# Unit - 5

1) Periodic fun$^n$ - A fun$^n$ returning to the same value at regular intervals

① $\cos 3x = \frac{2\pi}{3}$
② $\sin 5x = \frac{2\pi}{5}$
③ $\cos 5x = \frac{2\pi}{5}$
④ $\sin 7x = \frac{2\pi}{7}$
⑤ $\cos 7x = \frac{2\pi}{7}$
⑥ $\sin 9x = \frac{2\pi}{9}$
⑦ $\cos nx = \frac{2\pi}{n}$
⑧ $\sin nx = \frac{2\pi}{n}$
⑨ $\cos 11x = \frac{2\pi}{11}$
⑩ $\sin 13x = \frac{2\pi}{13}$
⑪ $\sin 17x = \frac{2\pi}{17}$
⑫ $\cos 17x = \frac{2\pi}{17}$

A fun$^n$ $f(x)$ is called Periodic if it is defined for all $x$ and there exists some positive number $p$ such that $f(x+p) = f(x)$, $\forall x$, the number $p$ is called Period of $f(x)$

2) Find Euler's formula for fourier coefficients $(a_0, a_n, b_n)$

→ The fourier series is defined as $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ ← ①

→ for $a_0$

Integrating eqn ① w.r.t $x$ by taking limits $c$ to $c+2\pi$, we get

$$\int_c^{c+2\pi} f(x)\,dx = \int_c^{c+2\pi} \frac{a_0}{2}\,dx + \sum_{n=1}^{\infty}\left[\int_c^{c+2\pi}(a_n \cos nx + b_n \sin nx)\,dx\right]$$

$$\int_c^{c+2\pi} f(x)\,dx = \frac{a_0}{2}[x]_c^{c+2\pi} + \sum_{n=1}^{\infty}\left[a_n \int_c^{c+2\pi} \cos nx\,dx + b_n \int_c^{c+2\pi} \sin nx\,dx\right]$$

$$= \frac{a_0}{2}[2\pi] + \sum_{n=1}^{\infty}[a_n(0) + b_n(0)]$$

$$\int_c^{c+2\pi} f(x)\,dx = a_0\pi$$

$$\boxed{a_0 = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\,dx}$$

→ for $a_n$

multiply both sides of eqn ① with $\cos mx$ and then Integrating w.r.t $x$ between limits $c$ to $c+2\pi$, we get.

$$\int_c^{c+2\pi} f(x)\cos mx\,dx = \int_c^{c+2\pi} \frac{a_0}{2}\cos mx\,dx + \sum_{n=1}^{\infty}\left[\int_c (a_n \cos mx \cos nx + b_n \sin nx \cos mx)\,dx\right]$$

For $m=n$ $\quad \int_c^{c+2\pi} \cos mx\,dx = 0 \quad \int_c^{c+2\pi} \cos^2 mx = \pi \quad \int_c^{c+2\pi} \cos mx \sin nx\,dx = 0$

$$\int_c^{c+2\pi} f(x)\cos mx\,dx = 0 + \sum_{n=1}^{\infty} a_n\pi$$

$$\boxed{a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\cos nx\,dx}$$

For $b_n$

multiply both side of eqn ① with $\sin mx$ and then integrating w.r.t $x$ between limits $c$ to $c+2\pi$, we get.

$$\int_c^{c+2\pi} f(x)\sin mx\,dx = \int_c^{c+2\pi} \frac{a_0}{2}\sin mx + \sum_{n=1}^{\infty}\left(\int_c^{c+2\pi}(a_n \cos nx \sin mx + b_n \sin nx \sin mx)\,dx\right)$$

For $m=n$ $\quad \int_c^{c+2\pi} \sin mx = 0 \quad \int_c^{c+2\pi} \cos mx \sin mx = 0 \quad \int_c^{c+2\pi} \sin^2 mx = \pi$

$$\int_c^{c+2\pi} f(x)\sin mx\,dx = 0 + b_n\pi$$

$$\boxed{b_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\sin nx\,dx}$$

Q-3 Using Euler's formula find $a_0$ for $f(x) = \sqrt{1-\cos x}$, $-\pi \le x \le \pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}\sqrt{1-\cos x}\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\sqrt{1-\cos x}\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\sqrt{2}\sin\frac{x}{2}\,dx$$

$$= \frac{2\sqrt{2}}{\pi}\left[-\cos\frac{x}{2}\cdot 2\right]_0^{\pi}$$

$$= \frac{4\sqrt{2}}{\pi}\left[-\left(\cos\frac{\pi}{2} - \cos 0\right)\right]$$

$$\boxed{a_0 = \frac{4\sqrt{2}}{\pi}\left[0 - (-1)\right]} = \boxed{+\frac{4\sqrt{2}}{\pi}}$$

