Question Bank

Unit - 1

Q-1 (1) Linearly Independent funⁿ (2) LD
(3) Wronskian

→ suppose $f_1(x), f_2(x), \ldots f_n(x)$ possess at least $n-1$ derivates,

$$\begin{vmatrix} f_1 & f_2 & \ldots & f_n \\ f_1' & f_2' & \ldots & f_n' \\ \vdots & & & \\ f_1^{(n-1)} & f_2^{(n-1)} & \ldots & f_n^{(n-1)} \end{vmatrix}$$

(3) → The above determinant is called wronskian of the funⁿ.

(1) → If the determinant is not zero for at least one point in the interval I, then the set of functions is linearly independent funⁿ.

(2) → If the determinant is zero for any point, then it is called linearly dependent funⁿ

Q-2 check whether the given sets are LI or LD.

(1) $\{1, e^x, e^{-x}\}$

$$\begin{vmatrix} 1 & e^x & e^{-x} \\ 0 & e^x & -e^{-x} \\ 0 & e^x & e^{-x} \end{vmatrix}$$

$= e^x e^{-x} + e^x e^{-x}$

$= 1 + 1 = \underline{2}$

$\therefore$ LI

(2) $\{e^x, e^{-x}, \cosh x\}$

$$\begin{vmatrix} e^x & e^{-x} & \cosh x \\ e^x & -e^{-x} & \sinh x \\ e^x & e^{-x} & \cosh x \end{vmatrix}$$

$= e^x e^{-x} \begin{vmatrix} 1 & 1 & \cosh x \\ 1 & -1 & \sinh x \\ 1 & 1 & \cosh x \end{vmatrix}$ $= 1(-\cosh x - \sinh x) - 1(\cosh x - \cosh x)$
$+ 1(\sinh x + \cosh x)$
$= -\cosh x - \sinh x + \sinh x + \cosh x = 0$

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$\therefore$ LD

(3) $\{1, e^x\}$

$$\begin{vmatrix} 1 & e^x \\ 0 & e^x \end{vmatrix} = e^x$$

$\therefore$ LI

Q-3 (a) Using method of undetermined coefficients solve

(1) $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$

→ the AE is given by

$m^2 - 2m + 5 = 0$

$m = \frac{2 \pm \sqrt{4-4(5)}}{2} = \frac{2 \pm \sqrt{16}i}{2} = 1 \pm 2i$

$4-20 = -16$

$$\boxed{y_c = e^x(c_1 \cos 2x + c_2 \sin 2x)}$$

By method of undetermined coefficients

$y_p = A + Bx + Cx^2 + Dx^3$

$y'_p = B + 2Cx + 3Dx^2$

$y''_p = 2C + 6Dx$

$2C + 6Dx - 2(B + 2Cx + 3Dx^2) + 5(A + Bx + Cx^2 + Dx^3) = 5x^3 - 6x^2 + 6x$

$(2C - 2B + 5A) + x(6D - 4C + 5B) + x^2(-6D + 5C) + x^3(5D) = 5x^3 - 6x^2 + 6x$

Comparing both sides

$5D = 5$	$-6D + 5C = -6$	$6D - 4C + 5B = 6$	$2C - 2B + 5A = 0$
$\boxed{D = 1}$	$-6 + 5C = -6$	$6 - 4(0) + 5B = 6$	$2(0) - 2(0) + 5A = 0$
	$5C = 0$	$5B = 0$	$\boxed{A = 0}$
	$\boxed{C = 0}$	$\boxed{B = 0}$	

$$\boxed{y_p = x^3}$$

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(2) $\frac{d^2y}{dx^2} + 4 = 4e^{2x}$

→ AE: $m^2 + 4 = 0$

$m^2 = -4$

$m = \pm 2i$

$\boxed{CF = C_1\cos 2x + C_2 \sin 2x}$

$\cancel{y_p = e^{ax}}$

$\cancel{y'_p = ae^{ax}}$

$\cancel{y''_p = a^2}$

$y_p = ce^{2x}$

$y''_p = 4ce^{2x}$

$4ce^{2x} + 4ce^{2x} = 4e^{2x}$

$8ce^{2x} = 4e^{2x}$

comparing both sides.

$8c = 4$

$c = \frac{1}{2}$

$\boxed{y_p = \frac{e^{2x}}{2}}$

(3) $y'' + 2y' + 10y = 25x^2 + 3$

→ AE: $m^2 + 2m + 10 = 0$

$m = \frac{-2 \pm \sqrt{4 - 4(10)}}{2} = \frac{-2 \pm \sqrt{-36}}{2} = \frac{-2 \pm 6i}{2} = -1 \pm 3i$

$\boxed{y_c = e^{-x}(c_1\cos 3x + c_2 \sin 3x)}$

$y_p = A + Bx + cx^2$

$y'_p = B + 2cx$

$y''_p = 2c$

$2c + 2(B + 2cx) + 10(A + Bx + cx^2) = 25x^2 + 3$

$(2c + 2B + 10A) + x(4c + 10B) + 10cx^2 = 25x^2 + 3$

$10c = 25$

$c = 25/10$

$\boxed{c = 5/2}$

$4c + 10B = 0$

$2\cdot 4\left(\frac{5}{2}\right) = -10B$

$B = -\frac{2\times 5}{10} = -1$

$\boxed{B = -1}$

$2c + 2B + 10A = 3$

$2\left(\frac{5}{2}\right) + 2(-1) + 10A = 3$

$10A = 3 + 2 - 5 = 0$

$\boxed{A = 0}$

$\boxed{y_p = -x + \frac{5}{2}x^2}$

(4) ~~$y'' + 16y = se$~~ $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 6\sin 3x$

$\rightarrow$ AE: $m^2 + 4m + 4 = 0$

$(m+2)^2 = 0$

$m = -2, -2$

$$\boxed{y_c = (c_1 + c_2 x) e^{-2x}}$$

$y_p = c_1 \sin 3x + c_2 \cos 3x$

$y'_p = 3c_1 \cos 3x - 3c_2 \sin 3x$

$y''_p = -9c_1 \sin 3x - 9c_2 \cos 3x$

$-9c_1 \sin 3x - 9c_2 \cos 3x + 4(3c_1 \cos 3x - 3c_2 \sin 3x) + 4(c_1 \sin 3x + c_2 \cos 3x)$
$= 6\sin 3x$

$\sin 3x(-9c_1 - 12c_2 + 4c_1) + \cos 3x(-9c_2 + 12c_1 + 4c_2) = 6\sin 3x$

$-9c_1 - 12c_2 + 4c_1 = 6$ $\qquad$ $-9c_2 + 12c_1 + 4c_2 = 0$

$(-5c_1 - 12c_2 = 6)\ 12$ $\qquad$ $(12c_1 - 5c_2 = 0)\ 5$

$-60c_1 - 144c_2 = 72$

$60c_1 - 25c_2 = 0$

$-169c_2 = 72$

$c_2 = \frac{-72}{169}$

$12c_1 = 5c_2$

$c_1 = \frac{5c_2}{12}$

$= \frac{-5}{12} \times \frac{72}{169} = \frac{-30}{169}$

$$\boxed{y_p = \frac{-30}{169}\sin 3x - \frac{72}{169}\cos 3x}$$

Q-5 Solve by the method of Variation of Parameter

(1) $y'' + a^2 y = \sec ax$

$\rightarrow$ AE: $m^2 + a^2 = 0$

$m^2 = -a^2$

$m = \pm ai$

$$\boxed{CF = c_1 \cos ax + c_2 \sin ax}$$

$y_1 = \cos ax \qquad y_2 = \sin ax$

$$w = \begin{vmatrix} \cos ax & \sin ax \\ -a\sin ax & a\cos ax \end{vmatrix}$$

$= a\cos^2 ax + a\sin^2 ax$

$w = \underline{a}$

$u_1 = -\int \frac{\sin ax \cdot \sec ax}{a}$

$= -\frac{1}{a}\int \tan ax$

$= -\frac{1}{a^2}\log(\operatorname{cosec} ax)$

$u_2 = \int \frac{\cos ax \cdot \sec ax}{a}$

$= \frac{x}{a}$

$$\boxed{y_p = -\frac{1}{a^2}\log(\operatorname{cosec} ax)\cos ax + \frac{x}{a}\sin ax}$$

(2) $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$

AE: $m^2 - 6m + 9 = 0$

$\cancel{m = \frac{2 \pm \sqrt{4-4(9)}}{2a} = 2 \pm \sqrt{\ }}$

$(m-3)^2 = 0$

$m = 3, 3$

$$\boxed{CF = (c_1 + c_2 x)e^{3x}}$$

$y_1 = e^{3x} \qquad y_2 = xe^{3x}$

$$W = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & (1+3x)e^{3x} \end{vmatrix}$$

$= (e^{3x})^2[1 + \cancel{3x} - \cancel{3x}] = (e^{3x})^2$

$u_1 = -\int \frac{xe^{3x} \cdot \frac{e^{3x}}{x^2}}{(e^{3x})^2}$

$= -\int \frac{1}{x}$

$= -\log x$

$u_2 = \int \frac{e^{3x} \cdot \frac{e^{3x}}{x^2}}{(e^{3x})^2} = \int \frac{1}{x^2} = -\frac{1}{x}$

$y_p = -e^{3x}\log x - e^{3x}(1) = \boxed{-e^{3x}(\log x + 1)}$

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(3) $y'' + 9y + \sec 3x$ (4) $y'' + 16y = \sec 4x$

Same as (1)

Q-6 solve legendre's eqn.

