ASSIGNMENT

1) Order: The Order of a differential equation is the order of the highest derivative present in the equation.

Degree: The degree of a differential equation is the power of the highest order derivative after clearing the radical sign and fraction.

2)

1) order: 5
degree: 3

2) order: 1
degree: 1

3) order: 2
degree: 1

4) ~~order:~~
degree: 3

5) ~~order:~~
degree: 6

3)

1) $m = 2, 2$

$$C_1 e^{2x} + xC_2 e^{2x}$$

$$\Rightarrow y_c = (C_1 + xC_2) e^{2x}$$

2) $m = 1$

$$\Rightarrow y_c = C_1 e^{x}$$

3) $(D^2-2D+1)y=\cos 3x$

auxillary eq^n

$m^2-2m+1=0$

$m^2-m-m+1=0$

$m(m-1)-1(m-1)=0$

$m=1,1$

$y_c = (c_1+xc_2)e^x$

4) $m=1,2,3$

$y_c = c_1e^x + c_2e^{2x} + c_3e^{3x}$

5) $m=7,8$

$y_c = c_1e^{7x} + c_2e^{8x}$

6) $m=3,4$

$y_c = c_1e^{3x} + c_2e^{4x}$

7) $m=-3\pm\sqrt{5}$

$y_c = e^{-3x}(c_1\cosh\sqrt{5}x + c_2\sinh\sqrt{5}x)$

8) $m=-3,-2$

$y_c = c_1e^{-3x} + c_2e^{-2x}$

9) $m=\pm i$

$y_c = (c_1\cos x + c_2\sin x)$

10) $m=3,3,8,9$

$y_c = (c_1+xc_2)e^{3x} + c_3e^{8x} + c_4e^{9x}$

11) $(D^3-8)y=0$

2	1	0	0	-8
	0	2	4	8
	1	2	4	0

auxillary eq^n

$m^3-8=0$

$(m-2)(m^2+2m+4)=0$

$m=-1\pm\sqrt{3}i$

$y_c = c_1 e^{2x} + e^{-x}(c_2 \cos\sqrt{3}x + c_3 \sin\sqrt{3}x)$

12) Solve $y'' - 3y' - 4y = 0$

$(D^2 - 3D - 4)y = 0$

auxillary eq^n

$m^2 - 3m - 4 = 0$

$m^2 - 4m + m - 4 = 0$

$m(m-4) + 1(m-4) = 0$

$m = -1, 4$

$y_c = c_1 e^{-x} + c_2 e^{4x}$

13) Solve $y'' - 4y' - 5y = 0$

$(D^2 - 4D - 5)y = 0$

auxillary eq^n

$m^2 - 4m - 5 = 0$

$m^2 - 5m + m - 5 = 0$

$m(m-5) + 1(m-5) = 0$

$m = -1, 5$

$y_c = c_1 e^{-x} + c_2 e^{5x}$

14) Solve $y'' - y' = 0$

$(D^2 - D)y = 0$

auxillary eq^n

$m^2 - m = 0$

$m(m-1) = 0$

$m = 0, m = 1$

$y_c = c_1 + c_2 e^{x}$

Q.4)

1) $(D^3+2D^2-10D+4)y=0$

$y_p = \frac{1}{f(D)} R(x) \quad (\because R(x)=0)$

$y_p = 0$.