Q-4 Find $b_n$ for $f(x) = 2x$ $\quad -\pi \le x \le \pi$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} 2x\sin nx\,dx$$

$$= \frac{2}{\pi}\, 2\int_0^{\pi} x\sin nx\,dx$$

$$= \frac{4}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - 1\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{4}{\pi}\left[\pi\left(\frac{-\cos\pi}{n}\right)\right]$$

$$= \frac{4}{n}\,-(-1)^n \quad \Rightarrow \boxed{b_n = \frac{-4(-1)^n}{n}}$$

Q-5 Find $a_n$ for $f(x) = x^2$ $\quad -\pi \le x \le \pi$

$$\rightarrow a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2\cos nx\,dx$$

$$= \frac{2}{\pi}\left[x^2\frac{\sin nx}{n} - 2x\left(\frac{-\cos nx}{n^2}\right) + 2\left(\frac{-\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{2\pi}{n^2}(-1)^n\right] \Rightarrow \boxed{b_n = \frac{4}{n^2}(-1)^n}$$

Q-6 find $a_0$ for $f(x) = |x|$, $-\pi \le x \le \pi$

→ $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} x \, dx$

$= \frac{2}{\pi} \int_0^{\pi} x \, dx$

$= \frac{2}{\pi} \left[\frac{x^2}{2}\right]_0^{\pi}$

$= \frac{\pi^2}{\pi}$ $\boxed{a_0 = \pi}$

Q-7 write fourier series expansion for odd & even funⁿ.

→ for ~~odd~~ even $b_n = 0$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \, dx$$

for ~~even~~ odd $a_0 = a_n = 0$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \, dx$$

Q-8 find $a_0$ for given funⁿ $f(x) = \cos\frac{x}{2}$, $0 < x < 2$

$P = 2l = 2$

$l = 1$

$a_0 = \frac{1}{l} \int_0^{c+2l} f(x) dx$

$= \int_0^2 \cos\frac{x}{2} dx$

$= \left[\sin\frac{x}{2}\right]_0^2 \cdot 2 = [\sin 1 - \sin 0] 2 = 2(0.84)$

$\boxed{a_0 = 1.68}$

Q-9 find $a_0$ for $f(x) = 3x^2$ $-\pi \le x \le \pi$

$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} 3x^2$

$= \frac{3 \times 2}{\pi} \int_0^{\pi} x^2$

$= \frac{6}{\pi} \left[\frac{x^3}{3}\right]_0^{\pi} = \frac{2}{\pi} \pi^3 = 2\pi^2$ $\therefore \boxed{a_0 = 2\pi^2}$

Q-10 write fourier series expansion. for $f(x)$ in $0 < x < 2l$ & $-l < x < l$

→ ① for $0 < x < 2L$

$a_0 = \frac{1}{l} \int_0^{2l} f(x) \, dx$

$a_n = \frac{1}{l} \int_0^{2l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$

$b_n = \frac{1}{l} \int_0^{2l} f(x) \sin\left(\frac{n\pi x}{l}\right) dx$

② for $-L < x < L$

$a_0 = \frac{1}{l} \int_{-l}^{l} f(x) \, dx$

$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos\left(\frac{n\pi x}{l}\right) dx$

$b_n = \frac{1}{l} \int_{-l}^{l} f(x) \sin\left(\frac{n\pi x}{l}\right) dx$

Q-1

(1) $f(x) = e^x \quad 0<x<\pi$

$\rightarrow$ $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)\, dx$

$a_0 = \frac{1}{\pi} \int_0^{\pi} e^x\, dx$

$= \frac{1}{\pi} [e^x]_0^{\pi} = \boxed{\frac{1}{\pi}[e^{\pi} - 1]}$

$a_n = \frac{1}{\pi} \int_0^{\pi} e^x \cos nx\, dx$

$= \frac{1}{\pi} \left[\frac{e^x}{n^2+1} (\cos nx + n \sin nx)\right]_0^{\pi}$

$= \frac{1}{\pi} \left[ \frac{e^{\pi}}{1+n^2} [(-1)^n) - \frac{1}{1+n^2} (-1) \right]$

