig. 6.6 P-T variation for phase transformation at constant pressures

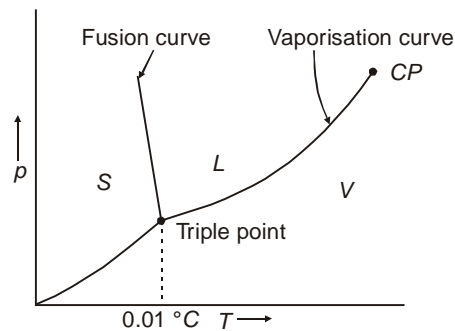
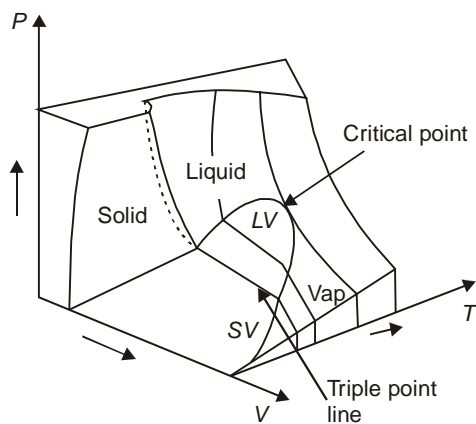
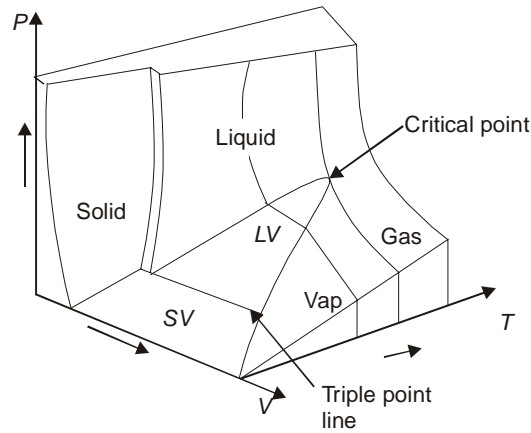


Fig. 6.7 P-T diagram for water

- (iii) **P-V-T surface:** This is the three dimensional variation describing three thermodynamic properties P , V and T . As we know that for defining a state at least two properties are needed, therefore on P - V - T surface also, by knowing any two the third can be seen and continuous variation of these properties is available. Here pressure, volume and temperature are taken on mutually perpendicular axis and surface obtained is depicted below with all salient points. P - V - T surface shall be different for different substances depending on their characteristics. Here P - V - T surface for two substances having opposite characteristics are given in Fig. 6.8.



(A) P-V-T surface for water (which expands upon freezing.)



(B) *P-V-T surface for Carbon dioxide (which contracts upon freezing.)*

Fig. 6.8 *P-V-T surface for water and CO₂*

6.5 THERMODYNAMIC RELATIONS INVOLVING ENTROPY

Entropy change during phase transformation process can be studied with temperature at any given pressure, based on discussions in article 6.3. Entropy changes for every state change are estimated and plotted on *T-S* diagram for 1 atm. pressure and ‘*m*’ mass of ice.

a – b: Temperature changes from –20°C to 0°C. Phase is solid

$$\text{Entropy change } \Delta S_{a-b} = S_b - S_a = \int_{T_a}^{T_b} \frac{dQ_{a-b}}{T}$$

Here
and

$$dQ_{a-b} = m \cdot c_{p, \text{ice}} \cdot dT$$

$$T_a = 253 \text{ K}, T_b = 273 \text{ K}.$$

or

$$\Delta S_{a-b} = \int_{T_a}^{T_b} \frac{m c_{p, \text{ice}} \cdot dT}{T}$$

b – c: Temperature does not change. It is constant at 0°C.

Entropy change,

$$\Delta S_{b-c} = S_c - S_b = \frac{\Delta Q_{b-c}}{T_b}$$

Here,

$$\frac{\Delta Q_{b-c}}{T_b} = \text{Latent heat of fusion/melting of ice at 1 atm}$$

$$T_b = 273 \text{ K}$$

c – d: Temperature rises from 0°C to 100°C. Phase is liquid.

Entropy change,

$$\Delta S_{c-d} = S_d - S_c = \int_{T_c}^{T_d} \frac{dQ_{c-d}}{T}$$

Here

$$dQ_{c-d} = m \cdot c_{p,\text{water}} \cdot dT \text{ and } T_c = 273 \text{ K}, T_d = 373 \text{ K}.$$

or,

$$\Delta S_{c-d} = \int_{T_c}^{T_d} \frac{m c_{p,\text{water}} dT}{T}$$

$d-e$: Temperature does not change. It is constant at 100°C .

Entropy change,

$$\Delta S_{d-e} = S_e - S_d = \frac{\Delta Q_{d-e}}{T_d}$$

Here, ΔQ_{d-e} = latent heat of vaporization at 1 atm.

and $T_d = 373 \text{ K}$.

$e-f$: Temperature rises from 100°C to 200°C . Phase is gas (steam)

Entropy change,

$$\Delta S_{e-f} = S_f - S_e = \int_{T_e}^{T_f} \frac{dQ_{e-f}}{T}$$

Here, $dQ_{e-f} = m \cdot c_{p,\text{steam}} \cdot dT$ and $T_e = 373 \text{ K}$, $T_f = 473 \text{ K}$

$$\Delta S_{e-f} = \int_{T_e}^{T_f} \frac{m c_{p,\text{steam}} \cdot dT}{T}$$

Above entropy change when plotted on T - S axis result as below.

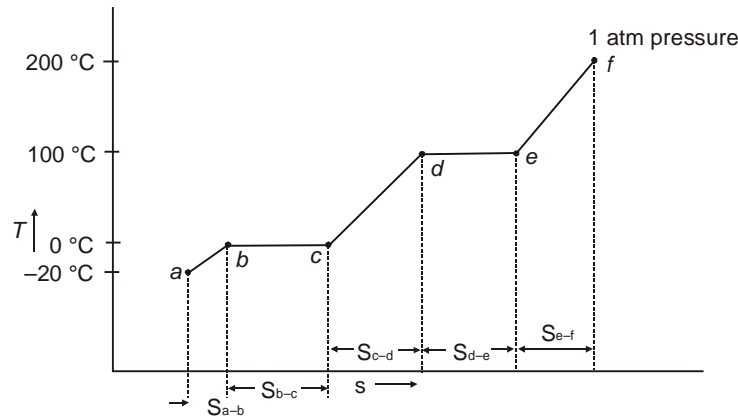
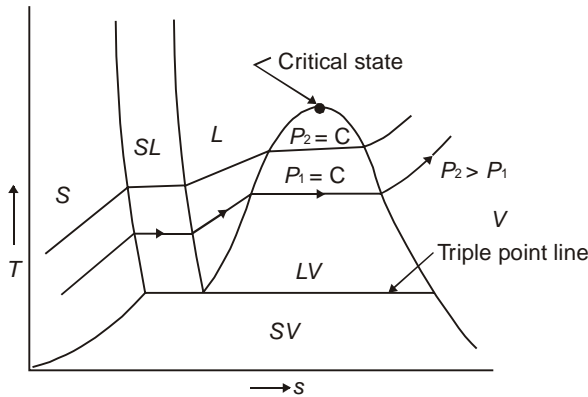
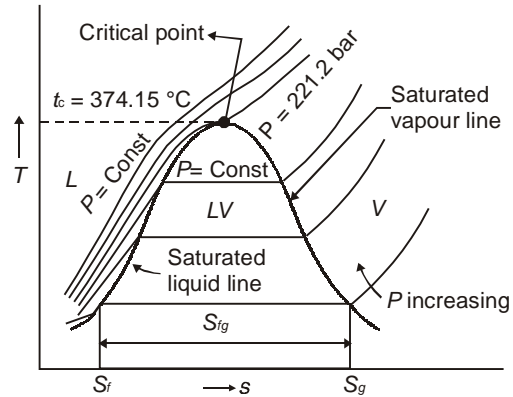


Fig. 6.9 Temperature-entropy variation for phase change at 1 atm.

On the similar lines, (as discussed above) the T - S variation may be obtained for water at other pressures. In the T - S diagram different important zones and lines are earmarked. Out of whole T - S diagram, generally the portion detailing liquid, liquid-vapour zone, vapour zone are of major interest due to steam undergoing processes and subsequently condensed and evaporated. Both T - S diagrams are shown here.

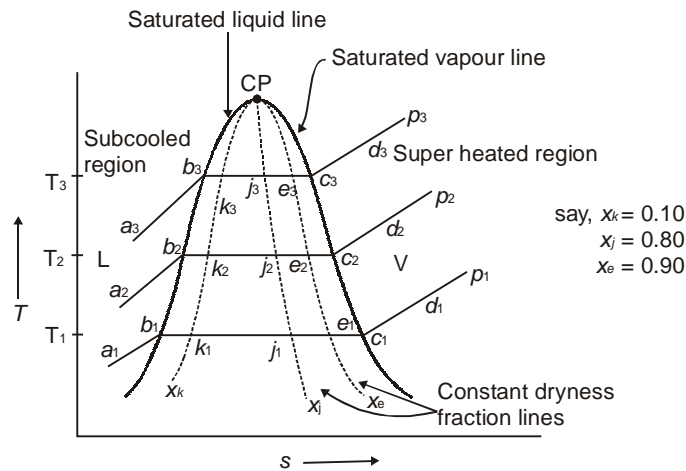
Fig. 6.10 T - s diagram for waterFig. 6.11 T - s diagram showing liquid and vapour regions

6.6 PROPERTIES OF STEAM

For thermodynamic analysis the following thermodynamic properties of steam are frequently used.

“Saturation temperature (T), saturation pressure (P), specific volume (v), enthalpy (h), entropy (s), internal energy (u)”

Let us look at T - S diagram below.

Fig. 6.12 T - s diagram

Thermodynamic properties and nomenclature used is indicated inside bracket along with property. Discussion is based on unit mass of steam/mixture. T - S diagram for 2-phases i.e. liquid and vapour has saturated liquid line and saturated vapour line meeting at critical point. Three constant pressure lines corresponding to pressure P_1 , P_2 and P_3 are shown. Let us take a constant pressure line for pressure p_1 which has states a_1 , b_1 , k_1 , j_1 , e_1 , c_1 , d_1 shown upon it. Region on the left of saturated liquid line is liquid region. Region enclosed between saturated liquid line and saturated vapour line is liquid-vapour mixture

region or also called wet region. Region on the right of saturated vapour line is vapour region. All the states lying on saturated liquid line are liquid (water) states shown as b_1, b_2, b_3 at different pressures. States a_1, a_2 and a_3 are the states lying in subcooled region. Compressed liquid or subcooled liquid exists at a_1, a_2 and a_3 at pressures p_1, p_2 and p_3 .

$$\begin{aligned} \text{Degree of sub cooling at } a_1 &= \text{Saturation temperature for pressure } p_1 - \text{Temperature at } a_1 \\ &= (T_1 - T_{a1}) \end{aligned}$$

where T_1, T_2, T_3 are saturation temperatures at pressures p_1, p_2 and p_3 .

At constant pressure p_1 when we move towards right of state b_1 then the phase transformation of water into steam (vapour) just begins. This conversion from liquid to vapour takes place gradually till whole liquid gets converted to vapour. Phase transformation into vapour gets completed at c_1, c_2, c_3 at pressures p_1, p_2 and p_3 respectively. States c_1, c_2, c_3 are called saturated vapour states and substance is completely in vapour phase at these points. Beyond state c_1 , at pressure p_1 it is vapour phase and sensible heating shall cause increase in temperature. State d_1 is the state of steam called superheated steam. Superheated steam exists at d_1, d_2 , and d_3 at pressures p_1, p_2 and p_3 . Degree of superheat at $d_1 = T_{d1} - T_1$

Similarly, degree of superheat at $d_2 = T_{d2} - T_2$,

degree of superheat at $d_3 = T_{d3} - T_3$.

As pressure is increased upto critical pressure then constant pressure line is seen to become tangential to critical point (CP) at which water instantaneously flashes into vapour.

In the wet region states k_1, j_1 and e_1 are shown at pressure p_1 .

At state b_1 mixture is 100% liquid

At state k_1 mixture has larger liquid fraction, say 90% liquid fraction and 10% vapour fraction.

At state j_1 mixture has say 20% liquid fraction and 80% vapour fraction.

At state e_1 mixture has say 10% liquid fraction and 90% vapour fraction.

At state c_1 mixture is 100% vapour.

Similarly, from explanation given above the states b_2, k_2, j_2, e_2, c_2 and b_3, k_3, j_3, e_3 and c_3 can be understood at pressures p_2 and p_3 respectively.

At any pressure for identifying the state in wet region, fraction of liquid and vapour must be known, for identifying state in subcooled region, degree of subcooling is desired and for identifying state in superheated region, degree of superheating is to be known.

For wet region a parameter called dryness fraction is used. Dryness fraction as defined earlier can be given as, x :

$$\text{Dryness fraction; } x = \frac{\text{Mass of vapour}}{\text{Mass of liquid} + \text{Mass of vapour}}$$

Dryness fraction values can be defined as follows;

Dryness fraction at state $b_1 = 0$

Dryness fraction at state $k_1 = 0.10$

$$\left\{ \text{as assumed} = \frac{0.10 \text{ (m)}}{(0.9 + 0.1) \text{ m}} \right\}$$

Dryness fraction at state $j_1 = 0.80$

Dryness fraction at state $e_1 = 0.90$

Dryness fraction at state $c_1 = 1.00$

Thus, saturated liquid line and saturated vapour line are locii of all states having 0 and 1 dryness fraction values.

At critical point dryness fraction is either 0 or 1.

For different pressures the loci of constant dryness fraction points may be obtained and it yields constant dryness fraction lines corresponding to $x_k(0.10)$, $x_j(0.80)$, $x_e(0.90)$, as shown by dotted lines.

Let us use subscript 'f' for liquid states and 'g' for vapour states.

Therefore, enthalpy corresponding to saturated liquid state = h_f .

Enthalpy corresponding to saturated vapour state = h_g .

Similarly, entropy may be given as s_f and s_g .

Specific volume may be given as v_f and v_g .

Internal energy may be given as u_f and u_g .

At any pressure for some dryness fraction x , the total volume of mixture shall comprise of volume occupied by liquid and vapour both.

$$\text{Total volume, } V = V_f + V_g.$$

Similarly,

Total mass, $m = m_f + m_g$, i.e. mass of liquid and mass of vapour put together substituting for volume,

$$m \cdot v = m_f \cdot v_f + m_g \cdot v_g.$$

where m is total mass and v is specific volume of mixture

$$v = \left(1 - \frac{m_g}{m}\right) \cdot v_f + \left(\frac{m_g}{m}\right) v_g$$

or

$$v = \left(1 - \frac{m_g}{m}\right) \cdot v_f + \left(\frac{m_g}{m}\right) \cdot v_g$$

or

$$v = \left(1 - \frac{m_g}{m}\right) \cdot v_f + \left(\frac{m_g}{m}\right) \cdot v_g$$

From definition,

$$x = \frac{m_g}{m_f + m_g} = \frac{m_g}{m}$$

or,

$$v = (1 - x) \cdot v_f + x \cdot v_g$$

or,

$$v = v_f + x \cdot (v_g - v_f)$$

or,

$$v = v_f + x \cdot v_{fg}$$

here v_{fg} indicates change in specific volume from liquid to vapour

Similarly, enthalpy, entropy and internal energy may be defined for such states in wet region.

i.e.

$$h = h_f + x (h_g - h_f)$$

or

$$h = h_f + x \cdot h_{fg}$$

here h_{fg} is difference in enthalpy between saturated liquid and vapour states. Actually h_{fg} is energy or heat required for vaporization or heat to be extracted for condensation i.e. latent heat.

Similarly, $s = s_f + x \cdot s_{fg}$ where $s_{fg} = s_g - s_f$

$$u = u_f + x \cdot u_{fg} \text{ where } u_{fg} = u_g - u_f.$$

6.7 STEAM TABLES AND MOLLIER DIAGRAM

Steam being pure substance has its unique and constant properties at different pressures and temperatures. Therefore, thermodynamic properties can be estimated once and tabulated for future use. Steam table is a tabular presentation of properties such as specific enthalpy, entropy, internal energy and specific volume at different saturation pressures and temperatures. Steam table may be on pressure basis or on temperature basis. The table on pressure basis has continuous variation of pressure and corresponding to it : saturation temperature (T_{sat}), enthalpy of saturated liquid (h_f), enthalpy of saturated vapour (h_g), entropy of saturated liquid (s_f), entropy of saturated vapour (s_g), specific volume of saturated liquid (v_f), specific volume of saturated vapour (v_g), internal energy of saturated liquid (u_f), internal energy of saturated vapour (u_g) are given on unit mass basis, i.e. as shown in table 6.2.

Similar to above the temperature based table which gives continuous variation of temperature and corresponding to it saturation pressure and other properties as h_f , h_g , h_{fg} , s_f , s_g , s_{fg} , v_f , v_g , u_f , u_g and u_{fg} are given.

Similarly, steam properties for superheated steam are also estimated and tabulated at some discrete pressures for varying degree of superheat. Super heated steam table are available for getting enthalpy, entropy, specific volume and internal energy separately. Example of superheated steam table for enthalpy is given here:

Table 6.2 Pressure based steam table

Pressure	Sat. Temp.	Enthalpy			Entropy			Specific volume		Internal energy		
	T_{sat} °C	h_f kJ/kg	h_g kJ/kg	h_{fg} kJ/kg	s_f kJ/kg°K	s_g kJ/kg°K	s_{fg} kJ/kg°K	v_f m ³ /kg	v_g m ³ /kg	u_f kJ/kg	u_g kJ/kg	u_{fg} kJ/kg

Table 6.3 Temperature based steam table

Temperature	Sat. Pressure	Enthalpy			Entropy			Specific volume		Internal energy		
°C	kPa	h_f kJ/kg	h_g kJ/kg	h_{fg} kJ/kg	s_f kJ/kg°K	s_g kJ/kg°K	s_{fg} kJ/kg°K	v_f m ³ /kg	v_g m ³ /kg	u_f kJ/kg	u_g kJ/kg	u_{fg} kJ/kg

Table 6.4 Superheated steam table for enthalpy

Pressure	Sat. temp. °C	Enthalpy values for varying degree of superheat (kJ/kg)						
kPa	(T_{sat})	T_1	T_2	T_3	T_4	T_5	T_6	T_7

Here $T_1, T_2, T_3, T_4, T_5, T_6, T_7 \dots$ are more than T_{sat} and have increasing value of degree of superheat. Steam tables as discussed above are available in appendix in this text book.

Mollier diagram is the enthalpy-entropy ($h-s$) diagram for steam. This diagram is obtained on the basis of following equation depending upon the phase transformation as discussed earlier.

$$Tds = dh - vdp. \text{ (First and second law combined)}$$

$$\text{For constant pressure } \left(\frac{dh}{ds} \right)_p = T$$

Enthalpy entropy diagram as obtained for all phases of water is as given in Figure 6.13 here.

Generally, liquid and vapour region is only of interest in engineering systems, so mostly used portion of h - s diagram is taken out and shown in Fig. 6.14. It is popularly known as *mollier diagram* or *mollier chart*.

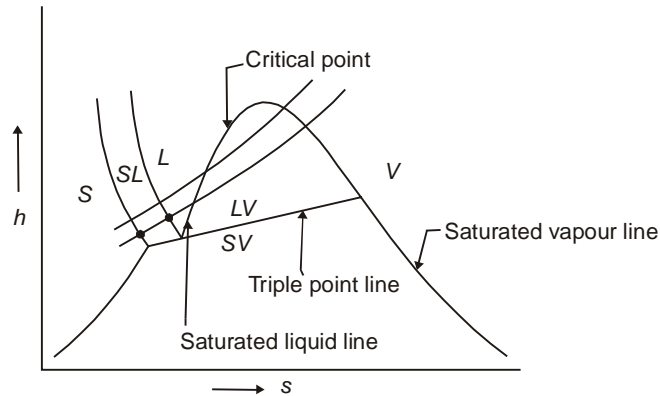


Fig. 6.13 Enthalpy-entropy diagram for all phases

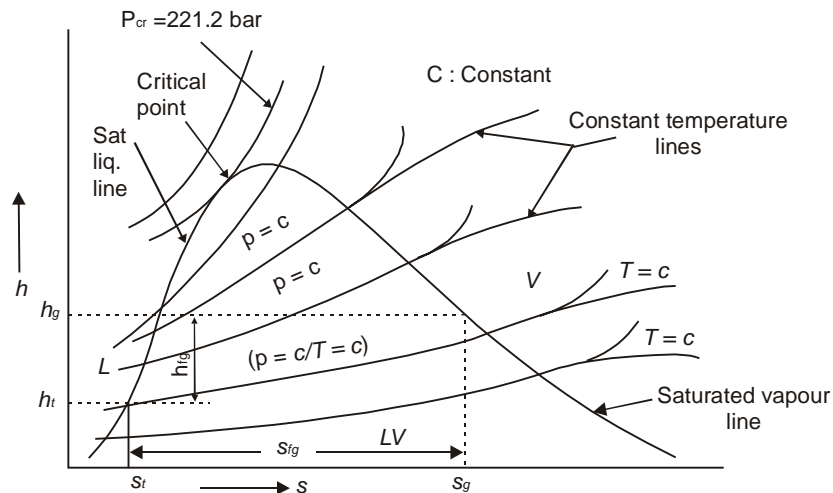


Fig. 6.14 h - s diagram (Mollier diagram)

Different significant lines such as saturated liquid line, saturated vapour line, isobaric lines, isothermal lines, constant specific volume lines, constant dryness fraction lines are shown upon Mollier diagram. Nature of variation of different lines can be explained from the real behaviour of substance and mathematical expression based on combination of first and second law.

Such as, why isobaric lines diverge from one another? This is due to the increase in saturation temperature with increase in pressure. Slope of isobar is equal to saturation temperature as shown in the beginning, therefore it also increases with increasing pressure.

Why isothermal lines are not visible in wet region? It is because constant temperature lines and constant pressure lines coincide upon in wet region. For every pressure there shall be definite saturation temperature which remains constant in wet region.

Mollier chart is also given in appendix, at the end of this book.

6.8 DRYNESS FRACTION MEASUREMENT

Dryness fraction is the basic parameter required for knowing the state of substance in liquid-vapour mixture region (wet region). For any pressure the dryness fraction varies from 0 to 1 in the wet region i.e. from saturated liquid to saturated vapour.

Dryness fraction being ratio of mass of vapour and total mass of substance can be conveniently estimated if these two mass values are known. It may also be termed as 'quality' of steam or 'dryness factor'.

$$\text{Dryness fraction} = \frac{\text{Mass of vapour}}{\text{Total mass i.e. (mass of vapour + mass of liquid)}}$$

Here we shall be looking into standard methods available for dryness fraction measurement. These are;

- (i) Throttling calorimeter
- (ii) Separating calorimeter
- (iii) Separating and throttling calorimeter
- (iv) Electrical calorimeter.

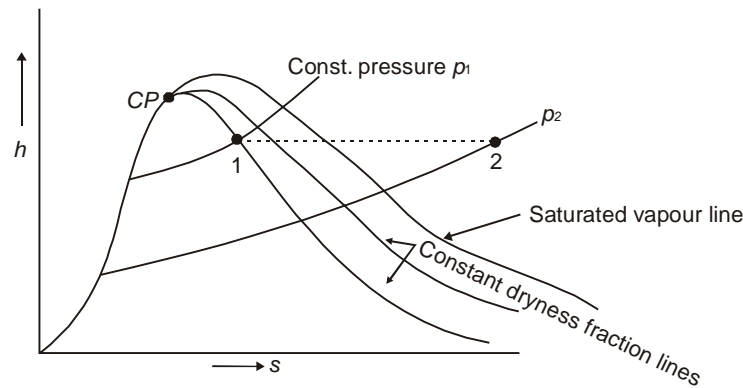


Fig. 6.15 Throttling process on $h-s$ diagram

(i) **Throttling calorimeter:** In this the throttling action is utilised for getting dryness fraction. If a mixture is throttled, then upon throttling its enthalpy remains constant before and after throttling. Let us look upon states of substance on $h-s$ diagram, before and after throttling.

Wet mixture being at state '1' initially, attains a new state '2' upon being throttled upto pressure p_2 . This state at the end of throttling lies in the superheated region such that,

$$h_1 = h_2$$

Say, dryness fraction at state 1 is x , then enthalpy at this point can be given as

$$h_1 = h_{f \text{ at } p_1} + x \times h_{fg \text{ at } p_1}$$

In the above expression h_f at p_1 and h_{fg} at p_1 can be seen from steam table if pressure of wet steam is known. Also the enthalpy at state 2 (end of throttling) can be seen from superheated steam table if pressure and temperature at '2' are known.

Substituting in, $h_1 = h_2$

$$h_f \text{ at } p_1 + x \times h_{fg} \text{ at } p_1 = h_2$$

Here h_2 , h_f at p_1 , h_{fg} at p_1 are all known as explained above.

$$\text{Therefore, } x = \frac{h_2 - h_f \text{ at } p_1}{h_{fg} \text{ at } p_1}$$

Now the arrangements are to be made for, (a) measurement of pressure of wet steam in the beginning, (b) throttling of wet mixture such that state at the end of throttling lies in superheated region, (c) measurement of pressure, temperature of throttled steam. Arrangement used in throttling calorimeter is as shown in Fig. 6.16.

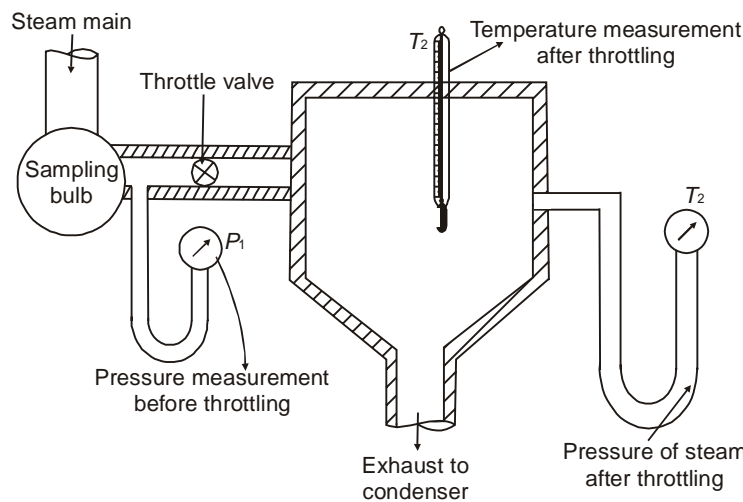


Fig. 6.16 Throttling calorimeter

(ii) **Separating calorimeter:** In this type of calorimeter the known mass of wet mixture is collected through a sampling bulb and sent to a separating chamber. Separating chamber has the series of obstacles, and zig-zag path inside it so that when mixture passes through them the liquid particles get separated due to sudden change in direction of flow and gravity action. Liquid thus separated out is collected in a collection tank and is measured. Thus by knowing the two mass values dryness fraction can be estimated as;

$$\text{Dryness fraction} = \frac{\{(\text{Total mass}) - (\text{Mass of liquid})\}}{\text{Total mass}}$$

Layout of separating calorimeter is given in Fig. 6.17.

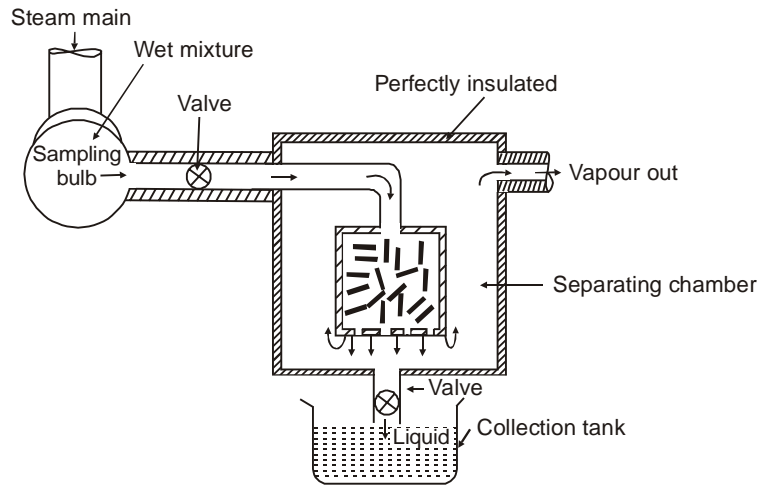


Fig. 6.17 Separating calorimeter

(iii) **Separating and throttling calorimeter:** Some times when the wet mixture is extremely wet then upon throttling state of steam is unable to become superheated. In such situations mixture is first passed through separating calorimeter to reduce liquid fraction in it and subsequently this less wet mixture is passed through throttling calorimeter. In such calorimeters the arrangement is as shown below.

Here excessively wet steam is first sent to separating calorimeter where its wetness is reduced by separating out some liquid fraction, say mass m_{f1} . Less wet steam is sent to throttling calorimeter and its dryness fraction estimated as x_2 . Then at the end this throttled steam (of superheated type) is made to pass through condenser. Mass of condensate is measured from that collected in condensate tank, say m_2 . Thus m_2 is total mass of steam sent from separating calorimeter to throttling calorimeter. As mass of liquid collected in collection tank of separating calorimeter is m_{f1} then total mass of wet steam under examination is $(m_{f1} + m_2)$. Dryness fraction at section 1–1 shall be;

$$x_1 = \frac{(\text{Mass of vapour at 1-1})}{\text{Total mass}}$$

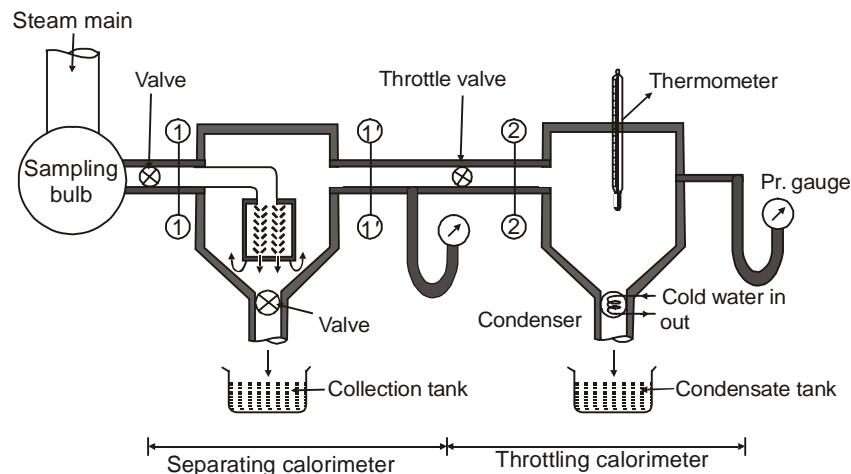


Fig. 6.18 Separating and throttling calorimeter

Mass of vapour at 1-1 shall be similar to mass of vapour entering at 2-2.

$$\text{Mass of vapour at 2-2} = x_2 \times m_2$$

$$\text{Hence, dryness fraction at 1-1, } x_1 = \frac{x_2 \cdot m_2}{m_{f1} + m_2}$$

Separating and throttling processes occurring are also shown on h - s diagram in Fig. 6.19.

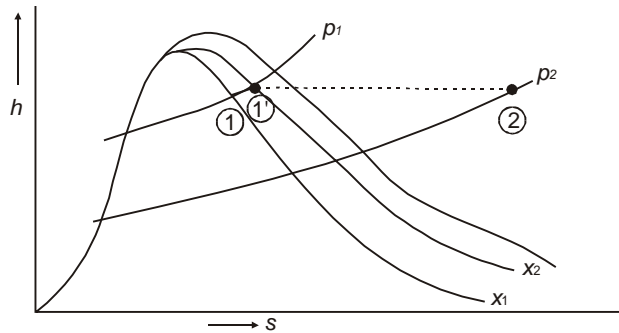


Fig. 6.19 Separating and throttling together on h - s diagram

(iv) **Electrical Calorimeter:** In electrical calorimeter too the principle employed is similar to that of throttling calorimeter. Here also wet mixture is brought to the superheated state by heating and not by throttling. For known amount of heat added and the final enthalpy for superheated steam being known, one can find out the initial enthalpy.

For mass m of mixture, heat Q_{add} added by heater, and the enthalpies before and after heating being h_1 , h_2 , steady flow energy equation may be written as; $mh_1 + Q_{\text{add}} = mh_2$.

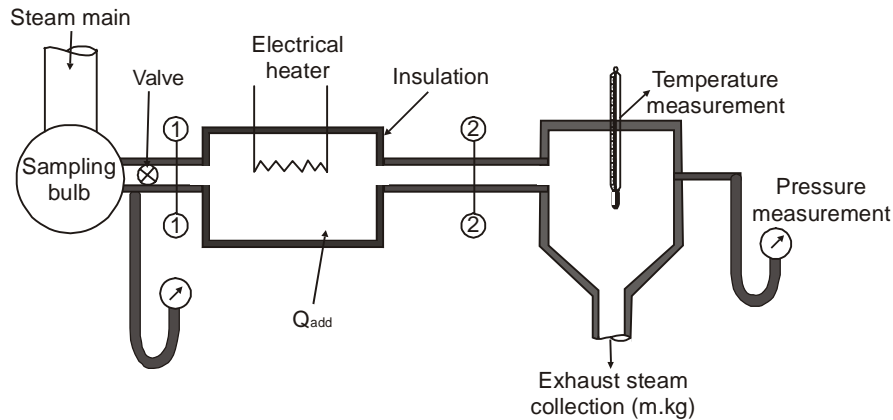


Fig. 6.20 Electrical calorimeter

Here, for electrical heater $Q_{\text{add}} = V.I$, where V and I are voltage and current.

h_2 is known, as mixture is brought to superheated state and pressure and temperature measured, locate enthalpy from superheated steam table, also $h_1 = h_{f \text{ at } p1} + x \cdot h_{fg \text{ at } p1}$, here $h_{f \text{ at } p1}$ & $h_{fg \text{ at } p1}$ can be seen from steam table.

Now h_1 being known, for known m , h_2 , Q_{add} , dryness fraction ' x ' can be easily obtained.

EXAMPLES

1. Derive the expressions for the following :

- (a) Work of evaporation or external work of evaporation
- (b) True latent heat
- (c) Internal energy of steam
- (d) Entropy of water
- (e) Entropy of evaporation
- (f) Entropy of wet steam
- (g) Entropy of superheated steam

Solution:

(a) *Work of evaporation:* This is the work done due to evaporation of water to steam as phase transformation from water to steam is accompanied by increase in volume. Work of evaporation can be estimated as; $= p (v_g - v_f)$, for unit mass

For very low pressures of steam generation, where $v_f \ll v_g$.

$$\text{Work of evaporation} = p \cdot v_g$$

Work of evaporation for wet steam with dryness fraction 'x'

$$= p \cdot x \cdot v_g.$$

(b) *True latent heat:* Latent heat causing the phase transformation from water to steam is accompanied with change in volume as well. Therefore, latent heat shall have two components i.e. (i) true latent heat causing phase change and (ii) work of evaporation due to volume increase. Mathematically, for unit mass, True latent heat $= h_{fg} - p(v_g - v_f)$.

(c) *Internal energy of steam:* For a given mass of steam the total heat energy with steam can be said to comprise of

- (i) Sensible heat
- (ii) True latent heat
- (iii) Work of evaporation

Out of above three the third component gets consumed in doing work. The internal energy of steam shall consist of first two components. For the enthalpy 'h' of steam,

$$\text{(for unit mass) Internal energy, } u = h - p(v_g - v_f)$$

neglecting v_f for low pressures,

$$u = h - p \cdot v_g.$$

for wet steam

$$u = h - p \cdot x v_g$$

for super heated steam,

$$h = h_g + c_{p \text{ superheat}} (T_{\text{superheat}} - T_{\text{sat}})$$

Hence

$$u = \{h_g + c_{p \text{ superheat}} \cdot (T_{\text{superheat}} - T_{\text{sat}})\} - p v_{\text{superheat}}$$

(d) *Entropy of water:* Entropy change of water as discussed in article 6.5 can be given as :

$$\int_1^2 ds = \int_{T_1}^{T_2} c_{p \text{ water}} \frac{dT}{T}$$

for constant specific heat of water

$$(s_2 - s_1) = c_{p \text{ water}} \ln \left(\frac{T_2}{T_1} \right)$$

Absolute entropy may be given in reference to absolute zero temperature

$$s = c_{p \text{ water}} \cdot \ln \left(\frac{T}{273.15} \right)$$

(e) *Entropy of evaporation:* During evaporation heat absorbed is equal to latent heat of evaporation. Therefore, for unit mass

$$s_{\text{evaporation}} = \frac{h_{fg}}{T_{\text{sat}}}$$

During incomplete evaporation (for wet steam)

$$s_{\text{evaporation}} = \frac{x \cdot h_{fg}}{T_{\text{sat}}}$$

(f) *Entropy of wet steam:*

Entropy of wet steam = Entropy of water + Entropy of evaporation

For unit mass,

$$s_{\text{wet}} = c_{p \text{ water}} \ln \frac{T_2}{T_1} + x \cdot \frac{h_{fg}}{T_2}$$

(g) *Entropy of super heated steam:* Entropy change during constant pressure heating for superheating unit mass of the steam.

$$= c_{p \text{ steam}} \ln \left(\frac{T_{\text{sup}}}{T_{\text{sat}}} \right)$$

Total entropy of super heated steam, starting with water at temperature T_1 .

$$s_{\text{superheat}} = c_{p \text{ water}} \ln \left(\frac{T_{\text{sat}}}{T_1} \right) + \frac{h_{fg}}{T_{\text{sat}}} + c_{p \text{ steam}} \cdot \ln \left(\frac{T_{\text{super heat}}}{T_{\text{sat}}} \right).$$

2. *Throttling calorimeter has steam entering to it at 10 MPa and coming out of it at 0.05 MPa and 100°C. Determine dryness fraction of steam.*

Solution:

During throttling, $h_1 = h_2$

At state 2, enthalpy can be seen for superheated steam at 0.05 MPa and 100°C.

Thus, $h_2 = 2682.5 \text{ kJ/kg}$

At state 1, before throttling

$$h_{f \text{ at } 10 \text{ MPa}} = 1407.56 \text{ kJ/kg}$$

$$h_{fg \text{ at } 10 \text{ MPa}} = 1317.1 \text{ kJ/kg}$$

$$h_1 = h_{f \text{ at } 10 \text{ MPa}} + x_1 h_{fg \text{ at } 10 \text{ MPa}} = h_2$$

$$2682.5 = 1407.56 + (x_1 \cdot 1317.1)$$

$$x_1 = 0.968$$

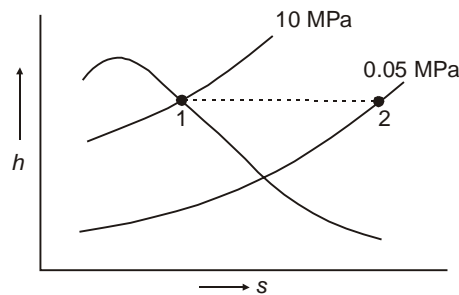


Fig. 6.21

Dryness fraction is **0.968**.

Ans.

3. Determine internal energy of steam if its enthalpy, pressure and specific volumes are 2848 kJ/kg, 12 MPa and 0.017 m³/kg.

Solution:

Internal energy,

$$\begin{aligned} u &= h - pv \\ &= (2848 - 12 \times 10^3 \times 0.017) \text{ kJ/kg} \\ &= 2644 \text{ kJ/kg} \end{aligned}$$

Internal energy = **2644 kJ/kg**

Ans.

4. Determine entropy of 5 kg of steam at 2 MPa and 300°C. Take specific heat of super heated steam as 2.1 kJ/kg.K.

Solution:

Steam state 2 MPa and 300°C lies in superheated region as saturation temperature at 2 MPa is 212.42°C and $h_{fg} = 1890.7$ kJ/kg.

Entropy of unit mass of superheated steam with reference to absolute zero.

$$= c_{p \text{ water}} \ln \left(\frac{T_{\text{sat}}}{273.15} \right) + \frac{h_{fg, 2\text{MPa}}}{T_{\text{sat}}} + c_{p \text{ superheat}} \ln \frac{T_{\text{super heat}}}{T_{\text{sat}}}$$

Substituting values

$$\begin{aligned} &= 4.18 \ln \left(\frac{485.57}{273.15} \right) + \left(\frac{1890.7}{485.57} \right) + \left[2.1 \ln \left(\frac{573.15}{485.57} \right) \right] \\ &= 6.646 \text{ kJ/kg.K.} \end{aligned}$$

Entropy of 5 kg of steam = 33.23 kJ/K

Entropy of steam = **33.23 kJ/K**

Ans.

5. Water in a pond boils at 110°C at certain depth in water. At what temperature the water shall boil if we intend to boil it at 50 cm depth from above mentioned level.

Solution:

Boiling point = 110°C, pressure at which it boils = 143.27 kPa (from steam table, sat. pressure for 110°C)

At further depth of 50 cm the pressure

$$\begin{aligned} &= 143.27 - ((10^3 \times 9.81 \times 0.50) \times 10^{-3}) \\ &= 138.365 \text{ kPa.} \end{aligned}$$

Boiling point at this depth = $T_{\text{sat}, 138.365}$

From steam table this temperature = $108.866 = 108.87^\circ\text{C}$

Boiling point = **108.87°C** Ans.

6. Water-vapour mixture at 100°C is contained in a rigid vessel of 0.5 m^3 capacity. Water is now heated till it reaches critical state. What was the mass and volume of water initially?

Solution:

In a rigid vessel it can be treated as constant volume process.

$$v_1 = v_2$$

Since final state is given to be critical state, then specific volume at critical point,

$$v_2 = 0.003155 \text{ m}^3/\text{kg}$$

At 100°C saturation temperature, from steam table

$$v_f = 0.001044 \text{ m}^3/\text{kg}, v_g = 1.6729 \text{ m}^3/\text{kg}$$

Thus for initial quality being x_1

$$v_1 = v_{f, 100^\circ\text{C}} + x_1 \cdot v_{fg, 100^\circ\text{C}}$$

$$\text{or } 0.003155 = 0.001044 + x_1 \times 1.671856$$

$$x_1 = 0.0012627$$

$$\text{Mass of water initially} = \text{Total mass} \cdot (1 - x_1)$$

$$\text{Total mass of fluid} = \frac{V}{v_2} = \frac{0.5}{0.003155} = 158.48 \text{ kg}$$

$$\text{Mass of water} = 158.48 \text{ kg}$$

$$\begin{aligned} \text{Volume of water} &= 158.48 \times 0.001044 \\ &= 0.1652 \text{ m}^3 \end{aligned}$$

Mass of water = **158.48 kg**, Ans.

Volume of water = **0.1652 m³**

7. Determine slope of an isobar at 2 MPa and 500°C on mollier diagram.

Solution:

On mollier diagram ($h - s$ diagram) the slope of isobaric line may be given as

$$\left(\frac{dh}{ds} \right)_{p = \text{const}} = \text{Slope of isobar}$$

From Ist and IInd law combined;

$$Tds = dh - vdp$$

for constant pressure

$$\left(\frac{dh}{ds} \right)_{p = \text{const}} = T$$

Here temperature $T = 773.15 \text{ K}$

$$\text{hence slope} = \left(\frac{dh}{ds} \right)_{p = \text{const}} = 773.15$$

Slope = **773.15** Ans.

8. Determine enthalpy, specific volume, entropy for mixture of 10% quality at 0.15 MPa.

Solution:

Given, $x = 0.10$

At 0.15 MPa, from steam table;

$$\begin{aligned}h_f &= 467.11 \text{ kJ/kg}, h_g = 2693.6 \text{ kJ/kg} \\v_f &= 0.001053 \text{ m}^3/\text{kg}, v_g = 1.1593 \text{ m}^3/\text{kg} \\s_f &= 1.4336 \text{ kJ/kg.K}, s_g = 7.2233 \text{ kJ/kg.K}\end{aligned}$$

Enthalpy at

$$x = 0.10$$

$$h = h_f + x.h_{fg} = 467.11 + \{0.10 \times (2693.6 - 467.11)\}$$

$$h = 689.759 \text{ kJ/kg}$$

Specific volume,

$$v = v_f + x.v_{fg}$$

$$= 0.001053 + \{0.10 \times (1.1593 - 0.001053)\}$$

$$v = 0.116877 \text{ m}^3/\text{kg}$$

Entropy,

$$s = s_f + x.s_{fg}$$

$$= 1.4336 + \{0.10 \times (7.2233 - 1.4336)\}$$

$$s = 2.01257 \text{ kJ/kg.K}$$

$$h = \mathbf{689.759 \text{ kJ/kg}}$$

$$v = \mathbf{0.116877 \text{ m}^3/\text{kg}}$$

$$s = \mathbf{2.01257 \text{ kJ/kg.K}}$$

Ans.

9. In a piston-cylinder arrangement the steam at 1.0 MPa, 80% dryness fraction, and 0.05 m³ volume is heated to increase its volume to 0.2 m³. Determine the heat added.

Solution:

Given;

Initial states, 1: $P_1 = 1.0 \text{ MPa}$, $V_1 = 0.05 \text{ m}^3$, $x_1 = 0.80$

Final state, 2: $V_2 = 0.2 \text{ m}^3$, $P_2 = 1 \text{ MPa}$

Work done during constant pressure process, $W = P_1(V_2 - V_1)$

$$W = 1000 (0.2 - 0.05)$$

$$W = 150 \text{ kJ}$$

$$\text{Mass of steam} = \frac{V_1}{v_1}$$

From steam table at P_1 ; $v_f = 0.001127 \text{ m}^3/\text{kg}$

$$v_g = 0.19444 \text{ m}^3/\text{kg}$$

$$u_f = 761.68 \text{ kJ/kg}$$

$$u_{fg} = 1822 \text{ kJ/kg}$$

so

$$v_1 = v_f + x_1 v_{fg}$$

$$v_1 = 0.15578 \text{ m}^3/\text{kg}$$

$$\text{Hence, mass of steam} = \frac{0.05}{0.15578} = 0.32097 \text{ kg}$$

$$\text{Specific volume at final state} = \frac{V_2}{\text{mass of steam}}$$

$$v_2 = 0.62311 \text{ m}^3/\text{kg}$$

Corresponding to this specific volume the final state is to be located for getting the internal energy at final state at 1 MPa

$$v_2 > v_{g1} \text{ MPa}$$

hence state lies in superheated region, from the steam table by interpolation we get temperature as;

State lies between temperature of 1000°C and 1100°C

so exact temperature at final state

$$\begin{aligned} &= 1000 + \frac{100 \times (0.62311 - 0.5871)}{(0.6335 - 0.5871)} \\ &= 1077.61^\circ\text{C} \end{aligned}$$

Thus internal energy at final state, 1 MPa, 1077.61°C;

$$u_2 = 4209.6 \text{ kJ/kg}$$

Internal energy at initial state,

$$u_1 = u_f + x_1 \cdot u_{fg} = 761.68 + 0.8 \times 1822$$

$$u_1 = 2219.28 \text{ kJ/kg}$$

From first law of thermodynamics;

$$Q - W = \Delta U$$

$$Q = (U_2 - U_1) + W$$

$$= m(u_2 - u_1) + W$$

$$= 0.32097 (4209.6 - 2219.28) + 150$$

$$Q = 788.83 \text{ kJ}$$

Heat added = 788.83 kJ

Ans.

10. Steam at 800 kPa and 200°C in a rigid vessel is to be condensed by cooling. Determine pressure and temperature corresponding to condensation.

Solution:

Here steam is kept in rigid vessel, therefore its' specific volume shall remain constant.

Specific volume at initial state, 800 kPa, 200°C, v_1 ,

It is superheated steam as $T_{\text{sat}} = 170.43^\circ\text{C}$ at 800 kPa.

From superheated steam table; $v_1 = 0.2404 \text{ m}^3/\text{kg}$.

At the beginning of condensation, specific volume = $0.2404 \text{ m}^3/\text{kg}$.

$$v_2 = 0.2404 \text{ m}^3/\text{kg}$$

This v_2 shall be specific volume corresponding to saturated vapour state for condensation.

Thus $v_2 = v_g = 0.2404 \text{ m}^3/\text{kg}$

Looking into steam table $v_g = 0.2404 \text{ m}^3/\text{kg}$ shall lie between temperatures 175°C ($v_g = 0.2168 \text{ m}^3/\text{kg}$) and 170°C ($v_g = 0.2428 \text{ m}^3/\text{kg}$) and pressures 892 kPa (175°C) and 791.7 kPa (170°C).

By interpolation, temperature at beginning of condensation

$$T_2 = 175 - \frac{(175 - 170) \times (0.2404 - 0.2168)}{(0.2428 - 0.2168)}$$

$$T_2 = 170.46^\circ\text{C}$$

$$\text{Similarly, pressure, } p_2 = 892 - \frac{(892 - 791.7) \times (0.2404 - 0.2168)}{(0.2428 - 0.2168)}$$

$$p_2 = 800.96 \text{ kPa}$$

Pressure and temperature at condensation = 800.96 kPa and 170.46°C

Ans.

11. Feed water pump is used for pumping water from 30°C to a pressure of 200 kPa. Determine change in enthalpy assuming water to be incompressible and pumping to be isentropic process.

Solution:

From 1st and 2nd law;

$$Tds = dh - vdp$$

for isentropic process, $ds = 0$

hence

$$dh = vdp$$

i.e.

$$(h_2 - h_1) = v_1 (p_2 - p_1)$$

Corresponding to initial state of saturated liquid at 30°C; from steam table;

$$p_1 = 4.25 \text{ kPa}, v_f = v_1 = 0.001004 \text{ m}^3/\text{kg}$$

Therefore

$$(h_2 - h_1) = 0.001004 (200 - 4.25)$$

$$(h_2 - h_1) = 0.197 \text{ kJ/kg}$$

Enthalpy change = 0.197 kJ/kg	Ans.
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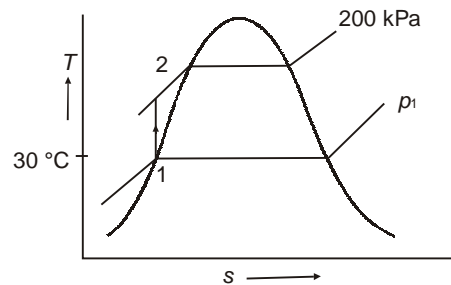


Fig. 6.22

12. A rigid vessel contains liquid-vapour mixture in the ratio of 3:2 by volume. Determine quality of water vapour mixture and total mass of fluid in vessel if the volume of vessel is 2 m³ and initial temperature is 150°C.

