ODE

1) Define order and degree of ODE

→ order - The order is the highest numbered derivative in the equation

→ Degree - Degree is the power of highest derivative in the eqn.

2) Find order & degree

① $\left(\frac{d^4x}{dt^4}\right)^6 + \left(\frac{d^5x}{dt^5}\right)^3 = m^4x$

→ order = 4 degree = 6

② $\frac{dy}{dx} + y = \sin x$

→ order = 1 degree = 1

③ $\left(\frac{d^2x}{dt^2}\right) + \left(\frac{dx}{dt}\right)^3 = \sin x$

→ O = 2 D = 1

④ Find Degree of $\left(\frac{d^3y}{dx^3}\right)^3 + \left(\frac{d^2y}{dx^2}\right)^5 + y = 0$

→ degree = 3

⑤ " " " $\left(\frac{d^3y}{dx^3}\right)^6 + \left(\frac{d^2y}{dx^2}\right)^5 + y = 0$

→ Degree = 6

Q-3 Find CF for all the cases.

① Find y_c if $m = 2, 2$

$\boxed{y_c = (c_1 + c_2x)e^{2x}}$

② Find y_c if $m = 1$

$\boxed{y_c = c_1e^x}$

③ Find y_c for $(D^2 - 2D + 1)\,y = \cos 3x$

→ $(m-1)^2 = 0$

$m = 1, 1$

$\boxed{y_c = (c_1 + c_2x)e^x}$

④ Find y_c if $m = 1, 2, 3$ ⇒ $\boxed{y_c = c_1e^x + c_2e^{2x} + c_3e^{3x}}$

⑤ if $m = 7, 8$ $\boxed{y_c = c_1e^{7x} + c_2e^{8x}}$

⑥ $m = 3, 4$ $\boxed{y_c = c_1e^{3x} + c_2e^{4x}}$

⑦ $m = -3 \pm \sqrt{5}$ $\boxed{y_c = e^{-3x}(c_1\cos\sqrt{5}x + c_2\sin\sqrt{5}x)}$

⑧ $m = -3, -2$ $\boxed{y_c = c_1e^{-3x} + c_2e^{-2x}}$

⑨ $m = \pm i$ $y_c = c_1\cos x + c_2\sin x$

(10) $m = 3, 3, 8, 9$

$$\boxed{y_c = (c_1 + c_2 x)e^{3x} + c_3 e^{8x} + c_4 e^{9x}}$$

3.8
$D^3 = 8$
$D = 2$

(11) $(D^3 - 8)y = 0$

$(m^3 - 8) = 0$

$(m^3 - 2^3) = 0$

$\cancel{(m-2)(m^2+2m+4)} = 0$

$(m-2)(m^2 + 2m + 4) = 0$

$(m-2)\cancel{(m+2)^2} \cancel{=0}$ $\quad m = \frac{-2 \pm \sqrt{4 - 4(4)}}{2} = \frac{-2 \pm \sqrt{-12}}{2} = -1 \pm \sqrt{3}\, i$

