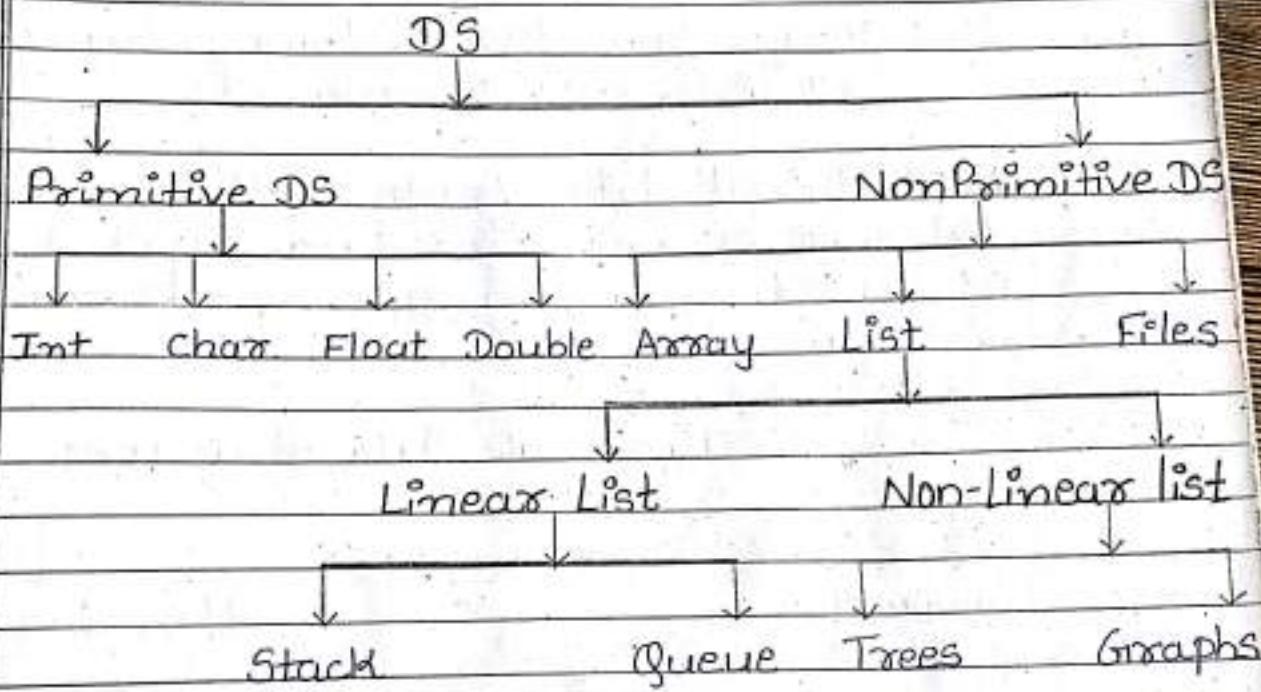


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Introduction

Data Structure is a particular way of organizing a data in a computer so it can be used efficiently.



Primitive Data Structure

Primitive Data Structure are provided by a programming language as a basic building block. They are also known as building types or basic type or primary data structure.

Ex: int, ch, float, double, etc

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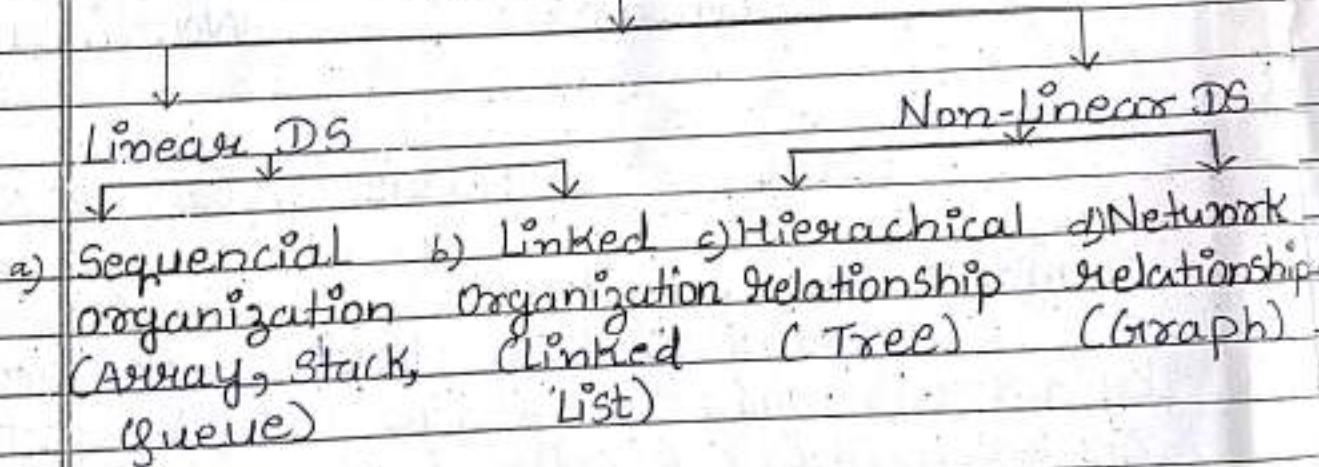
Non-Primitive Data Structure (Abstract DS)

The data structure which is derived from primitive data structure are known as Non-Primitive Data Structure.

Ex: Array, Stack, Queue, Union, Structure, Link List, Tree, Graph, etc

NOTE > This all data structure allows to perform different operation on data. We select this DS based on which type of operation is required.

Types of Data Structure



In linear Data Structure the elements are stored in sequential order it means if we know the address of first data item then we can retrive the second one & so on.

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- a) In sequential organization elements occupy continuous or consecutive memory location.
- b) In this data structure components are accessed in certain sequence but they are not necessarily stored in consecutive memory location. It uses a pointer which shows the relationship between the elements with the help of links.

STACK (LIFO)

Overflow Condition = $\text{TOP} \geq \text{Max}$

Underflow Condition = $\text{Top} = 0$

3	2	1
3	2	1
3	2	1
3	2	1

PUSH : To insert or add the element into the Stack

POP : This Operation delete the element from the Stack

PEEP : Peep Operation returns the value of top most element of the stack.

Application

- Reverse the String
- Expression Conversion

- Parsing

$\begin{matrix} \nearrow ab & \searrow ab \\ \text{Infix} \rightarrow \text{Prefix} & \text{Infix} \rightarrow \text{Postfix} \\ \text{Prefix} \rightarrow \text{Infix} & \text{Postfix} \rightarrow \text{Prefix} \\ \text{Postfix} \rightarrow \text{Infix} & \text{Prefix} \rightarrow \text{Postfix} \end{matrix}$

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- Disadvantage
 → slow access to other element

Queue (FIFO)

	1	2	3	4	5
$F = 0$		20	30	40	50
$R = 0$		F			

The elements in a queue are added at one end called rear, and removed from the other end called front.

Enqueue → means insertion or addition of element in a queue.

Dequeue → deletion or removal of an element in a queue

Simple Queue

Circular Queue

D Queue (Double ended Queue)

Underflow

$$F = 0$$

} For Simple Queue

Overflow

$$R \geq \text{max}$$

Underflow

$$F = R = 0$$

} For

Overflow

$$\text{If } R \geq \text{max} \& F = 1$$

} Circular Queue

Application

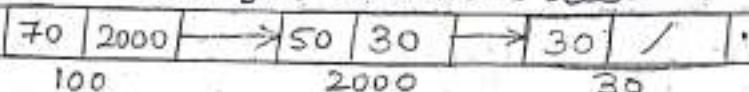
- CPU Task Scheduling
- Ground Robeering
- Keyboard Buffer
- Queue of Request at Web Server.

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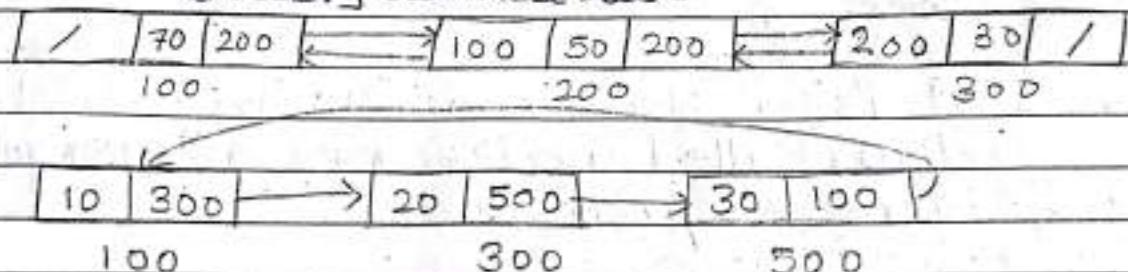
→ Interrupts are handle in the same order as Linked List .. they come.

1. Singly linked list
2. Doubly linked list

3. Circular linked list
- (Dynamic memory allocation)
- Value address /
Link of next
Singly linked list Value



Doubly linked list



Circular linked list

Linked list is a collection of item in which each item is linked with other and order is given by means of links from one item to another. Each item is denoted by node. which consist of two field.

Advantages

- It is a dynamic data structure in which the element can be added or deleted from anywhere in the list.
- In linked list ^{no fixed memory} need not to worry about how many element will be stored in the linked list.
- In linked list each element is allocated memory as it is added to the list

Disadvantage

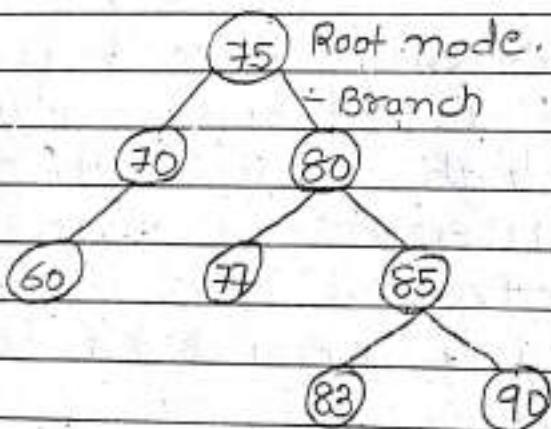
- It require more memory and slow search operations.
- Reverse traversing is difficult

Application

- It is used to implement Stack, Queue, Graph, Trees.

Tree

It is a Non-linear DS in which data items called nodes are arranged in some sequence.



Advantage.

Element can be located very quickly in tree.

Insertion is fast.

Disadvantage

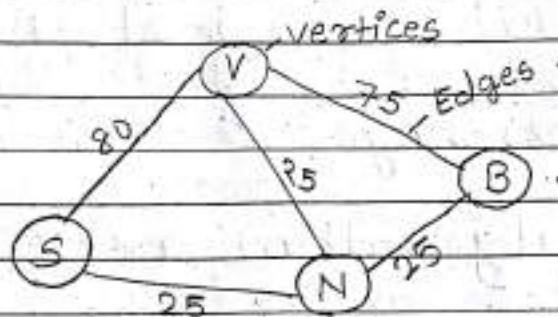
Deletion algorithm is complex.

Application

Telephone directory assistance, information stored in a tree so name & no. can be quickly find out.

TREE CONTAIN PURELY PARENT-CHILD RELATIONSHIP.

Graph



It is basically a collection of vertices and edges that connect this vertices. It is a generalization of tree structure where instead of having purely Parent & child relationship bet'n tree node my kind of complex relationship bet'n the vertices can be representative.

Tree has a constraint that no node can have many children but only one parent but in graph it doesn't have such type of restriction.

- Application
- Node can represent city & edges can represent road hence it can be used in GPS.
- A Graph can also used to represent Computer networks where nodes are called station & edges are network connection so it can be used in work station.

Disadvantage

- Some algorithm are slow & very complex.

Performance analysis & Asymptotic notation.

Performance Analysis of an algorithm is a process of calculating space required by that algorithm and time required by that algorithm. P.A of an algorithm is performed by using the foll. measures

- i) Space Complexity
- ii) Time Complexity

i) \Rightarrow Total amount of Computer memory required by an algorithm to complete its execution is called as Space Complexity of that algort.

For any algorithm memories required by full purpose

- i) Memory required to store program instruction or structure.
- ii) Memory required to store constant values.
- iii) Memory required to store variable.

Ex: void add (int n)

```

    {
        return n * n;
    }

```

for $n \Rightarrow 2$ bytes
 for return $\Rightarrow 2$ bytes
 State.