(1) $(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$

$\rightarrow$ Let $(2x+3) = e^z$ $\Rightarrow x = \frac{e^z-3}{2}$

$z = \log(2x+3)$

$[4D(D-1) - 4D - 12]\, y = \cancel{6}^{3}\left(\frac{e^z-3}{\cancel{2}}\right)$

$4D^2 - 8D - 12 = 3(e^z - 3)$

A.E: $4m^2 - 8m - 12 = 0$

$m^2 - 2m - 3 = 0$

$(m+1)(m-3) = 0$

$m = -1, 3$

$y_c = c_1 e^{-z} + c_2 e^{3x}$

$\boxed{y_c = c_1 (2x+3)^{-1} + c_2 (2x+3)^3}$

$y_p = \frac{1}{4D^2 - 8D - 12}(3e^z - 3)$

$= 3\left[\frac{e^z}{4D^2 - 8D - 12} - \frac{\cancel{3}^{3} e^0}{4D^2 - 8D - 12}\right]$

$D \to 1$ $\qquad$ $D \to 0$

$= 3\left[\frac{-e^z}{16} + \frac{1}{4}\right]$

$y_p = \frac{-3}{16} e^z + \frac{3}{4}$

$\boxed{y_p = \frac{-3}{16}(2x+3) + \frac{3}{4}}$

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② $(2x+3)^2 \frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 0$

→ y_c is same as ①

$y_p = 0$

③ $(x+1)^2 \frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} + y = 4\cos\log(x+1)$

→ Let $x+1 = e^z \Rightarrow x = e^z - 1$

$z = \log(x+1)$

$(D(D-1) + D + 1)y = 4\cos z$

$(D^2 - D + D + 1)y$

AE: $m^2 + 1 = 0$

$m = \pm i$

$$\boxed{y_c = c_1\cos x + c_2\sin x}$$

$y_p = \frac{1}{D^2+1} 4\cos z$

$= 4\left[\frac{1}{D^2+1}\cos z\right]$

$D^2 = -1$

$= 4\left[\frac{D}{2D}\cos z\right]$

$= \frac{4}{2}\cos z$

$$\boxed{y_p = 2\cos z}$$

Q-8 solve by Euler-cauchy method.

① $x^3y''' + 2x^2y'' + 2y = 10\left(x + \frac{1}{x}\right)$

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$x^3y''' + 2x^2y'' + 2y = 10\left(x + \frac{1}{x}\right)$

$\rightarrow$

Let $x = e^z \Rightarrow$

$z = \log x$

$[D(D-1)(D-2) + 2D(D-1) + 2]\,y = 10[e^z + e^{-z}]$

$(D^2-D)(D-2) + 2D^2 - 2D + 2$

$D^3 - 2D^2 - D^2 + \cancel{2D} + 2D^2 - \cancel{2D} + 2$

$D^3 - D^2 + 2$

AE: $m^3 - m^2 + 2$

$(m+1)(m^2 - 2m + 2) = 0$

-1	1	-1	0	2
	0	-1	2	-2
	1	-2	2	0

$m = \frac{2 \pm \sqrt{4 - 4(2)}}{2} = \frac{2 \pm \sqrt{-4}}{2} = \frac{2 \pm 2i}{2} = 1 \pm i$

$m = -1,\ 1 \pm i$

$CF = c_1 e^{z} + e^{z}(c_2 \cos z + c_3 \sin z)$

$$\boxed{y_c = c_1 x + x\left(c_2 \cos(\log x) + c_3 \sin(\log x)\right)}$$

$y_p = \frac{1}{D^3 - D^2 + 2}\, 10(e^z + e^{-z})$

$= 10\left[\frac{e^z}{D^3 - D^2 + 2} + \frac{e^{-z}}{D^3 - D^2 + 2}\right]$

$D \to 1 \qquad D \to -1$

$= 10\left[\frac{e^z}{1 - 1 + 2} + \frac{e^{-z}}{-1 - 1 + 2}\right]$

$= 10\left[\frac{e^z}{2} + \frac{z e^{-z}}{3D^2 - 2D}\right]$

$D \to -1$

$= 10\left[\frac{e^z}{2} + \frac{z e^{-z}}{3 + 2}\right]$

$= 5e^z + 2ze^{-z}$

$$\boxed{y_p = 5x + \frac{2\log x}{x}}$$

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(2) $x^3y''' + 2x^2y'' + 2y = 0$

→ Let $x = e^z$

$z = \log x$

$(D(D-1)(D-2) + 2D(D-1) + 2)\, y = 0$

$(D^2-D)(D-2) + 2(D^2-D) + 2$

$\cancel{D^3}$.

Same as (1)

$\underline{y_p = 0}$

(3) $x^2y'' - 3xy + 4y = x^2$

→ Let $x = e^z$

$z = \log x$.

$(D(D-1) \cancel{+} -3D + 4)\, y = e^{2z}$

$(D^2 - D - 3D + 4)$

$D^2 - 4D + 4$

AE: $\cancel{D}\ m^2 - 4m + 4 = 0$

$(m-2)^2 = 0$

$m = 2, 2$

$$\boxed{y_c = (c_1 + c_2x)e^{2x}}$$

$y_p = \frac{1}{D^2-4D+4} e^{2z}$

$D \to 2$

$= \frac{1}{\cancel{4}-\cancel{4}+4} e^{2z}$

$= \frac{1}{4} e^{2z}$

$$\boxed{y_p = \frac{1}{4} x^2}$$

(4) $x^2y'' - 2.5xy' - 2.0y = 0$

$\rightarrow$ let $x = e^z$

$z = \log x$

$\left(D(D-1) - \frac{5}{2}D - 2\right)y = 0$

$2D^2 - 2D - 5D - 4$

$2D^2 - 7D - 4$

A.E: $2m^2 - 7m - 4 = 0$

49
-32
17

$m = \frac{7 \pm \sqrt{49 - 4(8)}}{4}$

$= 7 \pm \sqrt{}$

$2m^2 - 8m + m - 4 = 0$

$2m(m-4) + 1(m-4) = 0$

$(m-4)(2m+1) = 0$

$m = -\frac{1}{2}, 4$

$$\boxed{y_c = c_1 e^{-1/2 x} + c_2 e^{4x}}$$

$y_p = \frac{1}{2D^2 - 7D - 4}(0)$

$\boxed{y_p = 0}$

Unit - 2

1) Define

(1) Power series - The Power series in the form $(x-x_0)$ is an infinite Series of the form

$\sum_{k=0}^{\infty} a_k (x-x_0)^k = a_0 + a_1(x-x_0) + a_2(x-x_0)^3 + \ldots$

where $a_0, a_1, a_2 =$ Constants

(2) Singular Point - A Point x_0 is said to be Singular if either $P(x)$ or $Q(x)$ or both are not analytic at x_0

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(3) Ordinary Point - A Point $x = x_0$ is said to be ordinary if either ~~both~~ $P(x)$ or $Q(x)$ or both are analytic at x_0

(4) ISP - If either $(x-x_0)P(x)$ and $(x-x_0)^2 Q(x)$ or both are analytic then the SP is called ISP

(5) RSP - ~~If~~ A SP $x = x_0$ is said to be RSP if $(x-x_0)P(x)$ and $(x-x_0)^2 Q(x)$ are not analytic

Q.2 Find ordinary Point for

(1) $y'' + y = 0$

$P(x) = 0$ $Q(x) = 1$

$P(x)$ and $Q(x)$ are analytic everywhere

$\therefore$ All Points are op

(2) $y'' + e^x y' + (\sin x)y = 0$

$P(x) = e^x$ $Q(x) = \sin x$

e^x and $\sin x$ are analytic everywhere

$\therefore$ All Points are OP

(3) $y'' + e^x y' + (\cos x) y = 0$

same as above

(4) $y'' - 2y = 0$

$P(x) = 0$ $Q(x) = -2$

$P(x)$ and $Q(x)$ are analytic everywhere

$\therefore$ All Points are OP.

(5) $y'' + e^{2x} y' + x^2 y = 0$

$P(x) = e^{2x}$ $Q(x) = x^2$

$P(x)$ and $Q(x)$ are analytic everywhere

$\therefore$ All Points are OP.

Q-(8) Find Singular Point for

(1) $y'' + \frac{1}{x-1} y' + \frac{1}{x-1} y = 0$

$\rightarrow$ $P(x) = \frac{1}{x-1}$ $\quad Q(x) = \frac{1}{x-1}$

$\therefore$ $P(x)$ and $Q(x)$ are not analytic at $x = 1$

$\therefore$ $x = 1$ is singular Point

(2) $(1-x^2) y'' - 6xy' - 4y = 0$

$\rightarrow$ $P(x) = \frac{-6x}{1-x^2}$ $\quad Q(x) = \frac{-4}{1-x^2}$

$P(x)$ and $Q(x)$ are not analytic at $x = \pm 1$

$\therefore$ $x = \pm 1$ is SP

(3) $y'' + \frac{1}{x-2} y' + \frac{1}{x-2} y = 0$

$\rightarrow$ $P(x) = \frac{1}{x-2}$ $\quad Q(x) = \frac{1}{x-2}$

$P(x)$ and $Q(x)$ are not analytic at $x = 2$

$\therefore$ $x = 2$ is SP

(4) $y'' + \frac{2}{x+3} y' - \frac{5}{x+3} y = 0$

$P(x) = \frac{2}{x+3}$ $\quad Q(x) = \frac{-5}{x+3}$

$P(x)$ and $Q(x)$ are not analytic at $x = -3$

$\therefore$ $x = -3$ is SP

Q-4 classify the singularities

(1) $(x^2+4) \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 12y = 0$

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$\rightarrow$ $P(x) = \frac{2x}{x^2+4}$ $\quad Q(x) = \frac{-12}{x^2+4}$

$P(x)$ and $Q(x)$ are not analytic at $x = \pm 2i$

$\therefore$ $x = \pm 2i$ is SP.

