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Syllabus

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Ch-1 Ordinary Linear Differential Eq^n of Second Order & Higher Or.

Ch-2 Series Solution of Ordinary diff. Eq^n

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Ch-4 Laplace Transform → 12 Mrks

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Chapter-1

Ordinary Linear Differential Equation of Second Order And Higher Order

Linear Homogenous Differential Equation

In which dependent Variable and all its derivatives occur in the 1st degree only and are not multipling together.

$$\frac{d^n y}{dx^n} + \frac{d^{n-1}y}{dx^{n-1}} + \dots + f(x) = 0 \rightarrow \text{Homogeneous parameter}$$

$D \Rightarrow$ Diff. Operator.

Sol^n y = Complimentory function.

Table to find the C.F

	Nature of roots	C.F (y_c)
1.	If roots are real & different. i.e $m = m_1, m_2, m_3, \dots$ $m = 1, 2$	$C.F = C_1e^{m_1x} + C_2e^{m_2x} + C_3e^{m_3x} + \dots$ G.S $\rightarrow y = y_c = \dots$ $y_c = C_1e^{(1)x} + C_2e^{(2)x}$.
2.	If roots are real & equal i.e $m = m_1, m_1, m_2$ $m = 2, 2, 3$	$C.F = (C_1 + xC_2)e^{m_1x} + C_3e^{m_2x}$ $y_c = e^{2x}(C_1 + xC_2) + C_3e^{3x}$
3.	If roots are imaginary $x = a \pm ib$ $m = 2 + 3i$;	$C.F = e^{ax}(C_1\cos bx + C_2\sin bx)$ $y_c = e^{2x}(C_1\cos 3x + C_2\sin 3x)$

4.	If roots are fractional i.e $m = a \pm \sqrt{b}$ $m = 3 \pm \sqrt{5}$	$C.F = e^{ax}(c_1 \cosh(bx) + c_2 \sinh(bx))$ $y_c = e^{3x}(c_1 \cosh(\sqrt{5}x) + c_2 \sinh(\sqrt{5}x))$
5.	If roots are equal & imaginary. $m = a \pm ib,\ a \pm ib$ $m = 4 \pm 3i,\ 4 \pm 3i$	$C.F = e^{ax}[(c_1 + xc_2)\cos bx + (c_3 + xc_4)\sin bx]$ $y_c = e^{4x}[(c_1 + xc_2)\cos 3x + (c_3 + xc_4)\sin 3x]$

Solve $\frac{dy}{dx} - 4xy = 0$

$$\int \frac{dy}{y} = 4\int x\,dx$$

$$\log y = 2x^2 + c$$

$$y = e^{2x^2 + c}$$

General eq^n

$$Dy + f(x) = 0$$

$$a_n D^n y + a_{n-1} D^{n-1} y + \dots = 0$$

$$(a_n D^n + a_{n-1} D^{n-1} + a_{n-2} D^{n-2} + \dots)y = 0$$

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$m = 1, 2, 3, 2, 3$

$y_c = c_1 e^{(1)x} + e^{2x}(c_2 + x c_3) +$
$e^{3x}(c_4 + x c_5)$

Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 0$

$\times$ $y'' - 3y' - 4y = 0$ $\quad \because \left(D = \frac{d}{dx}\right)$

$y'' - 3y' = 4y$

$y'' - 3y' = 4\left(\frac{y'' - 3y'}{4}\right)$

$(D^2 - 3D - 4)\, y = 0$

auxillary eq^n $m^2 - 3m - 4 = 0$

$m^2 - 4m + m - 4 = 0$

$m(m-4) + 1(m-4) = 0$

$m = -1, 4$

General sol^n

$\boxed{y = y_c = c_1 e^{-x} + c_2 e^{4x}}$

Solve $y'' - 5y' + 6y = 0$

$(\because D = d/dx)$

$(D^2 - 5D + 6)\, y = 0$

auxillary eq^n

$m^2 - 5m + 6 = 0$

$m^2 - 3m - 2m + 6 = 0$

$m(m-3) - 2(m-3) = 0$

$(m-2)(m-3) = 0$

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$$\begin{array}{c|cccc} 1 & 1 & 6 & 3 & -10 \\ & 0 & 1 & 7 & 10 \\ \hline & 1 & 7 & 10 & 0 \end{array}$$

$m = 3, 2$

General sol^n

$y = y_c = c_1 e^{2x} + c_2 e^{3x}$

Solve $y''' + 6y'' + 3y' - 10y = 0$

$(D^3 + 6D^2 + 3D - 10)\, y = 0 \quad (\because D = d/dx)$

auxillary eq^n

$m^3 + 6m^2 + 3m - 10 = 0$

$(m-1)(m^2 + 7m + 10) = 0$

$m^2 + 7m + 10 = 0$

$m^2 + 5m + 2m + 10 = 0$

$(m+5)(m+2)(m-1) = 0$

$m = -5, -2, 1$

General sol^n

$y = y_c = c_1 e^{-5x} + c_2 e^{-2x} + c_3 e^{x}$

Solve $(D^3 - 3D + 2)y = 0$

auxillary eq^n

$m^3 - 3m + 2 = 0$

$m^3 - 3m = -2$

$m(m^2 - 3) = -2$

$m = -2, \quad m^2 - 3 = -2$

$m^2 = 1$

$m = 1, +1, -2$

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General Sol^n

$y = y_c = (c_1 + x c_2) e^{x} + c_3 e^{2x}$

Solve $y''' - 3y' + 2y = 0$

$(D^3 - 3D + 2) y = 0$

auxillary eq^n

$m^3 - 3m + 2 = 0$

$m^3 - 2m - m + 2 = 0$

$(m-2)(m-1) = 0$

$m = 1, 2$

General sol^n

$y = y_c = c_1 e^{x} + c_2 e^{2x}$

Solve $(D^4 - D^3 - 9D^2 - 11D - 4) y = 0$

auxillary eq^n.

$11-4$ $m^4 - m^3 - 9m^2 - 11m - 4 = 0$

$m = -1.$

$m^3 - 2m^2 - 7m - 4 = 0$

$m = -1$

$m^2 - 3m \mp 4 = 0$

$m^2 - 4m + m \mp 4 = 0$

$(m-4)(m+1) = 0$

$m = -1, -1, -1, 4$

-1	1	-1	-9	-11	-4
	0	-1	2	7	4
	1	-2	-7	-4	0

-1	1	-2	-7	-4
	0	-1	3	4
	+1	-3	-4	0

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General eqn

$y = y_c = c_1 e^{4x} + e^{-x}(c_2 + x c_3 + x^2 c_4)$

$+ x$

The mind ones extended to the dimension of larger ~~area~~ ideas never returns to its original size.

Solve $(D^2+1)^3 (D^2+D+1)^2 y = 0$

$\Rightarrow ((D^2)^3 + (1)^3 + 3(D^2)^2(1) + 3(D^2)(1)^2) \cdot (D^4 + D^2 + 1 + 2D^3 + 2D^2 + 2D) y = 0$

$\Rightarrow (D^6 + 1 + 3D^4 + 3D^2) \cdot (D^4 + 2D^3 + 3D^2 + 2D + 1) y = 0$

$\Rightarrow (D^{10} + 2D^9 + 3D^8 + 2D^7 + D^6 + D^4 + 2D^3 + 3D^2 + 2D + 1 + 3D^8 + 6D^7 + 9D^6 + 6D^5 + 3D^4 + 3D^6 + 6D^5 + 9D^4 + 6D^3 + 3D^2) y = 0$

$\Rightarrow (D^{10} + 2D^9 + 6D^8 + 8D^7 + 13D^6 + 12D^5 + 13D^4 + 8D^3 + 6D^2 + 2D + 1) y = 0$

auxillary eqn

$m^{10} + 2m^9 + 6m^8 + 8m^7 + 13m^6 + 12m^5 + 13m^4 + 8m^3 + 6m^2 + 2m + 1 = 0$

$0 \pm i$

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$(m^2+1)^3 (D^2 m^2+m+1)^2 = 0$

$(m^2+1)^3 = 0 \qquad (m^2+m+1)^2 = 0$

$m^2+1 = 0 \qquad m^2+m+1 = 0$

$m^2 = -1$

$m = \pm i \qquad m = \dfrac{-b \pm \sqrt{b^2-4ac}\, i}{2a}$

$= \dfrac{-1 \pm \sqrt{3}\, i}{2}$

$m = i, i, i, -i, -i, -i, \dfrac{-1 \pm \sqrt{3} i}{2}, \dfrac{-1 \pm \sqrt{3} i}{2}$

General solⁿ.

$y = y_c = [(c_1 + xc_2 + x^2 c_3)\cos x + (c_4 + xc_5 + x^2 c_6)\sin x] +$
$e^{-x/2}\left[(c_7 + xc_8)\cos\left(\frac{\sqrt{3}}{2}\right)x + (c_9 + xc_{10})\sin\left(\frac{\sqrt{3}}{2}\right)x\right]$

Solve $y''' - y = 0$

$(D^3 - 1)y = 0$

auxillary eqⁿ

$m^3 - 1 = 0 \Rightarrow (m-1)(m^2+m+1)$

$m^3 = 1$

$m = 1, \cancel{1}, \cancel{1} \quad m = \dfrac{-1 \pm \sqrt{3} i}{2}$

General eqⁿ.

$y = y_c \quad (c_1 + xc_2 \cancel{+ x^2 c_3}) e^{x}$

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$y_c = c_1 e^{x} + (c_2 + x c_3)$

An idea which can be used once is a trick. If it can be used more than ones it's become method.

Complimentory function table

$f(m) = 0$

$m = 2, 3, 5, 6 \Rightarrow y_c = c_1 e^{2x} + c_2 e^{3x} + c_3 e^{5x} + c_4 e^{6x}$

$= 2, 2, 3, 3, 3, 7, 8 \Rightarrow y_c = (c_1 + x c_2) e^{2x} + (c_3 + x c_4 + x^2 c_5) e^{3x} + c_6 e^{7x} + c_7 e^{8x}.$

$m = a+ib, a-ib \Rightarrow y_c = e^{ax}[c_1 \cos bx + c_2 \sin bx]$

$m = a \pm ib, a \pm ib \Rightarrow y_c = e^{ax}[(c_1 + x c_2)\cos bx + (c_3 + x c_4)\sin bx]$

$m = a + \sqrt{b}, a - \sqrt{b} \Rightarrow y_c = e^{ax}[c_1 \cosh \sqrt{b} x + c_2 \sinh \sqrt{b} x]$

Q. Solve $y''' - 8y = 0$

$(D^3 - 8)y = 0$

$m^3 - 8 = 0$

$(m-2)(m^2+2m+4) = 0$

$m = 2$

$m = \frac{-2 \pm \sqrt{-4+8}}{2}$

$= \frac{-2 \pm \sqrt{12}i}{2}$

$= -1 \pm \sqrt{3}i$

General eq^n.

$y_c = c_1 e^{2x} + e^{-x}[c_2 \cos \sqrt{3}x +$
$c_3 \sin \sqrt{3}x]$

Solve $(D^2 + 6D + 4)y = 0$

auxillary eq^n

$m^2 + 6m + 4 = 0$

$m = \frac{-6 \pm \sqrt{36-16}}{2}$

$= \frac{-6 \pm \sqrt{20}}{2}$

$= \frac{-6 \pm 2\sqrt{5}}{2}$

$= -3 \pm \sqrt{5}$

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General eqⁿ

$$y = y_c = e^{-3x}\left[c_1 \cosh\sqrt{5}x + c_2 \sinh\sqrt{5}x\right]$$

Solve $(D^6 + 6D^4 + 9D^2)y = 0$

$$m^6 + 6D^4 + 9D^2 = 0$$

$$m^2(m^4 + 6m^2 + 9) = 0 \qquad m^2 = 0$$

$$m^4 + 6m^2 + 9 = 0 \qquad m = 0, 0$$

$$m^4 + 3m^2 + 3m^2 + 9 = 0$$

$$m^2(m^2+3) + 3(m^2+3) = 0$$

$$(m^2+3)(m^2+3) = 0.$$

$$m^2 = -3$$

$$m = \pm\sqrt{3}i$$

$$m = \pm\sqrt{3}i$$

General Solⁿ

$$y = y_c = (c_1 + xc_2) + ((c_3 + xc_4)\cos\sqrt{3}x + (c_5 + xc_6)\sin\sqrt{3}x)$$

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1. $y'' + y' - 6y = 0$

$(D^2 + D - 6)y = 0$

auxillary eq^n

$m^2 + m - 6 = 0$

$m^2 + 3m - 2m - 6 = 0$

$m(m+3) - 2(m+3) = 0$

$(m-2)(m+3) = 0$

$m = -3, 2$

General sol^n

$y = y_c = c_1 e^{-3x} + c_2 e^{2x}$

2. $y'' - 9y' + 20y = 0$

$(D^2 - 9D + 20)y = 0$

auxillary eq^n

$m^2 - 9m + 20 = 0$

$m^2 - 5m - 4m + 20 = 0$

$m(m-5) - 4(m-5) = 0$

$(m-4)(m-5) = 0$

$m = 4, 5$

General Sol^n

$y = y_c = c_1 e^{4x} + c_2 e^{5x}$

3. $y'' + y' = 0$

$(D^2 + D)y = 0$

auxillary eq^n

$m^2 + m = 0$

$m(m+1) = 0$

$m = -1, 0$

General Sol^n

$y = y_c = c_1 e^{-x} + c_2$

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4. $y'' = 4y$

$y'' - 4y = 0$

$(D^2 - 4)y = 0$

auxillary eq^n

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm 2$.

General sol^n

$y = y_c = c_1 e^{-2x} + c_2 e^{2x}$

5. $2y'' + y' - y = 0$

$(2D^2 + D - 1)y = 0$

auxillary eq^n

$2m^2 + m - 1 = 0$

$m = -1, \frac{1}{2}$

General Sol^n

$y = y_c = c_1 e^{-x} + c_2 e^{x/2}$

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6. $y'' + 4y' - 5y = 0$

$(D^2 + 4D - 5)y = 0$

auxillary eq^n.

$m^2 + 4m - 5 = 0$

$m^2 + 5m - m - 5 = 0$

$m(m+5) - 1(m+5) = 0$

$(m-1)(m+5) = 0$

$m = 1, -5, +1$

General sol^n

$y = y_c = C_1 e^{-5x} + C_2 e^{x}$

7. $4y'' - 12y' + 5y = 0$

$(4D^2 - 12D + 5)y = 0$.

auxillary eq^n

$4m^2 - 12m + 5 = 0$

$4m^2 - 10m - 2m + 5 = 0$

$2m(2m - 5) - (2m - 5) = 0$

$(2m-1)(2m-5) = 0$

$m = \frac{1}{2}$, $m = \frac{5}{2}$

General sol^n:

$y = y_c = C_1 e^{x/2} + C_2 e^{5x/2}$

8. $2y'' - 9y' = 0$

$2D^2 - 9D = 0$

$D(2D - 9) = 0$

$D = 0, \frac{9}{2}$

General sol^n.

$y = y_c = C_1 + C_2 e^{9x/2}$

9. $4y'' + 4y' - 3y = 0$

$(4D^2 + 4D - 3)y = 0$

auxillary eq^n.

$4m^2 + 4m - 3 = 0$

$4m^2 + 6m - 2m - 3 = 0$

$2m(2m+3) - 1(2m+3) = 0$

$m = -\frac{3}{2}, \frac{1}{2}$

General sol^n.

$y = y_c = C_1 e^{-\frac{3x}{2}} + C_2 e^{x/2}$.

10. $y'' + 9y' + 20y = 0$.

$(D^2 + 9D + 20)y = 0$.

auxillary eq^n

$m^2 + 9m + 20 = 0$

$m^2 + 4m + 5m + 20 = 0$

$m(m+4) + 5(m+4) = 0$

$(m+5)(m+4) = 0$

$m = -4, -5$

General sol^n.

$y = y_c = C_1 e^{-5x} + C_2 e^{-4x}$

11. $y'' + 2y' + y = 0$

$(D^2 + 2D + 1)y = 0$

auxillary eq^n

$m^2 + 2m + 1 = 0$

$m^2 + m + m + 1 = 0$

$m(m+1) + 1(m+1) = 0$.

$(m+1)(m+1)$

$m = -1, -1$

General sol^n

$y = y_c = (C_1 + xC_2)e^{-x}$

$m = \frac{-1 \pm 3i}{2}$

General Sol^n

$y = y_c = e^{-x/2}\left[c_1 \cos\frac{3x}{2} + c_2 \sin\frac{3x}{2}\right]$

30. $y'' + 5y' + 12.5y = 0$

$10y'' + 50y + 125y = 0$

$(10D^2 + 50D + 125)y = 0$

auxillary eq^n

$10m^2 + 50m + 125 = 0$

$m = \frac{-5 \pm 5i}{2}$

General Sol^n

$y = y_c = e^{-5x/2}\left[c_1 \cos\frac{5x}{2} + c_2 \sin\frac{5x}{2}\right]$

Solve $(D^4 + 2D^2 + 1)y = 0$

auxillary eq^n

$m^4 + 2m^2 + 1 = 0$

$(m^2+1)^2 = 0$

$m^2 = -1$

$m = \pm i$ (2times)

1	0	2	1
0			
1			

General sol^n

$y = y_c = \cancel{(c_1 \cos x + c_2 \sin x)}$

$= ((c_1 + x c_2)\cos x + (c_3 + x c_4)\sin x)$.

Solve $(D^4 - 1)y = 0$

auxillary eq^n

$m^4 - 1 = 0$

$(m^2+1)(m^2-1) = 0$

$m^2 = 1$

$m = \pm 1$

$m = \pm i$

General sol^n

$y = y_c = C_1 e^{-x} + C_2 e^{x} + (C_3 \cos x + C_4 \sin x)$

Solve $y''' \mp y = 0$

$(D^3 \mp 1)y = 0$

auxillary eq^n

$m^3 - 1 = 0$

1	1	0	0	-1
	0	1	1	1
	1	1	1	0

$(m-1)(m^2 + m + 1)$

$m = 1$

$m = \frac{-1 \pm \sqrt{1-4}}{2}$

$= \frac{-1 \pm \sqrt{3}i}{2}$

General sol^n

$y = y_c = C_1 e^{x} + (C_2 \cos\sqrt{3}x + C_3 \sin\sqrt{3}x)\, e^{-x/2}$

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Non-Homogeneous Differential Equation.