Total $\Rightarrow 4$ bytes

If algorithm require a fixed amount of space for all input values then that space complexity said to be Constant Space Complexity.

int sum (int A[], int n)

```

    {
        int sum = 0, i;
        for (i = 0; i < n; i++)
            sum = sum + A[i];
        return sum;
    }

```

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$$n \text{ Space} = 2 \text{ byte}$$

$$A[] \text{ Space} = 2 * n \text{ byte}$$

$$\text{Sum Space} = 2 \text{ byte}$$

$$i \text{ space} = 2 \text{ byte}$$

$$\text{return Space} = 2 \text{ byte}$$

$$\text{Total} = 2n + 8 \text{ byte}$$

If the amount of Space required by an algorithm is increase with increase of input value then the Space complexity is said to be Linear Space Complexity.

ii) \Rightarrow Total amount of time required by an algorithm to complete its execution is called as Time Complexity.

To calculate the time complexity we check only how our program is behaving for other diff. input values to perform all the operation like arithmetic, logical, assignment, any return value operation, etc.

If any program require fixed amount of time for all input values then its Time Complexity is Said to be Constant Time complexity and if amount

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of time required by an algorithm is increase with the increase in value then that type of complexity is said to be linear time complex.

1. Three types of asymptotic notation

- 1. Big oh (O) \rightarrow max. amount of time req. by alg. defines upperbound of alg.
- 2. Big Omega (Ω) \rightarrow min. amount
- 3. Big Theta (Θ) \rightarrow Avg.

- 1. It describes the worst case of an algo.

$$f(n) = O(g(n))$$

$f(n) \leq C * g(n)$ for every $(n \geq n_0)$,
 C - Some real constant

$f(n)$ = algo runtime

$g(n)$ = arbitrary time complexity you are trying to relate to your algo.

$$C * g(n)$$

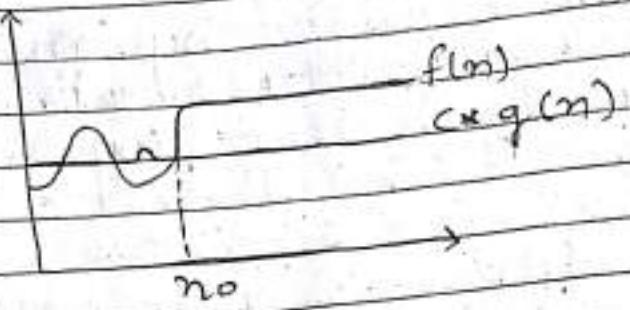
$$f(n)$$

n_0

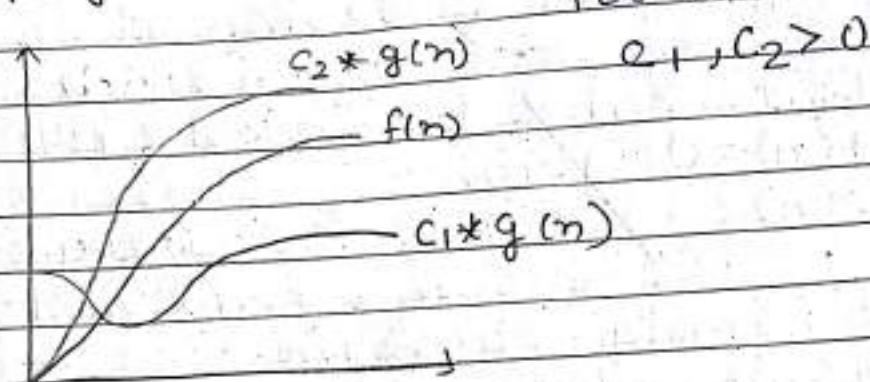
- 2. It describes the best case of an algo.

$$f(n) = \Omega(g(n))$$

if $f(n) \geq c * g(n)$ for every i/p size n ($n > n_0$)



3. $0 \leq c_1 * g(n) \leq f(n) \leq c_2 * g(n)$ for all $n \geq n_0$



Calculate the time Complexity

$\text{fun}(n)$

```
{
    for (i=1; i<=n; i++)
        {

```

```
            printf ("Hello");
```

}

$3n + 2$

$O(n)$

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fun(n)

{ $\Theta \frac{n+1}{2} \cdot \frac{n}{2}$
 for ($i=1$; $i < n$; $i = i + 2$)

{ $pt ("Hi") ; \frac{n}{2}$

} $\frac{3n+2}{2}$ $n = 1$

{ ① ② $n-1$ $n = 5$
 for ($i=1$; $i < n$; $i++$)

{ { $(n-1) \cdot (n+1)m \cdot (m-1)^2$

for ($j=1$; $j < m$; $j++$)

{

} $pt (i * j) ; (m-1) * n$

} $1 + n + m - 1 + m - 1 + n^2 - n$
 $n^2 + 2n + 1 + m^2 - n$

$3m^2 - n$

$O(m^2)$

$i = 1$

$n = 5$

$j = 1$

$j = 2$

$j = 3$

$j = 4$

$i = 4$

$= n-1$

$i = 2$

$m-1$

$i = 3$

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Write an Algorithm to insert an element in simple queue.

Procedure QINSERT (Q, F, R, N, Y)

F & R = It is a pointer which points to front of zeros elements of the Queue.

Q = queue.

N = Number of element in queue.

Y = element which is to be inserted

initially F & R set to zero

1. [Overflow?]

if $R \geq N$

then write ('overflow')

Return

2. [Increment rear Pointer]

$R \leftarrow R + 1$

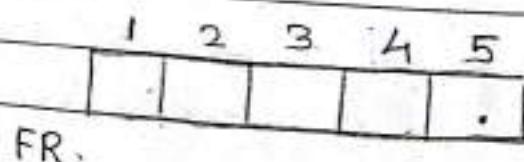
3. [Insert Element]

$Q[R] \leftarrow Y$

4. [Is front Pointer Property Set?]

1) $F = 0$ then $F \leftarrow 1$

Return



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(i) $R=0, Y=5, N=5, F=0$

1. [overflow?]

$$0 \neq 5$$

False

2. $R=1$

3. $Q[1]=5$

4. $F = 0$

$F=1$

1	2	3	4	5
5				

FR.

(ii) $R=1, Y=10, N=5, F=1$

1. $1 \geq 5$

False

2. $R=2$

3. $Q[2]=10$

4. $F \neq 0$

False

1	2	3	4	5
5	10			

F R

(iii) $R=2, Y=15, N=5, F=1$

1. $2 \geq 5$

False

2. $R=3$

3. $Q[3]=15$

4. $F = 0$

False

1	2	3	4	5
5	10	15		

F R

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(iv) $R = 3, Y = 20, N = 5, F = 1$

1. $3 \geq 5$
False

2. $R = 4$

3. $Q[4] = 20$

4. $F == 0$

False

1	2	3	4	
5	10	15	20	

F R.

(v) $R = 4, Y = 25, N = 5, F = 1$

1. $4 \geq 5$
False

2. $R = 5$

3. $Q[5] = 25$

4. $F == 0$

False

1	2	3	4	5
5	10	15	20	25

F R

(vi) $R = 5, Y = 30, N = 5, F = 1$

1. $5 \geq 5$

True

overflow

Draw a simple queue with 6 memory shell which has Front & Rear at zero position. Draw the simple queue structure & represent the position of front & rear pointer for each of the foll. operation.

Insert a, b, c, d, e, f, g

1 2 3 4 5 6

F R

1. $R=0, N=6, y=a, F=0$

(i) if $R \geq N$

$$\rightarrow 0 \geq 6$$

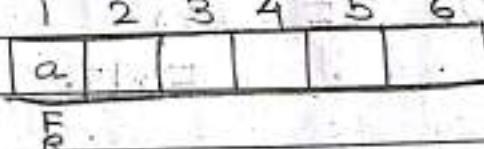
False

(ii) $R=1$

(iii) $Q[1] = a$

(iv) $F == 0$

$$\rightarrow F = 1$$



2. $R=1, N=6, y=b, F=1$

(i) if $R \geq N$

$$\rightarrow 1 \geq 6$$

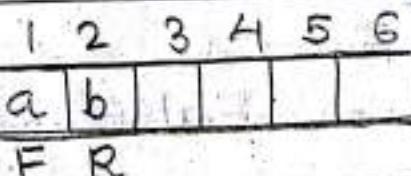
False

(ii) $R=2$

(iii) $Q[2] = b$

(iv) $F == 0$

False



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3. $R=2, N=6, Y=c, F=1$

(i) if $R \geq N$
 $2 \geq 6$.
 False

(ii) $R=3$ | 1 2 3 4 5 6
 (iii) $Q[3]=c$ | a b c | F R
 (iv) $F==0$ | | |
 False

4. $R=3, N=6, Y=d, F=1$

(i) if $R \geq N$
 $3 \geq 6$.
 False

(ii) $R=4$ | 1 2 3 4 5 6
 (iii) $Q[4]=d$ | a b c d | F R
 (iv) $F==0$ | | |
 False

5. $R=4, N=6, Y=e, F=1$

(i) if $R \geq N$
 $4 \neq 6$.

(ii) $R=5$ | 1 2 3 4 5 6
 (iii) $Q[5]=e$ | a b c d e | F R
 (iv) $F==0$ | | |
 False

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6. $R = 5, N = 6, Y = f, F = 1$

(i) if $R \geq N$

$5 \geq 6$

False

(ii) $R = 6$

(iii) $Q[6] = f$

(iv) $F == 0$

False

1 2 3 4 | 5 6

a b c d e f

F R

7. $R = 6, N = 6, Y = g, F = 1$

(i) if $R \geq N$

$6 \geq 6$

True

(ii)

Overflow

(iii)

(iv)

Deletion Algo (DEQUEUE)

Procedure QDELETE (Q, F, R)

Q = Queue

F & R = Pointers which points in front of rear

y = Temporary Variable

i. Underflow ?]

if $F = 0$

then write ('UNDERFLOW')

Return(0)

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2. [Delete element]

$$y \leftarrow Q[F]$$

3. [Queue empty?]

$$\text{if } F = R$$

$$\quad \text{then } F \leftarrow R \leftarrow 0$$

else

$$F \leftarrow F + 1$$

4. [Return element]

Return y

	1	2	3	4	5
	10	20	25	30	40
Deletion	F			R	

$$1. F = 1, R = 5$$

$$1 \neq 0$$

False

$$y = Q[1]$$

$$y = 10$$

$$1 \neq 5 \quad \text{False}$$

$$F = 1 + 1 = 2$$

$$y = 10$$

	1	2	3	4	5
	20	25	30	40	
	F			R	

$$2. F = 2, R = 5$$

$$2 \neq 0$$

False

$$y = Q[2]$$

$$y = 20$$

$$2 \neq 5 \quad \text{False}$$

$$F = 2 + 1 = 3$$

$$y = 20$$

		1	2	3	4	5
		25	30	40		
		F		R		

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3. $F = 3, R = 5$

$3 \neq \neq 0$

False

$y = Q[3]$

$y = 25$

$3 \neq \neq 5$ False

$F = 3 + 1 = 4$

$y = 25$

1	2	3	4	5
			30	40

F R

4. $F = 4, R = 5$

$4 \neq \neq 0$

False

$y = Q[4]$

$y = 30$

$4 \neq \neq 5$ False

$F = 4 + 1 = 5$

$y = 30$

1	2	3	4	5
				40

FR

5. $F = 5, R = 5$

$5 \neq \neq 0$

False

$y = Q[5]$

$y = 40$

$5 == 5$

True

$F = 0, R = 0$

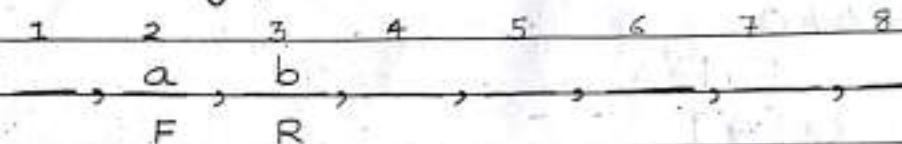
$y = 40$

1	2	3	4	5

FR

Consider the Simple Queue given below with 8 memory shell. which has $F=2$, $R=3$.
 Draw the Simple queue structure & represent the position of Front & Rear pointers for each of the foll operation.

- 1) Insert c & d
- 2) Delete one alphabet
- 3) Insert e & f
- 4) Delete one alphabet
- 5) Insert g & h.