$\rightarrow$ for $x_0 = 2i$

$(x-x_0)P(x) = (x-2i)\frac{2x}{x^2-(2i)^2} = \frac{\cancel{(x-2i)}2x}{\cancel{(x-2i)}(x+2i)}$ = finite

$(x-x_0)^2 Q(x) = (x-2i)^2 \cdot \frac{-12}{(x-2i)(x+2i)} = \frac{(x-2i)}{x+2i}(-12)$ = finite

$\therefore$ $x = 2i$ is RSP

$\rightarrow$ for $x_0 = -2i$

$(x-x_0)P(x) = x+2i \frac{2x}{x^2-(2i)^2} = \frac{\cancel{(x+2i)}2x}{\cancel{(x+2i)}(x-2i)} = \frac{2x}{x-2i}$ = finite

$(x-x_0)^2 Q(x) = (x+2i)^2 \frac{-12}{(x-2i)\cancel{(x+2i)}} = \frac{-12(x+2i)}{x-2i}$ = finite

$\therefore$ $x = -2i$ is RSP

② $(x+1)y'' - xy' - y = 0$

$\rightarrow$ $P(x) = \frac{-x}{x+1}$ $\quad Q(x) = \frac{-1}{x+1}$

$P(x)$ and $Q(x)$ are not analytic at $x = -1$

$\therefore$ $x = -1$ is SP.

$\rightarrow$ for $x = -1$

$(x-x_0)P(x) = \cancel{x+1}\frac{-x}{\cancel{x+1}} = -x$ = finite

$(x-x_0)^2 P(x) = (x+1)^2 \cdot \left(\frac{-1}{x+1}\right) = -(x+1)$ = finite

$\therefore$ $x = -1$ is RSP

Q-6 Solve the foll^n by Power series method.

(1) $\frac{d^2y}{dx^2} + y = 0$

→ $P(x) = 0 \quad q(x) = 1$

$y = a_k x^k$

$y' = k a_k x^{k-1}$

$y'' = k(k-1) a_k x^{k-2}$

$k(k-1) a_k x^{k-2} + a_k x^{k} = 0$
$\phantom{k(k-1) a_k x^{}}$ $k-2$

$\cancel{a} = \frac{a^{k}}{k(k-1)}$

$k(k-1) a_k x^{k-2} + a_{k-2} x^{k-2} = 0$

$a_k = \frac{-a_{k-2}}{k(k-1)}$

$n=2 \quad a_2 = \frac{-a_0}{2}$

$n=3 \quad a_3 = \frac{-a_1}{6}$

$n=4 \quad a_4 = \frac{-a_2}{12} = \frac{a_0}{24}$

$n=5 \quad a_5 = \frac{-a_3}{20} = \frac{a_1}{120}$

$y = a_0 + a_1 x - \frac{a_0 x^2}{2} - \frac{a_1 x^3}{6} + \frac{a_0 x^4}{24} + \frac{a_1 x^5}{120} \ldots$

$y = a_0\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}\right) + a_1\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\right)$

(2) $y' = 2xy$

→ $P(x) = 1 \quad Q(x) = -2x$

$y = a_k x^k$

$y' = k a_k x^{k-1}$

$k a_k x^{k-1} = 2x a_k x^k$

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$k a_k x^{k-1} = 2a_k x^{k+1}$

$k a_k \cancel{x^{k-1}} = 2a_{k-2} \cancel{x^{k-1}}$

$\cancel{a_{k-2} = \frac{k a_k}{2}}$

$a_k = \frac{2}{k} a_{k-2}$

$n=2$ $a_2 = \frac{2}{2} a_0 = a_0$

$n=3$ $a_3 = \frac{2}{3} a_1$

$n=4$ $a_4 = \frac{2}{4} a_2 = \frac{a_0}{2}$

$n=5$ $a_5 = \frac{2}{5} a_3 = \frac{4}{15} a_1$

$n=6$ $a_6 = \frac{2}{6} a_4 = \frac{a_0}{6}$

$n=7$ $a_7 = \frac{2}{7} a_5 = \frac{8}{105} a_1$

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$

$y = a_0\left(1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!}\right) + a_1\left(x + \frac{2x^3}{3} + \frac{4}{15}x^5 + \frac{8}{105}x^7 \dots\right)$

(3) $y'' = y'$

→ $P(x) = 1$ $Q(x) = 0$

$y = a_n x^n$

$y' = n a_n x^{n-1}$

$y'' = n(n-1) a_n x^{n-2}$

$\cancel{n}(n-1) a_n x^{n-2} = \cancel{n} a_n x^{n-1}$

$(n-1) a_n \cancel{x^{n-2}} = a_{n-1} \cancel{x^{n-2}}$

$a_n = \frac{a_{n-1}}{n-1}$

~~$n=1$ $a_1 = \frac{a_0}{0} = \infty$~~

$n=2$ $a_2 = a_1$

$n=3$ $a_3 = \frac{a_2}{2} = \frac{a_1}{2}$

$n=4$ $a_4 = \frac{a_3}{3} = \frac{a_1}{6}$

$n=5$ $a_5 = \frac{a_4}{4} = \frac{a_1}{24}$

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$

$$\boxed{y = a_0 + a_1\left(x + \frac{x^2}{1!} + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \dots\right)}$$

④ $y'' + xy' = 0$

$\rightarrow$ $P(x) = x \quad Q(x) = 0$

$y = a_n x^n$

$y' = n a_n x^{n-1}$

$y'' = n(n-1) a_n x^{n-2}$

$n(n-1) a_n x^{n-2} + x n a_n x^{n-1} = 0$

$n(n-1) a_n x^{n-2} + n a_n x^n = 0$

$\cancel{n}(n-1) a_n x^{n-2} + \cancel{n} a_{n-2} x^{n-2} = 0$

$(n-1) a_n \cancel{x^{n-2}} = -a_{n-2} \cancel{x^{n-2}}$

$a_n = \dfrac{-a_{n-2}}{n-1}$

$n=2 \quad a_2 = \dfrac{-a_0}{1}$

$n=3 \quad a_3 = \dfrac{-a_1}{2}$

$n=4 \quad a_4 = \dfrac{-a_2}{3} = \dfrac{a_0}{3}$

$n=5 \quad a_5 = \dfrac{-a_3}{4} = \dfrac{a_1}{8}$

$n=6 \quad a_6 = \dfrac{-a_4}{5} = \dfrac{-a_0}{15}$

$n=7 \quad a_7 = \dfrac{-a_5}{6} = \dfrac{-a_1}{48}$

$n=8 \quad a_8 = \dfrac{-a_6}{7} = \dfrac{a_0}{105}$

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$

$= a_0\left(1 - x^2 + \dfrac{x^4}{3} - \dfrac{x^6}{15} + \dfrac{x^8}{105}\right) + a_1\left(x - \dfrac{x^3}{2} + \dfrac{x^5}{8} - \dfrac{x^7}{48} \dots\right)$

⑤ $y'' - y = 0$

$\rightarrow$ $P(x) = 0 \quad Q(x) = -1$

$y = a_n x^n$

$y' = n a_n x^{n-1}$

$y'' = n(n-1) a_n x^{n-2}$

$y'' = y$

$n(n-1) a_n x^{n-2} = a_n x^n$

$n(n-1) a_n \cancel{x^{n-2}} = a_{n-2} \cancel{x^{n-2}}$

$$a_n = \frac{a_{n-2}}{n(n-1)}$$

$n=2$ $a_2 = \frac{a_0}{2!}$

$n=3$ $a_3 = \frac{a_1}{3!}$

$n=4$ $a_4 = \frac{a_0}{4(3)} = \frac{a_0}{4!}$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \dots$$

$$y = a_0\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots\right) + a_1\left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots\right)$$

(5) $y'' + 2xy = 0$

$\rightarrow$ $P(x) = 0$ $Q(x) = 2x$

$$y = a_n x^n$$
$$y' = n a_n x^{n-1}$$
$$y'' = n(n-1) a_n x^{n-2}$$

$$y'' = -2xy$$
$$n(n-1) a_n x^{n-2} = -2x\, a_n x^n$$
$$n(n-1) a_n \cancel{x^{n-2}} = -2 \cancel{x}\, a_n x^{n+1}$$
$$n(n-1) a_n \cancel{x^{n-2}} = -2\, a_{n-3} \cancel{x^{n-2}}$$
$$a_n = \frac{-2}{n(n-1)} a_{n-3}$$

$n=3$ $a_3 = \frac{-2}{3\times 2} a_0 = \frac{-a_0}{3}$

$n=4$ $a_4 = \frac{-\cancel{2}}{\cancel{4}^2 \times 3} a_1 = \frac{-a_1}{6}$

$n=5$ $a_5 = \frac{-\cancel{2}}{5\times \cancel{4}2} a_2 = \frac{-a_2}{10}$

$n=6$ $a_6 = \frac{-\cancel{2}}{\cancel{6}^3\times 5} a_3 = \cancel{\frac{-a_4}{15}} = \cancel{\frac{a_1}{15\times 6}}$ 90

$= \frac{-a_3}{15} = \frac{a_0}{45}$

$n=7$ $a_7 = \frac{-2}{7\times 6} a_4 = \frac{a_1}{7\times 3\times 6}$ $\frac{4}{126}$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

$$\boxed{y = a_0\left(1 - \frac{x^3}{3} + \frac{x^6}{45} + \dots\right) + a_1\left(x - \frac{x^4}{6} + \frac{x^7}{126} + \dots\right) + a_2\left(x^2 - \frac{x^5}{10} + \dots\right)}$$

Unit -3

1) Define order and degree for PDE.

→ The order of a PDE is determined by the highest derivative in the eqⁿ.

→ The degree is the Power of highest derivative in the eqⁿ.

Q-2 Find the order and degree.