2) $(D^2-4)y=0$

$y_p = \frac{1}{f(D)} R(x) \quad (\because R(x)=0)$

$= 0$

3) $(D^2-5D+6)y=0$

$y_p = \frac{1}{f(D)} R(x) \quad (\because R(x)=0)$

$= 0.$

Q.5 Solve $(D^3-5D^2+7D-3)y = e^{2x}\cosh x$

auxillary eq^n

$m^3-5m^2+7m-3=0$

$(m-1)(m^2-4m+3)=0$

1	1	-5	7	-3
	0	1	-4	3
	1	-4	3	0

$m^2-3m-m+3=0$

$m(m-3)-1(m-3)=0$

$m=1,1,3$

$y_c = (c_1+xc_2)e^x + c_3e^{3x}$

$y_p = \frac{1}{f(D)} R(x)$

$= \frac{e^{2x}\cosh x}{(D^3-5D^2+7D-3)}$

$= \frac{1}{(D^3-5D^2+7D-3)} e^{2x}\left[\frac{e^x+e^{-x}}{2}\right]$

$= \frac{1}{(D^3-5D^2+7D-3)}\left[\frac{e^{3x}+e^{x}}{2}\right]$

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$$= \frac{1}{2}\left[\frac{e^{3x}}{(D^3-5D^2+7D-3)} + \frac{e^{x}}{(D^3-5D^2+7D-3)}\right]$$

$$= \frac{1}{2}\left[\frac{xe^{3x}}{3D^2-10D+7} + \frac{xe^{x}}{3D^2-10D+7}\right]$$

$$= \frac{1}{2}\left[\frac{xe^{3x}}{4} + \frac{x^2e^{x}}{6D-10}\right]$$

$$= \frac{1}{2}\left[\frac{xe^{3x}}{4} + \frac{x^2e^{x}}{-4}\right]$$

$$y_p = \frac{xe^{3x}}{8} - \frac{x^2e^{x}}{8} = \frac{1}{8}\left[xe^{3x} - x^2e^{x}\right]$$

General sol^n

$$y = (c_1 + xc_2)e^{x} + c_3e^{3x} + \frac{1}{8}\left[xe^{3x} - x^2e^{x}\right]$$

Q.6 Solve $(D^2+4)y = e^{x} + \sin 2x$

auxillary eq^n

$m^2+4=0$

$m^2=-4$

$m=\pm 2i$

$y_c = (c_1\cos 2x + c_2\sin 2x)$

$$y_p = \frac{1}{f(D)} R(x)$$

$$= \frac{1}{(D^2+4)}\left[e^{x} + \sin 2x\right]$$

$$= \frac{e^{x}}{D^2+4} + \frac{\sin 2x}{D^2+4}$$

$$= \frac{e^{x}}{5} + \frac{x\sin 2x}{2D}$$

$$= \frac{e^{x}}{5} + \frac{x}{2}\left[-\frac{\cos 2x}{2}\right]$$

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$$y_p = \frac{e^x}{5} - \frac{x\cos 2x}{4}.$$

General solⁿ

$$y = (c_1\cos 2x + c_2\sin 2x) + \frac{e^x}{5} - \frac{x\cos 2x}{4}$$

Q.7 Solve $(D^2+2)y = e^x\cos 2x$

= auxillary eqⁿ

$$m^2+2=0$$

$$m^2=-2$$

$$m=\pm\sqrt{2}i$$

$$y_c = [c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x]$$

$$y_p = \frac{e^x\cos 2x}{(D^2+2)}$$

$$= e^x\left[\frac{\cos 2x}{(D+1)^2+2}\right]$$

$$= e^x\left[\frac{\cos 2x}{D^2+2D+3}\right]$$

$$= e^x\left[\frac{\cos 2x}{-4+2D+3}\right]$$

$$= e^x\left[\frac{\cos 2x}{2D-1}\right]$$

$$= e^x\left[\frac{(2D+1)\cos 2x}{4D^2-1}\right]$$

$$= \frac{e^x}{15}[2D\cos 2x + \cos 2x]$$

$$= \frac{e^x}{15}[-4\sin 2x + \cos 2x]$$

General solⁿ

$$y = (c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x) + \frac{e^x}{15}[-4\sin 2x + \cos 2x]$$

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8)

1) Solve $(D^2+4)y = e^{3x} + \cos 3x$

auxillary eqn

$m^2+4=0$

$m^2=-4$

$m = \pm 2i$

$y_c = (c_1 \cos 2x + c_2 \sin 2x)$

$$y_p = \frac{1}{D^2+4}\left[e^{3x} + \cos 3x\right]$$

$$= \frac{e^{3x}}{D^2+4} + \frac{\cos 3x}{D^2+4}$$

$$= \frac{e^{3x}}{13} + \frac{\cos 3x}{-5}$$

General soln.

$$y = (c_1\cos 2x + c_2 \sin 2x) + \frac{e^{3x}}{13} - \frac{\cos 3x}{5}.$$

2) Solve $(D^2+2D+1)y = e^{-x}$

auxillary eqn.