1.

a) $F=2, R=3, N=8, Y=c$

$3 \neq 8$

False

$R=3+1=4$

$Q[4]=Y$

$Y=C$

$2 \neq 0$

False

b) $F=2, R=4, N=8, Y=d$

$4 \neq 8$

False

$R=4+1=5$

$Y=Q[5]$

$Y=d$

$2 \neq 0$

False

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2. $F = 2, R = 5$

$2 \neq 0$

False

$y = Q[2]$

$y = a$

$2 \neq 5$

False

$F = 2+1 = 3$

$y = a$

3. a) $F = 3, R = 5, N = 8, y = e$

$5 \neq 8$

False

$R = 5+1 = 6$

$y = Q[6]$

$y = e$

$3 \neq 0$

False

b) $F = 3, R = 6, N = 8, y = f$

$6 \neq 8$

False

$R = 6+1 = 7$

$y = Q[7]$

$y = f$

$3 \neq 0$

False

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4. $F = 3, R = 7$

$3 \neq 0$

False

$y = Q[3]$

$y = b$

$3 \neq 7$

$F = 3 + 1 = 4$

$y = b$

5. a) $F = 4, R = 7, N = 8, y = g$

$7 \geq 8$

False

$R = 7 + 1 = 8$

$y = Q[8]$

$y = g$

$4 \neq 0$

False

b) $F = 4, R = 8, N = 8, y = h$

$8 \geq 8$

True

Overflow

Procedure ($QINSERT(F, R, Q, N, y)$)

$F \& R \rightarrow$ Front & Rear pointer

$Q \rightarrow$ Queue

$N \rightarrow$ No. of element in queue

$y \rightarrow$ element to be inserted

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1. [Reset Rear Pointer?]

if $R = N$,

then $R \leftarrow 1$

else

$R \leftarrow R + 1$

2. [Overflow]

if $F = R$

then write ('overflow')

Return

3. [Insert element]

$Q[R] \leftarrow y$

4. [Is Front Pointer]

0 1 2 3 4 5

$F = 0$

then $F \leftarrow 1$

$F = 1$

Return

$N = 4$, $y = A, B, C, D$

1 2 3 4

$0 \neq 4$, $R = 1$, $Q[1] = A$.

F, R

1 2 3 4

$1 \neq 4$, $R = 2$, $Q[2] = B$.

A

F

1 2 3 4

$2 \neq 4$, $R = 3$, $Q[3] = C$.

A B

$F R$

1 2 3

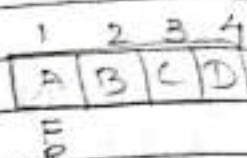
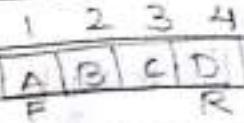
$3 \neq 4$, $R = 4$, $Q[4] = D$.

A B C

$F R$

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$A = A, R = 1, I = 1$
 'overflow'



Procedure $[QDELETE(F, R, Q, N)]$

1. [underflow]

if $F = 0$
 then write C 'UNDERFLOW'.
 Return(0).

2. [Delete element]

$y \leftarrow Q[F]$

3. [Queue empty?]

if $F = R$

$F \leftarrow R \leftarrow D$

Return(y)

4. [Increment front Pointer]

if $F = N$

$F \leftarrow 1$

else

$F \leftarrow F + 1$

return(y)

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Operation

1. Delete A.
2. Insert E
3. Delete B
4. Insert F
5. Delete C,D,E,F

1	2	3	4
A	B	C	D
F	.	R	.

1) $i \neq 0$
 $y = Q[1]$
 $A = y$
 $i \neq 4$.
 $F = F + 1$
 $F = 2$

1	2	3	4
	B	C	D
F	.	R	.

2) $4 = 4$ True
 $R = 1$.
 $2 \neq 1$ False
 $Q[1] = E$
 $i \neq 0$.

1	2	3	4
E	B	C	D
R	F	.	.

3) $2 \neq 0$
 $y = Q[2]$
 $y = B$
 $2 \neq 1$
 $F = F + 1$
 $F = 3$.

1	2	3	4
E	B	C	D
R	F	.	.

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4)

$$I \neq 4$$

$$R = R + 1$$

$$R = 2$$

$$Q[2] = F$$

$$3 \neq 0$$

	I	2	3	4
E		C	D	

R F

5) a)

$$3 \neq 0$$

$$Y = Q[3]$$

$$Y = C$$

$$3 \neq 2$$

$$F = F + 1$$

$$F = 4$$

b) $4 \neq 0$

$$Y = Q[4]$$

$$Y = D$$

$$4 \neq 2$$

$$F = N \quad 4 = 4 \text{ True}$$

$$F = 1$$

c) $1 \neq 0$

$$Y = Q[1]$$

$$Y = E$$

$$1 \neq 2$$

$$F \neq N \quad 1 \neq 4$$

$$F = F + 1$$

$$F = 2$$

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a) $2 \neq 0$

$y = Q[2]$

$y = F$

$2 = 2$

$F \leftarrow R \leftarrow 0$

$F = R = 0, F = 1$

$N = 6, F = 0, R = 0$

1. Insert 5, 10, 15, 20, 25

2. Delete 5 & 10

3. Insert 30, 35, 40

4. Delete 15, 20, 25

5. Insert 45, 50, 55

6. Delete 30, 35, 40, 45, 50, 55

1. a) $F = 0, R = 0, N = 6, Y = 5$

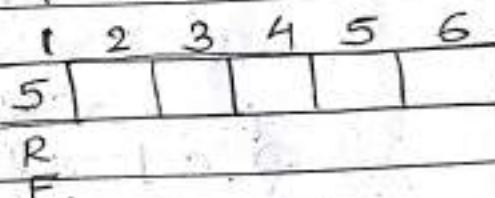
$0 \neq 6$

$R = R + 1$

$R = 1$

$Q[1] = 5$

$F = 0, F = 1$



b) $F = 1, R = 1, N = 6, Y = 10$

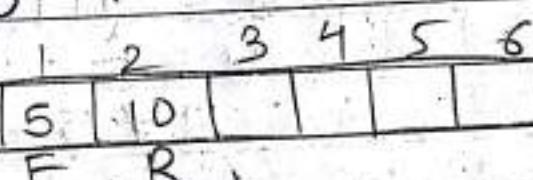
$1 \neq 6$

$R = R + 1$

$R = 2$

$Q[2] = 10$

$F = 1$



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c) $F = 1, R = 2, N = 6, Y = 15$

$$2 \neq 6$$

$$R = R + 1$$

$$= 3$$

$$Q[3] = 15$$

$$F = 1$$

d) $F = 1, R = 3, N = 6, Y = 20$

$$3 \neq 6$$

$$R = R + 1$$

$$= 4$$

$$Q[4] = 20$$

$$F = 1$$

e) $F = 1, R = 4, N = 6, Y = 25$

$$4 \neq 6$$

$$R = R + 1$$

$$= 5$$

$$Q[5] = 25$$

$$F = 1$$

2.

$$1 \neq 0$$

$$Y = Q[1]$$

$$Y = 5$$

$$1 \neq 5$$

$$1 \neq 6$$

$$F = 1 + 1$$

$$= 2$$

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$$2 \neq 0$$

$$y = Q[2]$$

$$y = 10$$

$$2 \neq 5$$

$$2 \neq 6$$

$$F = 2 + 1$$

$$= 3$$

3. $F = 3, R = 5, N = 6, Y = 30$

$$5 \neq 6$$

$$R = 6$$

$$3 \neq 6$$

$$Q[6] = 30$$

$$3 \neq 0$$

$F = 3, R = 6, N = 6, Y = 35$

$$6 = 6$$

$$R < 1$$

$$3 \neq 1$$

$$Q[1] = 35$$

$$3 \neq 0$$

$F = 3, R = 1, N = 6, Y = 40$

$$1 \neq 6$$

$$R = 2$$

$$3 \neq 2$$

$$Q[2] = 40$$

$$3 \neq 0$$

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4. $F = 3, R = 2, N = 6$
 $3 \neq 0$

$$Y = Q[3]$$

$$Y = 15$$

$$3 \neq 2$$

$$F = F + 1$$

$$F = 4$$

$$F = 4, R = 2, N = 6$$

 $4 \neq 0$

$$Y = Q[4]$$

$$Y = 20$$

$$4 \neq 2$$

$$F = F + 1$$

$$F = 5$$

$$F = 5, R = 2, N = 6$$

 $5 \neq 0$

$$Y = Q[5]$$

$$Y = 25$$

$$5 \neq 2$$

$$F = F + 1$$

$$F = 6$$

5. $F = 6, R = 2, N = 6$

$$2 \neq 6$$

$$R = 3$$

$$6 \neq 3$$

$$Q[3] =$$

(task scheduling)

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Priority Queue

A priority queue is an abstract data type in which each element is assigned a priority. The priority of the element will be used to determine the order in which this element will proceed. The general rule of processing the element of priority queue is

1. An element with higher priority is processed before an element with a lower priority.
2. Two elements with the same priority are processed based on First Come First Served basis.

A queue in which elements are inserted or removed according to some priority, is referred to as a priority queue.

Priority queue are widely used in operating system to execute the highest priority process first.

Implementation of Priority Queue

There are two ways

- i) Sorted list
- ii) Unsorted list

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Sorted list

$O(n) \Rightarrow$ Insertion
 $O(1) \Rightarrow$ Deletion

3	10	15	25	45
		↑		17

Unsorted list

$O(1) \Rightarrow$ Insertion
 $O(n) \Rightarrow$ Deletion

Representation of Priority Queue

1. Link Representation of Priority queue
2. Array Representation of Priority queue.

1. Link Representation

R_i	R_2	...	R_{i-1}	O_1	O_2	...	O_{i-1}	B_1	B_2	...	B_{k-1}
1	1	...	1	2	2	...	2	3	3	...	3

Priority 1 $R_1 R_2 \dots R_{i-1}$

Priority 2 $O_1 O_2 \dots O_{i-1}$

Priority 3 $B_1 B_2 \dots B_{k-1}$

If a Sequential Storage Structure is used for the priority queue then insertion of new element must be placed in the middle of the structure.

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This can require the moment of several items. It is better to split the priority queue into different different queue. Each having its own storage structure.

2. Array Representation

Priority Queue

Priority	Front	rear
1	3	3
2	1	3
3	4	5
4	4	1

Front & Rear Position

Prio. 1 2 3 4 5

1	A			
2	B	C	D	
3			E	F
4	I		G	H

1 2 3 4 5

p-1 A

1 2 3 4 5

p-2 B C D

When an array are used to implement a priority queue then a separate queue for each priority number is maintained. Each of this queue will be implemented using circular que

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of
split
sent
its

Here two dimensional array is used where each queue will be allocated the same amount of space.

To insert a new element with priority 'K' in the priority queue add the new element at the rear end of row 'K'.

K = row number as well as priority no. of that element.

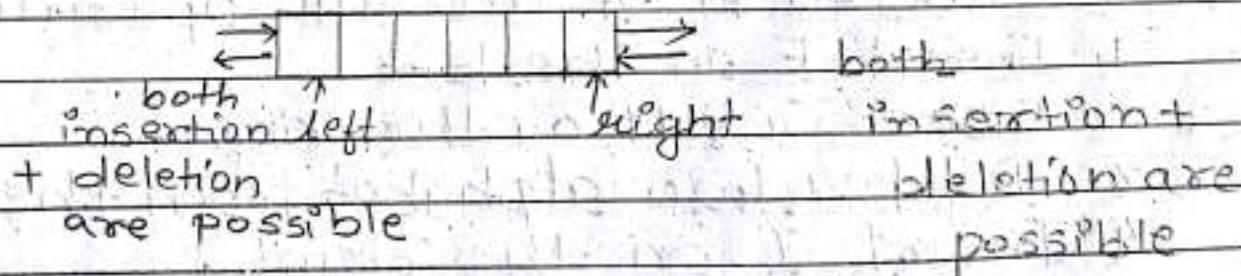
To delete an element we find the first non-empty queue & then process the front element of the non-empty queue.

Double Ended Queue (Dqueue)

Types

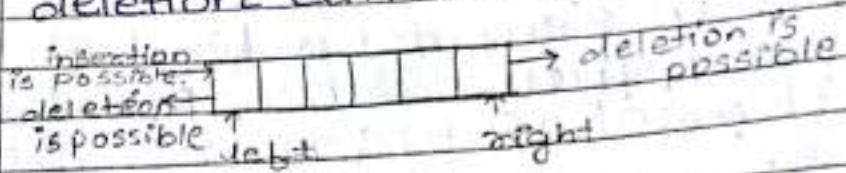
1. Input restricted dqueue
2. Output restricted dqueue

A dqueue is a linear list of elements in which element may be inserted or deleted from the both end which is called as double ended queue or head-tail link list.