$\boxed{a_n = \frac{\cancel{(-1)^n - 1)} \cancel{(e^{\pi})}}{\pi(1+n^2)}}$ $\quad \frac{e^{\pi}(-1)^n - 1}{\pi(1+n^2)}$

$b_n = \frac{1}{\pi} \int_0^{\pi} f(x) \sin nx\, dx$

$= \frac{1}{\pi} \int_0^{\pi} e^x \sin nx\, dx$

$= \frac{1}{\pi} \left[ \frac{e^x}{1+n^2} (\sin nx - n \cos nx) \right]_0^{\pi}$

$= \frac{1}{\pi} \left[ \frac{e^{\pi}}{1+n^2} (-n((-1)^n)) - \frac{1}{1+n^2} (-1) \right)$

$\boxed{b_n = \frac{n(1 - e^{\pi}(-1)^n)}{\pi(1+n^2)}}$

(2) $f(x) = 3e^{2x}, \quad 0 < x < 2\pi$

$\rightarrow$ $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)\, dx$

$a_0 = \frac{1}{\pi} \int_0^{2\pi} 3e^{2x}\, dx$

$= \frac{3}{\pi} \left[\frac{e^{2x}}{2}\right]_0^{2\pi} dx$

$\boxed{a_0 = \frac{3}{2\pi}(e^{4\pi} - 1)}$

$a_n = \frac{1}{\pi} \int_0^{2\pi} 3e^{2x} \cos nx\, dx$

$= \frac{3}{\pi} \left[ \frac{e^{2x}}{n^2+4} (2\cos nx + n \sin nx) \right]_0^{2\pi}$

$= \frac{3}{\pi} \left( \frac{e^{4\pi}}{n^2+4} (2(1)) - \frac{1}{n^2+4} (2(1)) \right)$

$\boxed{a_n = \frac{6}{\pi} \left[ \frac{e^{4\pi} - 1}{n^2+4} \right]}$

$b_n = \frac{1}{\pi} \int_0^{2\pi} 3e^{2x} \sin nx\, dx$

$= \frac{3}{\pi} \left[ \frac{e^{2x}}{4+n^2} (2 \sin nx - n \cos nx) \right]_0^{2\pi}$

$= \frac{3}{\pi} \left( \frac{e^{4\pi}}{4+n^2} (-n(1)) - \frac{1}{4+n^2} (-n(1)) \right)$

$\boxed{b_n = -\frac{3}{\pi} \left( \frac{(e^{4\pi} - 1)\, n}{n^2+4} \right)}$

Q-3 $f(x) = e^{ax}$ $\quad 0 < x < 2\pi$

$\rightarrow$ $a_0 = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\,dx$

$= \frac{1}{\pi}\int_0^{2\pi} e^{ax}\,dx$

$= \frac{1}{\pi}\left[\frac{e^{ax}}{a}\right]_0^{2\pi} dx$

$\boxed{a_0 = \frac{1}{\pi a}\left[e^{2a\pi}-1\right]}$

$a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\cos nx\,dx$

$= \frac{1}{\pi}\int_0^{2\pi} e^{ax}\cos nx\,dx$

$= \frac{1}{\pi}\left[\frac{e^{ax}}{n^2+a^2}\; a\cos nx + n\sin nx\right]_0^{2\pi}$

$= \frac{1}{\pi}\left[\frac{e^{2a\pi}}{n^2+a^2}\,a(1) - \frac{1}{n^2+a^2}\,a(1)\right]$

$\boxed{a_n = \frac{a}{\pi(n^2+a^2)}\left(e^{2a\pi}-1\right)}$

$b_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\sin nx\,dx$

$= \frac{1}{\pi}\int_0^{2\pi} e^{ax}\sin nx\,dx$

$= \frac{1}{\pi}\left[\frac{e^{ax}}{a^2+n^2}\; a\sin nx - n\cos nx\right]_0^{2\pi}$

$= \frac{1}{\pi}\left(\frac{e^{2a\pi}}{n^2+a^2}(-n) - \frac{1}{n^2+a^2}(-n)\right]$