$(a_n D^n + a_{n-1} D^{n-1} + a_{n-2} D^{n-2} + \dots + c)y = R(x)$.

$f(D)y = R(x)$.

General Sol^n.

$y = C.F + P.I$ (P.I = Particular Integral)

$= y_c + y_p$.

$y_c \Rightarrow f(m) = 0 \Rightarrow y_c = \text{--------}$.

$y_p \Rightarrow$

$y_p = \frac{1}{(D-a)} R(x) = e^{ax} \int R(x) e^{-ax} dx$

Solve $y'' - 3y' + 2y = e^x$

$(D^2 - 3D + 2)y = R(x) - e^x$

auxillary eq^n

$m^2 - 3m + 2 = 0$

$m^2 - 2m - m + 2 = 0$

$m(m-2) - 1(m-2) = 0$

$(m-1)(m-2) = 0$

$m = 1, 2$

General Sol^n

$y_c = C_1 e^x + C_2 e^{2x}$

$y_p = e^{2x} \int e^x \times e^{-2x} dx$

$= -e^x$.

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General Sol^n

$y = C_1 e^x + C_2 e^{2x} +$

$y_p = \frac{1}{f(D)} R(x) = \frac{1}{(D^2 - 3D + 2)} e^x$

$= \frac{1}{(D-1)(D-2)} e^x$.

$= \frac{-1}{(D-1)} \times e^x$.

$y_p = + e^x \int e^{+x} \times e^x dx$

$= e^x \int dx$

$= e^x (x)$

$= x e^x$.

$y_p = -x e^{2x}$.

General Sol^n

$y = C_1 e^x + C_2 e^{2x} - x e^x$

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$$\boxed{\frac{1}{f(D)}e^{ax} = \frac{1}{f(a)}e^{ax}}$$

$y_p = \frac{1}{D^2-3D+2} e^{x}$

$f(a)=0$
$f'(a) \neq 0$

$f(D) = f(1) = 0$

$= x \frac{1}{f'(D)} e^{ax} = x \frac{1}{2D-3} e^{x}$

$= \frac{1}{D-2} e^{x} - \frac{1}{D-1} e^{x}$

$= \left[\frac{1}{-1} e^{x}\right] = -xe^{x}$

Special case of $R(x)$

$R(x) = e^{ax}$

$y_p = \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax} \quad f(a) \neq 0$

$= x \frac{1}{f'(a)} e^{ax} \quad f(a)=0, \ f'(a) \neq 0$

$= x^2 \frac{1}{f''(a)} e^{ax} \quad f'(a)=0, \ f''(a) \neq 0$

Solve $(D^3 - 5D^2 + 7D - 3)y = e^{2x} \cosh x$.

auxillary eq^n

$m^3 - 5m^2 + 7m - 3 = 0$

$(m-1)(m^2 - 4m + 3) = 0$

1	-5	7	-3
0	1	-4	3
1	-4	3	0

$m^2 - 3m - m + 3 = 0$

$m(m-3) - 1(m-3) = 0$

$(m-1)(m-1)(m-3) = 0$

$m = 1, 1, 3$.

$y_c = (c_1 + xc_2)e^{x} + c_3 e^{3x}$

$y_p = \frac{1}{D^3 - 5D^2 + 7D - 3} e^{2x} \times \left[\frac{e^{x} + e^{-x}}{2}\right]$

$= \frac{1}{D^3 - 5D^2 + 7D - 3} \left[\frac{e^{2x+x} + e^{2x-x}}{2}\right]$

$= \frac{1}{D^3 - 5D^2 + 7D - 3} \frac{1}{2}\left[e^{3x} + e^{x}\right]$

$\Rightarrow \frac{1}{2} \frac{e^{3x}}{D^3 - 5D^2 + 7D - 3} + \frac{1}{2} \frac{e^{x}}{D^3 - 5D^2 + 7D - 3}$

$\Rightarrow \frac{1}{2} \times \frac{x}{3D^2 - 10D + 7} \times e^{3x} + \frac{1}{2} \frac{x e^{x}}{3D^2 - 10D + 7}$

$\Rightarrow \frac{xe^{3x}}{8} + \frac{1}{2} \frac{x^2 e^{x}}{6D - 10}$

$= \frac{xe^{3x}}{8} - \frac{x^2 e^{x}}{8}$ $= \frac{xe^{3x} - e^{x} x^2}{8}$

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General solⁿ

$$y = (c_1 + x c_2)e^x + c_3 e^{3x} + \frac{x e^{3x}}{8} - x^2 e^x$$

Solve $(D^3 - 5D^2 + 7D - 3)y = e^{3x} \sinh x$

⇒ auxillary eqⁿ :

$m^3 - 5m^2 + 7m - 3 = 0$

$(m-1)(m-1)(m-3) = 0$

$m = 1, 1, 3$.

$y_c = (c_1 + x c_2)e^x + c_3 e^{3x}$

⇒ $y_p = \frac{1}{f(D)} e^{3x} \sinh x$

$= \frac{1}{f(D)} e^{3x} \left[\frac{e^x - e^{-x}}{2} \right]$

$= \frac{1}{f(D)} \left[\frac{e^{4x} - e^{2x}}{2} \right]$

$= \frac{1}{2} \left[\frac{e^{4x}}{D^3 - 5D^2 + 7D - 3} - \frac{e^{2x}}{D^3 - 5D^2 + 7D - 3} \right]$

$= \frac{1}{2} \left[\frac{e^{4x}}{9} + \frac{e^{2x}}{1} \right]$

$= \frac{e^{4x}}{18} + \frac{e^{2x}}{2}$

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General solⁿ

$$y = (c_1 + x c_2)e^x + c_3 e^{3x} + \frac{e^{4x}}{18} + \frac{e^{2x}}{2}$$

Solve $y''' - y = (e^x + 1)^2$

$(D^3 - 1)y = (e^x + 1)^2 \;\; \because = e^{2x} + 2e^x + 1$

auxillary eqⁿ

$m^3 - 1 = 0$

$(m-1)(m^2 + m + 1) = 0$.

$m = 1, \frac{-1 \pm \sqrt{3}i}{2}$.

$y_c = c_1 e^x + e^{-x/2} \left[c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} x \right]$

$y_p = \frac{1}{(D^3 - 1)} \left[e^{2x} + 2e^x + 1 \right]$.

$= \frac{e^{2x}}{D^3 - 1} + 2 \frac{e^x}{D^3 - 1} + \frac{1}{D^3 - 1} \times e^{0x}$

$= \frac{e^{2x}}{7} + 2 \frac{x e^x}{3D^2 - 1} + -1$

$y_p = \frac{e^{2x}}{7} + x e^x - 1$

$y = \cancel{c_1 e^x + x c}$

$c_1 e^x + e^{-x/2} \left[c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} \right.$

$+ \frac{e^{2x}}{7} + x e^x - 1$

9-1

Solve $y'' - 6y' + 9y = 6e^{3x} - 5\log 2$

$\Rightarrow (D^2 - 6D + 9)y = 6e^{3x} - 5\log 2$.

$\Rightarrow$ auxillary eq^n.

$m^2 - 6m + 9 = 0$

$m^2 - 3m - 3m + 9 = 0$

$m(m-3) - 3(m-3) = 0$

$m = 3, 3$

$y_c = (C_1 + xC_2)e^{3x}$

$y_p = \frac{1}{f(D)}[\log e^{3x\log 6} - 5\log 2]$

$= \frac{e^{3x\log 6}}{D^2 - 6D + 9} - \frac{e^{\log(5\log 2)} \times e^{0x}}{D^2 - 6D + 9}$

$= \frac{x e^{3x\log 6}}{2D - 6} + - \frac{e^{\log(5\log 2)} \times e^{0x}}{\cancel{2D-6}\ 9}$

$\Rightarrow \frac{6x^2 e^{3x\cancel{\log 6}}}{2} - \frac{e^{\log(5\log 2)}}{9}$

$= \frac{x^2 \times 6^{3x}}{2} - \frac{5\log 2}{9}$

$y = (C_1 + xC_2)e^{3x} + \frac{x^2 6^{3x}}{2} - \frac{5\log 2}{9}$

Extra Question

1. $(D^2 + 3D + 2)y = 2e^{2x}$

auxillary eq^n

$m^2 + 3m + 2 = 0$

$m^2 + 2m + m + 2 = 0$

$m(m+2) + 1(m+2) = 0$

$(m+1)(m+2) = 0$

$m = -1, -2$

$y_c = c_1 e^{-x} + c_2 e^{-2x}$

$y_p = \frac{1}{f(D)} \times 2e^{2x}$

$= \frac{2}{D^2 + 3D + 2} e^{2x}$

$= \frac{2}{4 + 6 + 2} e^{2x}$

$= \frac{2}{12} e^{2x}$

$= \frac{e^{2x}}{6}$

General sol^n

$y = c_1 e^{-x} + c_2 e^{-2x} + \frac{e^{2x}}{6}$

2. $(D^2 + 3D + 2)y = \sinh 3x$

auxillary eq^n

$m^2 + 3m + 2 = 0$

$m = -1, -2$

$y_p = \frac{1}{f(D)}\left[\frac{e^{3x} - e^{-3x}}{2}\right]$

$$= \frac{e^{3x}}{2(D^2+3D+2)} - \frac{e^{-3x}}{2(D^2+3D+2)}$$

$$= \frac{e^{3x}}{40} - \frac{e^{-3x}}{4}$$

General sol^n

$$y = c_1e^{-x} + c_2e^{-2x} + \frac{e^{3x}}{40} - \frac{e^{-3x}}{4}$$

3. $(D^2+3D+2)y = \sinh 3x \cosh 3x$

auxillary eq^n

$m^2+3m+2=0$

$m = -1, -2$

$$y_p = \frac{1}{f(D)}\left[\frac{e^{3x}-e^{-3x}}{2}\right]\left[\frac{e^{3x}+e^{-3x}}{2}\right]$$

$$= \frac{1}{4f(D)}\left[e^{6x}-e^{-6x}\right]$$

$$= \frac{e^{6x}}{4(D^2+3D+2)} - \frac{e^{-6x}}{4(D^2+3D+2)}$$

$$= \frac{e^{6x}}{224} - \frac{e^{-6x}}{80}$$

General eq^n

$$y = c_1e^{-x} + c_2e^{-2x} + \frac{e^{6x}}{224} - \frac{e^{-6x}}{80}$$

4. $(D^2+3D+2)y = 6\log 2$

auxillary eq^n

$m^2+3m+2=0$

$m=-1,-2$

$$y_p = \frac{1}{f(D)} 6\log 2 \times e^{0x}$$

$$= \frac{6\log 2}{2}$$

$$= 3\log 2$$

General sol^n

$$y = c_1e^{-x} + c_2e^{-2x} + 3\log 2$$

5. $(D^2+3D+2)y = 2e^{2x}\sinh 2x$

auxillary eq^n

$m^2+3m+2=0$

$m=-1,-2$

$$y_p = \frac{1}{f(D)} \times \cancel{2}e^{2x}\left[\frac{e^{2x}-e^{-2x}}{\cancel{2}}\right]$$

$$= \frac{1}{f(D)}\left[e^{4x}-e^{0x}\right]$$

$$= \frac{e^{4x}}{D^2+3D+2} - \frac{e^{0x}}{D^2+3D+2}$$

$$= \frac{e^{4x}}{30} - \frac{1}{2}$$

General sol^n

$$y = c_1e^{-x} + c_2e^{-2x} + \frac{e^{4x}}{30} - \frac{1}{2}$$

6. $(D^2+2D+1)y = 2e^{2x}\sinh 2x$

auxillary eq^n

$m^2+2m+1=0$

$m=-1,-1$

$y_c = (c_1+xc_2)e^{-x}$

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$$y_p = \frac{1}{f(D)} \times \cancel{2} e^{2x}\left[\frac{e^{2x}-e^{-2x}}{\cancel{2}}\right]$$

$$= \frac{e^{4x}}{D^2+2D+1} - \frac{1}{D^2+2D+1}$$

$$= \frac{e^{4x}}{25} - 1$$

General sol^n

$$y = (c_1+xc_2)e^{-x} + \frac{e^{4x}}{25} - 1$$

7. $(D^2+2D+1)y = \sinh 3x$

auxillary eq^n

$m^2+2m+1=0$

$m=-1,-1$

$$y_p = \frac{1}{f(D)}\left[\frac{e^{3x}-e^{-3x}}{2}\right]$$

$$= \frac{e^{3x}}{2(D^2+2D+1)} - \frac{e^{-3x}}{2(D^2+2D+1)}$$

$$= \frac{e^{3x}}{32} - \frac{e^{-3x}}{8}$$

General sol^n

$$y = (c_1+xc_2)e^{-x} + \frac{e^{3x}}{32} - \frac{e^{-3x}}{8}$$

8. $(D^2+2D+1)y = \sinh 3x \cosh 3x$

auxillary eq^n

$m^2+2m+1=0$

$m=-1,-1$

$$y_p = \frac{1}{f(D)}\left[\frac{e^{3x}-e^{-3x}}{2}\right]\left[\frac{e^{3x}+e^{-3x}}{2}\right]$$

$$= \frac{1}{4f(D)}\left[e^{6x}-e^{-6x}\right]$$

$$= \frac{e^{6x}}{4(D^2+2D+1)} - \frac{e^{-6x}}{4(D^2+2D+1)}$$

$$= \frac{e^{6x}}{196} - \frac{e^{-6x}}{100}$$

General sol^n

$$y = (c_1+xc_2)e^{-x} + \frac{e^{6x}}{196} - \frac{e^{-6x}}{100}.$$

9. $(D^2+2D+1)y = 6\log 2$

auxillary eq^n

$m^2+2m+1=0$

$m=-1,-1$

$$y_p = \frac{1}{f(D)} 6\log 2 \times e^{0x}$$

$$= \frac{6\log 2 \times e^{0x}}{(D^2+2D+1)}$$

$$= 6\log 2$$

General sol^n

$$y = (c_1+xc_2)e^{-x} + 6\log 2$$

10. $(D^2-1)y = 2e^{2x}\sinh 2x$

auxillary eq^n

$m^2-1=0$

$m^2=1$

$m=\pm 1$

$y_c = c_1 e^{-x} + c_2 e^{x}$

$$y_p = \frac{1}{f(D)} \times \cancel{2} e^{2x}\left[\frac{e^{2x}-e^{-2x}}{\cancel{2}}\right]$$

$$= \frac{e^{4x}}{D^2-1} - \frac{1}{D^2-1}$$

$= \frac{e^{4x}}{15} + 1$

General Sol^n

$y = C_1e^{-x} + C_2e^{x} + \frac{e^{4x}}{15} + 1$

11. $(D^2-1)y = \sinh 3x$

auxillary eq^n

$m^2-1=0$

$m=-1,1$

$y_p = \frac{1}{f(D)}\left[\frac{e^{3x}-e^{-3x}}{2}\right]$

$= \frac{e^{3x}}{2(D^2-1)} - \frac{e^{-3x}}{2(D^2-1)}$

$= \frac{e^{3x}}{16} - \frac{e^{-3x}}{16}$

General Sol^n

$y = C_1e^{-x} + C_2e^{x} + \frac{e^{3x}}{16} - \frac{e^{-3x}}{16}$

12. $(D^2-1)y = \sinh 3x \cosh 3x$

auxillary eq^n

$m^2-1=0$

$m=-1,1$

$y_p = \frac{1}{f(D)}\left[\frac{e^{3x}-e^{-3x}}{2}\right]\left[\frac{e^{3x}+e^{-3x}}{2}\right]$

$= \frac{1}{4(D^2-1)}\left[e^{6x}-e^{-6x}\right]$

$= \frac{e^{6x}}{4(D^2-1)} - \frac{e^{-6x}}{4(D^2-1)}$

$= \frac{e^{6x}}{140} - \frac{e^{-6x}}{140}$

General Sol^n.

$y = C_1e^{-x} + C_2e^{x} + \frac{e^{6x}}{140} - \frac{e^{-6x}}{140}$.