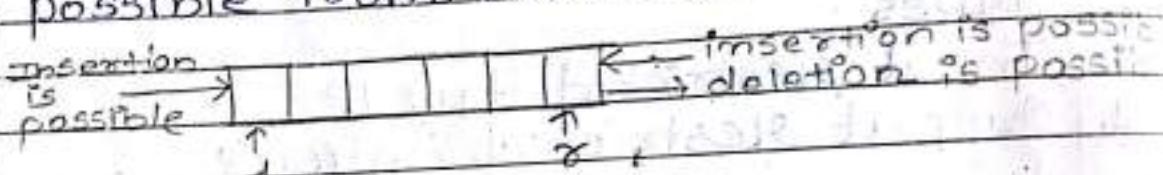


No element can added or deleted from the middle. In Computer memory, deque is implemented as circular array or circular doubly linked list.

1. \Rightarrow In input restricted deque insertion can be done only at one end, but deletion can be done from both ends.



2. \Rightarrow In this deque deletion can be possible at only one end, & insertion can be possible from both ends.



Consider a deque given below with 10 memory cells which has $l=1$ & $r=5$.

Draw the deque structure & represent position of left & right pointer for each of the foll. operation

1. Insert F on the left
2. Insert G & H on the right
3. Delete two alphabet from the left.
4. Insert I on the right.
5. Insert J on the left.

6. Delete two alphabet from right

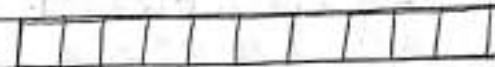
0	1	2	3	4	5	6	7	8	9
	A	B	C	D	E				I

$l = 1$ $r = 9$

1. Insert F on left

$l = -$

$l = 0$ $dq[l] = F$



2. Insert G & H on right

$l = +$

$l = 6$

$dq[r] = G$

$l = +$

$l = 7$

$dq[r] = H$

3. Delete 2 alphabet from left

$l = 0$

$F = y = dq[l]$

$l = 1$

$A = y = dq[l]$

4. Insert I on right

$l = +$

$l = 8$

$dq[r] = I$

5. Insert J on left

$l = -$

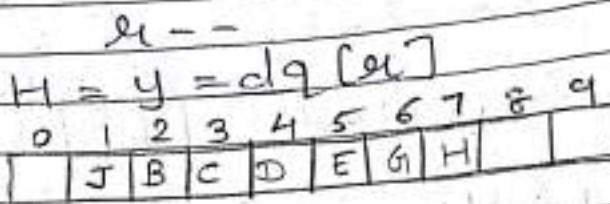
$l = 1$

$dq[l] = J$

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6. Delete two alphabet from right

$$y = dq[ri]$$



If there is only one element into dqeuue & if deletion operation is perform then after deletion set l & ri to '0'.

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Linked list

A link list is a simple term & it is a linear collection of data elements.

A list is a collection of items in which each item is linked with other & order is given by means of links from one item to another. Each item is denoted by node which consists of two fields

- 1) value of the node
- 2) Address of next node
- and the last node contains the null in its address path.

Advantages of Array

1. Array is very simple to use & easy to create.
2. No memory management is needed.
3. In Array we can access any element with the help of index directly.
4. Array are quick to loop.

Disadvantages of Array

1. Array is static in nature.
Memory is allocated at compile time.
2. Insertion & deletion of element is more complicated in comparison to the link list.

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Advantages of Linked List

1. It is a dynamic data structure thereby the size of the link list can grow & shrink in size during the execution of program.
2. In link list programmer doesn't worry about how many element will be stored in a link list.
3. In a link list each element is allocated memory as it is added to the list.

Disadvantages of Linked list

1. Lots of overhead. (lots of malloc call & pointer assignment)
2. We must traverse the entire list to go to the nth node.
3. It consume extra space to contain the address of next element of string. 1
2
3
4
5
6
7
8
9

Application of Linked List

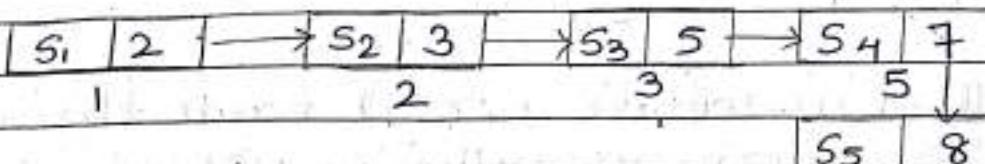
1. Stack, Queue, Graph, Tree are implemented using link list.
2. Undo functionality, Photoshop or word.
3. Consider the history section of web browser where it creates the link list of web pages visited so when you press the back

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bottom the previous node data is fatched.

"Every node contain a pointer to another node which is of the same type". It is called as self referential data type."

Representation of Link list in memory



	Roll No	Next .	
First →	s1	2	10

1	s2	3	First = 1
2	s3	5	

3 Avail →		Avail →	6
-----------	--	---------	---

4	s4	7	4 ↓
---	----	---	-----

5			9
---	--	--	---

6		6	↓
---	--	---	---

7	s5	8	9
---	----	---	---

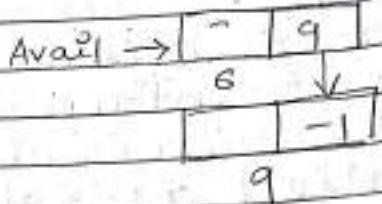
8	s6	10	-1
---	----	----	----

9			
---	--	--	--

10	s7	-1	
----	----	----	--

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	Roll No.	Next
1	S ₁	2
2	S ₂	3
3	S ₃	5
4	S ₈	-1
5	S ₄	7



Avail → 6

6	S ₅	8
7	S ₆	10
8		
9		
10	S ₇	4

When we delete a row from the linked list then operating system changes the status of memory occupied by it from memory available.

The Computer maintain a list of all the free memory shell. The list of available shell is called as free pool.

Linked List is a part

Link List as Pointer Variable

1) Which Store the address of the first node of the list.

Likewise for the free pool which is the linked list of all free memory shell we have a pointer variable 'Avail' which store the address of the first free space.

2) If we delete node then space occupied by that node must be given.

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back to the free pool. So memory can be reused by some other program.

- 3) The operating system does this task of adding & removing memory from the free pool.

This task is performed when O.S finds the CPU idle or the program are failing to use memory.

The O.S scan through all the memory shell & marks those cell that are being used by some other program. Then it collect all the cell which are not being used & at their address to free pool. This process is called garbage collection.

Singly Linked List

- 1) Algorithm for Insertion at beginning

FIRST = INSERT(X, FIRST)

X = A new element which is to be inserted.

FIRST = A pointer to the first element to linked list whose typically node contains info & Link.

New is a temporary variable.

Avail is a pointer to the top element of the availability stack

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(10, NULL)

1] [underflow?]

If Avail = Null
 then write ('AVAILABILITY STACK EMPTY,
 Return (FIRST)

2] [obtain address of next free node]
 New \leftarrow Avail

3] [Remove free node from availability stack]
 Avail \leftarrow Link (Avail)

4] [Initialize fields & new node & its link in the list]

INFO (New) \leftarrow x

LINK (New) \leftarrow FIRST

5] [Return address of new node]
 Return (New)

Insert 10, 20, 30, 40, 50, 60
 x First

1. First = (10, Null)

200 \neq NULL

New \leftarrow 200

Avail \leftarrow 250

INFO (New) \leftarrow 10

LINK (New) \leftarrow Null New = 200

First \leftarrow 200

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2. $\text{First} = (20, 200)$

$250 \neq \text{NULL}$

$\text{New} \leftarrow 250$

$\text{Avail} \leftarrow 300$

$\text{Info}(\text{New}) \leftarrow 20$

$\text{Link}(\text{New}) \leftarrow 200 \quad \text{NEW} = 250$

$\text{First} \leftarrow 250$

3. $\text{First} = (30, 250)$

$300 \neq \text{NULL}$

$\text{New} \leftarrow 300$

$\text{Avail} \leftarrow 350$

$\text{Info}(\text{New}) \leftarrow 30$

$\text{link}(\text{New}) \leftarrow 250 \quad \text{NEW} = 300$

$\text{First} \leftarrow 300$

4. $\text{First} = (40, 300)$

$350 \neq \text{NULL}$

$\text{New} \leftarrow 350$

$\text{Avail} \leftarrow 400$

$\text{Info}(\text{New}) \leftarrow 40$

$\text{link}(\text{New}) \leftarrow 300 \quad \text{NEW} = 350$

$\text{First} \leftarrow 350$

5. $\text{First} = (50, 350)$

$400 \neq \text{NULL}$

$\text{New} \leftarrow 400$

$\text{Avail} \leftarrow \text{NULL}$

$\text{Info}(\text{New}) \leftarrow 50$

$\text{link}(\text{New}) \leftarrow 350 \quad \text{NEW} = 400$

$\text{First} \leftarrow 400$

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2) Algorithm for insertion at end.

FIRST = INSERT (x, FIRST)

1] [underflow?]

AVAIL = NULL

then write ('Availability stack empty')

return (FIRST)

2] [obtain address of next free node]

NEW ← AVAIL

3] [Remove free node from availability stack]

AVAIL ← LINK (AVAIL)

4] [Initialize fields of new node]

INFO (NEW) ← x

LINK (NEW) ← NULL

5] [Is the list empty?]

If FIRST = NULL

then Return (NEW)

6] [Initialize search for the last node]

SAVE ← FIRST

7] [Search for end of LIST]

Repeat while LINK (SAVE) ≠ NULL

SAVE ← Link (SAVE)

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8) [Set LINK Field of last node to new]
LINK(SAVE) \leftarrow NEW

9) [Return FIRST node pointer]
Return(FIRST)

```
temp = first  
while (temp != NULL)  
{  
    pf ("%d", *temp);  
    temp = link(temp);  
}
```

Insert a mode in sorted order

FIRST = INSORD(X, FIRST)

D) [Underflow?]

if AVAIL = NULL
then write ('Availability stack underflow!')

Return(FIRST)

2) [Obtain address of next free node]

NEW \leftarrow AVAIL

3) [Remove node from availability stack]

AVAIL \leftarrow LINK(AVAIL)

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4) [Copy information contents into New node]
 $\text{INFO(NEW)} \leftarrow x$

5) [Is the list empty?]
if $\text{FIRST} = \text{NULL}$
then $\text{LINK(NEW)} \leftarrow \text{NULL}$
Return (NEW)

6) [Does the new node precede all others in the list?]
if $\text{INFO(NEW)} \leq \text{INFO(FIRST)}$
then $\text{LINK(NEW)} \leftarrow \text{FIRST}$
Return (NEW)

7) [Initialize temporary pointer]
 $\text{SAVE} \leftarrow \text{FIRST}$

8) [Search for Predecessor of new node]
Repeat while $\text{LINK(SAVE)} \neq \text{NULL}$ and
 $\text{INFO(LINK(SAVE))} < \text{INFO(NEW)}$
 $\text{SAVE} \leftarrow \text{LINK(SAVE)}$

9) [Set link fields of new node & its predecessor]
 $\text{LINK(NEW)} \leftarrow \text{LINK(SAVE)}$
 $\text{LINK(SAVE)} \leftarrow \text{NEW}$

10) [Return FIRST node pointer]
Return (FIRST)

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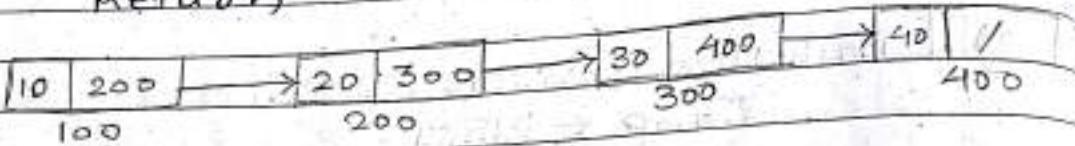
Delete an element from Singly linked list.

Procedure : DELETE (X, FIRST)

- 1) [Empty list?]
 If FIRST = NULL
 then write ('Underflow')
 Return .
- 2) [Initialize Search for X]
 Temp \leftarrow FIRST
- 3) [Find X]
 Repeat through Step-5 while.
 INFO(TEMP) \neq x and LINK(TEMP) \neq NULL
 \hookrightarrow (IF we take x as address then Temp \neq x)
 PRED \leftarrow TEMP .
- 4) [Update Predecessor Markers]
 TEMP \leftarrow LINK(TEMP)
- 5) [Move to next node]
 add below step if x is value
 $[T_1 \in TEMP]$
- 6) [End of the list?] \hookrightarrow (if x as address Temp \neq x)
 If INFO(TEMP) \neq x
 then write ('Node NOT FOUND')
 Return .