$\boxed{b_n = \frac{-n}{\pi(n^2+a^2)}\left(e^{2a\pi}-1\right)}$

Q-4 $f(x) = \frac{1}{2}(\pi - x)$ $\quad , 0 < x < 2\pi$

$a_0 = \frac{1}{\pi}\int_c^{c+2\pi} \frac{1}{2}(\pi - x)$

$= \frac{1}{2\pi}\left[\pi x - \frac{x^2}{2}\right]_0^{2\pi}$

$= \frac{1}{2\pi}\left(2\pi^2 - \frac{4\pi^2}{2}\right)$

$\boxed{a_0 = 0}$

$a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\cos nx\,dx$

$= \frac{1}{2\pi}\int_0^{2\pi} (\pi - x)\cos nx\,dx$

$= \frac{1}{2\pi}\left[(\pi - x)\frac{\sin nx}{n} - (-1)\left(\frac{-\cos nx}{n^2}\right)\right]_0^{2\pi}$

$= \frac{1}{2\pi}\left[-\frac{\cos nx}{n^2}\right]_0^{2\pi}$

$= \frac{-1}{2\pi n^2}\left(1 - 1\right)$

$\boxed{a_n = 0}$

$b_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\sin nx\,dx$

$= \frac{1}{2\pi}\int_0^{2\pi} (\pi - x)\sin nx\,dx$

$= \frac{1}{2\pi}\left[(\pi - x)\left(-\frac{\cos nx}{n}\right) - (-1)\left(\frac{-\sin nx}{n^2}\right)\right]_0^{2\pi}$

$= \frac{1}{2\pi n}\left((-\pi)(-(1)) - \pi(-1)\right)$

$= \frac{1}{2\pi n}\left(\pi + \pi\right)$

$= \frac{1}{2\pi n}(2\pi)$

$\boxed{b_n = \frac{1}{n}}$

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Q-5 $f(x) = x^3, -\pi < x < \pi$

~~let~~ $f(-x) = -x^3 = -f(x)$

$a_0 = a_n = 0$

$b_n = \frac{2}{\pi} \int_0^\pi x^3 \sin nx \, dx$

$= \frac{2}{\pi} \left[ x^3 \left(-\frac{\cos nx}{n}\right) - 3x^2 \left(-\frac{\sin nx}{n^2}\right) + 6x \left(\frac{\cos nx}{n^3}\right) \mp 6 \left(\frac{\sin nx}{n^4}\right) \right]_0^\pi$

$= \frac{2}{\pi} \left( -\pi^3 \frac{(-1)^n}{n} + 6\pi \frac{(-1)^n}{n^3} \right)$

$= -\pi^2 \frac{(-1)^n}{n} + \frac{6(-1)^n}{n^3} \quad \boxed{= \frac{2(-1)^n}{n} \left( -\pi^2 + \frac{6}{n^2} \right)}$

Q-6 $f(x) = x^2, \quad -2 \le x \le 2$

$L = 2$

$f(-x) = x^2 = f(x) \quad \therefore f(x)$ is even funⁿ.

$\boxed{\therefore b_n = 0}$

$a_0 = \frac{1}{l} \int_{-l}^{l} f(x) \, dx$

$= \frac{1}{2} \int_{-2}^{2} x^2 dx$

$= \frac{2}{2} \left[ \frac{x^3}{3} \right]_0^2$

$= \frac{2^3}{3} = \boxed{\frac{8}{3}}$

$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos nx \, dx$

$= \frac{1}{2} \int_{-2}^{2} x^2 \cos \frac{n\pi x}{2} dx$

$= \frac{2}{2} \left[ x^2 \sin \frac{nx}{n} - 2x \left( \frac{-\cos nx}{n^2} \right) + 2 \left( \frac{-\sin nx}{n^3} \right) \right]_0^2$

$-2x \left( -\cos \frac{n\pi x}{2} \right) \Big/ \left( \frac{n\pi}{2} \right)^2$

$= \frac{2(2) \cdot (-1)^n}{n^2 \frac{\pi^2}{4}}$

$\boxed{a_n = \frac{16(-1)^n}{\pi^2 n^2}}$

Q-7 $f(x) = x + |x|, -\pi < x < \pi$

$\rightarrow \quad a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} x + |x|$

$= \frac{1}{\pi} \left( \int_{-\pi}^{\pi} x + \int_{-\pi}^{\pi} |x| \right)$

$= \frac{1}{\pi} \left( 0 + 2 \left[ \frac{x^2}{2} \right]_0^\pi \right)$

$= \frac{2}{\pi} \left[ \frac{\pi^2}{2} \right] \quad \boxed{a_0 = \pi}$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (x + |x|) \cos nx \, dx$

$= \frac{1}{\pi} \left[ \int_{-\pi}^{\pi} x \cos nx + \int_{-\pi}^{\pi} |x| \cos nx \, dx \right]$