Solution:

From steam table at 150°C,

$$v_f = 0.001091 \text{ m}^3/\text{kg}, v_g = 0.3928 \text{ m}^3/\text{kg}$$

$$\text{Volume occupied by water} = 1.2 \text{ m}^3$$

$$\text{Volume of steam} = 0.8 \text{ m}^3$$

$$\text{Mass of water} \Rightarrow m_f = \frac{1.2}{0.001091} = 1099.91 \text{ kg}$$

$$\text{Mass of steam} \Rightarrow m_g = \frac{0.8}{0.3928} = 2.04 \text{ kg}$$

$$\text{Total mass in tank} = m_f + m_g = 1103.99 \text{ kg}$$

$$\text{Quality or Dryness fraction, } x = \frac{2.04}{1103.99} = 0.001848$$

Mass = 1103.99 kg , Quality = 0.001848	Ans.
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given

$$\begin{aligned}
 u_{f, 100 \text{ kPa}} &= 417.36 \text{ kJ/kg} \\
 v_{f, 100 \text{ kPa}} &= 0.001043 \text{ m}^3/\text{kg} \\
 h_{fg, 100 \text{ kPa}} &= 2258 \text{ kJ/kg}, \\
 u_{fg, 100 \text{ kPa}} &= 2088.7 \text{ kJ/kg}, \\
 v_{g, 100 \text{ kPa}} &= 1.6940 \text{ m}^3/\text{kg} \\
 x_1 &= 0.5, v_1 = 0.8475 \text{ m}^3/\text{kg}
 \end{aligned}$$

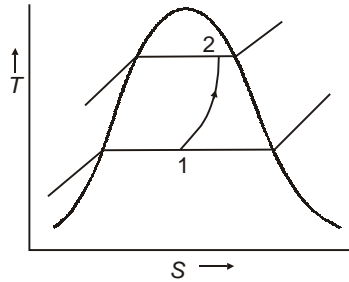


Fig. 6.24

Enthalpy at 1, $h_1 = 417.46 + 0.5 \times 2258 = 1546.46 \text{ kJ/kg}$

Thus, volume of vessel = $(100 \times 0.5) \times (0.8475)$

$$V = 42.375 \text{ m}^3$$

Internal energy in the beginning = $U_1 = m_1 \times u_1$

$$= 100 (417.36 + 0.5 \times 2088.7)$$

$$U_1 = 146171 \text{ kJ}$$

Let the mass of dry steam added be 'm', Final specific volume inside vessel, v_2

$$v_2 = v_{f, 1000 \text{ kPa}} + x_2 \cdot v_{fg, 1000 \text{ kPa}}$$

At 2000 kPa, from steam table,

$$v_{g, 2000 \text{ kPa}} = 0.09963 \text{ m}^3/\text{kg}$$

$$u_{g, 2000 \text{ kPa}} = 2600.3 \text{ kJ/kg}$$

$$h_{g, 2000 \text{ kPa}} = 2799.5 \text{ kJ/kg}$$

Total mass inside vessel = Mass of steam at 2000 kPa + Mass of mixture at 100 kPa

$$\frac{V}{v_2} = \frac{V}{v_{g, 2000 \text{ kPa}}} + \frac{V}{v_1}$$

$$\frac{42.375}{v_2} = \frac{42.375}{0.09963} + \frac{42.375}{0.8475}$$

$$v_2 = 0.089149$$

substituting

$$v_2 = v_{f, 1000 \text{ kPa}} + x_2 \cdot v_{fg, 1000 \text{ kPa}}$$

At 1000 kPa from steam table,

$$h_{f, 1000 \text{ kPa}} = 762.81 \text{ kJ/kg},$$

$$h_{fg, 1000 \text{ kPa}} = 2015.3 \text{ kJ/kg} \quad v_{f, 1000 \text{ kPa}} = 0.001127 \text{ m}^3/\text{kg}$$

$$v_{g, 1000 \text{ kPa}} = 0.19444 \text{ m}^3/\text{kg}$$

$$x_2 = 0.455$$

it gives

For adiabatic mixing

$$(100 + m) \cdot h_2 = 100 \times h_1 + m \times h_{g, 2000 \text{ kPa}}$$

$$(100 + m) \cdot (762.81 + 0.455 \times 2015.3) = \{100 \times (1546.46)\} + \{m \times 2799.5\}$$

It gives upon solving;

$$(100 + m) (1679.77) = 154646 + (2799.5)m$$

$$m = 11.91 \text{ kg}$$

Mass of dry steam at 2000 kPa to be added = **11.91 kg**
 Quality of final mixture = **0.455**

Ans.

15. In a condenser the following observations were made,

Recorded condenser vacuum = 71.5 cm of Mercury

Barometer reading = 76.8 cm of Mercury

Temperature of condensation = 35°C

Temperature of hot well = 27.6°C

Mass of condensate per hour = 1930 kg

Mass of cooling water per hour = 62000 kg

Inlet temperature = 8.51°C

Outlet temperature = 26.24°C

Determine the state of steam entering condenser.

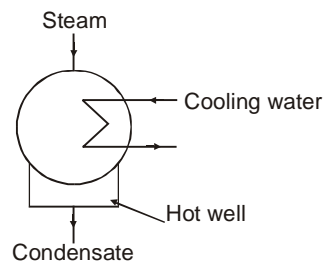


Fig. 6.25

Solution:

From Dalton's law of partial pressure the total pressure inside condenser will be sum of partial pressures of vapour and liquid inside.

$$\text{Condenser pressure} = \left\{ \frac{76.8 - 71.5}{(73.55)} \times 101.325 \right\} \text{ kPa}$$

$$= 7.30 \text{ kPa}$$

Partial pressure of steam corresponding to 35°C from steam table;

$$= 5.628 \text{ kPa}$$

Enthalpy corresponding to 35°C from steam table,

$$h_f = 146.68 \text{ kJ/kg}$$

$$h_{fg} = 2418.6 \text{ kJ/kg}$$

Let quality of steam entering be 'x'.

From energy balance;

$$m_w (26.24 - 8.51) \times 4.18 = 1930 (146.68 + x \times 2418.6 - 4.18 \times 27.6)$$

or $62000 (17.73 \times 4.18) = 1930 (31.31 + x \times 2418.6)$
 which gives $x = 0.97$

Dryness fraction of steam entering = **0.97** Ans.

16. In a vertical vessel of circular cross section having diameter of 20 cm water is filled upto a depth of 2 cm at a temperature of 150°C . A tight fitting frictionless piston is kept over the water surface and a force of 10 kN is externally applied upon the piston. If 600 kJ of heat is supplied to water determine the dryness fraction of resulting steam and change in internal energy. Also find the work done.

Solution:

Heating of water in vessel as described above is a constant pressure heating. Pressure at which process occurs

$$= \frac{\text{Force}}{\text{area}} + \text{atmospheric pressure}$$

$$= \left(\frac{10}{\frac{\pi}{4}(0.2)^2} + 101.3 \right) \text{ kPa}$$

$$= 419.61 \text{ kPa}$$

$$\text{Volume of water contained} = \frac{\pi}{4} \times (0.2)^2 \times (0.02)$$

$$\text{Volume} = 6.28 \times 10^{-4} \text{ m}^3$$

$$\text{Mass of water} = 6.28 \times 10^{-4} \times 10^3 = 0.628 \text{ kg}$$

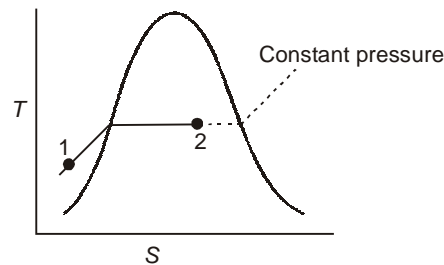


Fig. 6.26

Heat supplied shall cause sensible heating and latent heating.

Hence, Enthalpy change = Heat supplied

$$600 = \{(h_{f \text{ at } 419.6 \text{ kPa}} + x \cdot h_{fg \text{ at } 419.6 \text{ kPa}}) - (4.18 \times 150)\} \times 0.628$$

$$600 = \{(612.1 + x \cdot 2128.7) - 627\} \times 0.628$$

Dryness fraction $x = 0.456$

Dryness fraction of steam produced = **0.456** Ans.

Internal energy of water, initially

$$\begin{aligned} U_1 &= m h_1 - p_1 V_1 \\ &= (0.628 \times 4.18 \times 150) - (419.61 \times 6.28 \times 10^{-4}) \\ U_1 &= 393.5 \text{ kJ} \end{aligned}$$

Finally, internal energy of wet steam

$$U_2 = m h_2 - p_2 V_2$$

Here

$$\begin{aligned} V_2 &= m \cdot x \cdot v_g \text{ at } 419.61 \text{ kPa} \\ &= 0.456 \times 0.4435 \times 0.628 \\ &= 0.127 \text{ m}^3 \end{aligned}$$

Hence

$$\begin{aligned} U_2 &= (0.628 \times 1582.8) - (419.61 \times 0.127) \\ U_2 &= 940.71 \text{ kJ} \end{aligned}$$

$$\text{Change in internal energy} = U_2 - U_1$$

$$\text{Change in internal energy} = \mathbf{547.21 \text{ kJ Ans.}}$$

$$\text{Work done} = P \cdot (V_2 - V_1)$$

$$= 419.61 \times (0.127 - 6.28 \times 10^{-4})$$

$$\text{Work done} = 53.03 \text{ kJ}$$

$$\boxed{\text{Work done} = \mathbf{53.03 \text{ kJ}} \text{ Ans.}}$$

17. In a separating and throttling calorimeter the total quantity of steam passed was 40 kg and 2.2 kg of water was collected from separator. Steam pressure before throttling was 1.47 MPa and temperature and pressure after throttling are 120°C and 107.88 kPa. Determine the dryness fraction of steam before entering to calorimeter. Specific heat of superheated steam may be considered as 2.09 kJ/kg.K.

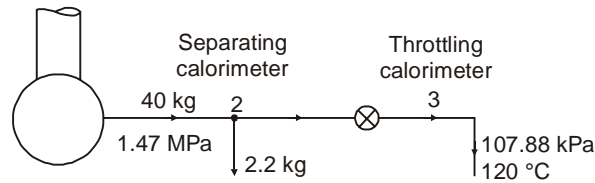


Fig. 6.27

Solution:

Consider throttling calorimeter alone,

$$\text{Degree of superheat} = 120 - 101.8 = 18.2^\circ\text{C}$$

$$\begin{aligned} \text{Enthalpy of superheated steam} &= (2673.95 + (18.2 \times 2.09)) \\ &= 2711.988 \text{ kJ/kg} \end{aligned}$$

$$\text{Enthalpy before throttling} = \text{Enthalpy after throttling}$$

$$840.513 + x_2 \cdot 1951.02 = 2711.988$$

or

$$x_2 = 0.9592$$

$$\text{For separating calorimeter alone, dryness fraction, } x_1 = \frac{40 - 2.2}{40}$$

$$x_1 = 0.945$$

$$\begin{aligned} \text{Overall dryness fraction} &= (x_1 \cdot x_2) = (0.945 \times 0.9592) \\ &= 0.9064 \end{aligned}$$

$$\boxed{\text{Dryness fraction : } \mathbf{0.9064} \text{ Ans.}}$$

18. A rigid vessel is divided into two parts A and B by means of frictionless, perfectly conducting piston. Initially, part A contains 0.4 m³ of air (ideal gas) at 10 bar pressure and part B contains 0.4 m³ of wet steam at 10 bar. Heat is now added to both parts until all the water in part B is evaporated. At this condition the pressure in part B is 15 bar. Determine the initial quality of steam in part B and the total amount of heat added to both parts. [U.P.S.C. 1995]

Solution:

Here heat addition to part *B* shall cause evaporation of water and subsequently the rise in pressure. Final, part *B* has dry steam at 15 bar. In order to have equilibrium the part *A* shall also have pressure of 15 bar. Thus, heat added

$$\begin{aligned} Q &= V(P_2 - P_1) \\ &= 0.4(15 - 10) \times 10^2 \\ Q &= 200 \text{ kJ} \end{aligned}$$

Final enthalpy of dry steam at 15 bar, $h_2 = h_{g \text{ at 15 bar}}$

$$h_2 = 2792.2 \text{ kJ/kg}$$

Let initial dryness fraction be x_1 . Initial enthalpy,

$$\begin{aligned} h_1 &= h_{f \text{ at 10 bar}} + x_1 \cdot h_{fg \text{ at 10 bar}} \\ h_1 &= 762.83 + x_1 \cdot 2015.3 \end{aligned}$$

Heat balance yields,

$$\begin{aligned} h_1 + Q &= h_2 \\ (762.83 + x_1 \cdot 2015.3) + 200 &= 2792.2 \\ x_1 &= 0.907 \end{aligned}$$

Heat added = 200 kJ
Initial quality = 0.907

Ans.

19. A piston-cylinder contains 3 kg of wet steam at 1.4 bar. The initial volume is 2.25 m^3 . The steam is heated until its' temperature reaches 400°C . The piston is free to move up or down unless it reaches the stops at the top. When the piston is up against the stops the cylinder volume is 4.65 m^3 . Determine the amounts of work and heat transfer to or from steam. [U.P.S.C. 1994]

Solution:

From steam table, specific volume of steam at 1.4 bar = $1.2455 \text{ m}^3/\text{kg}$

$$= v_{g \text{ at 1.4 bar}}$$

$$\text{Specific volume of wet steam in cylinder, } v_1 = \frac{2.25}{3} = 0.75 \text{ m}^3/\text{kg}$$

$$\text{Dryness fraction of initial steam, } x_1 = \frac{0.75}{1.2455} = 0.602$$

$$\begin{aligned} \text{Initial enthalpy of wet steam, } h_1 &= h_{f \text{ at 1.4 bar}} + x_1 \cdot h_{fg \text{ at 1.4 bar}} \\ &= 457.99 + (0.602 \times 2232.3) \Rightarrow h_1 = 1801.83 \text{ kJ/kg} \end{aligned}$$

$$\text{At } 400^\circ\text{C specific volume of steam, } v_2 = \frac{4.65}{3} = 1.55 \text{ m}^3/\text{kg}$$

For specific volume of $1.55 \text{ m}^3/\text{kg}$ at 400°C the pressure can be seen from the steam table. From superheated steam tables the specific volume of $1.55 \text{ m}^3/\text{kg}$ lies between the pressure of 0.10 MPa (specific volume $3.103 \text{ m}^3/\text{kg}$ at 400°C) and 0.20 MPa (specific volume $1.5493 \text{ m}^3/\text{kg}$ at 400°C). Actual pressure can be obtained by interpolation;

$$P_2 = 0.10 + \left(\frac{0.20 - 0.10}{(1.5493 - 3.103)} \right) \times (1.55 - 3.103)$$

$$P_2 = 0.199 \text{ MPa} \approx 0.20 \text{ MPa}$$

$$\text{Saturation pressure at 0.20 MPa} = 120.23^\circ\text{C}$$

$$\begin{aligned}\text{Finally the degree of superheat} &= 400 - 120.23 \\ &= 279.77^\circ\text{C}\end{aligned}$$

Final enthalpy of steam at 0.20 MPa and 400°C , $h_2 = 3276.6 \text{ kJ/kg}$

$$\begin{aligned}\text{Heat added during process} &= m(h_2 - h_1) \\ &= 3 \times (3276.6 - 1801.83) \\ \Delta Q &= 4424.31 \text{ kJ}\end{aligned}$$

$$\begin{aligned}\text{Internal energy of initial wet steam, } u_1 &= u_{f \text{ at 1.4 bar}} + x_1 \cdot u_{fg \text{ at 1.4 bar}} \\ u_1 &= 457.84 + (0.607 \times 2059.34) \\ u_1 &= 1707.86 \text{ kJ/kg}\end{aligned}$$

Internal energy of final state,

$$\begin{aligned}u_2 &= u_{\text{at 0.2 MPa, } 400^\circ\text{C}} \\ u_2 &= 2966.7 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{Change in internal energy } \Rightarrow \Delta U &= m(u_2 - u_1) \\ &= 3 \times (2966.7 - 1707.86) \\ \Delta U &= 3776.52 \text{ kJ}\end{aligned}$$

From first law of thermodynamics,

$$\begin{aligned}\text{Work done } \Delta W &= \Delta Q - \Delta U \\ &= 4424.31 - 3776.52\end{aligned}$$

$$\text{Work done, } \Delta W = 647.79 \text{ kJ}$$

Heat transfer = 4424.31 kJ
Work transfer = 647.79 kJ

Ans.

20. An insulated vessel is divided into two compartments connected by a valve. Initially, one compartment contains steam at 10 bar, 500°C , and the other is evacuated. The valve is opened and the steam is allowed to fill the entire volume, achieving a final pressure of 1 bar. Determine the final temperature, in $^\circ\text{C}$, the percentage of the vessel volume initially occupied by steam and the amount of entropy produced, in kJ/kg. K. [U.P.S.C. 1993]

Solution:

Here throttling process is occurring therefore enthalpy before and after expansion remains same. Let initial and final states be given by 1 and 2. Initial enthalpy, from steam table.

$$\begin{aligned}h_1 \text{ at 10 bar and } 500^\circ\text{C} &= 3478.5 \text{ kJ/kg} \\ s_1 \text{ at 10 bar and } 500^\circ\text{C} &= 7.7622 \text{ kJ/kg.K} \\ v_1 \text{ at 10 bar and } 500^\circ\text{C} &= 0.3541 \text{ m}^3/\text{kg}\end{aligned}$$

Finally pressure becomes 1 bar so the final enthalpy at this pressure (of 1 bar) is also 3478.5 kJ/kg which lies between superheat temperature of 400°C and 500°C at 1 bar. Let temperature be T_2 ,

$$\begin{aligned}h_{\text{at 1 bar, } 400^\circ\text{C}} &= 3278.2 \text{ kJ/kg} \\ h_{\text{at 1 bar, } 500^\circ\text{C}} &= 3488.1 \text{ kJ/kg}\end{aligned}$$

$$h_2 = 3478.5 = h_{\text{at 1 bar, } 400^\circ\text{C}} + \frac{(h_{\text{at 1 bar, } 500^\circ\text{C}} - h_{\text{at 1 bar, } 400^\circ\text{C}})}{(500 - 400)} (T_2 - 400)$$

$$3478.5 = 3278.2 + \left\{ \frac{3488.1 - 3278.2}{100} \right\} (T_2 - 400)$$

$$T_2 = 495.43^\circ\text{C},$$

$$\text{Final temperature} = \mathbf{495.43^\circ\text{C}} \quad \text{Ans.}$$

Entropy for final state,

$$s_2 = s_{\text{at 1 bar, } 400^\circ\text{C}} + \frac{(s_{\text{at 1 bar, } 500^\circ\text{C}} - s_{\text{at 1 bar, } 400^\circ\text{C}})}{(500 - 400)} (495.43 - 400)$$

$$s_2 = 8.5435 + \left\{ \frac{8.8342 - 8.5435}{100} \right\} \times (95.43)$$

$$s_2 = 8.8209 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Change in entropy, } \Delta s = 8.8209 - 7.7622$$

$$= 1.0587 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Change in entropy} = \mathbf{1.0587 \text{ kJ/kg} \cdot \text{K}} \quad \text{Ans.}$$

Final specific volume, $v_2 = v_{\text{at 1 bar, } 400^\circ\text{C}}$

$$+ \frac{(v_{\text{at 1 bar, } 500^\circ\text{C}} - v_{\text{at 1 bar, } 400^\circ\text{C}})}{(500 - 400)} \times (95.43)$$

$$= 3.103 + \left\{ \frac{3.565 - 3.103}{100} \right\} \times 95.43$$

$$v_2 = 3.544 \text{ m}^3/\text{kg}$$

$$\text{Percentage volume occupied by steam} = \frac{0.3541}{3.544} \times 100 = 9.99\%$$

$$\text{Percentage of vessel volume initially occupied by steam} = \mathbf{9.99\%} \quad \text{Ans.}$$

21. Determine the maximum work per kg of steam entering the turbine and the irreversibility in a steam turbine receiving steam at 2.5 MPa, 350°C and rejecting steam at 20 kPa, 0.92 dry. During the expansion the one-quarter of initial steam is bled at 30 kPa, 200°C. Consider the heat loss during expansion as 10 kJ/s and atmospheric temperature as 30°C.

Solution:

For the states shown on turbine in the figure, the steam table can be used to get following values:

At 2.5 MPa, 350°C,

$$h_1 = 3126.3 \text{ kJ/kg}, s_1 = 6.8403 \text{ kJ/kg} \cdot \text{K}$$

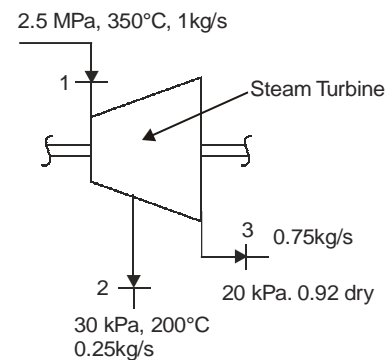


Fig. 6.28

At 30 MPa, 200°C,

$$h_2 = 2878.6 \text{ kJ/kg.}$$

$$s_2 = 8.5309 \text{ kJ/kg.K}$$

At 20 kPa, 0.92 dry,

$$h_f = 251.40 \text{ kJ/kg, } h_g = 2609.7 \text{ kJ/kg}$$

$$s_f = 0.8320 \text{ kJ/kg.K, } s_{fg} = 7.0766 \text{ kJ/kg.K}$$

⇒

$$h_3 = 251.40 + 0.92 \times (2609.7 - 251.40)$$

$$h_3 = 2421.04 \text{ kJ/kg}$$

$$s_3 = 0.8320 + (0.92 \times 7.0766)$$

$$s_3 = 7.3425 \text{ kJ/kg.K}$$

At atmospheric temperature,

$$h_{f \text{ at } 30^\circ\text{C}} = h_0 = 125.79 \text{ kJ/kg; } s_{f \text{ at } 30^\circ\text{C}} = s_0 = 0.4369 \text{ kJ/kg.K}$$

Availability of steam entering turbine,

$$\begin{aligned} A_1 &= (h_1 - h_0) - T_0(s_1 - s_0) \\ &= (3126.3 - 125.79) - 303(6.8403 - 0.4369) \\ &= 1060.28 \text{ kJ/kg} \end{aligned}$$

Availability of steam leaving turbine at state 2 & 3,

$$\begin{aligned} A_2 &= (h_2 - h_0) - T_0(s_2 - s_0) = (2878.6 - 125.79) \\ &\quad - 303(8.5309 - 0.4369) \end{aligned}$$

$$A_2 = 300.328 \text{ kJ/kg}$$

$$\begin{aligned} A_3 &= (h_3 - h_0) - T_0(s_3 - s_0) \\ &= (2421.04 - 125.79) - 303(7.3425 - 0.4369) \end{aligned}$$

$$A_3 = 202.85 \text{ kJ/kg}$$

Maximum work per kg of steam entering turbine for

$$\begin{aligned} W_{\max} &= 1 \times A_1 - m_2 A_2 - m_3 A_3 \\ W_{\max} &= A_1 - (0.25)A_2 - 0.75A_3 \\ &= 1060.28 - (0.25 \times 300.328) - (0.75 \times 202.85) \\ W_{\max} &= \mathbf{833.06 \text{ kJ/kg} \quad \text{Ans.}} \end{aligned}$$

Irreversibility,

$$\begin{aligned} I &= T_0 (m_2 \times s_2 + m_3 \times s_3 - m_1 s_1) - Q. \\ &= 303 (0.25 \times 8.5309 + 0.75 \times 7.3425 - 6.8403) - (-10) \end{aligned}$$

Irreversibility,

$$I = \mathbf{252.19 \text{ kJ/s} \quad \text{Ans.}}$$

22. Determine the change in availability due to throttling of steam from 6 MPa and 400°C to 5 MPa when surroundings are at 100 kPa and 20°C. The changes in KE and PE may be considered negligible.

Solution:

From steam tables

Initially at 6 MPa, 400°C, $h_1 = 3177.2 \text{ kJ/kg}$

$$s_1 = 6.5408 \text{ kJ/kg.K}$$

After throttling at 5 MPa,

$$h_2 = h_1; \text{ in view of throttling process.}$$

Hence at 5 MPa and

$$h_2 = 3177.2 \text{ kJ/kg}$$

Superheated Steam table gives,

$$\text{at 5 MPa, } h_{\text{at } 350^\circ\text{C}} = 3068.4 \text{ kJ/kg}$$

$$\text{at 5 MPa, } h_{\text{at } 400^\circ\text{C}} = 3195.7 \text{ kJ/kg}$$

Hence by interpolation at 5 MPa, enthalpy of 3177.2 kJ/kg will be at

$$T_2 = 350 + \frac{(400 - 350) \times (3177.2 - 3068.4)}{(3195.7 - 3068.4)}$$

$$T_2 = 392.7^\circ\text{C}$$

After throttling steam will be at 5 MPa, 392.7°C.

By interpolation Entropy, $s_2 = 6.6172 \text{ kJ/kg.K}$

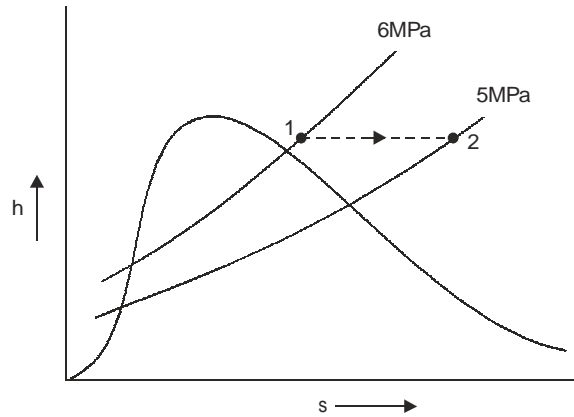


Fig. 6.29

At dead state of 20°C,

$$h_{f \text{ at } 20^\circ\text{C}} = h_0 = 83.96 \text{ kJ/kg}$$

$$s_{f \text{ at } 20^\circ\text{C}} = s_0 = 0.2966 \text{ kJ/kg.K}$$

Hence availability at state 1,

$$A_1 = (h_1 - h_0) - T_0 (s_1 - s_0) + \frac{1}{2} (C_1^2) + g (z_1 - z_0)$$

⇒

$$A_1 = (3177.2 - 83.96) - 293(6.5408 - 0.2966) + 0 + 0$$

$$A_1 = 1263.68 \text{ kJ/kg}$$

Availability of steam after throttling,

$$A_2 = (h_2 - h_0) - T_0 (s_2 - s_0) + \left(\frac{1}{2} C_2^2\right) + g (z_2 - z_0)$$

$$A_2 = (3177.2 - 83.96) - 293 (6.6172 - 0.2966)$$

$$A_2 = 1241.30 \text{ kJ/kg}$$

$$\text{Change in availability} = A_2 - A_1$$

$$= (1241.30 - 1263.68)$$

$$= -22.5 \text{ kJ/kg}$$

$$= \mathbf{22.5 \text{ kJ/kg, decrease.} \quad \text{Ans.}}$$

23. A parallel flow heat exchanger has hot water flowing at 95°C for heating cold water at 15°C to 45°C . Hot water flows at the rate of 800 gm/sec and the temperature of this hot water stream should not be less than 50°C at exit. Estimate the second law efficiency and rate of exergy destruction considering dead state temperature of 25°C .

Solution:

Let hot stream and cold stream be shown to enter at section 1-1 and leave at 2-2

Given;

$$T_{H1} = 95^\circ\text{C}, T_{H2} = 50^\circ\text{C}, m_H = 800 \text{ gm/s}$$

$$T_{C1} = 15^\circ\text{C}, T_{C2} = 45^\circ\text{C} \quad m_H = 0.8 \text{ kg/s}$$

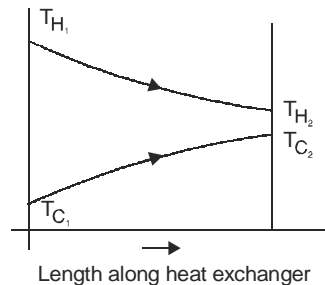
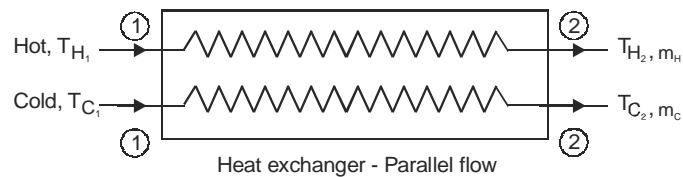


Fig. 6.30

For parallel flow heat exchanger as shown in figure;

$$m_H \times C_{p,H} (T_{H1} - T_{H2}) = m_c \times C_{p,c} (T_{C2} - T_{C1})$$

$$\Rightarrow 0.8 \times (95 - 50) = m_c (45 - 15)$$

$$\Rightarrow m_c = 1.2 \text{ kg/s}$$

$$\text{Second law efficiency} = \frac{(\text{Rate exergy increase in cold stream})}{(\text{Rate of exergy input to heat exchanger through hot stream})}$$

The exergy entering (input) through hot water stream,

$$A_{H1} = m_H \{ (h_{H1} - h_0) - T_0 (s_{H1} - s_0) \}$$

Using steam tables

At 25°C $h_0 = 104.89 \text{ kJ/kg}$, $s_0 = 0.3674 \text{ kJ/kg.K}$ saturated liquid yields.

At 95°C, $h_{H_1} = h_{f \text{ at } 95^\circ\text{C}} = 397.96 \text{ kJ/kg}$

(for hot stream)

$s_{H_1} = s_{f \text{ at } 95^\circ\text{C}} = 1.2500 \text{ kJ/kg.K}$

At 50°C, $h_{H_2} = h_{f \text{ at } 50^\circ\text{C}} = 209.33 \text{ kJ/kg.K}$

(for hot stream)

$s_{H_2} = s_{f \text{ at } 50^\circ\text{C}} = 0.7038 \text{ kJ/kg.K}$

At 45°C, $h_{c_2} = h_{f \text{ at } 45^\circ\text{C}} = 188.45 \text{ kJ/kg.K}$

(for cold stream)

$s_{c_2} = s_{f \text{ at } 45^\circ\text{C}} = 0.6387 \text{ kJ/kg.K}$

At 15°C, $h_{c_1} = h_{f \text{ at } 15^\circ\text{C}} = 62.99 \text{ kJ/kg.K}$

(for cold stream)

$s_{c_1} = s_{f \text{ at } 15^\circ\text{C}} = 0.2245 \text{ kJ/kg.K}$

Rate of exergy input through hot water stream,

$$A_{H_1} = 0.8 \times \{(397.96 - 104.89) - 298(1.25 - 0.3674)\}$$

$$A_{H_1} = 24.04 \text{ kJ/s}$$

Rate of exergy increase in cold stream;

$$\Delta A_c = m_c \{(h_{c_2} - h_{c_1}) - T_0(s_{c_2} - s_{c_1})\}$$

$$\Delta A_c = 1.2 \{(188.45 - 62.99) - 298(0.6387 - 0.2245)\}$$

$$\Delta A_c = 2.43 \text{ kJ/s}$$

$$\text{Second law efficiency} = \frac{\Delta A_c}{A_{H_1}} = \frac{2.43}{24.04} = 0.1011$$

or **10.11%** **Ans.**

Rate of exergy loss in hot stream,

$$\Delta A_H = m_H \{(h_{H_1} - h_{H_2}) - T_0(s_{H_1} - s_{H_2})\}$$

$$\Delta A_H = 0.8 \{(397.96 - 209.33) - 298(1.25 - 0.7038)\}$$

$$\Delta A_H = 20.69 \text{ kJ/s}$$

Hence exergy destruction

$$= \Delta A_H - \Delta A_c$$

$$= 20.69 - 2.43$$

$$= \mathbf{18.26 \text{ kJ/s}} \quad \mathbf{Ans.}$$

EXERCISE

- 6.1** Discuss generation of steam from ice at -5°C at 1 atm with the help of T - S and P - V diagrams.
- 6.2** What is meant by mollier diagram? Explain.
- 6.3** Write short notes on the following;
Sensible heating, Latent heating, Critical point, Triple point
- 6.4** Discuss different zones on T - V diagram for steam.

- 6.5** Derive the expression for enthalpy change during steam generation from feed water to superheated steam.
- 6.6** Discuss the throttling calorimeter for dryness fraction measurement.
- 6.7** Give a neat sketch of “separating and throttling calorimeter” for dryness fraction measurement.
- 6.8** Sketch the throttling and superheating processes on h - s and T - S diagrams.
- 6.9** Determine the final condition of steam if it is passed through a reducing valve which lowers the pressure from 2 MPa to 1 MPa. Assume initial state of steam to be 15% wet. [0.87]
- 6.10** Determine the final condition of steam, workdone, heat transferred and change in entropy if 0.5 kg of steam at 1 MPa and 0.8 dry is heated at constant pressure until its volume gets doubled. [408.6°C, 77.5 kJ, 453.5 kJ, 0.895 kJ/K]
- 6.11** Determine the state of substance if 3346 kJ of heat is added to wet steam in a closed rigid vessel of 3m³ volume containing 5 kg of wet steam at a pressure of 200 kPa till its pressure become 304 kPa. [Dry]
- 6.12** Complete the following table from steam table.

	Pressure (MPa)	Temperature (°C)	Enthalpy (kJ/kg)	Quality (x)	Specific volume (m ³ /kg)	Entropy (kJ/kg.K)
(a)	1	—	—	—	—	6.5865
(b)	—	250.4	—	0	—	—
(c)	10	—	—	0.8	—	—
(d)	20	700	—	—	—	—
(e)	15	800	—	—	—	—

- (a) 179.9°C, 762.8 kJ/kg, 1, 0.1944 m³/kg.
- (b) 4 MPa, 1087.31 kJ/kg, 1.252 m³/kg, 2.7964 kJ/kg.K
- (c) 311.06°C, 2461.33 kJ/kg, 0.01442 m³/kg, 5.1632 kJ/kg.K
- (d) 3809 kJ/kg, 1, 0.02113 m³/kg, 6.7993 kJ/kg.
- (e) 4092.4 kJ/kg, 1, 0.0321 m³/kg, 7.204 kJ/kg.K.
- 6.13** Determine the pressure in a rigid vessel and volume of rigid vessel if it contains 500 kg of water at 65°C. [25 kPa, 0.51 m³]
- 6.14** Estimate the change in volume of water and the total heat required for its' vaporization in a boiler producing saturated steam at 75 kPa. One kg feed water is supplied to boiler as saturated water. [2.22 m³, 2.28 MJ]
- 6.15** Determine enthalpy, entropy and specific volume for following cases
- (i) Steam at 4 MPa and 80% wet. (ii) Steam at 10 MPa and 550°C.
- (iii) Steam at 8 MPa and 295°C.
- Also estimate the above properties using Mollier diagram and quantify the percentage variation
- [1430.13 kJ/kg, 3.45 kJ/kg.K, 0.011 m³/kg]
- [3500.9 kJ/kg, 6.76 kJ/kg.K, 0.036 m³/kg]
- [2758 kJ/kg, 5.74 kJ/kg.K, 0.024 m³/kg]
- 6.16** Determine the temperature of steam at 20 MPa if its specific volume is 0.0155m³/kg. [520°C]
- 6.17** Steam undergoes reversible adiabatic expansion in steam turbine from 500 kPa, 300°C to 50 kPa. Determine the work output per kg of steam turbine and quality of steam leaving steam turbine. [357.64 kJ/kg, 0.98]
- 6.18** Steam flowing through two pipelines at 0.5 MPa are mixed together so as to result in a mixture flowing at 2.2 kg/s and mass flow ratio of two is 0.8. One stream has quality of 0.8. Determine the temperature of second stream so as to result in the final mixture having dryness fraction of 0.994. [300°C approx.]
- 6.19** A steam turbine operates with isentropic efficiency of 90%. Turbine handles 6 kg/s of steam at 0.980 MPa and 200°C and leaves at 0.294 MPa. Determine the power developed in hp and change of entropy from inlet to exit. [1660 hp, 0.050 kJ/kg.K]

- 6.20** A boiler is fed with water velocity of 2m/s, 1.96 MPa, 100°C. Steam is produced at 400°C temperature and comes out with velocity of 50 m/s. Determine the rate at which heat should be supplied per kg of steam for above operation of boiler. [2824.8 kJ/kg]
- 6.21** A steam nozzle is supplied steam at 1 MPa, 200°C and 100 m/s. Expansion upto 0.3 MPa occurs in the nozzle. Assuming isentropic efficiency of nozzle to be 0.9 determine final steam velocity.
- 6.22** Combined separating and throttling calorimeter is used to determine quality of steam. Following observations are made;
 Steam inlet pressure = 1.4 MPa
 Pressure after throttling = 0.1 MPa
 Temperature after throttling = 120°C
 Water collected in separator = 0.45 kg
 Steam condensed after throttling = 6.75 kg
 Take specific heat of superheated steam = 2.1 kJ/kg.K
 Also find the limiting quality of steam to be measured by above throttling calorimeter alone assuming that separating calorimeter is not there. [0.90, 0.94]
- 6.23** Steam at 400kPa, dryness fraction of 0.963 is isentropically compressed till it becomes dry saturated. This one kg steam is then heated isobarically till the initial volume is attained and subsequently steam is restored to initial state following isochoric cooling. Determine the net work and net heat interactions. Also show processes on T-s diagram. [29.93 kJ/kg, 29.93 kJ/kg]
- 6.24** Wet steam at 1 MPa, 0.125m³ volume and enthalpy of 1814 kJ is throttled up to 0.7 bar pressure. Determine the final state of steam, initial mass and dryness fraction considering $c_p = 2.1$ kJ/kg.K [101.57°C, 0.675kg, 0.953]
- 6.25** Steam initially at 5 bar, 0.6 dry is isochorically heated till its pressure becomes 10 bar. This 15 kg steam is expanded up to 3 bar following $pv^{1.3} = \text{constant}$. Subsequently steam is cooled at constant pressure till its dryness fraction becomes half of that existed after second process. Determine the heat, work and entropy change in three processes.
 [I process: 13.38 MJ, 0, 30,285 kJ/K.
 II process: - 1.25MJ, 2.73 MJ, - 2.99kJ/K
 III process: - 15.22 MJ, - 1.28 MJ, 37.4 kJ/K]
- 6.26** Determine the heat transfer and change in entropy in each process when steam at 20 bar, 250°C expands till it reaches 4 bar following $pv^{1.35} = \text{constant}$ and subsequently heated at constant volume till its pressure becomes 8 bar. [- 319.36 kJ/kg, & - 0.725 kJ/kg.K
 764.95kJ/kg & 1.65 kJ/kg.K]
- 6.27** A closed vessel of 0.6 m³ initially has steam at 15 bar, 250°C. Steam is blown off till pressure drops up to 4 bar. Subsequently vessel is cooled at constant pressure till it becomes 3 bar. Considering the expansion of gas to be isentropic during blow-off determine heat transferred during cooling process. [- 620.38 kJ]
- 6.28** Determine the heat transferred when steam is taken out isobarically from a boiler tank till boiler is left with 80% water only. Volume of boiler tank is 10m³ and initially it has equal volumes of steam and water at 10 bar. [1.75 × 10⁶ kJ]
- 6.29** Determine the temperature of steam at 1.5 MPa having mass of 50 gm and stored in vessel with volume of 0.0076 m³. Vessel is cooled until pressure in vessel becomes 1.1 MPa. Determine the temperature at which steam will be just dry saturated during cooling process. Also determine the final dryness fraction and total heat rejected. [250°C, 191.6°C, 0.85, 18.63 kJ]
- 6.30** Calculate the dryness fraction of steam after throttling when it is throttled from 1.4 MPa to 1 MPa & 423K.
 Also determine the final condition of steam if this pressure drop takes place in closed vessel of 0.56 m³ volume and heat is lost by conduction and radiation. [0.98, 0.298]

Availability and General Thermodynamic Relations

7.1 INTRODUCTION

In the present civilization the use of energy resources has increased tremendously. Fast depleting fossil fuel reserves have inevitably gathered the attention of one and all to think and devise for optimum energy utilization. In order to optimally use energy, the efforts are required for identification and elimination of the sources of inefficiency during its use, which obviously requires in depth study and analysis. A look into the laws of thermodynamics shows that the first law of thermodynamics bases upon the series of experiments done by James Joules, demonstrating the bidirectional numerical equivalence of converting work into heat while second law of thermodynamics exhibits a unidirectional equivalence between work and heat, i.e. for a given amount of heat the equivalent amount of work cannot be obtained whereas vice-a-versa may be there. Thus, the concept of quality of energy came into existence and work is considered as high grade of energy and heat as low grade of energy. Other forms of high grade energy are electrical energy, wind energy, tidal energy etc. and low grade energy may be heat from nuclear reactions, heat from combustion of fuel etc. Engineers have been using the first law of thermodynamics stating the energy conservation, therefore it could be concluded that energy can not be destroyed and exists with matters in all forms everywhere. It is now quite convincing to understand that the scarcity of energy resources and energy crisis is a paradox. Still in real life we find scarcity of energy, as in practice one is interested in the ability to feed, drive machines and occurrence of energy processes etc. Such discussions gave birth to the concept of 'available energy' and 'unavailable energy' or a concept of 'maximum work'.

This concept became very important in phenomenological thermodynamics, as it referred to the possibilities of performing work in real conditions. G. Gouy and A. Stodola pioneered in the studies pertaining to effect of ambient temperature upon the obtainable work and law of the loss of maximum work. The law of the loss of maximum work says that the work obtained is always less than the maximum obtainable work due to the irreversibility in thermal processes. Available energy concept came out of these propositions. Quality of energy, its convertibility into other forms and capability to perform work etc. are quantitatively defined using availability analysis. New term 'exergy' was introduced by Z. Rant in 1956 so as to differentiate it from energy. 'Exergy' analysis or 'availability' analysis has capability to identify and quantify the causes of thermodynamic imperfections in thermodynamic processes and thus indicate about the possibilities of improving the processes. It is preferred over energy analysis as energy analysis can not detect majority of thermodynamic imperfections. Such as the irreversible heat transfer, throttling and adiabatic combustion etc. do not have any energy loss but make the quality of energy inferior. Energy entering with fuel, electricity, flowing streams of matter and so on can be

accounted for in products and by products. Energy cannot be destroyed. The idea that something can be destroyed is useful but should not be applied to 'energy', however it could be applied to another variable 'exergy'. Moreover, it is exergy and not energy that properly gauges the quality (utility), say one kJ of electricity generated by a power plant versus one kJ in plant cooling water stream. Electricity obviously has greater quality and the greater economic value.

These phenomenon can be evaluated by second law analysis easily. Exergy analysis could be integrated with principles of engineering economics to determine the potential for cost effective improvement of existing systems. Exergy and costing principles can also be used at initial design stage to develop systems that are 'optimized in annualized cost', 'sparing in use of fossil fuels' and 'environmentally friendly'.

Let us see a electricity generating power cycle as shown.

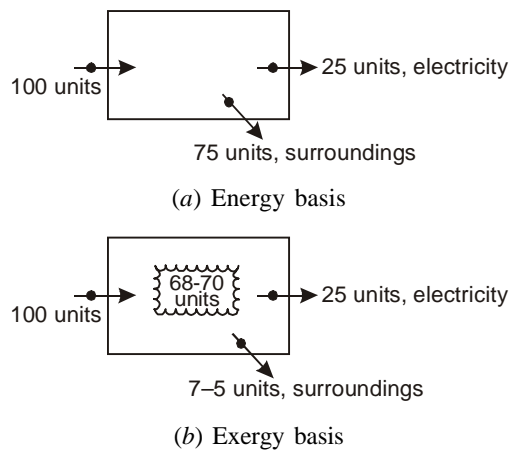


Fig. 7.1 Energy and exergy basis

Here Fig. 7.1(a) represents energy basis indicating that out of 100 energy units entering with fuel, 25 energy units are obtained as electricity and the remaining 75 units are discharged to surroundings. On exergy basis it may also be considered that 100 units of exergy enter with fuel are 25 units of exergy exit along with the electricity. For remaining 75 units it is seen that 68-70 units of exergy are destroyed within the plant due to irreversibilities and only 5-7 units are discharged to surroundings. Here it is worth noting from exergy basis that out of 75 units of energy considered to be discharged to surroundings in energy basis actually only 5-7 units are discharged to surroundings and the majority 68-70 units are lost due to irreversibilities. The loss of energy due to irreversibilities can be minimized by minimizing or eliminating the causes of irreversibility. Exergy analysis thus shows that significant performance improvement can come only by identifying and correcting the sources of inefficiency within system as the discharge is a minor area of concern.

7.2 AVAILABILITY OR EXERGY

From earlier discussions, it is obvious that energy can be conveniently categorised as low grade energy and high grade energy. Also, the second law of thermodynamics prohibits the complete conversion of low grade energy into high grade energy. The portion of low grade energy that can be converted is called 'available energy' or 'exergy' or 'availability' and the portion of energy not available for conversion is called 'unavailable energy' or 'anergy'. Mathematically;

$$\text{Anergy} = \text{Energy} - \text{Exergy}.$$

“Exergy can be quantified as the amount of work obtainable by bringing some matter to the state of thermodynamic equilibrium with common components of natural surroundings through reversible processes, thus involving interaction only with above mentioned components of nature.”

As per Moran and Sciubba (1994), the “exergy refers to the maximum theoretical work that can be extracted from a combined system comprising of ‘system’ and ‘environment’ as the system passes from a given state to equilibrium with the environment—that is, system changes its’ state to the dead state at which combined system possesses energy but no exergy.”

Rickert defined “exergy as the shaft work or electrical energy necessary to produce a material in its specified state from materials common in the environment in a reversible way, heat being exchanged only with environment.”

Exergy is an extensive property whose value is fixed by the state of system once the environment has been specified. Exergy can also be represented on an intensive basis i.e. per unit mass or per mole basis. For all states of the system exergy shall be numerically greater than or equal to zero.

$$\text{Exergy} \geq 0$$

Exergy as defined above is a measure of departure of the state of a system from that of environment. For state at temperature T and environment at temperature T_0 the difference ($T \sim T_0$) shall decide the value of exergy i.e. greater the difference, the greater shall be exergy value. This exergy can be of basically two types i.e. chemical exergy and thermomechanical exergy. Thermomechanical exergy can be further classified as physical, kinetic and potential exergy. Physical exergy is the work obtainable by taking the substance by reversible physical processes from its initial states pressure ‘ p ’ and temperature ‘ T ’ to the state determined by the temperature and pressure of environment. Kinetic exergy is equal to the kinetic energy, when the velocity is considered relative to the surface of the earth. Potential exergy is equal to the potential energy when it is evaluated with respect to the average level of the surface of the earth in the locality of the process under consideration.

Chemical exergy refers to the work that can be obtained by taking a substance state at environmental pressure and temperature to the state of thermodynamic equilibrium with environment and bring system to restricted dead state.

Thermomechanical exergy refers to the maximum theoretical work obtainable as system passes from some given state to the restricted dead state.

Thermal exergy is defined as the sum of ‘physical exergy’ and ‘chemical exergy’.

Rant defined, “exergy as that part of energy which could be fully converted into any other kind of energy”. Exergy is function of state parameters of matter under consideration and of the state parameters of common components of environment as exergy results from the possibility of interaction between matter under consideration and common components of environment.

‘Environment’ here refers to the region or part of surroundings whose intensive properties do not change significantly with the occurrence of processes under consideration, while ‘surroundings’ comprise of everything that is not included in system. Environment is considered to be large and homogeneous in terms of pressure and temperature. Environment is regarded free of irreversibilities. All significant irreversibilities are present in the system and its’ immediate surroundings. Irreversibilities present within system are called ‘internal irreversibilities’ while ‘external irreversibilities’ are those present in its’ immediate surroundings.

‘Dead state’ refers to the state at which system and the environment are at mechanical, thermal and chemical equilibrium. Thus neither there can be any spontaneous change within the system or within the environment, nor any spontaneous interaction between the two. Dead state being a limiting state is also called ‘restricted dead state’. At dead state the system is at same temperature and pressure as that of its’ surroundings and shall have no kinetic energy or potential energy relative to surroundings. A system

shall thus have zero exergy (availability) at dead state and yield maximum possible work only when it follows a reversible process from its' state to the state of its' surroundings (dead state). Exergy or availability thus quantifies the maximum theoretical work available without violation of any laws of thermodynamics.

An engine operating with heat reservoir at T_1 and supplying Q_1 amount of heat and the environment temperature being T_0 shall give maximum amount of work when it operates between T_1 and T_0

$$\text{Maximum efficiency, } \eta_{\max} = 1 - \frac{T_0}{T_1} = \eta_{\text{rev}}$$

$$\text{Maximum work} = Q_1 \cdot \eta_{\max} = \text{Availability}$$

Let us consider a 'combined system' and find work done. Combined system comprises of control system and environment. Contents of control system do not mix with environment or have any reaction with environment. Maximum work is available when control system changes its state from initial state to dead state.

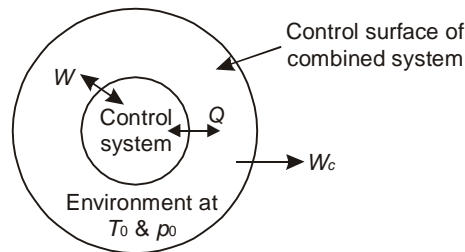


Fig. 7.2 Combined system

Let the control system have heat and work interaction Q and W with environment. Let us have only work interactions, W_c across the control surface of combined system. Let us use following nomenclature,

A = availability or exergy	Subscript:
E = energy, KE = kinetic energy,	c = combined system
PE = potential energy	e = environment
p = pressure	s = control system
Q = heat	0 = dead state
S = entropy	i = initial state
T = temperature	
U = internal energy	
W = work	

Here for combined system

$\Delta V_c = 0$, where $V_c = V_s + V_e$, although V_s of system or V_e of environment may change but total volume of combined system shall remain constant.

Total work interaction of combined system can be given by total energy change of combined system.

$$W_c = -\Delta E_c$$

Energy change of combined system = Energy change in control system + Energy change in environment

$$\Delta E_c = \Delta E_s + \Delta E_e$$

Energy change in control system, $\Delta E_s = \text{Energy of system at dead state i.e., final state} - \text{Energy of system at initial state}$

Energy of system at dead state,

as energy

$$E = U + KE + PE$$

$$E_{s, \text{ at dead state}} = U_0, \text{ as } KE = PE = 0$$

Energy of system at initial state, $E_{s, \text{ initially}} = E_{s, i}$

$$\Delta E_s = U_0 - E_{s, i}$$

Energy change in environment, shall be due to heat interaction and the work associated with its' volume change ($p dv$ work). For example expansion inside a piston cylinder arrangement shall have piston also displacing the volume of environment ($p dV$ work is boundary work). Change in extensive properties, internal energy, entropy, volume of environment can be given by first law of thermodynamics.

$$\Delta E_e = \Delta U_e = T_0 \Delta S_e - p_0 \Delta V_e$$

Hence, work interaction of combined system,

$$W_c = - \{ (U_0 - E_{s, i}) + (T_0 \Delta S_e - p_0 \Delta V_e) \}$$

Also, we have seen that for combined system

$$\Delta V_c = 0$$

$$\Delta V_s + \Delta V_e = 0$$

or,

$$\Delta V_s = - \Delta V_e$$

Here, $\Delta V_s = \text{change in volume of system} = (\text{Final volume of system at dead state} - \text{Initial volume})$

$$\Delta V_s = V_0 - V_{s, i}$$

Substituting in W_c ,

$$W_c = \{ (E_{s, i} - U_0) + p_0 (V_{s, i} - V_0) - T_0 \Delta S_e \}$$

Total entropy change of combined system shall be due to irreversibilities within the combined system;

$$\Delta S_c = \Delta S_s + \Delta S_e$$

$$\Delta S_c = (S_0 - S_{s, i}) + \Delta S_e$$

or,

$$\Delta S_e = (-S_0 + S_{s, i}) + \Delta S_c$$

Substituting ΔS_e in work, W_c ,

$$W_c = \{ (E_{s, i} - U_0) + p_0 (V_{s, i} - V_0) - T_0 ((-S_0 + S_{s, i}) + \Delta S_c) \}$$

For the combined system total entropy change shall be either zero for reversible process or more than zero for irreversible process;

Mathematically, $\Delta S_c \geq 0$

Hence,

$$W_c \leq \{ (E_{s, i} - U_0) + p_0 (V_{s, i} - V_0) - T_0 (S_{s, i} - S_0) \}$$

For reversible processes there will be no entropy generation, i.e. $\Delta S_c = 0$ and work shall be maximum only when the combined system is internally reversible from all respects.

Thus, $W_{c, \text{ max}} = \{ (E_{s, i} - U_0) + p_0 (V_{s, i} - V_0) - T_0 (S_{s, i} - S_0) \}$

In general terms for any initial state of system, which is having all reversible processes.

$$W_{c, \text{ max}} = \{ (E - U_0) + p_0 (V - V_0) - T_0 (S - S_0) \}$$

$$\text{Availability or exergy, } A = W_{c, \text{ max}} = \{ (E - U_0) + p_0 (V - V_0) - T_0 (S - S_0) \}$$

Availability or exergy cannot be less than zero as the maximum work interaction can not be less than zero.