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- 7) [DELETE X] If x is address $x = \text{FIRST}$
- If $x = \text{INFO}(\text{FIRST})$
then $\text{FIRST} \leftarrow \text{LINK}(\text{FIRST})$
- else
 $\text{LINK}(\text{PRED}) \leftarrow \text{LINK}(\text{TEMP})$
- 8) [\sim mode to availability area]
Return $\text{LINK}(x) \leftarrow \text{AVAIL}$
 $\text{Avail} \leftarrow T_i$
Return



Copy of Link List

The new list is to contain node whose information and pointer fields are denoted by field and ptr respectively. Address of first node in newly created list is to be placed in begin.

- 1) [Empty list?]

if $\text{FIRST} = \text{NULL}$
then Return (NULL)

- 2) [Copy FIRST-node]

if $\text{AVAIL} = \text{NULL}$
then write (Availability stack underflow)
Return(0).

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FRI SAT
= FIRST

3) else

NEW ← AVAIL
AVAIL ← LINK(AVAIL)
FIELD(NEW) ← INFO(FIRST)
BEGIN ← NEW

4) [Initialize traversal]

SAVE ← FIRST

5) [Move to next node if not at end of list].

Repeat through Step 6 while
LINK(SAVE) ≠ NULL

6) [Update predecessors and save pointers]

PRED ← NEW

SAVE ← LINK(SAVE)

7) [Copy node].

if AVAIL = NULL

then write ('Availability Stack Underflow')

Return(0)

else NEW ← AVAIL

AVAIL ← LINK(AVAIL)

FIELD(NEW) ← INFO(SAVE)

PTR(PRED) ← NEW

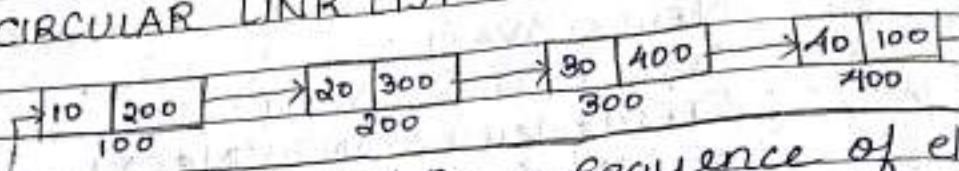
8) [Set line of list node & return]

PTR(NEW) ← NULL

RETURN(BEGIN)

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CIRCULAR LINK LIST



Circular link list is a sequence of elements in which every element has linked to its next element in the sequence and the last element has a link to its first element.

Advantages

In a circular link list every node is accessible from a given node that is from this given node all nodes can be reached by chaining through the list.

Disadvantages

The only downside of the circular link list is the complexity of iteration.

Algorithm

- Insert a node at the front/head of a circular link list

FIRST = CINSERT (x, FIRST)

alist → (newlist)

(newlist) → alist

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- ① [Underflow]
if Avail = Null
then write ('Availability stack underflow')
Return (FIRST)
- ② [Obtain address of next free node]
NEW \leftarrow AVAIL
- ③ [Remove free node from availability stack]
AVAIL \leftarrow LINK(AVAIL)
- ④ [Initialize fields of new node]
INFO(NEW) \leftarrow X
- ⑤ [Check for empty list]
if FIRST = NULL
then FIRST \leftarrow NEW.
LINK(NEW) \leftarrow FIRST
Return NEW.
- ⑥ [Set PTR]
PTR \leftarrow FIRST
- ⑦ [Check for last node]
while LINK(PTR) \neq FIRST
PTR \leftarrow LINK(PTR)
- ⑧ [Add new node at front]
LINK(NEW) \leftarrow FIRST
LINK(PTR) \leftarrow NEW. FIRST \leftarrow NEW
Return FIRST

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→ Insert a node at the end of circular link list

$\text{FIRST} = \text{CINSERT}(x, \text{FIRST})$

① [underflow]

if $\text{AVAIL} = \text{NULL}$
 then write ('Availability stack underflow')
 Return (FIRST)

② [obtain address of next free node]
 $\text{NEW} \leftarrow \text{Avail}$

③ [Remove free node from availability stack]
 $\text{Avail} \leftarrow \text{LINK}(\text{Avail})$

④ [Initialize fields of new node]
 $\text{INFO}(\text{NEW}) \leftarrow x$

⑤ [Check for empty list]

if $\text{FIRST} = \text{NULL}$

then $\text{FIRST} \leftarrow \text{NEW}$

$\text{LINK}(\text{NEW}) \leftarrow \text{FIRST}$

return NEW

⑥ [Set LINK pointer of new node]
 $\text{LINK}(\text{NEW}) \leftarrow \text{FIRST}$

⑦ [Set PTR]

$\text{PTR} \leftarrow \text{FIRST}$

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⑧ [check for first node]
while $\text{LINK}(\text{PTR}) \neq \text{FIRST}$
 $\text{PTR} \leftarrow \text{LINK}(\text{PTR})$

⑨ [Add new node at end]
 $\text{LINK}(\text{PTR}) \leftarrow \text{NEW}$
 $\text{RETURN } (\text{FIRST})$

→ Delete a node from front

$\text{FIRST} = \text{CDELETE } (\text{FIRST})$

① [check for empty list]
if $\text{FIRST} = \text{NULL}$
then write ('LIST IS EMPTY')
Return (FIRST)

② [check for only one in LIST]
if $\text{FIRST} = \text{LINK}(\text{FIRST})$
then Return (NULL)

③ [Set temp variable]
 $\text{temp} \leftarrow \text{FIRST}$

④ [check for last node]
while $\text{LINK}(\text{temp}) \neq \text{FIRST}$
then $\text{temp} \leftarrow \text{LINK}(\text{temp})$

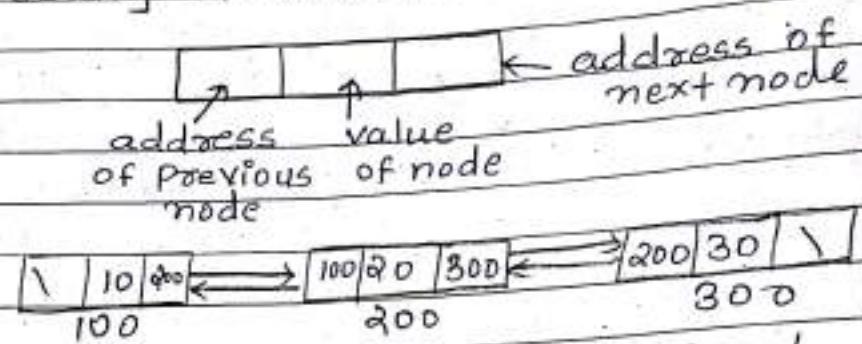
⑤ [Set Pointer]
 $\text{FIRST} \leftarrow \text{LINK}(\text{FIRST})$
 $\text{LINK}(\text{temp}) \leftarrow \text{FIRST}$

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- ⑥ [Return first node Pointer]
Return (FIRST)

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Doubly Linked list



A Doubly link list contain two link field. The links are used to denote predecessor and successor of node. The link denoting the predecessor of a node is called as left link & the node denoting the successor is called as right link.

Definition: A Doubly link list is a Sequence of element in which every element has a link to its previous & next element in the sequence

Advantages

- 1> We can traverse in both direction from starting to end as well as end to starting.
- 2> It is easy to reverse the link list

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3) If we are at one node then we can go to any node which is not possible in singly link list.

4)

Disadvantage.

- 1) It require more space per node because one extra field is required for pointer for previous node.
- 2) Element are access sequential.

Application.

- 1) To implement tree.
- 2) Browser catch which allows you to with the back button
- 3) A Music player which has next & previous button.
- 4) Undo & Redo functionality in word.

Insert a node into doubly link list.

Procedure : DoubINS(L, R, M, x)

L & R = pointer variable denoting the left most & Right most in the list respectively.

M = The insertion is to be performed to the left of a specified node whose address is given by the pointer variable M.

x = Element to be inserted

MF 90°

N = L

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Info → Information field of a node
LPTR → Left & Right link of a node
RPTR

Initially L & R = Null

1) [Obtain a new node from the availability stack]

New ← Node

2) [Copy information field.]

Info (New) ← x

3) [Insertion into an empty list?]

if R = Null

then LPTR (New) ← RPTR (New) ← NULL

L ← R ← New

Return

4) [Left-most insertion]

if M = L

if M = R

then LPTR (New) ← NULL RPTR (New) ←

RPTR (New) ← M LPTR (New) ←

LPTR (M) ← New RPTR (M) ←

L ← New

Return

Return

5) [Insertion in middle]

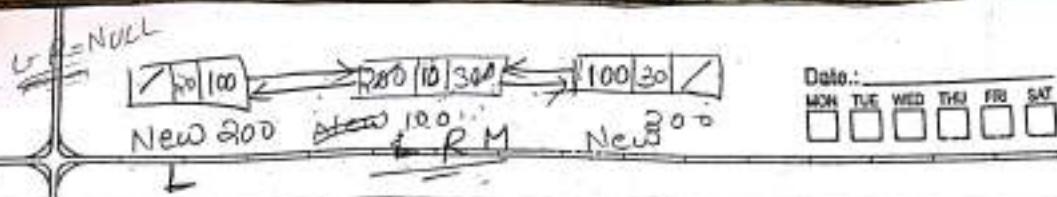
LPTR (New) ← LPTR (M)

RPTR (New) ← M

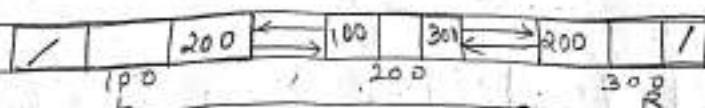
LPTR (M) ← New

RPTR (LPTR (New)) ← New

Return



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Delete an element from Doubly Link List.

Procedure : DOUBDEL (L, R, OLD)

OLD = Address of a node which is to be deleted.

① [Underflow?]

if R = NULL

then write ('Underflow')

Return

② [Delete node]

if L = R (single node in list)

then L ← R ← NULL

else if OLD = L

L ← RPTR(L) (left-most node of linked list)

LPTR(L) ← NULL

else if OLD = R (Right most node)

then R ← LPTR(R)

RPTR(R) ← NULL

else

RPTR(LPTR(OLD)) ← RPTR(OLD)

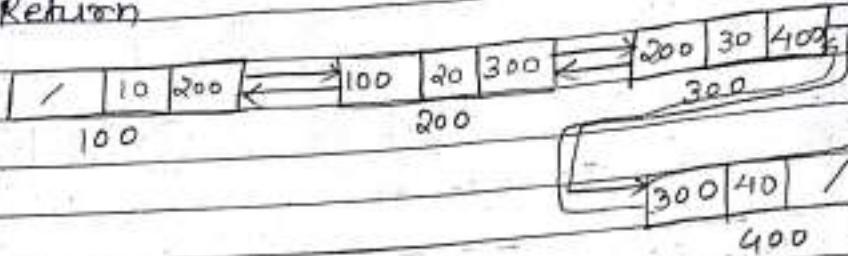
LPTR(RPTR(OLD)) ← LPTR(OLD)

③ [Return deleted node]

Restore(OLD)

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Return



Singl
Appli
Polymer

Advantages of Doubly linked List
for deletion of a node over
singly link list

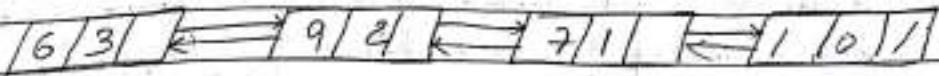
- 1) Deletion of a node in a singly link list we have to find the predecessor of the discarded group & the search was performed by chaning through successive node.
- 2) The Search was necessary in order to change the link of the predecessor node to a value that would point to the sucessor of the node which is to be deleted.
- 3) Such Search would be very time consuming depending on the number of deletion & no. of nodes in the linked list.
- 4) In doubly linked list no Such Search is required. Simply when the address of a node which is to be deleted & the predecessor & sucessor are the immediately node so doubly linked list are much more efficient w.r.t.

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Singly linked list.

→ Application of Linked List
Polynomial Representation.

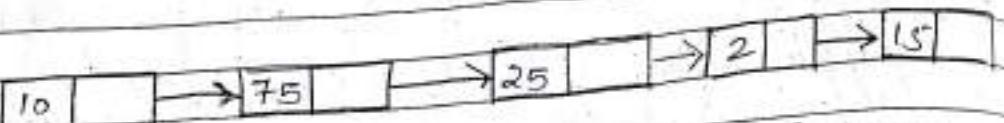
$$6x^3 + 9x^2 + 7x + 1$$



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Hashing

10
75
25
2
5



→ If I want to Search 15 in this array I have to traverse whole array then my searching Complexity is $O(n)$

→ The Complexity depends on the total no. of elements in that data structure so it is not desirable for large data set.