$= \frac{1}{\pi} \left[ 0 + 2 \int_0^\pi x \cos nx \, dx \right]$

$= \frac{2}{\pi} \left[ x \frac{\sin nx}{n} - 1 \left( -\frac{\cos nx}{n^2} \right) \right]_0^\pi$

$\boxed{a_n = \frac{2}{\pi n^2} \left( (-1)^n - 1 \right)}$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (x + |x|) \sin nx \, dx$

$= \frac{1}{\pi} \left( \int_{-\pi}^{\pi} x \sin nx + \int_{-\pi}^{\pi} |x| \sin nx \, dx \right)$

$= \frac{1}{\pi} \left( 2 \int_0^\pi x \sin nx + 0 \right)$

$= \frac{2}{\pi} \left[ x \left( -\frac{\cos nx}{n} \right) - (1) \left( -\frac{\sin nx}{n^2} \right) \right]_0^\pi$

$= \frac{2}{\pi} \left( \frac{\pi}{n} (-(-1)^n) \right)$

$\boxed{b_n = \frac{-2}{n} (-1)^n}$

⑧ $f(x) = x + x^2$ in $(-1, 1)$

$\rightarrow$ $a_0 = \frac{1}{\pi} \int_{-1}^{1} (x + x^2) dx.$

$= \frac{1}{\pi} \left( 0 + 2\left[\frac{x^3}{3}\right]_0^1 dx \right)$

$= \frac{2}{3\pi}(1 - 0) \Rightarrow \boxed{a_0 = \frac{2}{3\pi}}$

$(-1)(-1) = 1 \quad 1(-1)$

$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos(n\pi x) dx$

$= 1 \int_{-1}^{1} (x + x^2) \cos n\pi x \, dx$

$= \left[ 0 + 2\int_0^1 x^2 \cos n\pi x \, dx \right)$

$= 2\left[ x^2 \frac{\sin n\pi x}{n\pi} - 2x\frac{(-\cos n\pi x)}{(n\pi)^2} + 2\frac{(-\sin n\pi x)}{(n\pi)^3} \right]_0$

$= 2\left( \frac{2(1)(\cos n\pi)}{n^2\pi^2} \right)$

$\boxed{a_n = \frac{4}{n^2\pi^2}(-1)^n}$

$b_n = \frac{1}{l}\int_{-l}^{l} f(x) \sin n\pi x \, dx$

$= 1\int_{-1}^{1} (x + x^2) \sin n\pi x \, dx$

$= 2\int_0^1 x \sin n\pi x + 0$

$= 2\left( x\frac{(-\cos n\pi x)}{n\pi} - 1\left(\frac{-\sin n\pi x}{n^2\pi^2}\right) \right]_0^1$

$= 2\left( 1\left( \frac{-\cos n\pi}{n\pi} \right) \right)$

$\boxed{b_n = \frac{-2(-1)^n}{n\pi}}$

⑨ $f(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1. \end{cases}$

$a_0 = \frac{1}{\pi}\int_{-1}^{1} f(x) dx$

$= \frac{1}{\pi}\left[ \int_{-1}^{0} f(x) dx + \int_0^1 f(x) dx \right]$

$= \frac{1}{\pi}\left( \int_{-1}^{0} -1 \, dx + \int_0^1 1 \, dx \right)$

$= \frac{1}{\pi}\left( -[x]_{-1}^{0} + (x]_0^1 \right]$

$= \frac{1}{\pi}\left[ -(0-(-1)) + 1 \right]$

$= \frac{1}{\pi}(-1 + 1) \Rightarrow \boxed{a_0 = 0}$

$a_n = \frac{1}{\pi}\int_{-1}^{1} f(x) \cos nx \, dx$

$= \frac{1}{\pi}\left[ \int_{-1}^{0} f(x) \cos nx + \int_0^1 f(x) \cos nx \, dx \right]$

$= \frac{1}{\pi}\left[ -\int_{-1}^{0} \cos nx + \int_0^1 \cos nx \, dx \right)$

$= \frac{1}{\pi}\left[ -\left[\frac{\sin nx}{n}\right]_{-1}^{0} + \left[\frac{\sin nx}{n}\right]_0^1 \right]$

$= \frac{1}{n}\left[ -\left[ 0 - \frac{\sin -n}{n} + \frac{\sin n}{n} - 0 \right] \right]$