Also it can be given as,

$$W_c = A - T_0 \cdot \Delta S_c$$

Above expression shows that some work done by combined system gets lost i.e. the irreversibilities causing entropy production keep work below its' maximum value.

Availability or exergy is not conserved like energy. Exergy gets destroyed by irreversibilities when the control system changes to dead state and no work is done by combined system as in case of spontaneous change.

Availability destruction is proportional to entropy generation due to irreversibilities in processes. Irreversibility can be given as the product of dead state temperature and entropy generation due to irreversible process.

$$I = T_0 \cdot \Delta S_c$$

and

$$W_c = A - I$$

and

$$I = A - W_c$$

Above discussion indicates that the *maximum work shall be obtained when a process takes place in reversible manner*. But in fact almost all the processes in real life occur in irreversible manner, so some portion of energy is always unavailable. As irreversible processes are continuously increasing therefore unavailable energy is also gradually increasing. This phenomenon is also called *principle of degradation of energy or law of degradation of energy*.

$$\text{Availability, } A = \{(E - U_0) - T_0(S - S_0) + p_0(V - V_0)\}, \text{ kJ}$$

Availability per unit mass,

$$\omega = \{(e - u_0) - T_0(s - s_0) + p_0(v - v_0)\}, \text{ kJ/kg}$$

Availability or exergy is thus a measure of departure of state of system from that of environment. Thus, it is an attribute of system and environment together. However, once the environment is specified, a numerical value can be assigned to availability in terms of property values for system only. Hence, *exergy can be regarded as property of the system*.

7.3 AVAILABILITY ASSOCIATED WITH HEAT AND WORK

Let us consider a reversible heat engine having heat transfer from environment to control system and vice-a-versa.

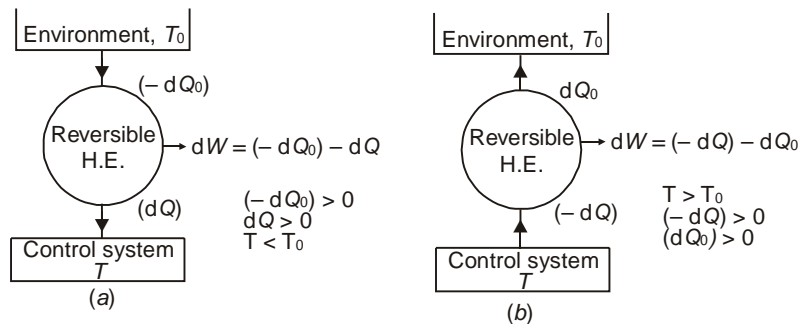


Fig. 7.3

Let us consider a reversible heat engine transferring heat δQ to the control system at temperature T from environment at temperature T_0 . From second law of thermodynamics,

$$\frac{\delta Q}{T} = \left(\frac{-\delta Q_0}{T_0} \right)$$

work,
$$\delta W = \left(\frac{T_0}{T} - 1 \right) \delta Q$$

Let us now consider a reversible heat engine transferring heat δQ from control system to environment at T_0 . From second law of thermodynamics,

$$\frac{\delta Q_0}{T_0} = \left(\frac{-\delta Q}{T} \right)$$

so work,
$$\delta W = \left(1 - \frac{T_0}{T} \right) (-\delta Q)$$

$$= \left(\frac{T_0}{T} - 1 \right) \delta Q$$

Availability associated with heat transfer : Let us consider a control system at dead state interacting with other system and there is heat interaction Q in control system. Let the final state of control system be given by 'f'. Due to heat interaction Q the control system may get heated up or cooled so that the final state is different from that of environment. Control system's temperature may increase from T_0 to T_f or decrease from T_f to T_0 but in every case availability shall increase. Availability of control system at final state gives maximum work that will be available from the combined system (control system + environment) as control system returns to dead state.

Work available from a reversible heat engine when control system gets heated or cooled by environment,

$$W_{\max} = \int_f^0 \left(\frac{T_0}{T} - 1 \right) \delta Q$$

Availability associated with heat transfer =
$$\int_0^f \left(1 - \frac{T_0}{T} \right) \delta Q$$

$$A_Q = \int \left(1 - \frac{T_0}{T} \right) \delta Q$$

If there is irreversibility present within control system due to internal irreversibilities, then availability change from initial to final state can be given as

$$A_Q = \int \left(1 - \frac{T_0}{T} \right) \delta Q - I$$

Since, here control system's initial state was dead state having zero availability so the change in availability

$$\Delta A_Q = \int \left(1 - \frac{T_0}{T} \right) \delta Q - I$$

Availability associated with work : Let us consider a control system initially at dead state. Control system has adiabatic compression occurring in it due to work interaction with some other system. – W work is done on control system and it attains some final state, ' f '. Availability in this case shall be the maximum work available from the combined system of control system and environment as control system returns to the dead state.

If the work W is done by the control system as it returns from final state ' f ' to dead state and the change in volume V_f to V_0 takes place by displacing the environment ($p dV$ work), then availability associated with work,

$$A_w = \Delta A_w = [W - p_0(V_f - V_0)]$$

In case no boundary work is there, then $V_f = V_0$

$$A_w = W = \Delta A_w$$

Here it is also the availability change as system is returning to dead state. In case there is availability loss due to internal irreversibilities then change in availability,

$$A_w = [W - p_0(V_f - V_0)] - I = \Delta A_w$$

Similarly, availability associated with kinetic energy and potential energy can be given as,

$$A_{KE} = \frac{1}{2} m V^2 \quad ; \text{availability with K.E.}$$

and $A_{PE} = mgz$; availability with P.E.

Generalized availability equation : A general availability equation for a control system having heat and work interactions with other systems can be obtained using earlier formulations. Let us consider a control system interacting with other systems and also having irreversibilities causing availability destruction in it. For elemental change during a process the energy balance can be given as, $dE = \delta Q - \delta W$.

$$\text{Total entropy change, } dS = \frac{\delta Q}{T} + \delta S_{\text{irrev}}$$

where T is temperature on control surface having δQ heat transfer and δS_{irrev} is entropy generated due to irreversibilities

Energy equation can be rewritten as,

$$dE + p_0 dV = \delta Q - \delta W + p_0 dV$$

Entropy equation can be rewritten as,

$$T_0 dS = \frac{\delta Q \cdot T_0}{T} + T_0 \cdot \delta S_{\text{irrev}}$$

Combining modified forms of energy and entropy equations by subtracting one from other,

$$dE + p_0 dV - T_0 dS = \delta Q - \delta W + p_0 dV - \frac{\delta Q \cdot T_0}{T} - T_0 \cdot \delta S_{\text{irrev}}$$

$$dE + p_0 dV - T_0 dS = \left(1 - \frac{T_0}{T}\right) \cdot \delta Q - (\delta W - p_0 \cdot dV) - T_0 \cdot \delta S_{\text{irrev}}$$

We have already seen in earlier article that the change in availability can be given as,

$$dA = dE + p_0 dV - T_0 dS$$

Hence,

$$dA = \left(1 - \frac{T_0}{T}\right) \delta Q - (\delta W - p_0 dV) - T_0 \delta S_{\text{irrev}}$$

Here, $T_0 \delta S_{\text{irrev}} = I$

For any process in control system between states 1 and 2, availability change can be given as,

$$\Delta A_{1-2} = \underbrace{\int_1^2 \left(1 - \frac{T_0}{T}\right) dQ}_{\text{Availability associated with heat transfer}} - \underbrace{(W_{1-2} - p_0 \Delta V_{1-2})}_{\text{Availability associated with work interaction}} - \underbrace{I}_{\text{Irreversibility}}$$

Generally, for any control mass in control system the availability change can be given as,

$$\boxed{\Delta A_{1-2} = \int_1^2 \left(1 - \frac{T_0}{T}\right) dQ - (W - p_0 \Delta V) - I}$$

Above availability change can also be given on per unit time basis.

7.4 EFFECTIVENESS OR SECOND LAW EFFICIENCY

Performance of engineering systems are generally measured using efficiency as defined by first law of thermodynamics. Efficiency as defined by first law uses energy for its' quantification. *Second law efficiency* or *effectiveness* or *exergetic efficiency* is an analogous parameter defined using availability.

Energy balance for a system with steady state yields,

$$\text{Energy in} = \text{Energy output} + \text{Energy loss}$$

Availability equation shall yield,

$$\text{Availability in} = (\text{Availability output} + \text{Availability loss} + \text{Availability destruction due to irreversibility})$$

Mathematically,

$$\text{(by first law), Efficiency } \eta = \frac{\text{Energy out in product (= Output)}}{\text{Energy in}}$$

$$\eta = \frac{\text{Energy input} - \text{Energy loss}}{\text{Energy input}}$$

$$\boxed{\eta = 1 - \frac{\text{Energy loss}}{\text{Energy input}}}$$

$$\text{(by second law), Effectiveness, } \varepsilon = \frac{\text{Availability output}}{\text{Availability in}}$$

$$\varepsilon = 1 - \left[\frac{\text{Availability loss} + \text{Availability destruction due to irreversibility}}{\text{Availability in}} \right]$$

Effectiveness can also be given as the ratio of thermal efficiency to the maximum possible thermal efficiency (reversible processes) under same conditions.

$$\varepsilon = \frac{\eta_{th}}{\eta_{th,rev}}$$

For work producing systems, effectiveness = $\left(\frac{\text{Useful work}}{\text{Reversible work or maximum work}} \right)$

For work absorbing systems, effectiveness = $\left(\frac{\text{Reversible work or maximum work}}{\text{Useful work}} \right)$

For refrigerators and heat pumps, effectiveness = $\left(\frac{\text{COP}}{\text{COP}_{rev.}} \right)$

In general terms, Second law efficiency or effectiveness = $\left(\frac{\text{Availability used}}{\text{Availability supplied}} \right)$

7.5 SECOND LAW ANALYSIS OF STEADY FLOW SYSTEMS

Let us consider a steady flow system as shown,

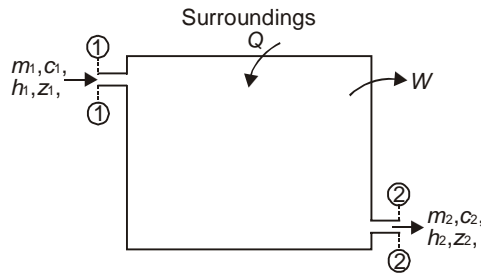


Fig. 7.4 Steady flow system

Section 1–1 and 2–2 refer to inlet and exit respectively.

Steady flow system interacts with surroundings at P_0 and T_0 . Steady flow energy equation can be given as,

$$Q + m_1 \left(h_1 + \frac{c_1^2}{2} + gz_1 \right) = W + m_2 \left(h_2 + \frac{c_2^2}{2} + gz_2 \right)$$

Entropy generated,

$$S_{gen} = m_2 s_2 - m_1 s_1 + \frac{Q_{surr}}{T_0}$$

Heat transfer from control system, $(-)$ = Heat gained by surrounding $(+)$; $-Q = Q_{surr}$, from above two equations, substitution for Q yields,

$$-T_0 S_{gen} + m_1 \left(h_1 + \frac{c_1^2}{2} + gz_1 - s_1 T_0 \right) = W + m_2 \left(h_2 + \frac{c_2^2}{2} + gz_2 - s_2 T_0 \right)$$

$$\text{or, } W = m_1 \left(h_1 + \frac{c_1^2}{2} + gz_1 - T_0 s_1 \right) - m_2 \left(h_2 + \frac{c_2^2}{2} + gz_2 - T_0 s_2 \right) - T_0 \cdot S_{\text{gen}}$$

W can be quantified as above. This W shall be actual work available from system. Here entropy generation due to irreversibilities in processes reduce W and so for fully reversible processes $S_{\text{gen}} = 0$ and we get maximum available work;

$$W_{\text{max}} = m_1 \left(h_1 + \frac{c_1^2}{2} + gz_1 - T_0 s_1 \right) - m_2 \left(h_2 + \frac{c_2^2}{2} + gz_2 - T_0 s_2 \right)$$

In general terms, actual work available in this kind of systems where boundary work ($p.dV$) is absent can be given as under,

$$W = \sum m_i \left(h_i + \frac{c_i^2}{2} + gz_i - T_0 \cdot s_i \right) - \sum m_e \left(h_e + \frac{c_e^2}{2} + gz_e - T_0 \cdot s_e \right) - T_0 \cdot S_{\text{gen}}$$

where subscript 'i' and 'e' refer to inlet and exit in system.

For no irreversibilities present or for reversible processes,

$$W_{\text{max}} = \sum m_i \left(h_i + \frac{c_i^2}{2} + gz_i - T_0 \cdot s_i \right) - \sum m_e \left(h_e + \frac{c_e^2}{2} + gz_e - T_0 \cdot s_e \right)$$

Expression 1

Change in availability,

$$\Delta A = \sum m_i \left\{ (h_i - h_0) + \frac{c_i^2}{2} + gz_i - T_0 (s_i - s_0) \right\} - \sum m_e \cdot \left\{ (h_e - h_0) + \frac{c_e^2}{2} + gz_e - T_0 (s_e - s_0) \right\}$$

on unit mass basis,

$$\Delta \omega = \{ T_0 \cdot \Delta s - \Delta h - \Delta K.E. - \Delta P.E. \} \text{ kJ/kg}$$

where $\Delta K.E.$ and $\Delta P.E.$ refer to kinetic energy and potential energy changes in system.

It indicates that the change in availability can be given by the difference of fluid stream availability at inlet and exit. "Fluid stream availability" can be defined in respect to dead state as, $c_0 = 0$, $z_0 = 0$

$$\psi = (h - h_0) + \frac{c^2}{2} + gz - T_0 (s - s_0) \text{ kJ/kg}$$

$$\psi = \frac{(u - u_0) + p_0 (v - v_0) - T_0 (s - s_0)}{2} + \frac{c^2}{2} + gz, \text{ kJ/kg.}$$

Here underlined terms are called physical exergy, $c^2/2$ is kinetic exergy and gz is potential exergy

$$\Delta A = W_{\text{max}} = \sum m_i \psi_i - \sum m_e \cdot \psi_e$$

Change of availability can be obtained using stream availability as described above. "Stream availability" is quantification of availability at a point.

Irreversibility rate in steady flow process can be given as, $I = T_0 \cdot S_{\text{gen}}$, (kW)

Exergy (Availability) and energy can be compared based upon their characteristics as given below.

<i>Exergy (availability)</i>	<i>Energy</i>
1. Exergy does not follow the law of conservation.	1. Energy follows the law of conservation.
2. It is function of states of the matter under consideration and the 'environment'.	2. It is function of the state of matter under consideration.
3. It is estimated with respect to the state of reference imposed by environment.	3. It may be calculated based upon the assumed state of reference.
4. Exergy always depends upon pressure.	4. In case of ideal gas energy does not depend upon pressure.
5. Exergy increases with temperature drop at low temperatures. For constant pressure processes exergy attains minimum value at the temperature of environment.	5. Energy increases with rise of temperature.
6. Exergy has positive value for ideal vacuum.	6. Energy is zero for an ideal vacuum.

7.6 GENERAL THERMODYNAMIC RELATIONS

Objective of this section is to develop mathematical relations for estimation of various thermodynamic properties such as u , h , s etc. for a compressible system. Thermodynamic properties such as pressure, volume and temperature (P , V , T) etc. can be directly measured experimentally while some other properties can not be measured directly and require thermodynamic relations for their determination. These thermodynamic relations are the basis for getting useful thermodynamic properties.

Important mathematical relations : To define state of a simple compressible system of known mass and composition one requires minimum two independent intensive properties. Thus, all intensive properties can be determined through functions of the two independent intensive properties such as,

$$p = p(T, v), u = u(T, v), h = h(T, v)...$$

Above are functions of two independent variables and can be given in general as, $z = z(x, y)$, where x , y are independent variables.

Exact differential : In earlier discussions we have seen that the differential of any property should be exact. Therefore, let us review calculus briefly.

Exact differential of any function z shall be as given below for z being continuous function of x and y .

$$dz = \left(\frac{\partial z}{\partial x} \right)_y dx + \left(\frac{\partial z}{\partial y} \right)_x dy$$

or,

$$dz = M \cdot dx + N \cdot dy$$

where, $M = \left(\frac{\partial z}{\partial x} \right)_y$, $N = \left(\frac{\partial z}{\partial y} \right)_x$ i.e. M is partial derivative of z with respect to x when variable y is held

constant and N is partial derivative of z with respect to y when variable x is held constant.

Here, since M and N have continuous first partial derivative therefore, order of differentiation is immaterial for properties and second partial derivative can be given as,

$$\frac{\partial}{\partial y} \left[\left(\frac{\partial z}{\partial x} \right)_y \right]_x = \frac{\partial}{\partial x} \left[\left(\frac{\partial z}{\partial y} \right)_x \right]_y$$

or,

$$\left(\frac{\partial M}{\partial y} \right)_x = \left(\frac{\partial N}{\partial x} \right)_y$$

Thus, the test of exactness for any property shall be,

$$\boxed{\left(\frac{\partial M}{\partial y} \right)_x = \left(\frac{\partial N}{\partial x} \right)_y}$$

Reciprocity relation and cyclic relation : Let us consider three variables x, y, z such that any two of these are independent variables. Thus, we can write

$$x = x(y, z); y = y(x, z)$$

In differential form, $dx = \left(\frac{\partial x}{\partial y} \right)_z \cdot dy + \left(\frac{\partial x}{\partial z} \right)_y \cdot dz; dy = \left(\frac{\partial y}{\partial x} \right)_z dx + \left(\frac{\partial y}{\partial z} \right)_x dz$

Combining above two relations we get

$$\left[1 - \left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial x} \right)_z \right] dx = \left[\left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x + \left(\frac{\partial x}{\partial z} \right)_y \right] dz$$

As x and z are independent variables so let us keep z constant and vary x , i.e. $dz = 0$ and $dx \neq 0$ which yields reciprocity relation as,

$$\left[1 - \left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial x} \right)_z \right] = 0$$

or,

$$\boxed{\left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial x} \right)_z = 1}$$

or,

$$\boxed{\left(\frac{\partial x}{\partial y} \right)_z = \frac{1}{\left(\frac{\partial y}{\partial x} \right)_z}} \quad \text{Reciprocity relation}$$

Similarly, let us keep x constant and vary z i.e. $dx = 0, dz \neq 0$ which shall be possible only when;

$$\left[\left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x + \left(\frac{\partial x}{\partial z} \right)_y \right] = 0$$

or

$$\boxed{\left(\frac{\partial x}{\partial y} \right)_z \left(\frac{\partial y}{\partial z} \right)_x \left(\frac{\partial z}{\partial x} \right)_y = -1} \quad \text{Cyclic relation}$$

7.6.1 Gibbs and Helmholtz Functions

For a simple compressible system of fixed chemical composition thermodynamic properties can be given from combination of first law and second law of thermodynamics as,

$$du = T \cdot ds - pdv$$

$$dh = T \cdot ds + vdp$$

Gibbs function (g) and Helmholtz function (f) are properties defined as below.

Gibbs function,

$$g = h - T \cdot s, \text{ on unit mass basis i.e. specific Gibb's function}$$

$$\text{also, } G = H - T \cdot S$$

Helmholtz function,

$$f = u - T \cdot s, \text{ on unit mass basis i.e. specific Helmholtz function}$$

$$\text{also, } F = U - T \cdot S$$

In differential form Gibbs function can be given as below for an infinitesimal reversible process

$$dg = dh - T \cdot ds - s \cdot dT$$

$$dg = vdp - sdT \text{ for a reversible isothermal process, } \int_1^2 dg = \int_1^2 vdp$$

$$\text{or, also } dG = Vdp - SdT \text{ for reversible isothermal process; } \int_1^2 dG = \int_1^2 Vdp$$

For a “reversible isobaric and isothermal process”, $dp = 0$, $dT = 0$ $dG = 0$

i.e. $G = \text{constant}$

‘Gibbs function’ is also termed as ‘Gibbs free energy’. For a reversible isobaric and isothermal process Gibbs free energy remains constant or Gibbs function of the process remains constant. Such reversible thermodynamic processes may occur in the processes involving change of phase, such as sublimation, fusion, vaporization etc., in which Gibbs free energy remains constant.

‘Helmholtz function’ is also called ‘Helmholtz free energy’. For any infinitesimal reversible process Helmholtz function can be given in differential form as,

$$df = du - T \cdot ds - sdT$$

$$\text{or, } df = -pdv - sdT$$

$$\text{or, } dF = -pdV - SdT$$

For a reversible isothermal process

$$df = -pdv$$

$$\text{or, } \int_1^2 df = - \int_1^2 p \cdot dv$$

$$\text{or, } \int_1^2 dF = - \int_1^2 pdV$$

For a reversible isothermal and isochoric process, $dT = 0$, $dV = 0$

$$df = 0$$

$$\text{or, } dF = 0$$

or,

$$F = \text{Constant}$$

Above concludes that the Helmholtz free energy remains constant during a reversible isothermal and isochoric process. Such processes may occur during chemical reactions occurring isothermally and isochorically.

7.6.2 Maxwell Relations

Differential equations of thermodynamic properties, u , h , f and g can be given as function of p , T , v , s as below:

$$\begin{aligned} du &= T \cdot ds - p dv \\ dh &= T \cdot ds + v dp \\ df &= -p dv - s \cdot dT \\ dg &= v dp - s \cdot dT \end{aligned}$$

Above equations can be used for defining the functions u , h , f , g based upon analogy with,

$$dz = M \cdot dx + N \cdot dy, \text{ for } z = z(x, y) \text{ and } \left(\frac{\partial M}{\partial y} \right)_x = \left(\frac{\partial N}{\partial x} \right)_y.$$

From above four equations for properties to be exact differentials, we can write functions;

$$\begin{aligned} u &= u(s, v) \\ h &= h(s, p) \\ f &= f(v, T) \\ g &= g(p, T) \end{aligned}$$

For differential of function ' u ' to be exact;

$$\left(\frac{\partial T}{\partial v} \right)_s = - \left(\frac{\partial p}{\partial s} \right)_v$$

For differential of function ' h ' to be exact; $\left(\frac{\partial T}{\partial p} \right)_s = \left(\frac{\partial v}{\partial s} \right)_p$

For differential of function ' f ' to be exact; $\left(\frac{\partial p}{\partial T} \right)_v = \left(\frac{\partial s}{\partial v} \right)_T$

For differential of function ' g ' to be exact; $\left(\frac{\partial v}{\partial T} \right)_p = - \left(\frac{\partial s}{\partial p} \right)_T$

Above four conditions for exact differentials of thermodynamic properties result into "Maxwell relations".

Thus, "Maxwell relations" are :

$$\begin{aligned} \left(\frac{\partial T}{\partial v} \right)_s &= - \left(\frac{\partial p}{\partial s} \right)_v \\ \left(\frac{\partial T}{\partial p} \right)_s &= \left(\frac{\partial v}{\partial s} \right)_p \\ \left(\frac{\partial p}{\partial T} \right)_v &= \left(\frac{\partial s}{\partial v} \right)_T \end{aligned}$$

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T$$

Maxwell relations have large significance as these relations help in estimating the changes in entropy, internal energy and enthalpy by knowing p , v and T . Some applications of these equations are discussed in subsequent articles.

7.6.3 Clapeyron Equation

Let us look upon phase change at fixed temperature and pressure and estimate changes in specific entropy, internal energy and enthalpy during phase change. Let us start with one of Maxwell relations;

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

From earlier discussions on pure substances we have seen that during phase transformation at some temperature the pressure is saturation pressure. Thus pressure is also independent of specific volume and can be determined by temperature alone. Hence, $p_{\text{sat}} = f(T_{\text{sat}})$

or
$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{dp}{dT}\right)_{\text{sat}}$$

Here $\left(\frac{dp}{dT}\right)_{\text{sat}}$ is the slope of saturation curve on pressure-temperature ($p - T$) diagram at some point determined by fixed constant temperature during phase transformation and is independent of specific volume.

Substituting in the Maxwell relation.

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{dp}{dT}\right)_{\text{sat}}$$

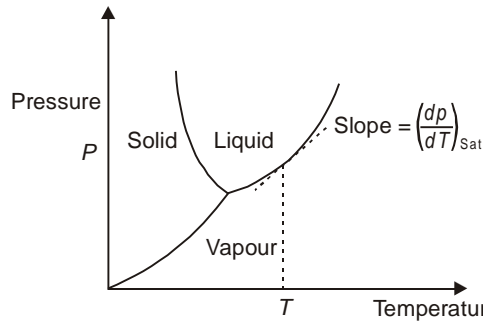


Fig. 7.5 Pressure-temperature diagram for pure substance

Thus, during vaporization i.e. phase transformation from liquid to vapour state, above relation can be given as,

$$\int_{\text{sat. liquid}}^{\text{dry vapour}} ds = \int_{\text{sat. liquid}}^{\text{dry vapour}} \left(\frac{dp}{dT}\right)_{\text{sat}} \cdot dv$$

Using notations for dry vapour and saturated liquid it can be given as,

$$(s_g - s_f) = \left(\frac{dp}{dT} \right)_{\text{sat}} \cdot (v_g - v_f)$$

or,

$$\left(\frac{dp}{dT} \right)_{\text{sat}} = \left(\frac{s_g - s_f}{v_g - v_f} \right)$$

$$\boxed{\left(\frac{dp}{dT} \right)_{\text{sat}} = \left(\frac{s_{fg}}{v_{fg}} \right)}$$

From differential form of specific enthalpy,

$$dh = T \cdot ds + v \cdot dp$$

for phase change occurring at constant pressure and temperature,

$$dh = T \cdot ds$$

for saturated liquid to dry vapour transformation,

$$(h_g - h_f) = T \cdot (s_g - s_f)$$

$$h_{fg} = T \cdot s_{fg}$$

Substituting $\frac{h_{fg}}{T}$ in place of entropy s_{fg} in $\left(\frac{dp}{dT} \right)_{\text{sat}}$

$$\boxed{\left(\frac{dp}{dT} \right)_{\text{sat}} = \left(\frac{h_{fg}}{T \cdot v_{fg}} \right)} \text{ Clapeyron equation}$$

Above equation is called Clapeyron equation. It can be used for determination of change in enthalpy during phase change i.e. h_{fg} from the p , v and T values which can be easily measured. Thus, Clapeyron equation can also be used for “sublimation process” or “melting occurring at constant temperature and pressure” just by knowing slope of saturation curve on p - T diagram, temperature and change in specific volume.

Hence, for initial state ‘1’ getting transformed into final state ‘2’ due to phase transformation at constant pressure and temperature, general form of Clapeyron equation:

$$\left(\frac{dp}{dT} \right)_{\text{sat}} = \left(\frac{h_{12}}{T \cdot v_{12}} \right)$$

At low pressure during liquid-vapour transformation it is seen that specific volume of saturated liquid state is very small as compared to dry vapour state, i.e. $v_f \ll v_g$. Also at low pressure the substance in vapour phase may be treated as perfect gas. Therefore, Clapeyron equation can be modified in the light of two approximations of “ v_f being negligible compared to v_g at low pressures” and “ideal gas

equation of state during vapour phase at low pressure, $\left(v_g = \frac{RT}{p} \right)$ ”.

Clapeyron equation thus becomes, Clausius-Clapeyron equation as given here,

$$\left(\frac{dp}{dT} \right)_{\text{sat}} = \frac{(h_g - h_f)}{(T \cdot v_g)}$$

$$\text{or,} \quad \left(\frac{dp}{dT} \right)_{\text{sat}} = \frac{h_g - h_f}{T \cdot (RT/p)}$$

$$\text{or,} \quad \left(\frac{dp}{dT} \right)_{\text{sat}} = \left(\frac{h_{fg} \cdot p}{RT^2} \right)$$

$$\text{or,} \quad \boxed{\left(\frac{dp}{p} \right)_{\text{sat}} = h_{fg} \cdot \left(\frac{dT}{RT^2} \right)_{\text{sat}}} \quad \text{Clausius-Clapeyron equation}$$

Integrating between two states 1 and 2

$$\boxed{\ln \left(\frac{p_2}{p_1} \right)_{\text{sat}} = \frac{h_{fg}}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)_{\text{sat}}}$$

Clausius-Clapeyron equation is thus a modified form of Clapeyron equation based upon certain approximations and is valid for low pressure phase transformations of liquid-vapour or solid-vapour type.

7.6.4 General Relations for Change in Entropy, Enthalpy, Internal Energy and Specific Heats

Let us now derive expressions for changes in entropy, enthalpy, internal energy and specific heats as a function of thermodynamic properties, p , v and T . For defining a state any two of the properties amongst the p , v , and T may be regarded as independent properties. Let us take (T, p) and (T, v) as two sets of independent properties for defining other dependent properties.

Temperature and Pressure (T, p) as Independent Properties : By considering T and p as independent properties, dependent property say entropy can be given as,

$$s = s(T, p)$$

Writing differential form of entropy function,

$$ds = \left(\frac{\partial s}{\partial T} \right)_p \cdot dT + \left(\frac{\partial s}{\partial p} \right)_T \cdot dp$$

From Maxwell relations the partial derivative $\left(\frac{\partial s}{\partial p} \right)_T$ can be substituted by $-\left(\frac{\partial v}{\partial T} \right)_p$ as,

$$\boxed{ds = \left(\frac{\partial s}{\partial T} \right)_p \cdot dT - \left(\frac{\partial v}{\partial T} \right)_p \cdot dp}$$

Similarly, specific enthalpy can be given as function of T and p ; $h = h(T, p)$

$$\text{Writing differential form; } dh = \left(\frac{\partial h}{\partial T} \right)_p dT + \left(\frac{\partial h}{\partial p} \right)_T \cdot dp$$

We have already seen that specific heat at constant pressure can be given as function of specific enthalpy and temperature at constant pressure.

$$C_p = \left(\frac{\partial h}{\partial T} \right)_p$$

Substituting C_p in dh ,

$$dh = C_p \cdot dT + \left(\frac{\partial h}{\partial p} \right)_T \cdot dp$$

From definition of enthalpy, first and second law combined,

$$dh = T \cdot ds + v dp$$

Substituting dh and ds from above,

$$C_p \cdot dT + \left(\frac{\partial h}{\partial p} \right)_T \cdot dp = T \cdot \left(\frac{\partial s}{\partial T} \right)_p dT - T \left(\frac{\partial v}{\partial T} \right)_p \cdot dp + v \cdot dp$$

or,

$$\left\{ \left(\frac{\partial h}{\partial p} \right)_T + T \left(\frac{\partial v}{\partial T} \right)_p - v \right\} dp = \left\{ T \cdot \left(\frac{\partial s}{\partial T} \right)_p - C_p \right\} dT$$

Here T and p are considered to be independent variables so let us keep pressure constant and vary temperature i.e. $dp = 0$, $dT \neq 0$. It yields in modified form of above underlined equation as,

$$\left\{ T \cdot \left(\frac{\partial s}{\partial T} \right)_p - C_p \right\} dT = 0$$

or,

$$C_p = T \cdot \left(\frac{\partial s}{\partial T} \right)_p$$

or,

$$\left(\frac{\partial s}{\partial T} \right)_p = \frac{C_p}{T}$$

Substituting $\left(\frac{\partial s}{\partial T} \right)_p$ in ds expression we get,

$$ds = \frac{C_p dT}{T} - \left(\frac{\partial v}{\partial T} \right)_p \cdot dp$$

Similarly, temperature can be kept constant and pressure varied independently as,

$$dT = 0, dp \neq 0$$

Above underlined equation gets modified as,

$$\left\{ \left(\frac{\partial h}{\partial p} \right)_T + T \left(\frac{\partial v}{\partial T} \right)_p - v \right\} dp = 0$$

or,

$$\left(\frac{\partial h}{\partial p} \right)_T = v - T \left(\frac{\partial v}{\partial T} \right)_p$$

Substituting $\left(\frac{\partial h}{\partial p} \right)_T$ in expression for dh we get

or,

$$dh = C_p dT + \left\{ v - T \left(\frac{\partial v}{\partial T} \right)_p \right\} dp$$

$$h_2 - h_1 = \int_{T_1}^{T_2} C_p dT + \int_{p_1}^{p_2} \left\{ v - T \left(\frac{\partial v}{\partial T} \right)_p \right\} dp$$

Temperature and Specific Volume (T, v) as independent property: Considering T and v as independent properties the dependent properties can be given as,

$$u = u(T, v)$$

Writing differential form of specific internal energy.

$$du = \left(\frac{\partial u}{\partial T} \right)_v dT + \left(\frac{\partial u}{\partial v} \right)_T dv$$

From definition of specific heat at constant volume, $C_v = \left(\frac{\partial u}{\partial T} \right)_v$

or,

$$du = C_v dT + \left(\frac{\partial u}{\partial v} \right)_T dv$$

Writing specific entropy as function of T and v ,

$$s = s(T, v)$$

$$ds = \left(\frac{\partial s}{\partial T} \right)_v dT + \left(\frac{\partial s}{\partial v} \right)_T dv$$

From Maxwell relations, $\left(\frac{\partial s}{\partial v} \right)_T = \left(\frac{\partial p}{\partial T} \right)_v$; substituting in above we get

$$ds = \left(\frac{\partial s}{\partial T} \right)_v dT + \left(\frac{\partial p}{\partial T} \right)_v dv$$

From I and II law combined, $du = T \cdot ds - p dv$

Substituting du and ds in above equation,

$$C_v dT + \left(\frac{\partial u}{\partial v} \right)_T dv = T \left(\frac{\partial s}{\partial T} \right)_v dT + T \cdot \left(\frac{\partial p}{\partial T} \right)_v dv - p dv$$

Rearranging terms,

$$\left\{ \left(\frac{\partial u}{\partial v} \right)_T - T \left(\frac{\partial p}{\partial T} \right)_v + p \right\} dv = \left\{ T \cdot \left(\frac{\partial s}{\partial T} \right)_v - C_v \right\} dT$$

As T and v are considered independent variables therefore let us keep T as constant and v as variable, i.e. $dT = 0$, $dv \neq 0$. It yields,

$$\left\{ \left(\frac{\partial u}{\partial v} \right)_T - T \left(\frac{\partial p}{\partial T} \right)_v + p \right\} dv = 0$$

or,

$$\left(\frac{\partial u}{\partial v} \right)_T = T \left(\frac{\partial p}{\partial T} \right)_v - p$$

Similarly, let us keep v constant and T as variable i.e $dv = 0$ $dT \neq 0$. It yields,

$$\left\{ T \left(\frac{\partial s}{\partial T} \right)_v - C_v \right\} dT = 0$$

or,
$$\left\{ T \left(\frac{\partial s}{\partial T} \right)_v - C_v \right\} = 0$$

or,
$$C_v = T \cdot \left(\frac{\partial s}{\partial T} \right)_v \quad \text{or,} \quad \left(\frac{\partial s}{\partial T} \right)_v = \frac{C_v}{T}$$

Let us now substitute $\left(\frac{\partial u}{\partial v} \right)_T$ in the differential function du which yields,

$$du = C_v \cdot dT + \left\{ T \cdot \left(\frac{\partial p}{\partial T} \right)_v - p \right\} dv$$

For any state change from 1 to 2 we can get change in internal energy, as

$$u_2 - u_1 = \int_{T_1}^{T_2} C_v dT + \int_{v_1}^{v_2} \left\{ T \cdot \left(\frac{\partial p}{\partial T} \right)_v - p \right\} dv$$

Also, let us substitute $\left(\frac{\partial s}{\partial T} \right)_v$ in the expression of entropy change ' ds '. It results in,

$$ds = \frac{C_v dT}{T} + \left(\frac{\partial p}{\partial T} \right)_v dv$$

Thus, number of expressions are available for getting the change in h , u and s , which are summarized as under.

$$dh = C_p dT + \left\{ v - T \left(\frac{\partial v}{\partial T} \right)_p \right\} dp$$

$$du = C_v dT + \left\{ T \left(\frac{\partial p}{\partial T} \right)_v - p \right\} dv$$

$$ds = \frac{C_p dT}{T} - \left(\frac{\partial v}{\partial T} \right)_p \cdot dp$$

$$ds = \frac{C_v dT}{T} + \left(\frac{\partial p}{\partial T} \right)_v \cdot dv$$

From above two expressions of entropy change, the difference between C_p and C_v values can be obtained as,

$$(C_p - C_v)dT = T \cdot \left(\frac{\partial p}{\partial T} \right)_v dv + T \left(\frac{\partial v}{\partial T} \right)_p dp$$

Let us write from equation of state, the function $p = p(T, v)$

Differential,
$$dp = \left(\frac{\partial p}{\partial T} \right)_v dT + \left(\frac{\partial p}{\partial v} \right)_T dv$$

Substituting dp in above equation of $(C_p - C_v) dT$ we get,

$$(C_p - C_v) dT = T \left(\frac{\partial p}{\partial T} \right)_v dv + T \left(\frac{\partial v}{\partial T} \right)_p \cdot \left(\frac{\partial p}{\partial T} \right)_v dT + T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial v} \right)_T dv$$

or,

$$\left[(C_p - C_v) - T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial T} \right)_v \right] dT = \left[T \left(\frac{\partial p}{\partial T} \right)_v + T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial v} \right)_T \right] dv$$

Since T and v are independent so let us keep T as constant and v as variable i.e $dT = 0$ $dv \neq 0$.

$$\left[T \left(\frac{\partial p}{\partial T} \right)_v + T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial v} \right)_T \right] dv = 0$$

or,
$$\left(\frac{\partial p}{\partial T} \right)_v = - \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial v} \right)_T$$

Similarly keeping v as constant and T variable i.e., $dv = 0$, $dT \neq 0$

$$\left\{ (C_p - C_v) - T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial T} \right)_v \right\} dT = 0$$

or,
$$(C_p - C_v) = T \left(\frac{\partial v}{\partial T} \right)_p \left(\frac{\partial p}{\partial T} \right)_v$$

Substituting for $\left(\frac{\partial p}{\partial T} \right)_v$ from above in $(C_p - C_v)$ we get,

$$(C_p - C_v) = -T \left(\frac{\partial v}{\partial T} \right)_p^2 \left(\frac{\partial p}{\partial v} \right)_T$$

In single phase region the specific volume can also be given as function of T & p and the differential of function v shall be,

$$v = v(T, p)$$

or,

$$dv = \left(\frac{\partial v}{\partial T} \right)_p dT + \left(\frac{\partial v}{\partial p} \right)_T dp$$

The above differential form of specific volume indicates that it depends upon partial derivatives of specific volume with respect to temperature and pressure. Partial derivatives of v with respect to temperature can be related to “volume expansivity” or “coefficient of volume expansion” as below,

$$\text{Volume expansivity, } \beta = \frac{1}{v} \left(\frac{\partial v}{\partial T} \right)_p$$

Partial derivative of specific volume with respect to pressure can be related to “isothermal compressibility”, α as below.

$$\text{Isothermal compressibility, } \alpha = -\frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_T$$

Inverse of isothermal compressibility is called “isothermal bulk modulus”,

$$B_T = -v \left(\frac{\partial p}{\partial v} \right)_T$$

Thus, volume expansivity gives the change in volume that occurs when temperature changes while pressure remains constant. Isothermal compressibility gives change in volume when pressure changes while temperature remains constant. These volume expansivity and isothermal compressibility are thermodynamic properties.

Similarly, the change in specific volume with change in pressure isentropically is also called “isentropic compressibility” or “adiabatic compressibility”, Mathematically $\alpha_s = \frac{1}{v} \left(\frac{\partial v}{\partial p} \right)_s$. Reciprocal of isentropic

compressibility is called “isentropic bulk modulus,” $B_s = -v \left(\frac{\partial p}{\partial v} \right)_s$

Substituting β and α in $(C_p - C_v)$ expression;

$$C_p - C_v = -T(\beta^2 \cdot v) \left(\frac{-1}{\alpha \cdot v} \right)$$

or,

$$C_p - C_v = \frac{v \cdot T \cdot \beta^2}{\alpha} \quad \text{Mayer relation}$$

Above difference in specific heat expression is called “Mayer relation” and helps in getting significant conclusion such as,

- The difference between specific heats is zero at absolute zero temperature i.e. specific heats at constant pressure and constant volume shall be same at absolute zero temperature ($T = 0$ K).
- Specific heat at constant pressure shall be generally more than specific heat at constant volume i.e., $C_p \geq C_v$. It may be attributed to the fact that ‘ α ’ the isothermal compressibility shall always be

+ve and volume expansivity ' β ' being squared in $(C_p - C_v)$ expression shall also be +ve. Therefore $(C_p - C_v)$, shall be either zero or positive value depending upon magnitudes of v , T , β and α .

- For incompressible substances having $dv = 0$, the difference $(C_p - C_v)$ shall be nearly zero.

Hence, specific heats at constant pressure and at constant volume are identical.

Let us obtain expression for ratio of specific heats. Earlier we have obtained C_p and C_v as below,

$$C_p = T \cdot \left(\frac{\partial s}{\partial T} \right)_p \quad \text{or,} \quad \frac{C_p}{T} = \left(\frac{\partial s}{\partial T} \right)_p$$

and

$$C_v = T \cdot \left(\frac{\partial s}{\partial T} \right)_v \quad \text{or} \quad \frac{C_v}{T} = \left(\frac{\partial s}{\partial T} \right)_v$$

By cyclic relation we can write for p , T and s properties;

$$\left(\frac{\partial s}{\partial T} \right)_p \cdot \left(\frac{\partial T}{\partial p} \right)_s \cdot \left(\frac{\partial p}{\partial s} \right)_T = -1$$

or,

$$\left(\frac{\partial s}{\partial T} \right)_p = \frac{-1}{\left(\frac{\partial T}{\partial p} \right)_s \cdot \left(\frac{\partial p}{\partial s} \right)_T}$$

Similarly for s , T and v properties we can write using cyclic relation;

$$\left(\frac{\partial s}{\partial T} \right)_v \cdot \left(\frac{\partial T}{\partial v} \right)_s \cdot \left(\frac{\partial v}{\partial s} \right)_T = -1$$

or,

$$\left(\frac{\partial s}{\partial T} \right)_v = \frac{-1}{\left(\frac{\partial T}{\partial v} \right)_s \cdot \left(\frac{\partial v}{\partial s} \right)_T}$$

Substituting in the relation for (C_p/T) and $\left(\frac{C_v}{T} \right)$.

$$\left(\frac{C_p}{T} \right) = \frac{-1}{\left(\frac{\partial T}{\partial p} \right)_s \cdot \left(\frac{\partial p}{\partial s} \right)_T}$$

and

$$\left(\frac{C_v}{T} \right) = \frac{-1}{\left(\frac{\partial T}{\partial v} \right)_s \cdot \left(\frac{\partial v}{\partial s} \right)_T}$$

Taking ratio of two specific heats,

$$\frac{C_p}{C_v} = \frac{\left(\frac{\partial T}{\partial v} \right)_s \cdot \left(\frac{\partial v}{\partial s} \right)_T}{\left(\frac{\partial T}{\partial p} \right)_s \cdot \left(\frac{\partial p}{\partial s} \right)_T}$$

$$\text{or,} \quad \left(\frac{C_p}{C_v} \right) = \left(\frac{\partial T}{\partial v} \right)_s \cdot \left(\frac{\partial v}{\partial s} \right)_T \cdot \left(\frac{\partial p}{\partial T} \right)_s \cdot \left(\frac{\partial s}{\partial p} \right)_T$$

$$\left(\frac{C_p}{C_v} \right) = \left\{ \left(\frac{\partial v}{\partial s} \right)_T \cdot \left(\frac{\partial s}{\partial p} \right)_T \right\} \cdot \left\{ \left(\frac{\partial p}{\partial T} \right)_s \cdot \left(\frac{\partial T}{\partial v} \right)_s \right\}$$

By chain rule of calculus we can write,

$$\left(\frac{\partial v}{\partial p} \right)_T = \left(\frac{\partial v}{\partial s} \right)_T \cdot \left(\frac{\partial s}{\partial p} \right)_T$$

and

$$\left(\frac{\partial p}{\partial v} \right)_s = \left(\frac{\partial p}{\partial T} \right)_s \cdot \left(\frac{\partial T}{\partial v} \right)_s$$

Upon substitution in specific heat ratio we get,

$$\left(\frac{C_p}{C_v} \right) = \left(\frac{\partial v}{\partial p} \right)_T \cdot \left(\frac{\partial p}{\partial v} \right)_s$$

$$\text{or,} \quad = \left\{ \frac{-1}{v} \left(\frac{\partial v}{\partial p} \right)_T \right\} \cdot \left\{ \frac{1}{\frac{-1}{v} \left(\frac{\partial v}{\partial p} \right)_s} \right\}$$

$$\frac{C_p}{C_v} = \frac{\alpha}{\alpha_s} = \frac{\text{Isothermal compressibility}}{\text{Isentropic compressibility}}$$

Thus, ratio of specific heats at constant pressure and constant volume can be given by the ratio of isothermal compressibility and isentropic compressibility.

7.6.5 Joule-Thomson Coefficient

Joule-Thomson coefficient is defined as the rate of change of temperature with pressure during an isenthalpic process or throttling process. Mathematically, Joule-Thomson coefficient (μ) can be given

$$\text{as, } \mu = \left(\frac{\partial T}{\partial p} \right)_h$$

It is defined in terms of thermodynamic properties and is itself a property. Joule-Thomson coefficient gives slope of constant enthalpy lines on temperature—pressure diagram. Thus, it is a parameter for characterizing the throttling process. Slope of isenthalpic line may be positive, zero or negative, i.e. $\mu > 0$, $\mu = 0$ and $\mu < 0$ respectively. Mathematically evaluating the consequence of μ we see,

- for $\mu > 0$, temperature decreases during the process.
- for $\mu = 0$, temperature remains constant during the process.
- for $\mu < 0$, temperature increases during the process.

Joule-Thomson expansion can be shown as in Fig. 7.6. Here gas or liquid is passed through porous plug for causing isenthalpic process. Valve put near exit is used for regulating pressure after constant enthalpy process i.e. p_2 .

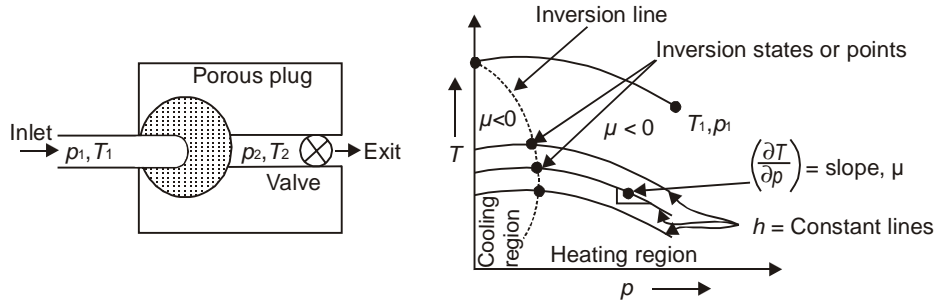


Fig. 7.6 Joule-Thomson expansion

If pressure p_2 is varied then the temperature variation occurs in the isenthalpic manner as shown in T - p diagram. This graphical representation of isenthalpic curve gives the Joule-Thomson coefficient by its slope at any point. Slope may be positive, negative or zero at different points on the curve. The points at which slope has zero value or Joule-Thomson coefficient is zero are called “inversion points” or “inversion states”. Temperature at these inversion states is called “inversion temperature”. Loci of these inversion states is called “inversion line”. Thus, inversion line as shown divides T - p diagram into two distinct region i.e. one on the left of line and other on the right of line. For the states lying on left of the inversion line temperature shall decrease during throttling process while for the states on right of inversion line throttling shall cause heating of fluid being throttled. Temperature at the intersection of inversion line with zero pressure line is called “maximum inversion temperature”.

7.6.6 Chemical Potential

In case of multicomponent systems such as non-reacting gas mixtures the partial molal properties are used for describing the behaviour of mixtures and solutions. Partial molal properties are intensive properties of the mixture and can be defined as,

$$X_i = \left(\frac{\partial X}{\partial n_i} \right)_{T, p, n_k}$$

where X is extensive property for multi component system in single phase.

$X = X(T, p, n)$ i.e. function of temperature, pressure and no. of moles of each component n_k refers to the all n values with varying k values and are kept constant except for n_i .

In multicomponent systems the partial molal Gibbs function for different constituents are called “chemical potential” for particular constituent. Chemical potential, μ can be defined for i th component as,

$$\mu_i = \left(\frac{\partial G}{\partial n_i} \right)_{T, p, n_k}$$

where G , n_i , n_k , T and P have usual meanings. Chemical potential being a partial molal property is intensive property.

Also, it can be given as,

$$G = \sum_{i=1}^j (n_i \cdot \mu_i)$$

Thus, for non reacting gas mixture the expression for internal energy, enthalpy, Helmholtz function can be given using G defined as above,

$$\text{Internal energy, } U = TS - pV + \sum_{i=1}^j n_i \mu_i$$

$$\text{Enthalpy, } H = TS + \sum_{i=1}^j n_i \cdot \mu_i$$

$$\text{Helmholtz function, } F = -pV + \sum_{i=1}^j n_i \cdot \mu_i$$

Writing differential of G considering it as function of $(T, p, n_1, n_2, \dots, n_j)$

$$G = G(T, p, n_1, n_2, n_3 \dots n_j)$$

$$dG = \left(\frac{\partial G}{\partial p} \right)_{T, n} dp + \left(\frac{\partial G}{\partial T} \right)_{p, n} dT + \sum_{i=1}^j \left(\frac{\partial G}{\partial n_i} \right)_{T, p, n_k} dn_i$$

From definition of Gibbs function $dG = Vdp - SdT$, for $T = \text{constant}$,

$$V = \left(\frac{\partial G}{\partial p} \right)_{T, n}$$

$$\text{for pressure as constant, } -S = \left(\frac{\partial G}{\partial T} \right)_{p, n}$$

Therefore,

$$dG = Vdp - SdT + \sum_{i=1}^j (\mu_i \cdot dn_i)$$

Also from $G = \sum_{i=1}^j (n_i \cdot \mu_i)$ we can write differential as,

$$dG = \sum_{i=1}^j (n_i \cdot d\mu_i) + \sum_{i=1}^j (\mu_i \cdot dn_i)$$

From two differential of function G we get,

$$Vdp - SdT = \sum_{i=1}^j (n_i \cdot d\mu_i)$$

Above equation is also called Gibbs-Duhem equation.

7.6.7 Fugacity

From earlier discussions for a single component system one can write,

$$G = n \cdot \mu$$

or, $\mu = \frac{G}{n} \Rightarrow$ Chemical potential for pure substance = Gibbs function per mole.

or $\bar{g} = \frac{G}{n} = \mu$

For Gibbs function written on unit mole basis,

For constant temperature $\bar{v} = \left(\frac{\partial \mu}{\partial p} \right)_T$

If single component system is perfect gas then, $\bar{v} = \frac{\bar{R}T}{p}$

or, $\left(\frac{\partial \mu}{\partial p} \right)_T = \frac{\bar{R}T}{p}$

or

$$\mu_T = \bar{R}T \ln p + \text{constant}$$

Here chemical potential may have any value depending upon the value of pressure. Above mathematical formulation is valid only for perfect gas behaviour being exhibited by the system. For a real gas above mathematical equation may be valid if pressure is replaced by some other property called 'fugacity'.

Fugacity was first used by Lewis.