13. $(D^2-1)y = 6\log 2$

auxillary eq^n

$m^2-1=0$

$m=-1,1$

$y_p = \frac{1}{f(D)} 6\log 2 \times e^{0x}$

$= \frac{1}{D^2-1} 6\log 2$

$= -6\log 2$

General Sol^n

$y = C_1e^{-x} + C_2e^{x} + 6\log 2$

14. $(D^2-1)D^2 y = 2e^{2x}\sinh 2x$

auxillary eq^n

$m^2(m^2-1)=0$

$m=0,0 \quad , \quad m^2=1$

$m=\pm 1$

$y_c = C_1 + C_2 + C_3e^{-x} + C_4e^{x}$

$y_p = \frac{1}{f(D)} \cancel{2} e^{2x}\left[\frac{e^{2x}-e^{-2x}}{\cancel{2}}\right]$

$= \frac{e^{4x}}{D^2(D^2-1)} - \frac{1}{D^2(D^2-1)}$

$= \frac{e^{4x}}{240} - \frac{x^2}{12D^2-2}$

General Sol^n

$y = C_1 + C_2 + C_3e^{-x} + C_4e^{x} + \frac{e^{4x}}{240} + \frac{x^2}{2}$

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15. $D^2(D^2-1)y = \sinh 3x$

auxillary eq^n

$m^2(m^2-1)=0$

$m=0,0,-1,1$

$$y_p = \frac{1}{f(D)}\left[\frac{e^{3x}-e^{-3x}}{2}\right]$$

$$= \frac{e^{3x}}{2(D^2\times(D^2-1))} - \frac{e^{-3x}}{2(D^2(D^2-1))}$$

$$= \frac{e^{3x}}{144} - \frac{e^{-3x}}{144}$$

General sol^n

$$y = C_1 + C_2 + C_3e^{-x} + C_4e^{x} + \frac{e^{3x}}{144} - \frac{e^{-3x}}{144}$$

16. $D^2(D^2-1)y = \sinh 3x \cosh 3x$

auxillary eq^n

$m^2(m^2-1)=0$

$m=0,0,-1,1$

$$y_p = \frac{1}{f(D)}\left[\frac{e^{3x}-e^{-3x}}{2}\right]\left[\frac{e^{3x}+e^{-3x}}{2}\right]$$

$$= \frac{e^{6x}}{4(D^2(D^2-1))} - \frac{e^{-6x}}{4(D^2(D^2-1))}$$

$$= \frac{e^{6x}}{5040} - \frac{e^{-6x}}{5040}$$

General sol^n

$$y = C_1 + C_2 + C_3e^{-x} + C_4e^{x} + \frac{e^{6x}}{5040} - \frac{e^{-6x}}{5040}$$

17. $D^2(D^2-1)y = 6\log 2$

auxillary eq^n

$m^2(m^2-1)=0$

$m=0,0,-1,1$

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$$y_p = \frac{1}{D^2(D^2-1)} 6\log 2 \times e^{0x} + \frac{x^2\, 6\log 2}{12D^2-2}$$

$= 0$

General sol^n

$$y = C_1 + C_2 + C_3e^{-x} + C_4e^{x} \pm \frac{x^2 6\log 2}{-2}$$

Special Case $R(x) =$

1. $\frac{e^{ax}}{f(D)} = \frac{1}{f(a)} e^{ax}$ $\quad f(a) \neq 0$

$= \frac{x}{f'(a)} e^{ax}$ $\quad f'(a) \neq 0$

2. $\frac{1}{\phi(D^2)} \sin ax/\cos ax = \frac{1}{\phi(-a^2)} \sin ax/\cos ax$

$\phi(-a^2) \neq 0$

$= \frac{x}{\phi(-a^2)} \sin ax$

Solve $(D^2+9)y = \cos 4x$

auxillary eq^n

$(D^2+9)y = 0$

$m^2+9=0$

$m^2=-9$

$m = \pm 3i$

$y_c = (C_1\cos 3x + C_2\sin 3x)$

$$y_p = \frac{1}{-a^2+9} \cos 4x$$

$D = ai$

$$y_p = -\frac{\cos 4x}{7}$$

General Sol^n

$$y = (c_1 \cos 3x + c_2 \sin 3x) - \frac{\cos 4x}{7}$$

Solve $\{y'' - y' - 2y\} - \sin 2x = 0$

$$y'' - y' - 2y = \sin 2x$$

$$(D^2 - D - 2)y = \sin 2x$$

auxillary eq^n

$$m^2 - m - 2 = 0$$

$$m^2 - 2m + m - 2 = 0$$

$$m(m-2) + 1(m-2) = 0$$

$$m = -1, 2$$

$$y_p = \frac{1}{(D^2 - D - 2)} \sin 2x$$

$$= \frac{1}{-a^2 - a - 2} \sin 2x$$

$$= \frac{1}{-4 - D - 2} \sin 2x$$

$$= \frac{1}{-D - 6} \sin 2x$$

$$= \frac{-1}{(D+6)} \sin 2x$$

$$= (-1) \frac{1}{D+6} \times \frac{D-6}{D-6} \times \sin 2x$$

$$= (-1) \frac{(D-6)}{D^2 - 36} \sin 2x$$

$$= \frac{6-D}{D^2 - 36} \sin 2x$$

$$= \frac{-1}{40} (6 - D) \sin 2x$$

$$= -\frac{6}{40} \sin 2x + \frac{1}{40} D \sin 2x$$

$$= -\frac{3}{20} \sin 2x + \frac{1}{20} \cos 2x$$

Solve $(D^2 + 4)y = e^x + \sin 2x$

auxillary eq^n

$$m^2 + 4 = 0$$

$$m^2 = -4$$

$$m = \pm 2i$$

$$y_c = (c_1 \cos 2x + c_2 \sin 2x)$$

$$y_p = \frac{1}{(D^2 + 4)} [e^x + \sin 2x]$$

$$= \frac{e^x}{D^2 + 4} + \frac{\sin 2x}{-a^2 + 4}$$

$$= \frac{e^x}{5} + \frac{\sin 2x \times x}{-2d + 0}$$

$$= \frac{e^x}{5} + \frac{x \sin 2x}{-2d} \times \frac{d}{d}$$

$$= \frac{e^x}{5} - \frac{x}{2} \frac{D \sin 2x}{D^2}$$

$$= \frac{e^x}{5} + \left(-\frac{2x}{8}\right) \times 2\cos 2x$$

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General Sol^n

$y = (c_1 \cos 2x + c_2 \sin 2x) + \frac{e^x}{5} - \frac{x}{4}\cos 2x$

Solve $(D^2+4)y = \sin^2 x$

$(D^2+4)y = \frac{(1-\cos 2x)}{2}$

$y_p = \frac{1}{(D^2+4)}\left[\frac{1}{2} - \frac{\cos 2x}{2}\right]$

$= \frac{1}{2(D^2+4)} \times e^{0x} - \frac{1}{2(-a^2+4)}\cos 2x$

$= \frac{1}{8} - \frac{1}{2(-2D)} \times x \cos 2x$

$= \frac{1}{8} + \frac{x}{4D} \times \frac{D}{D}\cos 2x$

$= \frac{1}{8} + \frac{x}{4\times -4}\left[D\cos 2x\right]$

$= \frac{1}{8} \mp \frac{x}{\cancel{16}\,8} \times \cancel{2}\sin 2x$

$= \frac{1}{8}\left[1 \mp x\sin 2x\right]$

General Sol^n

$y = (c_1 \cos 2x + c_2 \sin 2x) + \frac{1}{8}\left[1 \mp x \sin 2x\right]$

Solve $(D^2+9)(D^2+1)y = \cos 3x$

auxillary eq^n

$(m^2+9)(m^2+1) = 0$

$m = \pm 3i, \pm i$

$y_c = (c_1 \cos 3x + c_2 \sin 3x) + (c_3 \cos x + c_4 \sin x)$

$y_p = \frac{1}{(D^2+9)(D^2+1)}\cos 3x$

$= \frac{1}{(D^2+9)}\left[\frac{\cos 3x}{D^2+1}\right]$

$= \frac{1}{D^2+9}\left[\frac{\cos 3x}{-8}\right]$

$= \frac{-1}{8}\left[\frac{\cos 3x}{-2D}\right]$

$= \frac{1}{16}\left[\frac{D}{D^2}\cos 3x\right]$

$= \frac{1}{16}\left[\frac{1}{-\cancel{9}\,3} \times -\cancel{3}\sin 3x\right]$

$= \frac{\sin 3x}{48}$

(1) $(D^2+2D+1)y = \sin 3x$

$\Rightarrow$ auxillary eqn

$m^2+2m+1=0$

$m^2+m+m+1=0$

$m(m+1)+1(m+1)=0$

$m=-1,-1$

$y_c = (C_1+xC_2)e^{-x}$

$y_p = \frac{1}{f(D)} \sin 3x$

1. $(D^2+4)y = \sin 3x$

$y_c = C_1\cos 2x + C_2\sin 2x$

$y_p = \frac{\sin 3x}{D^2+4}$

$= -\frac{\sin 3x}{5}$

General soln

$y = C_1\cos 2x + C_2\sin 2x - \frac{\sin 3x}{5}$

2. $(D^2+4)y = \cos 4x$

$y_c = C_1\cos 2x + C_2\sin 2x$

$y_p = \frac{\cos 4x}{D^2+4}$

$= -\frac{\cos 4x}{12}$

General soln

$y = C_1\cos 2x + C_2\sin 2x - \frac{\cos 4x}{12}$

3. $(D^2+9)(D^2+1)y = \sin 3x$

$y_c = (C_1\cos 3x + C_2\sin 3x) + (C_3\cos x + C_4\sin x)$

$y_p = \frac{\sin 3x}{(D^2+9)(D^2+1)}$

$= \frac{1}{(D^2+9)}\left[\frac{\sin 3x}{D^2+1}\right]$

$= \frac{1}{(D^2+9)}\left[-\frac{\sin 3x}{8}\right]$

$= -\frac{1}{8}\left[\frac{\sin 3x}{D^2+9}\right]$

$= -\frac{1}{8}\left[\frac{\sin 3x}{-2D}\right]$

$= \frac{1}{8}\left[\frac{D}{2D^2} \cos\sin 3x\right]$

$= \frac{1}{16}\left[\frac{1}{-9} \times 3\cos 3x\right]$

$= -\frac{\cos 3x}{48}$

4. $(D^2+9)(D^2+1)y = \cos 4x$

$y_c = (C_1\cos 3x + C_2\sin 3x) + (C_3\cos x + C_4\sin x)$

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$$y_p = \frac{\cos 4x}{(D^2+9)(D^2+1)}$$

$$= \frac{1}{(D^2+9)}\left[\frac{\cos 4x}{D^2+1}\right]$$

$$= \frac{1}{(D^2+9)}\left[\frac{-\cos 4x}{15}\right]$$

$$= -\frac{1}{15}\left[\frac{\cos 4x}{D^2+9}\right]$$

$$= -\frac{1}{15}\left[\frac{\cos 4x}{-7}\right]$$

$$= \frac{\cos 4x}{105}$$

General Sol^n.

$$y = (C_1\cos 3x + C_2\sin 3x) + (C_3\cos x + C_4\sin x) + \frac{\cos 4x}{105}$$

5. $(D^2 - D - 2)y = \sin 3x$

$$y_c = C_1 e^{-x} + C_2 e^{2x}$$

$$y_p = \frac{\sin 3x}{(D^2 - D - 2)}$$

$$= \frac{\sin 3x}{-9 - D - 2}$$

$$= \frac{\sin 3x}{-D - 11}$$

$$= -\frac{1}{(D+11)}\sin 3x$$

$$= -\frac{D-11}{(D^2-121)}\sin 3x$$

$$= \frac{11-D}{D^2-121}\sin 3x$$

$$= \frac{-11}{130}\sin 3x + \frac{D}{130}\sin 3x$$

$$= \frac{-11}{130}\sin 3x + \frac{3}{130}\cos 3x$$

6. $(D^2 - D - 2)y = \cos 4x$

$$y_c = C_1 e^{-x} + C_2 e^{2x}$$

$$y_p = \frac{\cos 4x}{(D^2 - D - 2)}$$

$$= \frac{\cos 4x}{-16 - D - 2}$$

$$= -\frac{1}{(D+18)}\cos 4x$$

$$= \frac{18 - D}{D^2 - 324}(\cos 4x)$$

$$= \frac{-18}{340}\cos 4x \mp \frac{4}{340}\sin 4x$$

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General Soln

$y = C_1 e^{-x} + C_2 e^{2x} - \frac{18}{340} \cos 4x - \frac{4}{340} \sin 4x$

* Special Case

$\frac{1}{f(D)} R(x)$ where $R(x) = x^m$

$= [1 - \phi(D)]^{-1}$

$\frac{1}{(D^3+2)^2} = \frac{1}{D^2+4D+4}$

$= \frac{1}{4(D^2/4 + D + 1)}$

$= \frac{1}{4} [1 - (-D^2/4 - D)]^{-1}$

General eqn

$(1+x)^a = 1 + ax + \frac{a(a-1)}{2!} x^2 + \frac{a(a-1)(a-2)}{3!} x^3 + \dots + \frac{a(a-1)\dots(a-(n-1))}{n!} x^n$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \dots$

$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots$

$(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$

Solve $(D^2 - 4)y = x^2$

auxillary eqn

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm 2$

$y_c = C_1 e^{-2x} + C_2 e^{2x}$

$y_p = \frac{1}{D^2 - 4} \times x^2$

$= \frac{1}{4\left(\frac{D^2}{4} - 1\right)} x^2$

$= \cancel{\frac{1}{-4}(1}$

$= \frac{-1}{4} \left(\frac{1}{1 - D^2/4} \right) x^2$

$= -\frac{1}{4} (1 - D^2/4)^{-1} x^2$

$= \frac{-1}{4} \left[1 + \frac{D^2}{4} + \frac{D^4}{16} + \frac{D^6}{64} + \dots \right] x^2$

$= \cancel{\frac{-1}{4} \left[x^2 + \frac{D^2 x^2}{4} + \frac{D^4 x^2}{16} \right]}$

$= \frac{-1}{4} [x^2 +$

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$= -\frac{1}{4}\left[x^2 + \frac{1}{4}(2) + 0 + 0\right]$

$= -\frac{1}{4}\left[x^2 + \frac{1}{2}\right]$

Solve $(D^3 - D^2 - 6D)y = x^2 + 1$

auxillary eq^n

$m^3 - m^2 - 6m = 0$

$\cancel{(m-2)}\quad m(m^2 - m - 6) = 0$

$m = 0$

$m^2 - 3m + 2m - 6 = 0$

$m = 3, -2$

$y_c = C_1 + C_2 e^{-2x} + C_3 e^{3x}$

$y_p = \frac{1}{(D^3 - D^2 - 6D)} \times (x^2 + 1)$

$= \frac{1}{-6D\left(1 - \frac{D^2}{6} + \frac{D}{6}\right)} \times x^2 + 1$

$= \frac{1}{-6D}\left(1 - \left(D^2/6 - D/6\right)\right)^{-1}(x^2+1)$

$= \frac{-1}{6D}\left[1 + \left(D^2/6 \mp D/6\right) + \left(D^2/6 - D/6\right)^2 + \cdots\right](x^2+1)$

$= -\frac{1}{6D}\left[1 + \frac{1}{6}(D^2 - D) + \frac{1}{36}\cancel{D^2}(D^2 - D)^2 + \cdots\right](x^2+1)$

$= \frac{-1}{6D}\left[x^2 + 1 + \frac{1}{3} - \frac{x}{3} + \frac{1}{18}\right]$

$= \frac{-1}{6}\int\left(x^2 + 1 + \frac{1}{3} - \frac{x}{3} + \frac{1}{18}\right)dx$

$= \frac{-1}{6}\left[\frac{x^3}{3} + x + \frac{x}{3} - \frac{x^2}{6} + \frac{x}{18}\right]$

$= \frac{-1}{6}\left[\frac{x^3}{3} - \frac{x^2}{6} + \frac{18x + 6x + x}{18}\right]$

$= \frac{-1}{6}\left[\frac{x^3}{3} - \frac{x^2}{6} + \frac{25x}{18}\right]$

General Sol^n

$y = C_1 + C_2 e^{-2x} + C_3 e^{3x} - \frac{1}{6}\left[\frac{x^3}{3} - \frac{x^2}{6} + \frac{25x}{18}\right]$

Special Case

$R(x) = e^{ax}(V(x)) = \frac{1}{f(D)} R(x)$

$= e^{ax}\frac{1}{f(D+a)} V(x)$

Solve $(D^2+3D+2)y = e^{2x}\sin x$

auxillary eq^n

$m^2+3m+2=0$

$m^2+2m+m+2=0$

$m(m+2)+1(m+2)=0$

$(m+1)(m+2)=0$

$m=-1,-2$

$y_c = C_1e^{-x}+C_2e^{-2x}$

$y_p = e^{2x}\dfrac{1}{D^2+3D+2}\sin x$

$= e^{2x}\dfrac{1}{(D+1)(D+2)}\sin x$ not required

$= e^{2x}\dfrac{1}{(D+1)}\left[\dfrac{\sin x}{D+2}\right]$

$= e^{2x}\dfrac{1}{(D+2)^2+3(D+2)+2}\sin x$

$= e^{2x}\dfrac{1}{D^2+7D+12}\sin x$

$= e^{2x}\dfrac{1}{(-1+7D+12)}\sin x$

$= e^{2x}\dfrac{1}{(7D+11)}\sin x$

$= e^{2x}\dfrac{(7D-11)}{49D^2-121}\sin x$

$= e^{2x}\left[\dfrac{7D\sin x}{-72}\right]+e^{2x}\dfrac{11}{72}\sin x$

$= e^{2x}\left[\dfrac{-7\cos x}{72}+\dfrac{11}{72}\sin x\right]$

General sol^n

$y = C_1e^{-x}+C_2e^{-2x}+e^{2x}\left[\dfrac{-7}{72}\cos x+\dfrac{11}{72}\sin x\right]$

Solve $(D^2-2D+1)y = x^2e^{3x}$

auxillary eq^n

$m^2-2m+1=0$

$m^2-m-m+1=0$

$m(m-1)-1(m-1)=0$

$m=+1,+1$

$y_c = (C_1+xC_2)e^{x}$

$y_p = e^{3x}\dfrac{1}{D^2-2D+1}x^2$

$= e^{3x}\dfrac{1}{(D+3)^2-2(D+3)+1}x^2$

$= e^{3x}\dfrac{1}{D^2+4D+4}x^2$

$= e^{3x}\left[\dfrac{x^2}{4\left[\frac{D^2}{4}+D+1\right]}\right]$

$= \dfrac{e^{3x}}{4}\left[\left[1+(D^2/4+D)^{-1}\right]x^2\right]$

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$= \frac{e^{3x}}{4}\left[1-\left(\frac{D^2}{4}+D\right)+\left(\frac{D^2}{4}+D\right)^2-\dots\right]x^2$

$= \frac{e^{3x}}{4}\left[x^2-\left(\frac{2}{4}+2x\right)+2\right]$

$= \frac{e^{3x}}{4}\left[x^2 \mp 2x+\frac{3}{2}\right]$

General Solⁿ

$y = (c_1+xc_2)e^x+\frac{e^{3x}}{4}\left[x^2\mp 2x+\frac{3}{2}\right]$

Solve $(D^2-1)y = \cosh x \cos x$

auxillary eqⁿ.

$m^2-1=0$

$m^2=1$

$m=\pm 1$

$y_c = c_1e^{-x}+c_2e^{x}$

$y_p = \frac{1}{(D^2-1)}\cosh x \cos x$

$= \frac{1}{[D^2-1]}\left[\frac{e^x+e^{-x}}{2}\right]\cos x$

$= \frac{1}{2(D^2-1)}\left[e^x\cos x + e^{-x}\cos x\right]$

$= \frac{e^x}{2}\left[\frac{\cos x}{D^2-1}\right]+\frac{e^{-x}}{2}\left[\frac{\cos x}{D^2-1}\right]$

$= \frac{e^x}{2}\times\frac{\overset{\cos x}{\cancel{x\sin x}}}{2}+\frac{e^{-x}}{2}\times+\frac{x\cos x}{2}$

$= -\frac{\cancel{x}e^x\cos x}{4} \mp \frac{\cancel{x}e^{-x}\cos x}{4}$

General Solⁿ.