~~$m = 2, 2, 2$~~

$$\boxed{CF = c_1 e^{2x} + (c_2 \cos\sqrt{3}x + c_3 \sin\sqrt{3}x)\, e^{-x}}$$

(12) $y'' - 3y' - 4y = 0$

$(m^2 - 3m - 4) = 0$

$(m-4)(m+1) = 0$

$\therefore m = -1, 4$

$$\boxed{CF = c_1 e^{-x} + c_2 e^{4x}}$$

(13) $y'' - 4y' - 5y = 0$

$m^2 - 4y - 5 = 0$

$(m-5)(m+1) = 0$

$m = -1, 5$ $\quad \boxed{CF = c_1 e^{-x} + c_2 e^{5x}}$

(14) $y'' - y' = 0$

$m^2 - m = 0$

$m(m-1) = 0$

$m = 0, 1$ $\quad \boxed{CF = c_1 e^0 + c_2 e^x}$

2	1	2	-10	4
	0	2	8	-4
	1	4	-2	0

Q-4 (1) Find y_p $\quad (D^3 + 2D^2 - 10D + 4)y = 0$

AE: $m^3 + 2m^2 - 10m + 4 = 0$

$(m-2)(m^2 + 4m - 2) = 0$

$(m=2)$ & $m = \frac{-4 \pm \sqrt{24}}{2} = -2 \pm \sqrt{6}\, i$

$16 - 4(-2)$
$16 + 8 = 24$

$$\boxed{CF = c_1 e^{2x} + (c_2 \cos\sqrt{6}x + c_3 \sin\sqrt{6}x)\, e^{-2x}}$$

$PI = \frac{1}{f(D)} R^x$

$$\boxed{PI = 0}$$

(2) $(D^2 - 4)y = 0$

AE: $m^2 - 4 = 0$

$m = \pm 2$

$$\boxed{CF = c_1 e^{2x} + c_2 e^{-2x}}$$

(3) $(D^2-5D+6)y=0$

$\rightarrow$ AE: $m^2-5m+6=0$

$\cancel{(m-6)(m+1)=0}$ $\qquad$ $(m-2)(m-3)=0$

$\cancel{m=-1,6}$ $\qquad$ $m=2,3$

$\boxed{Cf = c_1e^{-x}+c_2e^{6x}}$ $\qquad$ $\boxed{Cf = c_1e^{2x}+c_2e^{3x}}$

Q-5 $(D^3-5D^2+7D-3)y = e^{2x}\cosh x$

$$\begin{array}{c|cccc} 1 & 1 & -5 & 7 & -3 \\ & 0 & 1 & -4 & 3 \\ \hline & 1 & -4 & 3 & 0 \end{array}$$

$\rightarrow$ AE: $m^3-5m^2+7m-3=0$

$(m-1)(m^2-4m+3)=0$

$(m-1)(m-1)(m-3)=0$

$m=1,1,3$

$\boxed{Cf=(c_1+c_2x)e^{x}+c_3e^{3x}}$

$PI = \frac{1}{f(D)}R(x)$

$= \frac{1}{(D^3-5D^2+7D-3)} e^{2x}\cosh x$

$D\to D+2$

$= e^{2x}\frac{1}{(D+2)^3-5(D+2)^2+7(D+2)-3}\cosh x$

$= e^{2x}\left[\frac{1}{D^3+2^3+3D^2(2)+3\times4D-5(D^2+2D+4)+7D+14-3}\cosh x\right]$

$= e^{2x}\left[\frac{\cosh x}{D^3+8+6D^2+12D-5D^2-10D-20+7D+14-3}\right]$

$= e^{2x}\left(\frac{\cosh x}{D^3+D^2+9D-15}\right)$

$PI = \frac{1}{D^3-5D^2+7D-3}e^{2x}\cosh x$

$= e^{2x}\left(\frac{e^{x}+e^{-x}}{2}\right)$

$= \frac{e^{3x}+e^{x}}{2}$

$= \frac{1}{2}\left(\frac{e^{3x}}{D^3-5D^2+7D-3}+\frac{e^{x}}{D^3-5D^2+7D-3}\right)$

$D\to D+3$ $\qquad$ $D\to 1$

$= \frac{1}{2}\left(\frac{e^{3x}}{27-45+21-3}+\frac{e^{x}}{1-5+7-3}\right)$

$=$ ~~$D\to D+3$~~ $\qquad$ ~~$D\to D+3$~~

$= \frac{1}{2}\left(\frac{x e^{3x}}{3D^2-10D+7}+\frac{xe^{x}}{3D^2-10D+7}\right)$