Fugacity denoted by ' \mathcal{F} ' can be substituted for pressure in above equation,

$$\mu = \bar{R}T \ln \mathcal{F} + \text{Constant}$$

For constant pressure using $\bar{v} = \left(\frac{\partial \mu}{\partial p} \right)_T$ and above equation, we get

$$\bar{R}T \left(\frac{\partial \ln \mathcal{F}}{\partial p} \right)_T = \bar{v}$$

Thus, for a limiting case when ideal gas behaviour is approached the fugacity of a pure component shall equal the pressure in limit of zero pressure.

$$\lim_{p \rightarrow 0} \left(\frac{\mathcal{F}}{p} \right) = 1$$

For an ideal gas $\mathcal{F} = p$

For real gas, equation of state can be given using compressibility factor as,

$$p \bar{v} = Z \bar{R}T$$

or, $\bar{v} = \frac{Z \bar{R}T}{p}$

Substituting the fugacity function,

$$\frac{Z \bar{R}T}{p} = \bar{R}T \left(\frac{\partial \ln \mathcal{F}}{\partial p} \right)_T$$

$$\left(\frac{\partial \ln \mathcal{F}}{\partial p} \right)_T = \frac{Z}{p}$$

or,
$$\left(\frac{\partial \ln \mathcal{F}}{\partial \ln p} \right)_T = Z. \text{ Here as } p \rightarrow 0 \text{ the } Z \rightarrow 1$$

Also we have seen

$$\mu_T = \bar{R} T \ln \mathcal{F} + \text{constant}$$

or,
$$(d\mu)_T = \bar{R} T d(\ln \mathcal{F})_T$$

or,
$$dg_T = \bar{R} T d(\ln \mathcal{F})_T$$

Integrating between very low pressure p^* and high pressure p .

$$\int_g^{g^*} dg_T = \int_{\mathcal{F}}^{\mathcal{F}^*} \bar{R} T d(\ln \mathcal{F})_T$$

or
$$g = g^* + RT \ln \left(\frac{\mathcal{F}}{\mathcal{F}^*} \right)$$

Here for very low pressure, $\mathcal{F}^* = p^*$

or,
$$g = g^* + RT \ln \left(\frac{\mathcal{F}}{p} \right)$$

When low pressure is 1 atm then the ratio $\left(\frac{\mathcal{F}}{\mathcal{F}^*} \right)$ is called “activity”.

EXAMPLES

1. Steam at 1.6 MPa, 300°C enters a flow device with negligible velocity and leaves at 0.1 MPa, 150°C with a velocity of 150 m/s. During the flow heat interaction occurs only with the surroundings at 15°C and steam mass flow rate is 2.5 kg/s. Estimate the maximum possible output from the device.

Solution:

Let us neglect the potential energy change during the flow.

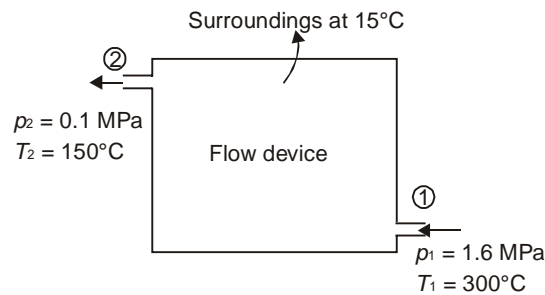


Fig. 7.7

Applying S.F.E.E., neglecting inlet velocity and change in potential energy,

$$W_{\max} = (h_1 - T_0 s_1) - \left(h_2 + \frac{C_2^2}{2} - T_0 s_2 \right)$$

$$W_{\max} = (h_1 - h_2) - T_0(s_1 - s_2) - \frac{C_2^2}{2}$$

From steam tables,

$$h_1 = h_{\text{at } 1.6 \text{ MPa, } 300^\circ\text{C}} = 3034.8 \text{ kJ/kg}$$

$$s_1 = s_{\text{at } 1.6 \text{ MPa, } 300^\circ\text{C}} = 6.8844 \text{ kJ/kg} \cdot \text{K}$$

$$h_2 = h_{\text{at } 0.1 \text{ MPa, } 150^\circ\text{C}} = 2776.4 \text{ kJ/kg}$$

$$s_2 = s_{\text{at } 0.1 \text{ MPa, } 150^\circ\text{C}} = 7.6134 \text{ kJ/kg} \cdot \text{K}$$

Given;

$$T_0 = 288 \text{ K}$$

$$W_{\max} = (3034.8 - 2776.4) - 288(6.8844 - 7.6134) - \frac{(150)^2}{2} \times 10^{-3}$$

$$= 457.1 \text{ kJ/kg}$$

$$\text{Maximum possible work} = 2.5 \times 457.1 \text{ kJ/s} = 1142.75 \text{ kW}$$

Maximum possible work = 1142.75 kW	Ans.
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2. Two tanks A and B contain 1 kg of air at 1 bar, 50°C and 3 bar, 50°C when atmosphere is at 1 bar, 15°C. Identify the tank in which stored energy is more. Also find the availability of air in each tank.

Solution:

In these tanks the air stored is at same temperature of 50°C. Therefore, for air behaving as perfect gas the internal energy of air in tanks shall be same as it depends upon temperature alone. But the availability shall be different.

Both the tanks have same internal energy	Ans.
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Availability of air in tank

$$A = \{E - U_0\} + p_0(V - V_0) - T_0(S - S_0)$$

$$= m \{ (e - u_0) + p_0(v - v_0) - T_0(s - s_0) \}$$

$$= m \left\{ c_v(T - T_0) + p_0 \left(\frac{RT}{p} - \frac{RT_0}{p_0} \right) - T_0 \left(c_p \ln \frac{T}{T_0} - R \ln \frac{p}{p_0} \right) \right\}$$

$$A = m \left\{ c_v(T - T_0) + R \left(\frac{p_0}{p} T - T_0 \right) - T_0 c_p \ln \frac{T}{T_0} + T_0 R \ln \frac{p}{p_0} \right\}$$

For tank A, $m = 1 \text{ kg}$, $c_v = 0.717 \text{ kJ/kg} \cdot \text{K}$, $T = 323 \text{ K}$, $R = 0.287 \text{ kJ/kg} \cdot \text{K}$, $T_0 = 288 \text{ K}$, $p_0 = 1 \text{ bar}$, $c_p = 1.004 \text{ kJ/kg} \cdot \text{K}$

$$\text{Availability}_A = 1 \{ 0.717 (323 - 288) + 0.287 (1 \times 323 - 288) - (288 \times 1.004) \ln \left(\frac{323}{288} \right) + 288 \times 0.287 \ln 1 \}$$

$$= 1.98 \text{ kJ}$$

For tank B, $T = 323 \text{ K}$, $p = 3 \text{ bar}$

$$\text{Availability}_B = 1 \left\{ 0.717 (323 - 288) + 0.287 \left(\frac{1}{3} \times 323 - 288 \right) - \left(288 \times 1.004 \ln \frac{323}{288} \right) + \left(288 \times 0.287 \ln \frac{3}{1} \right) \right\} = 30.98 \text{ kJ}$$

Availability of air in tank B is more than that of tank A.

Availability of air in tank A = 1.98 kJ Availability of air in tank B = 30.98 kJ	Ans.
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3. 15 kg/s steam enters a perfectly insulated steam turbine at 10 bar, 300°C and leaves at 0.05 bar, 0.95 dry with velocity of 160 m/s. Considering atmospheric pressure to be 1 bar, 15°C. Determine (a) power output, (b) the maximum power for given end states, (c) the maximum power that could be obtained from exhaust steam. Turbine rejects heat to a pond having water at 15°C.

Solution:

From steam tables,

Enthalpy at inlet to turbine, $h_1 = 3051.2 \text{ kJ/kg}$

$s_1 = 7.1229 \text{ kJ/kg} \cdot \text{K}$

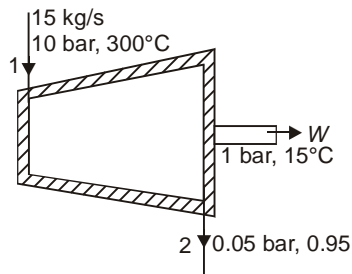


Fig. 7.8

Enthalpy at exit of turbine, $h_2 = h_{\text{at } 0.05 \text{ bar, } 0.95 \text{ dry}}$

$$s_2 = s_f \text{ at } 0.05 \text{ bar} + (0.95 \times s_{fg} \text{ at } 0.05 \text{ bar})$$

$$s_2 = 0.4764 + (0.95 \times 7.9187)$$

$$s_2 = 7.999 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Similarly, } h_2 = h_f \text{ at } 0.05 \text{ bar} + (0.95 \times h_{fg} \text{ at } 0.05 \text{ bar})$$

$$= 137.82 + (0.95 \times 2423.7)$$

$$h_2 = 2440.34 \text{ kJ/kg}$$

Neglecting the change in potential energy and velocity at inlet to turbine, the steady flow energy equation may be written as to give work output.

$$w = (h_1 - h_2) - \frac{V_2^2}{2}$$

$$w = (3051.2 - 2440.34) - \left(\frac{(160)^2}{2} \times 10^{-3} \right)$$

$$w = 598.06 \text{ kJ/kg}$$

$$\text{Power output} = m \cdot w = 15 \times 598.06 = 8970.9 \text{ kW}$$

Power output = 8970.9 kW

Ans.

Maximum work for given end states,

$$w_{\max} = (h_1 - T_0 \cdot s_1) - \left(h_2 + \frac{V_2^2}{2} - T_0 \cdot s_2 \right)$$

$$w_{\max} = (3051.2 - 288 \times 7.1229) - \left(2440.34 + \frac{(160)^2 \times 10^{-3}}{2} - 288 \times 7.999 \right)$$

$$w_{\max} = 850.38 \text{ kJ/kg}$$

$$W_{\max} = m \cdot w_{\max} = 15 \times 850.38 = 12755.7 \text{ kW}$$

Maximum power output = 12755.7 kW
--

Ans.

Maximum power that could be obtained from exhaust steam shall depend upon availability with exhaust steam and the dead state. Stream availability of exhaust steam,

$$\begin{aligned} A_{\text{exhaust}} &= \left(h_2 + \frac{V_2^2}{2} - T_0 s_2 \right) - (h_0 - T_0 s_0) \\ &= (h_2 - h_0) + \frac{V_2^2}{2} - T_0 (s_2 - s_0) \end{aligned}$$

Approximately the enthalpy of water at dead state of 1 bar, 15°C can be approximated to saturated liquid at 15°C.

$$h_0 = h_{f \text{ at } 15^\circ\text{C}} = 62.99 \text{ kJ/kg}$$

$$s_0 = s_{f \text{ at } 15^\circ\text{C}} = 0.2245 \text{ kJ/kg} \cdot \text{K}$$

Maximum work available from exhaust steam

$$= (2440.34 - 62.99) + \left(\frac{(160)^2}{2} \times 10^{-3} \right) - 288 (7.999 - 0.2245)$$

$$A_{\text{exhaust}} = 151.1 \text{ kJ/kg}$$

Maximum power that could be obtained from exhaust steam

$$= m \times A_{\text{exhaust}}$$

$$= 15 \times 151.1 = 2266.5 \text{ kW}$$

Maximum power from exhaust steam = 2266.5 kW

Ans.

4. 5 kg of steam, initially at elevation of 10 m and velocity of 25 m/s undergoes some process such that finally it is at elevation of 2m and velocity of 10 m/s. Determine the availability corresponding to the initial and final states. Also estimate the change in availability assuming environment temperature and pressure at 25°C and 100 kPa respectively. Thermodynamic properties u , v , s are as under.

Dead state of water

$$\begin{aligned}u_0 &= 104.86 \text{ kJ/kg} \\v_0 &= 1.0029 \times 10^{-3} \text{ m}^3/\text{kg} \\s_0 &= 0.3673 \text{ kJ/kg} \cdot \text{K}\end{aligned}$$

Initial state

$$\begin{aligned}u_1 &= 2550 \text{ kJ/kg} \\v_1 &= 0.5089 \text{ m}^3/\text{kg} \\s_1 &= 6.93 \text{ kJ/kg} \cdot \text{K}\end{aligned}$$

Final state

$$\begin{aligned}u_2 &= 83.94 \text{ kJ/kg} \\v_2 &= 1.0018 \times 10^{-3} \text{ m}^3/\text{kg} \\s_2 &= 0.2966 \text{ kJ/kg} \cdot \text{K}\end{aligned}$$

Solution:

Availability at any state can be given by,

$$A = m \left[(u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \right]$$

Availability at initial state,

$$\begin{aligned}A_1 &= 5 \left[(2550 - 104.86) \times 10^3 + 100 \times 10^3 (0.5089 - 1.0029 \times 10^{-3}) \right. \\&\quad \left. - 298(6.93 - 0.3673) \times 10^3 + \frac{(25)^2}{2} + (9.81 \times 10) \right]\end{aligned}$$

$$A_1 = 2704.84 \text{ kJ.}$$

= 2704.84 kJ Availability at initial state

Ans.

Availability at final state

$$\begin{aligned}A_2 &= 5 \left[(83.94 - 104.86) \times 10^3 + 100 \times 10^3 (1.0018 \times 10^{-3} - 1.0029 \right. \\&\quad \left. \times 10^{-3}) - 298(0.2966 - 0.3673) \times 10^3 + \frac{(10)^2}{2} + (9.81 \times 2) \right]\end{aligned}$$

$$A_2 = 1.09 \text{ kJ}$$

Availability at final state = 1.09 kJ
--

Ans.

$$\begin{aligned}\text{Change in availability: } A_2 - A_1 &= 1.09 - 2704.84 \\&= -2703.75 \text{ kJ}\end{aligned}$$

Hence availability decreases by 2703.75 kJ

Ans.

5. For a steady flow process as shown below, prove that irreversibility, $I = T_0 S_{\text{gen}}$, where T_0 and P_0 are temperature and pressure at dead state.

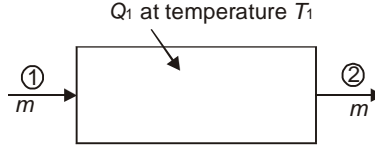


Fig. 7.9

Solution:

Let us assume changes in kinetic and potential energy to be negligible. Let us use subscript 1 for inlet and 2 for outlet.

From first law of thermodynamics;

$$Q_1 + m_1 h_1 = m_2 h_2, \text{ here } m_1 = m_2 = m$$

or $Q_1 = m(h_2 - h_1)$

From second law of thermodynamics,

$$S_{\text{gen}} + \frac{Q_1}{T_1} + m s_1 = m s_2; m(s_2 - s_1) = \frac{Q_1}{T_1} + S_{\text{gen}}$$

From availability considerations in control volume,

$$\left(1 - \frac{T_0}{T_1}\right) Q_1 + m a_1 = m a_2 + I$$

$$\text{or, } \left(1 - \frac{T_0}{T_1}\right) Q_1 + m\{(h_1 - h_0) - T_0(s_1 - s_0)\} = m\{(h_2 - h_0) - T_0(s_2 - s_0)\} + I$$

or,

Upon substituting from above equations,

$$m(s_2 - s_1) = \frac{Q_1}{T_1} + \frac{I}{T_0}$$

or it can be given that

$$\frac{I}{T_0} = S_{\text{gen}}$$

or,

$$\boxed{I = T_0 \cdot S_{\text{gen}}} \quad \text{Hence proved.}$$

6. Exhaust gases leave an internal combustion engine at 800°C and 1 atmosphere, after having done 1050 kJ of work per kg of gas in engine. (c_p of gas = $1.1 \text{ kJ/kg} \cdot \text{K}$). The temperature of surrounding is 30°C

(i) How much available energy per kg of gas is lost by throwing away the exhaust gases?

(ii) What is the ratio of the lost available exhaust gas energy to engine work?

Solution:

$$\text{Loss of available energy} = \text{Irreversibility} = T_0 \cdot \Delta S_c$$

Here, $T_0 = 303 \text{ K}$ = Temperature of surroundings

$$\Delta S_c = \Delta S_s + \Delta S_e$$

$$\text{Change in entropy of system} = \frac{1050}{(273+800)} = 0.9786 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Change in entropy of surroundings} = \frac{-c_p \cdot (800 - 30)}{(273 + 30)}$$

$$= \frac{-1.1 \times 770}{303}$$

$$= -2.7954 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Loss of available energy} = 303 (-2.7954 + 0.9786)$$

$$= -550.49 \text{ kJ/kg}$$

$$\text{Loss of available energy} = 550.49 \text{ kJ/kg}$$

$$\text{Ratio of lost available exhaust gas energy to engine work} = \frac{550.49}{1050}$$

$$= \boxed{\frac{0.524}{1}} \text{ Ans.}$$

7. 10 kg of water undergoes transformation from initial saturated vapour at 150°C, velocity of 25 m/s and elevation of 10 m to saturated liquid at 20°C, velocity of 10 m/s and elevation of 3 m. Determine the availability for initial state, final state and change of availability considering environment to be at 0.1 MPa and 25°C and $g = 9.8 \text{ m/s}^2$.

Solution:

Let us consider velocities and elevations to be given in reference to environment. Availability is given by

$$A = m \left[(u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \frac{C^2}{2} + gz \right]$$

Dead state of water,

$$u_0 = 104.88 \text{ kJ/kg}$$

$$v_0 = 1.003 \times 10^{-3} \text{ m}^3/\text{kg}$$

$$s_0 = 0.3674 \text{ kJ/kg} \cdot \text{K}$$

For initial state of saturated vapour at 150°C,

$$u_1 = 2559.5 \text{ kJ/kg}, v_1 = 0.3928 \text{ m}^3/\text{kg}, s_1 = 6.8379 \text{ kJ/kg} \cdot \text{K}$$

For final state of saturated liquid at 20°C,

$$u_2 = 83.95 \text{ kJ/kg}, v_2 = 0.001002 \text{ m}^3/\text{kg}, s_2 = 0.2966 \text{ kJ/kg} \cdot \text{K}$$

Substituting in the expression for availability

Initial state availability,

$$A_1 = 10 \times [(2559.5 - 104.88) + (0.1 \times 10^3 \times (0.3928 - 0.001003)) - (298.15 \times (6.8379 - 0.3674)) + \left(\frac{(25)^2}{2} \times 10^{-3} \right) + (9.81 \times 10 \times 10^{-3})]$$

$$A_1 = 5650.28 \text{ kJ}$$

Final state availability

$$A_2 = 10[(83.95 - 104.88 + (0.1 \times 10^3 \times (0.001002 - 0.001003)) - (298.15 \times (0.2966 - 0.3674)) + \left(\frac{(10)^2}{2} \times 10^{-3} \right) + (9.81 \times 3 \times 10^{-3})]$$

$$\begin{aligned}
 A_2 &= 2.5835 \text{ kJ} \approx 2.58 \text{ kJ} \\
 \text{Change in availability, } \Delta A &= A_2 - A_1 \\
 &= 2.58 - 5650.28 \\
 &= -5647.70 \text{ kJ}
 \end{aligned}$$

Initial availability = 5650.28 kJ Final availability = 2.58 kJ Change in availability = Decrease by 5647.70 kJ	Ans.
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8. A steam turbine has steam flowing at steady rate of 5 kg/s entering at 5 MPa and 500°C and leaving at 0.2 MPa and 140°C. During flow through turbine a heat loss of 600 kJ/s occurs to the environment at 1 atm and 25°C. Determine

- (i) the availability of steam at inlet to turbine,
- (ii) the turbine output
- (iii) the maximum possible turbine output, and
- (iv) the irreversibility

Neglect the changes in kinetic energy and potential energy during flow.

Solution:

Let inlet and exit states of turbine be denoted as 1 and 2.

At inlet to turbine,

$$p_1 = 5 \text{ MPa}, T_1 = 500^\circ\text{C}, h_1 = 3433.8 \text{ kJ/kg}, s_1 = 6.9759 \text{ kJ/kg} \cdot \text{K}$$

At exit from turbine,

$$p_2 = 0.2 \text{ MPa}, T_2 = 140^\circ\text{C}, h_2 = 2748 \text{ kJ/kg}, s_2 = 7.228 \text{ kJ/kg} \cdot \text{K}$$

At dead state,

$$p_0 = 101.3 \text{ kPa}, T_0 = 25^\circ\text{C}, h_0 = 104.96 \text{ kJ/kg}, s_0 = 0.3673 \text{ kJ/kg} \cdot \text{K}$$

Availability of steam at inlet, $A_1 = m[(h_1 - h_0) - T_0(s_1 - s_0)]$

$$A_1 = 5 [(3433.8 - 104.96) - 298.15 (6.9759 - 0.3673)]$$

$$A_1 = 6792.43 \text{ kJ}$$

Availability of steam at inlet = 6792.43 kJ	Ans.
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Applying first law of thermodynamics

$$Q + mh_1 = mh_2 + W$$

$$W = m(h_1 - h_2) - Q$$

$$= 5(3433.8 - 2748) - 600$$

$$W = 2829 \text{ kJ/s}$$

Turbine output = 2829 kW	Ans.
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Maximum possible turbine output will be available when irreversibility is zero.

$$W_{\text{rev}} = W_{\text{max}} = A_1 - A_2$$

$$= m [(h_1 - h_2) - T_0(s_1 - s_2)]$$

$$= 5[(3433.8 - 2748) - 298.15 (6.9759 - 7.228)]$$

$$W_{\text{max}} = 3053.18 \text{ kJ/s}$$

Maximum output = 3053.18 kW	Ans.
------------------------------------	-------------

Irreversibility can be estimated by the difference between the maximum output and turbine output.

$$I = W_{\max} - W = 224.18 \text{ kJ/s}$$

Irreversibility = 224.18 kW

Ans.

9. Show that the sublimation line and vaporization lines have different slopes at triple point on the phase diagram of water.

Solution:

It is desired to show that slope of sublimation line shown by 0–1 is different than vaporization line 1 – 2.

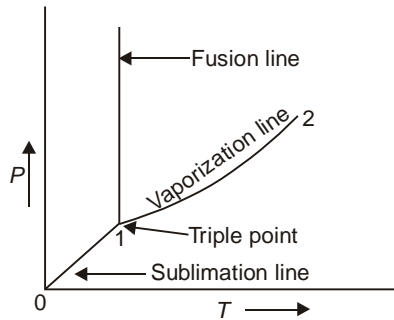


Fig 7.10

To show the slope let us find $\frac{dp}{dT}$ values at triple point 1. Here i, f, g subscripts refer to ice, water and steam states,

By Clapeyron equation.

$$\left(\frac{dp}{dT}\right)_{0-1} - \left(\frac{dp}{dT}\right)_{1-2} = \frac{s_{ig}}{v_{ig}} - \frac{s_{fg}}{v_{fg}}$$

For triple point state $s_{ig} = s_{if} + s_{fg}$

and

$$v_{ig} = v_{if} + v_{fg}$$

Substituting in above slope difference expression

$$\begin{aligned} \left(\frac{dp}{dT}\right)_{0-1} - \left(\frac{dp}{dT}\right)_{1-2} &= \left(\frac{s_{if} + s_{fg}}{v_{if} + v_{fg}}\right) - \frac{s_{fg}}{v_{fg}} \\ &= \frac{s_{if} \cdot v_{fg} + s_{fg} \cdot v_{fg} - s_{fg} v_{if} - s_{fg} v_{fg}}{(v_{if} + v_{fg}) \cdot v_{fg}} \\ &= \frac{s_{if} \cdot v_{fg} - s_{fg} \cdot v_{if}}{(v_{if} + v_{fg}) \cdot v_{fg}} \end{aligned}$$

It is seen that $v_{if} \ll v_{fg}$ but the order of s_{if} being less than s_{fg} is not very small as compared to $v_{if} \ll v_{fg}$. Neglecting smaller terms by order of magnitude

$$\left(\frac{dp}{dT}\right)_{0-1} - \left(\frac{dp}{dT}\right)_{1-2} = \frac{s_{if}}{v_{fg}}$$

Here, s_{if} and v_{fg} both are positive quantities so the ratio $\left(\frac{s_{if}}{v_{fg}}\right)$ is also positive and hence difference of

slopes between sublimation line and vaporization line is positive. Thus, it shows that slope of sublimation line and vaporization line are different.

10. Obtain the expression for change in internal energy of gas obeying the Vander Waals equation of state.

Solution:

Van der Waals equation of state can be given as under,

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT \Rightarrow p = \frac{RT}{v - b} - \frac{a}{v^2} \Rightarrow \frac{RT}{v - b} = p + \frac{a}{v^2}$$

Differentiating this equation of state, partially w.r.t. T at constant volume,

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v - b}$$

General expression for change in internal energy can be given as under,

$$du = C_v dT + \left\{ T \left(\frac{\partial p}{\partial T} \right)_v - p \right\} dv$$

Substituting in the expression for change in internal energy

$$du = C_v \cdot dT + \left\{ T \cdot \frac{R}{(v - b)} - p \right\} dv$$

Substituting for $\left(\frac{RT}{v - b}\right)$ is expression of du ,

$$du = C_v \cdot dT + \left\{ p + \frac{a}{v^2} - p \right\} dv$$

$$du = C_v \cdot dT + \frac{a}{v^2} dv$$

The change in internal energy between states 1 and 2,

$$\int_1^2 du = u_2 - u_1 = \int_1^2 C_v dT - a \left(\frac{1}{v_2} - \frac{1}{v_1} \right)$$

$$\boxed{u_2 - u_1 = \int_1^2 C_v \cdot dT - a \left(\frac{1}{v_2} - \frac{1}{v_1} \right)} \quad \text{Ans.}$$

11. 500 kJ of heat is removed from a constant temperature heat reservoir maintained at 835 K. Heat is received by a system at constant temperature of 720 K. Temperature of the surroundings, the lowest available temperature is 280 K. Determine the net loss of available energy as a result of this irreversible heat transfer.
[U.P.S.C. 1992]

Solution:

Here, $T_0 = 280$ K, i.e surrounding temperature.

$$\text{Availability for heat reservoir} = T_0 \cdot \Delta S_{\text{reservoir}}$$

$$= 280 \times \frac{500}{835}$$

$$= 167.67 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Availability for system} = T_0 \cdot \Delta S_{\text{system}}$$

$$= 280 \times \frac{500}{720}$$

$$= 194.44 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Net loss of available energy} = (167.67 - 194.44)$$

$$= -26.77 \text{ kJ/kg} \cdot \text{K}$$

Loss of available energy = 26.77 kJ/kg · K

Ans.

12. Steam flows through an adiabatic steady flow turbine. The enthalpy at entrance is 4142 kJ/kg and at exit 2585 kJ/kg. The values of flow availability of steam at entrance and exit are 1787 kJ/kg and 140 kJ/kg respectively, dead state temperature T_0 is 300 K, determine per kg of steam, the actual work, the maximum possible work for the given change of state of steam and the change in entropy of steam. Neglect changes in kinetic and potential energy. [U.P.S.C. 1993]

Solution:

Here dead state is given as 300 K and the maximum possible work for given change of state of steam can be estimated by the difference of flow availability as given under:

$$W_{\text{max}} = 1787 - 140 = 1647 \text{ kJ/kg}$$

$$\text{Actual work from turbine, } W_{\text{actual}} = 4142 - 2585$$

$$\text{Actual work} = 1557 \text{ kJ/kg}$$

Actual work = 1557 kJ/kg

Maximum possible work = 1647 kJ/kg

Ans.

13. What shall be second law efficiency of a heat engine having efficiency of 0.25 and working between reservoirs of 500°C and 20°C?

Solution:

$$\text{Reversible engine efficiency, } \eta_{\text{rev}} = 1 - \frac{T_{\text{min}}}{T_{\text{max}}} = 1 - \frac{293}{773} = 0.6209$$

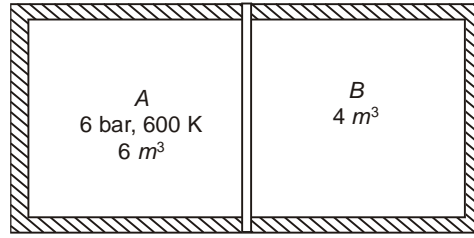
$$\text{Second law efficiency} = \frac{\eta}{\eta_{\text{rev}}} = \frac{0.25}{0.6209} = 0.4026 \text{ or } 40.26\%$$

= 40.26%

Ans.

14. An adiabatic cylinder of volume 10 m³ is divided into two compartments A and B each of volume 6 m³ and 4 m³ respectively, by a thin sliding partition. Initially the compartment A is filled with air at 6 bar and 600 K, while there is a vacuum in the compartment B. Air expands and fills both the compartments. Calculate the loss in available energy. Assume atmosphere is at 1 bar and 300 K.

[U.P.S.C. 1997]

Solution:**Fig 7.11**

Here $T_0 = 300 \text{ K}$, $P_0 = 1 \text{ bar}$
 $V_A = 6 \text{ m}^3$, $V_B = 4 \text{ m}^3$
 $P_1 = 6 \text{ bar}$, $T_1 = 600 \text{ K}$

Initially, $V_1 = V_A = 6 \text{ m}^3$, and finally, $V_2 = V_A + V_B = 10 \text{ m}^3$

Expansion occurs in adiabatic conditions.

Temperature after expansion can be obtained by considering adiabatic expansion.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{\gamma-1}$$

$$T_2 = 600 \left(\frac{6}{10} \right)^{(1.4-1)} = 489.12 \text{ K}$$

$$\text{Mass of air, } m = \frac{PV_1}{RT_1} = \frac{6 \times 10^5 \times 6}{287 \times 600} = 20.91 \text{ kg}$$

$$\text{Change in entropy of control system, } (S_2 - S_1) = mc_v \ln \frac{T_2}{T_1} + mR \ln \frac{V_2}{V_1}$$

$$\Delta S_s = (S_2 - S_1) = 20.91 \left\{ 0.718 \times \ln \left(\frac{489.12}{600} \right) + 0.287 \ln \left(\frac{10}{6} \right) \right\}$$

$$= -2.01 \times 10^{-3} \text{ kJ/K}$$

Here, there is no change in entropy of environment, $\Delta S_e = 0$

$$\text{Total entropy change of combined system} = \Delta S_c = \Delta S_s + \Delta S_e$$

$$= -2.01 \times 10^{-3} \text{ kJ/K}$$

$$\text{Loss of available energy} = \text{Irreversibility} = T_0 \cdot \Delta S_c$$

$$= 300 \times (-2.01 \times 10^{-3})$$

$$= -0.603 \text{ kJ}$$

$$\boxed{\text{Loss of available energy} = \mathbf{0.603 \text{ kJ}} \text{ Ans.}}$$

15. Prove that ideal gas equation satisfies the cyclic relation.

Solution:

Ideal gas equation, $Pv = RT$

Let us consider two variable (v, T) to be independent and P as dependent variable.

$$P = P(v, T) = \frac{RT}{v}$$

By cyclic relation,

$$\left(\frac{\partial P}{\partial v} \right)_T \cdot \left(\frac{\partial v}{\partial T} \right)_P \cdot \left(\frac{\partial T}{\partial P} \right)_v = -1$$

Let us find the three partial derivatives separately and then substitute.

$$\left(\frac{\partial P}{\partial v} \right)_T = \frac{-RT}{v^2}, \left(\frac{\partial v}{\partial p} \right)_p = \frac{R}{P}, \left(\frac{\partial T}{\partial P} \right)_v = \frac{v}{R}$$

Substituting

$$\left(\frac{-RT}{v^2} \right) \left(\frac{R}{P} \right) \left(\frac{v}{R} \right) = \frac{-RT}{Pv} = -1 \text{ Hence proved.}$$

16. A heat engine is working between 700°C and 30°C. The temperature of surroundings is 17°C. Engine receives heat at the rate of 2×10^4 kJ/min and the measured output of engine is 0.13 MW. Determine the availability, rate of irreversibility and second law efficiency of engine.

Solution:

$$\begin{aligned} \text{Availability or reversible work, } W_{\text{rev}} &= \eta_{\text{rev}} \cdot Q_1 = \left(1 - \frac{303}{573} \right) \times 2 \times 10^4 \\ &= 1.38 \times 10^4 \text{ kJ/min} \end{aligned}$$

$$\begin{aligned} \text{Rate of irreversibility, } I &= W_{\text{rev}} - W_{\text{useful}} \\ &= \left(\frac{1.38 \times 10^4}{60} - 0.13 \times 10^3 \right) = 100 \text{ kJ/s} \end{aligned}$$

$$\begin{aligned} \text{Second law efficiency} &= \frac{W_{\text{useful}}}{W_{\text{rev}}} \\ &= \frac{0.13 \times 10^3}{\left(\frac{1.38 \times 10^4}{60} \right)} = 0.5652 \text{ or } 56.52\% \end{aligned}$$

Availability = 1.38×10^4 kJ/min, Rate of irreversibility = 100 kW, Second law efficiency = 56.52%	Ans.
---	-------------

17. A rigid tank contains air at 1.5 bar and 60°C. The pressure of air is raised to 2.5 bar by transfer of heat from a constant temperature reservoir at 400°C. The temperature of surroundings is 27°C. Determine per kg of air, the loss of available energy due to heat transfer. [U.P.S.C. 1998]

Solution:

$$\text{Loss of available energy} = \text{Irreversibility} = T_0 \cdot \Delta S_c$$

$$T_0 = 300 \text{ K}, \quad \Delta S_c = \Delta S_s + \Delta S_e$$

$$\text{Change in entropy of system} = \Delta S_s$$

Change in entropy of environment/surroundings = ΔS_e

Here heat addition process causing rise in pressure from 1.5 bar to 2.5 bar occurs isochorically. Let initial and final states be given by subscript 1 and 2.

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}, T_1 = 333 \text{ K}, T_2 = ?, P_1 = 1.5 \text{ bar}, P_2 = 2.5 \text{ bar}$$

$$T_2 = \frac{2.5 \times 333}{1.5} = 555 \text{ K}$$

Heat addition to air in tank,

$$Q = m \cdot c_p \cdot \Delta T = 1 \times 1.005 \times (555 - 333)$$

$$Q = 223.11 \text{ kJ/kg}$$

$$\Delta S_s = \frac{Q}{T_1} = \frac{223.11}{333} = 0.67 \text{ kJ/kg} \cdot \text{K}$$

$$\Delta S_e = \frac{-Q}{T_{\text{reservoir}}} = \frac{-223.11}{673} = -0.332 \text{ kJ/kg} \cdot \text{K}$$

$$\Delta S_c = 0.67 - 0.332$$

$$\Delta S_c = 0.338 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Loss of available energy} = 300 \times (0.338)$$

$$= 101.4 \text{ kJ/kg}$$

Loss of available energy = 101.4 kJ/kg

Ans.

18. Using the Maxwell relation derive the following $T \cdot ds$ equation, $T \cdot ds = C_p \cdot dT - T \left(\frac{\partial v}{\partial T} \right)_p dp$
 [U.P.S.C. 1998]

Solution:

Let

$$s = s(T, p)$$

$$ds = \left(\frac{\partial s}{\partial T} \right)_p \cdot dT + \left(\frac{\partial s}{\partial p} \right)_T \cdot dp$$

or,

$$T \cdot ds = T \cdot \left(\frac{\partial s}{\partial T} \right)_p \cdot dT + T \cdot \left(\frac{\partial s}{\partial p} \right)_T \cdot dp$$

Using Maxwell's relation, $\left(\frac{\partial s}{\partial p} \right)_T = - \left(\frac{\partial v}{\partial T} \right)_p$

and

$$T \cdot \left(\frac{\partial s}{\partial T} \right)_p = C_p$$

Substitution yields,

$T \cdot ds = C_p \cdot dT - T \cdot \left(\frac{\partial v}{\partial T} \right)_p \cdot dp$

 Hence proved

19. Determine the enthalpy of vaporization of water at 200°C using Clapeyron equation. Compare it with tabulated value.

Solution:

$$\text{Clapeyron equation says, } h_{fg} = T \cdot v_{fg} \cdot \left(\frac{dp}{dT} \right)_{\text{sat}}$$

From steam tables

$$v_{fg} = (v_g - v_f)_{\text{at } 200^\circ\text{C}} = (0.12736 - 0.001157) = 0.126203 \text{ m}^3/\text{kg}$$

Let us approximate,

$$\begin{aligned} \left(\frac{dp}{dT} \right)_{\text{sat, } 200^\circ\text{C}} &= \left(\frac{\Delta p}{\Delta T} \right)_{\text{sat, } 200^\circ\text{C}} = \frac{P_{\text{sat at } 205^\circ\text{C}} - P_{\text{sat at } 195^\circ\text{C}}}{(205 - 195)} \\ &= \frac{(1.7230 - 1.3978)}{10} = 0.03252 \text{ MPa/}^\circ\text{C} \end{aligned}$$

Substituting in Clapeyron equation,

$$\begin{aligned} h_{fg} &= (273 + 200) \times 0.126203 \times 0.03252 \times 10^3 \\ &= 1941.25 \text{ kJ/kg} \end{aligned}$$

Calculated enthalpy of vaporization = **1941.25 kJ/kg.**

Enthalpy of vaporization from steam table = **1940.7 kJ/kg.** **Ans.**

20. Determine h_{fg} of R-12 refrigerant at -10°C using both Clapeyron equation and the Clapeyron-Clausius equation. Give the deviation in %. Take

$$P_{\text{sat at } -5^\circ\text{C}} = 260.96 \text{ kPa}$$

$$P_{\text{sat at } -15^\circ\text{C}} = 182.60 \text{ kPa.}$$

$$v_g \text{ at } -10^\circ\text{C} = 0.07665 \text{ m}^3/\text{kg}$$

$$v_f \text{ at } -10^\circ\text{C} = 0.00070 \text{ m}^3/\text{kg}$$

$$R = 0.06876 \text{ kJ/kg} \cdot \text{K}$$

$$h_{fg} \text{ at } -10^\circ\text{C} = 156.3 \text{ kJ/kg from tables.}$$

Solution:

By Clapeyron equation

$$\begin{aligned} h_{fg} &= T \cdot v_{fg} \left\{ \frac{dP}{dT} \right\}_{\text{sat}} \\ &= T \cdot (v_g - v_f) \left\{ \frac{\Delta P}{\Delta T} \right\} \\ &= (-5 + 273) \times (0.07665 - 0.0007) \times \left(\frac{P_{\text{sat at } -5^\circ\text{C}} - P_{\text{sat at } -15^\circ\text{C}}}{(-5 - (-15))} \right) \\ &= 268 \times 0.07595 \times \frac{(260.96 - 182.60)}{10} \\ h_{fg} &= 159.49 \text{ kJ/kg} \end{aligned}$$

By Clapeyron-Clausius equation,

$$\ln \left(\frac{P_2}{P_1} \right)_{\text{sat}} = \frac{h_{fg}}{R} \left[\frac{1}{T_1} - \frac{1}{T_2} \right]_{\text{sat}}$$

$$\ln \left(\frac{P_{\text{sat at } -5^\circ\text{C}}}{P_{\text{sat at } -15^\circ\text{C}}} \right) = \frac{h_{fg}}{R} \left[\frac{1}{T_{\text{sat at } -15^\circ\text{C}}} - \frac{1}{T_{\text{sat at } -5^\circ\text{C}}} \right]$$

$$\ln \left(\frac{260.96}{182.60} \right) = \frac{h_{fg}}{0.06876} \times \left[\frac{1}{(-15 + 273)} - \frac{1}{(-5 + 273)} \right]$$

$$\Rightarrow h_{fg} = 169.76 \text{ kJ/kg}$$

% deviation from Clapeyron equation

$$= \left(\frac{169.76 - 159.49}{159.49} \right) \times 100$$

$$= 6.44\%$$

h_{fg} by Clapeyron equation = **159.49 kJ/kg**

h_{fg} by Clapeyron-Clausius equation = **169.76 kJ/kg**

% deviation in h_{fg} value by Clapeyron-Clausius equation compared to the value from Clapeyron equation = **6.44%**

Ans.

21. Determine the volume expansivity and isothermal compressibility of steam at 300 kPa and 300°C.

Solution:

$$\text{Volume expansivity} = \frac{1}{v} \left[\frac{\partial v}{\partial T} \right]_P$$

$$\text{Isothermal compressibility} = -\frac{1}{v} \left[\frac{\partial v}{\partial P} \right]_T$$

Let us write $\frac{\partial v}{\partial T} = \frac{\Delta v}{\Delta T}$ and $\frac{\partial v}{\partial P} = \frac{\Delta v}{\Delta P}$. The differences may be taken for small pressure and temperature changes.

Volume expansivity,

$$= \frac{1}{v} \left[\frac{\partial v}{\partial T} \right]_{300 \text{ kPa}}$$

$$= \frac{1}{v_{\text{at } 300 \text{ kPa, } 300^\circ\text{C}}} \left[\frac{v_{350^\circ\text{C}} - v_{250^\circ\text{C}}}{(350 - 250)} \right]_{300 \text{ kPa}}$$

$$= \frac{1}{0.8753} \left[\frac{0.9534 - 0.7964}{100} \right]_{300 \text{ kPa}}$$

$$\text{Volume expansivity} = \mathbf{1.7937 \times 10^{-3} \text{ K}^{-1}} \quad \mathbf{Ans.}$$

$$\begin{aligned}
 &= \frac{-1}{v_{\text{at } 300\text{kPa}, 300^\circ\text{C}}} \left[\frac{v_{350\text{kPa}} - v_{250\text{kPa}}}{(350 - 250)} \right]_{300^\circ\text{C}} \\
 &= \frac{-1}{0.8753} \left[\frac{0.76505 - 1.09575}{100} \right] \\
 &= 3.778 \times 10^{-3} \text{ kPa}^{-1}
 \end{aligned}$$

Isothermal compressibility = **3.778 × 10⁻³ kPa⁻¹** **Ans.**

22. An evacuated tank of 0.5 m³ is filled by atmospheric air at 1 bar till the pressure inside tank becomes equal to atmospheric temperature. Considering filling of tank to occur adiabatically determine final temperature inside the tank and also the irreversibility and change in entropy. Take atmospheric temperature as 25°C.

Solution:

Filling of the tank is a transient flow (unsteady flow) process. For the transient filling process, considering subscripts 'i' and 'f' for initial and final states,

$$\begin{aligned}
 h_i &= u_f \\
 c_p T_i &= c_v T_f \\
 T_f &= \frac{c_p}{c_v} T_i \\
 T_f &= \frac{1.005}{0.718} \times 298.15
 \end{aligned}$$

Inside final temperature, $T_f = \mathbf{417.33 \cdot K}$ **Ans.**

Change in entropy $\Delta S_{\text{gen}} = (S_f - S_i) + \Delta S_{\text{surr}}$

$$\begin{aligned}
 &= c_p \ln \frac{T_f}{T_i} + 0 \\
 &= 1.005 \times \ln \frac{417.33}{298.15}
 \end{aligned}$$

Change in entropy $\Delta S_{\text{gen}} = \mathbf{0.3379 \text{ kJ/kg} \cdot \text{K}}$ **Ans.**

$$\begin{aligned}
 \text{Irreversibility, } I &= T_0 \cdot \Delta S_{\text{gen}} \\
 &= 298.15 \times 0.3379
 \end{aligned}$$

Irreversibility, $I = \mathbf{100.74 \text{ kJ/kg}}$ **Ans.**

23. A closed vessel stores 75 kg of hot water at 400°C. A heat engine transfers heat from the hot water to environment which is maintained at 27°C. This operation of heat engine changes temperature of hot water from 400°C to 27°C over a finite time. Determine the maximum possible work output from engine. Take specific heat of water as 4.18 kJ/kg·K.

Solution:

Here the combined closed system consists of hot water and heat engine. Here there is no thermal reservoir in the system under consideration. For the maximum work output, irreversibility = 0

$$\text{Therefore, } \frac{d}{dt} (E - T_0 S) = W_{\max}$$

$$\text{or } W_{\max} = (E - T_0 S)_1 - (E - T_0 S)_2$$

$$\begin{aligned} \text{Here } E_1 = U_1 = m c_p T_1, E_2 = U_2 = m c_p T_2 \\ T_1 = 400 + 273 = 673 \text{ K}, T_2 = 27 + 273 = 300 \text{ K} = T_0 \end{aligned}$$

Therefore,

$$\begin{aligned} W_{\max} &= m c_p (T_1 - T_2) - T_0 (S_1 - S_2) \\ &= m c_p (T_1 - T_2) - T_0 \cdot m \cdot c_p \left(\ln \frac{T_1}{T_2} \right) \\ &= 75 \times 4.18 \times \left\{ (673 - 300) - 300 \times \ln \left(\frac{673}{300} \right) \right\} \\ &= 40946.6 \text{ kJ} \end{aligned}$$

Maximum work = 40946.6 kJ

Ans.

24. In a steam turbine the steam enters at 50 bar, 600°C and 150 m/s and leaves as saturated vapour at 0.1 bar, 50 m/s. During expansion, work of 1000 kJ/kg is delivered. Determine the inlet stream availability, exit stream availability and the irreversibility. Take dead state temperature as 25°C.

Solution:

$$h_1 = h_{\text{at 50 bar, 600°C}} = 3666.5 \text{ kJ/kg}, s_1 = s_{\text{at 50 bar, 600°C}} = 7.2589 \text{ kJ/kg} \cdot \text{K}$$

$$h_2 = h_{g \text{ at 0.1 bar}} = 2584.7 \text{ kJ/kg}, s_2 = s_{g \text{ at 0.1 bar}} = 8.1502 \text{ kJ/kg} \cdot \text{K}$$

$$\begin{aligned} \text{Inlet stream availability} &= \left(h_1 + \frac{c_1^2}{2} \right) - T_0 s_1 \\ &= \left(3666.5 + \frac{(150)^2 \times 10^{-3}}{2} \right) - (298 \times 7.2589) \\ &= 1514.59 \text{ kJ/kg} \end{aligned}$$

Input stream availability is equal to the input absolute availability.

$$\begin{aligned} \text{Exit stream availability} &= \left(h_2 + \frac{c_2^2}{2} \right) - T_0 s_2 \\ &= \left(2584.7 + \frac{(50)^2 \times 10^{-3}}{2} \right) - (298 \times 8.1502) \\ &= 157.19 \text{ kJ/kg} \end{aligned}$$

Exit stream availability is equal to the exit absolute availability.

$$W_{\text{rev}} = 1514.59 - 157.19 = 1357.4 \text{ kJ/kg}$$

$$\text{Irreversibility} = W_{\text{rev}} - W = 1357.4 - 1000 = 357.4 \text{ kJ/kg}$$

This irreversibility is in fact the availability loss.

Inlet stream availability = **1514.59 kJ/kg**

Exit stream availability = **157.19 kJ/kg**

Irreversibility = **357.4 kJ/kg**

Ans.

EXERCISE

- 7.1 Define 'available energy' and 'unavailable energy'.
- 7.2 What do you understand by second law efficiency? How does it differ from first law efficiency?
- 7.3 What is meant by a dead state? Discuss its' importance.
- 7.4 Define availability. Obtain an expression for availability of closed system.
- 7.5 Differentiate between useful work and maximum useful work in reference to the availability.
- 7.6 What do you understand by Gibbs function? How does it differ from the availability function?
- 7.7 Describe the Helmholtz function.
- 7.8 What are Maxwell relations? Discuss their significance?
- 7.9 Describe Clapeyron equation.
- 7.10 What do you understand by Joule-Thomson coefficient? Explain.
- 7.11 Describe chemical potential.
- 7.12 Write short notes on the following:
 - (i) Clapeyron-Clausius equation,
 - (ii) Volume expansivity
 - (iii) Fugacity,
 - (iv) Second law analysis of engineering systems.
- 7.13 Determine the loss of availability when 1 kg air at 260°C is reversibly and isothermally expanded from 0.145 m³ initial volume to 0.58 m³ final volume. [70.56 kJ/kg]
- 7.14 Determine the entropy generation and decrease in available energy when a heat source of 727°C transfers heat to a body at 127°C at the rate of 8.35 MJ/min. Consider the temperature of sink as 27°C. [12.54 kJ/K · min, 3762 kJ]
- 7.15 Determine the available energy of furnace having the gases getting cooled from 987°C to 207°C at constant temperature while the temperature of surroundings is 22°C. [-518.1 kJ/kg]
- 7.16 Determine the available amount of energy removed and the entropy increase of universe when 5 kg air at 1.38 bar, 500 K is cooled down to 300 K isobarically. The temperature of surroundings may be taken as 4°C. [-268.7 kJ, 3.316 kJ/K]
- 7.17 Determine the entropy change, unavailable energy and available energy for the process in which 2 kg air is heated isobarically so as to cause increase in its temperature from 21°C to 315°C. Take $T_0 = 10^\circ\text{C}$. [1.393 kJ/K, 394.2 kJ, 196.6 kJ]
- 7.18 Steam enters in a steam turbine at 60 bar, 500°C and leaves at 0.1 bar, 0.89 dry with a flow rate of 3.2652 × 10⁴ kg/hr. Determine the loss of available energy. [1286.2 kJ/s]
- 7.19 Determine the available portion of heat removed from 2.5 kg air being cooled from 2.1 bar, 205°C to 5°C at constant volume. The heat is rejected to surroundings at - 4°C. [- 97.2 kJ]

- 7.20** Prove that heat is an inexact differential. $\{Q(T, s)\}$.
- 7.21** Derive an expression for change in entropy of a gas obeying Vander Waals equation of state.
- 7.22** Determine the coefficient of thermal expansion and coefficient of isothermal compressibility for a gas obeying Vander Waals equation of state.
- 7.23** Determine the second law efficiency of a heat engine operating between 700°C and 30°C. The heat engine has efficiency of 0.40. [55.74%]
- 7.24** Determine the amount of heat that can be converted to the useful work if total heat at 1000 kJ is available at 500°C. The temperature of environment is 17°C. [624.84 kJ]
- 7.25** Determine the change in availability of air contained in an insulated vessel of $20 \times 10^3 \text{ cm}^3$. The initial state of air is 1 bar, 40°C. The air is heated so as to arrive at temperature of 150°C. The temperature of surrounding environment may be considered as 20°C. [0.027 kJ]
- 7.26** Determine the enthalpy of vaporization of water at 50°C using the Clapeyron equation. [2396.44 kJ/kg]
- 7.27** Determine the % variation in the enthalpy of vaporization of steam at 500 kPa using Clapeyron equation as compared to value in steam table. [0.201%]
- 7.28** Air enters a compressor at 40°C, 500 kPa for being compressed upto 2000 kPa. Consider the compression to be at constant temperature without internal irreversibilities. Air flows into compressor at 6 kg/min. Neglecting the changes in kinetic energy and potential energy determine the availability transfers accompanying heat and work and irreversibility. Take $T_0 = 25^\circ\text{C}$, $P_0 = 1 \text{ bar}$. The control volume may be taken as under,
- (i) Control volume comprises of compressor alone
 - (ii) Control volume comprises of compressor and its immediate surroundings so that heat transfer occurs at T_0 .
- [(i) -0.597 kJ/s , -12.46 kJ/s , 0.597 kJ/s
(ii) 0 kJ/s , -12.46 kJ/s , 0.597 kJ/s]
- 7.29** Steam expands in a cylinder from 22 bar, 450°C to 4.5 bar, 250°C. The environment may be considered to be at 1 bar, 25°C. Determine
- (i) the availability of steam at inlet, exit and change in availability.
 - (ii) the irreversibilities in the expansion process. [196 kJ, 132 kJ, -64 kJ , 5.5 kJ]
- 7.30** In a steam power cycle steam enters at 60 bar, 500°C into turbine and leaves at 0.04 bar. The isentropic efficiency of turbine is 85% and that of pump is 70%. Considering the environment to have $T_0 = 25^\circ\text{C}$, $P_0 = 1 \text{ bar}$, determine the second law efficiency of constituent components i.e. boiler, turbine, condenser and pump. [67.6%, 99%, 84.4%, 65%]

Vapour Power Cycles

8.1 INTRODUCTION

Thermodynamic cycles can be primarily classified based on their utility such as for power generation, refrigeration etc. Based on this thermodynamic cycles can be categorized as;

- (i) Power cycles,
- (ii) Refrigeration and heat pump cycles.

(i) **Power cycles:** Thermodynamic cycles which are used in devices producing power are called power cycles. Power production can be had by using working fluid either in vapour form or in gaseous form. When vapour is the working fluid then they are called vapour power cycles, whereas in case of working fluid being gas these are called gas power cycles. Thus, power cycles shall be of two types,

- (a) Vapour power cycle,
- (b) Gas power cycle.

Vapour power cycles can be further classified as,

1. Carnot vapour power cycle
2. Rankine cycle
3. Reheat cycle
4. Regenerative cycle.

Gas power cycles can be classified as,

1. Carnot gas power cycle
2. Otto cycle
3. Diesel cycle
4. Dual cycle
5. Stirling cycle
6. Ericsson cycle
7. Brayton cycle

Here in the present text Carnot, Rankine, reheat and regenerative cycles are discussed.