$y = c_1e^{-x}+c_2e^{x}\mp\left[\frac{xe^x\cos x}{4}+\frac{xe^{-x}\cos x}{4}\right]$

$\frac{1}{0}\quad\frac{2}{0}\quad\frac{0}{1}\quad\frac{0}{2}\quad\frac{0}{0}$

1. Solve $(D^2-2D+1)y = x^2e^x$

auxillary eqⁿ

$m^2-2m+1=0$

$m^2-m-m+1=0$

$m(m-1)-1(m-1)=0$

$m=1,1$

$y_c = (c_1+xc_2)e^x$

$y_p = e^x\frac{1}{D^2-2D+1}\times x^2$

$= e^x\frac{1}{(D+1)^2-2(D+1)+1}x^2$

$= e^x\frac{x^2}{D^2}$

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$$= e^{x} \times \frac{1}{D}\int x^2 dx = e^{x} \frac{1}{D} \times \frac{x^3}{3}$$

$$= e^{x} \times \frac{1}{3}\int x^3 dx = \frac{1}{12} x^4 e^{x}$$

General soln

$$y = (c_1 + x c_2) e^{x} + \frac{1}{12} x^4 e^{x}$$

2. $(D^2 - 3D + 2)y = x e^{x}$

auxillary eqn

$$m^2 - 3m + 2 = 0$$

$$m^2 - 2m - m + 2 = 0$$

$$m(m-2) - 1(m-2) = 0$$

$$m = 1, 2$$

$$y_c = c_1 e^{x} + c_2 e^{2x}$$

$$y_p = e^{x} \times \frac{x}{D^2 - 3D + 2}$$

$$= e^{x} \times \frac{x}{(D+1)^2 - 3(D+1) + 2}$$

$$= e^{x} \times \frac{x}{D(D-1)}$$

$$= e^{x} \times \frac{1}{-D} (1-D)^{-1} x$$

$$= -\frac{e^{x}}{D} [1 + D + D^2 + D^3 + \dots] x$$

$$= -\frac{e^{x}}{D} [x + 1]$$

$$= -e^{x} \int (x+1) dx$$

$$= -e^{x} \left[\frac{x^2}{2} + x\right]$$

General soln

$$y = c_1 e^{x} + c_2 e^{2x} - e^{x} \left(\frac{x^2}{2} + x\right)$$

3. $(D^3 - 3D - 2)y = 540 x^3 e^{-x}$

auxillary eqn.

$$m^3 - 3m - 2 = 0$$

$$(m+1)(m^2 - m - 2) = 0$$

$$m^2 - m - 2 = 0$$

$$m^2 - 2m + m - 2 = 0$$

$$m(m+2) - 1(m+2) = 0$$

$$m = -1, -1, 2$$

$$y_c = c_1 e^{-x} + x c_2 e^{-x} + c_3 e^{2x}$$

$$= (c_1 + x c_2) e^{-x}$$

$$y_p = e^{-x} \times 540 \times \frac{x^3}{D^3 - 3D - 2}$$

$$= 540 e^{-x} \times \frac{x^3}{(D-1)^3 - 3(D-1) - 2}$$

$$= 540 e^{-x} \times \frac{x^3}{D^3 - 3D^2}$$

$$= 540 e^{-x} \times \frac{x^3}{D^2 (D-3)}$$

$$= 540 e^{-x} \times \frac{x^3}{-3D^2\left(1 - \frac{D}{3}\right)}$$

$$= -180 e^{-x} \times \frac{(1 - D/3) x^3}{D^2}$$

$$= -\cancel{180} e^{-x} \frac{\left(x^3 - \frac{\cancel{3} x^2}{\cancel{3}}\right)}{\cancel{D^2}}$$

$= \cancel{-180e^{-x} \times \frac{1}{D} \int (x^3 - x^2) dx}$

$= \cancel{-180e^{-x} \times \frac{1}{D} \left[\frac{x^4}{4} - \frac{x^3}{3}\right]}$

$= \cancel{-180e^{-x} \times \left[\frac{x^5}{20} - \frac{x^4}{12}\right]}$

$=$

$= -180e^{-x} \times \frac{1}{D^2} \left(1 + \frac{D}{3} + \frac{D^2}{9} + \frac{D^3}{27} + \frac{D^4}{81} + \cdots\right) x^3$

$= -180e^{-x} \times \frac{1}{D^2} \left[x^3 + \frac{3x^2}{3} + \frac{6x}{9} + \frac{6}{27}\right]$

$= -180e^{-x} \times \frac{1}{D} \int \left[x^3 + 3x^2 + \frac{2}{3}x + \frac{2}{9}\right] dx$

$= -180e^{-x} \times \frac{1}{D} \left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{3} + \frac{2}{9}x\right]$

$= -180e^{-x} \int \left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{3} + \frac{2}{9}x\right] dx$

$= -180e^{-x} \left[\frac{x^5}{20} + \frac{x^4}{12} + \frac{x^3}{9} + \frac{x^2}{9}\right]$

$= -e^{-x} [9x^5 + 15x^4 + 20x^3 + 20x^2]$

General soln.

$y = (c_1 + xc_2)e^{-x} + c_3 e^{2x} - e^{-x}$
$(9x^5 + 15x^4 + 20x^3 + 20x^2)$

4. $(D-2)^3 y = xe^{2x}$

auxillary eqn

$(m-2)^3 = 0$

$m - 2 = 0$

$m = 2$ (3 times)

$y_c = (c_1 + xc_2 + x^2 c_3)e^{2x}$

$y_p = e^{2x} \times \frac{x}{(D-2)^3}$

$= e^{2x} \times \frac{x}{(D+2-2)^3}$

$= e^{2x} \times \frac{x}{D^3}$

$= e^{2x} \times \frac{1}{D^2} \int x dx$

$= e^{2x} \times \frac{1}{D^2} \left[\frac{x^2}{2}\right]$

$= \frac{e^{2x}}{2} \times \frac{1}{D} \int x^2 dx$

$= \frac{e^{2x}}{2} \times \frac{1}{D} \times \frac{x^3}{3}$

$= \frac{e^{2x}}{6} \times \int x^3 dx$

$= \frac{e^{2x}}{6} \times \frac{x^4}{4}$

$= \frac{1}{24} x^4 e^{2x}$

General soln

$y = (c_1 + xc_2 + x^2 c_3)e^{2x} + \frac{1}{24} x^4 e^{2x}$

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$$y_p = \frac{1}{f(D)} x\, V(x)$$
$$= x \frac{1}{f(D)} V(x) - \frac{f'(D)}{(f(D))^2} V(x)$$

1. Solve $(D^2+9)y = x\sin x$

auxillary eq^n

$m^2+9=0$

$m^2=-9$

$m=\pm 3i$

$y_c = (c_1\cos 3x + c_2\sin 3x)$

$$y_p = \frac{1}{(D^2+9)} x \times \sin x$$
$$= x\left[\frac{\sin x}{D^2+9}\right] - \frac{2D}{(D^2+9)^2}\sin x$$
$$= x\left[\frac{\sin x}{-1+9}\right] - 2D\left[\frac{\sin x}{(-1+9)^2}\right]$$
$$= \frac{x}{8}\sin x - \frac{2}{64} D\sin x$$
$$= \frac{x\sin x}{8} - \frac{\cos x}{32}$$

General Sol^n

$$y = (c_1\cos 3x + c_2\sin 3x) + \frac{x}{8}\sin x - \frac{\cos x}{32}$$

2. $(D^2-2D+1)y = x\sin x$

auxillary eq^n

$m^2-2m+1=0$

$m^2-m-m+1=0$

$m(m-1)-1(m-1)=0$

$m=1,1$

$y_c = (c_1+xc_2)e^x$

$$y_p = \frac{1}{(D^2-2D+1)} x\sin x$$
$$= x\left[\frac{\sin x}{D^2-2D+1}\right] - \frac{(2D-2)\sin x}{[D^2-2D+1]^2}$$
$$= x\left[\frac{\sin x}{-2D}\right] - \frac{2D}{(-2D)^2}\sin x + \frac{2}{(-2D)^2}\sin x$$
$$= -\frac{x}{2}\times -\cos x - \frac{1}{2}\times -\cos x + \frac{1}{2D} - \cos x$$
$$= \frac{x}{2}\cos x + \frac{1}{2}\cos x - \frac{1}{2}\sin x$$
$$= \frac{1}{2}(x\cos x + \cos x - \sin x)$$

General Sol^n

$$y = (c_1+xc_2)e^x + \frac{1}{2}(x\cos x + \cos x - \sin x)$$

3. $(D^2+4)y = x\sin 2x$

auxillary eq^n.

$m^2+4=0$

$m^2=-4$

$m=\pm 2i$

$y_c = (c_1\cos 2x + c_2\sin 2x)$

$$y_p = \frac{1}{(D^2+4)} x\sin 2x$$
$$= x\left[\frac{\sin 2x}{D^2+4}\right] - \frac{2D}{(D^2+4)^2}\sin 2x$$
$$= x \times \frac{x\sin 2x}{2D} - \frac{2D\,x}{2(D^2+4)\times 2D}\sin 2x$$
$$= \frac{x^2}{2}\int \sin 2x\,dx - \frac{x^2}{2\times 2D}\sin 2x$$

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$$= -\frac{x^2}{4}\cos 2x + \frac{x^2}{8}\cos 2x$$

$$= -\frac{x^2}{8}\cos 2x$$

General sol^n

$$y = (c_1\cos 2x + c_2\sin 2x) - \frac{x^2}{8}\cos 2x$$

$$e^{iax} = \cos ax + i\sin ax$$

$\cos ax$ = Real Part of e^{iax}

$\sin ax$ = I.P of e^{iax}

4. $(D^2+1)y = x^2\sin 2x$

$$m^2+1=0$$

$$m^2=-1$$

$$m=\pm i$$

$$y_c = (c_1\cos x + c_2\sin x)$$

$$y_p = \frac{1}{D^2+1} \times x^2\sin 2x$$

$$= \text{I.P of } \left(\frac{x^2 \times e^{i2x}}{(D^2+1)}\right)$$

$$= \text{I.P of } \left(\frac{x^2 \times e^{i2x}}{(D+2i)^2+1}\right)$$

$$= \text{I.P of } \left(e^{i2x} \frac{1}{D^2+4Di-3} x^2\right)$$

$$= (\cos 2x + i\sin 2x)\left[e^{i2x} \times -\frac{1}{3}\left(1+\left(\frac{D^2}{3}+\frac{4Di}{3}\right)+\left(\frac{D^2}{3}+\frac{4Di}{3}\right)^2+\dots\right)x^2\right]$$

$$= (\cos 2x + i\sin 2x)\left[-\frac{e^{i2x}}{3}\left[x^2+\frac{2}{3}+\frac{8}{3}xi+\frac{16\times 2i^2}{9}\right]\right]$$

$$= (\cos 2x + i\sin 2x)\left[-\frac{e^{i2x}}{3}\left[x^2+\frac{2}{3}+\frac{8}{3}xi-\frac{32}{9}\right]\right]$$

$$= -\frac{1}{3}\,\text{I.P}\left[e^{i2x}\left[x^2+\frac{8}{3}xi-\frac{14}{3}\right]\right]$$

$$= -\frac{1}{3}(\cos 2x + i\sin 2x)\left(x^2+\frac{8}{3}xi-\frac{14}{3}\right)$$

$$= -\frac{x^2}{3}\sin 2x - \frac{8}{9}x\cos 2x + \frac{14}{9}\sin 2x$$

Homework

General sol^n

$$y = (c_1\cos x + c_2\sin x) - \frac{x^2}{3}\sin 2x - \frac{8}{9}x \times \cos 2x + \frac{14}{9}\sin 2x$$

H.W 1. $(D^2+9)y = x\sin x$

$$y_p = \frac{1}{(D^2+9)} x\sin x$$

$$= \text{I.P of }\left[e^{ix}\left[\frac{x}{(D+i)^2+9}\right]\right]$$

$$= \text{I.P of }\left[e^{ix}\left[\frac{x}{D^2+2Di+8}\right]\right]$$

$$= \text{I.P of }\left[\frac{e^{ix}}{8}\frac{x}{[1+(D^2/8+Di/4)]}\right]$$

$$= \frac{1}{8}\,\text{I.P of }\left[e^{ix}[1+(D^2/8+Di/4)]^{-1}x\right]$$

$$= \frac{1}{8}\,\text{I.P of }\left[e^{ix}\left[1-\left(\frac{D^2}{8}+\frac{Di}{4}\right)+\left(\frac{D^2}{8}+\frac{Di}{4}\right)^2-\dots\right]x\right]$$

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$= \frac{1}{8}$ I.P of $\left[e^{ix}\left[x - \frac{i}{4}\right]\right]$

$= \frac{1}{8}(\cos x + i\sin x)\left(x - \frac{i}{4}\right)$

$= \frac{x}{8}\sin x - \frac{\cos x}{32}$

2. $(D^2 - 2D + 1)y = x\sin x$

$y_p = \frac{1}{(D^2 - 2D + 1)} x\sin x$

$=$ I.p of $\left[e^{ix}\left[\frac{x}{(D+i)^2 - 2(D+i) + 1}\right]\right]$

$=$ I.P of $\left[e^{ix}\left[\frac{x}{D^2 + 2Di - \cancel{1} - 2D - 2i + \cancel{1}}\right]\right]$

$=$ I.p of $\left[e^{ix}\left[\frac{x}{(D^2 - 2D) + (2D - 2)i}\right]\right]$

$=$ I.P of $\left[e^{ix}\left[\frac{x}{D^2 + 2D(i-1) - 2i}\right]\right]$

$=$ I.P of $e^{ix}\left[\frac{x}{2i\left[\frac{D^2}{2i} + \frac{2D}{2i}(i-1) - 1\right]}\right]$

$= \frac{\text{I.P of } e^{ix}}{-2i}\left[\frac{x}{1 - \left(\frac{D^2}{2i} + \frac{2D}{2i}(i-1)\right)}\right]$

$= \frac{-1}{2i}$ I.P of $e^{ix}\left[1 + \frac{D^2}{2i} + \frac{2D}{2i}(i-1)\right]x$

$= \frac{-1}{2i}$ I.P of $e^{ix}\left[x + \frac{\cancel{2}}{\cancel{2}i}(i-1)\right]$

$= \frac{(-\cancel{2}i)}{42}$ I.P of $e^{ix}\left[x + (-i)(i-1)\right]$

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$=$ I.P of $e^{ix}\left[\frac{i}{2}(x + i + 1)\right]$

$= \frac{i}{2}[\cos x + i\sin x](x + i + 1)$

$= \frac{i}{2}\left[x\cos x + i\cos x + \cos x + xi\sin x + \sin x + i\sin x\right]$

$= \frac{x}{2}\cos x + \frac{\cos x}{2} +$

$= [\cos x + i\sin x] \times \frac{1}{2}[xi + i - 1]$

$= \frac{x}{2}\cos x + \frac{1}{2}\cos x - \frac{\sin x}{2}$

3. $(D^2 + 4)y = x\sin 2x$

$y_p = \frac{1}{D^2 + 4} x\sin 2x$

$=$ I.P of $e^{i2x}\left[\frac{x}{(D + i2)^2 + 4}\right]$

$=$ I.P of $e^{i2x}\left[\frac{x}{D^2 + 4Di}\right]$

$=$ I.P of $e^{i2x}\left[\frac{x}{4Di\left[\frac{D}{4i} + 1\right]}\right]$

$= \frac{1}{4Di}$ I.P of $e^{i2x}\left[[1 + D/4i]^{-1}x\right]$

$= \frac{1}{4Di}$ I.P of $e^{i2x}\left[1 - \frac{D}{4}i + \frac{D^2 i^2}{16}\right]x$

$= -\frac{i}{4D}$ I.P of $e^{i2x}\left[x - \frac{i}{4}\right]$

$= \frac{-i}{4}$ I.P of $e^{i2x}\left[\frac{x^2}{2} - \frac{xi}{4}\right]$

$= \frac{1}{4}$ I.P of $e^{i2x}\left[-x^2 i - \frac{1}{4}x\right]$

$= \frac{1}{4}[\cos 2x + i\sin 2x]\left[-\frac{x^2 i}{2} - \frac{1}{4}x\right]$

$= \frac{-x^2 \cos 2x}{4} - \frac{1}{16}x\sin 2x$

$= -\frac{x^2}{8}\cos 2x - \frac{x}{16}\sin 2x$

$\sinh ax = \frac{e^{aix} - e^{-aix}}{2}$

$\sin ax = \frac{e^{iax} \pm e^{-iax}}{2}$

$e^{iax} = \cos ax + i\sin ax$
$e^{-iax} = \cos ax - i\sin ax$

Method of undetermined Coefficient

1. $e^x = \{e^x\}$
2. $\sin x / \cos x = \{\sin x, \cos x\}$
3. $x^m = \{mx^{m-1} + (m-2)x^{m-2} + \dots + Cx^0\}$

1. $\frac{d^2y}{dx^2} - 4y = e^x + \sin 2x$

$y_c = c_1 e^{-2x} + c_2 e^{2x}$
$y_p = c_3 e^x + c_4 \sin 2x + c_5 \cos 2x$

$y = y_c + y_p$
$= c_1 e^{-2x} + c_2 e^{2x} + c_3 e^x + c_4 \sin 2x + c_5 \cos 2x$

$y' = -2c_1 e^{-2x} + 2c_2 e^{2x} + c_3 e^x + 2c_4 \cos 2x - 2c_5 \sin 2x$

$y'' = 4c_1 e^{-2x} + 4c_2 e^{2x} + c_3 e^x + - 4c_4 \cos\sin 2x - 4c_5 \cos 4x$

$\Rightarrow \cancel{4c_1 e^{-2x}} + 4c_2 \cancel{e^{2x}} + c_3 e^x \cancel{+ 4c_4 \sin} 2x - 4c_5 \cos 4x - 4c_1 \cancel{e^{-2x}} - 4c_2 \cancel{e^{2x}} - 4c_3 e^x - 4\cancel{c_4 \sin} 2x - 4c_5 \cos 2x = e^x + \sin 2x$