$= \frac{1}{n}(-1 + 1) \quad \boxed{a_n = 0}$

$b_n = \frac{1}{\pi}\int_{-1}^{1} f(x) \sin nx \, dx$

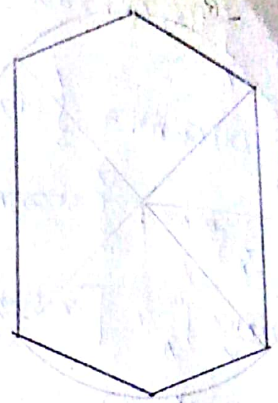
$= \frac{1}{\pi}\left[ \int_{-1}^{0} f(x) \sin n x\pi + \int_0^1 f(x) \sin n x \pi \, dx \right]$

$= \frac{1}{\pi}\left[ -1\int_{-1}^{0} \sin n x\pi + \int_0^1 \sin n x \pi \, dx \right]$

$= \frac{1}{\pi}\left[ -1\left(\frac{-\cos n x\pi}{n\pi}\right]_{-1}^{0} + \left(\frac{-\cos n x\pi}{n\pi}\right]_0^1 \right]$

$= \frac{1}{\pi}\left[ \frac{1 - \cos n\pi}{n\pi} + -\frac{\cos n}{n\pi} + \frac{1}{n} \right)$

$\boxed{b_n = \frac{2}{n\pi}\left(1 - (-1)^n\right)}$



$$f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$
$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right]$$
$$= \frac{1}{\pi}\left(-k\,[x]_{-\pi}^{0} + k\,(x]_{0}^{\pi}\right]$$
$$= \frac{1}{\pi}\left[-k\,[0+\pi) + k\,[\pi-0]\right)$$
$$= \frac{1}{\pi}\left[-k\pi + k\pi\right] \Rightarrow \boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$
$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_{0}^{\pi} f(x)\cos nx\,dx\right]$$
$$= \frac{1}{\pi}\left[-k\int_{-\pi}^{0}\cos nx\,dx + k\int_{0}^{\pi}\cos nx\,dx\right]$$
$$= \frac{1}{\pi}\left[-k\left(\frac{\sin nx}{n}\right)_{-\pi}^{0} + k\left(\frac{\sin nx}{n}\right]_{0}^{\pi}\right]$$
$$= \frac{1}{\pi}\left[-k\,(0) + k(0)\right)$$
$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$
$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx + \int_{0}^{\pi} f(x)\sin nx\,dx\right]$$
$$= \frac{1}{\pi}\left[-k\int_{-\pi}^{0}\sin nx + k\int_{0}^{\pi}\sin nx\,dx\right]$$
$$= \frac{1}{\pi}\left(-k\left[-\frac{\cos nx}{n}\right]_{-\pi}^{0} + k\left[-\frac{\cos nx}{n}\right]_{0}^{\pi}\right)$$
$$= \frac{1}{\pi}\left[\frac{k}{n}\left(1-(-1)^n\right) + \frac{k}{n}\left(1-(-1)^n\right)\right]$$
$$= \frac{1}{\pi}\left[\frac{k}{n}\left(1-(-1)^n + 1 - (-1)^n\right)\right)$$
$$= \frac{1}{\pi}(0) \qquad \boxed{\cancel{b_n = 0}} \qquad \boxed{b_n = \frac{2k}{n\pi}\left(1-(-1)^n\right)}$$

(ii) $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$

$\rightarrow$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$$
$$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right]$$
$$= \frac{1}{\pi}\left(-\pi\int_{-\pi}^{0} dx + \int_{0}^{\pi} x\,dx\right)$$
$$= \frac{1}{\pi}\left[-\pi\,[x]_{-\pi}^{0} + \left[\frac{x^2}{2}\right]_{0}^{\pi}\right)$$
$$= \frac{1}{\pi}\left(-\pi^2 + \frac{\pi^2}{2}\right]$$
$$= \pi\cdot\left(-1 + \frac{1}{2}\right)$$
$$= -\frac{\pi}{2}$$
$$\boxed{\therefore a_0 = -\frac{\pi}{2}}$$

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$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$

$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cos nx + \int_{0}^{\pi} f(x)\cos nx\right]$

$= \frac{1}{\pi}\left[-\pi\int_{-\pi}^{0}\cos nx + \int_{0}^{\pi} x\cos nx\right]$

$= \left[-\pi\left[\frac{\sin nx}{n}\right]_{-\pi}^{0} + \left[x\frac{\sin nx}{n} - \left(\frac{-\cos nx}{n^2}\right)\right]_0^{\pi}\right]$