$m^2+2m+1=0$

$m^2+m+m+1=0$

$m(m+1)+1(m+1)=0$

$m=-1,-1$

$y_c = (c_1 + x c_2)e^{-x}$

$$y_p = \frac{1}{(D^2+2D+1)} e^{-x}$$

$$= \frac{x e^{-x}}{2D+2}$$

$$= \frac{x^2 e^{-x}}{2}$$

General soln.

$$y = (c_1 + x c_2)e^{-x} + \frac{x^2 e^{-x}}{2}$$

3) Solve $(D^2+4)y = e^x + \sin x$

auxillary eq^n

$m^2+4 = 0$

$m^2 = -4$

$m = \pm 2i$

$y_c = (C_1 \cos 2x + C_2 \sin 2x)$

$$y_p = \frac{1}{D^2+4}\left[e^x + \sin x\right]$$

$$= \frac{e^x}{D^2+4} + \frac{\sin x}{D^2+4}$$

$$= \frac{e^x}{5} + \frac{\sin x}{3}$$

General Sol^n

$$y = (C_1 \cos 2x + C_2 \sin 2x) + \frac{e^x}{5} + \frac{\sin x}{3}$$

4) Solve $(D^2+2D+1)y = e^{3x} + x^2$

auxillary eq^n

$m^2+2m+1 = 0$

$m^2+m+m+1 = 0$

$m(m+1) + 1(m+1) = 0$

$m = -1, -1$

$y_c = (C_1 + xC_2)e^{-x}$

$$y_p = \frac{1}{D^2+2D+1}\left[e^{3x} + x^2\right]$$

$$= \frac{e^{3x}}{D^2+2D+1} + \frac{x^2}{D^2+2D+1}$$

$$= \frac{e^{3x}}{16} + \frac{1}{1(1+(D^2+2D))} x^2$$

$$= \frac{e^{3x}}{16} + (1+(D^2+2D))^{-1} x^2$$

$$= \frac{e^{3x}}{16} + [\cancel{x^2}\, 1 - D^2 - 2D + (D^4 + 4D^3 + 4D^2 + \ldots] x^2$$

$$= \frac{e^{3x}}{16} + [x^2 - 2 - 4x + 8]$$

$= \frac{e^{3x}}{16} + [x^2 - 4x + 6]$

General Sol^n

$y = \frac{e^{3x}}{16} + [x^2 - 4x + 6] + (c_1 + x c_2) e^{-x}$

5) Solve $(D^2 - 3D + 2)y = \cosh x$.

auxillary eq^n

$m^2 - 3m + 2 = 0$

$m^2 - 2m - m + 2 = 0$

$m(m-2) - 1(m-2) = 0$

$m = 1, 2$

$y_c = c_1 e^{x} + c_2 e^{2x}$

$y_p = \frac{1}{(D^2 - 3D + 2)} \cosh x$

$= \frac{1}{(D^2 - 3D + 2)} \left[\frac{e^{x} + e^{-x}}{2} \right]$

$= \frac{e^{x}}{2(D^2 - 3D + 2)} + \frac{e^{-x}}{2(D^2 - 3D + 2)}$

$= \frac{x e^{x}}{2(2D - 3)} + \frac{e^{-x}}{12}$

$= -\frac{x e^{x}}{2} + \frac{e^{-x}}{12}$

General Sol^n

$y = c_1 e^{x} + c_2 e^{2x} - \frac{x e^{x}}{2} + \frac{e^{-x}}{12}$.