(ii) **Refrigeration and heat pump cycles:** Thermodynamic cycles used for refrigeration and heat pump are under this category. Similar to power cycles, here also these cycles can be classified as “air cycles” and “vapour cycles” based on type of working fluid used.

8.2 PERFORMANCE PARAMETERS

Some of commonly used performance parameters in cycle analysis are described here.

Thermal efficiency: Thermal efficiency is the parameter which gauges the extent to which the energy input to the device is converted to net work output from it.

$$\text{Thermal efficiency} = \frac{\text{Net work in cycle}}{\text{Heat added in cycle}}$$

Heat rate: Heat rate refers to the amount of energy added by heat transfer to cycle to produce unit net work output. Usually energy added may be in kcal, unit of net work output in kW h and unit of heat rate may be in kcal/kW h. It is inverse of thermal efficiency.

Back work ratio: Back work ratio is defined as the ratio of pump work input (–ve work) to the work produced (+ve work) by turbine.

$$\text{Back work ratio} = \frac{W_{\text{pump}}}{W_{\text{turbine}}}$$

Generally, back work ratio is less than one and as a designer one may be interested in developing a cycle which has smallest possible back-work ratio. Small back-work ratio indicates smaller pump work (–ve work) and larger turbine work (+ve work).

Work ratio: It refers to the ratio of net work to the positive work.

$$\text{Mathematically, work ratio} = \frac{W_{\text{net}}}{W_{\text{turbine}}}$$

Specific steam consumption: It indicates the steam requirement per unit power output. It is generally given in kg/kW. h and has numerical value lying from 3 to 5 kg/kW. h

$$\text{Specific steam consumption} = \frac{3600}{W_{\text{net}}}, \text{ kg/kW.h}$$

8.3 CARNOT VAPOUR POWER CYCLE

Carnot cycle has already been defined earlier as an ideal cycle having highest thermodynamic efficiency. Let us use Carnot cycle for getting positive work with steam as working fluid. Arrangement proposed for using Carnot vapour power cycle is as follows.

- 1 – 2 = Reversible isothermal heat addition in the boiler
- 2 – 3 = Reversible adiabatic expansion in steam turbine
- 3 – 4 = Reversible isothermal heat rejection in the condenser
- 4 – 1 = Reversible adiabatic compression or pumping in feed water pump

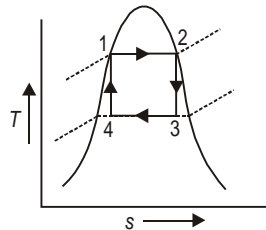


Fig 8.1 Carnot vapour power cycle

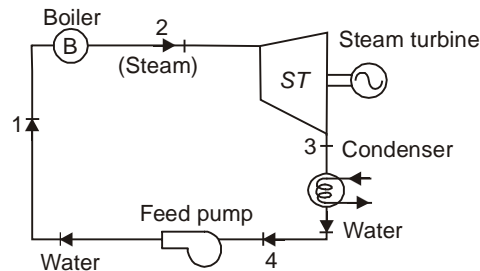


Fig 8.2 Arrangement for Carnot cycle

Assuming steady flow processes in the cycle and neglecting changes in kinetic and potential energies, thermodynamic analysis may be carried out.

$$\text{Thermal efficiency} = \frac{\text{Net work}}{\text{Heat added}}$$

Net work = Turbine work – Compression/Pumping work

For unit mass flow,

$$W = (h_2 - h_3) - (h_1 - h_4)$$

$$\text{Heat added, } Q_{\text{add}} = (h_2 - h_1)$$

$$\eta_{\text{Carnot}} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

$$= 1 - \frac{h_3 - h_4}{h_2 - h_1}$$

Here heat rejected, $Q_{\text{rejected}} = (h_3 - h_4)$

or $\eta_{\text{Carnot}} = 1 - \frac{Q_{\text{rejected}}}{Q_{\text{add}}}$

Also, heat added and rejected may be given as function of temperature and entropy as follows:

$$Q_{\text{add}} = T_1 \times (s_2 - s_1)$$

$$Q_{\text{rejected}} = T_3 \times (s_3 - s_4)$$

Also, $s_1 = s_4$ and $s_2 = s_3$

Therefore, substituting values: $\eta_{\text{Carnot}} = 1 - \frac{T_3}{T_1}$

or $\eta_{\text{Carnot}} = 1 - \frac{T_{\text{minimum}}}{T_{\text{maximum}}}$

Let us critically evaluate the processes in Carnot cycle and see why it is not practically possible.

1–2: Reversible Isothermal Heat Addition

Isothermal heat addition can be easily realised in boiler within wet region as isothermal and isobaric lines coincide in wet region. But the superheating of steam can't be undertaken in case of Carnot cycle as beyond saturated steam point isothermal heat addition can't be had inside boiler. This fact may also be understood from T – S diagram as beyond 2 the constant pressure line and constant temperature lines start diverging. It may be noted that boiler is a device which generates steam at constant pressure.

2–3: Reversible Adiabatic Expansion

Saturated steam generated in boiler at state '2' is sent for adiabatic expansion in steam turbine upto state 3. During this expansion process positive work is produced by steam turbine and a portion of this work available is used for driving the pump.

3–4: Reversible Isothermal Heat Rejection

Heat release process is carried out from state 3 to 4 in the condenser. Condenser is a device in which constant pressure heat rejection can be realized. Since expanded steam from steam turbine is available in wet region at state 3. Therefore, constant temperature heat rejection can be had as constant temperature and constant pressure lines coincide in wet region.

Heat rejection process is to be limited at state 4 which should be vertically below state 1. Practically it is very difficult to have such kind of control.

4-1: Reversible Adiabatic Compression (Pumping)

Carnot cycle has reversible adiabatic compression process occurring between 4 and 1, which could be considered for pumping of water into boiler.

In fact it is very difficult for a pump to handle wet mixture which undergoes simultaneous change in its phase as its pressure increases.

Above discussion indicates that Carnot vapour power cycle is merely theoretical cycle and cannot be used for a practical working arrangement. Also the maximum efficiency of Carnot cycle is limited by maximum and minimum temperatures in the cycle. Highest temperature attainable depends upon metallurgical limits of boiler material.

8.4 RANKINE CYCLE

Rankine cycle is a thermodynamic cycle derived from Carnot vapour power cycle for overcoming its limitations. In earlier discussion it has been explained that Carnot cycle cannot be used in practice due to certain limitations. Rankine cycle has the following thermodynamic processes.

1 – 2 = Isobaric heat addition (in boiler)

2 – 3 = Adiabatic expansion (in turbine)

3 – 4 = Isobaric heat release (in condenser)

4 – 1 = Adiabatic pumping (in pump)

$T - S$, $h - S$ and $P - V$ representations are as shown below.

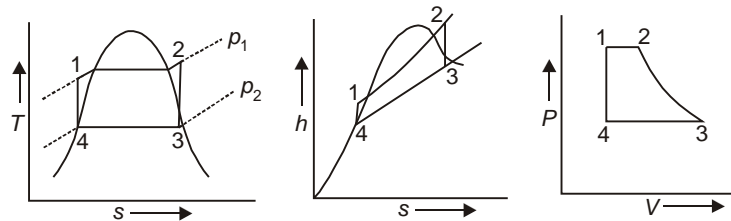


Fig. 8.3 $T-s$, $h-S$ and $P-V$ representations of Rankine cycle

Practical arrangement in a simple steam power plant working on Rankine cycle is shown ahead.

Thus in Rankine cycle, isothermal heat addition and heat rejection processes have been replaced by isobaric processes. Realization of 'isobaric heat addition' and 'heat rejection' in 'boiler' and 'condenser' respectively is in conformity with nature of operation of these devices. Isobaric heat addition can be had in boiler from subcooled liquid to superheated steam without any limitations.

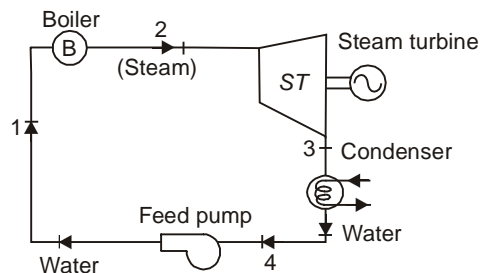


Fig. 8.4 Simple steam power plant layout

Let us understand the arrangement.

- 1 – 2: High pressure water supplied by feed pump is heated and transformed into steam with or without superheat as per requirement. This high pressure and temperature steam is sent for expansion in steam turbine. Heat added in boiler, for unit mass of steam.

$$Q_{\text{add}} = (h_2 - h_1)$$

- 2 – 3: Steam available from boiler is sent to steam turbine, where it's adiabatic expansion takes place and positive work is available. Expanded steam is generally found to lie in wet region. Expansion of steam is carried out to the extent of wet steam having dryness fraction above 85% so as to avoid condensation of steam on turbine blades and subsequently the droplet formation which may hit hard on blade with large force.

$$\text{Turbine work, for unit mass, } W_{\text{turbine}} = (h_2 - h_3).$$

- 3 – 4: Heat rejection process occurs in condenser at constant pressure causing expanded steam to get condensed into saturated liquid at state 4.

$$\text{Heat rejected in condenser for unit mass, } Q_{\text{rejected}} = (h_3 - h_4)$$

- 4 – 1: Condensate available as saturated liquid at state 4 is sent to feed pump for being pumped back to boiler at state 1.

$$\text{For unit mass, Pump work } W_{\text{pump}} = h_1 - h_4.$$

Here pumping process is assumed to be adiabatic for the sake of analysis whereas it is not exactly adiabatic in the pump.

From first and second law combined together;

$$dh = T \cdot ds + v \cdot dp.$$

Here in this adiabatic pumping process. $ds = 0$

Therefore

$$dh = v \cdot dp.$$

or

$$(h_1 - h_4) = v_4 (p_1 - p_4)$$

or

$$(h_1 - h_4) = v_4 (p_1 - p_3). \{ \text{as } p_3 = p_4 \}$$

$$W_{\text{pump}} = v_4 (p_1 - p_3)$$

Rankine cycle efficiency can be mathematically given by the ratio of net work to heat added.

$$\eta_{\text{Rankine}} = \frac{W_{\text{turbine}} - W_{\text{pump}}}{Q_{\text{add}}}$$

$$\eta_{\text{Rankine}} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

In the above expression, the enthalpy values may be substituted from steam table, mollier charts and by analysis for getting efficiency value.

Rankine cycle efficiency may be improved in the following ways:

- By reducing heat addition in boiler, which could be realized by preheating water entering into it.
- By increasing steam turbine expansion work, i.e. by increasing expansion ratio within limiting dryness fraction considerations.
- By reducing feed pump work.
- By using heat rejected in condenser for feed water heating. etc.

Irreversibilities and losses in Rankine cycle: In actual Rankine cycle there exist various irreversibilities and losses in its' constituent components and processes in them. In Rankine cycle the major irreversibility is encountered during the expansion through turbine. Irreversibilities in turbine significantly reduce the

expansion work. Heat loss from turbine to surroundings, friction inside turbine and leakage losses contribute to irreversibilities. Due to this irreversible expansion there occurs an increase in entropy as compared to no entropy change during reversible adiabatic expansion process. This deviation of expansion from ideal to actual process can be accounted for by isentropic turbine efficiency. Ideal expansion in steam turbine is shown by 2–3 on T – S representation. Actual expansion process is shown by 2–3'.

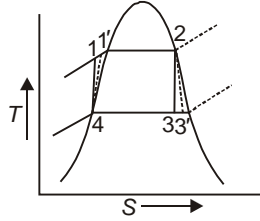


Fig. 8.5 Rankine cycle showing non-ideal expansion and pumping process

$$\text{Isentropic turbine efficiency, } \eta_{\text{isen}, t} = \frac{W_{t, \text{actual}}}{W_{t, \text{ideal}}} = \frac{W_{2-3'}}{W_{2-3}}; \text{ Actually, } W_{2-3'} < W_{2-3}$$

or,

$$\eta_{\text{isen}, t} = \left(\frac{h_2 - h_{3'}}{h_2 - h_3} \right)$$

Another important location for irreversibilities is the pump. During pumping some additional work is required to overcome frictional effects. Ideally pumping is assumed to take place with no heat transfer during pumping whereas actually it may not be so. Thus the pumping process as shown by ideal process 4–1 gets modified to 4–1' which is accompanied by increase in entropy across the pump. Isentropic efficiency of pump is a parameter to account for non-idealities of pump operation. Isentropic efficiency of pump is defined by;

$$\eta_{\text{isen}, p} = \frac{W_{p, \text{ideal}}}{W_{p, \text{actual}}} = \frac{W_{4-1}}{W_{4-1'}}; \text{ Actually, } W_{4-1} < W_{4-1'}$$

or,

$$\eta_{\text{isen}, p} = \left(\frac{h_1 - h_4}{h_{1'} - h_4} \right)$$

Thus, it indicates that actually pump work required shall be more than ideal pump work requirement.

Apart from the turbine and pump irreversibilities explained above there may be other sources of inefficiency too. These turbine and pump irreversibilities accounted for by isentropic efficiency of turbine and pump are called external irreversibility. Sources of internal irreversibilities are heat transfer from system to surroundings, frictional pressure loss in rest of components etc. There also occurs the steam pressure drop due to friction between pipe surface and working fluid.

8.5 DESIRED THERMODYNAMIC PROPERTIES OF WORKING FLUID

Working fluid being used in vapour power cycles must have following desirable properties. Generally water is used as working fluid in vapour power cycles as it is easily available in abundance and satisfies most of requirements. Other working fluids may be mercury, sulphur dioxide and hydrocarbons etc.

- (i) Working fluid should be cheap and easily available.

- (ii) Working fluid should be non-toxic, non-corrosive and chemically stable.
- (iii) Fluid must have higher saturation temperature at moderate pressures as it shall yield high efficiency because most of heat will be added at high temperature. Thus, mean temperature of heat addition shall be high even at moderate pressure.
- (iv) Working fluid should have smaller specific heat so that sensible heat supplied is negligible and Rankine cycle approaches to Carnot cycle. In case of fluid having small specific heat hatched portion shown in Fig. 8.6 will be absent or minimum.
- (v) Saturated vapour line should be steep enough so that state after expansion has high dryness fraction.
- (vi) Working fluid density should be high so that the size of plant becomes smaller.
- (vii) Working fluid should have its' critical temperature within metallurgical limits.
- (viii) It should show significant decrease in volume upon condensation.
- (ix) Working fluid should have its' freezing point much below atmospheric pressure so that there is no chance of freezing in condenser.

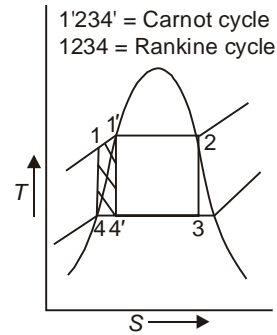


Fig. 8.6 Carnot cycle and Rankine cycle emphasizing for small specific heat of fluid

8.6 PARAMETRIC ANALYSIS FOR PERFORMANCE IMPROVEMENT IN RANKINE CYCLE

Let us carry out study of the influence of thermodynamic variables upon Rankine cycle performance.

(i) *Pressure of steam at inlet to turbine:* Steam pressure at inlet to turbine may be varied for same temperature of steam at inlet. Two different pressures of steam at inlet to turbine, also called throttle pressure are shown in Fig. 8.7. Comparative study shows that for back pressure and steam inlet temperatures being same the increase in steam inlet pressure from p'_1 to p_1 is accompanied by the reduction in net heat added as shown by hatched area $A_{2'3'3'7}$ and increase in net heat added by the amount shown by area $A_{1'1'2'71'}$. Generally two areas $A_{2'3'3'7}$ and $A_{1'1'2'71'}$ are nearly same which means that the increment in net heat added due to increasing throttle pressure from p_1 to p_1 is accompanied by decrease in net heat addition and the net heat added remains same as at lower throttle pressure p_1 . But increase in throttle pressure to p_1 also causes reduction in the heat rejected. At pressure p_1 , heat rejected is given by area $A_{43'6'54}$ while at pressure p_1 heat rejected is given by area A_{43654}

$$A_{43654} < A_{43'6'54}$$

$$(\text{Heat rejected})_{\text{cycle } 1234} < (\text{Heat rejected})_{\text{cycle } 1'2'3'4'}$$

$$\text{Cycle efficiency is given by } \eta_{\text{cycle}} = 1 - \frac{\text{Heat rejected}}{\text{Heat added}}$$

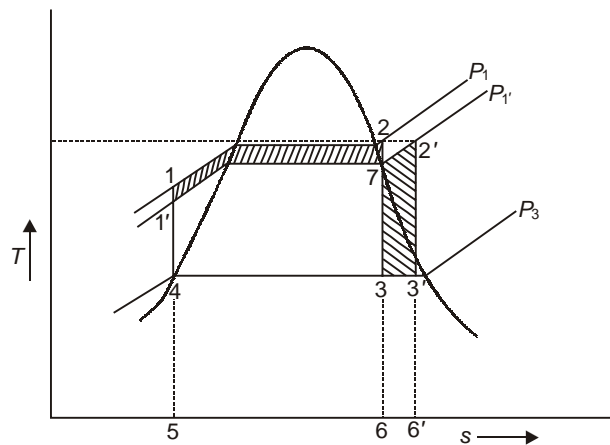


Fig. 8.7 Rankine cycle showing two different throttle pressures

Hence, it is obvious that increasing steam pressure at inlet to steam turbine is accompanied by increase in cycle thermal efficiency. But this increase in pressure increases wetness of steam as shown by states 3 and 3' i.e. $x_3 < x_{3'}$, where x is dryness fraction. This increase in wetness of expanding steam decreases the adiabatic efficiency of turbine and also increases the chances of erosion of steam turbine blades. Therefore, as there are two contrary consequences of increasing throttle pressure so a compromise is to be had. Normally, to avoid erosion of turbine blades the minimum dryness fraction at turbine exhaust should not go below 0.88.

(ii) *Temperature of steam at inlet to turbine:* Increasing temperature of steam at inlet to turbine may also be called as superheating of steam at inlet to turbine. The two Rankine cycles having different degree of superheating are shown in Fig. 8.8.

For two different temperatures of steam at inlet to turbine i.e. T_2 and T_2' , while $T_2 < T_2'$, the comparison of two cycles 12341 and 12'3'41 shows the effect of increasing temperature at inlet to turbine.

Rankine cycle represented on T - s diagram show that increasing temperature from T_2 to T_2' causes increase in net work by amount as shown by area $A_{22'33'2}$. This increase in steam temperature is also accompanied by increased heat addition as shown by area $A_{22'6'6'2}$. It is seen that this ratio of increase in net work to increase in heat addition is more than similar ratio for rest of cycle so the net effect is to improve the cycle thermal efficiency. It may also be stated that this increase in steam temperature from

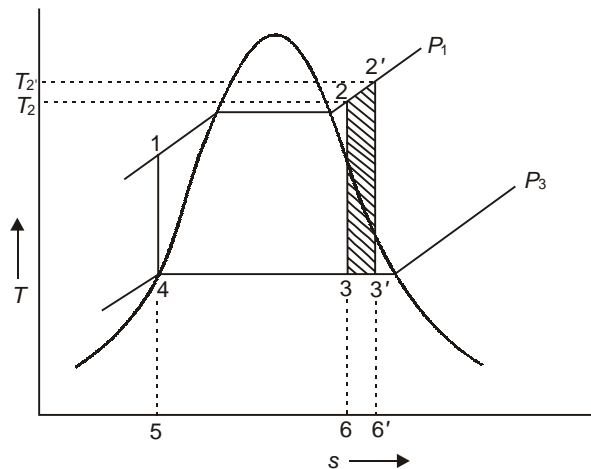


Fig. 8.8 Effect of increasing temperature at inlet to turbine

T_2 to T_2' , i.e. increase in degree of superheat increases mean temperature of heat addition, which increases thermal efficiency. With increased steam temperature the state of steam after expansion becomes more dry i.e. increase in temperature from 2 to 2' makes steam more dry after expansion i.e. from 3 to 3' having dryness fraction $x_{3'} > x_3$. This hotter steam supply to turbine is also advantageous from the specific work point of view. The work done per unit mass gets increased by superheating steam at inlet to turbine. Therefore, one is always interested in realizing highest possible temperature of steam, provided it is within metallurgical temperature limits. Highest practical steam temperature at turbine inlet presently is 650°C.

(iii) *Pressure at the end of expansion:* Let us see the influence of pressure at the end of expansion from steam turbine. This pressure may also be called exhaust pressure or back pressure or condenser pressure. Rankine cycle with two different exhaust pressures is shown in Fig. 8.9, while the maximum pressure and temperature remains same.

With the lowering of back pressure from p_3 to $p_{3'}$, Rankine cycles get modified from 12341 to 1'2'3'4'1'. This reduction in back pressure causes increment in net work as shown by area $A_{1'1433'4'5'1'}$ and also the heat addition increases by the amount shown by area $A_{1'166'1'}$. It is seen that the two areas are such that the thermal efficiency of cycle increases by lowering back pressure as increase in heat addition is more than increase in heat rejection. This lowering back pressure is accompanied by increase in wetness of steam from 3 to 3' i.e. dryness fraction $x_3 > x_{3'}$. Practically there exists limitation of dryness fraction after expansion to avoid erosion of turbine blades so lowering exhaust pressure is limited by it, inspite of showing improvement in thermal efficiency.

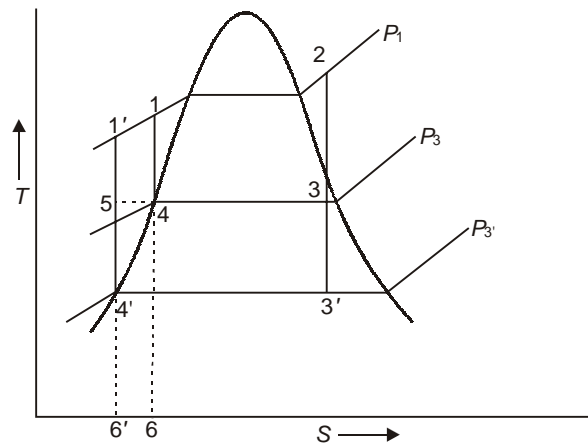


Fig. 8.9 Effect of varying exhaust pressure

(iv) *Temperature of feed water at inlet to boiler:* Temperature of feed water at inlet to boiler may be increased by employing some means such as feed water heating. This increase in feed water temperature reduces the heat requirement in boiler for getting desired state at inlet to steam turbine. Thus, with the reduced heat addition the thermal efficiency gets increased.

Different approaches practically used for improving the Rankine cycle performance have resulted into modified forms of Rankine cycle also called as Reheat cycle, Regenerative cycle etc.

8.7 REHEAT CYCLE

Schematic of reheat cycle is as shown in Fig. 8.10. Reheat cycle is based on the simple fact of realizing high efficiency with higher boiler pressure and yet avoid low quality of steam at turbine exhaust.

Here steam generated in boiler is supplied to high pressure steam turbine at state 2 and is expanded upto state 3. This steam is sent to boiler for being reheated so that its temperature gets increased, normally this temperature after reheating may be equal to temperature at inlet of high pressure steam turbine. Steam after reheating is supplied to subsequent turbine at state 4, say to low pressure steam turbine. Steam is now expanded upto the exhaust pressure say state '5'. Expanded steam is subsequently sent to condenser and condensate at state '6' is pumped back to the boiler employing feed pump at state '1'. Thus, it is possible to take advantage of high steam pressure at inlet to steam turbine as the problem of steam becoming excessively wet with increasing steam pressure could be regulated by reheating during the expansion. Expansion occurs in two stages one beginning at high pressure and other occurring at low pressure with reheating in between. The principal advantage of reheat is to increase the quality of steam at turbine exhaust.

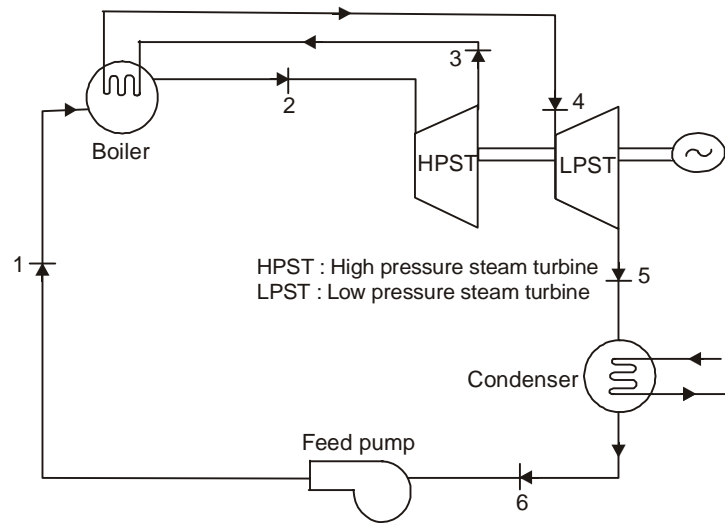


Fig 8.10 Reheat cycle

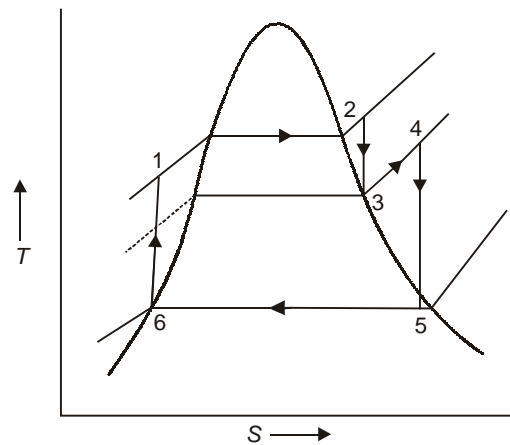


Fig 8.11 T-S representation for reheat cycle

Secondary advantage of reheating is marginal improvement in thermal efficiency when steam pressure is above 100 bar. At low steam pressure reheating does not show gain in cycle thermal efficiency and even the efficiency may be less than that of Rankine cycle due to mean temperature of heat addition being lower. Generally, with modern high pressure boilers and supercritical boilers reheating is essentially employed. Reheating is disadvantageous from economy of plant perspective as the cost of plant increases due to arrangement for reheating and increased condensation requirements due to increased dryness fraction of steam after expansion.

Thermodynamic analysis of reheat cycle as shown on T-S diagram may be carried out for estimation of different parameters as below,

$$\text{Total turbine work output} = W_{\text{HPST}} + W_{\text{LPST}}$$

$$\text{Net work, } W_{\text{net}} = (\text{Total turbine work output}) - (\text{Pump work})$$

$$W_{\text{net}} = W_{\text{HPST}} + W_{\text{LPST}} - W_p$$

where different works for m_s mass of steam are,

$$\text{HP steam turbine, } W_{\text{HPST}} = m_s \cdot (h_2 - h_3)$$

$$\text{LP steam turbine, } W_{\text{LPST}} = m_s \cdot (h_4 - h_5)$$

$$\text{Feed Pump, } W_p = (h_1 - h_6) \cdot m_s$$

$$W_{\text{net}} = \{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)\} \cdot m_s$$

$$\text{Heat supplied for } m_s \text{ mass of steam; } Q_{\text{add}} = (h_2 - h_1) \cdot m_s + m_s \cdot (h_4 - h_3)$$

$$\text{Cycle thermal efficiency, } \eta_{\text{Reheat}} = \frac{W_{\text{net}}}{Q_{\text{add}}}$$

$$\eta_{\text{Reheat}} = \frac{\{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)\}}{\{(h_2 - h_1) + (h_4 - h_3)\}}$$

$$\text{Specific work output, } W_{\text{reheat}} = \{(h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6)\}$$

Generally not more than two stages of reheat are practically employed. Theoretically, the improvement in efficiency due to second reheat is about half of that which results from single reheat. Also more number of reheat stages shall result into superheated steam at turbine exhaust. Thus, mean temperature of heat rejection gets raised and efficiency drops.

8.8 REGENERATIVE CYCLE

Regenerative cycle is a modified form of Rankine cycle in which it is devised to increase mean temperature of heat addition so that cycle gets close to Carnot cycle in which all heat addition occurs at highest possible temperature. In regenerative cycle the feed water is heated up so as to reduce the heat addition in boiler and heat addition occur at hotter feed water temperature. Theoretically regenerative cycle arrangement is as shown in Fig. 8.12.

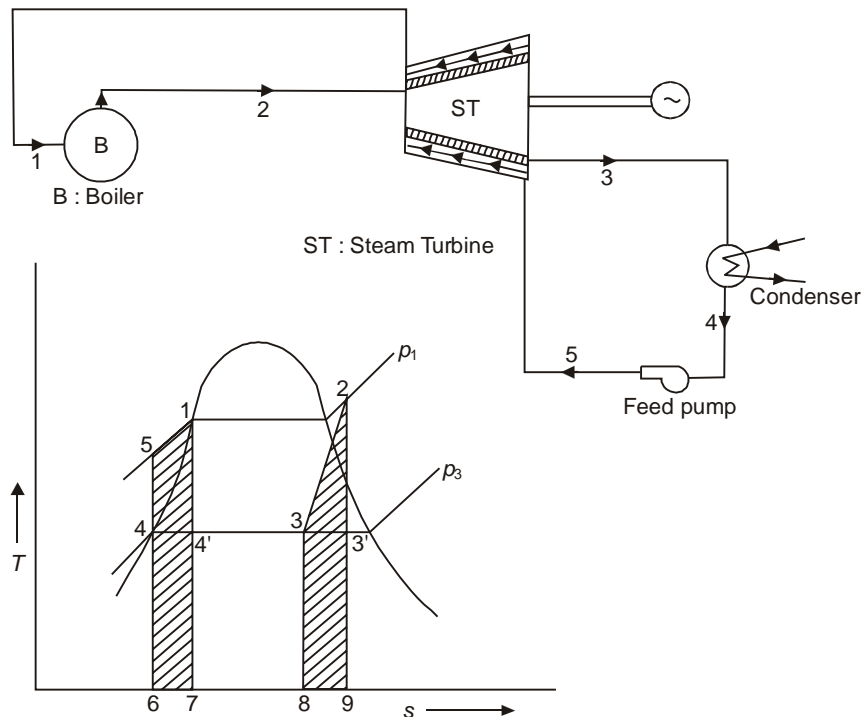


Fig 8.12 Schematic for theoretical regenerative cycle and T-s representation.

Theoretical arrangement shows that the steam enters the turbine at state 2 (temperature T_2) and expands to (temperature T_3) state 3. Condensate at state 5 enters the turbine casing which has annular space around turbine. Feed water enters turbine casing at state 5 and gets infinitesimally heated upto state 1 while flowing opposite to that of expanding steam. This hot feed water enters into boiler where steam generation occurs at desired state, say 2. Feed water heating in steam turbine casing is assumed to occur reversibly as the heating of feed water occurs by expanding steam with infinitesimal temperature difference and is called “regenerative heating”. This cycle is called regenerative cycle due to regenerative heating employed in it. Regenerative heating refers to the arrangement in which working fluid at one state is used for heating itself and no external heat source is used for this purpose. Here feed water picks up heat from steam expanding in steam turbine, thus the expansion process in steam turbine shall get modified from 2-3' ideally to 2-3. Heat picked up by feed water for getting heated up from state 5 to 1 is shown by hatched area A_{17651} on T - S diagram. Under ideal conditions for cent per cent heat exchange effectiveness the two areas i.e. A_{29832} indicating heat extraction from steam turbine and A_{17651} indicating heat recovered by feed water shall be same. Thus, T - S representation of regenerative cycle indicates that the cycle efficiency shall be more than that of Rankine cycle due to higher average temperature of heat addition.

But there exists serious limitation regarding realization of the arrangement described above. Limitations are due to impossibility of having a steam turbine which shall work as both expander for getting work output and heat exchanger for feed water heating. Also with the heat extraction from steam turbine the state of expanded steam at exhaust pressure shall be extremely wet, hence not desired. Due to these limitations the regenerative cycle is realized employing the concept of bleeding out steam from turbine and using it for feed water heating in feed water heaters.

Feed Water Heaters: The feed water heater refers to the device in which heat exchange occurs between two fluids i.e. steam and feed water either in direct contact or indirect contact. Direct contact feed water heater is the one in which bled steam and feed water come in direct contact. These are also called open feed water heater.

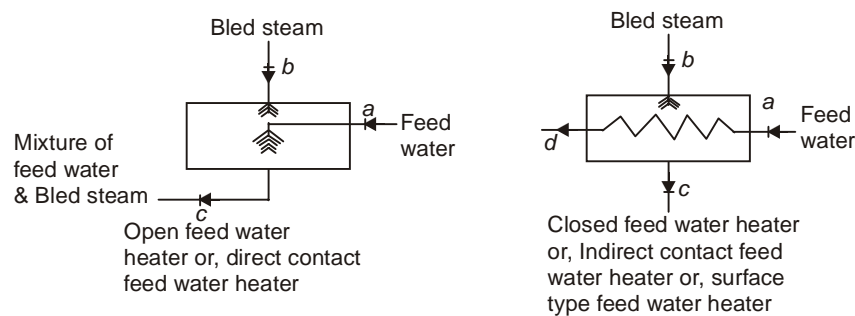


Fig. 8.13 Feed water heaters

In open feed water heater two fluids i.e., bled steam and feed water are at same pressure and adiabatic mixing is assumed to take place. Normally, it is considered that the mixture leaves open feed water heater as saturated liquid. Energy balance upon it shall be as follows,

$$m_a \cdot h_a + m_b \cdot h_b = (m_a + m_b) \cdot h_c$$

where subscripts a, b and c are for feed water, bled steam and mixture of the two as shown in Figure 8.13.

Indirect contact feed water heater as shown in Figure 8.13 is the one in which two fluid streams i.e. bled steam and feed water do not come in direct contact, but the heat exchange between two streams

occurs indirectly through metal interface. These are also called closed feed water heaters. In these feed water heaters since two fluids do not contact each other so they may be at different pressures. In these the arrangement comprises of steel, copper or brass tubes of solid drawn type placed in a shell. The heat transfer takes place through tube surface. Feed water flows inside the tube and is heated by extracted steam from outside. Steam enters in the shell and comes in contact with the tubes and then condenses over tubes. Steam condenses and trickles down and is collected in shell. Figure 8.14 gives schematic of surface type feed water heater. Performance of feed water heater is quantified using a parameter called “terminal temperature difference”. Terminal temperature difference (T.T.D.) refers to the difference of temperature between temperature of feed water outlet and saturation temperature of steam entering the heater.

Terminal temperature difference = (Feed water outlet temperature – Saturation temperature of steam entering heater)

T.T.D. has its value lying around 5–8°C. T.T.D. shall be zero in desuperheater type heaters where superheated steam is used for feed water heating upto saturation temperature of steam.

In feed water heaters where steam pressure is quite high, the condensate from heater is expanded in an expander called ‘flash tank’ or ‘drain expander’ or ‘drain cooler’. In this flash tank some portion of condensate gets converted into steam which is further used for heating feed water.

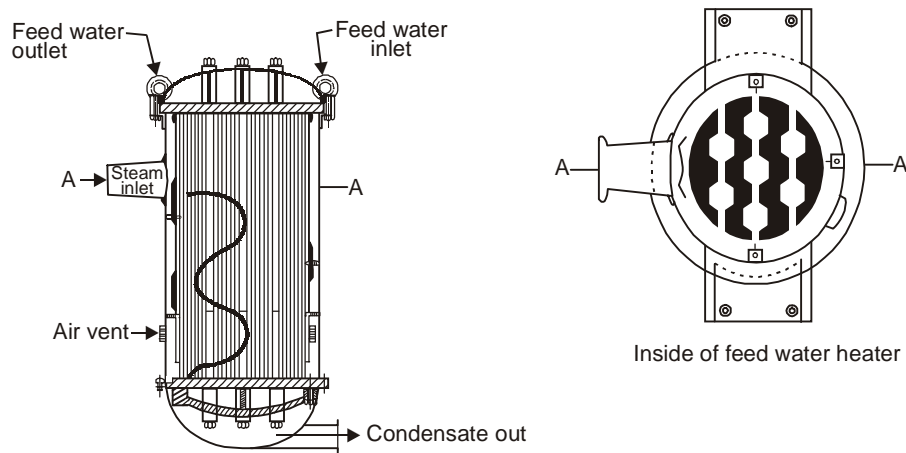


Fig. 8.14 Surface type feed heater

Direct contact heaters or open type heaters are more efficient than indirect contact type due to direct contact between two fluids. Feed water can achieve saturation temperature corresponding to the pressure of heating steam. In this case the terminal temperature difference is zero. During heating the non-condensable gases dissolved in water get released and are thrown out through vent passage. Deaerator is a type of open feed water heater. Schematic is shown in subsequent article on deaerator.

Deaerator: Deaerator is a type of open type feed water heater employed for the removal of dissolved oxygen and carbon dioxide from the feed water. The dissolved oxygen when not removed gets disintegrated into nascent oxygen at high temperature and pressure and forms iron oxide upon coming in contact with metal. This iron oxide formed causes pitting on the metal surface. At high temperature and pressure, dissolved CO_2 combines with metal and forms carbolic acid which causes mild pitting on

metal surfaces. Therefore, it becomes necessary to remove dissolved gases from the feed water. Although feed water treatment plant is there but still the impurities may creep in along with the make up water added to compensate for loss of water from system due to leakages at valve, pipe flanges, steam valve spindles and boiler blow down etc. Normally added water is 3-5% of total boiler feed.

Deaeration of water is based on the principle of decreasing partial pressure of gas for removal of dissolved gases. Henry's law and Dalton's law of partial pressure may be considered in this reference for understanding the phenomenon. Henry's law states that, "the mass of gas dissolved in a definite mass of liquid at a given temperature is directly proportional to partial pressure of gas in contact with liquid". It is good for the gases having no chemical reaction with water.

Decrease in partial pressure of gas in water is achieved by increasing the vapour pressure by heating the water. Here feed water is heated by low pressure steam for heating it upto its saturation temperature. Feed water entering deaerator is broken into small particles so as to increase contact area for better heat exchange with high temperature steam. Constructional detail of deaerator is shown in Fig. 8.15 which has basically deaerator head and storage tank. Water enters deaerator head from top on to a distributor plate. The water trickles down from upper most tray to the bottom trays through tiny holes in these trays. Steam enters storage tank from one end and enters deaerator head after passing through water collected in tank. Steam gradually heats feed water flowing downward with its' portion getting condensed and remaining steam flowing along with liberated gases out of deaerator head. Steam and gas mixture vented out from the top of deaerator head may be used for preheating make up water or feed water entering deaerator, if economical. The deaerated water is collected in storage tank below the deaerator head. A bubbler line is also provided in deaerator tank for fast heating of deaerator system during start up of unit. Bubbler line is a perforated pipe laid at bottom of storage tank through which steam for heating the water is supplied.

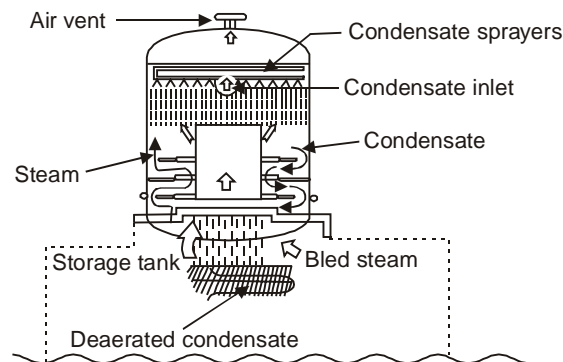


Fig. 8.15 Deaerator

Feed heater arrangements: In regenerative cycle, feed water heaters of different types are employed. There are some generic arrangements frequently used in these cycles. The arrangements are discussed below with two bleed points from where m_1 and m_2 masses of steam are bled out at pressures p_1 and p_2 and expansion occurs upto pressure p_3 . Total steam flowing is taken as 1 kg.

- (i) **Surface type heaters method:** This employs surface type feed water heaters and the arrangement for them is as shown in Fig. 8.16.

Here two surface heaters are used for showing the arrangement. Condensate of the bled steam is drained out using drain pump and sent in the main line at high pressure. This arrangement is also called as drain pump method.

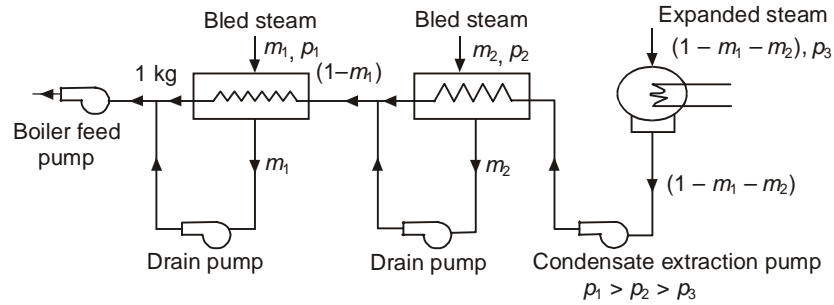


Fig. 8.16 Arrangement in drain pump method or surface type feed water heaters method

- (ii) *Open type heater method:* In this arrangement the open type feed water heaters are employed as shown. Here due to contact of two fluids, there occurs mixing of bled steam with water and is taken out using pump for being sent to next open feed water heater.
- (iii) *Surface type heaters with hot well:* This arrangement employs a hot well with surface type heaters. Bled steam condensate leaving surface type heaters is sent to hot well from where it is picked up by pump and flown through heaters for getting heated up. Arrangement is shown in Fig. 8.18.

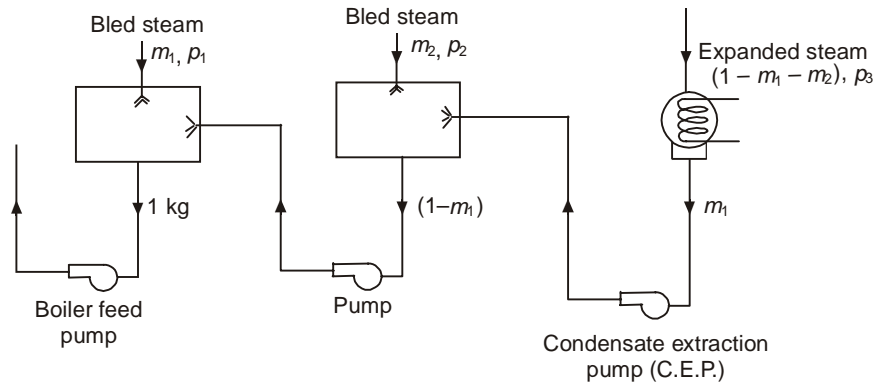


Fig. 8.17 Arrangement with open feed water heaters

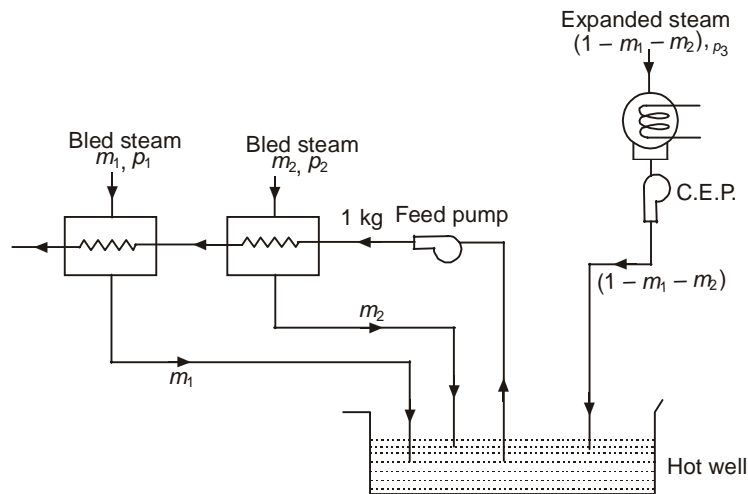


Fig 8.18 Arrangement with surface type feed water heaters and hotwell

- (iv) *Cascade method*: This arrangement is shown in Fig. 8.19. Here bled steam condensate is throttled and cascaded into low pressure surface heaters. Bled steam condensate from last heater is sent to hotwell from where it is picked up and pumped through surface type heaters. For lowering the pressure of condensate before mixing the traps may also be used. A trap allows the liquid to be throttled to a lower pressure and also traps the vapour.

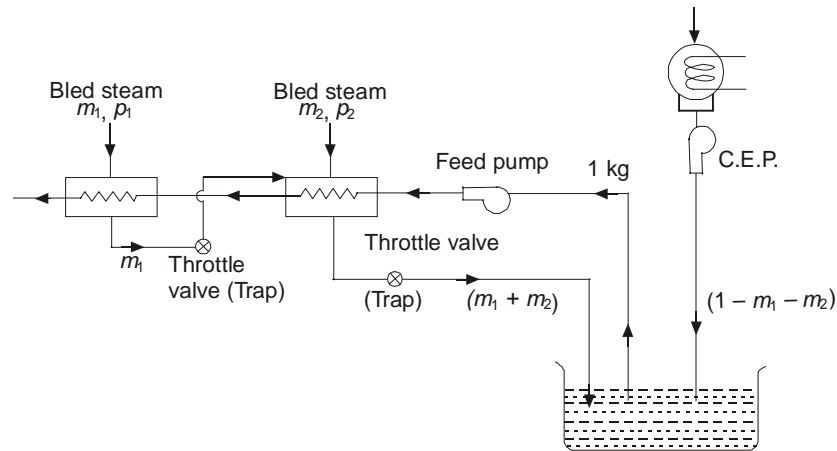


Fig. 8.19 Cascade method

Thermodynamics of regenerative cycle: Schematic of regenerative cycle with single feed water heater of open type is shown in Fig. 8.20. Arrangement shows that steam is bled out from turbine at state 6 and fed into feed heater.

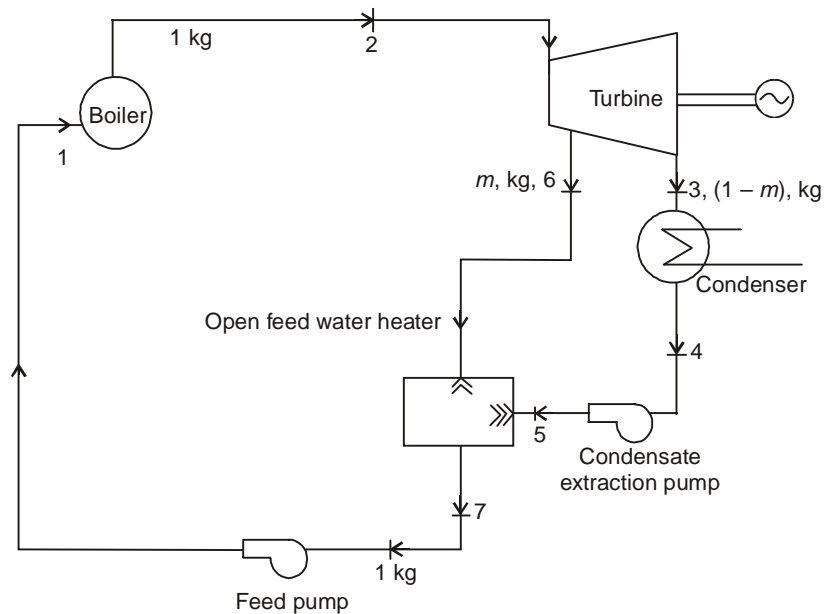


Fig. 8.20 Schematic for regenerative cycle with one open feed water heater

Feed water leaving at state 7 as shown on T - S diagram is being pumped upto boiler pressure through feed pump. T - S diagram indicates that the amount of heat picked up by feed water is shown by hatched area $A_{1751'}$. In case of absence of bleeding and feed heater the feed water will enter into boiler at state $1'$ as compared to state 1 when regenerative heating is employed. Thus, advantage of hotter feed water to boiler can be realized by bleeding expanding steam from turbine. Regeneration can be seen in the feed water heating as the bled steam gets mixed with feed water at state 5 thus resulting into hot feed water.

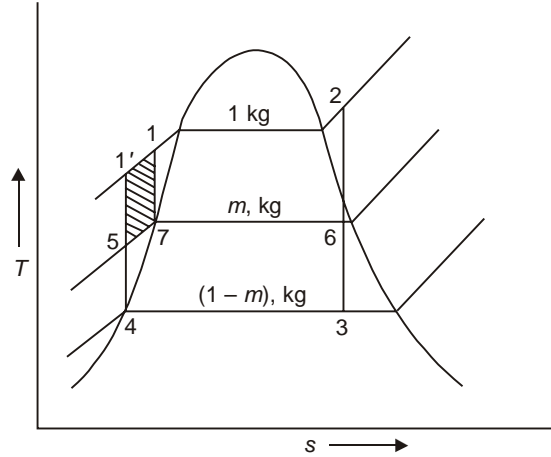


Fig. 8.21 T - s representation for regenerative cycle with one open feed water heater

As described earlier the bleeding offers advantage in terms of increased cycle efficiency due to increased mean temperature of heat addition. Hotter feed water also offers advantage in terms of reduced thermal stresses in boiler due to reduced temperature difference and less tendency of condensation of sulphur dioxide. Bleeding of steam causes reduced mass flow in condenser thereby reducing size of condenser. Bleeding is also disadvantageous because the work done per unit mass of steam gets reduced, thus increasing cost of the plant. Boiler capacity is to be increased for a given output.

Here it can be concluded that if the number of feed heaters be increased then their could be substantial increase in feed water temperature, thus offering a cycle having high mean temperature of heat addition, close to Carnot cycle. But it shall be accompanied by reduced work output and increased cost of the plant. Generally, the number of feed water heaters employed lies between 3 to 8 with average temperature rise in each heater being 10–15°C. For example, if there are six heaters then first two may be surface type or indirect contact type followed by open type or direct contact feed water heater which shall also act as deaerator followed by three surface type feed water heaters.