$\Rightarrow -3c_3 e^x - 8c_5 \cos 2x - 8c_4 \sin 2x = e^x + \sin 2x$

$\Rightarrow -3c_3 = 1$
$c_3 = -\frac{1}{3}$

$c_5 = 0$
$-8c_4 = 1$
$c_4 = -\frac{1}{8}$

$$\boxed{y_p = -\frac{e^x}{3} - \frac{\sin 2x}{8}}$$

2. $y'' + y' - 12y = e^{3x}$

$y_c = c_1 e^{-4x} + c_2 e^{3x}$

$y_p = c_3 e^{3x}$
$= c_3 \times x e^{3x}$

$y'_p = c_3 [3x e^{3x} + e^{3x}]$

$= 3c_3 x e^{3x} + c_3 e^{3x}$

$y''_p = 3c_3 [3x e^{3x} + e^{3x}] + 3c_3 e^{3x}$

$= 9c_3 x e^{3x} + 3c_3 e^{3x} + 3c_3 e^{3x}$
$= 9c_3 x e^{3x} + 6c_3 e^{3x}$

$= 9c_3 x e^{3x} + 6c_3 e^{3x} + 3c_3 x e^{3x} + c_3 e^{3x} - 12 c_3 x e^{3x} = e^{3x}$

$= 7c_3 e^{3x} = e^{3x}$

$c_3 = \frac{1}{7}$

$$\boxed{y_p = \frac{x e^{3x}}{7}}$$

General Sol^n
$y = c_1 e^{-4x} + c_2 e^{3x} + \frac{x e^{3x}}{7}$

3. $y'' - 9y = 4 + 5\sinh 3x$
auxillary eq^n
$m^2 - 9 = 0$
$m^2 = 9$
$m = \pm 3$
$y_c = c_1 e^{-3x} + c_2 e^{3x}$

$y_p = c_3 + c_4 x \sinh 3x + c_5 x \cosh 3x$

$y'_p = c_4 \sinh 3x + 3c_4 x \cosh 3x + c_5 \cosh 3x + 3c_5 x \sinh 3x$

$y''_p = 3c_4 \cosh 3x + 3c_4 \cosh 3x + 9c_4 x \sinh 3x + 3c_5 \sinh 3x + 3c_5 \sinh 3x + 9c_5 x \cosh 3x$

$y'' - 9y$
$= 6c_4 \cosh 3x + \cancel{9c_4 x \sinh 3x} + 6c_5 \sinh 3x + \cancel{9c_5 x \cosh 3x} - 9c_3 - \cancel{9c_4 x \sinh 3x} - \cancel{9c_5 x \cosh 3x}$
$= 4 + 5\sinh 3x$

$= c_3 = -\frac{4}{9}, \quad c_5 = \frac{5}{6}$

$$\boxed{y_p = -\frac{4}{9} + \frac{5}{6} x \cosh 3x}$$

General Sol^n
$y = c_1 e^{-3x} + c_2 e^{3x} - \frac{4}{9} + \frac{5}{6} x \cosh 3x$

A. $y'' - 9y = 4 + 5\sinh x$

$\Rightarrow$ $y_c = c_1 e^{3x} + c_2 e^{-3x}$

$y_p = c_3 + c_4 \sinh x + c_5 \cosh x$

$y'_p = c_4 \cosh x + c_5 \sinh x$

$y''_p = c_4 \sinh x + c_5 \cosh x$

$\Rightarrow$ $c_4 \sinh x + c_5 \cosh x - 9c_3 - 9c_4 \sinh x - 9c_5 \cosh x = 4 + 5\sinh x$

$\Rightarrow$ $-8c_4 \sinh x - 8c_5 \cosh x - 9c_3 = 4 + 5\sinh x$

$c_4 = -\frac{5}{8}$

$= \quad c_3 = -\frac{4}{9}$

$= \quad \boxed{y_p = -\frac{4}{9} - \frac{5}{8}\sinh x}$

General soln

$y = c_1 e^{3x} + c_2 e^{-3x} - \frac{4}{9} - \frac{5}{8}\sinh x$

Mid : 2

Method of Variation of Paramete[illegible]

* Wronskin Method.

1) $= \begin{vmatrix} 2x^2+2x & 2x^2-2x \\ 4x+2 & 4x-2 \end{vmatrix}$

$= (4x-2)(2x^2+2x) - (4x+2)(2x^2-2x)$

$= 8x^3 + 8x^2 - 4x^2 - 4x - 8x^3 - 4x^2 + 8x^2 + 4x$

$= 8x^2$

L.I.

2) $\sin x, \cos x$

$= \begin{vmatrix} \sin x & \cos x \\ \cos x & -\sin x \end{vmatrix}$

$= -(\sin^2 x + \cos^2 x)$

$= -1$

L.I.

3) $\sinh x, e^x, e^{-x}$

$= \begin{vmatrix} \sinh x & e^x & e^{-x} \\ \cosh x & e^x & -e^{-x} \\ \sinh x & e^x & e^{-x} \end{vmatrix}$

$= 0$

L.D

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4) $\cosh x, e^x \quad = 1$

$= \begin{vmatrix} \cosh x & e^x \\ \sinh x & e^x \end{vmatrix}$

$= e^x(\cosh x - \sinh x)$

$= e^x\left[\frac{e^x + e^{-x} - e^x + e^{-x}}{2}\right]$

$= e^x \times e^{-x}$

$= e^0$

$= 1$

5) $e^x, 3e^x$

$= \begin{vmatrix} e^x & 3e^x \\ e^x & 3e^x \end{vmatrix}$ L.D.

$= 0$

$\Rightarrow y_1(x), \; y_2(x)$

$w(y_1, y_2)x = w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$\Rightarrow \; y_1, y_2, y_3$

$w = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}$

$y_p = -y_1\int\frac{y_2(R(x))}{w}dx + y_2\int\frac{y_1 R(x)}{w}dx$

$y_p = u_1(x)y_1 + u_2(x)y_2$

$u_1(x) = -\int\frac{y_2 R(x)}{w}dx$

$u_2(x) = \int\frac{y_1 R(x)}{w}dx$

Q $y'' + 4y = \sec 2x$

auxillary eqn

$m^2 + 4 = 0$

$m = \pm 2i$

$y_c = c_1\sin 2x + c_2\cos 2x$

$y_1 = \sin 2x, \quad y_2 = \cos 2x$

$w = \begin{vmatrix} \sin 2x & \cos 2x \\ 2\cos 2x & -2\sin 2x \end{vmatrix}$

$= -2\sin^2 2x - 2\cos^2 2x$

$= |-2|$

$= 2$

$u_1(x) = -\int\frac{\cos 2x \sec 2x}{2}dx$

$= -\frac{x}{2}$

$u_2(x) = \int\frac{\sin 2x \sec 2x}{2}$

$= +\frac{1}{4}\log(\sec 2x)$

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$$y_p = \frac{x}{2}\sin 2x - \frac{1}{4}\log(\sec 2x)\cos 2x$$

$$G.S = y_c + y_p$$

$$\Rightarrow y_p = u_1y_1 + u_2y_2 + u_3y_3$$

$$u_1 = \int \frac{\begin{vmatrix} 0 & y_2 & y_3 \\ 0 & y_2' & y_3' \\ R(x) & y_3'' & y_3'' \end{vmatrix}}{w} dx$$

$$u_2 = \int \frac{\begin{vmatrix} y_1 & 0 & y_3 \\ y_1' & 0 & y_3' \\ y_1'' & R(x) & y_3'' \end{vmatrix}}{w} dx$$

$$u_3 = \int \frac{\begin{vmatrix} y_1 & y_2 & 0 \\ y_1' & y_2' & 0 \\ y_1'' & y_2'' & R(x) \end{vmatrix}}{w} dx$$

$$\Rightarrow y_p = u_1y_1 + u_2y_2$$

$$u_1 = \int \frac{\begin{vmatrix} 0 & y_2 \\ R(x) & y_2' \end{vmatrix}}{w} dx$$

Q. $y'' + y = x\sin x$

Given, $(D^2+1)y = x\sin x$

auxillary eq^n

$m^2+1 = 0$

$m^2 = -1$

$m = \pm i$

$y_c = C_1\cos x + C_2\sin x$

$y_1 = \cos x$

$y_2 = \sin x$

$$w = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$= \cos^2 x + \sin^2 x$

$= 1$

$$u_1(x) = -\int \frac{y_2 R(x)}{w} dx$$

$$= -\int \sin^2 x \times x\, dx$$

$$= -\int x\left(\frac{1-\cos 2x}{2}\right) dx$$

$$= -\int \frac{x}{2} dx + \frac{1}{2}\int \cos 2x\, dx$$

$$= -\frac{x^2}{4} + \frac{1}{4}\sin 2x$$

$$u_2(x) = \int \frac{y_1 R(x)}{w} dx$$

$$= \int \cos x \times x\sin x\, dx$$

$$= \int \frac{x}{2}\sin 2x\, dx$$

$$= \frac{1}{2}\left[-\frac{x\cos 2x}{2} + \frac{1}{4}\sin 2x\right]$$

$= -\frac{x}{4}\cos 2x - \frac{1}{4}\sin 2x$

$y_p = -\frac{x^2}{4} - \frac{x}{4}\cos 2x$

General solⁿ

$y = c_1\cos x + c_2\sin x - \frac{x^2}{4} - \frac{x}{4}\cos 2x$

Cauchy Euler eqⁿ

$x^n\frac{d^ny}{dx^n} + x^{n-1}\frac{d^{n-1}y}{dx^{n-1}} + \dots + x^0 y = R(x)$

Assume $x = e^z$ $\quad D = d/dx$

$z = \log x$ $\quad \theta = d/dz$

$f(\theta)y = R(e^z) = P(z)$ $\quad = x\, d/dx$

$f(m) = 0$

let $m = 1, 2, 3$

$y_c = c_1e^z + c_2e^{2z} + c_3e^{3z}$

$= c_1x + c_2x^2 + c_3x^3$

$y_p = \frac{1}{f(D)}P(z)$

Q $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$

⇒ let $x = e^z$

$z = \log x$

$(\theta(\theta-1) - \theta + 2)y = \theta e^z \times z$

$(\theta^2 - \theta - \theta + 2)y = 0$

4-4x2

$m^2 - 2m + 2 = 0$

$m = \frac{+2 \pm \sqrt{4}i}{2}$

$= \frac{2 \pm 2i}{2}$

$= 1 \pm i$

$y_c = e^x[c_1\cos x + c_2\sin x]$

$= e^z[c_1\cos z + c_2\sin z]$

$y_p = x[c_1\cos(\log x) + c_2\sin(\log x)]$

$y_p = \frac{1}{f(\theta)}R(z)$

$= \frac{1}{\theta^2 - 2\theta + 2} \times e^z \times z$

$= e^z\left[\frac{1}{(\theta+1)^2 - 2(\theta+1) + 2}\right]z$

$= e^z\left[\frac{1}{\theta^2 + \cancel{2\theta} + 1 - \cancel{2\theta} - \cancel{2} + \cancel{2}}\right]z$

$= e^z[1+\theta^2]^{-1}z$

$= e^z[1 - \theta^2 + \theta^4 \dots]z$

$= e^z z$

$y_p = x\log x$

General solⁿ

$y = x(c_1\cos(\log x) + c_2\sin(\log x)) + x\log x$

$3\theta^2 - 2\theta$

Solve $x^4 \frac{d^3y}{dx^3} + 2x^3\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 1$

$\Rightarrow x\left[x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y\right] = \frac{1}{x}$

$\Rightarrow \left[\theta(\theta-1)(\theta-2)y + 2\theta(\theta-1)y - \theta y + y\right] = \frac{1}{x}$

$\Rightarrow [(\theta^2-\theta)(\theta-2) + 2\theta^2 - 2\theta - \theta + 1]y = \frac{1}{e^z}$

$\Rightarrow [\theta^3 - \cancel{2\theta^2} - \theta^2 + \cancel{2\theta} + \cancel{2\theta^2} - \cancel{2\theta} - \theta + 1]y = \frac{1}{e^z}$

$= [\theta^3 - \theta^2 - \theta + 1]y = \frac{1}{e^z}$.

$m^3 - m^2 - m + 1$

$m = 1$

1	1	-1	-1	1
	0	1	0	-1
	1	0	-1	0

$m^2 - 1 = 0$

$m^2 = 1$

$m = \pm 1$

$m = 1, 1, -1$

$y_c = (C_1 + e^z C_2)e^z + C_3 e^{-z}$

$y_c = (C_1 + xC_2)x + C_3$

$y_p = \frac{1}{f(\theta)} R(z)$

$= \frac{1}{[\theta^3 - \theta^2 - \theta + 1]} e^{-z}$

$= \frac{1}{-1 - 1 + 1 + 1} \cdot = x\,\frac{1}{(3+2-1)} e^{-z}$

$= \frac{-x\log x}{4}$.

H.W

1. $x^2y'' - 4xy' + 6y = x^4$

$= \left[x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y\right] = x^4$

$=$ Let $x = e^z$

$z = \log x$

$= [\theta(\theta-1) - 4\theta + 6]y = [e^z]^4 = e^{4z}$

$=$ auxillary eq^n

$= \theta^2 - 5\theta + 6 = 0$

$m^2 - 5m + 6 = 0$

$m^2 - 3m - 2m + 6 = 0$

$m(m-3) - 2(m-3) = 0$

$m = 2, 3$.

$y_c = C_1 e^{2z} + C_2 e^{3z}$

$y_p = \frac{1}{f(\theta)} R(z)$

$= \frac{1}{[\theta^2 - 5\theta + 6]} e^{4z}$

$= \frac{1}{[(\theta+4)^2 - 5[\theta+4] + 6]} e^{4z}$

$= \frac{e^{4z}}{16 - 20 + 6}$

$= \frac{1}{2}e^{4z} = \frac{1}{2}x^4$.

General sol^n $= y = C_1x^2 + C_2x^3 + \frac{1}{2}x^4$

2. $x^2y'' - 3xy' + 4y = 2x^2$

$= \left[x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y\right] = 2x^2$

Let $x = e^z$

$z = \log x$

$= [\theta(\theta-1) - 3\theta + 4]y = 2e^{2z}$

$= [\theta^2 - 4\theta + 4]y = 2e^{2z}$

auxillary eq^n

$m^2 - 4m + 4 = 0$

$m^2 - 2m - 2m + 4 = 0$

$m(m-2) - 2(m-2) = 0$

$m = 2, 2$

$y_c = (c_1 + z c_2)e^{2z}$

$y_p = \frac{1}{f(\theta)} R(z)$

$= 2 \frac{1}{[\theta^2 - 4\theta + 4]} e^{2z}$

$= 2 \frac{e^{2z}}{[4 - 8 + 4]}$

$= 2z \frac{e^{2z}}{2\theta - 4}$

$= \cancel{2} z^2 \frac{e^{2z}}{2}$

$y_p = z^2 e^{2z} = 2\log x\, x^2$

log

General sol^n $y = (c_1 + x c_2)e^{2x} x^2$

$+ x^2(\log x)^2$

3. $x^2y'' + 7xy' + 13y = \log x$

$\left[x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 13y\right] = \log x$

Let $x = e^z$

$z = \log x$

$[\theta(\theta-1) + 7\theta + 13]y = z$

$[\theta^2 + 6\theta + 13]y = z$

auxillary eq^n

$m^2 + 6m + 13 = 0$

$m = -3 \pm 2i$

$y_c = \cancel{c_1 \cos (2 \ldots) e^{z}}$

$= e^{-3z}(c_1 \cos 2z + c_2 \sin 2z)$

$y_p = \frac{1}{f(\theta)} R(z)$

$= \frac{z}{[\theta^2 + 6\theta + 13]}$

$= \frac{1}{13} \frac{z}{\left[1 + \left[\frac{\theta^2}{13} + \frac{6\theta}{13}\right]\right]}$

$= \frac{1}{13} \left[1 + [\theta^2/13 + 6\theta/13]\right]^{-1} z$

$= \frac{1}{13} \left[1 \mp \frac{\theta^2}{13} \mp \frac{6\theta}{13} + \left[\frac{\theta^2}{13} + \frac{6\theta}{13}\right]^2\right] z$

$= \frac{1}{13} \left[z - \frac{6}{13}\right]$

$= y_p = \frac{1}{13}\left[\log x - \frac{6}{13}\right]$

General soln $y = x^{-3}(C_1\cos(2\log x) + C_2\sin(2\log x)) + \frac{1}{13}\left[\log x - \frac{6}{13}\right]$

4. $x^3y''' + 3x^2y'' + xy' + y = x + \log x$

$$\left[x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y\right] = x + \log x$$

= Let $x = e^z$

$z = \log x$

$= [\theta(\theta-1)(\theta-2) + 3\theta(\theta-1) + \theta + 1]y = e^z + z$

$= [\theta^3 - 2\cancel{\theta^2} - \cancel{\theta^2} + \cancel{2\theta} + \cancel{3\theta^2} - \cancel{3\theta} + \cancel{\theta} + 1]y = e^z + z$

$= [\theta^3 + 1]\, y = e^z + z$

auxillary eqn

$m^3 + 1 = 0$

$(m+1)(m^2 - m + 1) = 0$

-1	1	0	0	1
	0	-1	+1	-1
	1	-1	+1	0

$m^2 - m + 1 = 0$

$m = \frac{1 \pm \sqrt{3}i}{2}$

$y_c = C_1e^{-z} + e^{z/2}\left[C_2\cos\frac{\sqrt{3}}{2}z + C_3\sin\frac{\sqrt{3}}{2}z\right]$

$y_p = \frac{1}{f(\theta)} R(z)$

$= \frac{1}{[\theta^3+1]}[e^z + z]$

$= \left[\frac{e^z}{\theta^3+1} + \frac{z}{\theta^3+1}\right]$

$= \frac{e^z}{2} + [1+\theta^3]^{-1}z$

$= \frac{e^z}{2} + [z + 0]$

$y_p = \frac{e^z}{2} + z = \frac{x}{2} + \log x$

General soln : $y = C_1x^{-1} + \sqrt{x}\left[C_2\cos\left(\frac{\sqrt{3}}{2}\log x\right) + C_3\sin\left(\frac{\sqrt{3}}{2}\log x\right)\right] + \frac{x}{2} + \log x$

5. $x^3y''' + 2x^2y'' + 3xy' - 3y = x^2 + x$

$$\left[x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} - 3y\right] = x^2 + x$$

Let $x = e^z$

$z = \log x$

$\Rightarrow [\theta(\theta-1)(\theta-2) + 2\theta(\theta-1) + 3\theta - 3]y = e^{2z} + e^z$