(1) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

O - 2 D - 1

(2) $\left(\frac{\partial^3 u}{\partial x^3}\right)^2 + \frac{\partial^2 u}{\partial y^2} = 0$

O - 3 D - 2

(3) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 42$

O - 1 D - 2

(4) $\left(\frac{\partial u}{\partial x}\right)^4 = y\left(\frac{\partial u}{\partial y}\right)$

O - 1 D - 4

(5) $\left(\frac{\partial u}{\partial x}\right)^4 + \left(\frac{\partial^2 u}{\partial y^2}\right)^5$

O - 2 D - 5

(6) $\left(\frac{\partial^3 u}{\partial x^3}\right)^2 + \left(\frac{\partial^2 u}{\partial y^2}\right)^5 = 102$

O - 3 D - 2

Q-3 write the follⁿ

(1) one dimensional Heat eqn - $c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$

(2) one dimensional wave eqⁿ - $\frac{\partial^2 u}{\partial t^2} = c \frac{\partial^2 u}{\partial x^2}$

(3) Two dimensional Laplace eqn - $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(4) Two dimensional wave eqⁿ - $\frac{\partial^2 u}{\partial t^2} = c\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2}\right)$

(5) Three dimensional Laplace eqⁿ - $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

Q-4 classify the following PDE.

(1) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ (~~Ellipse~~ Parabola)

(2) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (Elliptic)

(3) $\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 u}{\partial t^2} = 0$ (Hyperbolic)

(4) $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$ (Parabolic)

(5) $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$ (Hyperbolic)

Q-5 form a PDE by eliminating arbitary ~~for~~ funⁿ

(1) $f(xy+z^2,\ x+y+z)=0$

$u = xy+z^2 \qquad v = x+y+z$

$f(u,v)=0$

Diff w.r.t x $= \dfrac{\partial f}{\partial u}(y+2zp) + \dfrac{\partial f}{\partial v}(1+p) = 0$

$$\frac{\partial f}{\partial u}(y+2zp) = -\frac{\partial f}{\partial v}(1+p) \quad \text{—— (1)}$$

Diff w.r.t y $= \dfrac{\partial f}{\partial u}(x+2zq) + \dfrac{\partial f}{\partial v}(1+q) = 0$

$$\frac{\partial f}{\partial u}(x+2zq) = -\frac{\partial f}{\partial v}(1+q) \text{ —— (2)}$$

Dividing (1) and (2), we get

$$\frac{y+2zp}{x+2zq} = \frac{1+p}{1+q}$$

$$y+yq+2zp+\cancel{2zpq} = x+px+2zq+\cancel{2zpq}$$

$$\boxed{(x-2z)p + (y-2z)q = y-x}$$

(2) $f(x+y+z,\ x^2+y^2+z^2)=0$

$f(u,v)=0$

Diffⁿ w.r.t x. $\dfrac{\partial f}{\partial u}(1+p) + \dfrac{\partial f}{\partial v}(2x+2zp) = 0$

$$\frac{\partial f}{\partial u}(1+p) = -\frac{\partial f}{\partial v}(2x+2zp) = 0 \quad \text{—— (1)}$$

Diffⁿ w.r.t y $\dfrac{\partial f}{\partial u}(1+q) + \dfrac{\partial f}{\partial v}(2y+2zq) = 0$

$$\frac{\partial f}{\partial u}(1+q) = -\frac{\partial f}{\partial v}(2y+2zq) = 0 \quad \text{—— (2)}$$

Dividing (1) and (2)

$$\frac{1+p}{1+q} = \frac{x+zp}{y+zq}$$

$y+py+zq+\cancel{zpq} = x+xq+zp+z$

$$\boxed{p(y-z)+q(z-x) = x-y}$$

③ $z = f\left(\frac{x}{y}\right)$

→ Diff wrt x : $p = \frac{\partial z}{\partial x} = f'\left(\frac{x}{y}\right)\frac{1}{y}$ —— ①

Diff wrt y : $q = \frac{\partial z}{\partial y} = f'\left(\frac{x}{y}\right)\frac{-x}{y^2}$ —— ②

Dividing ① and ②

$\frac{p}{q} = \frac{1/y}{-x/y^2} = \frac{-y}{x}$

$\boxed{px + qy = 0}$

④ $z = y + f(x + \ln y)$

$p = f'(x+\ln y)$ $\quad q = 1 + f'(x+\ln y)\cdot\frac{1}{y}$

$\frac{p}{q} = \underline{f'(x+\ln y)}$

⑤ $z = f(x+6y) + g(x-6y)$

→

$p = f'(x+6y) + g'(x-6y)$

$q = f'(x+6y)\cdot 6 - 6g'(x-6y)$

$r = \frac{\partial^2 z}{\partial x^2} = f''(x+6y) + g''(x-6y)$

$s = \frac{\partial^2 z}{\partial y^2} = 36 f''(x+6y) + 36 g''(x+6y)$

$= 36 r$

$s = 36 r$

$$\boxed{\frac{\partial^2 z}{\partial y^2} = 36\frac{\partial^2 z}{\partial x^2}}$$

Q-6

①

→

②

→

③

→

④

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Q-6 Form a PDE by eliminating arbitrary constants

(1) $z = (x-a)^2 + (y-b)^2$

$\rightarrow$ $p = \frac{\partial z}{\partial x} = 2(x-a)$ $\Rightarrow x-a = \frac{p}{2}$

$q = \frac{\partial z}{\partial y} = 2(y-b)$ $\Rightarrow y-b = \frac{q}{2}$

$z = \left(\frac{p}{2}\right)^2 + \left(\frac{q}{2}\right)^2$

$\boxed{4z = p^2 + q^2}$

(2) $z = (x-a)(y-b)$

$\rightarrow$ $p = (y-b)$ $q = (x-a)$

$\boxed{z = pq}$

(3) $z = ax + by$

$\rightarrow$ $p = a$ $q = b$

$\boxed{z = px + qy}$

(4) $z = ax + by + c$

$P = a$ $q = b$

$\boxed{z = px + qy}$

Q-8 Solve the foll^n PDE by direct Integration

(1) $\frac{\partial^2 z}{\partial x \partial y} = 2x - 3y^2$

$\rightarrow$ $\int \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \int (2x - 3y^2)$

$\int \frac{\partial z}{\partial y} = \int (x^2 - 3xy^2 + f(y))$

$z = x^2 y - x^2 y^2 + f'(y) + g(x)$ $= \boxed{x^2 y + xy^2 + \phi(y) + g(x)}$

(2) $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$

$\rightarrow \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \sin x \sin y$

$\int \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \int \sin x \sin y$

$\frac{\partial z}{\partial y} = -\cos x \sin y + f(y)$

$\int \frac{\partial z}{\partial y} = \int(\cos x \sin y + f(y))$

$z = \cos x \cos y + f'(y) + g(x)$

$\boxed{z = \cos x \cos y + \phi(y) + g(x)}$

(3) $\frac{\partial^2 y}{\partial x \partial y} = \cos x \cos y$

$\rightarrow \int \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \int(\cos x \cos y$

$\int \frac{\partial z}{\partial y} = \int(\sin x \cos y + f(y))$

$z = \sin x \sin y + f'(y) + g(x)$

$\boxed{z = \sin x \sin y + \phi(y) + g(x)}$

Q-9 (1) Solve the PDE $p + q^2 = 1$

$P = 1 \quad Q = q \quad R = 1$

AE: $\frac{dx}{1} = \frac{dy}{q} = \frac{dz}{1}$

$\int \frac{dx}{1} = \int \frac{dz}{1}$

$x = z + c_1$

$x - z = c_1$

$\int \frac{dx}{1} = \int \frac{dy}{q}$

$qx = y + c_2$

$c_2 = qx - y$

GS: $\phi(u,v) = \phi(c_1, c_2)$

$\boxed{\phi(x - z, qx - y) = 0}$

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(2) $p^2 + q^2 = npq$

$P = p \quad Q = q \quad R = npq$

AE: $\frac{dx}{p} = \frac{dy}{q} = \frac{dz}{npq}$

$\int \frac{dx}{p} = \int \frac{dy}{q}$

$qx = py + c_1$

$qx - py = c_1$

$\int \frac{dx}{p} = \int \frac{dz}{npq}$

$nqx = z + c_2$

$c_2 = nqx - z$

GS: $\phi(u, v)$

$\Rightarrow \phi(c_1, c_2) = 0$

$\boxed{\phi(qx - py, nqx - z) = 0}$

Section - 2

1) Define Laplace transform

→ If $f(t)$ be a given function that is defined for $\forall t \geq 0$ then the laplace transform of $f(t)$ is denoted by $L\{f(t)\}$, $f(s)$, $F(s)$ and it is defined as

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

2) State the first shifting Property of LT

→ If $L\{f(t)\} = f(s)$ then $L\{e^{at} f(t)\} = F(s-a)$, $s > 0$

3) state the Linearity Property of LT

→ If $L\{f(t)\} = F(s)$ and $L\{g(t)\} = G(s)$ then.

$$L\{af(t) \pm bg(t)\} = aL\{f(t)\} \pm bL\{g(t)\}$$

$$= aF(s) \pm bG(s)$$

4) Prove that if $L\{f(t)\} = F(s)$ then $L\{f'(t)\} = sL\{f(t)\} - f(0)$

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Proof By the def^n of LT, we have

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

$$L\{f'(t)\} = \int_0^\infty \underset{u}{e^{-st}}\, \underset{v}{f'(t)}\, dt$$

$$\int uv\, dx = uv_1 - u'v_2 + u''v_3 + \dots$$

$$L\{f'(t)\} = \left[e^{-st} [f(t)]_0^\infty - \int_0^\infty -se^{-st} f(t) dt \right]$$

$$= \left[(e^{-st} f(t))_0^\infty - s \int_0^\infty e^{-st} f(t)\, dt \right]$$

$$= \left[e^{-\infty} f(\infty) - e^0 f(0) \right] + s\, L\{f(t)\}$$

$$\boxed{L\{f'(t)\} = s L\{f(t)\} - f(0)}$$

5) Write.