$D\to D+3$ $\qquad$ $D\to D+3$

$= \frac{1}{2}\left(\frac{xe^{3x}}{} + \frac{xe^{x}}{}\right)$

$= \frac{1}{2}\left(\frac{xe^{3x}}{4}+\frac{x}{0}e^{x}\right)$

$= \frac{1}{2}\left(\frac{x}{4}e^{3x}+\frac{x^2e^{x}}{6x-10}\right)$

$D\to 1$

$= \frac{1}{2}\left(\frac{x}{4}e^{3x}+\frac{x^2}{-4}e^{x}\right)$

$\boxed{y_p = \frac{1}{8}(xe^{3x}-x^2e^{x})}$

6) $(D^2+4)y = e^x + \sin 2x$

$\rightarrow$ AE: $m^2+4=0$

$m^2 = -4$

$m = \cancel{\pm\sqrt{2}i}$ $\pm 2i$

$\boxed{CF = c_1 \cos 2x + c_2 \sin 2x}$

PI: $\frac{1}{f(D)} R(x)$

$= \frac{1}{D^2+4} e^x + \frac{1}{D^2+4} \sin 2x$

$D \to 1$ $\quad D^2 \to -4$

$= \frac{1}{5} e^x + \frac{1}{-4+4} \sin 2x$

$= \frac{1}{5} e^x + \frac{x \times D \sin 2x}{2D \times D}$

$D^2 \to -4$

$= \frac{1}{5} e^x + \frac{x \cdot D \sin 2x}{2(-4)}$

$= \frac{1}{5} e^x - \frac{x}{8}(2\cos 2x)$

$\boxed{y_p = \frac{1}{5} e^x - \frac{x}{4} \cos 2x}$

7) $(D^2+2)y = e^x \cos 2x$

$\rightarrow$ AE: $m^2+2=0$

$m^2 = -2$

$m = \pm\sqrt{2}i$

$\boxed{CF = c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x}$

PI $= \frac{1}{f(D)} R(x)$

$= \frac{1}{D^2+2} e^x \cos 2x$

$D \to D+1$

$= e^x \frac{1}{(D+1)^2+2} \cos 2x$

$= e^x \frac{1}{D^2+2D+3} \cos 2x.$

$= \cancel{e^x}$ $\quad D^2 \to -4$

$= e^x \frac{1}{-4+2D+3} \cos 2x$

$= e^x \frac{1}{2D-1} \cos 2x.$

$= e^x \frac{2D+1}{4D^2-1} \cos 2x$

$D^2 \to -4$

$= e^x \, (2D+1)\cos 2x$

$= \frac{e^x}{-17}\left(2(-2\sin 2x) + \cos 2x\right)$

$\boxed{y_p = \frac{e^x}{-17}\left(-4\sin 2x + \cos 2x\right)}$

8) ① $(D^2+4)y = e^{3x} + \cos 3x$

→ AE: $m^2+4=0$

$m^2=-4$

$m = \pm 2i$

$\boxed{CF = c_1\cos 2x + c_2 \sin 2x}$

$PI = \frac{1}{f(D)} R(x)$

$= \frac{1}{D^2+4}(e^{3x}+\cos 3x)$

$= \frac{1}{D^2+4} e^{3x} + \frac{1}{D^2+4}\cos 3x$

$D \to 3 \qquad D^2 \to -9$

$= \frac{1}{13} e^{3x} + \frac{1}{-9+4}\cos 3x$

$\boxed{y_p = \frac{1}{13}e^{3x} - \frac{1}{5}\cos 3x}$

② $(D^2+2D+1)y = e^{-x}$

→ AE: $m^2+2m+1=0$

$(m+1)^2=0$

$m=-1,-1$

$\boxed{CF = (c_1+c_2x)e^{-x}}$

$PI = \frac{1}{f(D)} e^{-x}$