$[\theta^3 - \cancel{3\theta^2} - \theta^2 + \cancel{2\theta} + \cancel{2\theta^2} - \cancel{2\theta} + 3\theta - 3]y = e^{2z} + e^z$

$[\theta^3 - \theta^2 + 3\theta - 3]\, y = e^{2z} + e^z$

auxillary eqn

$m^3 - m^2 + 3m - 3 = 0$

$(m-1)(m^2+3) = 0$

1	1	-1	3	-3
	0	1	0	3
	1	0	3	0

$m^2 = -3$

$m = \pm\sqrt{3}i$

$y_c = C_1e^z + [C_2\cos\sqrt{3}z + C_3\sin\sqrt{3}z]$

$y_p = \frac{1}{R(\theta)} R(z)$

$= \frac{1}{[\theta^3 - \theta^2 + 3\theta - 3]}[e^{2z} + e^z]$

$= \frac{e^{2z}}{[\theta^3-\theta^2+3\theta-3]} + \frac{e^z}{[\theta^3-\theta^2+3\theta-3]}$

$= \frac{e^{2z}}{7} + z\frac{e^z}{4}$

$= y_p = \frac{x^2}{7} + \frac{x\log x}{4}$

General soln

$y = C_1x + [C_2\cos[\sqrt{3}\log x] + C_3\sin(\sqrt{3}\log x)] + \frac{x^2}{7} + \frac{x\log x}{4}$

6. $x^2y'' + 5xy' + 4y = x\log x$

$\left[x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y\right] = x\log x$

Let $x = e^z$

$z = \log x$

$= [\theta(\theta-1) + 5\theta + 4]y = e^z \times z$

$= [\theta^2 - \theta + 5\theta + 4]y = ze^z$

$= [\theta^2 + 4\theta + 4]y = ze^z$

$= [\theta^2 + 2\theta + 2\theta + 4]y = ze^z$

auxillary eqn

$m^2 + 2m + 2m + 4 = 0$

$m(m+2) + 2(m+2) = 0$

$(m+2)(m+2) = 0$

$m = -2, -2$

$y_c = (C_1 + zC_2)e^{-2z}$

$y_p = \frac{1}{f(\theta)} R(z)$

$= \frac{1}{[\theta^2+4\theta+4]} ze^z$

$= e^z\left[\frac{1}{(\theta+1)^2+4(\theta+1)+4}\right]z$

$= e^z\left[\frac{1}{\theta^2+2\theta+1+4\theta+4+4}\right]z$

$= e^z\left[\frac{1}{\theta^2+6\theta+9}\right]z$

$= \frac{e^z}{9}\left[1+\left[\frac{\theta^2}{9}+\frac{2\theta}{3}\right]\right]^{-1}z$

$= \frac{e^z}{9}\left[1 - \frac{\theta^2}{9} - \frac{2}{3}\theta + \left[\frac{\theta^2}{9}+\frac{2\theta}{3}\right]^2 \ldots\right]z$

$= \frac{e^z}{9}\left[z - \frac{2}{3}\right]$

$y_p = \frac{x}{9}\left[\log x - \frac{2}{3}\right]$

General soln

$y = (C_1 + \log x\, C_2)x^{-2} + \frac{x}{9}\log x - \frac{2}{27}x$

7. $x^3y''' - y'' + 2y' - 2y = x^3 + 3x$

$\left[x^3\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 2y\right] = x^3 + 3x$

$\Rightarrow$ Let $x = e^z$

$z = \log x$

$[\theta$

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8. $y'' - \frac{6}{x^2} y = x \log x$

$\Rightarrow x^2 y'' - 6y = x^3 \log x$

$\left[x^2 \frac{d^2y}{dx^2} - 6y \right] = x^3 \log x$

Let $x = e^z$

$z = \log x$

$[\theta(\theta-1) - 6] y = e^{3z} \times z$

$= [\theta^2 - \theta - 6] y = z e^{3z}$

= Auxillary eqn

$m^2 - m - 6 = 0$

$m^2 - 3m + 2m - 6 = 0$

$m(m-3) + 2(m-3) = 0$

$m = -2, 3$

$y_c = c_1 e^{-2z} + c_2 e^{3z}$

$y_p = \frac{1}{f(\theta)} R(z)$

$= \frac{1}{[\theta^2 - \theta - 6]} z e^{3z}$

$= e^{3z} \left[\frac{1}{(\theta+3)^2 - (\theta+3) - 6} \right] z$

$= e^{3z} \left[\frac{1}{\theta^2 + 5\theta} \right] z$

$= \frac{e^{3z}}{5\theta} \left[1 + \frac{\theta}{5} \right]^{-1} z$

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$(D^2-2D-3)y = e^{3x}$

auxillary eqn

$m^2-2m-3=0$

$m^2-3m+m-3=0$

$m(m-3)+1(m-3)=0$

$m=-1,3$

$y_c = C_1 e^{-x} + C_2 e^{3x}$

$y_p = \frac{1}{f(D)} Q(x)$

$= \frac{1}{(D^2-2D-3)} e^{3x}$

$= \frac{e^{3x}}{[9-6-3]} = \frac{x e^{3x}}{2D-2}$

$= x e$

$y_p = \frac{1}{(D^2-2D-3)} x^2 \sin 5x$

$= e^{i5x} \times \frac{1}{[(D+i5)^2-2(D+i5)-3]} x^2$

$= e^{i5x} \times \frac{1}{[D^2+10Di-25-2D-10i-3]} x^2$

$= e^{i5x} \times \frac{x^2}{[D^2+10Di-2D-10i-28]}$

$= e^{i5x} \times \frac{x^2}{[(D^2-2D-28)+i(}$

Legendre's equation

$a_0(ax+b)^3\frac{d^3y}{dx^3} + a_1(ax+b)^2\frac{d^2y}{dx^2} + a_2(ax+b)$

$\frac{dy}{dx} + a_3 y = R(x)$

Assume $(ax+b) = e^z$

$(ax+b)^3\frac{d^3y}{dx^3} = a^3\theta(\theta-1)(\theta-2)y$

$(ax+b)^2\frac{d^2y}{dx^2} = a^2\theta(\theta-1)y$

$(ax+b)\frac{dy}{dx} = a\theta y$

Ex:1 $(x+1)^2\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = 4\cos\log(1+x)$

Let $(x+1) = e^z$

$z = \log(x+1)$

$\Rightarrow (\theta(\theta-1) + \theta + 1)y = 4\cos(1+x)$

auxillary eqn

$(\theta^2 - \theta + \theta + 1)y = 4\cos(1+x)$

$m^2+1=0$

$m^2=-1$

$m=\pm i$

$y_c = C_1\cos z + C_2 \sin z$

$y_p = \frac{1}{\theta^2+1} 4\cos z$

$= 4 \frac{1 \times z}{2\theta} \cos z$

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$\log\left(\frac{a}{b}\right)^{-1}$ $\quad 2x = e^z - 3$ $\quad x = -\log\left(\frac{a}{b}\right)$ $\quad \log\left(\frac{a}{b}\right)^{-1}$

$$= 2\frac{z}{\theta}\cos e^z$$

$$= 2z\frac{\sin e^z}{\cancel{e^z}}$$

$$= \frac{2\log(1+x)\sin x(\log(x+1))}{x}$$

General Sol^n

$$y = C_1\cos(\log(x+1)) + C_2\sin(\log(x+1)) + 2\log(1+x)\sin x(\log(x+1))$$

Ex:2 $(2x+3)^2y'' + (2x+3)y' - 2y = 24x^2 / 6x$

$\Rightarrow$ Let $(2x+3) = e^z$

$$z = \log(2x+3)$$

$$((2)^2\theta(\theta-1) + 2\theta - 2)y = 24\left(\frac{e^z-3}{2}\right)^2$$

$$[4\theta^2 - 4\theta + 2\theta - 2]y = 6(e^z-3)^2$$

auxillary eq^n

$$4m^2 - 2m - 2 = 0$$

$$2m^2 - m - 1 = 0$$

$$2m^2 - 2m + m - 1 = 0$$

$$2m(m-1) + 1(m-1) = 0$$

$$m = -\frac{1}{2}, 1$$

$$y_c = C_1e^{-z/2} + C_2e^z$$

$$= C_1e^{-\frac{\log(2x+3)}{2}} + C_2e^{\log(2x+3)}$$

$= C_1$

$e^{\left(-\frac{1}{2}\right)}$ $\quad (2x+3)^{-1/2}$

$$y_p = \frac{1}{[4\theta^2 - 2\theta - 2]} \times 6(e^z-3)^2$$

$$= \frac{3}{[2\theta^2 - \theta - 1)}[e^{2z} - 6e^z + 9]$$

$$= 3\left[\frac{e^{2z}}{8-2-1}\right] - 18\left[\frac{e^z}{2-1-1}\right] + (-\cancel{18}\ 27)$$

$$= \frac{3e^{2z}}{5} - 18 \times \frac{ze^z}{4\theta-1} - \cancel{18}\ 27$$

$$= \frac{3}{5}e^{2z} - 6ze^z - \cancel{18}\ 27$$

$$= \frac{3}{5}(2x+3)^2 - 6\log(2x+3)(2x+3) - 27$$

General Sol^n

$$y = \frac{C_1}{(2x+3)^{1/2}} + C_2(2x+3) + \frac{3}{5}(2x+3)^2 - 6\log(2x+3)(2x+3) - 27$$

$$(2x+3)^2y'' + (2x+3)y' - 2y = 6x$$

Let $(2x+3) = e^z$

$$z = \log(2x+3)$$

$$y_c = C_1e^{-z/2} + C_2e^z$$

$$y_p = \frac{1}{[4\theta^2 - 2\theta - 2]} 3\left[\frac{e^z - 3}{\cancel{2}}\right]$$

$$= 3\left[\frac{e^z}{4\theta^2 - 2\theta - 2} - \frac{3}{4\theta^2 - 2\theta - 2}\right]$$

$= 3\left[\frac{ze^z}{8\theta-2}\right] + \frac{9}{2}$

$= \frac{ze^z}{2} + \frac{9}{2}$

$= \frac{\log(2x+3) \times (2x+3)}{2} + \frac{9}{2}$

General sol^n

$y = \frac{c_1}{(2x+3)^{1/2}} + c_2(2x+3) + \frac{(2x+3)\log(2x+3)}{2} + \frac{9}{2}$.

4-8-18

Date: 31-8-18

Chapter-2

Power Series Solution of Ordinary differential eq^n.

Power Series

A Power Series in power of $(x-x_0)$ is an infinite series of the form.

$$f(x) = \sum_{K=0}^{\infty} a_K (x-x_0)^K.$$

$$= a_0 + a_1(x-x_0) + a_2(x-x_0)^2 + \dots + a_n(x-x_0)^n + \dots$$

$a_0, a_1, a_2 \dots$ Constant Co-efficient of $(x-x_0)^K$.

x = variable

x_0 = centre of series.

If centre of series $x_0 = 0$ then the power series in power of x is given by

$$f(x) = \sum_{K=0}^{\infty} a_K x^K \quad \text{Machlaurian Series}$$

$$= a_0 + a_1 x + a_2 x^2 + \dots a_n x^n + \dots$$

$$\sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^n(x_0) = f(x) \text{ Tailor Series}$$

Analytic

A fun^n $f(x)$ defined on an interval, the point $x = x_0$ is called analytic at $x = x_0$ if it's tailor series is equal to $f(x) = \sum_{n=0}^{\infty} \frac{(x-x_0)^n}{n!} f^n(x_0)$

NOTE: $f(x)$ at $x = x_0 \rightarrow f(x_0)$ = finite $\rightarrow$ Analytic $\rightarrow x_0$ = O.P
infinite $\rightarrow$ Not Ana $\rightarrow x_0$ = S.P

NOTE

1. The fun^n e^x, $\sin x$, $\cos x$, $\sinh x$, $\cosh x$ are analytic everywhere.
2. A rational fun^m is analytic everywhere if it's denomenator not equal to zero

Ordinary Point And Singular Point

→ consider the linear differential equation $y'' + P(x)y' + Q(x)y = 0$ the point $x = x_0$ is called ordinary point if $P(x)$ & $Q(x)$ are analytic at $x = x_0$.

→ The point $x = x_0$ is called Singular point if $P(x)$ or $Q(x)$ are not analytic at $x = x_0$

i> Regular Singular Point

If both $(x-x_0)P(x)$ and $(x-x_0)^2 Q(x)$ are analytic at $x = x_0$ is called regular Singular Point.

ii> Irregular Singular Point

If either $(x-x_0)P(x)$ or $(x-x_0)^2 Q(x)$ ~~the~~ po are not analytic at $x = x_0$ the point is called Irregular singular Point.

Ex : 1 $(1-x^2)\frac{d^2y}{dx^2} - 6x\frac{dy}{dx} - 4y = 0$

$$\frac{d^2y}{dx^2} - \frac{6x}{(1-x^2)}\frac{dy}{dx} - \frac{4}{(1-x^2)}y = 0$$

$\therefore P(x) = -\frac{6x}{1-x^2} = \frac{-6x}{(1-x)(1+x)}$

$= \frac{6-6x-6}{(1-x)(1+x)} = \frac{6(1-\cancel{x})}{(1-\cancel{x})(1+x)} - \frac{6}{(1-x)(1+x)}$

$= \frac{6}{1+x} - \frac{6}{1-x^2}$.

$\therefore$ for $x = 0 \rightarrow$ ordinary point.
$x = \pm 1 \rightarrow$ Singular Point.

$\therefore Q(x) = \frac{-4x}{1-x^2}$

Function is analytic except $x = \pm 1$.
The point is ordinary except at $x = \pm 1$.

For $x = +1$

$(x-x_0)P(x) = \frac{-6}{1-1} = -\frac{(\cancel{x-1})6x}{(1-\cancel{x})(1+x)} = -\frac{(\cancel{x-1})^2 4x}{(1-\cancel{x})(1+x)}$

$= \frac{6}{2} = 3$ $\quad = \frac{4}{2} = 2$.

$\therefore$ regular Singular point

For $x = -1$

$(x-x_0)P(x) = -\frac{(\cancel{x+1})6x}{(1-x)(1+\cancel{x})}$, $\quad (x-x_0)Q(x) = -\frac{(\cancel{x+1})^2 4x}{(1-x)(1+x)}$

$= \frac{-6}{2} = -3$ $\quad = \frac{-4}{-2} = -2$

$\therefore$ regular singular point

Ex: 2 $(x^2+4)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = 0$

$\frac{d^2y}{dx^2} + \frac{2x}{x^2+4}\frac{dy}{dx} - \frac{12}{x^2+4}y = 0$

$P(x) = \frac{2x}{x^2+4}$

$Q(x) = \frac{-12}{x^2+4}$

at $x = \pm 2i$ singular point

$\Rightarrow$ Function is analytic except $x = \pm 2i$

For $x = 2i$

$(x-x_0)P(x) = \frac{(x-2i)\times 2x}{x^2+4}$

$= \frac{\cancel{(x-2i)}\times 2x}{\cancel{(x-2i)}(x+2i)}$

$= \frac{4i}{4i} = 1$

$(x-x_0)^2 Q(x)$

$= \frac{(x-2i)^2 \times -12}{\cancel{(x-2i)}(x+2i)}$

$= 0$

$\therefore$ regular singular point

For $x = -2i$

$(x-x_0)p(x) = \frac{\cancel{(x+2i)}\times 2x}{(x-2i)\cancel{(x+2i)}}$

$= \frac{-4i}{-4i} = 1$

$(x-x_0)^2 Q(x)$

$= \frac{(x+2i)^2 \cdot -12}{(x-2i)\cancel{(x+2i)}}$

$= 0$

$\therefore$ regular Singular point.

Ex: 3 $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 3y = 0$

$\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} - \frac{3}{x^2}y = 0$

$P(x) = -\frac{1}{x}$

$Q(x) = \frac{-3}{x^2}$

For $x = 0 \Rightarrow$ function is not analytic

$\therefore$ Function is analytic except at $x = 0$

For $x = 0$

$(x-x_0)P(x) = \cancel{x}\times\frac{-1}{\cancel{x}}$

$= -1$

$(x-x_0)^2 Q(x) = \cancel{x}^2 \times \frac{-3}{\cancel{x^2}}$

$= \frac{-3}{x}$

$\therefore$ irregular singular point

Ex: 4 $(x^2-9)^2\frac{d^2y}{dx^2} + (x-3)\frac{dy}{dx} + 2y = 0$

$\Rightarrow \frac{d^2y}{dx^2} + \frac{(x-3)}{(x+9)^2(x-3)^2}\frac{dy}{dx} + \frac{2}{(x^2-9)^2}y = 0$

$P(x) = \frac{(x-3)}{(x^2-9)^2}\frac{dy}{dx} = \frac{dy}{(x+3)^2(x-3)dx}$

$Q(x) = \frac{2}{(x^2-9)^2}$

$(x^2-9)^2 = 0$

$x^2 = 9$

$x = \pm 3$ (2times)

$\therefore$ Function is ordinary everywhere except ± 3.

Ex:5 $y'' + y = 0$

$$\frac{d^2y}{dx^2} + y = 0$$

$P(x) = 0$

$Q(x) = 1$

$\therefore$ Every points are ordinary

Ex:6 $y'' + e^x y' + \sin(x^2) y = 0$

$$\frac{d^2y}{dx^2} + e^x \frac{dy}{dx} + \sin(x^2) y = 0$$

$P(x) = e^x$

$Q(x) = \sin(x^2)$

$e^x = 0$ $\sin(x^2)$

Is ~~finite~~ at every point

$\therefore$ Every points are ordinary

Ex: 7 $y'' + \frac{1}{x-1} y' + \frac{1}{x-2} y = 0$

$$\frac{d^2y}{dx^2} + \frac{1}{x-1}\frac{dy}{dx} + \frac{1}{x-2} y = 0$$

$\Rightarrow$ $P(x) = \frac{1}{x-1}$

$Q(x) = \frac{1}{x-2}$

$x = 1, 2$

$\Rightarrow$ Function is Ordinary every where except 1, 2

Power Series Solution near Ordinary point

- let $x=0$ be an ordinary point of equation.