→ To deal with this issue Hashing is introduced. Hashing is one approach in which time required to search an element doesn't depend on the no. of elements using hashing data structure an element is searched with constant time Complexity.

Hashtable

Hash Table is a data structure in which Keys or elements are mapped to an array position by a hash function.

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Hashing works like

$$H(\text{key}) = \text{location}$$

H = hash funⁿ

Key = Value to be stored

location = Index · Value which is derived from hash funⁿ

196, 182, 144, 147, 293, 1395, 1495

Hash table Size = 10

Hash funⁿ = Key mod n

Hash Value	Key	actual element	196 mod 10
v1			= 6
v2	182		182 mod 10
v3	293		= 2
v4	144		0(1)
v5	1395		
v6	196		
v7	147		
v8			
v9			

Key → Hash funⁿ → hash value
 ↑ elements ↑ index or location
 which is to be stored or hash value

Here at position 5 1395 is already stored thru so if we insert 1495 then old value is overwritten so this is the problem of collision.

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112
24

Hashing : The process of mapping the keys to appropriate location in a hash table is called hashing.

Suppose we have a set of strings {"abc", "def", "ghi"} & 4 would like to store in hash table.
 Suppose hash table has a size = 5.

0	1	2	3	4	a	b	c	d	e	f	g	h	i
def	abc			ghi									
abc = 6	6 mod 5 = 1				a								
def = 15	15 mod 5 = 0				b	2							
ghi = 24	24 mod 5 = 4				c	3							
					d	4							
					e	5							
					f	6							
					g	7							
					h	8							
					i	9							

Hash Function

A Hash funⁿ is simply a mathematical formula which applied to a key produce an integer which can be used as a index or position for the key in the Hash Table. The main aim of a Hash funⁿ is that element should be relatively randomly & uniformly distributed. To achieve a good hashing mechanism it is important to have a good hash funⁿ with foll. characteristics:

1. Easy +

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2.8 → T

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- 1. Easy to Compute
- 2. Determinism
- 3. Uniformity

1. \Rightarrow It means the cost of computing a Hash funⁿ must be small.

Ex: If binary search algorithm can search an element from a sorted table of n items $\log(n)$ key comparison then the cost of Hash funⁿ must be less than performing a $\log(n)$ key comparison.

2. \Rightarrow The Hash procedure must be deterministic. This means where the same Hash value must be generated for given input value.

3. \Rightarrow It should provide a uniform distribution across the Hash table. A good Hash funⁿ must map the keys as evenly as possible.

\Rightarrow Hashing are implemented by two steps.
1) An element is converted into integer by using a Hash funⁿ. This integer is used as index or location to store the original element.

2) Then the element is stored into the Hash Table which can be quickly retrieved using index.

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There are different Hash functions.

1. Division Method
2. Multiplication Method
3. Mid Square method.
4. Folding method.

1. In division method hash funⁿ

$$h(z) = z \text{ mod } m$$

if m & z is even then $h(z)$ is even
 if m is even? then $h(z)$ is odd
 z is odd]

if even keys are more than odd keys
 then the division method will not
 spread the Hash Value uniformly. So
 it is best to choose m to be a prime
 number because making m a prime
 no. increases the likely hood that
 the keys are mapped with a uniformity
 in the output range of values.

Calculate the Hash Values of Key 1, 2, 3, 4,
 5, 462 & $m = 97$.

$$\begin{array}{r} 12 \\ h(1234) = 97 | 1234 \\ 97 \\ 264 \\ 194 \\ 70 \end{array}$$

70 is the index

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$$h(5462) = 97 \begin{array}{r} 56 \\ 5462 \\ - 485 \\ \hline 612 \\ - 582 \\ \hline 30 \end{array}$$

30 is the index

2. →
1. choose a Constant A such that $0 < A < 1$
 2. Multiply the Key K with A.
 3. Extract the Fractional Part of KA.
 4. Multiply the result of Step-3 by m and take the floor.

Hash function given by $h(x) = \lfloor m(KA \text{ mod } 1) \rfloor$
 $(KA \text{ mod } 1)$ gives the fractional part of KA

$m = \text{total number of indices in the hash table}$,

$$A = 0.6180339887$$

Ex: Given a Hash table of size 1000 map the key 12345 to a appropriate location in a hash table.

$$m = 1000$$

$$\text{key} = 12345$$

$$\begin{aligned} h(12345) &= \lfloor m(KA \text{ mod } 1) \rfloor \\ &= \lfloor (1000)((12345)(0.6180339887)) \text{ mod } 1 \rfloor \\ &= \lfloor (1000)(7629.629591 \text{ mod } 1) \rfloor \\ &= \lfloor (1000)(0.629591) \rfloor \\ &= \lfloor 629.591 \rfloor \\ &= 629 \end{aligned}$$

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- 3.) 1. Square the Value of Key i.e k^2
 2. Extract the middle r bits of the result
 Obtain in step-1.
 3. In the mid Square method same bit must be chosen from all the key so the hash funⁿ is

$$h(k) = S \Rightarrow S \text{ is obtained by selecting } r \text{ bits from } k^2$$

If we most significant digit the most of the time MSD are same for most of the no. So if we select middle bits then chances of collision is less.

Calculate the Hash Value of Key 1234, 5642 using the mid-square middle method. The hash table has 100 memory location.

$$r=99$$

$$r=2$$

$$h(1234) = k^2 = 1522756 \quad h(k) = 27$$

$$h(5642) = k^2 = 31832164 \quad h(k) = 21$$

- 4.) 1. Divide the Key Value into the no. of Parts. i.e divide k into k_1, k_2, \dots, k_n where each part has the same no. of digits except the last part which may have lesser digit than the

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other parts.

2. Add the individual part that is obtain the sum of k_1, k_2, \dots, k_n . The hash value is produce by ignoring the last carry if any.
3. If the Hash Table has size of 1000 then to address this 1000 location we need atleast 3 digits. therefore each part of the key must have 3 digit except last part which may have lesser digit

Ex: Given a Hash table of 1000 location calculate the hash value using folding method for keys 5678, 321 & 34567

key	5678	321	34567
parts	56, 78	32, 1	34, 56, 7
sum	134	33	97
Hash value	34	33	97

ignore the last carry

Collision

Collision Resolution techniques

- 1) open addressing → Linear probing
→ quadratic probing
→ Double Hashing
- 2) chaining
- Closed hashing → Rehashing

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Once a collision takes place open addressing or closed hashing computes new position using a probe sequence and next record is stored in that position. Hash table contains two types of value.

1) Data value & 2) Sentinel Values
 sentinel value indicate that the location contains no values.

The process of examining memory location in the hash table is called as probing.

L.P. If a value is already stored at a location generated by $h(K)$ then the full hash funⁿ is used to resolve the collision.

Linear Probing

$$h(K, i) = [h'(K) + i] \bmod m$$

\downarrow \downarrow it is the size of
 $K \bmod m$ probe no. varies hash table
 from 0 to $m - K - 1$

Example: Consider a Hash Table of Size 10 using linear probing insert the key 72, 27, 36, 24, 63, 81, 92 and 101

0 1 2 3 4 5 6 7 8 9 $i = 0$

①	-1	-1	-1	-1	-1	-1	-1	-1	-1
---	----	----	----	----	----	----	----	----	----

② Insert 72

$$\begin{aligned}
 h(K, i) &= [h'(K) + i] \bmod m \\
 &= [K \bmod m + i] \bmod m
 \end{aligned}$$

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$$\begin{aligned}
 &= [72 \bmod m + 0] \bmod 10 \\
 &= [2+0] \bmod 10 \\
 &= [2 \bmod 10] \quad \begin{array}{ccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ -1 & -1 & 2 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{array} \\
 &= 2
 \end{aligned}$$

③ Insert 87

$$\begin{aligned}
 h(k, i) &= [h'(k) + i] \bmod m \\
 &= [k \bmod m + i] \bmod m \\
 &= [27 \bmod 10 + 0] \bmod 10 \\
 &= [7+0] \bmod 10 \\
 &= [7] \bmod 10 \\
 &= 7
 \end{aligned}$$

④ Insert 36

$$\begin{aligned}
 h(k, i) &= [h'(k) + i] \bmod m \\
 &= [k \bmod m + i] \bmod m \\
 &= [36 \bmod 10 + 0] \bmod 10 \\
 &= [6] \bmod 10 \\
 &= 6
 \end{aligned}$$

⑤ Insert 24

$$\begin{aligned}
 h(k, i) &= [h'(k) + i] \bmod m \\
 &= [k \bmod m + i] \bmod m \\
 &= [24 \bmod 10 + 0] \bmod 10 \\
 &= [4] \bmod 10 \\
 &= 4
 \end{aligned}$$

⑥ Insert 63

$$\begin{aligned}
 h(k, i) &= [h'(k) + i] \bmod m \\
 &= [k \bmod m + i] \bmod m \\
 &= [63 \bmod 10 + 0] \bmod m
 \end{aligned}$$

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$$= [3] \bmod 10 \\ = 3$$

(c) $T_{\text{Insert}}(K, i)$

$$\begin{aligned} (d) \quad & 8 \text{ Insert } 81 \\ h(K, i) &= [h'(K) + i] \bmod m \\ &= [K \bmod m + i] \bmod m \\ &= [81 \bmod 10 + 0] \bmod 10 \\ &= [81] \bmod 10 \\ &= 81 \end{aligned}$$

(e) $T_{\text{Insert}}(92)$

$$\begin{aligned} h(K, i) &= [h'(K) + i] \bmod m \\ &= [K \bmod m + i] \bmod m \\ &= [92 \bmod 10 + 0] \bmod 10 \\ &= [2] \bmod 10 \\ &= 2 \end{aligned}$$

$$\begin{aligned} h(K, i) &= [h'(K) + i] \bmod m \\ &= [K \bmod m + i] \bmod m \\ &= [92 \bmod 10 + 1] \bmod 10 \\ &= [2+1] \bmod 10 \\ &= 3 \end{aligned}$$

$$\begin{aligned} h(K, i) &= [92 \bmod 10 + 2] \bmod 10 \\ &= [2+2] \bmod 10 \\ &= [4] \bmod 10 \\ &= 4 \end{aligned}$$

$$\begin{aligned} h(K, i) &= [92 \bmod 10 + 3] \bmod 10 \\ &= [2+3] \bmod 10 \\ &= [5] \bmod 10 \\ &= 5 \end{aligned}$$

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⑨ Insert 101

$$\begin{aligned}
 h(k, i) &= [h'(k) + i] \bmod m \\
 &= [k \bmod m + i] \bmod m \\
 &= [101 \bmod 10 + 0] \bmod 10 \\
 &= [1] \bmod 10 \\
 &= 1
 \end{aligned}$$

$i++$

$$\begin{aligned}
 h(k, i) &= [101 \bmod 10 + 1] \bmod 10 \\
 &= [1 + 1] \bmod 10 \\
 &= [2] \bmod 10 \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 h(k, i) &= [101 \bmod 10 + 2] \bmod 10 \\
 &= [1 + 2] \bmod 10 \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 h(k, i) &= [101 \bmod 10 + 3] \bmod 10 \\
 &= [1 + 3] \bmod 10 \\
 &= 4
 \end{aligned}$$

$$\begin{aligned}
 h(k, i) &= [101 \bmod 10 + 4] \bmod 10 \\
 &= [1 + 4] \bmod 10 \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 h(k, i) &= [101 \bmod 10 + 5] \bmod 10 \\
 &= [1 + 5] \bmod 10 \\
 &= 6
 \end{aligned}$$

$$\begin{aligned}
 h(k, i) &= [101 \bmod 10 + 6] \bmod 10 \\
 &= [1 + 6] \bmod 10 \\
 &= 7
 \end{aligned}$$

$$\begin{aligned}
 h(k, i) &= [101 \bmod 10 + 7] \bmod 10 \\
 &= [1 + 7] \bmod 10 \\
 &= 8
 \end{aligned}$$

Problem of linear Probing
 Insert the key (7, 77, 177, 187) &
 $m=10$

1. Insert 7

$$\begin{aligned}
 h(K, i) &= [h'(K) + i] \bmod m \\
 &= [K \bmod m + i] \bmod m \\
 &= [7 \bmod 10 + 0] \bmod 10 \\
 &= [7] \bmod 10 \\
 &= 7
 \end{aligned}$$