$= \frac{1}{D^2+2D+1} e^{-x}$

$D \to -1$

$= \frac{1}{1-2+1} e^{-x}$

$= \frac{1}{2D+2} e^{-x}$

$D \to -1$

$= \frac{1}{-2+2} e^{-x}$

$\boxed{PI = \frac{1}{2} e^{-x}}$

③ $(D^2+4)y = e^x + \sin x$

→ AE: $m^2+4=0$

$m^2=-4$

$m = \pm 2i$

$\boxed{CF = c_1\cos 2x + c_2\sin 2x}$

$PI: \frac{1}{f(D)} R(x)$

$: \frac{1}{D^2+4} e^x + \frac{1}{D^2+4}\sin x$

$D \to 1 \qquad D^2 \to -1$

$= \frac{1}{1+4} e^x + \frac{1}{-1+4}\sin x$

$\boxed{PI = \frac{1}{5}e^x - \frac{1}{3}\sin x}$

④ $(D^2+2D+1)y = e^{3x} + x^2$

$\rightarrow$ AE: $m^2+2m+1 = 0$

$(m+1)^2 = 0$

$m = -1, -1$

$\boxed{CF = (c_1 + c_2 x)e^{-x}}$

PI: $\frac{1}{f(D)} R(x)$

$= \frac{1}{D^2+2D+1} e^{3x} + x^2$

$= \frac{1}{D^2+2D+1} e^{3x} + \frac{1}{D^2+2D+1} x^2$

$D \to 3$ $\quad \frac{1}{(1+(D^2+2D))} x^2$

$= \frac{1}{9+6+1} e^{3x} + (1+(D^2+2D))^{-1} x^2$

$= \frac{1}{16} e^{3x} + (1-(D^2+2D)+(D^2+2D)^2) x^2$

$(D^4+4D^3+4D^2)$

$= \frac{1}{16} e^{3x} + (x^2 - 2 + 4x + 8)$

$\boxed{y_p \quad \frac{1}{16} e^{3x} + (x^2+4x+6)}$

⑤ $(D^2-3D+2)y = \cosh x$

$\rightarrow$ AE: $m^2-3m+2 = 0$

$(m-2)(m-1) = 0$

$m = 1, 2$

$\boxed{CF = c_1 e^x + c_2 e^{2x}}$

$PI = \frac{1}{f(D)} R(x)$

$= \frac{1}{D^2-3D+2} \cosh x$

$= \frac{1}{D^2-3D+2} \frac{e^x + e^{-x}}{2}$

$= \frac{1}{2}\left(\frac{1}{D^2-3D+2} e^x + \frac{1}{D^2-3D+2} e^x\right)$

$D \to 1$ $\quad D \to -1$

$= \frac{1}{2}\left(\frac{1}{1-3+2} e^x + \frac{1}{-1-3+2} e^x\right)$

$= \frac{1}{2}\left(\frac{1}{2D-3} e^x + \frac{1}{-2} e^x\right)$

$D \to 1$

$= \frac{1}{2}\left(\frac{1}{2-3} e^x + \frac{1}{-2} e^x\right.$

$\boxed{y_p = \frac{1}{2}\left(-e^x + \frac{1}{-2} e^x\right.}$

(6) $(D^2-4D+3)y = 2e^x$

$\rightarrow$ AE: $m^2-4m+3=0$

$(m-3)(m-1)=0$

$m=1,3$ $\boxed{CF = c_1e^x + c_2e^{3x}}$

$PI = \frac{1}{f(D)} R(x)$

$= \frac{1}{D^2-4x+3} 2e^x$

$D \rightarrow 1$

$= 2\left(\frac{1}{1-4+3} e^x\right)$

$= 2\left(\frac{1}{2D-4} e^x\right)$

$D \rightarrow 1$

$= 2\left(\frac{1}{2-4} e^x\right)$

$= \frac{2}{-2} e^x$ $\boxed{y_p = -e^x}$