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + Q(x) y = 0$$

Working procedure

1. let $y = \sum_{K=0}^{\infty} a_K (x)^K$

$$y' = \sum_{K=1}^{\infty} a_K K (x)^{K-1}$$

$$= a_1 + 2a_2 x + 3a_3 x^2 + \dots + n a_n x^{n-1}$$

$$y'' = \sum_{K=2}^{\infty} a_K K(K-1)(x)^{K-2}$$

$$= 2a_2 + 6a_3 x + 4a_4 x^2 \cdot 3 + \dots + a_n n(n-1) x^{(n-2)}$$

2. Put Value of $y, \frac{dy}{dx}, \frac{d^2y}{dx^2}$ in eq ①

3. Collect the power of x and equate the Coefficient of power of $x=0$

4. Equating to zero the co-efficient of x^n that relationship is called recurrance relation

Q→ Solve $\frac{d^2y}{dx^2} + x^2 y = 0$

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + Q(x)y = 0$$

from above equation $p(x)=0$
$Q(x) = x^2$

All points are ordinary
$x=0$ is ordinary point

$$y = \sum_{k=0}^{\infty} a_k x^k =$$

$$y' = \sum_{k=1}^{\infty} a_k \cdot k x^{k-1} =$$

$$y'' = \sum_{k=2}^{\infty} a_k \cdot k(k-1)x^{k-2}$$

From eq^n ①

$$\sum_{k=2}^{\infty} a_k k(k-1)(x)^{k-2} + x^2 \sum_{k=0}^{\infty} a_k x^k = 0$$

$$2a_2 + 6a_3 x + 12a_4 x^2 + \dots + (n+2)(n+1) a_{n+2} x^n + \dots + x^2[a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_{n-2}x^{n-2}] = 0$$

$$\therefore 2a_2 + 6a_3 x + 12a_4 x^2 + \dots + (n+2)(n+1) a_{n+2} x^n + \dots + a_0 x^2 + a_1 x^3 + a_2 x^4 + a_3 x^5 + \dots + a_{(n-2)} x^n = 0$$

$$\therefore 2a_2 + 6a_3 x + (12a_4 + a_0)x^2 + (20a_5 + a_1)x^3 + \dots + ((n+2)(n+1)a_{n+2} + a_{n-2})x^n = 0$$

By equating coefficient

$2a_2 = 0$, $6a_3 = 0$, $12a_4 + a_0 = 0$
$a_2 = 0$, $a_3 = 0$, $a_4 = \frac{-a_0}{12} = -\frac{a_0}{4 \cdot 3}$

$20a_5 + a_1 = 0$, $a_{n+2} = \frac{-a_{(n-2)}}{(n+2)(n+1)}$
$a_5 = -\frac{a_1}{20}$,

$a_6 = -\frac{1}{6 \cdot 5} a_2 = 0$, $a_7 = -\frac{1}{7 \cdot 6} a_3 = 0$

$a_8 = -\frac{1}{8 \cdot 7} a_4 = \frac{1}{8 \cdot 7 \cdot 4 \cdot 3} a_0$, $a_9 = -\frac{1}{10 \cdot 9} a_6 = 0$

$a_{10} = -\frac{1}{10 \cdot 9} a_6 = 0$

$$y = a_0 + a_1 x + 0 + 0 - \frac{a_0}{4 \cdot 3}x^4 - \frac{a_1}{5 \cdot 4}x^5 + \frac{1}{8 \cdot 7 \cdot 4 \cdot 3}x^8 + \dots$$

Date:

Q → $y'' = y'$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0 \quad \rightarrow (1)$$

$P(x) = -1$

$Q(x) = 0$

All points are ordinary ∴ $x = 0$

$$y = \sum_{k=0}^{\infty} a_k x^k = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots + a_n x^n + \cdots + \infty$$

$$y' = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots + n a_n x^{n}$$

$$y'' = 2a_2 + 6a_3 x + 12a_4 x^2 + \cdots + n(n-1)a_n x$$

⇒ $(a_1 + 2a_2) + (2a_2 + 6a_3)x + (3a_3 + 12a_4)x^2$

⇒ $[2a_2 + 6a_3 x + 4\cdot 3 a_4 x^2 + 5\cdot 4 a_5 x^3 + 6\cdot 5 a_6 x^4 + \cdots] - [a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + 5a_5 x^4 + \cdots]$

⇒ $(2a_2 - a_1) + (6a_3 - 2a_2)x + (4\cdot 3a_4 - 3a_3)x^2 + \cdots + (a_{n+2}(n+2)(n+1) - a_{n+1}(n+1))x^n = 0$

⇒ $2a_2 - a_1 = 0$, $6a_3 - 2a_2 = 0$, $12a_4 = 3a_3$

$a_1 = 2a_2$, $6a_3 = 2a_2$, $a_3 = 4a_4$

$a_2 = 3a_3$, $a_4 = \frac{a_1}{4\cdot 3\cdot 2}$

$\frac{a_1}{6} = a_3$

a_{n+2} ... $(n+1) = a_{n+1}(n+1)$

$= (n+1)$ ...

$a_5 = \frac{a_1}{5\cdot 4\cdot 3\cdot 2\cdot 1}$

$a_6 = \frac{a_1}{6!}$

$a_n = \frac{a_1}{n!}$

$$y = a_0 + a_1 x + \frac{a_1}{2!}x^2 + \frac{a_1}{3!}x^3 + \cdots + \frac{a_1}{n!}x^n + \cdots$$

$$= a_0 + a_1\left(\frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)$$

$$= a_0 + a_1 e^x$$

Q → $y'' + y = 0$

$$\frac{d^2y}{dx^2} + y = 0 \quad \rightarrow (1)$$

$P(x) = 0$

$Q(x) = 1$

All points are ordinary

∴ $x = 0$ is ordinary point.

$(2\cdot 1 a_2 + 3\cdot 2\cdot a_3 x + 4\cdot 3 a_4 x^2 + \cdots + (n+2)(n+1)a_{n+2}x^n + \cdots) + (a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots) = 0$

⇒ $(a_0 + 2a_2) + (3\cdot 2a_3 + a_1)x + (4\cdot 3a_4 + a_2)x^2 + \cdots + (a_{n+2}(n+2)(n+1) + a_n) + \cdots = 0$

$-\frac{a_0}{2\cdot 1} = a_2$, $a_4 = \frac{-a_2}{4\cdot 3}$

$a_3 = -\frac{a_1}{3\cdot 2\cdot 1}$, $= \frac{a_0}{4\cdot 3\cdot 2\cdot 1}$

$$a_{n+2} = \frac{-a_n}{(n+2)(n+1)}$$

$$a_5 = -\frac{a_3}{5\cdot4} = \frac{a_1}{5\cdot4\cdot3\cdot2\cdot1} = \frac{a_1}{5!}$$

$$a_6 = -\frac{a_4}{6\cdot5} = \frac{-a_0}{6\cdot5\cdot4\cdot3\cdot2\cdot1} = -\frac{a_0}{6!}$$

$$(x^2+1)y'' + xy' - xy = 0$$

$\Rightarrow$ ~~(x~~

$$y'' + \frac{x}{x^2+1}y' - \frac{x}{x^2+1}y = 0$$

$$P(x) = \frac{x}{x^2+1} \qquad x^2+1=0$$

$$x^2 = -1$$

$$Q(x) = \frac{-x}{x^2+1} \qquad x = \pm i$$

At every point the function is analytical
$\therefore$ Every point are ordinary except $x = \pm i$

Here $x=0$ is O.P

let $y = \sum_{k=0} a_k x^k$

$y' =$

$$(x^2+1)[2a_2 + 3\cdot2a_3x + 4\cdot3a_4x^2 + \cdots]$$
$$+ x[a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + \cdots]$$
$$- x[a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots]$$

$$\Rightarrow [2a_2x^2 + 3\cdot2a_3x^3 + 4\cdot3a_4x^4 + \cdots + (n+2)(n+1)a_{n+2}x^{n+2} + \cdots] + [2a_2 + 3\cdot2a_3x + 4\cdot3a_4x^2 + \cdots + (n+2)(n+1)a_{n+2}x^n + \cdots]$$
$$+ [a_1x + 2a_2x^2 + 3a_3x^3 + 4a_4x^4 + \cdots + (n+1)a_{n+1}x^{n+1} + \cdots] -$$

$$[a_0x + a_1x^2 + a_2x^3 + a_3x^4 + \cdots + a_nx^{n+1} + \cdots]$$

$$= 2a_2 + (3\cdot2a_3 + a_1 - a_0)x + (2a_2 + 4\cdot3a_4 + 2a_2 - a_1]x^2 + [3\cdot2a_3 + 5\cdot4a_5 + 3a_3 - a_2]x^3$$
$$+ [4\cdot3a_4 + 6\cdot5a_6 + 4a_4 - a_3]x^4 + \cdots$$
$$+ [n(n+1)a_{n+1}x^{n+1} + (n+3)(n+2)a_{n+3}x^{n+1}$$
$$+ (n+1)a_{n+1} - a_n x]x^{n+1} = 0$$

$$= a_2 = 0 \qquad 4a_2 + 4\cdot3a_4 - a_1 = 0$$

$$3\cdot2a_3 + a_1 = a_0 \qquad a_1 = 4\cdot3a_4$$

$$3\cdot2a_3 + 5\cdot4a_5 + 3a_3 - a_2 = 0$$

$$4\cdot3a_4 + 6\cdot5a_6 + 4a_4 - a_3 = 0$$

$$16a_4 + 6\cdot5a_6 - \frac{a_0 + a_1}{3\cdot2} = 0$$

$$a_3 = \frac{a_0 - a_1}{3\cdot2}$$

$$[n(n+1) + (n+1)]a_{n+1} + (n+3)(n+2)a_{n+3} - a_n = 0$$

$$(n+1)^2 a_{n+1} - a_n = 0 \qquad a_1 = 0$$

$$a_{n+1} = \frac{a_n}{(n+1)^2} \qquad a_0 = 0$$

$$a_3 = \frac{a_2}{(2+1)^2} = 0$$

$$(n+1)a_{n+1} - a_n = 0$$

$$a_{n+1} = \frac{a_n}{n+1}$$

$$a_3 = \frac{a_2}{3} = 0$$

$$a_4 = \frac{a_3}{4} = 0$$

Chapter-3

Partial Diff. eq^n & it's applicatio

Partial Differential equation

Differential eq^n which contains one or more partial Derivatives is known as Partial Diff. eq^n.

Formation of Partial Diff. eq^n

It can be form by foll. method

i) By elimination of arbitary constant $z = ax^2 + by^2$

ii) By elimination of arbitary function.

$z = f(ax+by) + \phi(ax-by)$

$\frac{\partial z}{\partial x} = p \qquad \frac{\partial z}{\partial y} = q \qquad \frac{\partial^2 z}{\partial x^2} = r$

$\frac{\partial^2 z}{\partial x y} = s \qquad \frac{\partial^2 z}{\partial y^2} = t$

By elimination of arbitary constant

1. $z = ax + by$

$\frac{\partial z}{\partial x} = a = p$

$\frac{\partial z}{\partial y} = b = q$

$z = px + qy$

2. $z = ax^2 + by^2$

$\frac{\partial z}{\partial x} = 2ax \qquad a = \frac{p}{2x}$

$\frac{\partial^2 z}{\partial x^2} = 2a = r$

$\frac{\partial z}{\partial y} = 2by \qquad b = \frac{q}{2x}$

$\frac{\partial^2 z}{\partial y^2} = 2b = t$

$z = \frac{r}{2}x^2 + \frac{t}{2}y^2$

$2z = px + qy$

3. $z = (x-a)^2 + (y-b)^2$

$\frac{\partial z}{\partial x} = 2(x-a)$

$p = 2x - 2a$

$2x = p + 2a$

$x = \frac{p+2a}{2} \qquad a = \frac{2x-p}{2}$

$\frac{\partial z}{\partial y} = 2(y-b)$

$2y = \frac{q+2b}{2} \qquad b = \frac{2y-q}{2}$

$z = \left(\cancel{x} - \frac{p+\cancel{2a}-\cancel{2a}}{2}\right)^2 + \gamma$

$= \left(\frac{\cancel{2x} - \cancel{2x} + p}{2}\right)^2 + \left(\frac{\cancel{2y} - \cancel{2y} + q}{2}\right)^2$

$4z = p^2 + q^2$

4. $z = (x-a)(y-b)$

$\frac{\partial z}{\partial x} = (1)(y-b) = p$

$\frac{\partial z}{\partial y} = (1)(x-a) = q$

$z = qp$.

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$z = ax + by + ab$

$\frac{\partial z}{\partial x} = a = p$

$\frac{\partial z}{\partial y} = b = q$

$z = px + qy + pq$

$z = (x-a)^2 + (y-b)^3$

$\frac{\partial z}{\partial x} = 2(x-a) = p$

$\frac{\partial z}{\partial y} = 3(y-b)^2 = q$

$\frac{\partial^2 z}{\partial y^2} = 6(y-b) = t$

$z = \left(\frac{p}{2}\right)^2 + \left(\frac{t}{6}\right)^3 = \left(\frac{p}{2}\right)^2 + \left(\frac{q}{3}\right)^{3/2}$

$z = \frac{p^2}{4} + \frac{t^3}{216}$

$z = ax^2y^2 + bxy$

$\frac{\partial z}{\partial x} = 2axy^2 + by = p$

$\frac{\partial^2 z}{\partial x^2} = 2ay^2 + 0 = r \qquad ay^2 = \frac{r}{2}$

$\frac{\partial^2 z}{\partial y} = 2ax^2y + bx$

$\frac{\partial^2 z}{\partial x^2} = 2ax^2 = t$

$= rx + by = p$

$= b = \frac{p - rx}{y}$

$= z = \frac{t}{2x^2} \times x^2 y^2 + \frac{p - rx}{y} \times xy$

$= \frac{ty^2}{2} + px - rx^2$

$= \frac{r^2 x^2}{2} + px - rx^2$

Partial differential equation by elemination by arbitrary constant

1. $z = f(x+ay) + \phi(x-ay)$

$p = \frac{\partial z}{\partial x} = f'(x+ay) + \phi'(x-ay)$

$q = \frac{\partial z}{\partial y} = af'(x+ay) + (-a)\phi'(x-ay)$

$r = \frac{\partial^2 z}{\partial x^2} = f''(x+ay) + \phi''(x-ay)$

$t = \frac{\partial^2 z}{\partial y^2} = a^2 f''(x+ay) + a^2\phi''(x-ay)$

$t = a^2 r$

2. $z = f(x+6y) + \phi(x-6y)$

$\frac{\partial z}{\partial x} = f'(x+6y) + \phi'(x-6y)$

$\frac{\partial z}{\partial y} = 6f'(x+6y) - 6\phi'(x-6y)$

$\frac{\partial^2 z}{\partial x^2} = f''(x+6y) + \phi''(x-6y)$

$\frac{\partial^2 z}{\partial y^2} = 36(f''(x+6y) + \phi''(x-6y))$

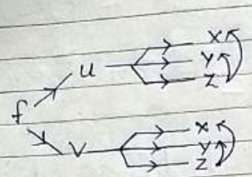
$= 36r$

Ex: $f(x+y+z,\ x^2+y^2+z^2) = 0$ where $z = f(x,y)$

$f(x^2+y^2+z^2) = 0$

$f(x+y+z) = 0$

$f(u,v) = 0$



$p = \frac{\partial f}{\partial x}$

$q = \frac{\partial f}{\partial y}$

$$P = \frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\left(\frac{\partial fu}{\partial x} + \frac{\partial u}{\partial z}\left(\frac{\partial z}{\partial x}\right)\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z}\left(\frac{\partial z}{\partial x}\right)\right)$$

$$q = \frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\left(\frac{\partial z}{\partial y}\right)\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}\left(\frac{\partial z}{\partial y}\right)\right)$$

$p = \frac{\partial z}{\partial x}$ Let $x+y+z = u$

$q = \frac{\partial z}{\partial y}$ $x^2+y^2+z^2 = v$

$\frac{\partial u}{\partial x} = 1,\ \frac{\partial u}{\partial y} = 1,\ \frac{\partial u}{\partial z} = 1$

$\frac{\partial v}{\partial x} = 2x,\ \frac{\partial v}{\partial y} = 2y,\ \frac{\partial v}{\partial z} = 2z$

$f(u,v) = 0 \longrightarrow$ ① diff eq① w.r.t x

$$0 = \frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x}\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial f}{\partial u}(1+p) = -\frac{\partial f}{\partial v}(2x+2zp) \longrightarrow ②$$

diff. eq① w.r.t y

$$0 = \frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial y}\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial y}\right)$$

$$\frac{\partial f}{\partial u}(1+q) = -\frac{\partial f}{\partial v}(2y+2zq) \rightarrow ③$$

eq② ÷ eq③

$$\frac{1+p}{1+q} = \frac{2x+2zp}{2y+2zq}$$

$(1+p)(2y+2zq) = (2x+2zp)(1+q)$

$2y+2zq+2yp+\cancel{2zpq} = 2x+2zp+2xq+\cancel{2zpq}$

$2(x-y)+2z(p-q)+2(xq-yp) = 0$

$x-y+zp-zq+xq-yp = 0$

$x(1+q) - y(1+p) + z(p-q) = 0$

$f(xy+z^2,\ x+y+z) = 0$ where $z = f(x,y)$

= $p = \frac{\partial z}{\partial x}$ Let $xy+z^2 = u$

$q = \frac{\partial z}{\partial y}$ $x+y+z = v$

$\frac{\partial u}{\partial x} = y,\ \frac{\partial u}{\partial y} = x,\ \frac{\partial u}{\partial z} = 2z$

$\frac{\partial v}{\partial x} = 1,\ \frac{\partial v}{\partial y} = 1,\ \frac{\partial v}{\partial z} = 1$