1. $L(t^n) = \dfrac{n!}{s^{n+1}} = \dfrac{\Gamma n+1}{s^{n+1}} = \dfrac{n\Gamma n}{s^{n+1}}$
2. $L(t^4) = \dfrac{4!}{s^5} = \dfrac{24}{s^5}$
3. $L(t^3) = \dfrac{3!}{s^4} = \dfrac{6}{s^4}$
4. $L(t^2) = \dfrac{2!}{s^3} = \dfrac{2}{s^3}$
5. $L(\sin 3t) = \dfrac{3}{s^2+9}$
6. $L(\cos 7t) = \dfrac{s}{s^2+49}$
7. $L(\cosh 3t) = \dfrac{s}{s^2-9}$
8. $L(\sinh 4t) = \dfrac{4}{s^2-16}$
9. $L(\cosh 3t) = \dfrac{s}{s^2-9}$
10. $L(\cosh 8t) = \dfrac{s}{s^2-64}$
11. $L(\sinh 4t) = \dfrac{4}{s^2-16}$
12. $L(\cos 4t) = \dfrac{s}{s^2+16}$
13. $L(\sinh 5t) = \dfrac{5}{s^2-25}$
14. $L(\cos 7t) = \dfrac{s}{s^2+49}$
15. $L(e^{2x}) = \dfrac{1}{s-2}$
16. $L(e^{-3x}) = \dfrac{1}{s+3}$
17. $L(e^{4x}) = \dfrac{1}{s-4}$
18. $L(e^{-7x}) = \dfrac{1}{s+7}$

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6) Define Inverse LT

→ If $L\{f(t)\} = F(s)$ then $f(t)$ is called ILT of $F(s)$ and it is denoted by $L^{-1}\{F(s) = f(t)$

7) State Change of Scale Property of ILT

→ If $L^{-1}\{F(s)\} = f(t)$ then $L^{-1}\{F(as)\} = \frac{1}{a} f\left(\frac{t}{a}\right)$

8) State Linearity Property of ILT.

→ If $L^{-1}\{F(s)\} = f(t)$ and $L^{-1}\{G(s)\} = g(t)$ then

$L^{-1}\{aF(s) \pm bG(s)\} = aL^{-1}\{F(s)\} \pm bL^{-1}\{G(s)\}$

$= af(t) \pm bg(t)$

9) Find the values of fou^n.

① $L^{-1}\left(\frac{1}{s^n}\right) = \frac{t^{n-1}}{(n-1)!} = \frac{t^{n-1}}{\Gamma n}$

② $L^{-1}\left\{\frac{1}{s^3}\right) = \frac{t^2}{2!}$

③ $L^{-1}\left(\frac{1}{s^2}\right) = t$

④ $L^{-1}\left(\frac{1}{s-2}\right) = e^{2t}$

⑤ $L^{-1}\left(\frac{1}{s^2+4}\right) = \frac{1}{2} \sin 2t$

⑥ $L^{-1}\left(\frac{1}{s^2+49}\right) = \frac{1}{7} \sin 7t$

11) State convolution theorem

→ If $L^{-1}\{F(s)\} = f(t)$ and $L^{-1}\{G(s)\} = g(t)$ then

$L^{-1}\{F(s)\,G(s)\} = \int_0^t f(u)\, g(t-u)\, du$

$= \int_0^t f(t-u)\, g(u)\, du$

$= f(t) * g(t)$

12) State Convolution.

→ Convolution of a funⁿ $f(t)$ & $g(t)$ is denoted as $f(t) * g(t)$ and it is defined as

$$f(t) * g(t) = \int_0^t f(u)\, g(t-u)\, du$$

$$= \int_0^t f(t-u)\, g(u)\, du$$

$$= g(t) * f(t)$$

13) Find the value of $1*1$ where $*$ denotes convolution Product.

→ Here

$f(t) = 1$ $\quad$ $g(t) = 1$

$f(u) = 1$ $\quad$ $g(u) = 1$.

$f(t-u) = 1$

$$f(t) * g(t) = \int_0^t f(t-u)\, g(u)\, du$$

$$= \int_0^t 1.1\, du = [u]_0^t$$

$$= t - 0$$

$$\boxed{1 * 1 = t}$$

14) State and Prove change of scale Property for ILT

→ If $L^{-1}\{F(s)\} = f(t)$ then $L^{-1}\{F(as)\} = \frac{1}{a} f\left(\frac{t}{a}\right)$.

Proof By the linearity Property of LT, we have

$$L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$$

Taking ILT on both the sides we get.

$$L^{-1}L\{f(at)\} = \frac{1}{a} L^{-1}\left\{F\left(\frac{s}{a}\right)\right\}$$

$$L^{-1}\left\{F\left(\frac{s}{a}\right)\right\} = a\, f(at)\}$$

$$L^{-1}\left\{F\left(\frac{a^2 s}{a}\right)\right\} = \frac{a}{a^2} f\left(\frac{at}{a^2}\right) \Rightarrow L^{-1}\{F(as)\} = \frac{1}{a} f\left(\frac{t}{a}\right)$$

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15) State and Prove linearity Property of ILT
→ If $L^{-1}\{F(s)\} = f(t)$ and $L^{-1}\{G(s)\} = g(t)$ then
$L^{-1}\{aF(s) \pm bG(s)\} = aL^{-1}\{F(s)\} \pm bL^{-1}\{G(s)\}$
$= af(t) \pm bg(t)$

Proof
→ From the linearity Property of LT, we have
$L\{af(t) \pm bg(t)\} = aL\{f(t)\} \pm bL\{g(t)\}$
$= aF(s) \pm bG(s)$

→ Taking ILT on both the sides. then
$LL^{-1}\{af(t) \pm bg(t)\} = L^{-1}\{aF(s) \pm bG(s)\}$

$\boxed{af(t) \pm bg(t) = L^{-1}\{aF(s) \pm bG(s)\}}$

16) State and Prove second Shifting Property of ILT
→ If $L\{f(t)\} = F(s)$ then $L\{f(t-a)u(t-a)\} = e^{-as}F(s)$
$= e^{-as}L\{f(t)\}$

→ If $L^{-1}\{F(s)\} = f(t)$ then $L^{-1}\{e^{-as}F(s)\} = L\{f(t-a)u(t-a)\}$

17) Find the Value of $foll^{n}$.

① $L(e^{at}t\sin at)$
→ By $diff^{n}$ by s Property
$L\{t\sin at\} = -\frac{d}{ds}L\{\sin at\}$

$= \frac{-d}{ds}\left[\frac{a}{s^2+a^2}\right]$

$= +\frac{2as}{(s^2+a^2)^2}$

By First shifting theorem
$L\{e^{at}t\sin at\} = \frac{2a(s-a)}{((s-a)^2+a^2)^2} = \frac{2a(s-a)}{(s^2-2as+a^2+a^2)^2} = \boxed{\frac{2a(s-a)}{(s^2-2as+2a^2)^2}}$

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② $L\{t^2 \sinh at\}$

→ By diff^n by s Property

$\therefore L\{t^2 \sinh at\} = (-1)^2 \frac{d^2}{ds^2} L\{\sinh at\}$

$= \frac{d^2}{ds^2}\left[\frac{a}{s^2-a^2}\right]$

$= \frac{d}{ds}\left[\frac{a(-2s)}{(s^2-a^2)^2}\right]$

$= -2a \frac{d}{ds}\left[\frac{s}{(s^2-a^2)^2}\right]$

$= -2a\left[\frac{(s^2-a^2)^2 - 2(2s)(s^2-a^2)s}{(s^2-a^2)^4}\right]$

$= -2a\left[(s^2-a^2)\right]\left[\frac{s^2-a^2-4s^2}{(s^2-a^2)^4}\right]$

$= \frac{-2a(-a^2-3s^2)}{(s^2-a^2)^3}$

$= \boxed{\frac{a(3s^2+a^2)}{(s^2-a^2)^3}}$

③ $L(\sin at + at\cos at)$

→ By linearity property

$L[\sin at + at\cos at) = L(\sin at) + a\,L(t\cos at)$

$L(\sin at) = \frac{a}{s^2+a^2}$

$L(t\cos at) = \frac{-d}{ds} L\{\cos at\}$ (∵ By diff^n by s Propert

$= \frac{-d}{ds}\left(\frac{s}{s^2+a^2}\right)$

$= -\left[\frac{(s^2+a^2) - s(2s)(\cancel{s^2+a^2})}{(s^2+a^2)^2}\right]$

$= -\left[\frac{s^2+a^2-2s}{(s^2+a^2)^2}\right] = -\left(\frac{s^2+a^2}{(s^2+a^2)^2} - \frac{2s}{(s^2+a^2)^2}\right]$

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b. $L(t\cos at) = \frac{-1}{s^2+a^2} + \frac{2s}{(s^2+a^2)^2}$

$L(\sin at + at\cos at) = L(\sin at) + a\,L(t\cos at)$

$= \cancel{\frac{a}{s^2+a^2}} \cancel{-\frac{a}{s^2+a^2}} + \frac{2as}{(s^2+a^2)^2}$

$= \boxed{\frac{2as}{(s^2+a^2)^2}}$

(2) $L(t^2\sin^2 4t)$

→ By diff^n by s Property.