(7) $(D+2)^2 y = e^{-2x}\sin x$

$\rightarrow$ AE: $(m+2)^2=0$

$m=-2,-2$

$\boxed{CF = (c_1+c_2x)e^{-2x}}$

$PI = \frac{1}{F(D)} R(x)$

$= \frac{1}{(D+2)^2} e^{-2x}\sin x$

$D \rightarrow D-2$

$= e^{-2x} \frac{1}{(D-2+2)^2} \sin 2x$

$= e^{-2x} \frac{1}{D^2} \sin 2x$

$D^2 \rightarrow -4$

$= e^{-2x} \frac{1}{-4} \sin 2x$

$\boxed{y_p = \frac{-e^{-2x}}{4}\sin 2x}$

(8) $(D^2-2D-3)y = 2e^x + 10\sin x$

$\rightarrow$ AE: $m^2-2m-3=0$

$(m-3)(m+1)=0$

$m=-1,3$

$\boxed{CF = c_1e^{-x} + c_2e^{3x}}$

$$PI = \frac{1}{f(D)} R(x)$$

$$= \frac{1}{D^2-2D+3}(2e^x + 10\sin x)$$

$$= \frac{2e^x}{D^2-2D+3} + \frac{10\sin x}{D^2-2D+3}$$

$D \to 1$ $\quad$ $D^2 \to -1$

$$= \frac{2e^x}{1-2+3} + \frac{10\sin x}{-1-2D+3}$$

$$= \frac{2e^x}{2D-2} + \frac{10\sin x}{-2D-2}$$

$D \to 1$

$$= \frac{2xe^x}{2(1)-2} - \frac{(2D-2)\,10\sin x}{4D^2-4}$$

$D^2 \to -1$

$$= \frac{2x^2e^x}{2} - \frac{(2D-2)10\sin x}{-8}$$

$$= x^2e^x + \frac{10}{4}(\cos x - \sin x)$$

$$\boxed{y_p = x^2e^x + \frac{5}{2}(\cos x - \sin x)}$$

(9) $(D^2+D)y = x^2+2x+4$

$\to$ AE: $m^2+m=0$

$m(m+1)=0$

$m=0,1$

$$\boxed{CF = C_1 + C_2e^x}$$

$$P_I : \frac{1}{f(D)} R(x)$$

$$= \frac{1}{D^2+D} x^2+2x+4$$

$$= \frac{1}{D(1+D)} x^2+2x+4$$

$$= \frac{1}{D}(1+D)^{-1} x^2+2x+4$$

$$= \frac{1}{D}(1-D+D^2) x^2+2x+4$$

$$= \frac{1}{D}(x^2+2x+4 - 2x - 2 + 2)$$

(10) $(D^3-5D^2+7D-3)y = e^{2x}\cosh x$

AE: $m^3-5m^2+7m-3=0$

$(m-1)(m^2-4m+3)=0$

$(m-1)(m-1)(m-3)=0$

$m=1,1,3$

1	1	-5	7	-3
	0	1	-4	3
	1	-4	3	0

$\boxed{CF = (c_1+c_2x)e^{x} + c_3e^{3x}}$

$R(x) = e^{2x}\cosh x$

$= e^{2x}\left(\frac{e^{x}+e^{-x}}{2}\right)$

$= \frac{1}{2}(e^{3x}+e^{x})$

$y_p = \frac{1}{f(D)} R(x)$

$= \frac{1}{D^3-5D^2+7D-3}\left(\frac{1}{2}(e^{3x}+e^{x})\right)$

$= \frac{1}{2}\left(\frac{e^{3x}}{D^3-5D^2+7D-3} + \frac{e^{x}}{D^3-5D^2+7D-3}\right)$

$D\to 3 \qquad D\to 1$

$= \frac{1}{2}\left(\frac{e^{3x}}{27-45+21-3} + \frac{e^{x}}{1-5+7-3}\right)$

$= \frac{1}{2}\left(\frac{xe^{3x}}{3D^2-10D+7} + \frac{xe^{x}}{3D^2-10D+7}\right)$