2. Insert 77

$$\begin{aligned}
 h(K, i) &= [h'(K) + i] \bmod m \\
 &= [K \bmod m + i] \bmod m \\
 &= [77 \bmod 10 + 0] \bmod 10 \\
 &= [77] \bmod 10 \\
 &= 7
 \end{aligned}$$

$i++$

$$\begin{aligned}
 h(K, i) &= [77 \bmod 10 + 1] \bmod 10 \\
 &= [7 + 1] \bmod 10 \\
 &= 8
 \end{aligned}$$

3. Insert 177

$$\begin{aligned}
 h(K, i) &= [h'(K) + i] \bmod m \\
 &= [177 \bmod 10 + 0] \bmod 10 \\
 &= [7 + 0] \bmod 10 \\
 &= 7
 \end{aligned}$$

$i++$

$$\begin{aligned}
 h(K, i) &= [177 \bmod 10 + 1] \bmod 10 \\
 &= [7 + 1] \bmod 10
 \end{aligned}$$

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$$= 8$$

$i++$

$$h(k, i) = [177 \bmod 10 + 2] \bmod 10$$

$$= [7 + 2] \bmod 10$$

$$= 9$$

4. Insert 187

$$h(k, i) = [h'(k) + i] \bmod m$$

$$= [k \bmod m + i] \bmod m$$

$$= [187 \bmod 10 + 0] \bmod 10$$

$$= [7] \bmod 10$$

$$= 7$$

$i++$

$$h(k, i) = [187 \bmod 10 + 1] \bmod 10$$

$$= [7 + 1] \bmod 10$$

$$= 8$$

$i++$

$$h(k, i) = [187 \bmod 10 + 2] \bmod 10$$

$$= [7 + 2] \bmod 10$$

$$= 9$$

$i++$

$$h(k, i) = [187 \bmod 10 + 3] \bmod 10$$

$$= [7 + 3] \bmod 10$$

$$= [10] \bmod 10$$

$$= 0$$

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Here one of the portion of hash table is filled with keys where other positions remain unused so this is known as primary cluster problem. To avoid primary clustering other techniques such as quadratic probing & double hashing are used.

Searching a Value using linear probing

Searching in a linear probing is same as storing. While searching for a value in a Hash table the array index is recomputed & the key of the element stored at that location is compared with the value that has to be searched. If the match is found the search operation is successive so the time complexity is $O(1)$. If the key doesn't match then the search function begins a sequential search until the value is found.

- i) the search function encounters a vacant location in the table indicating the value is not present.
- ii) Search function terminates because it reaches to the end of the table & the value is not present.

In a worst case the search operation has $O(n-1)$ comparison so the run time of comparison algo is $O(n)$.

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worst case encounters when after scanning all the $n-1$ elements the value is either present at the last location or not present in the table.

Here with the increase in the no. of collision the distance between the array index computed by the Hash funⁿ & the actual location of the element increase rather thereby increase in the search time.

Quadratic Probing

$$h(k, i) = [h'(k) + c_1 i + c_2 i^2] \bmod m$$

$$= [k \bmod m + c_1 i + c_2 i^2] \bmod m$$

c_1, c_2 = It is a constant such that

$$c_1, c_2 \neq 0$$

In quadratic probing eliminates the primary clustering problem of linear probing because it does quadr. search. Using quadratic probing insert the key

Using quadratic probing insert the keys. 72, 27, 36, 24, 63, 81, 10, 1, 92

$$c_1 = 1 \text{ & } c_2 = 3 \text{ & } m = 10$$

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

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Insert 72

$$\begin{aligned}
 h(K, i) &= [h'(K) + c_1 i + c_2 i^2] \bmod m \\
 &= [K \bmod m + c_1 i + c_2 i^2] \bmod m \\
 &= [72 \bmod 10 + 0 + 0] \bmod 10 \\
 &= [2] \bmod 10 \\
 &= 2
 \end{aligned}$$

Insert 87

$$\begin{aligned}
 h(K, i) &= [h'(K) + c_1 i + c_2 i^2] \bmod m \\
 &= [K \bmod m + c_1 i + c_2 i^2] \bmod m \\
 &= [27 \bmod 10 + 0 + 0] \bmod 10 \\
 &= [7] \bmod 10 \\
 &= 7
 \end{aligned}$$

Insert 36

$$\begin{aligned}
 h(K, i) &= [h'(K) + c_1 i + c_2 i^2] \bmod m \\
 &= [36 \bmod 10 + 0 + 0] \bmod 10 \\
 &= [6] \bmod 10 \\
 &= 6
 \end{aligned}$$

Insert 24

$$\begin{aligned}
 h(K, i) &= [h'(K) + c_1 i + c_2 i^2] \bmod m \\
 &= [24 \bmod 10 + 0 + 0] \bmod 10 \\
 &= [4] \bmod 10 \\
 &= 4
 \end{aligned}$$

Insert 63

$$\begin{aligned}
 &= [63 \bmod 10 + 0 + 0] \bmod 10 \\
 &= [3] \bmod 10 \\
 &= 3
 \end{aligned}$$

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Insert 81

$$= [81 \bmod 10 + 0 + 0] \bmod 10$$

$$= [1] \bmod 10$$

$$= 1$$

Insert 101

$$= [101 \bmod 10 + 0 + 0] \bmod 10$$

$$= [1] \bmod 10$$

$$= 1$$

$$= [101 \bmod 10 + 1 + 3] \bmod 10$$

$$= [5] \bmod 10$$

$$= 5$$

Insert 92

$$i=0 = [92 \bmod 10 + 0 + 0] \bmod 10$$

$$= [2] \bmod 10$$

$$= 2$$

$$i=1 = [92 \bmod 10 + 1 + 3] \bmod 10$$

$$= [6] \bmod 10$$

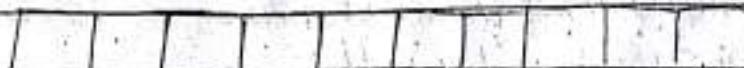
$$= 6$$

$$i=2 = [92 \bmod 10 + 2 + 10] \bmod 10$$

$$= [16 \bmod 10] \bmod 10$$

$$= [6] \bmod 10$$

$$= 6$$



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Even though hash table is empty we can't store some of the value in hash table.

Secondary Clustering

Although quadratic probing is free from primary clustering still encounter a secondary clustering it means there is a collision between two keys the same prob sequence will follow for both. If the quadratic probing the problem of multiple collision increases as the table becomes full.

Ex: Consider a hash table of size 10. Suppose the probe sequence for key 15 is 15, 17, 19, 21, 23, 25, 27, 29, 31, 33. Then the probe sequence for key 31 will be 31, 33, 35, 37, 39, 41, 43, 45, 47, 49.

Double Hashing

Double Hashing uses one hash value then repeatedly step forward at interval until an empty location reach.

The interval is decided using a second independent hash function hence the name Double Hashing

$$h(K, i) = (h_1(K) + i h_2(K)) \bmod m$$

m - size of the hash table.

$h_1(K)$ & $h_2(K)$ = two hash functions

$$h_1(K) = K \bmod m$$

$$h_2(K) = K \bmod m'$$

i = Prob sequence varies from 0 to $m-1$

m' = chosen to be less than m

$$m' = m-1 \text{ or } m-2$$

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Advantages of Double Hashing

- Double Hashing minimize repeated collision & the effect of clustering i.e. double hashing is free from Problem associated with primary clustering as well as secondary clustering.

Ex: Consider a Hash Table of size 10 using Double Hashing. Insert the keys 72, 27, 36, 24, 81, 92 & 101 into the hash table. Take $h_1 \cdot h_2 = (K) \bmod 10$

$$h_2 = (K) \bmod 8$$

0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

$$\text{i) } h(72, 0) = [k \bmod m + i * k \bmod m'] \bmod m$$

$$= [72 \bmod 10 + (0) * (72) \bmod 8] \bmod 10$$

$$= [2] \bmod 10$$

$$= 2$$

ii) Insert 27

$$h(27, 0)$$

$$= [27 \bmod 10 + (0) * (27) \bmod 8] \bmod 10$$

$$= [7] \bmod 10$$

$$= 7$$

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$$\begin{aligned}
 \text{(iii)} \quad & h(36, 0) \\
 &= [36 \bmod 10 + (0) * (36 \bmod 8)] \bmod 10 \\
 &= [6] \bmod 10 \\
 &= 6
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & h(24, 0) \\
 &= [24 \bmod 10 + (0) * (24 \bmod 8)] \bmod 10 \\
 &= [4] \bmod 10 \\
 &= 4
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & h(81, 0) \\
 &= [81 \bmod 10 + (0) * (81 \bmod 8)] \bmod 10 \\
 &= [1] \bmod 10 \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad & h(92, 0) \\
 &= [92 \bmod 10 + (0) * (92 \bmod 8)] \bmod 10 \\
 &= [2] \bmod 10 \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad & h(63, 0) \\
 &= [63 \bmod 10 + (0) * (63 \bmod 8)] \bmod 10 \\
 &= [3] \bmod 10 \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii)} \quad & h(101, 0) \\
 &=
 \end{aligned}$$

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$$\begin{aligned}
 & h(92, 1) \\
 &= [92 \bmod 10 + 1 * 92 \bmod 8] \bmod 10 \\
 &= [2 + 4] \bmod 10 \\
 &= [6] \bmod 10 \\
 &= 6 \quad i++
 \end{aligned}$$

$$\begin{aligned}
 & h(92, 2) \\
 &= [92 \bmod 10 + 2 * 92 \bmod 8] \bmod 10 \\
 &= [2 + 8] \bmod 10 \\
 &= [10] \bmod 10 \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 & \text{Viii)} \quad h(101, 0) \\
 &= [101 \bmod 10 + 0 * 101 \bmod 8] \bmod 10 \\
 &= [1] \bmod 10 \\
 &= 1 \quad i++
 \end{aligned}$$

$$\begin{aligned}
 & h(101, 1) \\
 &= [101 \bmod 10 + 1 * 101 \bmod 8] \bmod 10 \\
 &= [1 + 5] \bmod 10 \\
 &= [6] \bmod 10 \\
 &= 6 \quad i++
 \end{aligned}$$

$$\begin{aligned}
 & h(101, 2) \\
 &= [101 \bmod 10 + 2 * 101 \bmod 8] \bmod 10 \\
 &= [1 + 10] \bmod 10 \\
 &= [11] \bmod 10 \\
 &= 1 \quad i++
 \end{aligned}$$

$$\begin{aligned}
 & h(101, 3) \\
 &= [101 \bmod 10 + 3 * 101 \bmod 8] \bmod 10 \\
 &= [1 + 15] \bmod 10 \\
 &= [16] \bmod 10 \\
 &= 6
 \end{aligned}$$

	0	1	2	3	4	5	6	7	8	9
92	81	72	63	24	-1	36	27	-1	-1	

Although Double hashing is a very efficient algorithm it always requires m to be a prime number.
 Now take $m = 11$ & $m' = 7$

(i)

$$\begin{aligned}
 & h(72, 0) \\
 &= [72 \bmod 11 + (0) * 72 \bmod 7] \bmod 11 \\
 &= [6] \bmod 11 \\
 &= 6
 \end{aligned}$$

(ii)

$$\begin{aligned}
 & h(27, 0) \\
 &= [27 \bmod 11 + (0) * 27 \bmod 7] \bmod 11 \\
 &= [5] \bmod 11 \\
 &= 5
 \end{aligned}$$

(iii)

$$\begin{aligned}
 & h(36, 0) \\
 &= [36 \bmod 11 + (0) * 36 \bmod 7] \bmod 11 \\
 &= [3] \bmod 11 \\
 &= 3
 \end{aligned}$$

$$\begin{aligned}
 & (iv) h(24) \\
 &= [24] \\
 &= [2] \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 & (v) h(6) \\
 &= [6] \\
 &= [8] \\
 &= 8
 \end{aligned}$$

$$\begin{aligned}
 & (vi) h(1) \\
 &= [1] \\
 &= [1]
 \end{aligned}$$

$$\begin{aligned}
 & (vii) h(1) \\
 &= [1] \\
 &= [1]
 \end{aligned}$$

=
 =
 =
 =

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$$\begin{aligned}
 \text{(iv)} \quad & h(24, 0) \\
 = & [24 \bmod 11 + (0) * 24 \bmod 7] \bmod 11 \\
 = & [2] \bmod 11 \\
 = & 2
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & h(63, 0) \\
 = & [63 \bmod 11 + (0) * 63 \bmod 7] \bmod 11 \\
 = & [8] \bmod 11 \\
 = & 8
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad & h(81, 0) \\
 = & [81 \bmod 11 + (0) * 81 \bmod 7] \bmod 11 \\
 = & [4] \bmod 11 \\
 = & 4
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii)} \quad & h(92, 0) \\
 = & [92 \bmod 11 + (0) * 92 \bmod 7] \bmod 11 \\
 = & [4] \bmod 11 \\
 = & 4 \\
 & \dagger + +
 \end{aligned}$$

$$\begin{aligned}
 = & [92 \bmod 11 + (1) * 92 \bmod 7] \bmod 11 \\
 = & [4 + 1] \bmod 11 \\
 = & [5] \bmod 11 \\
 = & 5 \\
 & \dagger + +
 \end{aligned}$$

$$\begin{aligned}
 = & [92 \bmod 11 + (2) * 92 \bmod 7] \bmod 11 \\
 = & [4 + 2] \bmod 11 \\
 = & [6] \bmod 11 \\
 = & 6
 \end{aligned}$$