$L(t^2\sin^2 4t) = (-1)^2 \frac{d^2}{ds^2} L(\sin^2 4t)$

$L(\sin^2 4t) = L\left(\frac{1-\cos 8t}{2}\right)$

$= \frac{1}{2}\{L(1) - L(\cos 8t)\}$

$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+64}\right]$

$L(t^2\sin^2 4t) = \frac{1}{2}\frac{d^2}{ds^2}\left(\frac{1}{s} - \frac{s}{s^2+64}\right)$

$= \frac{1}{2}\left[\frac{d}{ds}\left(\frac{-1}{s^2} - \left(\frac{(s^2+64) - s(2s)}{(s^2+64)^2}\right)\right.\right.$

$= \frac{1}{2}\left[\frac{d}{ds}\left(\frac{-1}{s^2} - \frac{1}{s^2+64} + \frac{2s^2}{(s^2+64)^2}\right]\right.$

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$= \frac{1}{2}\left[\frac{2}{s^3} + \frac{2s}{(s^2+64)^2} + 2\left[\frac{2s(s^2+64)^2 - 2s^2(2s)(s^2+64)}{(s^2+64)^4}\right]\right.$

$= \frac{2}{2}\left[\frac{1}{s^3} + \frac{s}{(s^2+64)^2} + \left(\frac{\cancel{(s^2+64)}\left(2s(s^2+64) - 4s^3\right)}{(s^2+64)^{\cancel{4}}}\right)\right]$

$= \left(\frac{1}{s^3} + \frac{s}{(s^2+64)^2} + \left(\frac{2s^3 + 128s - 4s^3}{(s^2+64)^3}\right)\right.$

$$= \boxed{\frac{1}{s^3} + \frac{s}{(s^2+64)^2} + \frac{128s - 2s^3}{(s^2+64)^3}}$$

(5) $L\left(\frac{e^{-at} - e^{-bt}}{t}\right)$

→ By linearity property

$$L\left(\frac{e^{-at}}{t} - \frac{e^{-bt}}{t}\right) = L\left(\frac{e^{-at}}{t}\right) - L\left(\frac{e^{-bt}}{t}\right)$$

By divided by t Property

$$L\left(\frac{e^{-at}}{t}\right) - L\left(\frac{e^{-bt}}{t}\right) = \int_s^\infty L(e^{-at})\,ds - \int_s^\infty L(e^{-bt})\,ds$$

$$= \int_s^\infty \frac{1}{s+a}\,ds - \int_s^\infty \frac{1}{s+b}\,ds$$

$$= [\log(s+a)]_s^\infty - [\log(s+b)]_s^\infty$$

$$= \infty - \log(s+a) - \infty + \log(s+b)$$

$$= \boxed{\log\left(\frac{s+b}{s+a}\right)}$$

(6) $L\left(\frac{\sin\omega t}{t}\right)$

→ By divided by t Property.

$$L\left(\frac{\sin\omega t}{t}\right) = \int_s^\infty L(\sin\omega t)\,ds$$

$$= \int_s^\infty \frac{\omega}{s^2+\omega^2}\,ds$$

$$= \omega \int_s^\infty \frac{1}{s^2+\omega^2}\,ds$$

$$= \frac{\omega}{\omega}\left[\tan^{-1}\left(\frac{s}{\omega}\right)\right]_s^\infty$$

$$= \tan^{-1}\infty - \tan^{-1}\frac{s}{\omega}$$

$$= \pi/2 - \tan^{-1} s/\omega = \boxed{\cot^{-1}\frac{s}{\omega}}$$

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(7) $L\left(\int_0^t t^4 + \sin 3t\right)$

$\rightarrow L\left(\int_0^t t^4\right) + L\left(\int_0^t \sin 3t\right)$

By divided by s property

$= \frac{1}{s} L\{t^4\} + \frac{1}{s} L\{\sin 3t\}$

$= \frac{1}{s} \frac{4!}{s^5} + \frac{1}{s} \frac{3}{s^2+9}$

$= \boxed{\frac{1}{s}\left[\frac{24}{s^5} + \frac{3}{s^2+9}\right]}$

(8) $L(e^{3t} t)$

$\rightarrow$ By multipl^n by t^n property

$L(e^{3t} t) = (-1) \frac{d}{ds} L\{e^{3t}\}$

$= (-1) \frac{d}{ds} \frac{1}{s-3}$

$= (-1) \frac{(-1)}{(s-3)^2}$

$\boxed{L(e^{3t} t) = \frac{1}{(s-3)^2}}$

(9) $L($~~tsin3t~~$)(t\sin 3t)$

$\rightarrow$ By mul^n by t^n property

$L(t\sin 3t) = \frac{-d}{ds} L\{\sin 3t\}$

$= \frac{-d}{ds}\left(\frac{3}{s^2+9}\right)$

$= -3 \frac{d}{ds} (s^2+9)^{-1}$

$= \frac{-3(-2s)}{(s^2+9)^2} = \boxed{\frac{6s}{(s^2+9)^2}}$

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(10) $L(t\cos 5t)$

→ By mult^n by t^n Property.

$$L(t\cos 5t) = \frac{-d}{ds} L\{\cos 5t\}$$

$$= \frac{-d}{ds}\left(\frac{s}{s^2+25}\right)$$

$$= -\left[\frac{(s^2+25) - s(2s)}{(s^2+25)^2}\right]$$

$$= -\left[\frac{s^2+25-2s^2}{(s^2+25)^2}\right]$$

$$= \boxed{\frac{s^2-25}{(s^2+25)^2}}$$

(11) $L\left(\frac{1-\cos 2t}{t}\right)$

→ By divided by t Property

$$L\left(\frac{1-\cos 2t}{t}\right) = \int_s^\infty L(1-\cos 2t)\,ds$$

$$= \int_s^\infty L(1) - L(\cos 2t)\,ds$$

$$= \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+4}\right) ds$$

$$\int_s^\infty \frac{1}{s} = [\log s]_s^\infty = \log\infty - \log s$$
$$= -\log s$$

$$\int_s^\infty \frac{s}{s^2+4} \quad \text{let } s^2+4 = t$$
$$2s\,ds = dt$$
$$s\,ds = dt/2$$

$s \to s \quad s \to \infty$

$t \to s^2+4 \quad t \to \infty$

$$= \int_{s^2+4}^\infty \frac{dt}{2t}$$

$$= \frac{1}{2}[\log t]_{s^2+4}^\infty = -\frac{1}{2}\log(s^2+4)$$

$$L\left(\frac{1-\cos 2t}{t}\right) = -\log s + \frac{1}{2}\log(s^2+4)$$

$$= \boxed{\log\left(\frac{(s^2+4)^{1/2}}{s}\right)}$$

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12) $L[e^{-3t}(\cos 4t + 3\sin 4t + 2t^5)]$

$\rightarrow$ $L(e^{-3t}\cos 4t) + 3L(e^{-3t}\sin 4t) + 2L(e^{-3t}t^5)$

$= \left(\frac{s}{s^2+16}\right)_{s\to s+3} + 3\left(\frac{4}{s^2+16}\right)_{s\to s+3} + 2\left(\frac{5!}{s^6}\right)_{s\to s+3}$

$= \left(\frac{s+3}{(s+3)^2+16}\right) + 3\left(\frac{4}{(s+3)^2+16}\right) + 2\left(\frac{120}{(s+3)^6}\right)$

$= \boxed{\frac{s+15}{(s+3)^2+16} + \frac{240}{(s+3)^6}}$

13) $L(\sin^2 kt)$

$\rightarrow$ ~~$L(\sin^2 kt) = L\left(\frac{1-\cos 2kt}{2}\right)$~~

~~$= \frac{1}{2}[L(1) - L(\cos 2kt)]$~~

~~$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4k^2}\right]$~~

$\rightarrow$ By Mul^n by s Property

$L\{f''(t)\} = s^2 L\{f(t)\} - sf(0) - f'(0)$

$f(t) = \sin^2 kt \qquad f(0) = 0$

$f'(t) = 2\sin kt \cos kt = \sin 2kt \quad f'(0) = 0$

$f''(t) = 2\cos kt$

~~2cos~~ $L\{2\cos kt\} = s^2 L\{\sin^2 kt\} - 0 - 0$

$\frac{\cancel{s}\,2}{s^2+k^2} = \cancel{s^2} L\{\sin^2 kt\}$

$\boxed{L(\sin^2 kt) = \frac{2}{s(s^2+k^2)}}$

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$L(4e^{5t} + 6t^3 - 3\sin 4t + 2\cos 2t)$

$4L(e^{5t}) + 6L(t^3) - 3L(\sin 4t) + 2L(\cos 2t)$

$4\frac{1}{s-5} + 6\left(\frac{3!}{s^4}\right) - \frac{3\times 4}{s^2+16} + \frac{2s}{s^2+4}$

$= \boxed{\frac{4}{s-5} + \frac{36}{s^4} - \frac{12}{s^2+16} + \frac{2s}{s^2+4}}$

$L(t\,u(t-a))$

By Corollary 1, we have

$L\{f(t)\,u(t-a)\} = e^{-as} L\{f(t+a)\}$

$f(t) = t$

$f(t+a) = t+a$

$L\{f(t)\,u(t-a)\} = e^{-as} L(t+a)$

$= e^{-as}[L\{t\} + aL\{1\}]$

$= e^{-as}\left(\frac{1}{s^2} + \frac{a}{s}\right)$

$= \boxed{\frac{e^{-as}(1+as)}{s^2}}$

$L(e^{-3t}u(t-2))$

By corollary 1, $L\{f(t)u(t-a)\} = e^{-as}L\{f(t+a)\}$

$f(t) = e^{-3t}$ $\quad a = 2$

$f(t+2) = e^{-3(t+2)} = e^{-3t-6}$

$L\{f(t)\,u(t-a)\} = e^{-2s} L\{e^{-3t-6}\}$ $\quad e^{-3t}.e^{-6}$

$= e^{-2s}\frac{e^{-6}}{s+3}$

$= \boxed{\frac{e^{-2s-6}}{s+3}}$

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Q. 18 Find the values of foll^n.