$D\to 3 \qquad D\to 1$

$= \frac{1}{2}\left(\frac{xe^{3x}}{27-30+7} + \frac{xe^{x}}{3-10+7}\right)$

$= \frac{1}{2}\left(\frac{x^2e^{3x}}{9D-10} + \frac{x^2e^{x}}{9D-10}\right)$

$D\to 3 \qquad D\to 1$

$\boxed{y_p = \frac{1}{2}\left(\frac{x^2e^{3x}}{20} - x^2e^{x}\right)}$

(11) $(D^2-D-2)y = \sin 2x$

AE: $m^2-m-2=0$

$(m-2)(m+1)=0$

$m=-1,2$

$\boxed{cf = c_1e^{-x}+c_2e^{2x}}$

$PI = \frac{1}{D^2-D-2}\sin 2x$

$D^2\to -4$

$= \frac{1}{-4-D-2}\sin 2x$

$= -\frac{1}{D+6}\sin 2x$

$= -\frac{D-6}{D^2-36}\sin 2x$

$D^2\to -4$

$= \frac{D-6}{40}\sin 2x$

(12) $(D^2-2D+1)\,y = \cos 3x$

$\rightarrow$ AE: $m^2-2m+1=0$

$(m-1)^2=0$

$m=1,1$

$\boxed{Cf = (c_1+c_2x)e^x}$

$PI = \dfrac{1}{D^2-2D+1}\cos 3x$

$D^2 \to -9$

$= \dfrac{1}{-9-2D+1}\cos 3x$

$= \dfrac{1}{-2D-8}\cos 3x$

$= \dfrac{-1}{2}\,\dfrac{(D-4)}{(D+4)(D-4)}\cdot\cos 3x$

$= \dfrac{-1}{2}\,\dfrac{D-4}{D^2-16}\cos 3x$

$D^2 \to -9$

$= \dfrac{-1}{2}\,\dfrac{D-4}{-25}\cos 3x$

$\boxed{y_p = \frac{1}{50}(-3\sin 3x - 4\cos 3x)}$

(13) $(D^2+4)\,y = e^x + \sin 2x$

$\rightarrow$ AE: $m^2+4=0$

$m^2=-4$

$m=\pm 2i$

$\boxed{Cf = c_1\cos 2x + c_2\sin 2x}$

$PI = \dfrac{1}{D^2+4}(e^x+\sin 2x)$

$= \dfrac{1}{D^2+4}e^x + \dfrac{1}{D^2+4}\sin 2x$

$D\to 1 \qquad D^2\to -4$

$= \dfrac{1}{1+4}e^x + \dfrac{xD}{2D^2}\sin 2x$

$D^2\to -4$

$= \frac{1}{5}e^x + \dfrac{xD}{-8}\sin 2x$

$= \frac{1}{5}e^x - \frac{x}{8}(2\cos 2x)$

$\boxed{y_p = \frac{1}{5}e^x - \frac{x}{4}\cos 2x}$

(14) $(D^2-4)\,y = x^2$

$\rightarrow$ AE: $m^2-4=0$

$m^2=4$

$m=\pm 2$

$\boxed{Cf = c_1 e^{2x} + c_2 e^{-2x}}$

$$PI = \frac{1}{D^2-4} \cdot x^2$$

$$= \frac{1}{-4\left(1-\frac{D^2}{4}\right)} x^2$$

$$= \frac{1}{-4}\left(1-\frac{D^2}{4}\right)^{-1} x^2$$

$$= \frac{1}{-4}\left(1+\frac{D^2}{4}+\frac{D^4}{16}\right) x^2$$

$$= \frac{1}{-4}\left(x^2+\frac{2}{4}\right)$$

$$\boxed{y_p = \frac{-1}{4}\left(x^2+\frac{1}{2}\right)}$$

(15) $(D^2+16)y = x^4+e^{3x}+\cos 3x$

$\rightarrow$ AE: $m^2+16=0$

$m^2=-4$

$m=\pm 2i$

$$\boxed{CF = c_1\cos 2x + c_2\sin 2x}$$

$$y_p = \frac{1}{D^2+16}x^4 + \frac{1}{D^2+16}e^{3x} + \frac{1}{D^2+16}\cos 3x$$

$D \to 3$ $\qquad D^2 \to -9$

$$= \frac{1}{16\left(1+\frac{D^2}{16}\right)}x^4 + \frac{1}{9+16}e^{3x} + \frac{1}{-9+16}\cos 3x$$

$$= \frac{1}{16}\left(1+\frac{D^2}{16}\right)^{-1}x^4 + \frac{1}{25}e^{3x} + \frac{1}{7}\cos 3x$$

$$= \frac{1}{16}\left(1-\frac{D^2}{16}+\frac{D^4}{256}\right)x^4 + \frac{1}{25}e^{3x} + \frac{1}{7}\cos 3x$$

$$= \frac{1}{16}\left(x^4-\frac{12x^2}{16}+\frac{24}{256}\right) + \frac{1}{25}e^{3x} + \frac{1}{7}\cos 3x$$

$$\boxed{y_p = \frac{1}{16}\left(x^4-\frac{3}{4}x^2+\frac{3}{32}\right) + \frac{1}{25}e^{3x} + \frac{1}{7}\cos 3x}$$

(16) $(D^2-2D+1)y = e^{3x}x^2$

$\rightarrow$ AE: $m^2-2m+1=0$

$(m-1)^2=0$

$m=1,1$

$$\boxed{CF = (c_1+c_2x)e^x}$$

$$PI = \frac{1}{D^2-2D+1}e^{3x}x^2$$

$D \to D+3$

$$= e^{3x}\frac{1}{(D+3)^2-2(D+3)+1}x^2$$

$$= \frac{e^{3x}}{D^2+6D+9-2D-6+1}x^2$$

$$= \frac{e^{3x}}{D^2+4D+4}x^2$$

$$= \frac{e^{3x}}{4\left(1+\frac{D^2+4D}{4}\right)}x^2$$

$$= \frac{e^{3x}}{4}\left(1+\frac{D^2+4D}{4}\right)^{-1}x^2$$

$$= \frac{e^{3x}}{4}\left[\left(1 - \left(\frac{D^2+4D}{4}\right) + \left(\frac{D^2+4D}{4}\right)^2\right) x^2\right]$$

$$\frac{D^4+8D^3+16D^2}{16}$$

$$= \frac{e^{3x}}{4}\left(x^2 - \frac{2}{4} - \frac{4(2x)}{4} + \frac{16}{16}(2)\right)$$