For the regenerative cycle considered, with unit mass of steam leaving boiler and ' m ' kg of steam bled out for feed water heating:

$$\text{Steam turbine work} = (h_2 - h_6) + (1 - m) \cdot (h_6 - h_3)$$

$$\text{Pump work} = (1 - m) \cdot (h_5 - h_4) + 1 \cdot (h_1 - h_7)$$

$$\text{Net work} = \{(h_2 - h_6) + (1 - m) \cdot (h_6 - h_3)\} - \{(1 - m) \cdot (h_5 - h_4) + (h_1 - h_7)\}$$

$$\text{Heat added} = 1 \cdot (h_2 - h_1)$$

$$\text{Hence, regenerative cycle efficiency} = \frac{\text{Net work}}{\text{Heat added}}$$

$$\eta_{\text{regenerative}} = \frac{\{(h_2 - h_6) + (1 - m)(h_6 - h_3)\} - \{(1 - m)(h_5 - h_4) + (h_1 - h_7)\}}{(h_2 - h_1)}$$

Example: Regenerative cycle with two surface type heaters, (Fig. 8.22)

Let us carry out thermodynamic analysis for 1 kg of steam generated in boiler at pressure p_1 and masses of bled steam be m_6 and m_7 at pressure p_6 and p_7 .

$$\text{Steam turbine work} = \{1 \cdot (h_2 - h_6) + (1 - m_6) \cdot (h_6 - h_7) + (1 - m_6 - m_7) \cdot (h_7 - h_3)\}$$

$$\text{Total pumping work} = \{(1 - m_6 - m_7) \cdot (h_5 - h_4) + m_7 \cdot (h_{10} - h_9) + m_6 \cdot (h_{14} - h_8)\}$$

Bled steam leaving surface heaters 1 and 2 are at state 8 and 9 which are saturated liquid states at respective pressure, i.e., $h_8 = h_{f \text{ at } p_6}$, $h_9 = h_{f \text{ at } p_7}$

$$\text{Heat added in boiler} = 1 \cdot (h_2 - h_1)$$

Applying heat balance on two surface heaters we get,

On surface heater 1,

$$m_6 \cdot h_6 + (1 - m_6) \cdot h_{12} = m_6 \cdot h_8 + (1 - m_6) \cdot h_{13}$$

On surface heater 2,

$$m_7 \cdot h_7 + (1 - m_6 - m_7) \cdot h_5 = m_7 \cdot h_9 + (1 - m_6 - m_7) \cdot h_{11}$$

At the point of mixing of output from surface heater and bled condensate the heat balance yields.

$$1 \cdot h_1 = (1 - m_6) \cdot h_{13} + m_6 \cdot h_{14}$$

$$h_1 = (1 - m_6) \cdot h_{13} + m_6 \cdot h_{14}$$

and

$$(1 - m_6 - m_7) \cdot h_{11} + m_7 \cdot h_{10} = (1 - m_6) \cdot h_{12}$$

Upon the pumps 1, 2 and 3;

On pump 1,

$$v_4(p_1 - p_3) = h_5 - h_4$$

and

$$h_5 = v_4(p_1 - p_3) + h_4$$

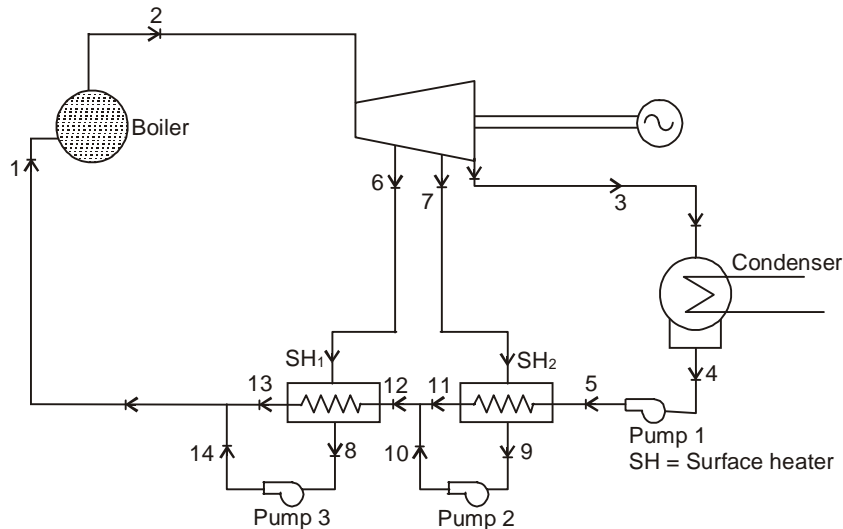


Fig. 8.22 (a) Schematic of regenerative cycle with two surface type heaters

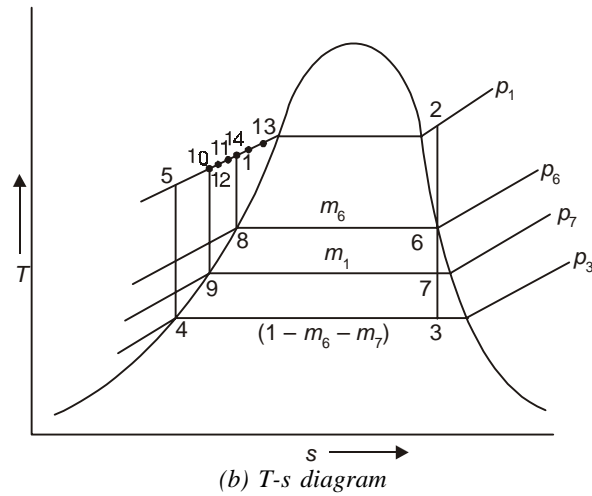


Fig. 8.22 Schematic and T-S representation for regenerative cycle with two surface type heaters

On pump 2,

$$v_9(p_1 - p_7) = h_{10} - h_9$$

$$h_{10} = v_9(p_1 - p_7) + h_9$$

On pump 3,

$$v_8(p_1 - p_6) = h_{14} - h_8$$

$$h_{14} = v_8(p_1 - p_6) + h_8$$

Above different enthalpy expressions can be used for getting enthalpy values at salient points of interest in order to get the net work and cycle efficiency of this arrangement.

8.9 BINARY VAPOUR CYCLE

Generally, water is used as working fluid in vapour power cycle as it is found to be better than any other fluid if looked from the point of view of desirable characteristics of working fluid. Water is poor in respect to the following desired characteristics of working fluid.

Fluid should have critical temperature well above the highest temperature set by metallurgical limits of construction material. Fluid should have a saturation pressure at the maximum cycle temperature that poses no strength problems and a saturation pressure at the minimum cycle temperature that poses no difficulty of leakage from atmosphere.

In respect to above properties water is found to exhibit poor characteristics as; Water has critical temperature of 374°C which is about 300 °C less than the temperature limits set by metallurgical properties. The saturation pressure of water is quite high even at moderate temperatures so it does not have desirable properties at higher temperatures. Therefore in high temperature region a substance which has low saturation pressure should be used and the fluid should have its' critical temperature well above metallurgical limits of about 600°C.

Therefore, it can be concluded that no single working fluid satisfies all the desirable requirements of working fluid, different working fluids may have different attractive features in them, but not all. So let us think of striking a combination of any two working fluids which are well suited together such as mercury and water. In such cases two vapour cycles operating on two different working fluids are put together and the arrangement is called binary vapour cycle.

Mercury has comparatively small saturation pressures at high temperature values but due to excessively low pressure values and large specific volume values at low temperatures it cannot be used alone

as working fluid. Mercury also does not wet the surface in contact so there is inefficient heat transfer although 0.002% of solution of magnesium and potassium is added to give it wetting property of steel. Steam is used with mercury for overcoming some limitations of mercury. Thus in combination of mercury-steam, the mercury is used for high pressures while steam is used for low pressure region.

Layout for mercury-steam binary vapour cycle is shown on Fig. 8.23 along with its depiction on T - S diagram. Here, mercury vapour are generated in mercury boiler and sent for expansion in mercury turbine and expanded fluid leaves turbine and enters into condenser. From condenser the mercury condensate is pumped back into the mercury boiler. In mercury condenser the water is used for extracting heat from mercury so as to condense it. The amount of heat liberated during condensation of mercury is too large to evaporate the water entering mercury condenser. Thus, mercury condenser also acts as steam boiler. For superheating of steam an auxilliary boiler may be employed or superheating may be realized in the mercury boiler itself.

MT : Mercury turbine

ST : Steam turbine

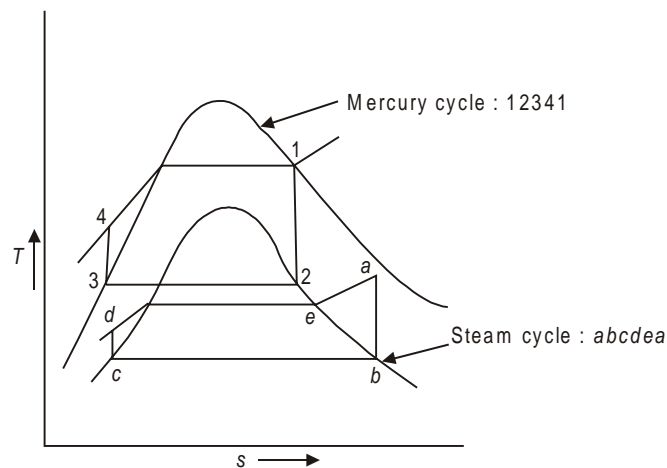
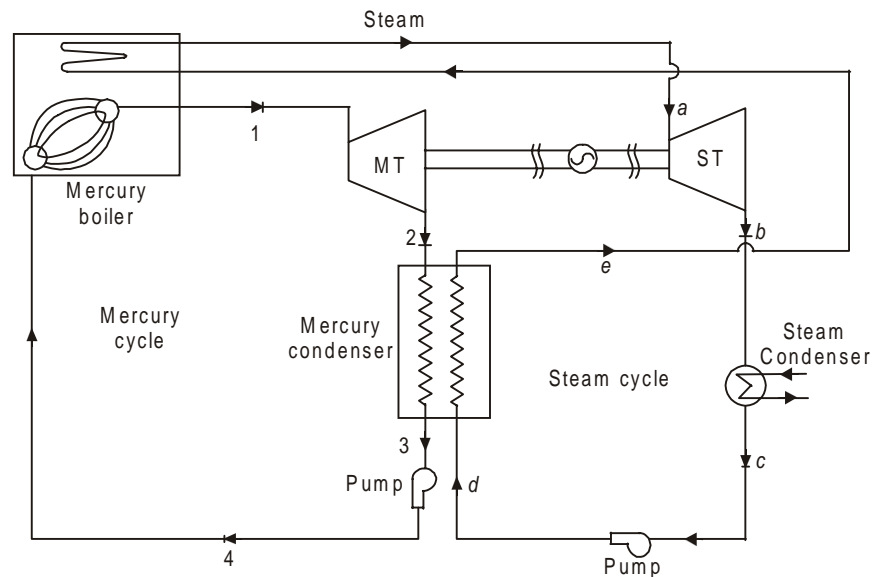


Fig. 8.23 Schematic of binary vapour cycle

$$\text{Net work from cycle shall be, } W_{\text{net}} = W_{\text{MT}} + W_{\text{ST}} - \Sigma W_{\text{pump}}$$

$$\text{Work from mercury turbine, } W_{\text{MT}} = m_{\text{MT}} \cdot (h_1 - h_2)$$

$$\text{Work from steam turbine, } W_{\text{ST}} = m_{\text{ST}} \cdot (h_a - h_b)$$

$$\text{Pump work} = m_{\text{MT}} \cdot (h_4 - h_3) + m_{\text{ST}} \cdot (h_d - h_c)$$

$$\text{Heat added to the cycle, } Q_{\text{add}} = m_{\text{MT}} \cdot (h_1 - h_4) + m_{\text{ST}} \cdot (h_a - h_e)$$

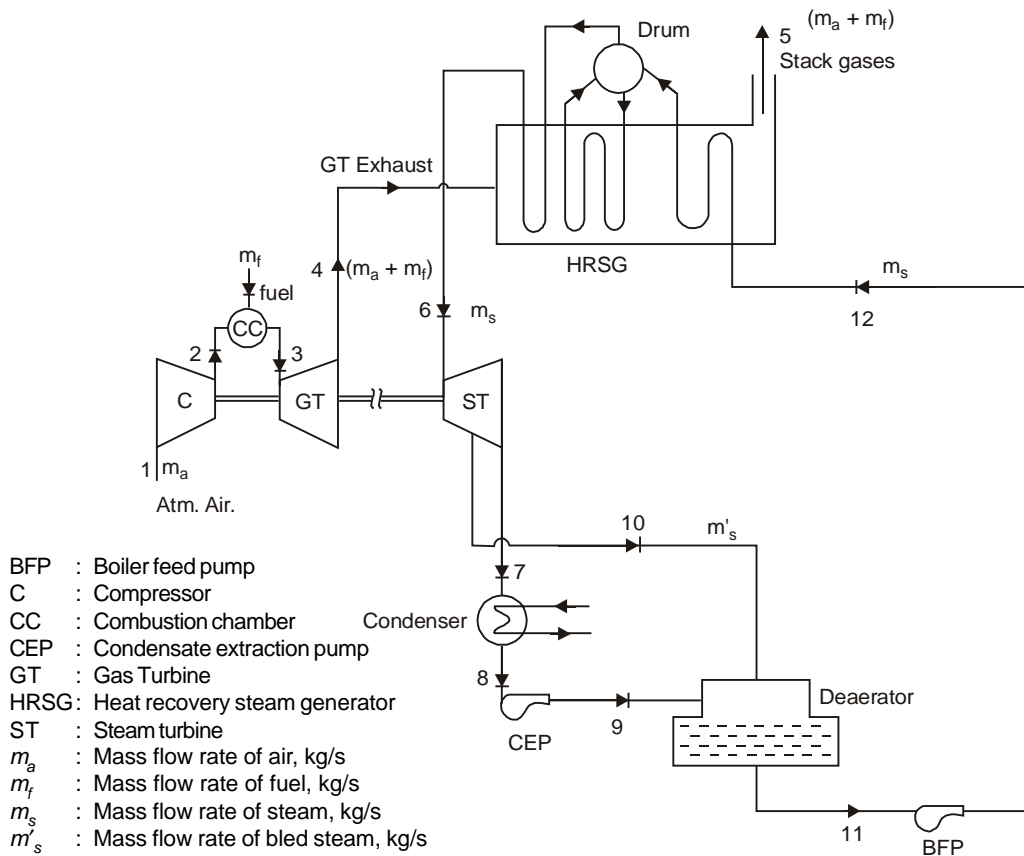
$$\text{Binary Cycle efficiency, binary} = \frac{W_{\text{net}}}{Q_{\text{add}}}$$

$$= \frac{\{m_{\text{MT}}(h_1 - h_2) + m_{\text{ST}}(h_a - h_b) - m_{\text{MT}}(h_4 - h_3) - m_{\text{ST}}(h_d - h_c)\}}{\{m_{\text{MT}}(h_1 - h_4) + m_{\text{ST}}(h_a - h_e)\}}$$

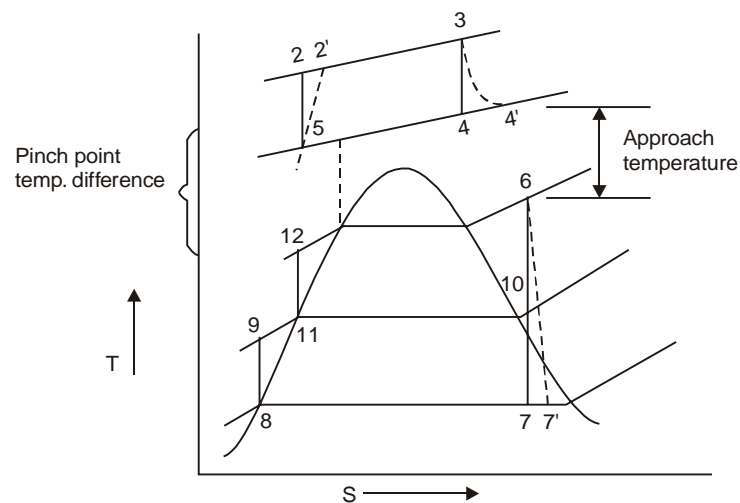
8.10 COMBINED CYCLE

Combined cycle refers to the combination of two cycles operating in synergy. The thermodynamic cycles operating together in the form of combined cycles have capability to operate in isolation too for producing work output. These different cycles have to operate on different fluids. Among different combined cycles the gas/steam combination is popular. The gas/steam combined cycles have combination of Brayton cycle and Rankine cycle. Exhaust gases from gas turbine in Brayton cycle are sent to heat recovery steam generator (HRSG) or waste heat recovery boiler (WHRB) for generation of steam to be expanded in steam turbine in Rankine cycle. High temperature cycle in combined cycle is called topping cycle and low temperature cycle is called bottoming cycle. Thus, in combined cycle the heat rejected by higher temperature cycle is recovered, in lower temperature cycle such as in heat recovery steam generator for generation of steam which subsequently runs steam turbine and augments the work output. In different combined cycles the topping cycles could be Otto cycle, Brayton cycle and Rankine cycle while Rankine cycle is generally used as bottoming cycle.

Fig. 8.24 shows the layout of a typical gas/steam combined cycle. Combined cycle could have various arrangements depending upon the alterations in topping cycle and bottoming cycle arrangements. In the shown layout there is simple gas turbine cycle, compression of air occurs between states 1 and 2. Subsequently heat addition and expansion occurs in combustion chamber and gas turbine through processes 2-3 and 3-4 respectively. Exhaust gases from gas turbine enter into heat recovery steam generator (HRSG) at state 4 and leave at state 5. Steam generated at state 6 from HRSG is sent to steam turbine for expansion and thus steam turbine work output augments work output of gas turbine. Expanded steam enters condenser at state 7 and condensate is sent back to HRSG at state 12 after passing it through deaerator. For m_a , m_f and m_s being flow rates of air, fuel and steam respectively thermodynamic analysis is carried out as under.



(a) Layout of gas/steam combined cycle



(b) T-s diagram representation for combined cycle.

Fig. 8.24 (a) Layout of gas/steam combined cycle
 (b) T-S diagram representation for combined cycle

Thus the work requirement in compressor,

$$W_c = m_a (h_2 - h_1)$$

Heat addition in combustion chamber, for fuel having calorific value CV

$$Q_{\text{add}} = m_f \times CV$$

Energy balance upon combustion chamber yields,

$$m_a \times h_2 + m_f \times CV = (m_a + m_f) h_3$$

Work available from gas turbine

$$W_{GT} = (m_a + m_f) (h_3 - h_4)$$

Net work from topping cycle, $W_{\text{topping}} = W_{GT} - W_c$

Work available from steam turbine, for bled steam mass flow rate for deaeration being m_s .

$$W_{ST} = \{m_s (h_6 - h_7) + (m_s - m'_s) (h_7 - h_{10})\}$$

Pump works

$$W_{CEP} = (m_s - m'_s) (h_9 - h_8)$$

$$W_{BFP} = m_s (h_{12} - h_{11})$$

Net work available from bottoming cycle

$$W_{\text{bottoming}} = W_{ST} - W_{CEP} - W_{BFP}$$

Hence total work output from combined cycle

$$W_{\text{combined}} = W_{\text{topping}} + W_{\text{bottoming}}$$

Thermal efficiency of combined cycle,

$$\eta_{\text{combined}} = \frac{W_{\text{combined}}}{Q_{\text{add}}}$$

Thermal efficiency of topping cycle (gas turbine cycle),

$$\eta_{\text{topping}} = \frac{W_{\text{topping}}}{Q_{\text{add}}}$$

We can see that work output of gas turbine cycle is less than combined cycle work output, while the heat addition remains same. Thus, thermal efficiency of combined cycle is more than gas turbine cycle (topping cycle),

$$\text{As, } W_{\text{combined}} > W_{\text{topping}} \Rightarrow \eta_{\text{combined}} > \eta_{\text{topping}}$$

8.11 COMBINED HEAT AND POWER

Combined heat and power refers to the arrangement in which cycle produces work (power) along with heat utilization for process heating. There exist number of engineering applications where both power and process heat are simultaneously required. Such arrangement is also called cogeneration. Cogeneration may be defined as the arrangement of producing more than one useful form of energy. Food processing

industry and chemical industry are the industries where steam is required for different processes and cogeneration is an attractive option for getting electricity along with process steam. Schematic of a cogeneration plant is shown in Fig. 8.25.

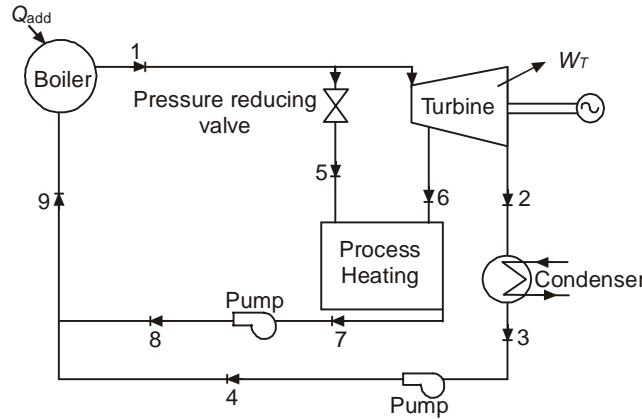


Fig. 8.25 Schematic for cogeneration

Cogeneration arrangement is popularly used in cold countries for district heating where, in this arrangement the power plant supplies electricity along with steam for process needs, such as space heating and domestic water heating.

Thermodynamic analysis of the cogeneration arrangement shows;

$$\text{Heat added in boiler } Q_{\text{add}} = m_1 (h_1 - h_9)$$

$$\text{Heat used in process heating } Q_{\text{process}} = \{m_5 (h_5 - h_7) + m_6 (h_6 - h_7)\}$$

$$\text{Turbine work: } W_T = \{(m_1 - m_5) \cdot (h_1 - h_6) + (m_1 - m_5 - m_6) \cdot (h_6 - h_2)\}$$

$$\text{Pump work: } W_P = \{(m_5 + m_6) \cdot (h_8 - h_7) + (m_1 - m_5 - m_6) \cdot (h_4 - h_3)\}$$

When there is no process heating then; $m_5 = 0$ and $m_6 = 0$

8.12 DIFFERENT STEAM TURBINE ARRANGEMENT

In certain applications simple steam turbines are unable to meet specific requirements. Back pressure turbine, pass out or extraction turbine and mixed pressure turbines are such special purpose turbines whose details are given ahead.

(a) *Back pressure turbine*: Back pressure turbine is the one in which steam is not expanded upto lowest pressure in steam turbine, instead steam leaves the turbine at higher pressure which is appropriate for the process steam/heating requirement. Thus, in back pressure turbine expansion is limited to high back pressure and steam leaving turbine goes for process heating. Generally, steam leaving turbine at high back pressure will be superheated. Since steam is to be used for process heating so the rate of heat transfer should be high. Superheated steam is not suitable for heating because of small rate of heat transfer therefore superheated steam should be desuperheated and brought to saturated steam state as saturated steam has high rate of heat transfer and also the control of temperature is convenient.

Thus, back pressure turbine has the provision of desuperheating as shown in Fig. 8.26. The steam leaving turbine enters into desuperheater where it is transformed into saturated steam. Saturated steam is subsequently sent for process heating where it gets condensed and condensate is sent back to the boiler through pump. A by pass valve is also provided so that if there is no power requirement then whole steam may be sent for process heating through desuperheater by closing turbine valve and opening by pass valve.

(b) *Pass out or extraction turbine:* Pass out turbine refers to the steam turbine having provision for extraction of steam during expansion. Such provision is required because in combined heat and power requirement the steam available from back pressure turbine may be more than required one or the power produced may be less than the required value. Pass out turbine has arrangement for continuous extraction of a part of steam at the desired pressure for process heating and left out steam goes into low pressure section of turbine through a pressure control valve. In the low pressure section of turbine, a control mechanism is provided so that the speed of turbine and pressure of steam extracted remains constant irrespective of the variations in power produced and process heating.

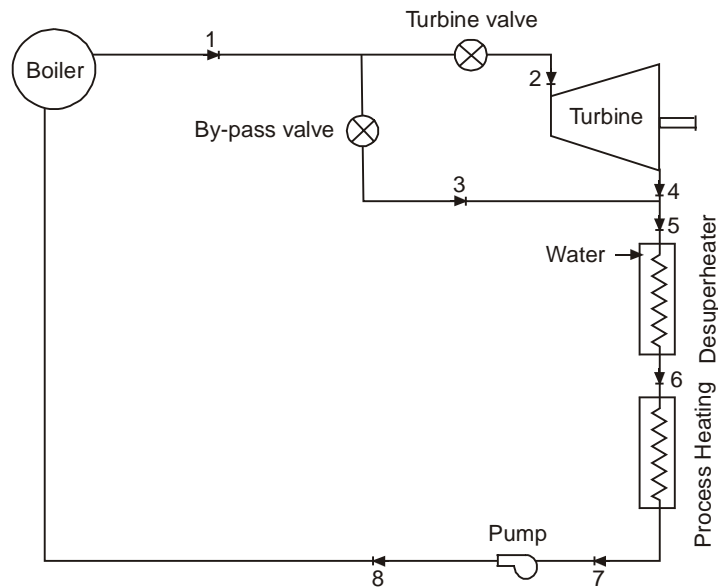


Fig. 8.26 Back pressure turbine

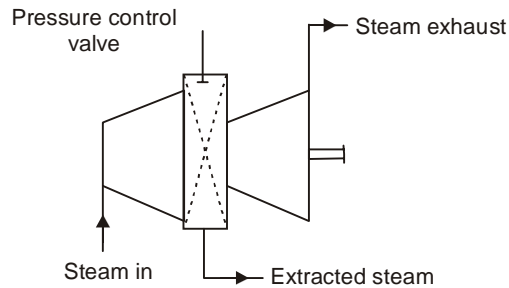


Fig. 8.27 Pass out turbine

The pass out turbines have to operate under widely varying load so its efficiency is quite poor. For facilitating the operation of pass out turbine from no extraction to full steam extraction conditions, nozzle control governing or throttle control governing are used.

(c) *Mixed pressure turbine:* These are the turbines which have capability of admitting steam at more than one pressures and subjecting multiple pressure steam streams to expand. Generally, mixed pressure turbines utilize high pressure steam from a boiler and also low pressure steam from exhaust of a non-condensing engine or some auxiliary of the plant.

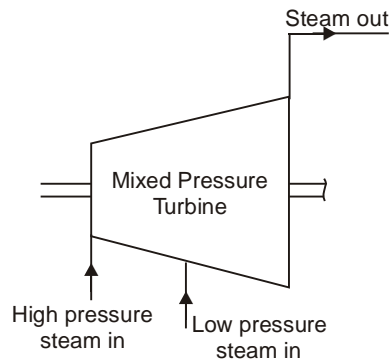


Fig 8.28 Mixed pressure turbine

Mixed pressure turbines are preferred when steam at single pressure is not available in desired quantity for producing required power. These mixed pressure turbines actually have more than one turbines in one cylinder.

EXAMPLES

1. A Carnot cycle works on steam between the pressure limits of 7 MPa and 7 kPa. Determine thermal efficiency, turbine work and compression work per kg of steam.

Solution:

T-s representation for the Carnot cycle operating between pressure of 7 MPa and 7 kPa is shown in Fig. 8.29

$$\begin{aligned}\text{Enthalpy at state 2, } h_2 &= h_g \text{ at 7 MPa} \\ h &= 2772.1 \text{ kJ/kg}\end{aligned}$$

$$\begin{aligned}\text{Entropy at state 2, } s_2 &= s_g \text{ at 7 MPa} \\ s_2 &= 5.8133 \text{ kJ/kg} \cdot \text{K}\end{aligned}$$

Enthalpy and entropy at state 3,

$$\begin{aligned}h_3 &= h_f \text{ at 7 MPa} = 1267 \text{ kJ/kg} \\ s_3 &= s_f \text{ at 7 MPa} = 3.1211 \text{ kJ/kg} \cdot \text{K}\end{aligned}$$

For process 2–1, $s_1 = s_2$. Let dryness fraction at state 1 be x_1 .

$$\begin{aligned}s_1 &= s_2 = s_f \text{ at 7 kPa} + x_1 \cdot s_{fg} \text{ at 7 kPa} \\ 5.8133 &= 0.5564 + x_1 \cdot 7.7237 \\ x_1 &= 0.6806\end{aligned}$$

$$\begin{aligned}\text{Enthalpy of state 1, } h_1 &= h_f \text{ at 7 kPa} + x_1 \cdot h_{fg} \text{ at 7 kPa} \\ &= 162.60 + (0.6806 \times 2409.54) \\ h_1 &= 1802.53 \text{ kJ/kg}\end{aligned}$$

Let dryness fraction at state 4 be x_4 ,

$$\begin{aligned}\text{For process 4–3, } s_4 &= s_3 = s_f \text{ at 7 kPa} + x_4 \cdot s_{fg} \text{ at 7 kPa} \\ 3.1211 &= 0.5564 + x_4 \cdot 7.7237\end{aligned}$$

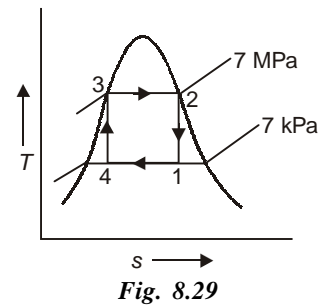


Fig. 8.29

$$x_4 = 0.3321$$

$$\begin{aligned}\text{Enthalpy at state 4, } h_4 &= h_{f \text{ at } 7 \text{ kPa}} + x_4 \cdot h_{fg \text{ at } 7 \text{ kPa}} \\ &= 162.60 + (0.3321 \times 2409.54)\end{aligned}$$

$$h_4 = 962.81 \text{ kJ/kg}$$

$$\text{Thermal efficiency} = \frac{\text{Net work}}{\text{Heat added}}$$

$$\text{Expansion work per kg} = h_2 - h_1 = (2772.1 - 1802.53) = 969.57 \text{ kJ/kg}$$

$$\begin{aligned}\text{Compression work per kg} &= h_3 - h_4 = (1267 - 962.81) \\ &= 304.19 \text{ kJ/kg (+ve)}\end{aligned}$$

$$\begin{aligned}\text{Heat added per kg} &= h_2 - h_3 = (2772.1 - 1267) \\ &= 1505.1 \text{ kJ/kg (-ve)}\end{aligned}$$

$$\begin{aligned}\text{Net work per kg} &= (h_2 - h_1) - (h_3 - h_4) = 969.57 - 304.19 \\ &= 665.38 \text{ kJ/kg}\end{aligned}$$

$$\text{Thermal efficiency} = \frac{665.38}{1505.1} = 0.4421 \text{ or } 44.21\%$$

$$\text{Thermal efficiency} = \mathbf{44.21\%}$$

$$\text{Turbine work} = \mathbf{969.57 \text{ kJ/kg (+ve)}}$$

$$\text{Compression work} = \mathbf{304.19 \text{ kJ/kg (-ve)}}$$

Ans.

2. A steam power plant uses steam as working fluid and operates at a boiler pressure of 5 MPa, dry saturated and a condenser pressure of 5 kPa. Determine the cycle efficiency for (a) Carnot cycle (b) Rankine cycle. Also show the T-s representation for both the cycles.

Solution:

From steam tables:

$$\text{At 5 MPa} \quad h_{f, 5\text{MPa}} = 1154.23 \text{ kJ/kg}, s_{f, 5\text{MPa}} = 2.92 \text{ kJ/kg} \cdot \text{K}$$

$$h_{g, 5\text{MPa}} = 2794.3 \text{ kJ/kg}, s_{g, 5\text{MPa}} = 5.97 \text{ kJ/kg} \cdot \text{K}$$

At 5 kPa

$$h_{f, 5\text{kPa}} = 137.82 \text{ kJ/kg}, s_{f, 5\text{kPa}} = 0.4764 \text{ kJ/kg} \cdot \text{K}$$

$$h_{g, 5\text{kPa}} = 2561.5 \text{ kJ/kg}, s_{g, 5\text{kPa}} = 8.3951 \text{ kJ/kg} \cdot \text{K}$$

$$v_{f, 5\text{kPa}} = 0.001005 \text{ m}^3/\text{kg}$$

As process 2-3 is isentropic, so $s_2 = s_3$

and

$$s_3 = s_{f, 5\text{kPa}} + x_3 \cdot s_{fg, 5\text{kPa}} = s_2 = s_{g, 5\text{MPa}}$$

$$x_3 = 0.694$$

Hence enthalpy at 3,

$$h_3 = h_{f, 5\text{kPa}} + x_3 \cdot h_{fg, 5\text{kPa}}$$

$$h_3 = 1819.85 \text{ kJ/kg}$$

Enthalpy at 2,

$$h_2 = h_{g, 5\text{MPa}} = 2794.3 \text{ kJ/kg}$$

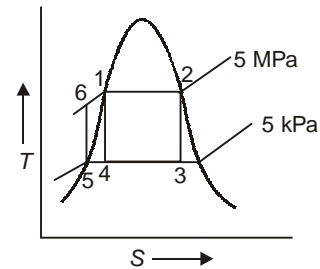


Fig. 8.30

Carnot cycle : 1-2-3-4-1

Rankine cycle : 1-2-3-5-6-1

Process 1-4 is isentropic, so $s_1 = s_4$

$$s_1 = 2.92 = 0.4764 + x_4 \cdot (8.3951 - 0.4764)$$

$$x_4 = 0.308$$

Enthalpy at 4,

$$h_4 = 137.82 + (0.308 \times (2561.5 - 137.82))$$

$$h_4 = 884.3 \text{ kJ/kg}$$

Enthalpy at 1,

$$h_1 = h_f \text{ at } 5 \text{ MPa}$$

$$h_1 = 1154.23 \text{ kJ/kg}$$

Carnot cycle (1-2-3-4-1) efficiency:

$$\begin{aligned} \eta_{\text{carnot}} &= \frac{\text{Net work}}{\text{Heat added}} \\ &= \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)} \\ &= \frac{\{(2794.3 - 1819.85) - (1154.23 - 884.3)\}}{(2794.3 - 1154.23)} \end{aligned}$$

$$\eta_{\text{carnot}} = 0.4295$$

or

$$\eta_{\text{carnot}} = \mathbf{42.95\% \quad Ans.}$$

In Rankine cycle, 1-2-3-5-6-1

$$\text{Pump work, } h_6 - h_5 = v_{f,5}(p_6 - p_5)$$

$$= 0.001005 (5000 - 5)$$

$$h_6 - h_5 = 5.02$$

$$h_5 = h_f \text{ at } 5 \text{ kPa} = 137.82 \text{ kJ/kg}$$

$$\text{Hence } h_6 = 137.82 + 5.02 = 142.84 \text{ kJ/kg}$$

$$h_6 = 142.84 \text{ kJ/kg}$$

$$\text{Net work in Rankine cycle} = (h_2 - h_3) - (h_6 - h_5)$$

$$= 974.45 - 5.02$$

$$= 969.43 \text{ kJ/kg}$$

$$\text{Heat added} = h_2 - h_6$$

$$= 2794.3 - 142.84$$

$$= 2651.46 \text{ kJ/kg}$$

$$\text{Rankine cycle efficiency} = \frac{969.43}{2651.46}$$

$$\eta_{\text{Rankine}} = 0.3656$$

or

$$\eta_{\text{Rankine}} = \mathbf{36.56\% \quad Ans.}$$

3. A steam turbine plant operates on Rankine cycle with steam entering turbine at 40 bar, 350°C and leaving at 0.05 bar. Steam leaving turbine condenses to saturated liquid inside condenser. Feed pump pumps saturated liquid into boiler. Determine the net work per kg of steam and the cycle efficiency assuming all processes to be ideal. Also show cycle on T-s diagram. Also determine pump work per kg of steam considering linear variation of specific volume.

Solution:

From steam table

$$h_2 = h_{\text{at 40 bar, 350°C}} = 3092.5 \text{ kJ/kg}$$

$$s_2 = s_{\text{at 40 bar, 350°C}} = 6.5821 \text{ kJ/kg} \cdot \text{K}$$

$$h_4 = h_f \text{ at 0.05 bar} = 137.82 \text{ kJ/kg}$$

$$s_4 = s_f \text{ at 0.05 bar} = 0.4764 \text{ kJ/kg}$$

$$v_4 = v_f \text{ at 0.05 bar} = 0.001005 \text{ m}^3/\text{kg}$$

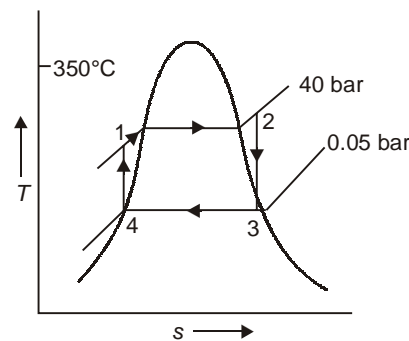


Fig. 8.31

Let dryness fraction at state 3 be x_3 ,

For ideal process, 2-3, $s_2 = s_3$

$$s_2 = s_3 = 6.5821 = s_f \text{ at 0.05 bar} + x_3 \cdot s_{fg} \text{ at 0.05 bar}$$

$$6.5821 = 0.4764 + x_3 \cdot 7.9187$$

$$x_3 = 0.7711$$

$$h_3 = h_f \text{ at 0.05 bar} + x_3 \cdot h_{fg} \text{ at 0.05 bar}$$

$$= 137.82 + (0.7711 \times 2423.7)$$

$$h_3 = 2006.74 \text{ kJ/kg}$$

For pumping process

$$h_1 - h_4 = v_4 \cdot \Delta p = v_4 \times (p_1 - p_4)$$

$$h_1 = h_4 + v_4 \times (p_1 - p_4)$$

$$= 137.82 + (0.001005 \times (40 - 0.05) \times 10^2)$$

$$h_1 = 141.84 \text{ kJ/kg}$$

$$\text{Pump work per kg of steam} = (h_1 - h_4) = 4.02 \text{ kJ/kg}$$

$$\text{Net work per kg of steam} = (\text{Expansion work} - \text{Pump work}) \text{ per kg of steam}$$

$$= (h_2 - h_3) - (h_1 - h_4)$$

$$\begin{aligned}
 &= 1081.74 \text{ kJ/kg} \\
 \text{Cycle efficiency} &= \frac{\text{Net work}}{\text{Heat added}} \\
 &= \frac{1081.74}{(h_2 - h_1)} \\
 &= \frac{1081.74}{(3092.5 - 141.84)} \\
 &= 0.3667 \text{ or } 36.67\%
 \end{aligned}$$

Net work per kg of steam = 1081.74 kJ/kg Cycle efficiency = 36.67% Pump work per kg of steam = 4.02 kJ/kg	Ans.
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4. A steam power plant running on Rankine cycle has steam entering HP turbine at 20 MPa, 500°C and leaving LP turbine at 90% dryness. Considering condenser pressure of 0.005 MPa and reheating occurring upto the temperature of 500°C determine,

- the pressure at which steam leaves HP turbine
- the thermal efficiency

Solution:

Let us assume that the condensate leaves condenser as saturated liquid and the expansion in turbine and pumping processes are isentropic.

From steam tables,

$$\begin{aligned}
 h_2 &= h_{\text{at } 20 \text{ MPa, } 500^\circ\text{C}} = 3238.2 \text{ kJ/kg} \\
 s_2 &= 6.1401 \text{ kJ/kg} \cdot \text{K} \\
 h_5 &= h_{\text{at } 0.005 \text{ MPa, } 0.90 \text{ dry}} \\
 h_5 &= h_{f \text{ at } 0.005 \text{ MPa}} + 0.9 \times h_{fg \text{ at } 0.005 \text{ MPa}} \\
 &= 137.82 + (0.9 \times 2423.7) \\
 h_5 &= 2319.15 \text{ kJ/kg} \\
 s_5 &= s_{f \text{ at } 0.005 \text{ MPa}} + 0.9 \times s_{fg \text{ at } 0.005 \text{ MPa}} \\
 &= 0.4764 + (0.9 \times 7.9187) \\
 s_5 &= 7.6032 \text{ kJ/kg} \cdot \text{K} \\
 h_6 &= h_{f \text{ at } 0.005 \text{ MPa}} = 137.82 \text{ kJ/kg}
 \end{aligned}$$

It is given that temperature at state 4 is 500°C and due to isentropic process $s_4 = s_5 = 7.6032 \text{ kJ/kg} \cdot \text{K}$. The state 4 can be conveniently located on Mollier chart by the intersection of 500°C constant temperature line and entropy value of 7.6032 kJ/kg · K and the pressure and enthalpy obtained. But these shall be approximate.

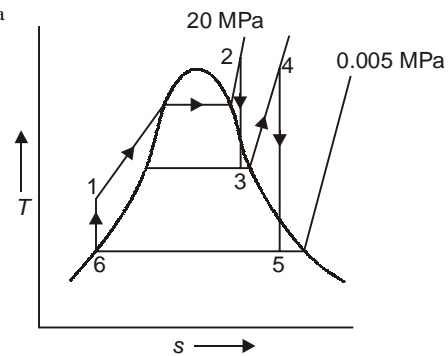


Fig. 8.32

The state 4 can also be located by interpolation using steam table. The entropy value of $7.6032 \text{ kJ/kg} \cdot \text{K}$ lies between the superheated steam states given under, $p = 1.20 \text{ MPa}$, $s_{\text{at } 1.20 \text{ MPa and } 500^\circ\text{C}} = 7.6759 \text{ kJ/kg} \cdot \text{K}$

$$p = 1.40 \text{ MPa}, s_{\text{at } 1.40 \text{ MPa and } 500^\circ\text{C}} = 7.6027 \text{ kJ/kg} \cdot \text{K}$$

By interpolation state 4 lies at pressure

$$\begin{aligned} &= 1.20 + \frac{(1.40 - 1.20)}{(7.6027 - 7.6759)} (7.6032 - 7.6759) \\ &= 1.399 \text{ MPa} \approx 1.40 \text{ MPa} \end{aligned}$$

Thus, steam leaves HP turbine at 1.4 MPa

$$\text{Enthalpy at state 4, } h_4 = 3474.1 \text{ kJ/kg}$$

For process 2-3, $s_2 = s_3 = 6.1401 \text{ kJ/kg} \cdot \text{K}$. The state 3 thus lies in wet region as $s_3 < s_{g \text{ at } 1.40 \text{ MPa}}$. Let dryness fraction at state 3 be x_3 .

$$\begin{aligned} s_3 &= s_{f \text{ at } 1.4 \text{ MPa}} + x_3 \cdot s_{fg \text{ at } 1.4 \text{ MPa}} \\ 6.1401 &= 2.2842 + x_3 \cdot 4.1850 \\ x_3 &= 0.9214 \end{aligned}$$

$$\begin{aligned} h_3 &= h_{f \text{ at } 1.4 \text{ MPa}} + x_3 \cdot h_{fg \text{ at } 1.4 \text{ MPa}} \\ &= 830.3 + (0.9214 \times 1959.7) = 2635.97 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Enthalpy at 1, } h_1 &= h_6 + v_6(p_1 - p_6) \\ &= h_{f \text{ at } 0.005 \text{ MPa}} + v_{f \text{ at } 0.005 \text{ MPa}} (20 - 0.005) \times 10^3 \\ &= 137.82 + (0.001005 \times 19.995 \times 10^3) \\ h_1 &= 157.91 \text{ kJ/kg} \end{aligned}$$

$$\begin{aligned} \text{Net work per kg of steam} &= (h_2 - h_3) + (h_4 - h_5) - (h_1 - h_6) \\ &= 1737.09 \text{ kJ/kg} \end{aligned}$$

$$\text{Heat added per kg of steam} = (h_2 - h_1) = 3080.29 \text{ kJ/kg}$$

$$\text{Thermal efficiency} = \frac{\text{Net work}}{\text{Heat added}} = \frac{1737.09}{3080.29} = 0.5639 \text{ or } 56.39\%$$

Pressure of steam leaving HP turbine = **1.40 MPa** **Ans.**
Thermal efficiency = **56.39%**

5. In a steam turbine installation running on ideal Rankine cycle steam leaves the boiler at 10 MPa and 700°C and leaves turbine at 0.005 MPa. For the 50 MW output of the plant and cooling water entering and leaving condenser at 15°C and 30°C respectively determine

- (a) the mass flow rate of steam in kg/s
- (b) the mass flow rate of condenser cooling water in kg/s

(c) the thermal efficiency of cycle

(d) the ratio of heat supplied and rejected (in boiler and condenser respectively).

Neglect K.E. and P.E. changes.

Solution:

From steam table

At inlet to turbine, $h_2 = h_{\text{at } 10 \text{ MPa, } 700^\circ\text{C}}$

$$h_2 = 3870.5 \text{ kJ/kg}$$

$$s_2 = 7.1687 \text{ kJ/kg} \cdot \text{K}$$

For process 2-3, $s_2 = s_3$ and $s_3 < s_{f \text{ at } 0.005 \text{ MPa}}$ so state 3 lies in wet region. Let dryness fraction at state 3 be x_3 .

$$s_3 = 7.1687 = s_{f \text{ at } 0.005 \text{ MPa}} + x_3 \cdot s_{fg \text{ at } 0.005 \text{ MPa}}$$

$$7.1687 = 0.4764 + (x_3 \times 7.9187)$$

$$x_3 = 0.845$$

$$\begin{aligned} h_3 &= h_{f \text{ at } 0.005 \text{ MPa}} + x_3 \cdot h_{fg \text{ at } 0.005 \text{ MPa}} \\ &= 137.82 + (0.845 \times 2423.7) \end{aligned}$$

$$h_3 = 2185.85 \text{ kJ/kg}$$

$$h_4 = h_{f \text{ at } 0.005 \text{ MPa}} = 137.82 \text{ kJ/kg}$$

For pumping process, $(h_1 - h_4) = v_4 \times (p_1 - p_4)$

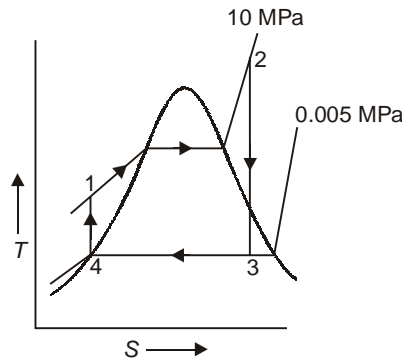


Fig. 8.33

$$v_4 = v_{f \text{ at } 0.005 \text{ MPa}} = 0.001005 \text{ m}^3/\text{kg}$$

$$h_1 = 137.82 + (0.001005 \times (10 - 0.005)) \times 10^2$$

$$h_1 = 138.82 \text{ kJ/kg}$$

Net output per kg of steam,

$$\begin{aligned} w_{\text{net}} &= (h_2 - h_3) - (h_1 - h_4) \\ &= (3870.5 - 2185.85) - (138.82 - 137.82) \end{aligned}$$

$$w_{\text{net}} = 1683.65 \text{ kJ/kg}$$

$$\text{Mass flow rate of steam, } m_s = \frac{50 \times 10^3}{1683.65} = 29.69 \text{ kg/s}$$

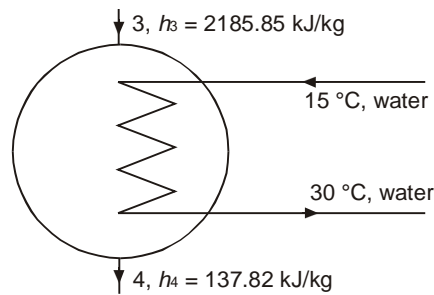


Fig. 8.34

By heat balance on condenser, for mass flow rate of water being m_w kg/s.

$$(h_3 - h_4) \times m_s = m_w \cdot C_{p, w} (T_{w, out} - T_{w, in})$$

$$29.69 \times (2185.85 - 137.82) = m_w \times 4.18 \quad (15)$$

$$m_w = 969.79 \text{ kg/s}$$

The heat added per kg of steam $q_{add} = (h_2 - h_1) = 3731.68 \text{ kJ/kg}$

$$\text{Thermal efficiency} = \frac{w_{net}}{q_{add}} = \frac{1683.65}{3731.68} = 0.4512 \text{ or } 45.12\%$$

$$\text{Ratio of heat supplied and rejected} = \frac{(h_2 - h_1)}{(h_3 - h_4)} = 1.822$$

Mass of flow rate of steam = **29.69 kg/s**
 Mass flow rate of condenser cooling water = **969.79 kg/s**
 Thermal efficiency = **45.12%**
 Ratio of heat supplied and rejected = **1.822**

Ans.

6. A regenerative Rankine cycle has steam entering turbine at 200 bar, 650°C and leaving at 0.05 bar. Considering feed water heaters to be of open type determine thermal efficiency for the following conditions;

- (a) there is no feed water heater
- (b) there is only one feed water heater working at 8 bar
- (c) there are two feed water heaters working at 40 bar and 4 bar respectively.

Also give layout and T-s representation for each of the case described above.

Solution:

Case (a) When there is no feed water heater

$$\text{Thermal efficiency of cycle} = \frac{(h_2 - h_3) - (h_1 - h_4)}{(h_2 - h_1)}$$

From steam tables,

$$h_2 = h_{\text{at } 200 \text{ bar, } 650^\circ\text{C}} = 3675.3 \text{ kJ/kg}$$

$$s_2 = s_{\text{at } 200 \text{ bar, } 650^\circ\text{C}} = 6.6582 \text{ kJ/kg} \cdot \text{K}$$

$$h_4 = h_{f \text{ at } 0.05 \text{ bar}} = 137.82 \text{ kJ/kg}$$

$$v_4 = v_{f \text{ at } 0.05 \text{ bar}} = 0.001005 \text{ m}^3/\text{kg}$$

$$h_{f \text{ at } 0.05 \text{ bar}} = 137.82 \text{ kJ/kg}, h_{fg \text{ at } 0.05 \text{ bar}} = 2423.7 \text{ kJ/kg}$$

$$s_{f \text{ at } 0.05 \text{ bar}} = 0.4764 \text{ kJ/kg} \cdot \text{K}, s_{fg \text{ at } 0.05 \text{ bar}} = 7.9187 \text{ kJ/kg} \cdot \text{K}$$

For process 2–3, $s_2 = s_3$. Let dryness fraction at 3 be x_3 .

$$s_3 = 6.6582 = s_{f \text{ at } 0.05 \text{ bar}} + x_3 \cdot s_{fg \text{ at } 0.05 \text{ bar}}$$

$$6.6582 = 0.4764 + x_3 \cdot 7.9187$$

$$x_3 = 0.781$$

$$\begin{aligned} h_3 &= h_{f \text{ at } 0.05 \text{ bar}} + x_3 \cdot h_{fg \text{ at } 0.05 \text{ bar}} \\ &= 2030.73 \text{ kJ/kg} \end{aligned}$$

For pumping process 4–1,

$$h_1 - h_4 = v_4 \cdot \Delta p$$

$$h_1 - 137.82 = 0.001005 \times (200 - 0.05) \times 10^2$$

$$h_1 = 157.92 \text{ kJ/kg}$$

$$\text{Thermal efficiency of cycle} = \frac{(3675.3 - 2030.73) - (157.92 - 137.82)}{(3675.3 - 157.92)}$$

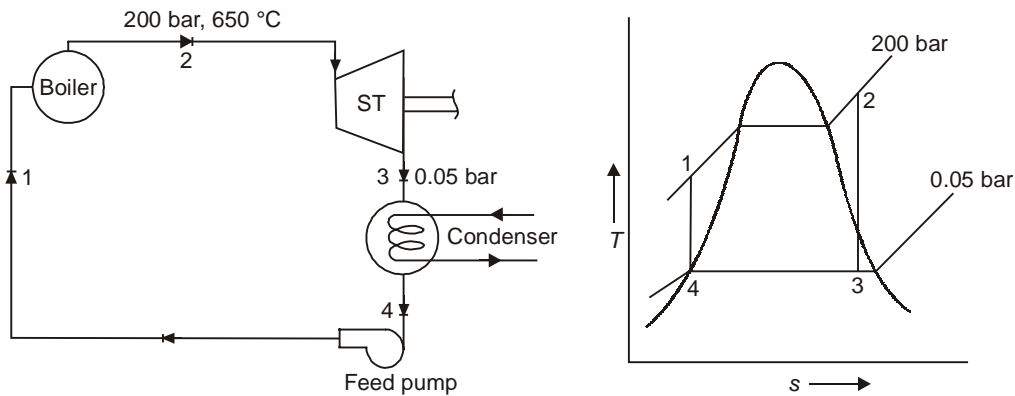


Fig. 8.35 Layout and T-s diagram, (Q 6.a)

$$= 0.4618 \text{ or } 46.18\%$$

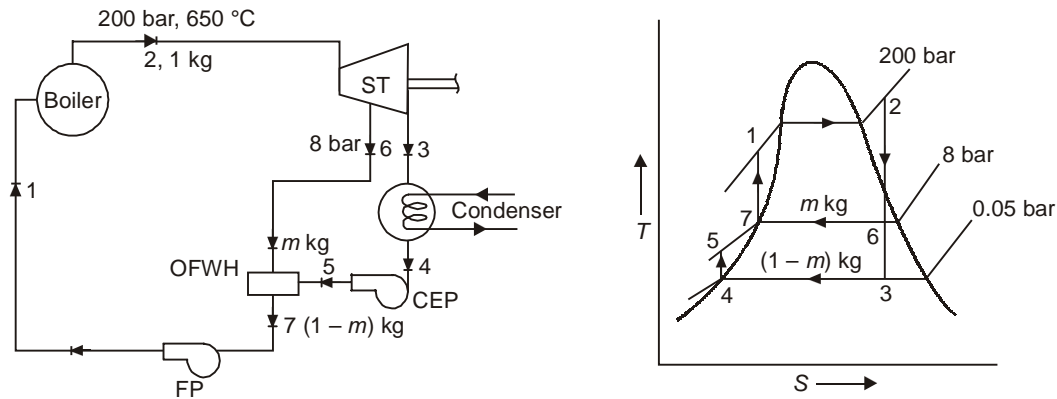
Case (b) When there is only one feed water heater working at 8 bar

Here, let mass of steam bled for feed heating be m kg

For process 2–6, $s_2 = s_6 = 6.6582 \text{ kJ/kg} \cdot \text{K}$

Let dryness fraction at state 6 be x_6

$$s_6 = s_{f \text{ at } 8 \text{ bar}} + x_6 \cdot s_{fg \text{ at } 8 \text{ bar}}$$



CEP = Condensate Extraction Pump
 FP = Feed Pump
 ST = Steam Turbine
 OFWH = Open Feed Water Heater

Fig. 8.36 Layout and T - s diagram, (Q 6,b)

From steam tables, $h_{f \text{ at 8 bar}} = 721.11 \text{ kJ/kg}$

$v_{f \text{ at 8 bar}} = 0.001115 \text{ m}^3/\text{kg}$, $h_{fg \text{ at 8 bar}} = 2048 \text{ kJ/kg}$

$s_{f \text{ at 8 bar}} = 2.0462 \text{ kJ/kg} \cdot \text{K}$, $s_{fg \text{ at bar}} = 4.6166 \text{ kJ/kg} \cdot \text{K}$

Substituting entropy values, $x_6 = 0.999$

$h_6 = h_{f \text{ at 8 bar}} + x_6 \cdot h_{fg \text{ at 8 bar}} = 721.11 + (0.999 \times 2048) = 2767.06 \text{ kJ/kg}$

Assuming the state of fluid leaving open feed water heater to be saturated liquid at 8 bar.