(1) $L^{-1}\left(\frac{5}{(s+2)^5}\right)$

$= 10\, e^{2t}\, \frac{t^4}{4!}$

$= \frac{10^5}{4\times3\times2}\, e^{2t} t^4 = \boxed{\frac{5}{12} e^{2t} t^4}$

(2) $L^{-1}\left(\frac{s+1}{s^2-6s+25}\right)$

$= L^{-1}\left(\frac{s+1}{s^2-6s+9+16}\right)$

$= L^{-1}\left(\frac{s-3+4}{(s-3)^2+16}\right)$

$= L^{-1}\left(\frac{s-3}{(s-3)^2+16}\right) + 4\, L^{-1}\left(\frac{\cancel{s-3+16}\ 1}{(s-3)^2+16}\right)$

$= e^{3t}\cos 4t + \frac{4}{4} e^{3t}\sin 4t)$

$= \boxed{e^{3t}(\cos 4t + \sin 4t)}$

(3) $L^{-1}\left(\frac{2}{s^2-4}\right)$

$\rightarrow \frac{2}{2}\sinh 2t = \boxed{\sinh 2t}$

(4) $L^{-1}\left(\log\left(\frac{s+a}{s+b}\right)\right)$

$F(s) = \log\left(\frac{s+a}{s+b}\right) = \log(s+a) - \log(s+b)$

$L^{-1}\left(\frac{d}{ds}F(s)\right) = L^{-1}\left(\frac{d}{ds}(\log(s+a) - \log(s+b))\right)$

$= L^{-1}\left(\frac{1}{s+a} - \frac{1}{s+b}\right)$

$$= e^{-at} - e^{-bt}$$

$$f(t) = \frac{-1}{t} L^{-1}\left(-\frac{d}{ds} F(s)\right)$$

$$= \frac{-1}{t}\left(e^{-at} - e^{-bt}\right) = \boxed{\frac{e^{-bt} - e^{-at}}{t}}$$

(5) $L^{-1}\left[\frac{1}{s(s+a)^3}\right]$

→ By division by s Property $L^{-1}\left(\frac{F(s)}{s}\right) = \int_0^t f(t)\,dt$

$$f(s) = \frac{1}{(s+a)^3}$$

$$L^{-1}(f(s)) = e^{-at}\frac{t^2}{2!} = \frac{1}{2} e^{-at} t^2 = f(t)$$

$$\int_0^t f(t)\,dt = \frac{1}{2}\int_0^t e^{-at} t^2$$

$$= \frac{1}{2}\left[t^2 \frac{e^{-at}}{-a} - 2t\frac{e^{-at}}{a^2} + 2\frac{e^{-at}}{-a^3}\right]_0^t$$

$$= \boxed{\frac{1}{2}\left[-\frac{t^2}{a} e^{-at} - \frac{2t}{a} e^{-at} - \frac{2e^{-at}}{a^3} + \frac{2}{a^3}\right]}$$

(6) $L^{-1}\left[\log\left(1 + \frac{w^2}{s^2}\right)\right] = L^{-1}\left[\log\left(\frac{s^2+w^2}{s^2}\right)\right]$

→

$$= \log(s^2+w^2) - \log s^2$$

$$\log(s^2+w^2) - 2\log s$$

$$\Rightarrow \frac{d}{ds} F(s) = \frac{2s}{s^2+w^2} - \frac{2}{s}$$

$$L^{-1}\left(\frac{d}{ds} F(s)\right) = L^{-1}\left(\frac{2s}{s^2+w^2}\right) - 2L^{-1}\left(\frac{1}{s}\right)$$

$$= 2\cos wt - 2$$

$$F(t) = \frac{-1}{t} L^{-1}\left(\frac{d}{ds} F(s)\right) = \boxed{\frac{-2}{t}(\cos wt - 1)}$$

(7) $L^{-1}\left(\frac{3s-12}{s^2+8}\right)$

$3L^{-1}\left(\frac{s}{s^2+8}\right) \cancel{+} -12\,L^{-1}\left(\frac{1}{s^2+8}\right)$

$2\sqrt{2}$ $= 3\cos 2\sqrt{2}t - \frac{\cancel{12}^{6}}{2\sqrt{2}}\sin 2\sqrt{2}t$

$= 3\cos 2\sqrt{2}t - 3\sqrt{2}\sin 2\sqrt{2}t$

$= \boxed{3\left(\cos 2\sqrt{2}t - \sqrt{2}\sin 2\sqrt{2}t\right)}$

(8) $L^{-1}\left(\frac{e^{-\frac{\pi s}{2}} + e^{-\frac{3\pi s}{2}}}{(s^2+1)}\right)$

A-20 Solve by method of Partial fun^n.

1) $L^{-1}\left(\frac{5s+3}{(s-1)(s^2-2s+5)}\right)$

$\rightarrow \frac{5s+3}{(s-1)(s^2-2s+5)} = \frac{A}{s-1} + \frac{Bs+C}{s^2-2s+5}$

$5s+3 = A(s^2-2s+5) + (Bs+C)(s-1)$

$5s+3 = s^2(A+B) + s(-2A-B+C) + (5A-C)$

$A+B=0$
$A=-B$
$B=-A$

$-2A-B+C=5$
$-2A-A+C=5$

$5A - \cancel{C} = 3$
$-A + \cancel{C} = 5$
$4A = 8$
$\boxed{A=2}$
$\boxed{B=-2}$

$-A+C=5$
$-2+C=5$
$\boxed{C=7}$

$L^{-1}\left(\frac{5s+3}{(s-1)(s^2-2s+5)}\right) = L^{-1}\left(\frac{2}{s-1} + \frac{-2s+7}{s^2-2s+5}\right)$

$= L^{-1}\left(\frac{2}{s-1} - 2\frac{(s-2)+2}{(s-2)^2+1^2} + 7\frac{1}{(s-2)^2+1^2}\right)$

$= 2e^t - 2e^{2t}\cos t - 4e^{2t}\sin t + 7e^{2t}\sin t$

$= 2e^t - 2e^{2t}\cos t + 3e^{2t}\sin t$

$= \boxed{2e^t + e^{2t}(3\sin t - 2\cos t)}$

2) $L^{-1}\left(\frac{s+3}{(s^2+6s+13)^2}\right)$

$\rightarrow$

$\frac{s+3}{(s^2+6s+13)^2} = \frac{As+B}{(s^2+6s+13)} + \frac{Cs+D}{(s^2+6s+13)^2}$

$(s+3) = (As+B)(s^2+6s+13) + Cs+D$

$s+3 = s^3(A) + s^2(6A+B) + s(13A+6B+C) + (13B+D)$

$\boxed{A=0}$

$6A+B=0$
$\boxed{B=0}$

$13A+6B+C=1$
$\boxed{C=1}$

$13B+D=0$
$D=3$

(4)

$$L^{-1}\left(\frac{s+3}{(s^2+6s+13)^2}\right) = L^{-1}\left(\frac{s+3}{[(s+3)^2+4]^2}\right)$$

$$= e^{-3t}\, L^{-1}\left(\frac{s}{(s^2+2^2)^2}\right)$$

$$= e^{-3t}\,\frac{1}{2(2)}\, t\sin 2t$$

$$= \boxed{\frac{t}{4}e^{-3t}\sin 2t}$$

(5) $L^{-1}\left(\dfrac{s}{(s-1)(s^2+2s+2)}\right)$

$$\to \frac{s}{(s-1)(s^2+2s+2)} = \frac{A}{s-1} + \frac{Bs+C}{s^2+2s+2}$$

$$s = A(s^2+2s+2) + (Bs+C)(s-1)$$

$$s = s^2(A+B) + s(2A-B+C) + (2A-C)$$

$A+B=0$ $\quad$ $2A-B+C=1$ $\quad$ $2A-C=0$

$B=-A$ $\quad$ $3A+C=1$ $\quad$ $3A+C=1$

$5A=1$

$\boxed{A=1/5}$ $\quad$ $\boxed{B=-1/5}$

$2A-C=0$ $\quad$ ~~$3A+C=0$~~

$2A=C$ $\quad$ ~~$C=-3A=-\frac{3}{5}$~~

$C=\frac{2}{5}$ $\quad$ $\boxed{\cancel{C=-\frac{3}{5}}}$

$$\frac{s}{(s-1)(s^2+2s+2)} = \frac{1}{5(s-1)} - \frac{1}{5}\,\frac{(s-2)}{s^2+2s+2}$$

$$L^{-1}[F(s)] = \frac{1}{5}\left[L^{-1}\left(\frac{1}{s-1}\right) - L^{-1}\left(\frac{s+1}{(s+1)^2+1^2}\right) + 3L^{-1}\left(\frac{1}{(s+1)^2+1^2}\right)\right]$$

$$= \frac{1}{5}\left[e^{t} - e^{-t}\cos t + 3e^{-t}\sin t\right]$$

$$= \frac{1}{5}e^{t} - \frac{1}{5}e^{-t}(\cos t - 3\sin t)$$

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(4) $L^{-1}\left(\frac{s+3}{(s^2+6s+13)^2}\right)$

$\rightarrow L^{-1}\left(\frac{s+3}{((s+3)^2+2^2)^2}\right)$

$e^{-3t} L^{-1}\left(\frac{s}{(s^2+2^2)^2}\right)$

$= \boxed{e^{-3t}\frac{t}{4}\sin 2t}$

same as (2)

(5) $L^{-1}\left(\frac{4s+5}{(s-1)^2(s+2)}\right)$

$\rightarrow$

$$\frac{4s+5}{(s-1)^2(s+2)} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s+2}$$

$$4s+5 = A(s-1)(s+2) + B(s+2) + C(s-1)^2$$

Let $s=1$

$4(1)+5 = B(3)$

$9 = 3B$

$\boxed{B=3}$

Let $s=-2$

$4(-2)+5 = C(3)^2$

$-8+5 = 9C$

$\boxed{C=-1/3}$

Let $s=0$

$5 = A(-1)(2) + 3(2) - \frac{1}{3}(1)$

$5 = -2A + 6 - \frac{1}{3}$

$2A = 1 - \frac{1}{3} = \frac{2}{3}$

$\boxed{A=\frac{1}{3}}$

$$L^{-1}\left(\frac{4s+5}{(s-1)^2(s+2)}\right) = L^{-1}\left(\frac{1}{3(s-1)} + \frac{3}{(s-1)^2} - \frac{1}{3(s+2)}\right)$$

$$= \cancel{\frac{e^t}{3} + 3e^t t - \frac{e^{-2t}}{3}} = \boxed{\frac{e^t}{3} + 3e^t t - \frac{e^{-2t}}{3}}$$

$$\boxed{\cancel{\frac{1}{3}(3e^t + 3e^t t + e^{-2t})}}$$

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(6) $L^{-1}\left(\frac{6}{(s+2)(s-4)}\right)$

$\rightarrow$ $\frac{6}{(s+2)(s-4)} = \frac{A}{s+2} + \frac{B}{s-4}$

$6 = A(s-4) + B(s+2)$

$6 = s(A+B)(-4A+2B)$

$A+B=0$ $\quad\quad$ $-4A+2B=6$

$B=-A$ $\quad\quad$ $\cancel{4A}+4B=0$

$6B=6$

$\boxed{B=1}$

$\boxed{A=-1}$

$L^{-1}\left(\frac{6}{(s+2)(s-4)}\right) = L^{-1}\left(\frac{-1}{s+2} + \frac{1}{s-4}\right)$

$= \boxed{-e^{-2t} + e^{4t}}$

Q-21 Use Convolution theorem.