$= \left[0 + \frac{(-1)^n - 1}{n^2}\right)$

$\boxed{a_n = \frac{(-1)^n - 1}{n^2\pi}}$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \ldots$

$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx + \int_{0}^{\pi} f(x)\sin nx\,dx\right]$

$= \frac{1}{\pi}\left[-\pi\int_{-\pi}^{0}\sin nx + \int_{0}^{\pi} x\sin nx\,dx\right]$

$= \frac{1}{\pi}\left[-\pi\left(-\frac{\cos nx}{n}\right)_{-\pi}^{0} + \left(x\left(-\frac{\cos nx}{n}\right) - \left(\frac{-\sin nx}{n^2}\right)\right)\right]$

$= \frac{1}{\pi}\left[\frac{\pi}{n}(1-(-1)^n + \frac{\pi}{n}(-(-1)^n)\right]$

$\boxed{b_n = \frac{1}{\cancel{n}}\left(\frac{1-2(-1)^n}{n}\right)}$

12) $P=3 \quad f(x) = 2x - x^2$ range $(0,3)$

$\rightarrow \quad P = 2l = 3$

$l = 3/2$

$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$

$= \frac{2}{3}\int_0^3 2x - x^2\,dx$

$= \frac{2}{3}\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^3$

$= \frac{2}{3}(3^2 - 3^2)$

$\boxed{a_0 = 0}$

$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}\,dx$

$= \frac{2}{3}\int_0^{3} (2x - x^2)\cos\frac{n\pi x}{3/2}\,dx$

$= \frac{2}{3}\left[2\left[\frac{x\left(\sin n\pi x\cdot 2/3\right)}{2n\pi/3} - 1\frac{\left(-\cos\frac{n\pi x\cdot 3}{2}\right)}{\left(\frac{2n\pi}{3}\right)^2}\right]\right.$

$-\left[\frac{x^2\sin\frac{n\pi x 3}{2}}{3n\pi/2} - 2x\frac{\left(-\cos\frac{3n\pi x}{2}\right)}{\left(\frac{3n\pi}{2}\right)^2} + 2\frac{\left(-\sin\frac{3n\pi x}{2}\right)}{\left(\frac{3n\pi}{2}\right)^3}\right]_0^3$

$\cancel{= \frac{2}{3}\left(\frac{6\times 2((-1)^n - 1)}{9n^2\pi^2 (2/3)^2} + 4\frac{\cos(-1)((-1)^n - 0)}{9n^2\pi^2 (2/3)^2}\right)}$

$\cancel{= \frac{2}{3}\left[\frac{-9 + 16}{9n^2\pi^2}\right]}$

$\cancel{= \frac{2}{3}(2}$

$= \frac{2}{3}\left[\frac{2(9)}{4\pi^2 n^2} - \frac{6\cdot 2(1)}{4\pi^2 n^2} - \frac{2(3)\times 9}{24 n^2\pi^2}\right.$

$= \cancel{\frac{2}{3}}\ \boxed{a_n = \frac{-9}{n^2\pi^2}}$

$b_n = \frac{2}{3}\int_0^3 (2x - x^2)\sin\frac{2n\pi x}{3/2}\,dx.$

$= \frac{2}{3}\left[\int_0^3 2x\sin\frac{2\pi n x}{3} - \int_0^3 x^2\sin\frac{2\pi n x}{3}\,dx\right]$

$= \frac{2}{3}\left(2\left[\frac{x\left(-\cos\frac{2\pi n x}{3}\right)}{\frac{2\pi n}{3}} - 1\frac{\left(-\sin\frac{2\pi n x}{3}\right)}{\left(\frac{2\pi n}{3}\right)^2}\right]_0^3\right.$

$-\left[x^2\frac{\left(-\cos\frac{2\pi n x}{3}\right)}{\left(\frac{2\pi n}{3}\right)} - 2x\frac{\left(-\sin\frac{2\pi n x}{3}\right)}{\left(\frac{2\pi n}{3}\right)^2} + 2\frac{\left(\cos\frac{2\pi n x}{3}\right)}{\left(\frac{2\pi n}{3}\right)^3}\right.$

$= \frac{2}{3}\left(\frac{6(-1)\times 3}{2\pi n} + \frac{9\times 3}{2\pi n} + \cancel{\frac{18}{4n^2\pi^2}} + \cancel{\frac{18}{4n^2\pi^2}}\right.$