6) Solve $(D^2 - 4D + 3)y = 2e^{x}$

auxillary eq^n

$m^2 - 4m + 3 = 0$

$m^2 - 3m - m + 3 = 0$

$m(m-3) - 1(m-3) = 0$

$(m-1)(m-3) = 0$

$m = 1, 3$

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$y_c = C_1 e^{x} + C_2 e^{3x}$

$y_p = \frac{1}{(D^2-4D+3)} \times 2e^{x}$

$= 2\left[\frac{x e^{x}}{2D-4}\right]$

$= \frac{2xe^{x}}{-2}$

$= -xe^{x}$

General solⁿ

$y = C_1 e^{x} + C_2 e^{3x} - xe^{x}$

7) Solve $(D+2)^2 y = e^{-2x}\sin x$

auxillary eqⁿ

$(m+2)^2 = 0$

$m = -2, -2$

$y_c = (C_1 + xC_2)e^{-2x}$

$y_p = \frac{e^{-2x}\sin x}{(D+2)^2}$

$= e^{-2x}\left[\frac{\sin x}{((D-2)+2)^2}\right]$

$= e^{-2x}\left[\frac{\sin x}{D^2}\right]$

$= e^{-2x}\sin x$

General solⁿ

$y = (C_1 + xC_2)e^{-2x} + e^{-2x}\sin x$

8) Solve $(D^2-2D-3)y = 2e^{x} + 10\sin x$

auxillary eqⁿ

$m^2 - 2m - 3 = 0$

$m^2 - 3m + m - 3 = 0$

$m(m-3) + 1(m-3) = 0$

$m = -1, 3$

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$y_c = c_1 e^{-x} + c_2 e^{3x}$

$y_p = \frac{1}{(D^2-2D-3)}[2e^x + 10\sin x]$

$= \frac{2e^x}{D^2-2D-3} + 10\frac{\sin x}{D^2-2D-3}$

$= \frac{2e^x}{1-2-3} + 10\frac{\sin x}{-2D-4}$

$= \frac{-e^x}{2} + (-10)(2D-4)\sin x$

$= \frac{-e^x}{2} - 5\frac{(D-1)\sin x}{D^2-1}$

$= \frac{-e^x}{2} - 5x\frac{(D-1)\sin x}{2D}$

$= \frac{-e^x}{2} - \frac{5}{2}x\cos x + x\frac{5}{2}\sin x$

General sol^n.

$y = c_1 e^{-x} + c_2 e^{3x} - \frac{e^x}{2} - \frac{5}{2}x[\cos x - \sin x]$

9) Solve $(D^2+D)y = x^2+2x+4$

auxillary eq^n

$m^2+m=0$

$m(m+1)=0$

$m=0, m=-1$

$y_c = c_1 + c_2 e^{-x}$

$y_p = \frac{1}{D^2+D}[x^2+2x+4]$

$= \frac{x^2}{D(1+D)} + 2\frac{x}{D(1+D)} + \frac{4e^{0x}}{(D^2+D)}$

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$= \frac{1}{D}[1+D]^{-1}x^2 + \frac{2}{D}[1+D]^{-1}x + 4x$

$= \frac{1}{D}[1+D+D^2+\ldots]x^2 + \frac{2}{D}[1+D+D^2+\ldots]x + 4x$

$= \frac{1}{D}[x^2+2x+2] + \frac{2}{D}[x+1] + 4x$

$= \frac{x^3}{3} + x^2 + 2x + x^2 + 2x + 4x$

$= \frac{x^3}{3} + 2x^2 + 8x$

General Solⁿ.

$y = C_1 + C_2e^{-x} + \frac{x^3}{3} + 2x^2 + 8x$

10) Solve $(D^3-5D^2+7D-3)y = e^{2x}\cosh x$

auxillary eqⁿ.

$m^3-5m^2+7m-3 = 0$

$(m-1)(m^2-4m+3) = 0$

$m = 1, 1, 3$

$y_c = (C_1 + xC_2)e^{x} + C_3e^{3x}$

$y_p = \frac{1}{f(D)}[R(x)]$

1	1	-5	7	-3
	0	1	-4	3
	1	-4	3	0

⇒ Refer Q-5

11) Solve $(D^2-D-2)y = \sin 2x$

auxillary eqⁿ

$m^2-m-2=0$

$m^2-2m+m-2=0$

$m(m-2)+1(m-2)=0$

$m = -1, 2$

$y_c = C_1e^{-x} + C_2e^{2x}$

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$$y_p = \frac{\sin 2x}{(D^2-D-2)}$$