$$= \frac{e^{3x}}{4}\left(x^2 - \frac{1}{2} - 2x + 2\right)$$

$2 - \frac{1}{2}$

$$\boxed{y_p = \frac{e^{3x}}{4}\left(x^2 - 2x + \frac{3}{2}\right)}$$

17) $(D^2 - 5D + 6)y = e^{2x}\sin 2x$

$\rightarrow$ AE: $D^2 - 5D + 6 = 0$

$(D-2)(D-3) = 0$

$D = 2, 3$

$$\boxed{CF = c_1 e^{2x} + c_2 e^{3x}}$$

$$PI = \frac{1}{D^2 - 5D + 6} e^{2x}\sin 2x$$

$D \rightarrow D+2$

$$= e^{2x} \frac{1}{(D+2)^2 - 5(D+2) + 6} \sin 2x$$

$$= e^{2x} \frac{1}{D^2 + 2D + 4 - 5D - 10 + 6} \sin 2x$$

$$= e^{2x} \frac{1}{D^2 - 3D} \sin 2x$$

$D^2 \rightarrow -4$

$$= e^{2x} \frac{1}{-4 - 3D} \sin 2x$$

$$= -e^{2x} \frac{1\,(3D-4)}{(3D+4)(3D-4)} \sin 2x$$

$$= -e^{2x} \frac{(3D-4)}{9D^2 - 16} \sin 2x$$

$D^2 \rightarrow -4$

$$= -e^{2x} \frac{(3D-4)}{-36-16} \sin 2x$$

$$= \frac{e^{2x}}{52}\left(3(2\cos 2x) - 4\sin 2x\right)$$

$$\boxed{y_p = \frac{-e^{2x}}{52}\left(6\cos 2x - 4\sin 2x\right)}$$

18) $(D^2 + 4)y = x\sin 2x$

AE: $D^2 + 4 = 0$

$D^2 = -4$ $\quad D = \pm 2i$

$$\boxed{y_c = c_1\cos 2x + c_2\cos 2x}$$

$$y_p = \frac{1}{D^2+4} x\sin 2x$$

$$= \frac{x \sin 2x}{D^2+4} - \frac{2D}{(D^2+4)^2}\sin 2x$$

$D^2 \to -4$ $\qquad$ $D^2 \to -4$

$$= \frac{x^2 \sin 2x}{2D} - \frac{\cancel{2}D x}{\cancel{2}(D^2+4)(2D)} \sin 2x$$

$$= \frac{x^2}{2}\left(\frac{-\cos 2x}{2}\right) - 2D \cdot \left\{ \frac{x}{D^2+4}\left(-\frac{\cos 2x}{2}\right) \right\}$$

$$= \frac{-x^2 \cos 2x}{4} + \frac{D}{4}\left\{ \frac{x}{D^2+4} \cos 2x \right\}$$

$D^2 \to -4$

$$= \frac{-x^2}{4}\cos 2x + \frac{D}{4}\left\{ \frac{x2}{2D} \cos 2x \right\}$$

$$= -\frac{x^2}{4}\cos 2x + \frac{D}{4}\left\{ \frac{x^2}{2}\left(\frac{\sin 2x}{2}\right) \right\}$$

$$= -\frac{x^2}{4}\cos 2x + \cancel{\frac{x^2}{8}(2\cos 2x)}$$

$$= -\frac{x^2}{4}\cos 2x + \frac{1}{16}\left[2x^2 \cos 2x + 2x \sin 2x\right]$$

$$= \frac{-x^2}{4}\cos 2x + \frac{1}{8}x^2\cos 2x + \frac{1}{8}x \sin 2x$$

$$\boxed{y_p = \frac{x}{8}\sin 2x - \frac{x^2}{8}\cos 2x}$$

19) $(D^2-2D+1)y = xe^x \sin x$

$\to$ CF: $m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1, 1$

$$\boxed{CF = (c_1 + c_2 x)e^x}$$

$$PI = \frac{1}{D^2-2D+1} e^x x\sin x$$

$D \to D+1$

$$= e^x \frac{1}{(D+1)^2 - 2(D+1)+1} x \sin x$$

$$= e^x \frac{1}{D^2+\cancel{2D}+1-\cancel{2D}-2+1} x \sin x$$

$$= e^x \frac{1}{D^2} x \sin x$$

$$= e^x \left[\frac{x \sin x}{D^2} - \frac{x(2D)}{D^4}\sin x\right]$$

$D^2 \to -1$ $\qquad$ $D^2 \to -1$

$$\boxed{y_p = e^x\left[-x\sin x - 2x(\cos x)\right]}$$

Unit - 5

1) Periodic funⁿ - A funⁿ returning to the same value at regular intervals

① $\cos 3x = \frac{2\pi}{3}$

② $\sin 5x = \frac{2\pi}{5}$

③ $\cos 5x = \frac{2\pi}{5}$

④ $\sin 7x = \frac{2\pi}{7}$

⑤ $\cos 7x = \frac{2\pi}{7}$

⑥ $\sin 9x = \frac{2\pi}{9}$

⑦ $\cos nx = \frac{2\pi}{n}$

⑧ $\sin nx = \frac{2\pi}{n}$

⑨ $\cos 11x = \frac{2\pi}{11}$

⑩ $\sin 13x = \frac{2\pi}{13}$

⑪ $\sin 17x = \frac{2\pi}{17}$

⑫ $\cos 17x = \frac{2\pi}{17}$

A funⁿ $f(x)$ is called Periodic if it is defined for all x and there exists some positive number p such that $f(x+p) = f(x)$, $\forall x$, the number p is called Period of $f(x)$

2) Find Euler's formula for fourier coefficients (a_0, a_n, b_n)

→ The fourier series is defined as $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ —— ①