= $f(u,v) = 0 \rightarrow$ diff. eq① w.r.t x

$$\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x}\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial x}\right) = 0$$

$$\frac{\partial f}{\partial u}(y+2zp) + \frac{\partial f}{\partial v}(1+p) = 0$$

$$\frac{\partial f}{\partial v}(1+p) = -\frac{\partial f}{\partial u}(y+2zp) \rightarrow (2)$$

diff eq (1) w.r.t y

$$\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial y}\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial y}\right) = 0$$

$$\frac{\partial f}{\partial u}(x+2zq) + \frac{\partial f}{\partial v}(1+q) = 0$$

$$\frac{\partial f}{\partial v}(1+q) = -\frac{\partial f}{\partial u}(x+2zq) \rightarrow (3)$$

div (2) ÷ (3)

$$\frac{1+p}{1+q} = \frac{y+2zp}{x+2zq}$$

$$(1+p)(x+2zq) = (1+q)(y+2zp)$$

$$x + 2zq + xp + \cancel{2zqp} = y + 2zp + yq + \cancel{2zpq}$$

$$x(1+p) - y(1+q) + 2z(q-p) = 0$$

H.W

Ex $f(x+y+z,\ x^2+y^2-z^2) = 0$

Solution of Partial diff. equation

1) Method of direct integration

Ex $3\frac{\partial^2 v}{\partial x^2} = 4y$

$$\frac{\partial^2 v}{\partial x^2} = \frac{4}{3}y$$

$$\frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x}\right) = \frac{4}{3}y$$

$$\frac{\partial v}{\partial x} = \int \frac{4}{3}y\,\partial x$$

$$\frac{\partial v}{\partial x} = \frac{4}{3}yx + f(y)$$

$$v = \int\left(\frac{4}{3}yx + f(y)\right)\partial x$$

$$\boxed{v = \frac{4}{3}y\frac{x^2}{2} + f(y)x + g(y)}$$

Ex: $\frac{\partial^2 z}{\partial x\partial y} = 3x^2 - 2y$

$$= \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = 3x^2 - 2y$$

$$\frac{\partial z}{\partial y} = 3\int x^2\partial x - 2y\int \partial x$$

$$\frac{\partial z}{\partial y} = x^3 - 2yx + f(y)$$

$$z = x^3\int\partial y - 2x\int y\partial y + \int f(y)\partial y$$

$$= x^3y - 2xy^2 + \phi(y) + g(x)$$

where $\phi(y) = \int f(y)\partial y$

Ex $\frac{\partial^2 z}{\partial x^2} + z = 0$ where $z = e^y$

$\frac{\partial^2 z}{\partial x^2} = -z$

$\frac{\partial^2 (e^y)}{\partial x^2} = -e^y$

$\frac{\partial}{\partial x}\left(\frac{\partial (e^y)}{\partial x}\right) = -e^y$

$\frac{\partial (e^y)}{\partial x} = -e^y x + f(y)$

$e^y = -e^y \frac{x^2}{2} + f(y)x + g(y)$

$2z = f(y)x - x^2 e^y + g(y)$

Ex $\frac{\partial^2 z}{\partial x \partial y} = \cos x \cos y$

$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \cos x \cos y$

$\frac{\partial z}{\partial y} = \cos y \sin x + f(y)$

$z = \sin x \sin y + \phi(y) + g(x)$

Ex $\frac{\partial^2 z}{\partial x \partial y} = e^{-y} \cos x$

$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = e^{-y} \cos x$

$\frac{\partial z}{\partial y} = e^{-y} \sin x + f(y)$

$\frac{\partial z}{\partial y} = -e^{-y} \sin x + \phi(y) + g(x)$

Ex: $\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos y$

$= \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \sin x \cos y$

$\frac{\partial z}{\partial y} = -\cos x \cos y + f(y)$

$z = -\cos x \sin y + \phi(y) + g(x)$

Ex $\frac{\partial^2 z}{\partial y^2} = 6x^3 y$

$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = 6x^3 y$

$\frac{\partial z}{\partial y} = 6x^3 \times \frac{y^2}{2} + f(x)$

$z = \cancel{6}x^3 \times \frac{y^3}{\cancel{6}} + f(x)y + g(x)$

Ex: $\frac{\partial^2 z}{\partial x \partial y} = \cos(2x+3y)$

$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \cos(2x+3y)$

$\frac{\partial z}{\partial y} = \frac{\sin(2x+3y)}{2\cancel{x+3y}} \times 2 + f(y)$

$z = \frac{-\cos(2x+3y)}{2 \cdot 3 \cancel{(2x+3y)^2}} + \phi(y) + g(x)$

Linear Partial Equation of 1^{st} Order

$P_p + Q_q = R$ → 1^{st} order L.D.E

The general sol^n of above eq^n can be form as

$$\boxed{\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}}$$

Sol^n $f(c_1, c_2) = 0$

(i) $xP + yQ = 3Z$

$xp + yq = 3Z$

$P = x$, $Q = y$, $R = 3Z$

general form

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{3Z}$$

$$\int \frac{dx}{x} = \int \frac{dy}{y} + c$$

$\ln x = \ln y + c$

$\ln x - \ln y = c$

$\ln \frac{x}{y} = c$

$$\frac{x}{y} = e^c = c_1$$

$$\int \frac{dy}{y} = \int \frac{dz}{3Z} + c$$

$$\ln y = \frac{1}{3} \ln z + c$$

$3 \ln y = \ln z + 3c$

$\ln y^3 - \ln z = 3c$

$$\ln \frac{y^3}{z} = 3c$$

$$\frac{y^3}{z} = e^{3c} = c_2$$

$f(x/y, y^3/z) = 0$

$$\int \frac{dx}{x} = \int \frac{dz}{3Z} + c$$

$$\ln x = \frac{1}{3} \ln z + c$$

$3 \ln x = \ln z + 3c$

$\ln x^3 - \ln z = 3c$

$\ln \left(\frac{x^3}{z}\right) = 3c$

$x^3/z = e^{3c}$

(ii) $yzp - xzq = xy$

$P = yz$, $Q = -xz$, $R = xy$

$$\Rightarrow \frac{dx}{yz} = \frac{dy}{-xz} = \frac{dz}{xy}$$

$$\Rightarrow \frac{dx}{yz} = \frac{dy}{-xz}$$

$\Rightarrow -x\,dx = y\,dy$

$= -\int x\,dx = \int y\,dy + c$

$$= -\frac{x^2}{2} = \frac{y^2}{2} + c$$

$\Rightarrow x^2 + y^2 = -2c = c_1$

$$\Rightarrow \int \frac{dy}{-xz} = \int \frac{dz}{xy} + c$$

$$\Rightarrow -\frac{y^2}{2} = \frac{z^2}{2} + c$$

Date:

$$z^2 + y^2 = -2C = C_2$$
$$f(x^2+y^2,\ y^2+z^2) = 0$$

(iii) $\tan x\, p + \tan y\, q = \tan z$

$P = \tan x,\quad Q = \tan y,\quad R = \tan z$

$\rightarrow \quad \frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}.$

$$\int \frac{dx}{\tan x} = \int \frac{dy}{\tan y} + C$$
$$\int \cot x\, dx = \int \cot y\, dy + C$$
$$\log(\sin x) = \log(\sin y) + c$$
$$\log(\sin x) - \log(\sin y) = C$$
$$\log\left(\frac{\sin x}{\sin y}\right) = C$$
$$\frac{\sin x}{\sin y} = e^C = c_1$$

$\rightarrow \quad \frac{dx}{\tan x} = \frac{dz}{\tan z}$

$$\int \cot x\, dx = \int \cot z\, dz + C$$
$$\log(\sin x) = \log(\sin z) + c$$
$$\log(\sin x) - \log(\sin z) = C$$
$$\log\left(\frac{\sin x}{\sin z}\right) = C$$
$$\frac{\sin x}{\sin z} = e^c = C_2$$

$\rightarrow \quad f(c_1, c_2) = f\left(\frac{\sin x}{\sin y},\ \frac{\sin x}{\sin z}\right) = 0$

(iv) $xp + yq = z$

(v) $2p + 3q = 1$

Date: 18/6/18

Chapter - 5

Fourier Series & Fourier Integral

→ Introduction
→ Euler's Formula
→ Fourier Series for even & odd function
→ Fourier Series for a function having period 2L.
→ Half Range Fourier Series.
→ Fourier Integral Theorem.

- periodic function: A function $f(x)$ is called periodic if it is defined for all x and their exist some positive number P $f(x+P) = f(x)$, $\forall x$ that the number P is called a Period of $f(x)$.

NOTES
1. $\sin x$, $\cos x$, $\sec x$ and $\operatorname{cosec} x$ are periodic function with period 2π and $\tan x$, $\cot x$ are functions with period π.
2. The functions $\sin nx$ & $\cos nx$ are periodic function with period $2\pi/n$.
3. $x, x^2, x^3, e^x, \log x$ are non-periodic function.

- Some Important Results

1. $\int_c^{c+2\pi} \cos nx\, dx = 0 \quad ; n \neq 0$

$$z^2 + y^2 = -2c = c_2$$

$$f(x^2+y^2, y^2+z^2) = 0$$

(iii) $\tan x\, p + \tan y\, q = \tan z$

$P = \tan x$, $Q = \tan y$, $R = \tan z$

$\rightarrow$ $$\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}.$$

$$\int \frac{dx}{\tan x} = \int \frac{dy}{\tan y} + c$$

$$\int \cot x\, dx = \int \cot y\, dy + c$$

$$\log(\sin x) = \log(\sin y) + c$$

$$\log(\sin x) - \log(\sin y) = c$$

$$\log\left(\frac{\sin x}{\sin y}\right) = c$$

$$\frac{\sin x}{\sin y} = e^c = c_1$$

$\rightarrow$ $$\frac{dx}{\tan x} = \frac{dz}{\tan z}$$

$$\int \cot x\, dx = \int \cot z\, dz + c$$

$$\log(\sin x) = \log(\sin z) + c$$

$$\log(\sin x) - \log(\sin z) = c$$

$$\log\left(\frac{\sin x}{\sin z}\right) = c$$

$$\frac{\sin x}{\sin z} = e^c = c_2$$

$\rightarrow$ $$f(c_1, c_2) = f\left(\frac{\sin x}{\sin y}, \frac{\sin x}{\sin z}\right) = 0$$

(iv) $xp + yq = z$

(v) $2p + 3q = 1$

Non-linear Partial Differentiation

The non-linear eqn can be written in the form of $f(p, q) = 0$

Let $p = a$

$q = b$

$f(a, b) = 0$

$b = f(a)$

G.S

$z = ax + by + c$

$z = ax + f(a)y + c$

Ex:1 $p^3 - q^3 = 0$

Let $p = a$

$q = b$.

$a^3 - b^3 = 0$

$b^3 = a^3$

$b^3 = f(a^3)$ $\qquad b = a$

G.S

$z = ax + by + c$

$= ax + ay + c$

Ex:2 $p^2 + q^2 = npq$

Let $p = a$

$q = b$

$a^2 + b^2 = nab$.

$a^2 + b^2 - nab = 0$

$b^2 = nab - a^2$.

$a^2 - \frac{2nab}{2} + b^2 = 0$

$a^2 - 2nab + b^2 + nab = 0$

$(a - b)^2 + nab = 0$.

$$b = \frac{na \pm \sqrt{n^2a^2 - 4a^2}}{2}$$

$$= \frac{na \pm \sqrt{(n^2a^2 - 2a^2)}}{2}$$

$$z = ax + by + c$$

$$= ax + \left(\frac{na \pm \sqrt{n^2a^2 - 4a^2}}{2}\right) y + c$$

Ex : 3 $p^2 + q^2 = 2pq$

$p^2 + q^2 - 2pq = 0$

$(p-q)^2 = 0$

$p - q = 0$

Let $p = a$ & $q = b$

$a - b = 0$

$b = a$

$z = ax + by + c$

$= ax + ay + c$

$f(z, pq) = 0$

Let $q = ap$

Ex : 1 $zpq = p + q$

Let $q = ap$

$zpap = p + ap$

$zap^2 = p(1+a)$

$zap = 1 + a$

$p = \frac{1+a}{za}$

$dz = \frac{1+a}{za} dx + \frac{1+a}{z} dy$

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$= \int z\,dz = \int \frac{1+a}{a} dx + \int 1 + a\,dy + c$

$= \frac{z^2}{2} = \frac{1+a}{a} \cdot x + 1 + a \cdot y + c$

$= az^2 = 2(1+a)x + 2a(1+a)y + c$

Ex : 2 $p^2 + q^2 = 1$

$p^2 + q^2 - 1 = 0$

Let $p = a$

$q = b$

$a^2 + b^2 - 1 = 0$

$b^2 = 1 - a^2$

$b = (1-a^2)^{1/2} = \pm(1-a^2)$

$z = ax + by + c$

$= ax + (1-a^2)^{1/2} y + c$

$= ax + (\pm(1-a^2)) y + c$

Separation of Variable Method.

- The Partial differential equation in the form of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0 \rightarrow ①$ can be solved by separation of variable method.
- The general solⁿ of eqⁿ ① is

$u(x,y) = u = X(x)\,Y(y) \rightarrow ②$

$u(x,y)$ can be partially differentiate w.r.t x & y

$\frac{\partial u}{\partial x} = X'(x)Y(y)$ $\quad \frac{\partial^2 u}{\partial x^2} = X''(x)Y(x)$

$\frac{\partial u}{\partial y} = X(x)Y'(y)$ $\quad \frac{\partial^2 u}{\partial y^2} = X(x)Y''(y)$

$X'(x) = \frac{\partial X}{\partial x}$ $\quad u = XY$

$\frac{\partial u}{\partial x} = X'Y \qquad \frac{\partial^2 u}{\partial x^2} = X''Y$

$\frac{\partial u}{\partial y} = XY' \qquad \frac{\partial^2 u}{\partial y^2} = XY''$

Q.1 $x\frac{\partial u}{\partial x} - 2y\frac{\partial u}{\partial y} = 0$

$xX'Y - 2yXY' = 0$

$xX'Y = 2yXY'$

$\frac{x}{\partial x} \times \frac{\partial X}{X} = \frac{2y}{\partial y}\frac{\partial Y}{Y} = K$

$\frac{x}{\partial x} \times \frac{\partial X}{X} = K$

$\int \frac{\partial X}{X} = K \int \frac{\partial x}{x} + \ln c$

$\ln X = K \ln x + \ln c$

$\ln X = \ln x^K \cdot c$

$X = x^K \cdot c$

$\frac{2y}{\partial y}\frac{\partial Y}{Y} = K$

$\int \frac{\partial Y}{Y} = \frac{K}{2}\int \frac{\partial y}{y} + \ln c_1$

$\ln Y = \frac{K}{2}\ln y + \ln c_1$

$\ln Y = \ln y^{K/2} \cdot c_1$

$Y = y^{K/2} \cdot c_1$

General Sol^n $u = XY = c \cdot c_1 \cdot x^K \cdot y^{K/2}$

$= A x^K \cdot y^{K/2}$

where $(A = c \cdot c_1)$

Q.2 $2x\frac{\partial u}{\partial x} + 7y\frac{\partial u}{\partial y} = 0$

$2xX'Y + 7yXY' = 0$

$2xX'Y = -7yXY'$

$\frac{2x}{\partial x}\frac{\partial X}{X} = -\frac{7y}{\partial y}\frac{\partial Y}{Y} = K$

$\frac{2x}{\partial x}\frac{\partial X}{X} = K$

$\int \frac{\partial X}{X} = \frac{K}{2}\int \frac{\partial x}{x} + \ln c$

$\ln X = \frac{K}{2}\ln x + \ln c$

$\ln X = \ln x^{K/2} \cdot c$

$X = x^{K/2} \cdot c$

$-\frac{7y}{\partial y} \cdot \frac{\partial Y}{Y} = K$

$\int \frac{\partial Y}{Y} = -\frac{K}{7}\int \frac{\partial y}{y} + \ln c_1$

$\ln Y = -\frac{K}{7}\ln y + \ln c_1$

$\ln Y = \ln y^{-K/7} \cdot c_1$

$Y = y^{-K/7} \cdot c_1$

General Sol^m $u = XY = x^{K/2} \cdot c \cdot y^{-K/7} \cdot c_1$

$= c \cdot c_1 \cdot x^{K/2} \cdot y^{-K/7}$

$= A x^{K/2} y^{-K/7}$

where $(A = c \cdot c_1)$

Q.3 $3y\frac{\partial u}{\partial y} - 5x\frac{\partial u}{\partial x} = 0$

$3yXY' - 5xX'Y = 0$

$5xX'Y = 3yXY'$

$5\frac{x}{\partial x} \times \frac{\partial X}{X} = \frac{3y}{\partial y} \times \frac{\partial Y}{Y} = K$

$\int \frac{\partial X}{X} = \frac{K}{5}\int \frac{\partial x}{x} + \ln C$

$\ln X = \frac{K}{5}\ln x + \ln C$

$\ln X = \ln x^{K/5} \cdot C$

$X = x^{K/5} \cdot C$

$\frac{3y}{\partial y} \times \frac{\partial Y}{Y} = K$

$\int \frac{\partial Y}{Y} = \frac{K}{3}\int \frac{\partial y}{y} + \ln C_1$

$\ln Y = \frac{K}{3}\ln y + \ln C_1$

$\ln Y = \ln y^{K/3} \cdot C_1$

$Y = y^{K/3} \cdot C_1$

General sol^n $u = x^{K/5} \cdot C \cdot y^{K/3} \cdot C_1$

$= A x^{K/5} \cdot y^{K/3}$

(where $A = C \cdot C_1$)

Q.4 $ax\frac{\partial u}{\partial x} + by\frac{\partial u}{\partial y} = 0$

$ax\frac{\partial u}{\partial x} = -by\frac{\partial u}{\partial y}$

$axX'Y = -byXY'$

$= \frac{ax}{\partial x}\frac{\partial X}{X} = -\frac{by}{\partial y}\frac{\partial Y}{Y} = K$

$\frac{ax}{\partial x}\frac{\partial X}{X} = K$

$\int \frac{\partial X}{X} = \frac{K}{a}\int \frac{\partial x}{x} + \ln C$

$\ln X = \frac{K}{a}\ln x + \ln C$

$\ln X = \ln x^{K/a} \cdot C$

$X = x^{K/a} \cdot C$

$-\frac{by}{\partial y}\frac{\partial Y}{Y} = K$

$\int \frac{\partial Y}{Y} = -\frac{K}{b}\int \frac{\partial y}{y} + \ln C_1$

$\ln Y = -\frac{K}{b}\ln y + \ln C_1$

$\ln Y = \ln y^{-K/b} \cdot C_1$

$Y = y^{-K/b} \cdot C_1$

General sol^n $u = XY = x^{K/a} \cdot C \cdot y^{-K/b} \cdot C_1$

$= A x^{K/a} \cdot y^{-K/b}$

(where $A = C \cdot C_1$)