$h_7 = h_{f \text{ at 8 bar}} = 721.11 \text{ kJ/kg}$.

For process 4-5, $h_5 = h_4 + v_4 \times (8 - 0.05) \times 10^2 = 137.82 + (0.001005 \times 7.95 \times 10^2) = 138.62 \text{ kJ/kg}$

Applying energy balance at open feed water heater,

$$m \times h_6 + (1 - m) \times h_5 = 1 \times h_7$$

$$(m \times 2767.06) + ((1 - m) \times 138.62) = 721.11$$

$$m = 0.2216 \text{ kg}$$

For process 7-1, $h_1 = h_7 + v_7 (200 - 8) \times 10^2$; here $h_7 = h_{f \text{ at 8 bar}}$, $v_7 = v_{f \text{ at 8 bar}}$

$$h_1 = h_7 + v_{f \text{ at 8 bar}} (200 - 8) \times 10^2 = 721.11 + (0.001115 \times 192 \times 10^2)$$

$$h_1 = 742.518 \text{ kJ/kg}$$

$$\text{Thermal efficiency of cycle} = \frac{(h_2 - h_6) + (1 - m) \cdot (h_6 - h_3) - \{(1 - m)(h_5 - h_4) + (h_1 - h_7)\}}{(h_2 - h_1)}$$

$$= \frac{(3675.3 - 2767.06) + (1 - 0.2216) \times (2767.06 - 2030.73) - \{(1 - 0.2216) \times (138.62 - 137.82) + (742.518 - 721.11)\}}{(3675.3 - 742.518)}$$

Thermal efficiency of cycle = 0.4976 or 49.76%

Case (c) When there are two feed water heaters working at 40 bar and 4 bar

Here, let us assume the mass of steam at 40 bar, 4 bar to be m_1 kg, and m_2 kg respectively.

For process 2–10–9–3, $s_2 = s_{10} = s_9 = s_3 = 6.6582$ kJ/kg · K

At state 10. $s_{10} > s_g$ at 40 bar (6.0701 kJ/kg · K) so state 10 lies in superheated region at 40 bar pressure. From steam table by interpolation, $T_{10} = 370.36^\circ\text{C}$ so, $h_{10} = 3141.81$ kJ/kg

Let dryness fraction at state 9 be x_9 so,

$$s_9 = 6.6582 = s_{f \text{ at 4 bar}} + x_9 \cdot s_{fg \text{ at 4 bar}}$$

$$6.6582 = 1.7766 + x_9 \times 5.1193$$

$$x_9 = 0.9536$$

$$h_9 = h_{f \text{ at 4 bar}} + x_9 \times h_{fg \text{ at 4 bar}} = 604.74 + 0.9536 \times 2133.8$$

$$h_9 = 2639.53 \text{ kJ/kg}$$

Assuming the state of fluid leaving open feed water heater to be saturated liquid at respective pressures i.e.

$$h_{11} = h_{f \text{ at 4 bar}} = 604.74 \text{ kJ/kg}, v_{11} = 0.001084 \text{ m}^3/\text{kg} = v_{f \text{ at 4 bar}}$$

$$h_{13} = h_{f \text{ at 40 bar}} = 1087.31 \text{ kJ/kg}, v_{13} = 0.001252 \text{ m}^3/\text{kg} = v_{f \text{ at 40 bar}}$$

For process 4–8, i.e. in CEP.

$$h_8 = h_4 + v_4 \times (4 - 0.05) \times 10^2$$

$$= 137.82 + (0.001005 \times 3.95 \times 10^2)$$

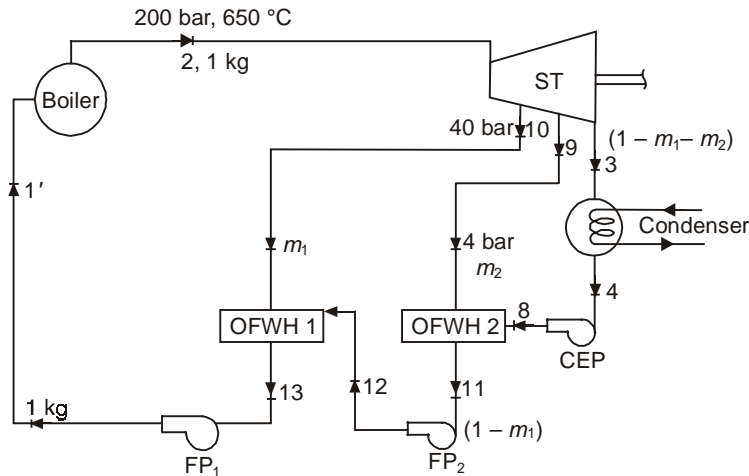
$$h_8 = 138.22 \text{ kJ/kg}$$

For process 11–12, i.e. in FP_2 ,

$$h_{12} = h_{11} + v_{11}(40 - 4) \times 10^2$$

$$= 604.74 + (0.001084 \times 36 \times 10^2)$$

$$h_{12} = 608.64 \text{ kJ/kg}$$



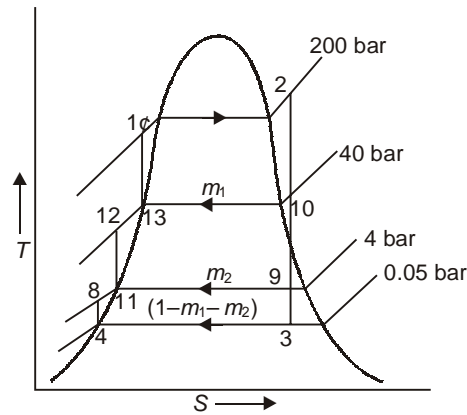


Fig. 8.37 Layout and T-s diagram. (Q6.c)

For process 13-1' i.e. in FP₁, $h'_1 = h_{13} + v_{13}(200 - 40) \times 10^2$
 $= 1087.31 + (0.001252 \times 160 \times 10^2)$
 $h'_1 = 1107.34 \text{ kJ/kg}$
 $(m_1 \times 3141.81) + (1 - m_1) \times 608.64 = 1087.31$
 $m_1 = 0.189 \text{ kg}$

Applying energy balance an open feed water heater 1 (OFWH1)

$$(m_1 \times h_{10}) + (1 - m_1) \times h_{12} = 1 \times h_{13}$$

$$(m_1 \times 3141.81) + (1 - m_1) \times 608.64 = 1087.31$$

$$m_1 = 0.189 \text{ kg}$$

Applying energy balance an open feed water heater 2 (OFWH2)

$$m_2 \times h_9 + (1 - m_1 - m_2) h_8 = (1 - m_1) \times h_{11}$$

$$(m_2 \times 2639.53) + (1 - 0.189 - m_2) \times 138.22 = (1 - 0.189) \times 604.74, m_2 = 0.151 \text{ kg}$$

Thermal efficiency of cycle,

$$\eta = \frac{\{(h_2 - h_{10}) + (1 - m_1)(h_{10} - h_9) + (1 - m_1 - m_2)(h_9 - h_3)\} - \{W_{\text{CEP}} + W_{\text{FP}_1} + W_{\text{FP}_2}\}}{(h_2 - h_{1'})}$$

$$W_{\text{CEP}} = (1 - m_1 - m_2) (h_8 - h_4) = 0.264 \text{ kJ/kg steam from boiler}$$

$$W_{\text{FP}_1} = (h_{1'} - h_{13}) = 20.03 \text{ kJ/kg of steam from boiler}$$

$$W_{\text{FP}_2} = (1 - m_1) (h_{12} - h_{11}) = 3.16 \text{ kJ/kg of steam from boiler}$$

$$W_{\text{CEP}} + W_{\text{FP}_1} + W_{\text{FP}_2} = 23.454 \text{ kJ/kg of steam from boiler}$$

$$\eta = \frac{\{(3675.3 - 3141.81) + (1 - 0.189)(3141.81 - 2639.53) + (1 - 0.189 - 0.151)(2639.53 - 2030.73)\} - \{23.454\}}{(3675.3 - 1107.34)}$$

$$= 0.5137 \text{ or } 51.37\%$$

<p>Cycle thermal efficiency $\eta_a = 46.18\%$ $\eta_b = 49.76\%$ $\eta_c = 51.37\%$</p>	Ans.
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Hence it is obvious that efficiency increases with increase in number of feed heaters.

7. A reheat cycle has steam generated at 50 bar, 500°C for being sent to high pressure turbine and expanded upto 5 bar before supplied to low pressure turbine. Steam enters at 5 bar, 400°C into low pressure turbine after being reheated in boiler. Steam finally enters condenser at 0.05 bar and subsequently feed water is sent to boiler. Determine cycle efficiency, specific steam consumption and work ratio.

Solution:

From steam table,

$$h_2 = h_{\text{at 50 bar, 500°C}} = 3433.8 \text{ kJ/kg}$$

$$s_2 = s_{\text{at 50 bar, 500°C}} = 6.9759 \text{ kJ/kg} \cdot \text{K}$$

$$s_3 = s_2 = 6.9759 \text{ kJ/kg} \cdot \text{K}$$

Since $s_3 > s_g$ at 5 bar so state 3 lies in superheated region at 5 bar,

By interpolation from steam tables,

$$T_3 = 183.14^\circ\text{C at 5 bar, } h_3 = 2818.03 \text{ kJ/kg}$$

$$h_4 = h_{\text{at 5 bar, 400°C}} = 3271.9 \text{ kJ/kg}$$

$$s_4 = s_{\text{at 5 bar, 400°C}} = 7.7938 \text{ kJ/kg} \cdot \text{K}$$

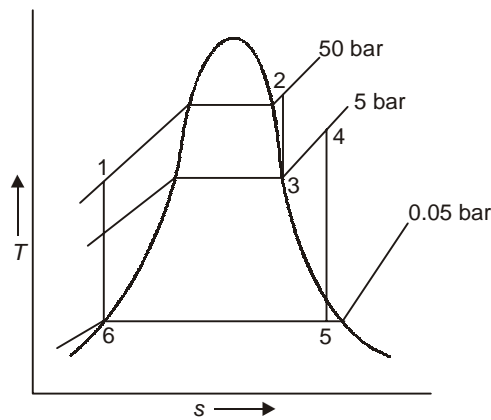


Fig. 8.38

For expansion process 4-5, $s_4 = s_5 = 7.7938 \text{ kJ/kg} \cdot \text{K}$

Let dryness fraction at state 5 be x_5 .

$$s_5 = s_f \text{ at 0.05 bar} + x_5 \times s_{fg} \text{ at 0.05 bar}$$

$$7.7938 = 0.4764 + x_5 \times 7.9187$$

$$x_5 = 0.924$$

$$h_5 = h_f \text{ at 0.05 bar} + x_5 \times h_{fg} \text{ at 0.05 bar}$$

$$h_5 = 137.82 + 0.924 \times 2423.7 = 2377.32 \text{ kJ/kg}$$

$$h_6 = h_f \text{ at 0.05 bar} = 137.82 \text{ kJ/kg}$$

$$h_6 = v_f \text{ at 0.05 bar} = 0.001005 \text{ m}^3/\text{kg}$$

For process 6-1 in feed pump, $h_1 = h_6 + v_6 \times (50 - 0.05) \times 10^2$

$$h_1 = 137.82 + 0.001005 \times (49.95 \times 10^2)$$

$$h_1 = 142.84 \text{ kJ/kg}$$

$$\begin{aligned}\text{Cycle efficiency} &= \frac{W_{\text{net}}}{Q_{\text{add}}} \\ W_T &= (h_2 - h_3) + (h_4 - h_5) \\ &= (3433.8 - 2818.03) + (3271.9 - 2377.32) \\ &= 1510.35 \text{ kJ/kg} \\ W_{\text{pump}} &= (h_1 - h_6) = 142.84 - 137.82 = 5.02 \text{ kJ/kg} \\ W_{\text{net}} &= W_T - W_{\text{pump}} = 1505.33 \text{ kJ/kg} \\ Q_{\text{add}} &= (h_2 - h_1) = 3433.8 - 142.84 = 3290.96 \text{ kJ/kg} \\ \text{Cycle efficiency} &= \frac{1505.33}{3290.96} = 0.4574 \text{ or } 45.74\%\end{aligned}$$

We know, 1 hp = 0.7457 kW

$$\text{Specific steam consumption} = \frac{0.7457 \times 3600}{1505.33} = 1.78 \text{ kg/hp} \cdot \text{hr}$$

$$\text{Work ratio} = \frac{\text{Net work}}{\text{Positive work}} = \frac{W_{\text{net}}}{W_T} = \frac{1505.33}{1510.35} = 0.9967$$

Cycle efficiency = **45.74%**, Specific steam consumption = **1.78 kg/hp. hr.**
Work ratio = **0.9967.**

Ans.

8. In a steam power plant the high pressure turbine is fed with steam at 60 bar, 450°C and enters low pressure turbine at 3 bar with a portion of steam bled out for feed heating at this intermediate pressure. Steam finally leaves low pressure turbine at 0.05 bar for inlet to condenser. Closed feed heater raises the condensate temperature to 115°C. Bled steam leaving closed feed heater is passed through trap to mix with condensate leaving condenser. Consider actual alternator output to be 30 MW, boiler efficiency as 90% and alternator efficiency of 98%. Determine,

- (a) the mass of steam bled for feed heating,
- (b) the capacity of boiler in kg/hr.
- (c) the overall thermal efficiency of plant

Also give layout and T-s diagram.

Solution:

From steam tables,

At state 2, $h_2 = 3301.8 \text{ kJ/kg}$, $s_2 = 6.7193 \text{ kJ/kg} \cdot \text{K}$

$h_5 = h_{f \text{ at } 0.05 \text{ bar}} = 137.82 \text{ kJ/kg}$, $v_5 = v_{f \text{ at } 0.05 \text{ bar}} = 0.001005 \text{ m}^3/\text{kg}$

Let mass of steam bled for feed heating be $m \text{ kg/kg}$ of steam generated in boiler. Let us also assume that condensate leaves closed feed water heater as saturated liquid i.e.

$h_8 = h_{f \text{ at } 3 \text{ bar}}$
 $h_8 = 561.47 \text{ kJ/kg}$

For process 2–3–4, $s_2 = s_3 = s_4 = 6.7193 \text{ kJ/kg} \cdot \text{K}$

Let dryness fraction at state 3 and state 4 be x_3 and x_4 respectively.

$$s_3 = 6.7193 = s_{f \text{ at } 3 \text{ bar}} + x_3 \cdot s_{fg \text{ at } 3 \text{ bar}}$$

$$\begin{aligned}
 &= 1.6718 + x_3 \times 5.3201 \\
 x_3 &= 0.949 \\
 s_4 &= 6.7193 = s_{f \text{ at } 0.05 \text{ bar}} + x_4 \cdot s_{fg \text{ at } 0.05 \text{ bar}} \\
 &= 0.4764 + x_4 \times 7.9187 \\
 x_4 &= 0.788
 \end{aligned}$$

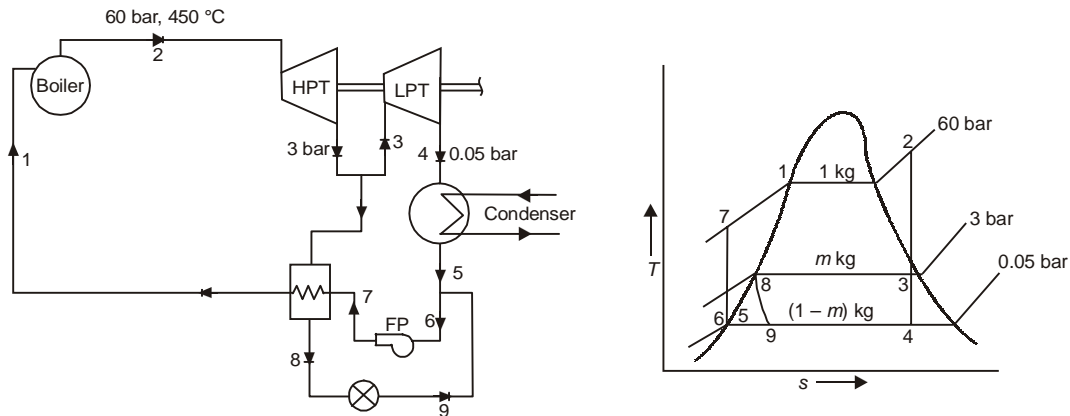


Fig. 8.39 Layout and T-s diagram

Thus,

$$\begin{aligned}
 h_3 &= h_{f \text{ at } 3 \text{ bar}} + x_3 \cdot h_{fg \text{ at } 3 \text{ bar}} = 561.47 + (0.949 \times 2163.8) = 2614.92 \text{ kJ/kg} \\
 h_4 &= h_{f \text{ at } 0.05 \text{ bar}} + x_4 \cdot h_{fg \text{ at } 0.05 \text{ bar}} = 137.82 + (0.788 \times 2423.7) = 2047.6 \text{ kJ/kg}
 \end{aligned}$$

Assuming process across trap to be of throttling type so, $h_8 = h_9 = 561.47 \text{ kJ/kg}$. Assuming $v_5 = v_6$,

$$\text{Pumping work } (h_7 - h_6) = v_5 \cdot (60 - 0.05) \times 10^2$$

$$(h_7 - h_6) = 6.02 \text{ kJ/kg}$$

For mixing process between condenser and feed pump,

$$(1 - m) \cdot h_5 + m \cdot h_9 = 1 \cdot h_6$$

$$(1 - m) \times 137.82 + m \times 561.47 = h_6$$

$$h_6 = 137.82 + m \times 423.65$$

Therefore,

$$h_7 = h_6 + 6.02 = 143.84 + m \times 423.65$$

Applying energy balance at closed feed water heater;

$$m \times h_3 + (1 - m)h_7 = m \cdot h_8 + (4.18 \times 115)$$

$$(m \times 2614.92) + (1 - m) (143.84 + m \times 423.65) = m \times 561.47 + 480.7, m = 0.144 \text{ kg}$$

Steam bled for feed heating = **0.144 kg/kg steam generated.** **Ans.**

The net power output

$$\begin{aligned}
 W_{\text{net}} &= (h_2 - h_3) + (1 - m) (h_3 - h_4) - (1 - m) \times (h_7 - h_6) \\
 &= 1167.28 \text{ kJ/kg steam generated}
 \end{aligned}$$

$$\text{Mass of steam required to be generated} = \frac{30 \times 10^3}{0.98 \times W_{\text{net}}}$$

$$= \frac{30 \times 10^3}{0.98 \times 1167.28} = 26.23 \text{ kg/s}$$

$$= 94428 \text{ kg/hr}$$

Capacity of boiler required = **94428 kg/hr** **Ans.**

$$\text{Overall thermal efficiency} = \frac{W_{\text{net}}}{Q_{\text{add}}}$$

$$Q_{\text{add}} = \frac{(h_2 - h_1)}{0.90} = \frac{(3301.8 - 4.18 \times 115)}{0.90}$$

$$= 3134.56 \text{ kJ/kg}$$

$$\text{Overall thermal efficiency} = \frac{1167.28}{3134.56} = 0.3724 \text{ or } 37.24\%$$

Overall thermal efficiency = **37.24%** **Ans.**

9. A steam power plant has expansion occurring stages in three stages with steam entering first stage at 30 bar, 400°C and leaving first stage at 6 bar for being sent to second stage with some steam being bled out for feed heating in closed feed water heater. Steam leaves second stage at 1 bar and enters third stage with some more steam being bled out for feed heating in closed feed water heater. Steam finally leaves third stage at 0.075 bar after complete expansion and enters condenser. Condensate temperature is 38°C at inlet to second heater, temperature of feed water after first heater and second heater is 150°C and 95°C respectively. Bled steam is condensed to saturated liquid with no undercooling in each of feed heater. Drain from first heater is passed through steam trap into second feed heater and combined drain from second heater is pumped by drain pump into feed line after second heater. Considering efficiency ratio of turbine as 0.8 and turbine output of 15 MW determine the capacity of drain pump. Neglect drain pump work.

Solution:

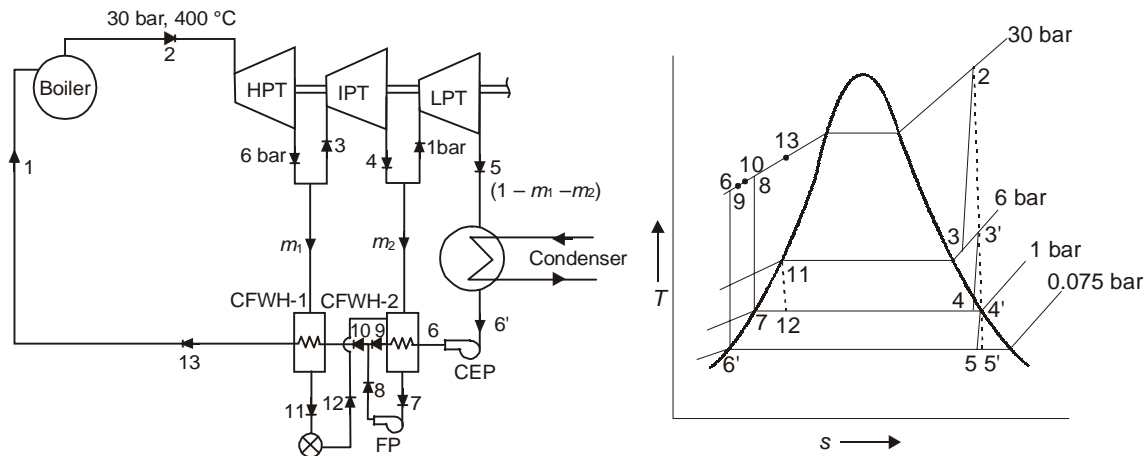


Fig. 8.40 Layout and T-s diagram

At inlet to first turbine stage, $h_2 = 3230.9$ kJ/kg, $s_2 = 6.9212$ kJ/kg · K

For ideal expansion process $s_2 = s_3$

By interpolation, $T_3 = 190.97^\circ\text{C}$ from superheated steam tables at 6 bar

$$h_3 = 2829.63 \text{ kJ/kg}$$

actual state at exit of first stage $h'_3 = h_2 - 0.8 \times (h_2 - h_3)$

$$h'_3 = 2909.88 \text{ kJ/kg}$$

Actual state 3' shall be at 232.78°C , 6 bar, so $s'_3 = 7.1075$ kJ/kg · K

For second stage $s'_3 = s_4$; By interpolation, $s_4 = 7.1075$

$$= s_{f \text{ at 1 bar}} + x_4 \cdot s_{fg \text{ at 1 bar}}$$

$$7.1075 = 1.3026 + x_4 \cdot 6.0568$$

$$x_4 = 0.958$$

$$h_4 = h_{f \text{ at 1 bar}} + x_4 \cdot h_{fg \text{ at 1 bar}}$$

$$= 417.46 + (0.958 \times 2258)$$

$$h_4 = 2580.62 \text{ kJ/kg}$$

Actual enthalpy at exit from second stage, $h_{4'} = h_4 - 0.8 (h_4 - h_5)$

$$h_{4'} = 2646.47 \text{ kJ/kg}$$

Actual dryness fraction, $x_{4'} \Rightarrow h_{4'} = h_{f \text{ at 1 bar}} + x_{4'} \cdot h_{fg \text{ at 1 bar}}$

$x_{4'} = 0.987$, Actual entropy, $s_{4'} = 7.2806$ kJ/kg · K

For third stage, $s_{4'} = s_5 = 7.2806 = s_{f \text{ at 0.075 bar}} + x_5 \cdot s_{fg \text{ at 0.075 bar}}$

$$x_5 = 0.8735$$

$$h_5 = 2270.43 \text{ kJ/kg}$$

Actual enthalpy at exit from third stage, $h_{5'} = h_{4'} - 0.8(h_{4'} - h_5)$

$$h_{5'} = 2345.64 \text{ kJ/kg}$$

Let mass of steam bled out be m_1 and m_2 kg at 6 bar, 1 bar respectively.

By heat balance on first closed feed water heater, (see schematic arrangement)

$$h_{11} = h_{f \text{ at 6 bar}} = 670.56 \text{ kJ/kg}$$

$$m_1 \times h_{3'} + h_{10} = m_1 \cdot h_{11} + 4.18 \times 150$$

$$(m_1 \times 2829.63) + h_{10} = (m_1 \cdot 670.56) + 627$$

$$h_{10} + (2159.07) m_1 = 627$$

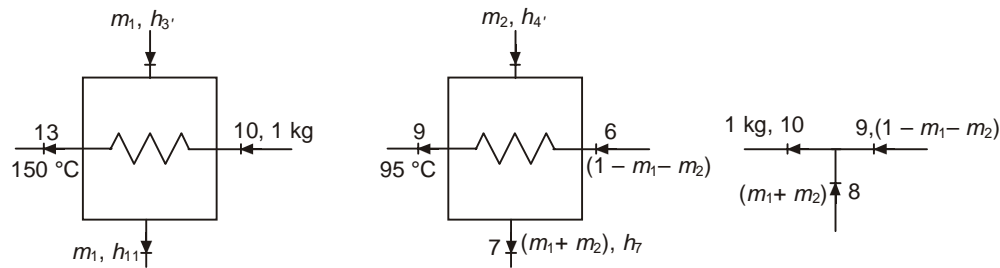
By heat balance on second closed feed water heater, (see schematic arrangement)

$$h_7 = h_{f \text{ at 1 bar}} = 417.46 \text{ kJ/kg}$$

$$m_2 \cdot h_{4'} + (1 - m_1 - m_2) \times 4.18 \times 38 = (m_1 + m_2) \cdot h_7 + 4.18 \times 95 \times (1 - m_1 - m_2)$$

$$(m_2 \cdot 2646.47) + (1 - m_1 - m_2) \times 158.84 = ((m_1 + m_2) \cdot 417.46) + (397.1 \times (1 - m_1 - m_2))$$

$$m_2 \times 2467.27 - m_1 \times 179.2 - 238.26 = 0$$

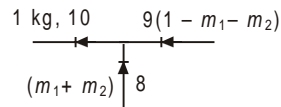


Heat balance at point of mixing.

$$h_{10} = (m_1 + m_2) \cdot h_8 + (1 - m_1 - m_2) \times 4.18 \times 95$$

Neglecting pump work, $h_7 = h_8$

$$h_{10} = m_2 \times 417.46 + (1 - m_1 - m_2) \times 397.1$$



Substituting h_{10} and solving we get, $m_1 = 0.1293$ kg

$$m_2 = 0.1059 \text{ kg/kg of steam generated.}$$

Turbine output per kg of steam generated,

$$w_T = (h_2 - h_3) + (1 - m_1) (h_3' - h_4) + (1 - m_1 - m_2) \cdot (h_4' - h_5')$$

$$w_T = 780.446 \text{ kJ/kg of steam generated.}$$

$$\text{Rate of steam generation required} = \frac{15 \times 10^3}{780.446} = 19.22 \text{ kg/s}$$

$$\text{or} \quad = 69192 \text{ kg/hr}$$

$$\begin{aligned} \text{Capacity of drain pump i.e. FP shown in layout} &= (m_1 + m_2) \times 69192 \\ &= 16273.96 \text{ kg/hr} \end{aligned}$$

$$\boxed{\text{Capacity of drain pump} = 16273.96 \text{ kg/hr}} \quad \text{Ans.}$$

10. A steam power plant has steam entering at 70 bar, 450°C into HP turbine. Steam is extracted at 30 bar and reheated upto 400°C before being expanded in LP turbine upto 0.075 bar. Some portion of steam is bled out during expansion in LP turbine so as to yield saturated liquid at 140°C at the exit of open feed water heater. Considering HP and LP turbine efficiencies of 80% and 85% determine the cycle efficiency. Also give layout and T-s diagram.

Solution:

At inlet to HP turbine, $h_2 = 3287.1$ kJ/kg, $s_2 = 6.6327$ kJ/kg.K

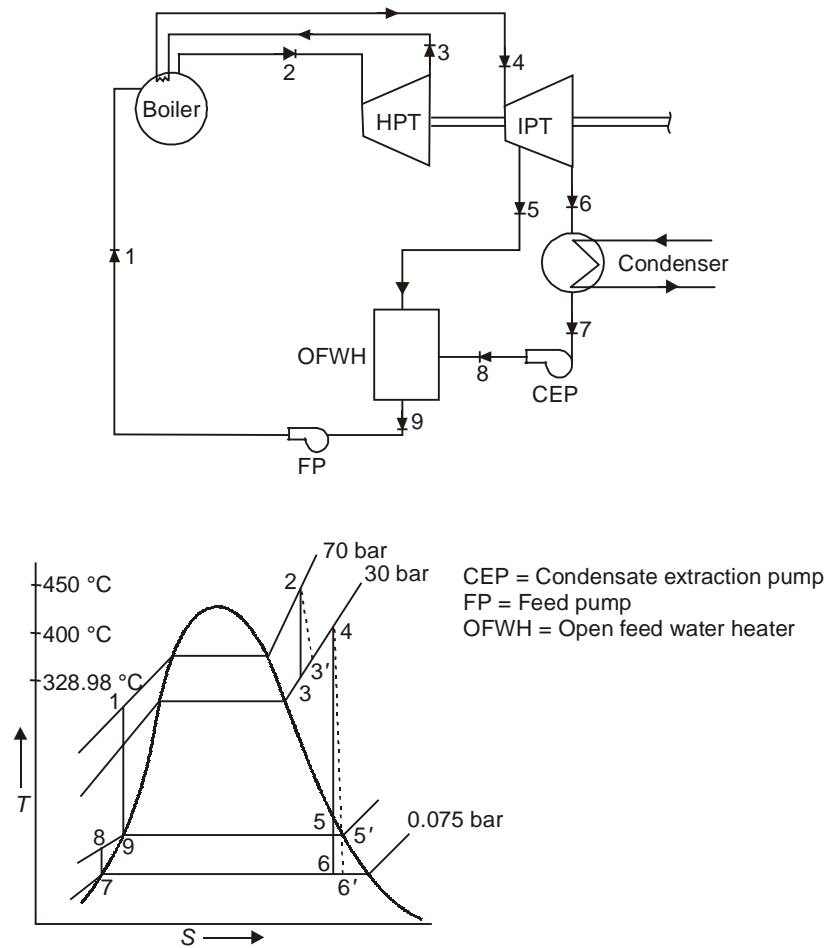


Fig. 8.41 Layout and T-S diagram

By interpolation state 3 i.e. for isentropic expansion between 2 – 3 lies at 328.98°C at 30 bar.

$$h_3 = 3049.48 \text{ kJ/kg.}$$

$$\text{Actual enthalpy at } 3', h_{3'} = h_2 - 0.80 (h_2 - h_3)$$

$$h_{3'} = 3097 \text{ kJ/kg}$$

$$\text{Enthalpy at inlet to LP turbine, } h_4 = 3230.9 \text{ kJ/kg, } s_4 = 6.9212 \text{ kJ} \cdot \text{K}$$

For ideal expansion from 4-6, $s_4 = s_6$. Let dryness fraction at state 6 be x_6 .

$$s_6 = 6.9212 = s_f \text{ at } 0.075 \text{ bar} + x_6 \cdot s_{fg} \text{ at } 0.075 \text{ bar}$$

$$x_6 = 0.827$$

$$h_6 = h_f \text{ at } 0.075 \text{ bar} + x_6 \cdot h_{fg} \text{ at } 0.075 \text{ bar} = 2158.55 \text{ kJ/kg}$$

For actual expansion process in LP turbine.

$$h_{6'} = h_4 - 0.85 (h_4 - h_6)$$

$$h_{6'} = 2319.4 \text{ kJ/kg}$$

Ideally, enthalpy at bleed point can be obtained by locating state 5 using $s_5 = s_4$. The pressure at bleed point shall be saturation pressure corresponding to the 140°C i.e. from steam table $p_5 = 3.61$ bar. Let dryness fraction at state 5 be x_5 .

$$s_5 = 6.9212 = s_f \text{ at } 140^\circ\text{C} + x_5 \cdot s_{fg} \text{ at } 140^\circ\text{C} \Rightarrow x_5 = 0.99$$

$$h_5 = h_f \text{ at } 140^\circ\text{C} + x_5 \cdot h_{fg} \text{ at } 140^\circ\text{C} \Rightarrow h_5 = 2712.38 \text{ kJ/kg}$$

$$\text{Actual exthalpy } h_{5'} = h_4 - 0.85 (h_4 - h_5) = 2790.16 \text{ kJ/kg}$$

$$\text{Enthalpy at exit of open feed water heater, } h_9 = h_f \text{ at } 30 \text{ bar} = 1008.42 \text{ kJ/kg}$$

$$\text{Specific volume at inlet of CEP, } v_7 = 0.001008 \text{ m}^3/\text{kg},$$

$$\text{Enthalpy at inlet of CEP, } h_7 = 168.79 \text{ kJ/kg}$$

For pumping process 7-8

$$h_8 = h_7 + v_7 (3.61 - 0.075) \times 10^2$$

$$h_8 = 169.15 \text{ kJ/kg}$$

Applying energy balance at open feed water heater. Let mass of bled steam be m kg per kg of steam generated.

$$m \times h_5 + (1 - m) \cdot h_8 = h_9$$

$$(m \times 2790.16) + ((1 - m) \cdot 169.15) = 1008.42$$

$$m = 0.32 \text{ kg/kg of steam generated}$$

For process on feed pump, 9 – 1, $v_9 = v_f \text{ at } 140^\circ\text{C} = 0.00108$

$$h_1 = h_9 + v_9 \times (70 - 3.61) \times 10^2$$

$$h_1 = 1015.59 \text{ kJ/kg}$$

Net work per kg of steam generated,

$$\begin{aligned} W_{\text{net}} &= (h_2 - h_3) + (h_4 - h_5') + (1 - m) \cdot (h_5' - h_6') \\ &\quad - \{(1 - m) (h_8 - h_7) + (h_1 - h_9)\} \\ &= 181.1 + 440.74 + 320.117 - \{0.2448 + 7.17\} \end{aligned}$$

$$W_{\text{net}} = 934.54 \text{ kJ/kg steam generated}$$

Heat added per kg of steam generated,

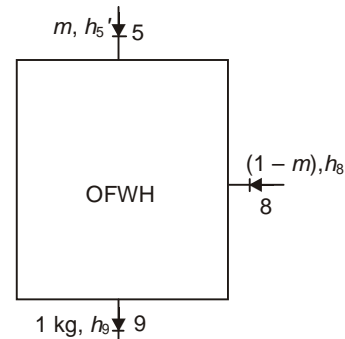
$$q_{\text{add}} = (h_2 - h_1) + (h_4 - h_3)$$

$$q_{\text{add}} = 2262.51 + 133.9 = 2396.41 \text{ kJ/kg of steam generated}$$

$$\text{Thermal efficiency, } \eta = \frac{W_{\text{net}}}{q_{\text{add}}} = \frac{934.54}{2396.41}$$

$$\eta = 0.3899 \text{ or } 38.99\%$$

Thermal efficiency = 38.99%	Ans.
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II. A steam power plant works on regenerative cycle with steam entering first turbine stage at 150 bar, 500°C and getting expanded in three subsequent stages upto the condenser pressure of 0.05 bar. Some steam is bled out between first and second stage for feed heating in closed feed water heater at 10 bar with the saturated liquid condensate being pumped ahead into the boiler feed water line. Feed water leaves closed feed water heater at 150 bar, 150°C. Steam is also taken out between second and third stages at 1.5 bar for being fed into an open feed water heater working at that pressure. Saturated liquid at 1.5 bar leaves open feed water heater for being sent to closed feed water heater. Considering mass flow rate of 300 kg/s into the first stage of turbine determine cycle thermal efficiency and net power developed in kW. Also give lay out and T-s representation.

Solution:

Enthalpy of steam entering ST_1 , $h_2 = 3308.6 \text{ kJ/kg}$, $s_2 = 6.3443 \text{ kJ/kg} \cdot \text{K}$

For isentropic expansion 2-3-4-5, $s_2 = s_3 = s_4 = s_5$

Let dryness fraction of states 3, 4 and 5 be x_3 , x_4 and x_5

$$\Rightarrow s_3 = 6.3443 = s_{f \text{ at } 10 \text{ bar}} + x_3 \cdot s_{fg \text{ at } 10 \text{ bar}}$$

$$x_3 = 0.945$$

$$h_3 = 2667.26 \text{ kJ/kg}$$

$$\Rightarrow s_4 = 6.3443 = s_{f \text{ at } 1.5 \text{ bar}} + x_4 \cdot s_{fg \text{ at } 1.5 \text{ bar}}$$

$$x_4 = 0.848 \Rightarrow h_4 = 2355.18 \text{ kJ/kg}$$

$$\Rightarrow s_5 = 6.3443 = s_{f \text{ at } 0.05 \text{ bar}} + x_5 \cdot s_{fg \text{ at } 0.05 \text{ bar}}$$

$$x_5 = 0.739 \cdot h_5 = 1928.93 \text{ kJ/kg}$$

$$h_6 = h_{f \text{ at } 0.05 \text{ bar}} = 137.82 \text{ kJ/kg}$$

$$v_6 = 0.001005 \text{ m}^3/\text{kg}$$

$$= v_{f \text{ at } 0.05 \text{ bar}}$$

$$h_7 = h_6 + v_6 (1.5 - 0.05) \times 10^2 = 137.96 \text{ kJ/kg}$$

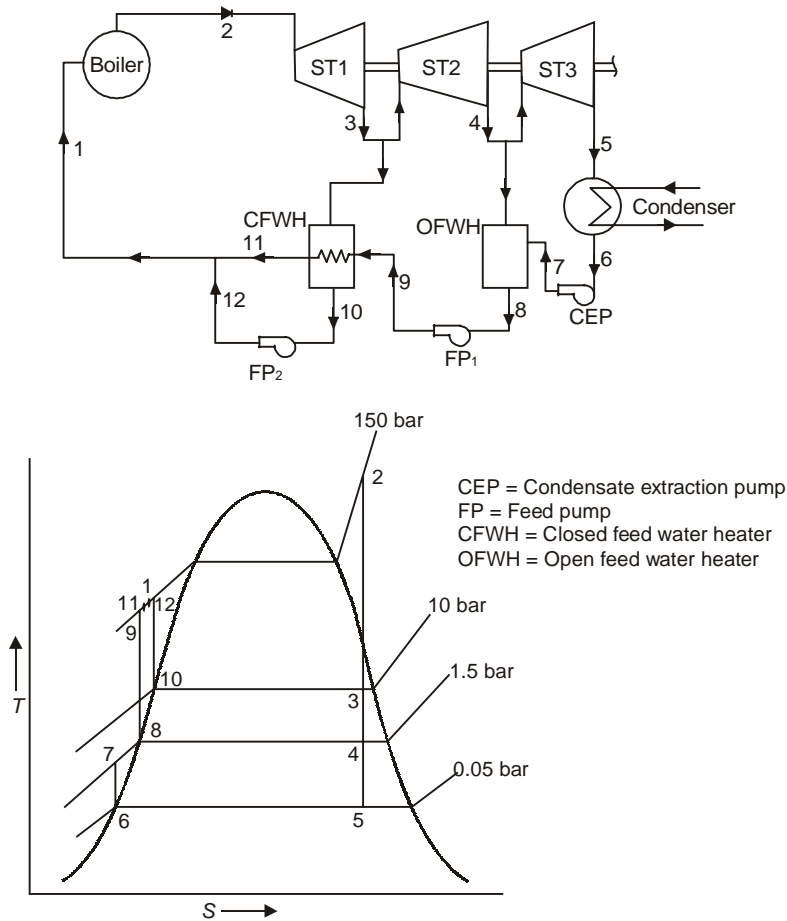


Fig. 8.42 Layout and T-s diagram

$$h_8 = h_f \text{ at 1.5 bar} = 467.11 \text{ kJ/kg},$$

$$v_8 = 0.001053 \text{ m}^3/\text{kg} = v_f \text{ at 1.5 bar}$$

$$h_9 = h_8 + v_8(150 - 1.5) \times 10^2 = 482.75 \text{ kJ/kg}$$

$$h_{10} = h_f \text{ at 150 bar} = 1610.5 \text{ kJ/kg}$$

$$v_{10} = 0.001658 \text{ m}^3/\text{kg} = v_f \text{ at 150 bar}$$

$$h_{12} = h_{10} + v_{10}(150 - 10) \times 10^2 = 1633.71 \text{ kJ/kg}$$

Let mass of steam bled out at 10 bar, 1.5 bar be m_1 and m_2 per kg of steam generated.

Heat balance on closed feed water heater yields,

$$m_1 \cdot h_3 + (1 - m_1) h_9 = m_1 \cdot h_{10} + (1 - m_1) \times 4.18 \times 150$$

$$(m_1 \times 2667.26) + (1 - m_1) \times 482.75 = (m_1 \times 1610.5) + (627 \cdot (1 - m_1))$$

$$m_1 = 0.12 \text{ kg/kg of steam generated.}$$

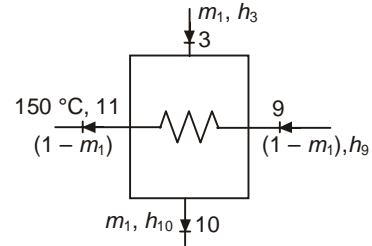
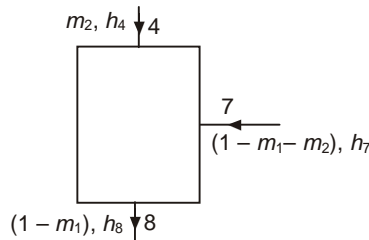
Heat balance on open feed water can be given as under

$$m_2 \cdot h_4 + (1 - m_1 - m_2) \cdot h_7 = (1 - m_1) \cdot h_8$$

$$(m_2 \times 2355.18) + (1 - m_1 - m_2) \times 137.96 = (1 - m_1) \times 467.11$$

$$(m_2 \times 2355.18) + (1 - 0.12 - m_2) \times 137.96 = (1 - 0.12) \times 467.11$$

$$m_2 = 0.13 \text{ kg/kg of steam}$$



For mass flow rate of 300 kg/s $\Rightarrow m_1 = 36 \text{ kg/s}$, $m_2 = 39 \text{ kg/s}$

For mixing after closed feed water heater,

$$h_1 = (4.18 \times 150) \cdot (1 - m_1) + m_1 \times h_{12} = 747.81 \text{ kJ/kg}$$

Net work output per kg of steam generated = $w_{ST1} + w_{ST2} + w_{ST3} - \{w_{CEP} + w_{FP} + w_{FP2}\}$

$$w_{\text{net}} = (h_2 - h_3) + (1 - m_1)(h_3 - h_4) + (1 - m_1 - m_2)(h_4 - h_5) - \{(1 - m_1 - m_2) \cdot (h_7 - h_6) + (1 - m_1) \cdot (h_9 - h_8) + (m_1 \cdot (h_{12} - h_{10}))\}$$

$$w_{\text{net}} = 641.34 + 274.63 + 319.69 = \{0.105 + 13.76 + 2.7852\}$$

$$w_{\text{net}} = 1219.00 \text{ kJ/kg of steam generated.}$$

Heat added per kg of steam generated. $q_{\text{add}} = (h_2 - h_1) = 2560.79 \text{ kJ/kg}$

$$\text{Cycle thermal efficiency, } \eta = \frac{w_{\text{net}}}{q_{\text{add}}} = 0.4760 \text{ or } 47.6\%$$

$$\text{Net power developed in kW} = 1219 \times 300 = 365700 \text{ kW}$$

Cycle thermal efficiency = **47.6%**

Net power developed = **365700 kW**

Ans.

12. A steam power plant has expansion of steam leaving boiler at 100 bar, 500°C occurring in three stages i.e. HPT, IPT and LPT (high pressure, intermediate pressure and low pressure turbine) upto condenser pressure of 0.075 bar. At exit of HPT some steam is extracted for feed heating in closed feed water heater at 20 bar and remaining is sent to IPT for subsequent expansion upto 4 bar. Some more quantity of steam is extracted at 4 bar for feed heating in open feed water heater and remaining steam is allowed to expand in low pressure turbine upto condenser pressure. Feed water leaves closed feed water heater at 100 bar and 200°C. The condensate leaving as saturated liquid at 20 bar is trapped into open feed water heater. The state of liquid leaving open feed water heater may be considered saturated liquid at 4 bar. For a net power output of 100 MW determine thermal efficiency and steam generation rate in boiler.

(a) Modify the above arrangement by introducing reheating of steam entering IPT at 20 bar upto 400°C. Obtain thermal efficiency of modified cycle and compare it with non-reheat type arrangement.

Solution:

At inlet to HPT, $h_2 = 3373.7$ kJ/kg, $s_2 = 6.5966$ kJ/kg · K

For isentropic expansion between 2-3-4-5, $s_2 = s_3 = s_4 = s_5$

State 3 lies in superheated region as $s_3 > s_g$ at 20 bar. By interpolation from superheated steam table, $T_3 = 261.6^\circ\text{C}$. Enthalpy at 3. $h_3 = 2930.57$ kJ/kg.

Since $s_4 < s_g$ at 4 bar so states 4 and 5 lie in wet region.

Let dryness fraction at state 4 and 5 be x_4 and x_5 .

$$s_4 = 6.5966 = s_{f \text{ at 4 bar}} + x_4 \cdot s_{fg \text{ at 4 bar}}$$

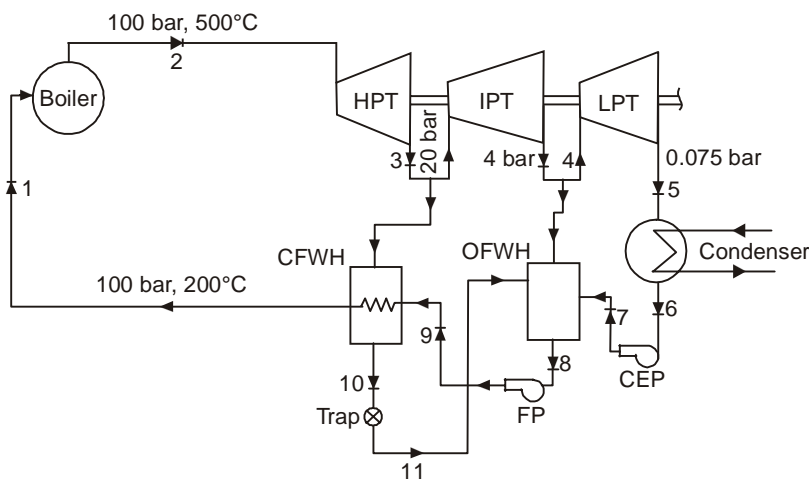
$$x_4 = 0.941$$

$$h_4 = h_{f \text{ at 4 bar}} + x_4 \cdot h_{fg \text{ at 4 bar}} = 2612.65 \text{ kJ/kg}$$

for state 5,

$$s_5 = 6.5966 = s_{f \text{ at 0.075 bar}} + x_5 \cdot s_{fg \text{ at 0.075 bar}}$$

$$x_5 = 0.784$$



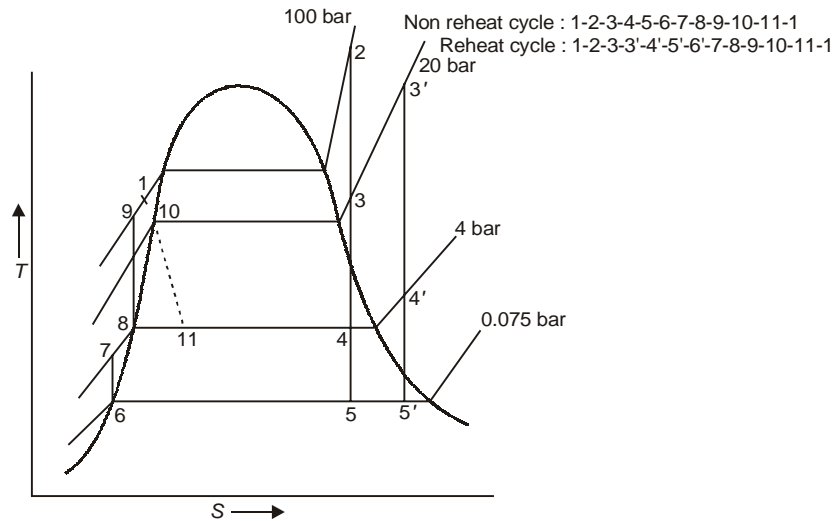


Fig. 8.43 Layout and T - s diagram

$$h_5 = h_f \text{ at } 0.075 \text{ bar} + x_5 \cdot h_{fg} \text{ at } 0.075 \text{ bar} = 2055.09 \text{ kJ/kg}$$

Let mass of steam bled at 20 bar, 4 bar be m_1 and m_2 per kg of steam generated respectively.

$$h_{10} = h_f \text{ at } 20 \text{ bar} = 908.79 \text{ kJ/kg}, h_8 = h_f \text{ at } 4 \text{ bar} = 604.74 \text{ kJ/kg}$$

$$\text{At trap } h_{10} = h_{11} = 908.79 \text{ kJ/kg}$$

At condensate extraction pump, (CEP), $h_7 - h_6 = v_6 (4 - 0.075) \times 10^2$

$$v_6 = v_f \text{ at } 0.075 \text{ bar} = 0.001008 \text{ m}^3/\text{kg}$$

$$h_6 = h_f \text{ at } 0.075 \text{ bar} = 168.79 \text{ kJ/kg} \Rightarrow h_7 = 169.18 \text{ kJ/kg}$$

At feed pump, (FP), $h_9 - h_8 = v_8 (20 - 4) \times 10^2$

$$h_8 = h_f \text{ at } 4 \text{ bar} = 604.74 \text{ kJ/kg}$$

$$v_8 = v_f \text{ at } 4 \text{ bar} = 0.001084 \text{ m}^3/\text{kg}$$

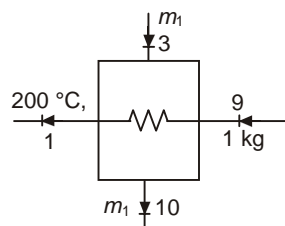
$$h_9 = 604.74 + (0.001084 \times 16 \times 10^2) = 606.47 \text{ kJ/kg}$$

Let us apply heat balance at closed feed water heater,

$$m_1 \cdot h_3 + h_9 = m_1 \cdot h_{10} + 4.18 \times 200$$

$$(m_1 \times 2930.57) + 606.47 = (m_1 \times 908.79) + 836$$

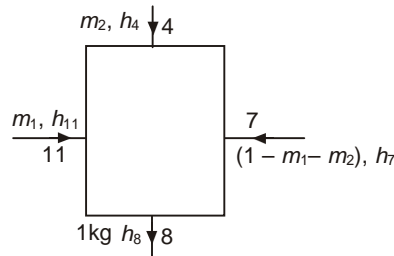
$$m_1 = 0.114 \text{ kg}$$



Applying heat balance at open feed water,

$$m_1 h_{11} + m_2 \cdot h_4 + (1 - m_1 - m_2) \cdot h_7 = h_8$$

$$(m_1 \cdot 908.79) + (m_2 \times 2612.65) + ((1 - m_1 - m_2) \cdot 169.18) = 604.74, m_2 = 0.144 \text{ kg}$$



Net work per kg steam generated,

$$w_{\text{net}} = (h_2 - h_3) + (1 - m_1) \cdot (h_3 - h_4) + (1 - m_1 - m_2) \cdot (h_4 - h_5) - \{(1 - m_1 - m_2) \cdot (h_7 - h_6) + (h_9 - h_8)\}$$

$$= 443.13 + 281.67 + 413.7 - \{0.288 + 1.73\}$$

$$w_{\text{net}} = 1136.48 \text{ kJ/kg}$$

Heat added per kg steam generated,

$$q_{\text{add}} = (h_2 - h_1) = (3373.7 - 4.18 \times 200) = 2537.7 \text{ kJ/kg}$$

$$\text{Thermal efficiency} = \frac{w_{\text{net}}}{q_{\text{add}}} = 0.4478 \text{ or } 44.78\%$$

$$\text{Steam generation rate} = \frac{100 \times 10^3}{w_{\text{net}}} = 87.99 \text{ kg/s}$$

Thermal efficiency = 44.78%
Steam generation rate = 87.99 kg/s

Ans.