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$$\begin{aligned}
 &= [q_2 \bmod 11 + 3 * q_2 \bmod 7] \bmod 11 \\
 &= [4 + 3] \bmod 11 \\
 &= [7] \bmod 11 \\
 &= 7
 \end{aligned}$$

(viii) $h(101, 0)$

$$\begin{aligned}
 &= [101 \bmod 11 + 0 * 101 \bmod 7] \bmod 11 \\
 &= [2] \bmod 11 \\
 &= 2
 \end{aligned}$$

$$\begin{aligned}
 &\quad i++ \\
 &= [101 \bmod 11 + 1 * 101 \bmod 7] \bmod 11 \\
 &= [2 + 3] \bmod 11 \\
 &= [5] \bmod 11 \\
 &= 5
 \end{aligned}$$

$h(101, 2)$

$$\begin{aligned}
 &= [101 \bmod 11 + 2 * 101 \bmod 7] \bmod 11 \\
 &= [2 + 6] \bmod 11 \\
 &= [8] \bmod 11 \\
 &= 8
 \end{aligned}$$

$h(101, 3)$

$$\begin{aligned}
 &= [2 + 9] \bmod 11 + 3 * 101 \bmod 7] \bmod 11 \\
 &= [11] \bmod 11 \\
 &= 0
 \end{aligned}$$

0	1	2	3	4	5	6	7	8	9
101	-1	24	36	81	27	72	92	63	-1

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Rehashing

When the hash table becomes nearly full the no. of collision increases which by degrading the performance of insertion & search operation. In such cases a better option is to create a new hash table with size double of the original hash table.

All the entries in the original hash table will then have to be move to the new hash table. This is done by taking each entry & computing its new hash value and then inserting into the new hash table. Consider the t

Consider the hash table of size 5,

$$h(x) = x \% 5$$

0	1	2	3	4
26	31	43	17	

Size - 10

$$h(26) = 26 \% 10$$

$$= 26 \text{ mod } 10 \quad | \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$$

$$= 6 \quad | \quad |$$

$$h(31) = 31 \% 10$$

$$= 31 \text{ mod } 10 \quad | \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$$

$$= 1 \quad | \quad |$$

$$h(43) = 43 \% 10$$

$$= 43 \text{ mod } 10$$

$$= 3 \quad | \quad |$$

$$0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$$

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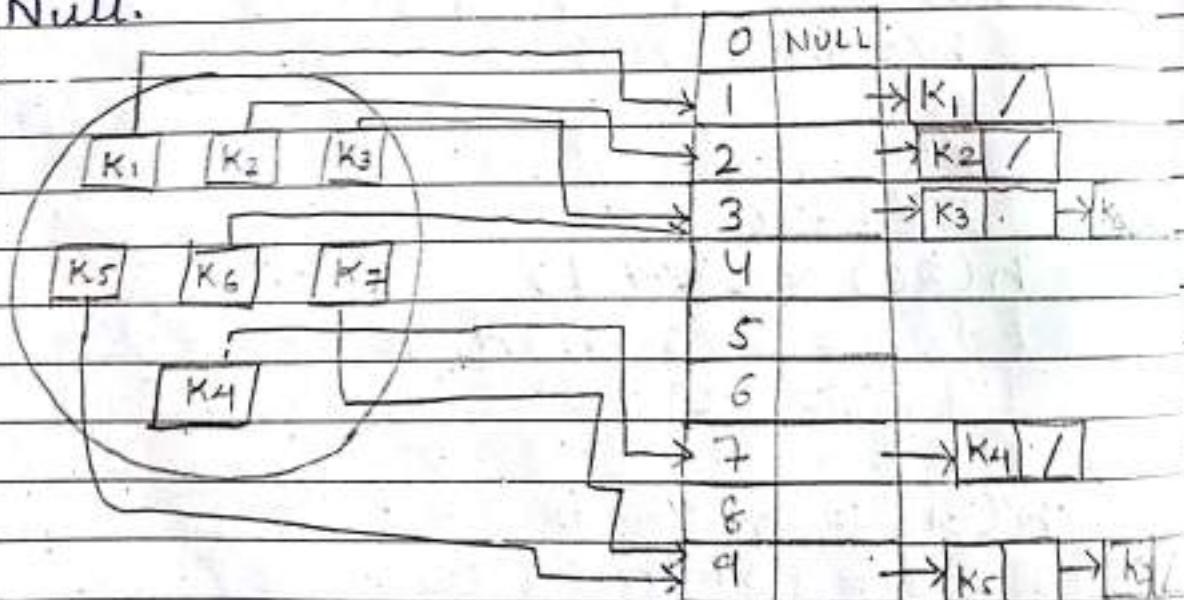
$$h(17) = 17 \bmod 10$$

$$= 7$$

0	1	2	3	4	5	6	7	8	9
31		43			26	17			

Collision Resolution By Chaining

In chaining each location in hash table stores a pointer in the link list that contain all the key values that are hash to that location. i.e (location 1 in the hash table points to the head of the link list of all the key values that has to 1). If no key values has to 1 then location in the hash table contains Null.



Operations on a Chain Hash Table

Searching for a value in a chain hash table is as simple as scanning a link list for an entry in a given tree.

Insertion operation append a key end, or front of the link list pointed by the hash location.

Deleting a key require searching a list & removing the element.

Chain hash table with list link list are widely used. The code for this algorithm is exactly the same as that for inserting, deleting & searching a value in a singly link list. The cost of insertion of a key in hash table is of $O(1)$.

& cost of deletion is $O(m)$

$m = \text{no. of elements in the list}$ of that location. In a worst case searching complexity is $O(m)$.

$n = \text{no. of keys stored in the chain hash table}$. This case occurs when all the key values are inserted into the link list of the same location.

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Insert a key 7, 24, 18, 52, 36, 54, 11, 23, 60.

Chain Hash Table of 9 memory location,
use $h(k) = k \bmod m$

0	$\rightarrow [18] /$	$\rightarrow 36 /$	$\rightarrow 54 /$	$h(7) = 7 \bmod 9$
1				$= 7$
2	$\rightarrow [11] /$			$h(24) = 24 \bmod 9$
3				$= 6$
4				$h(18) = 18 \bmod 9$
5	$\rightarrow [23] /$			$= 0$
6	$\rightarrow 24 /$	$\rightarrow 60 /$		$h(52) = 52 \bmod 9$
7	$\rightarrow 7 /$	$\rightarrow 52 /$		$= 7$
8				

$$h(36) = 36 \bmod 9 \\ = 0$$

$$h(54) = 54 \bmod 9 \\ = 0$$

$$h(11) = 11 \bmod 9 \\ = 2$$

$$h(23) = 23 \bmod 9 \\ = 5$$

$$h(60) = 60 \bmod 9 \\ = 6$$

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Advantages An

1. It remains effective when the no. of keys to be stored is much higher than the no. of location in the hash table. However the no. of keys increases the performance of chain hash table decrease linearly.
2. For ex: A chain hash table with 1000 memory location & 100000 stored key will give 5-10 time less performance as compared to the performance of chain hash table. But chain hash table is still 1000 time faster than simple hash table.
3. The other advantages of chaining of collision resolution is that it's performance unlike quadratic probing doesn't degrade when the table is more than half full.
4. This technique is absolute free from clustering.

Disadvantages

1. It inherits the disadvantages of link list. To store even a key value the space overhead of the next pointer in each entry can be significant.
2. Traversing a link list has poor cache performance making the processor cache ineffective.

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Advantages & Disadvantages of Hashing

1. The main advantage of hash table over other data structure is speed. It is more advantages when the no. of keys is large.
2. Being able to look up & insert the data in order of O(1), which is independent of the size of the data structure.
3. Hash table can be difficult to implement.
4. Choosing a effective hash funⁿ for specific application is more an art than a science.
5. Hashing is not effective when the no. of entries is very small.
6. It becomes ineffective when there are many collision.

Application of Hashing

1. Hash table are widely used in situation where large amount of data have to be access to quickly search & retrieve information.
2. Hashing is used as a symbol table.
3. Hash table can be use to store massive amount of information. For ex: To store Drivers license record, given the Drivers license no. hash table helps to quickly

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4. get the information about the drivers.
 5. Hashing is used in telephone database to quickly find the person's telephone no.
- Some database stores a separate file known as indexes when data has to be retrieve from a file. The key information is first found in the appropriate index file which reference the exact memory location of data in database file. This key info. in the index file is often stored as a hash value.

Symbol table

Symbol table is an important data structure created & maintained by compiler in order to store the information about the occurrence of various entities such as variable name, obj., classes and interface, etc.

A Symbol Table may serve the following purpose depending upon the language.

- 1) To store the name or symbols of all entities in structured form.
- 2) To verify if variable is declared or not.
- 3) To implement type checking by verifying assignments & expressions in the source code are semantically correct.

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- correct or not.
- 4) To determine the scope of name.
A Symbol table is simply a hash table which can be either linear or hash table.
It maintains the entry for each... in the foll. format.
<Symbol name, type, attribute>
↓
Ex: static int interest
<interest, int, static>

Implementation

If compiler is to handle a small amount of data then symbol table can be implemented as an unordered list which is easy to code but it is only suitable for small tables only.

A Symbol table can be implemented in one of the foll. ways:

- 1) Linear list (sorted or unsorted)
- 2) BST
- 3) Hash table

Symbol table are mostly implemented as hash table, where the source code symbol itself is treated as a key for the hash function and return the value is the information about the symbol.

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Ex:

```
int value = 10;  
void pro_one()  
{
```

```
    int one_1;  
    int one_2;  
{
```

```
    int one_3; } inner-scope 1  
    int one_4; }
```

}

```
    int one_5;  
{
```

```
    int one_6; } inner-scope 2  
    int one_7; }
```

}

}

```
void pro_two()  
{
```

```
    int two_1;  
    int two_2;
```

{

```
    int two_3; } inner-scope 3  
    int two_4; }
```

}

```
    int two_5;
```

}

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Global Symbol Table

Value	Var	Init
Proc-one	Proc.	Void
Proc-two	Proc.	Void
one.1	Var	int
one.2	Var	int
one.3	Var	int
one.4	Var	int
one.5	Var	int
two.1	Var	int
two.2	Var	int
two.3	Var	int
two.4	Var	int
two.5	Var	int
one.6	Var	int
one.7	Var	int
two.6	Var	int
two.7	Var	int

Runtime of Linear Search & Binary Search.

It's complexity is order $O(n)$. For binary search algorithm it is mandatory for the target array to be sorted.

$$\begin{aligned}
 \text{mid} &= \text{low} + (\text{high} - \text{low})/2 \\
 &= 0 + (9-0)/2 \\
 &= 4
 \end{aligned}$$

0	1	2	3	4	5	6	7	8	9
10	14	19	26	31	31	33	35	42	44
5	6	7	8	9					
31	33	35	42	44					

$$\begin{aligned}
 &= 5 + (9-5)/2 \\
 &= 5+2 = 7
 \end{aligned}$$

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$$\begin{array}{|c|c|} \hline 3 & 1 & 3 & 3 \\ \hline \end{array} = 5 + (6-5)/2
 = 5 + 1/2
 = 5$$

- Suppose we start an array of length 8 then incorrect guesses reduce the size of the position to 4, 2, 1
- Once, the reasonable position contain just one element then no further guess occurs & the last one extra guess b/w the one element position is either correct or incorrect
- with an array of length 8, Binary search needs at most 4 comparison, $n+1$ guesses is needed for any
- To represent this thing one mathem. funⁿ is there which is $\log n$.
- ∴ The complexity is binary search is $O(\log n)$