$= \frac{2}{3}\left(\frac{-18 + 27}{2n\pi}\right)$

$= \frac{2}{3}\left(\frac{9}{2n\pi}\right)$

$\boxed{b_n = \frac{3}{n\pi}}$

13) obtain cosine series for $f(x) = e^x$, $-1<x<1$

$\rightarrow$ $a_0 = \frac{2}{l} \int_0^l f(x)\, dx$

$= \frac{2}{l} [e^x]_0^l \quad = \boxed{\frac{2}{l}[e^l - 1]}$

$a_n = \frac{2}{l} \int_0^l f(x) \cos\frac{n\pi x}{l}\, dx$

$= \frac{2}{l} \left( e^{-x} \cos\frac{n\pi x}{l}\, dx \right)$

$= \frac{2}{l} \left( \left[ \frac{e^x}{1+\left(\frac{n\pi}{l}\right)^2} \left( \cos\frac{n\pi x}{l} + \frac{n\pi}{l} \sin\frac{n\pi x}{l} \right] \right]_0^l \right)$

$= \frac{2}{l} \left( \frac{l^2 e^l}{l^2+n^2\pi^2} \left[ e^l (-1)^n - 1 \right) \right.$

$= \cancel{\frac{2}{l} \frac{l^2 e^x}{l^2+n^2\pi^2}}$ $\boxed{a_n = \frac{2l\left(e^l(-1)^n - 1\right)}{l^2+n^2\pi^2}}$

14) obtain sine series for $f(x) = 2x$; $-1 \leq x \leq 1$

$\rightarrow$ Here $f(x) = 2x$

$f(-x) = -f(x) \quad \therefore f(x)$ is odd funⁿ

$\therefore a_0 = a_n = 0$

$b_n = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$

$= \frac{2}{1} \int_0^1 2x \sin\left(\frac{n\pi x}{1}\right) dx$

$= 4 \left[ x\left(-\frac{\cos n\pi x}{n\pi}\right) - 1 \left(-\frac{\sin n\pi x}{(n\pi)^2}\right) \right]_0^1$

$= 4 \left( \frac{-(-1)^n}{n\pi} \right) = \boxed{\frac{-4(-1)^n}{n\pi}}$

15) find cosine integral of $f(x) = e^{-kx}$ $\quad x>0 \quad k>0$

16) sine integral of $f(x) = e^{-kx}$ $\quad x>0 \quad k>0$

18) $\rightarrow$ Fourier cosine integral

$B(\lambda) = 0$

$f(x) = \int_0^\infty A(\lambda) \cos\lambda x\, d\lambda$

$A(\lambda) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \cos\lambda x\, dx$

$= \frac{2}{\pi} \int_0^\infty f(x) \cos\lambda x\, dx$

$$= \frac{2}{\pi}\left[ \int_0^\infty e^{-kx} \cos \lambda x \, dx \right]$$

$$= \frac{2}{\pi}\left[ \frac{e^{-kx}}{k^2+\lambda^2} \left(-k \cos \lambda x + \lambda \sin \lambda x\right)\right]_0^\infty$$

$$= \frac{2}{\pi (k^2+\lambda^2)}\left[ e^{-\infty} (-k \cos \infty + \lambda \sin \infty) - e^0 (-k \cos 0 + \lambda \sin 0)\right]$$

$$= \frac{2}{\pi (k^2+\lambda^2)} \left[ 0 - 1(-k) \right] \Rightarrow \frac{2k}{\pi (k^2+\lambda^2)}$$

$$\boxed{A(\lambda) = \frac{2k}{\pi (k^2+\lambda^2)}}$$

Fourier sine Integral

$$A(\lambda) = 0$$

$$f(x) = \int_0^\infty B(\lambda) \sin \lambda x \, d\lambda$$

$$B(\lambda) = \frac{1}{\pi} \int_{-\infty}^{\infty} f(x) \sin \lambda x \, dx$$

$$= \frac{2}{\pi} \int_0^\infty f(x) \sin \lambda x \, dx$$

$$= \frac{2}{\pi} \int_0^\infty e^{-kx} \sin \lambda x \, dx$$

$$= \frac{2}{\pi} \left[ \frac{e^{-kx}}{k^2+\lambda^2} \left(-k \sin \lambda x - \lambda \cos \lambda x\right)\right]_0^\infty$$

$$= \frac{2}{\pi (k^2+\lambda^2)} \left[ 0 + \lambda \right] =$$

$$\boxed{B(\lambda) = \frac{2\lambda}{\pi (k^2+\lambda^2)}}$$

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