(a) For the reheating introduced at 20 bar up to 400°C: The modified cycle representation is shown on T-s diagram by 1-2-3-3'-4'-5'-6-7-8-9-10-11

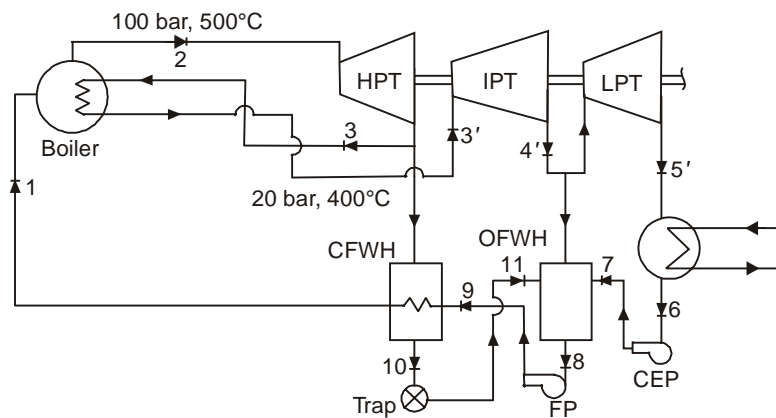


Fig. 8.44 Reheat cycle

At state 2,

$$h_2 = 3373.7 \text{ kJ/kg},$$

$$s_2 = 6.5966 \text{ kJ/kg} \cdot \text{K}$$

At state 3,

$$h_3 = 2930.57 \text{ kJ/kg}$$

At state 3',

$$h_{3'} = 3247.6 \text{ kJ/kg}$$

$$s_{3'} = 7.1271 \text{ kJ/kg} \cdot \text{K}$$

At state 4' and 5' $s_{3'} = s_{4'} = s_{5'} = 7.1271 \text{ kJ/kg} \cdot \text{K}$

From steam tables by interpolation state 4' is seen to be at 190.96°C at 4 bar

$$h_{4'} = 2841.02 \text{ kJ/kg}$$

Let dryness fraction at state 5' be $x_{5'}$.

$$s_{5'} = 7.1271 = s_f \text{ at } 0.075 \text{ bar} + x_{5'} \cdot s_{fg} \text{ at } 0.075 \text{ bar}$$

\Rightarrow

$$x_{5'} = 0.853$$

$$h_{5'} = h_f \text{ at } 0.075 \text{ bar} + x_{5'} \cdot h_{fg} \text{ at } 0.075 \text{ bar}$$

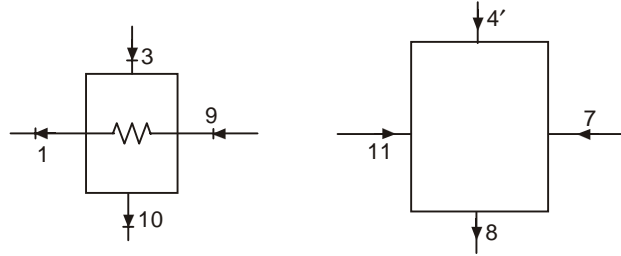
$$h_{5'} = 2221.11 \text{ kJ/kg}$$

Let mass of bled steam at 20 bar and 4 bar be $m_{1'}$, $m_{2'}$ per kg of steam generated. Applying heat balance at closed feed water heater.

$$m_{1'} \cdot h_3 + h_9 = m_{1'} \cdot h_{10} + 4.18 \times 200$$

\Rightarrow

$$m_{1'} = 0.114 \text{ kg}$$



Applying heat balance at open feed water heater

$$m_{1'} \cdot h_{11} + m_{2'} \cdot h_{4'} + (1 - m_{1'} - m_{2'}) \cdot h_7 = h_8$$

$$(0.114 \times 908.79) + (m_{2'} \cdot 2841.02) + (1 - 0.114 - m_{2'}) \cdot 169.18 = 604.74$$

$$m_{2'} = 0.131 \text{ kg}$$

Net work per kg steam generated

$$w_{\text{net}} = (h_2 - h_3) + (1 - m_{1'}) \cdot (h_{3'} - h_{4'}) + (1 - m_{1'} - m_{2'}) (h_{4'} - h_{5'}) - \{(1 - m_{1'} - m_{2'}) \cdot (h_7 - h_6) + (h_9 - h_8)\}$$

$$w_{\text{net}} = 443.13 + 360.22 + 468.03 - \{0.293 + 1.73\}$$

$$w_{\text{net}} = 1269.36 \text{ kJ/kg}$$

Heat added per kg steam generated, $q_{\text{add}} = (h_2 - h_1) + (1 - m_{1'}) (h_{3'} - h_3)$

$$= 2537.7 + 280.88$$

$$q_{\text{add}} = 2818.58 \text{ kJ/kg}$$

Thermal efficiency $\eta = \frac{w_{\text{net}}}{q_{\text{add}}} = 0.4503$ or 45.03%

% Increase in thermal efficiency due to reheating $= \left(\frac{0.4503 - 0.4478}{0.4478} \right) \times 100$
 $= 0.56\%$

Thermal efficiency of reheat cycle = 45.03% % Increase in efficiency due to reheating = 0.56%	Ans.
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13. In a binary vapour cycle working on mercury and steam, the mercury vapour is generated dry saturated at 8.45 bar and expanded upto 0.07 bar in mercury turbine. The condenser of mercury cycle is used for generating steam at 40 bar, 0.98 dry. The steam is superheated separately upto 450°C and then supplied into steam turbine for being expanded upto 0.075 bar. Two closed feed water heaters are used by bleeding out steam at 8 bar and 1 bar so as to provide feed water leaving at 150°C and 90°C respectively. Condensate leaves feed water heater as saturated liquid at respective pressures and is mixed with the hot feed water leaving the respective feed heater. The turbine running on mercury has capability of converting 85% of available heat into work. The enthalpies of mercury may be taken as,

enthalpy of dry saturated vapour at 8.45 bar = 349 kJ/kg

enthalpy after isentropic expansion to 0.07 bar = 234.5 kJ/kg

enthalpy of saturated liquid at 0.07 bar = 35 kJ/kg

Assume feed water to enter at 150°C into mercury condenser. Neglect pump work for getting efficiency. Determine the steam generation rate per kg of mercury and efficiency of cycle.

Solution:

For mercury cycle,

Isentropic heat drop = 349 – 234.5 = 114.5 kJ/kg Hg

Actual heat drop = 0.85 × 114.5 = 97.325 kJ/kg Hg

Heat rejected in condenser = [349 – 97.325 – 35]

= 216.675 kJ/kg

Heat added in boiler = 349 – 35 = 314 kJ/kg

For steam cycle,

Enthalpy of steam generated = $h_{\text{at 40 bar, 0.98 dry}} = 2767.13$ kJ/kg

Enthalpy of steam at inlet to steam turbine $h_2 = h_{\text{at 40 bar, 450°C}} = 3330.3$ kJ/kg

Entropy of steam at inlet to steam turbine, $s_2 = 6.9363$ kJ/kg · K

Therefore, heat added in condenser of mercury cycle

$= h_{\text{at 40 bar, 0.98 dry}} - h_{\text{feed at 40 bar}}$
 $= 2767.13 - 4.18 \times 150 = 2140.13$ kJ/kg

Therefore, mercury required per kg of steam $= \frac{2140.13}{\text{Heat rejected in condenser}}$
 $= \frac{2140.13}{216.675} = 9.88$ kg per kg of steam

For isentropic expansion, $s_2 = s_3 = s_4 = s_5 = 6.9363$ kJ/kg · K

State 3 lies in superheated region, by interpolation the state can be given by, temperature 227.07°C at 8 bar, $h_3 = 2899.23$ kJ/kg

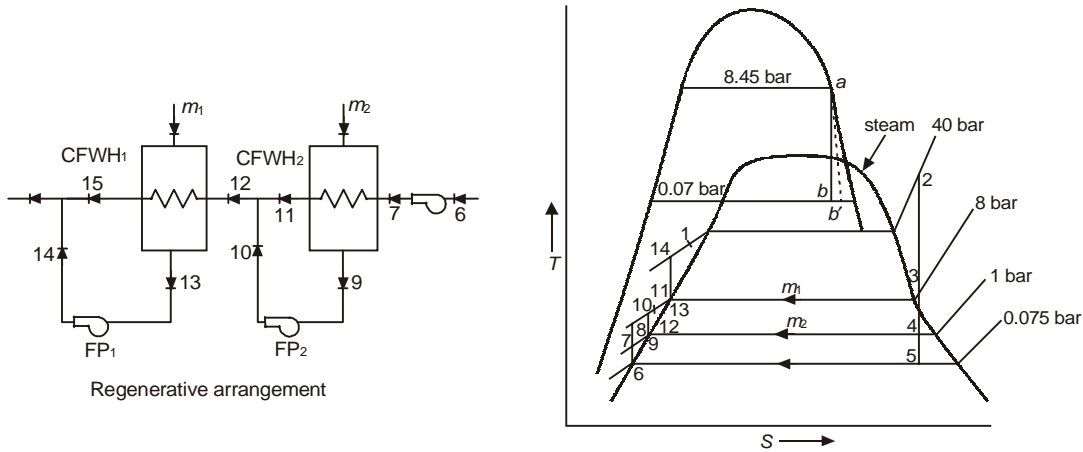


Fig. 8.45 T-s diagram showing expansion and bleeding

State 4 lies in wet region, say with dryness fraction x_4 ,

$$s_4 = 6.9363 = s_{f \text{ at 1 bar}} + x_4 \times s_{fg \text{ at 1 bar}} \Rightarrow x_4 = 0.93$$

$$h_4 = h_{f \text{ at 1 bar}} + x_4 \cdot h_{fg \text{ at 1 bar}} = 2517.4 \text{ kJ/kg}$$

Let state 5 lie in wet region with dryness fraction x_5 ,

$$s_5 = 6.9363 = s_{f \text{ at 0.075 bar}} + x_5 \cdot s_{fg \text{ at 0.075 bar}}$$

$$x_5 = 0.828$$

$$h_5 = 2160.958 \text{ kJ/kg}$$

Let mass of steam bled at 8 bar and 1 bar be m_1 and m_2 per kg of steam generated.

$$\begin{aligned} h_7 &= h_6 + v_6(1 - 0.075) \times 10^2 \\ &= h_{f \text{ at 0.075 bar}} + v_{f \text{ at 0.075 bar}}(1 - 0.075) \times 10^2 \\ &= 168.79 + 0.001008 \times (1 - 0.075) \times 10^2 \end{aligned}$$

$$h_7 = 168.88 \text{ kJ/kg}$$

$$h_9 = h_{f \text{ at 1 bar}} = 417.46 \text{ kJ/kg}$$

$$h_{13} = h_{f \text{ at 8 bar}} = 721.11 \text{ kJ/kg}$$

Applying heat balance on CFWH₁, $T_1 = 150^\circ\text{C}$ and also $T_{15} = 150^\circ\text{C}$

$$m_1 \times h_3 + (1 - m_1) \times h_{12} = m_1 \times h_{13} + (4.18 \times 150) \times (1 - m_1)$$

$$(m_1 \times 2899.23) + (1 - m_1) \times h_{12} = (m_1 \times 721.11) + 627 \cdot (1 - m_1)$$

Applying heat balance on CFWH₂, $T_{11} = 90^\circ\text{C}$

$$m_2 \times h_4 + (1 - m_1 - m_2) \times h_7 = m_2 \times h_9 + (1 - m_1 - m_2) \times 4.18 \times 90$$

$$(m_2 \times 2517.4) + (1 - m_1 - m_2) \times 168.88 = (m_2 \times 417.46) + 376.2(1 - m_1 - m_2)$$

Heat balance at mixing between CFWH₁ and CFWH₂,

$$(1 - m_1 - m_2) \times 4.18 \times 90 + m_2 \times h_{10} = (1 - m_1) \times h_{12}$$

$$376.2(1 - m_1 - m_2) + m_2 \times h_{10} = (1 - m_1) \times h_{12}$$

For pumping process 9–10, $h_{10} = h_9 + v_9(8 - 1) \times 10^2$

$$h_{10} = h_{f \text{ at bar}} + v_{f \text{ at 1 bar}} (7 \times 10^2)$$

$$h_{10} = 417.46 + 0.001043 \times 700 = 418.19 \text{ kJ/kg}$$

Solving above equations, we get

$$m_1 = 0.102 \text{ kg per kg steam generated}$$

$$m_2 = 0.073 \text{ kg per kg steam generated.}$$

Pump work in process 13–14, $h_{14} - h_{13} = v_{13} \times (40 - 8) \times 10^2 = 0.001252 \times 32 \times 10^2$

$$h_{14} - h_{13} = 4.006 \text{ kJ/kg}$$

$$\text{Total heat supplied} = (9.88 \times 314) + (3330.3 - 2767.13)$$

$$= 3665.49 \text{ kJ/kg of steam}$$

Net work per kg of steam,

$$\begin{aligned} w_{\text{net}} &= w_{\text{mercury}} + w_{\text{steam}} \\ &= \{9.88 \times 97.325\} + \{(h_2 - h_3) + (1 - m_1) \cdot (h_3 - h_4) \\ &\quad + (1 - m_1 - m_2) \cdot (h_4 - h_5) \\ &\quad - (1 - m_1 - m_2) (h_4 - h_6) \\ &\quad - m_2 (h_{10} - h_9) - m_1 (h_{14} - h_{13})\} \\ &= \{961.571\} + \{431.07 + 342.88 + 294.06 - 0.074 - 0.053 - 0.408\} \\ w_{\text{net}} &= 2029.046 \text{ kJ/kg} \end{aligned}$$

$$\text{Thermal efficiency of binary vapour cycle} = \frac{2029.046}{3665.49} = 0.5536 \text{ or } 55.36\%$$

Thermal efficiency = 55.36%

Ans.

14. A steam power plant has mixed pressure turbine of output 2500 hp with high pressure steam entering at 20 bar, 300°C and low pressure steam entering at 2 bar and dry saturated. The steam leaves turbine at 0.075 bar and efficiency ratio of both HP and LP stages are 0.8. The Willan's line for both are straight line and steam consumption at no load is 10% of full load. Determine the HP steam required for producing 1000 hp if low pressure steam is available at the rate of 1.5 kg/s.

Solution:

This is a mixed pressure turbine so the output of turbine shall be sum of the contributions by HP and LP steam streams.

For HP: At Inlet of HP steam, $\Rightarrow h_1 = 3023.5 \text{ kJ/kg}$, $s_1 = 6.7664 \text{ kJ/kg} \cdot \text{K}$

Ideally, $s_2 = s_1 = 6.7664$

$$s_2 = 6.7664 = s_{f \text{ at } 0.075 \text{ bar}} + x_3 \times s_{fg \text{ at } 0.075 \text{ bar}}$$

$$\Rightarrow x_3 = 0.806$$

$$h_{3\text{HP}} = h_{f \text{ at } 0.075 \text{ bar}} + x_3 \cdot h_{fg \text{ at } 0.075 \text{ bar}} = 2108.03 \text{ kJ/kg}$$

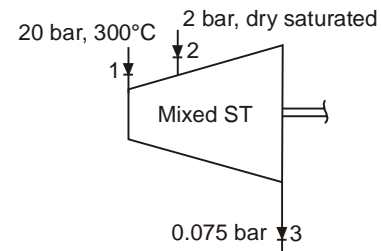
$$\text{Actual enthalpy drop in HP} = (h_1 - h_{3\text{HP}}) \times 0.8 = 732.38 \text{ kJ/kg}$$

For LP: At inlet of LP steam

$$h_2 = 2706.7 \text{ kJ/kg}, s_2 = 7.1271 \text{ kJ/kg} \cdot \text{K}$$

$$\text{Enthalpy at exit, } h_{3\text{LP}} = 2222.34 \text{ kJ/kg}$$

$$\text{Actual enthalpy drop in LP} = (h_2 - h_{3\text{LP}}) \times 0.8 = 387.49 \text{ kJ/kg}$$



$$\text{HP steam consumption at full load} = \frac{2500 \times 0.7457}{732.38} = 2.54 \text{ kg/s}$$

$$\text{HP steam consumption at no load} = 0.1 \times 2.54 = 0.254 \text{ kg/s}$$

$$\text{LP steam consumption at full load} = \frac{2500 \times 0.7457}{387.49} = 4.81 \text{ kg/s}$$

$$\text{LP steam consumption at no load} = 0.1 \times 4.81 = 0.481 \text{ kg/s}$$

The problem can be solved geometrically by drawing Willan's line as per scale on graph paper and finding out the HP steam requirement for getting 1000 hp if LP steam is available at 1.5 kg/s.

or,

Analytically the equation for Willan's line can be obtained for above full load and no load conditions for HP and LP separately.

Willan's line for HP: $y = mx + C$, here y = steam consumption, kg/s

x = load, hp

$$y_{\text{HP}} = m_{\text{HP}} \cdot x + C_{\text{HP}}$$

$$2.54 = m_{\text{HP}} \cdot 2500 + C_{\text{HP}}$$

$$0.254 = m_{\text{HP}} \cdot 0 + C_{\text{HP}} \Rightarrow C_{\text{HP}} = 0.254 \text{ and } m_{\text{HP}} = 9.144 \times 10^{-4}$$

\Rightarrow

$$y_{\text{HP}} = 9.144 \times 10^{-4} \cdot x_{\text{HP}} + 0.254$$

Willan's line for LP: $y_{\text{LP}} = m_{\text{LP}} \cdot x_{\text{LP}} + C_{\text{LP}}$

$$4.81 = m_{\text{LP}} \cdot 2500 + C_{\text{LP}}$$

$$0.481 = m_{\text{LP}} \cdot C_{\text{LP}} \Rightarrow C_{\text{LP}} = 0.481, m_{\text{LP}} = 1.732 \times 10^{-3}$$

$$y_{\text{LP}} = 1.732 \times 10^{-3} \cdot x_{\text{LP}} + 0.481$$

Total output (load) from mixed turbine, $x = x_{\text{HP}} + x_{\text{LP}}$

For load of 1000 hp to be met by mixed turbine, let us find out the load shared by LP for steam flow rate of 1.5 kg/s

$$1.5 = 1.732 \times 10^{-3} \cdot x_{\text{LP}} + 0.481$$

$$x_{\text{LP}} = 588.34 \text{ hp}$$

Since by 1.5 kg/s of LP steam only 588.34 hp output contribution is made so remaining $(1000 - 588.34 = 411.66 \text{ hp})$, 411.66 hp should be contributed by HP steam. By Willan's line for HP turbine,

$$y_{\text{HP}} = (9.144 \times 10^{-4} \times 411.66) + 0.254 = 0.63 \text{ kg/s}$$

So, HP steam requirement = 0.63 kg/s

HP steam required = **0.63 kg/s** Ans.

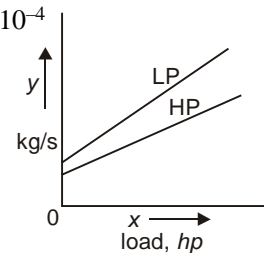


Fig. 8.46 Tentative representation of Willan's line

15. A steam power plant installation has steam leaving boiler at 40 bar, 300°C and expanding in HP turbine upto 2 bar. Half of steam leaving HP turbine is sent for process heating and remaining enters a separator where all the moisture is removed. Dry steam from separator is sent to low pressure LP turbine at 2 bar and gets expanded upto 0.075 bar.

The drain (moisture) of separator gets mixed with condensate from process heater and combined flow enters the hot well at 90°C. Trap is provided at exit of both process heater and separator. Condensate extraction pump extracts condensate from condenser and sends it to hot well at 40°C. Neglecting pump work and radiation losses etc. determine temperature of water leaving hotwell and heat transferred per kg in process heater. Also find out thermal efficiency of installation and give layout.

Solution:

Let us carry out analysis for 1 kg of steam generated in boiler.

Enthalpy at inlet to HPT, $h_2 = 2960.7 \text{ kJ/kg}$, $s_2 = 6.3615 \text{ kJ/kg} \cdot \text{K}$

State at 3 i.e. exit from HPT can be identified by $s_2 = s_3 = 6.3615 \text{ kJ/kg} \cdot \text{K}$

Let dryness fraction be x_3 , $s_3 = 6.3615 = s_{f \text{ at 2 bar}} + x_3 \cdot s_{fg \text{ at 2 bar}}$

$$\Rightarrow x_3 = 0.863 \quad h_3 = 2404.94 \text{ kJ/kg}$$

If one kg of steam is generated in boiler then at exit of HPT, 0.5 kg goes into process heater and 0.5 kg goes into separator

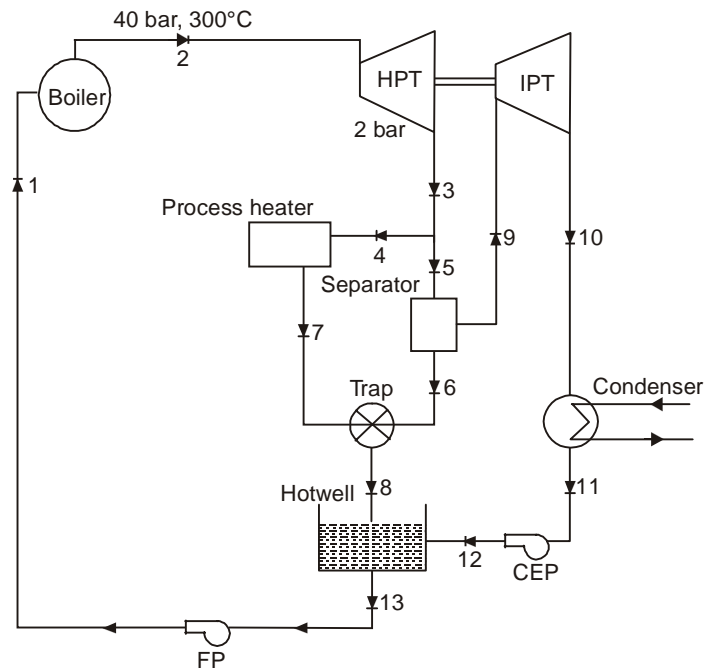


Fig. 8.47 Layout

Mass of moisture retained in separator $= (1 - 0.863) \times 0.5 = 0.0685 \text{ kg}$

Therefore, mass of steam entering LPT $= 0.5 - 0.0685 = 0.4315 \text{ kg}$

Total mass of water entering hot well at 8 (i.e. from process heater and drain from separator) $= 0.5685 \text{ kg}$

Let us assume the temperature of water leaving hotwell be $T^\circ\text{C}$. Applying heat balance for mixing;

$$(0.5685 \times 4.18 \times 90) + (0.4315 \times 4.18 \times 40) = (1 \times 4.18 \times T), T = 68.425^\circ\text{C}$$

Temperature of water leaving hotwell = **68.425°C** **Ans.**

Applying heat balanced on trap

$$0.5 \times h_7 + 0.0685 \times h_{f \text{ at 2 bar}} = (0.5685 \times 4.18 \times 90)$$

$$h_7 = 358.59 \text{ kJ/kg}$$

Therefore, heat transferred in process heater $= 0.5 \times (h_3 - h_7)$

$$= 1023.175 \text{ kJ/kg steam generated}$$

$$\text{Heat transferred per kg steam generated} = \mathbf{1023.175 \text{ kJ/kg steam generated}} \quad \text{Ans.}$$

For state 10 at exit of LPT, $s_{10} = s_3 = s_2 = 6.3615 \text{ kJ/kg} \cdot \text{K}$

Let dryness fraction be x_{10}

$$\begin{aligned} s_{10} &= 6.3615 = s_{f \text{ at } 0.075 \text{ bar}} + x_{10} \cdot s_{fg \text{ at } 0.075 \text{ bar}} \\ \Rightarrow x_{10} &= 0.754 \\ \Rightarrow h_{10} &= h_{f \text{ at } 0.075 \text{ bar}} + x_{10} \cdot h_{fg \text{ at } 0.075 \text{ bar}} \\ h_{10} &= 1982.91 \end{aligned}$$

Net work output, neglecting pump work per kg of steam generated,

$$\begin{aligned} w_{\text{net}} &= (h_2 - h_3) \times 1 + 0.4315 \times (h_3 - h_{10}) \\ &= 555.76 + 182.11 \\ w_{\text{net}} &= 737.87 \text{ kJ/kg steam generated} \end{aligned}$$

Heat added in boiler per kg steam generated, $q_{\text{add}} = (h_2 - h_1)$

$$\begin{aligned} &= (2960.7 - 4.18 \times 68.425) \\ q_{\text{add}} &= 2674.68 \text{ kJ/kg} \end{aligned}$$

$$\text{Thermal efficiency} = \frac{w_{\text{net}}}{q_{\text{add}}} = 0.2758 \text{ or } 27.58\%$$

$$\text{Thermal efficiency} = \mathbf{27.58\%} \quad \text{Ans.}$$

16. In a steam power plant operating on Rankine cycle, the steam enters the turbine at 70 bar and 550°C with a velocity of 30 m/s. It discharges to the condenser at 0.20 bar with a velocity 90 m/s. If the steam flow rate is 35 kg/s, find the thermal efficiency and net power produced. [U.P.S.C. 1992]

Solution:

From steam tables, $h_1 = 3530.9 \text{ kJ/kg}$, $s_1 = 6.9486 \text{ kJ/kg} \cdot \text{K}$

Assuming isentropic expansion in nozzle, $s_1 = s_2 = 6.9486 \text{ kJ/kg} \cdot \text{K}$

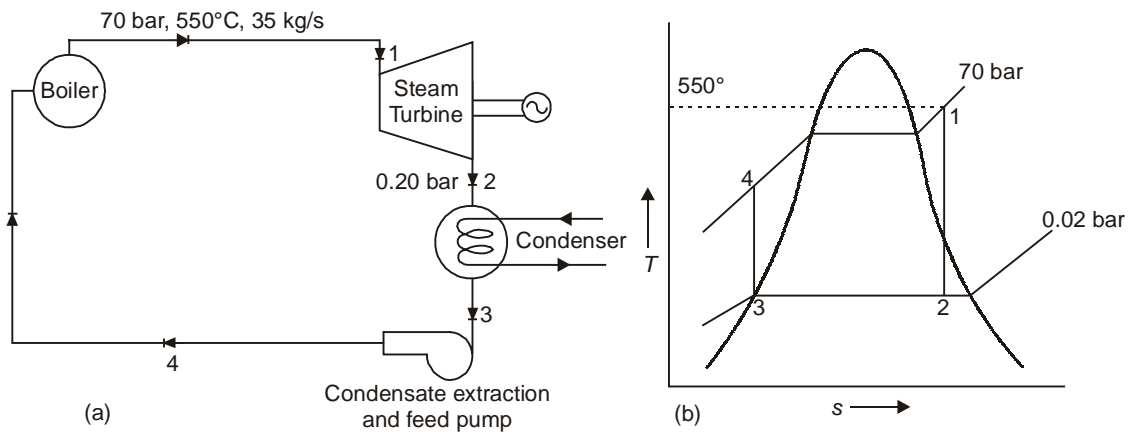


Fig. 8.48 Schematic and representation on T-s diagram

Let dryness fraction at state 2 be x_2 then;

$$s_2 = s_{f \text{ at } 0.2 \text{ bar}} + x_2 \times s_{fg \text{ at } 0.2 \text{ bar}}$$

$$6.9486 = 0.8320 + x_2 \cdot (7.0766)$$

Dryness fraction at state 2, $x_2 = 0.864$

Hence,

$$h_2 = h_{f \text{ at } 0.2 \text{ bar}} + x_2 \cdot h_{fg \text{ at } 0.2 \text{ bar}}$$

$$= 251.40 + (0.864 \times 2358.3)$$

$$h_2 = 2288.97 \text{ kJ/kg}$$

Considering pump work to be of isentropic type, $\Delta h_{34} = v_3 \times \Delta p_{34}$

From steam stable, $v_3 = v_{f \text{ at } 0.2 \text{ bar}} = 0.001017 \text{ m}^3/\text{kg}$
or

$$\Delta h_{34} = 0.001017 \times \frac{(70 - 0.20) \times 10^5}{10^3}$$

Pump work, $W_p = \Delta h_{34} = 7.099 \text{ kJ/kg}$

Turbine work, $W_T = \Delta h_{12} = (h_1 - h_2) = (3530.9 - 2288.97)$

$$W_T = 1241.93 \text{ kJ/kg}$$

$$\text{Net work} = W_T - W_p$$

$$= 1241.93 - 7.099$$

$$W_{net} = 1234.831 \text{ kJ/kg}$$

$$\text{Power produced} = \text{mass flow rate} \times W_{net}$$

$$= 35 \times 1234.831$$

$$= 43219.085 \text{ kJ/s}$$

Net Power = 43.22 MW

Ans.

Heat supplied in boiler = $(h_1 - h_4)$, kJ/kg

$$\text{Enthalpy at state 4, } h_4 = h_3 + \Delta h_{34}$$

$$= h_{f \text{ at } 0.2 \text{ bar}} + \Delta h_{34}$$

$$= 251.40 + 7.099$$

$$h_4 = 258.49 \text{ kJ/kg}$$

$$\text{Total Heat supplied to boiler} = 35 \times (3530.9 - 258.49)$$

$$= 114534.35 \text{ kJ/s}$$

$$\text{Thermal efficiency} = \frac{\text{Net work}}{\text{Heat supplied}}$$

$$= 0.3773$$

Thermal efficiency = 37.73%

Ans.

17. The following data refers to a steam turbine power plant employing one stage of regenerative feed heating:

State of steam entering HP stage : 10 MPa, 600°C

State of steam entering LP stage: 2 MPa, 400°C

Condenser pressure: 10 KPa

The correct amount of steam is bled for feed heating at exit from the HP stage. Calculate the mass of steam bled per kg of steam passing through the HP stage and the amount of heat supplied in the boiler per second for an output of 10 MW. Neglect pump work, [U.P.S.C. 1993]

Solution:

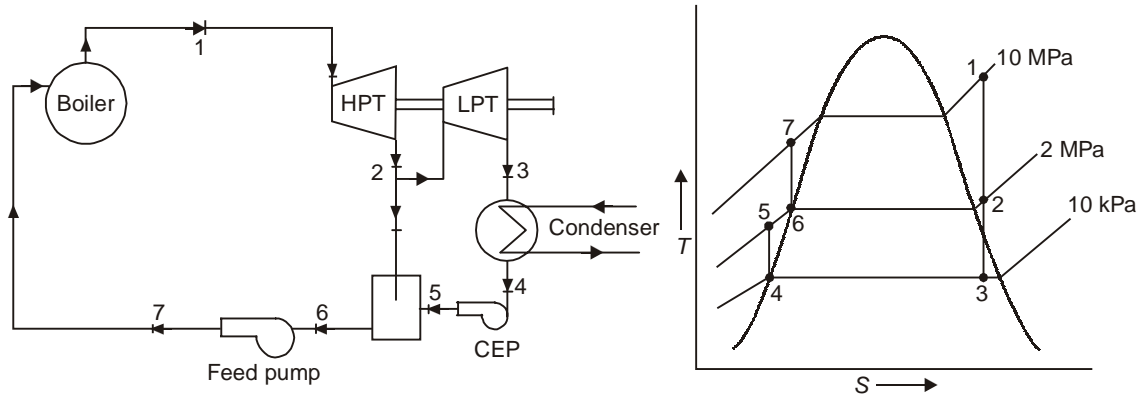


Fig. 8.49 Possible arrangement and T-s representation

From steam tables:

$$h_1 = 3625.3 \text{ kJ/kg}, s_1 = 6.9029 \text{ kJ/kg} \cdot \text{K}$$

Due to isentropic expansion, $s_1 = s_2 = s_3 = 6.9029 \text{ kJ/kg} \cdot \text{K}$

At state 2, i.e. at pressure of 2 MPa and entropy 6.9029 kJ/kg · K.

By interpolating state for s_2 between 2 MPa, 300°C and 2 MPa, 350°C from steam tables,

$$h_2 = 3105.08 \text{ kJ/kg}$$

For state 3, i.e. at pressure of 0.01 MPa entropy, s_3 lies in wet region as $s_3 < s_g$ at 0.01 MPa. Let dryness fraction be x_3 at this state

$$s_3 = s_f \text{ at } 0.01 \text{ MPa} + x_3 \cdot s_{fg} \text{ at } 0.01 \text{ MPa}$$

$$6.9029 = 0.6493 + x_3 \times 7.5009$$

$$x_3 = 0.834$$

$$\text{Enthalpy at state 3, } h_3 = h_f \text{ at } 0.01 \text{ MPa} + x_3 \cdot h_{fg} \text{ at } 0.01 \text{ MPa}$$

$$= 191.83 + (0.834 \times 2392.8)$$

$$h_3 = 2187.43 \text{ kJ/kg}$$

Let the mass of steam bled be m_b per kg of steam from exit of HP for regenerative feed heating.

Considering state at exit from feed heater being saturated liquid the enthalpy at exit of feed heater will be, $h_{f \text{ at } 2 \text{ MPa}}$

$$h_6 = h_{f \text{ at } 2 \text{ MPa}} = 908.79 \text{ kJ/kg}$$

For adiabatic mixing in feed heater, for one kg of steam leaving boiler, the heat balance yields,

$$(1 - m_b) \cdot h_5 + m_b \cdot h_2 = h_6$$

While neglecting pump work,

$$h_5 = h_4 = h_{f \text{ at } 0.01 \text{ MPa}} = 191.83 \text{ kJ/kg}$$

Substituting in heat balance on the feed heater,

$$(1 - m_b) \cdot 191.83 + m_b \cdot 3105.08 = 908.79$$

$$m_b = 0.246 \text{ kg per kg of steam entering HP turbine}$$

Steam bled per kg of steam passing through HP stage = **0.246 kg** Ans.

Let mass of steam leaving boiler be m kg/s.

$$\text{Output} = 10 \times 10^3 = m(h_1 - h_2) + m(1 - m_b)(h_2 - h_3)$$

$$10 \times 10^3 = m\{(3625.3 - 3105.08) + (1 - 0.246)(3105.08 - 2187.43)\}$$

$$m = 8.25 \text{ kg/s}$$

Neglecting pump work, $h_7 = h_6 = 908.79 \text{ kJ/kg}$

Heat supplied to boiler, $Q_{7-1} = m(h_1 - h_7)$

$$Q_{7-1} = 8.25 (3625.3 - 908.79)$$

$$= 22411.208 \text{ kJ/s}$$

Heat added = **22411.21 kJ/s** Ans.

18. Steam enters the first stage of a turbine at 100 bar, 500°C and expands isentropically to 10 bar. It is then reheated to 500°C and expanded in the second stage to the condenser pressure of 0.1 bar. Steam is bled from the first stage at 20 bar and fed to a closed feed water heater. Feed water leaves the closed heater at 100 bar, 200°C (enthalpy = 856.8 kJ/kg), while the condensate is supplied to the open heater into which steam is bled at 4 bar pressure. Saturated liquid at 4 bar exits from the open heater and enters the closed heater. The net output of the turbine is 50 MW. Assuming the turbine and pump processes to be isentropic, determine the mass of steam bled at each feed water heater per kg of steam entering the first stage, the mass of steam entering the first stage per second, and the thermal efficiency.

[U.P.S.C. 1995]

Solution:

From steam table, at inlet to first stage of turbine,

$$h_1 = h_{\text{at } 100 \text{ bar, } 500^\circ\text{C}} = 3373.7 \text{ kJ/kg}$$

$$s_1 = s_{\text{at } 100 \text{ bar, } 500^\circ\text{C}} = 6.5966 \text{ kJ/kg} \cdot \text{K}$$

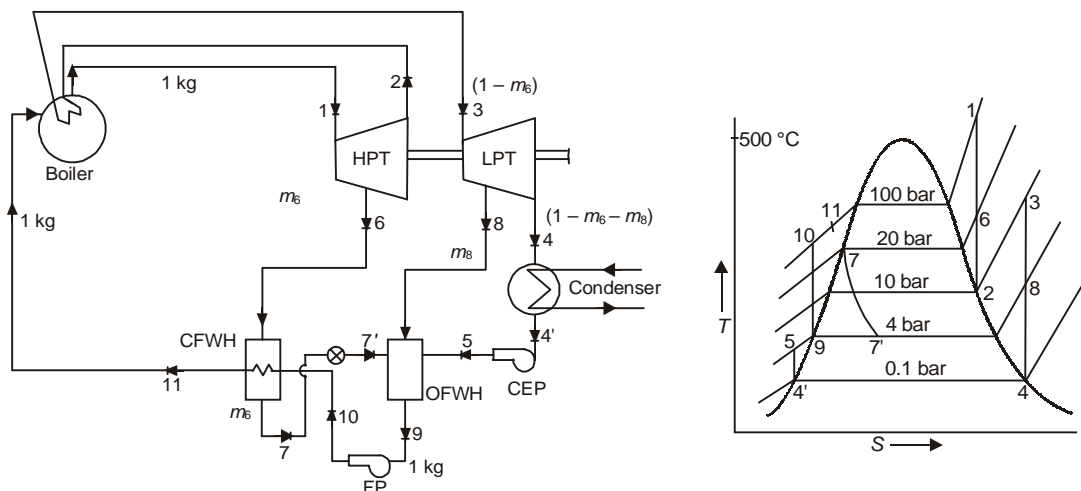


Fig. 8.50 Arrangement and T-s representation

Due to isentropic expansion, $s_1 = s_6 = s_2$ and $s_3 = s_8 = s_4$

State at 6 i.e. bleed state from HP turbine, Temperature by interpolation from steam table = 261.6°C

At inlet to second stage of turbine, $h_6 = 2930.572$ kJ/kg

$$h_3 = h_{\text{at 10 bar, 500°C}} = 3478.5 \text{ kJ/kg}$$

$$s_3 = s_{\text{at 10 bar, 500°C}} = 7.7622 \text{ kJ/kg} \cdot \text{K}$$

At exit from first stage of turbine i.e. at 10 bar and entropy of 6.5966 kJ/kg · K Temperature by interpolation from steam table at 10 bar and entropy of 6.5966 kJ/kg · K

$$T_2 = 181.8^\circ\text{C}$$

$$h_2 = 2782.8 \text{ kJ/kg}$$

State at 8, i.e. bleed state from second stage of expansion, i.e. at 4 bar and entropy of 7.7622 kJ/kg · K Temperature by interpolation from steam table, $T_8 = 358.98^\circ\text{C} \approx 359^\circ\text{C}$

$$h_8 = 3188.7 \text{ kJ/kg}$$

State at 4 i.e. at condenser pressure of 0.1 bar and entropy of 7.7622 kJ/kg · K the state lies in wet region. So let the dryness fraction be x_4 .

$$s_4 = s_f \text{ at 0.1 bar} + x_4 \cdot s_{fg} \text{ at 0.1 bar}$$

$$7.7622 = 0.6493 + x_4 \cdot 7.5009$$

$$x_4 = 0.95$$

$$h_4 = h_f \text{ at 0.1 bar} + x_4 \cdot h_{fg} \text{ at 0.1 bar}$$

$$= 191.83 + (0.95 \times 2392.8)$$

$$h_4 = 2464.99 \text{ kJ/kg}$$

Given, $h_{11} = 856.8$ kJ/kg, $h_9 = h_f \text{ at 4 bar}$

$$h_9 = 604.74 \text{ kJ/kg}$$

Considering pump work, the net output can be given as,

$$W_{\text{net}} = W_{\text{HPT}} + W_{\text{LPT}} - (W_{\text{CEP}} + W_{\text{FP}})$$

where $W_{\text{HPT}} = \{(h_1 - h_6) + (1 - m_6)(h_6 - h_2)\}$ per kg of steam from boiler.

$$W_{\text{LPT}} = \{(1 - m_6) + (h_3 - h_8)(1 - m_6 - m_8)(h_8 - h_4)\}$$
 per kg of steam from boiler.

For closed feed water heater, energy balance yields;

$$m_6 \cdot h_6 + h_{10} = m_6 \cdot h_7 + h_{11}$$

Assuming condensate leaving closed feed water heater to be saturated liquid,

$$h_7 = h_f \text{ at 20 bar} = 908.79 \text{ kJ/kg}$$

Due to throttline, $h_7 = h_{7'} = 908.79$ kJ/kg

For open feed water heater, energy balance yields,

$$m_6 \cdot h_7 + m_8 \cdot h_8 + (1 - m_6 - m_8) \cdot h_5 = h_9$$

For condensate extraction pump, $h_5 - h_{4'} = v_{4'} \cdot \Delta p$

$$h_5 - h_f \text{ at 0.1 bar} = v_f \text{ at 0.1 bar} \cdot (4 - 0.1) \times 10^2$$

$$h_5 - 191.83 = (0.001010) \times (3.9 \times 10^2)$$

$$h_5 = 192.224 \text{ kJ/kg}$$

For feed pump, $h_{10} - h_9 = v_9 \cdot \Delta p$

$$h_{10} - 604.74 = v_f \text{ at 4 bar} \times (100 - 4) \times 10^2$$

$$h_{10} - 604.74 = 0.001084 \times 96 \times 10^2$$

$$h_{10} = 615.15 \text{ kJ/kg}$$

Substituting in energy balance upon closed feed water heater,

$$m_6 \times 2701.2 + 615.15 = m_6 \times 908.79 + 856.8$$

$$m_6 = 0.135 \text{ kg per kg of steam from boiler.}$$

Substituting in energy balance upon feed water heater,

$$m_6 \cdot h_7 + m_8 \cdot h_8 + (1 - m_6 - m_8) \cdot h_5 = h_9$$

$$(0.135 \times 908.79) + (m_8 \times 3188.7) + (1 - 0.135 - m_8) \times 192.224 = 604.74$$

$$m_8 = 0.105 \text{ kg per kg of steam from boiler}$$

Let mass of steam entering first stage of turbine be m kg, then

$$W_{\text{HPT}} = m \{ (h_1 - h_6) + (1 - m_6) (h_6 - h_2) \}$$

$$= m \{ (3373.7 - 2930.572) + (1 - 0.135) \} (2930.572 - 2782.8)$$

$$W_{\text{HPT}} = m \{ 570.95 \}, \text{ kJ}$$

Also,

$$W_{\text{LPT}} = \{ (1 - m_6) (h_3 - h_8) + (1 - m_6 - m_8) \cdot (h_8 - h_4) \}, \text{ per kg of steam from boiler}$$

$$W_{\text{LPT}} = \{ (1 - 0.135) (3478.5 - 3188.7) + (1 - 0.135 - 0.105) \cdot (3188.7 - 2464.99) \}$$

$$W_{\text{LPT}} = m \{ 800.69 \} \text{ kJ}$$

Pump works (negative work)

$$W_{\text{CEP}} = m \cdot (1 - m_6 - m_8) (h_5 - h_4)$$

$$= m \cdot (1 - 0.135 - 0.105) \cdot (192.224 - 191.83)$$

$$W_{\text{CEP}} = \{ 0.299 m \}$$

$$W_{\text{FP}} = m \{ h_{10} - h_9 \}$$

$$= m \{ 615.45 - 604.74 \}$$

$$W_{\text{FP}} = \{ 10.71 m \}$$

Net output

$$W_{\text{net}} = W_{\text{HPT}} + W_{\text{LPT}} - W_{\text{CEP}} - W_{\text{FP}}$$

$$50 \times 10^3 = \{ 570.95 m + 800.69 m - 0.299 m - 10.71 m \}$$

$$m = 36.75 \text{ kg/s}$$

Heat supplied in boiler, $Q_{\text{add}} = m(h_1 - h_{11})$

$$= 36.75 (3373.7 - 856.8)$$

$$= 92496.075 \text{ kJ/s}$$

$$\text{Thermal efficiency} = \frac{W_{\text{net}}}{Q_{\text{add}}}$$

$$= \frac{50 \times 10^3}{92496.075}$$

$$= 0.54056 \text{ or } 54.06\%$$

Mass of steam bled at 20 bar = **0.135 kg per kg** of steam entering first stage

Mass of steam bled at 4 bar = **0.105 kg per kg** of steam entering first stage

Mass of steam entering first stage = **36.75 kg/s**

Thermal efficiency = **54.06%**

Ans.

19. A steam power plant installation has reheating and regenerative feed water heating employing a surface type feed heater and other contact type feed heater on high pressure side and low pressure side respectively. Steam enters HP turbine at 100 bar, 803 K and leaves high pressure turbine at 25 bar from where some steam is bled for feed heating in high pressure surface type heater and remaining is reheated up to 823 K and then expanded in low pressure turbine up to 0.05 bar pressure. The contact type feed heater is supplied with steam bled at 6 bar from LP steam turbine. There occurs throttling pressure loss of 3 bar in reheater. Surface type feed heater sends the drain to contact type feed heater from where the total feed is sent to surface type feed heater employing a boiler feed water pump as saturated water at pressure of 100 bar. Determine the amounts of steam bled off, overall thermal efficiency and specific steam consumption in kg/kwh. Considering tubing efficiency pump efficiency, generator efficiency, and mechanical efficiency as 0.85, 0.90 & 0.95 respectively and plant output as 120 MW. Consider discharges of drains at saturated liquid state at respective pressures in feed heaters. Also show how the processes on T-s and h-s diagrams along with line sketch of arrangement.

Solution: From steam table,

At 100 bar, 803 K the state of inlet steam

$$h_1 = 3450.02 \text{ kJ/kg}, s_1 = 6.6923 \text{ kJ/kg.K}$$

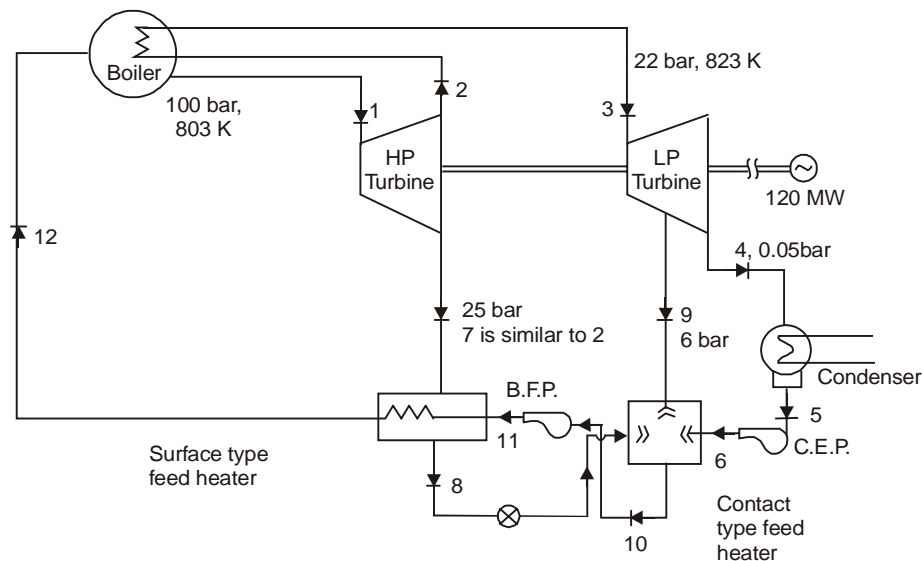
At inlet to LP steam turbine at 22 bar, 823K

$$h_3 = 3576.99 \text{ kJ/kg}, s_3 = 7.52411 \text{ kJ/kg.K}$$

For exit from HP turbine, $s_1 = s_2$

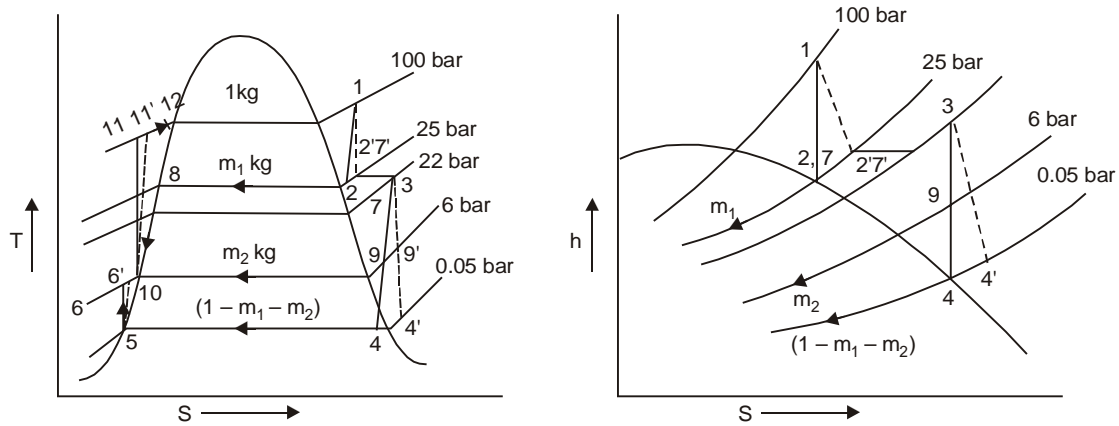
Using Mollier diagram

$$h_2 = 3010, \text{ kJ/kg.K.}$$



BFP : Boiler feed pump

CEP : Condensate extraction pump

**Fig. 8.51**

Let us consider 1 kg of steam generated in boiler and bled fractions be m_1 & m_2 .

$$\text{Considering turbine efficiency, } \eta_{\text{turb}} = \frac{h_1 - h_2'}{h_1 - h_2}$$

$$0.85 = \frac{3450.02 - h_2'}{(3450.02 - 3010)}$$

\Rightarrow

$$h_2' = 3076.003 \text{ kJ/kg}$$

From Mollier diagram, considering isentropic expansion in LP turbine

$$h_9 = 3175 \text{ kJ/kg}$$

$$h_4 = 2300 \text{ kJ/kg}$$

$$\text{Considering turbine efficiency, } 0.85 = \frac{h_3 - h_9'}{h_3 - h_9}$$

$$0.85 = \frac{(3576.99 - h_9')}{(3576.99 - 3175)}$$

\Rightarrow

$$h_9' = 3235.29 \text{ kJ/kg}$$

Also

$$0.85 = \frac{h_3 - h_4'}{h_3 - h_4} = \frac{(3576.99 - h_4')}{(3576.99 - 2300)}$$

\Rightarrow

$$h_4' = 2491.5 \text{ kJ/kg}$$

From steam table

$$h_5 = h_f \text{ at } 0.05 \text{ bar} = 137.82 \text{ kJ/kg,}$$

$$v_s = v_f \text{ at } 0.05 \text{ bar} = 0.001005 \text{ m}^3/\text{kg}$$