(1) $L^{-1}\left[\frac{1}{s^2(s-1)}\right]$

$\rightarrow$ $F(s) = \frac{1}{s^2}$ $\quad\quad$ $G(s) = \frac{1}{s-1}$

$f(t) = L^{-1}(F(s))$ $\quad\quad$ $g(t) = L^{-1}\left(\frac{1}{s-1}\right)$

$= t$ $\quad\quad$ $= e^t$

$f(t-u) = t-u$ $\quad\quad$ $g(u) = e^u$

$f(t) * g(t) = \int_0^t f(t-u)\, g(u)\, du$

$= \int_0^t \underset{u}{(t-u)}\, \underset{v}{e^u}\, du$

$= \left[(t-u)e^u - (-1)e^u\right]_0^t$

$= [0 + e^t - (te^0 + e^0)]$
$= \boxed{e^t - t - 1}$

② $t * e^t$

→ $f(t) = t \qquad g(t) = e^t$
$f(t-u) = t-u \qquad g(u) = e^u$

same as above ①

③ $1 * 1$
→ $f(t) = 1 \qquad g(t) = 1$
$f(t-u) = 1 \qquad g(u) = 1$

$f(t) * g(t) = \int_0^t f(t-u)\, g(u)\, du$

$= \int_0^t 1 \cdot 1\, du$

$= [u]_0^t = \boxed{t}$

Q22 Solve initial value Problems using LT

① $y'' + 3y' + 2y = e^t \; ; \; y(0) = 1 \quad y'(0) = 0$

→

$L(y') = s\bar{y} - y(0)$
$L(y'') = s^2\bar{y} - sy(0) - y'(0)$

Taking LT on both sides we get
$L(y'') + 3L(y') + 2L(y) = L(e^t)$
$s^2\bar{y} - sy(0) - \underline{y(0)} + 3s\bar{y} - 3y(0) + 2\bar{y} = \frac{1}{s-1}$

$s^2\bar{y} - s + 3s\bar{y} - 3 + 2\bar{y} = \frac{1}{s-1}$

$$s^2\bar{y} + 3s\bar{y} + 2\bar{y} - (s+3) = \frac{1}{s-1}$$

$$\bar{y}(s^2+3s+2) = \frac{1}{s-1} + s+3$$

$$= \frac{1+s^2+2s\mp 3}{s-1} = \frac{s^2+2s-2}{s-1}$$

$$\bar{y} = \frac{s^2+2s-2}{(s-1)(s+1)(s+2)}$$

$$\frac{s^2+2s-2}{(s-1)(s+1)(s+2)} = \frac{A}{s-1} + \frac{B}{s+1} + \frac{C}{s+2}$$

$$s^2+2s-2 = A(s+1)(s+2) + B(s-1)(s+2) + C(s-1)(s+1)$$

let $s=1$	let $s=-1$	let $s=-2$
$1+2-2 = A(2)(3)$	$1-2-2 = B(-2)(1)$	$4-4-2 = C(-3)(-1)$
$\boxed{A = \frac{1}{6}}$	$\boxed{B = \frac{3}{2}}$	$\boxed{C = \frac{-2}{3}}$

$$\bar{y} = \frac{1}{6(s-1)} + \frac{3}{2(s+1)} - \frac{2}{3(s+2)}$$

Taking ILT on both the sides

$$L^{-1}(\bar{y}) = \frac{1}{6}L^{-1}\frac{1}{s-1} + \frac{3}{2}L^{-1}\frac{1}{s+1} - \frac{2}{3}L^{-1}\left(\frac{1}{s+2}\right)$$

$$= \frac{1}{6}e^{t} + \frac{3}{2}e^{-t} - \frac{2}{3}e^{-2t}$$

$$\boxed{y = \frac{1}{6}\left(e^{t} + 9e^{-t} - 4e^{-2t}\right)}$$

(2) $y'' + a^2y = K\sin at$

$$L(y') = s\bar{y} - f(0)$$

$$L(y'') = s^2\bar{y} - sf(0) - f'(0)$$

Taking LT on both the sides

$L(y'') + a^2 L(y) = k\, L(\sin at)$

$s^2\bar{y} - s f(0) - f'(0) + a^2\bar{y} = \frac{ka}{s^2+a^2}$

Let $y(0) = A$ $\quad y'(0) = B$

$\bar{y}(s^2+a^2) - sA - B = \frac{ka}{s^2+a^2}$

$\bar{y} = \frac{ka}{(s^2+a^2)^2} + \frac{sA+B}{(s^2+a^2)}$

Taking ILT on both sides

$L^{-1}(\bar{y}) = L^{-1}\left\{\frac{ka}{(s^2+a^2)^2}\right\} + A\,L^{-1}\left(\frac{s}{s^2+a^2}\right) + B\,L^{-1}\left(\frac{1}{s^2+a^2}\right)$

$= \frac{ka}{2a^3}(\sin at - at\cos at) + A\cos at + \frac{B}{a}\sin at$

$$\boxed{y = \cos at\left(A - \frac{kt}{2a}\right) + \sin at\left(\frac{B}{a} + \frac{k}{2a^2}\right)}$$

(3) $\frac{d^2y}{dt^2} + 4y = 0$, $\quad y(0) = 1 \quad y'(0) = 6$

→ Taking LT on both the sides

$L(y'') + 4L(y) = 0$

$s^2\bar{y} - s f(0) - f'(0) + 4\bar{y} = 0$

$\bar{y}(s^2+4) - s(1) - 6 = 0$

$\bar{y} = \frac{s+6}{s^2+4}$

Taking ILT on both sides

$L^{-1}(\bar{y}) = L^{-1}\left(\frac{s}{s^2+4}\right) + 6\,L^{-1}\left(\frac{1}{s^2+4}\right)$

$$\boxed{y = \cos 2t + 3\sin 2t}$$

4 $y'' - 2y' - 8y = 0$, $y(0) = 3$, $y'(0) = 6$

Taking LT on both sides.

$L(y'') - 2L(y') - 8L(y) = 0$

$s^2\bar{y} - s y(0) - y'(0) - 2s\bar{y} + 2y(0) - 8\bar{y} = 0$

$\bar{y}(s^2 - 2s - 8) - s(3) - \cancel{6} + \cancel{2(3)} = 0$

$\bar{y}(s^2 - 2s + 8) = 3s \cancel{+18}$

$\bar{y} = \dfrac{3s + \cancel{18}}{s^2 - 2s + 8} = \dfrac{3s}{(s-4)(s+2)}$

Taking ILT on both the sides.

$L^{-1}(\bar{y}) = 3L^{-1}\left(\dfrac{s-2}{(s-2)^2 + 2^2}\right) + 6L^{-1}\left(\dfrac{\cancel{s-}1}{(s-2)^2 + 2^2}\right) + 18\left(\dfrac{1}{(s-2)^2 + 2^2}\right)$

$= 3L^{-1}\left(\dfrac{s-2}{(s-2)^2 + 2^2}\right) + 24\, L^{-1}\left(\dfrac{1}{(s-2)^2 + 2^2}\right)$

$= 3e^{2t}\cos 2t + \dfrac{24}{2} e^{2t} \sin 2t$

$= e^{2t}(3\cos 2t + 12 \sin 2t)$

$= \boxed{3e^{2t}(\cos 2t + 4\sin 2t)}$

$\dfrac{3s}{(s-4)(s+2)} = \dfrac{A}{s-4} + \dfrac{B}{s+2}$

$3s = A(s+2) + B(s-4)$

Let $s = 4$

$12 = A(6)$

$\boxed{A = 2}$

Let $s = -2$

$-6 = B(-6)$

$\boxed{B = 1}$

$\bar{y} = \frac{2}{s-4} + \frac{1}{s+2}$

taking ILT on both the sidy

$L^{-1}(\bar{y}) = 2L^{-1}\left(\frac{1}{s-4}\right) + L^{-1}\left(\frac{1}{s+2}\right)$

$\boxed{y = 2e^{4t} + e^{-2t}}$

(5) $y'' + 2y' + y = 6te^{-t}$

taking LT on both Sides , $y(0) = y'(0) = 0$

$s^2\bar{y} - sy(0) - y'(0) + 2s\bar{y} - y(0) + \bar{y} = 6L^{-1}(te^{-t})$

$\bar{y}(s^2+2s+1) = 6L^{-1}(te^{-t})$

$L^{-1}(te^{-t}) = 6\left[\frac{-d}{ds}F(s)\right]$

$= 6\left[\frac{-d}{ds}L(e^{-t})\right]$

$= 6\left[\frac{-d}{ds}\frac{1}{s+1}\right]$

$= \frac{6}{(s+1)^2}$

$\bar{y} = \frac{6}{(s+1)^2(s+1)^2} = \frac{6}{(s+1)^4}$

Taking ILT on both Sidy

$L^{-1}(\bar{y}) = 6L^{-1}\left(\frac{1}{(s+1)^4}\right)$

$= 6e^{-t}\frac{t^3}{3!}$

$\boxed{y = e^{-t}t^3}$