$$= \frac{\sin 2x}{-4-D-2}$$

$$= -\frac{\sin 2x}{(D+6)}$$

$$= -\frac{(D-6)}{(D^2-36)}\sin 2x$$

$$= \frac{(-D+6)\sin 2x}{-40}$$

$$= \frac{1}{40}\times 2\cos 2x - \frac{6}{40}\sin 2x$$

$$= \frac{\cos 2x}{20} - \cancel{6}\,\frac{3}{20}\sin 2x$$

General Sol^n

$$y = c_1 e^{-x} + c_2 e^{2x} + \frac{\cos 2x}{20} - \frac{3}{20}\sin 2x$$

12) Solve $(D^2-2D+1)y = \cos 3x$

auxcillary eq^n

$$m^2-2m+1=0$$

$$m^2-m-m+1=0$$

$$m(m-1)-1(m-1)=0$$

$$m=1,1$$

$$y_c = (c_1+xc_2)e^x$$

$$y_p = \frac{\cos 3x}{D^2-2D+1}$$

$$= \frac{\cos 3x}{-9-2D+1}$$

$$= \frac{\cos 3x}{-2(D+4)}$$

$$= \frac{-1}{2}\frac{(D-4)}{(D^2-16)}\cos 3x$$

$$= +\frac{1}{2\times 25}\cdot(D-4)\cos 3x$$

$= \frac{-3\sin 3x}{50} - \frac{2}{25}\cos 3x$

General Soln

$y = (c_1 + xc_2)e^x - \frac{3}{50}\sin 3x - \frac{2}{25}\cos 3x$

13) Refer Q-6.

14) Solve $(D^2-4)y = x^2$

auxillary eqn

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm 2$

$y_c = c_1 e^{-2x} + c_2 e^{2x}$

$y_p = \frac{1}{D^2-4}[x^2]$

$= -\frac{1}{4}\left(1 - \frac{D^2}{4}\right)^{-1} x^2$

$= -\frac{1}{4}\left[1 + \frac{D^2}{4} + \frac{D^4}{16} + \dots\right] x^2$

$= -\frac{1}{4}\left[x^2 + \frac{1}{2}\right]$

$= -\frac{x^2}{4} - \frac{1}{8}$

General Soln

$y = c_1 e^{-2x} + c_2 e^{2x} - \frac{x^2}{4} - \frac{1}{8}$

15) Solve $(D^2+16)y = x^4 + e^{3x} + \cos 3x$

auxillary eqn

$m^2 + 16 = 0$

$m^2 = -16$

$m = \pm 4i$

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$$y_c = (C_1\cos 4x + C_2\sin 4x)$$

$$y_p = \frac{[x^4 + e^{3x} + \cos 3x]}{D^2+16}$$

$$= \frac{x^4}{D^2+16} + \frac{e^{3x}}{D^2+16} + \frac{\cos 3x}{D^2+16}$$

$$= \frac{1}{16}\left[1+\frac{D^2}{16}\right]^{-1} x^4 + \frac{e^{3x}}{25} + \frac{\cos 3x}{7}$$

$$= \frac{1}{16}\left[1 - \frac{D^2}{16} + \frac{D^4}{256} + \ldots\right] x^4 + \frac{e^{3x}}{25} + \frac{\cos 3x}{7}$$

$$= \frac{1}{16}\left[x^4 - \frac{3}{4}x^2 + \frac{24}{256}\right] + \frac{e^{3x}}{25} + \frac{\cos 3x}{7}$$

General Sol^n

$$y = (C_1\cos 4x + C_2\sin 4x) + \frac{1}{16}\left[x^4 - \frac{3}{4}x^2 + \frac{24}{256}\right] + \frac{e^{3x}}{25} + \frac{\cos 3x}{7}$$

15) Solve $(D^2 - 2D + 1)y = e^{3x}x^2$

auxillary eq^n.

$$m^2 - 2m + 1 = 0$$

$$m = 1, 1$$

$$y_c = (C_1 + xC_2)e^x$$

$$y_p = \frac{e^{3x}x^2}{(D^2 - 2D + 1)}$$

$$= e^{3x}\left[\frac{x^2}{(D+3)^2 - 2(D+3) + 1}\right]$$

$$= e^{3x}\left[\frac{x^2}{D^2 + 4D + 4}\right]$$

$$= \frac{e^{3x}}{4}\left[1 + \left[\frac{D^2}{4} + D\right]\right]^{-1} x^2$$

$$= \frac{e^{3x}}{4}\left[1 - \frac{D^2}{4} - D + \frac{D^4}{16} + \frac{D^3}{2} + D^2 + \cdots\right]x^2$$

$$= \frac{e^{3x}}{4}\left[x^2 - \frac{1}{2} - 2x + 2\right]$$

$$= \frac{e^{3x}}{4}\left[x^2 - 2x + \frac{3}{2}\right]$$

General soln

$$y = (c_1 + x c_2)e^x + \frac{e^{3x}}{4}\left[x^2 - 2x + \frac{3}{2}\right]$$

17 ~~16~~) Solve $(D^2 - 5D + 6)y = e^{2x}\sin 2x$

auxillary eqn.

$$m^2 - 5m + 6 = 0$$

$$m^2 - 3m - 2m + 6 = 0$$

$$m(m-3) - 2(m-3) = 0$$

$$m = 2, 3$$

$$y_c = c_1 e^{2x} + c_2 e^{3x}$$

$$y_p = \frac{1}{f(D)} R(x)$$

$$= \frac{1}{D^2 - 5D + 6}\left[e^{2x}\sin 2x\right]$$

$$= e^{2x}\left[\frac{\sin 2x}{D(D-1)}\right]$$

$$= e^{2x}\left[\frac{(D+1)\sin 2x}{D(D^2-1)}\right]$$

$$= \frac{e^{2x}}{3}\left[\frac{(D+1)\sin 2x}{D}\right]$$

$$= \frac{e^{2x}}{3}\left[\frac{-1}{2}(D+1)\cos 2x\right]$$

$$= \frac{-e^{2x}}{6}\left[2\sin 2x + \cos 2x\right]$$

General Sol^n.

$$y = c_1 e^{2x} + c_2 e^{3x} - \frac{e^{2x}}{6}[2\sin 2x + \cos 2x]$$

18) Solve $(D^2+4)y = x\sin 2x$

auxillary eq^n.

$$m^2+4=0$$
$$m^2=-4$$
$$m=\pm 2i$$
$$y_c = (c_1\cos 2x + c_2 \sin 2x)$$

$$y_p = \frac{x\sin 2x}{(D^2+4)}$$

$$= x\left[\frac{\sin 2x}{D^2+4}\right] - \frac{(2D)}{(D^2+4)^2}\sin 2x$$

$$= x\left[\frac{x\sin 2x}{2D}\right] - \frac{2D\,x}{2(D^2+4)(2D)}\sin 2x$$

$$= -\frac{x^2}{4}\cos 2x + \frac{x^2}{4\times 2}\cos 2x$$

$$= -\frac{x^2}{4}\cos 2x + \frac{x^2}{8}\cos 2x.$$

General Sol^n

$$y = (c_1\cos 2x + c_2\sin 2x) - \frac{x^2}{4}$$
$$\cos 2x + \frac{x^2\cos 2x}{8}$$

19) Solve $(D^2-2D+1)y = xe^x\sin x$

= auxillary eq^n

$$m^2-2m+1$$
$$m=1,1$$
$$y_c = (c_1+xc_2)e^x$$

$$y_p = \frac{xe^x \sin x}{(D^2-2D+1)}$$

$$= e^x \left[\frac{x \sin x}{D^2} \right]$$

$$= e^x \left[x \left[\frac{\sin x}{D^2} \right] - \frac{2D}{(D^2)^2} \sin x \right]$$

$$= e^x \left[-x \sin x - 2\cos x \right]$$

General solⁿ

$$y = (C_1 + xC_2)e^x + e^x[-x\sin x - 2\cos x]$$