→ <u>for a_0</u>

Integrating eqn ① w.r.t x by taking limits c to $c+2\pi$, we get

$$\int_c^{c+2\pi} f(x)\,dx = \int_c^{c+2\pi} \frac{a_0}{2}\,dx + \sum_{n=1}^{\infty}\left[\int_c^{c+2\pi}(a_n \cos nx + b_n \sin nx)\,dx\right]$$

$$\int_c^{c+2\pi} f(x)\,dx = \frac{a_0}{2}[x]_c^{c+2\pi} + \sum_{n=1}^{\infty}\left[a_n \int_c^{c+2\pi} \cos nx\,dx + b_n \int_c^{c+2\pi} \sin nx\,dx\right]$$

$$= \frac{a_0}{2}[2\pi] + \sum_{n=1}^{\infty}[a_n(0) + b_n(0)]$$

$$\int_c^{c+2\pi} f(x)\,dx = a_0 \pi$$

$$\boxed{a_0 = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\,dx}$$

→ <u>for a_n</u>

multiply both sides of eqn ① with $\cos mx$ and then Integrating w.r.t x between limits c to $c+2\pi$, we get.

$$\int_c^{c+2\pi} f(x)\cos mx\,dx = \int_c^{c+2\pi}\frac{a_0}{2}\cos mx\,dx + \sum_{n=1}^{\infty}\left[\int_c^{c+2\pi}(a_n \cos nx \cos mx + b_n \sin nx \cos mx)\,dx\right]$$

For <u>$m=n$</u> $\int_c^{c+2\pi} \cos mx\,dx = 0$ $\int_c^{c+2\pi} \cos^2 mx = \pi$ $\int_c^{c+2\pi} \cos mx \sin nx\,dx = 0$

$$\int_c^{c+2\pi} f(x)\cos mx\,dx = 0 + \sum_{n=1}^{\infty} a_n \pi$$

$$\boxed{a_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\cos nx\,dx}$$

Multiply both side of eqn ① with $\sin mx$ and then integrating wrt x between limits c to $c+2\pi$, we get.

$$\int_c^{c+2\pi} f(x)\sin mx\,dx = \int_c^{c+2\pi} \frac{a_0}{2}\sin mx + \sum_{n=1}^{\infty}\left(\int_c^{c+2\pi}(a_n \cos nx \sin mx + b_n \sin nx \sin mx)\,dx\right)$$

For $m=n$ $\quad \int_c^{c+2\pi}\sin mx = 0 \quad \int_c^{c+2\pi}\cos mx \sin mx = 0 \quad \int_c^{c+2\pi}\sin^2 mx = \pi$

$$\int_c^{c+2\pi} f(x)\sin mx\,dx = 0 + b_n\pi$$

$$\boxed{b_n = \frac{1}{\pi}\int_c^{c+2\pi} f(x)\sin nx\,dx}$$

Q-3 Using Euler's formula find a_0 for $f(x)=\sqrt{1-\cos x}$, $-\pi \le x \le \pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}\sqrt{1-\cos x}\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\sqrt{1-\cos x}\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi}\sqrt{2}\sin\frac{x}{2}\,dx$$

$$= \frac{2\sqrt{2}}{\pi}\left[-\cos\frac{x}{2}\cdot 2\right]_0^{\pi}$$

$$= \frac{4\sqrt{2}}{\pi}\left[-(\cos\pi - \cos 0)\right]$$

$$\boxed{a_0 = \frac{4\sqrt{2}}{\pi}\left[1-(-1)^n\right]}$$

Q-4 Find b_n for $f(x)=2x$ $\quad -\pi \le x \le \pi$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} 2x\sin nx\,dx$$

$$= \frac{2}{\pi}\cdot 2\int_0^{\pi} x\sin nx\,dx$$

$$= \frac{4}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - 1\left(-\frac{\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{4}{\pi}\left[\pi\left(-\frac{\cos\pi}{n}\right)\right]$$

$$= \frac{4}{n} - (-1)^n \Rightarrow \boxed{b_n = \frac{-4(-1)^n}{n}}$$

Q-5 Find a_n for $F(x)=x^2$ $\quad -\pi \le x \le \pi$

$$\rightarrow a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos nx\,dx$$

$$= \frac{2}{\pi}\int_0^{\pi} x^2\cos nx\,dx$$

$$= \frac{2}{\pi}\left[x^2\frac{\sin nx}{n} - 2x\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{2\pi}{n^2}(-1)^n\right] \Rightarrow \boxed{b_n = \frac{4}{n^2}(-1)^n}$$

Q-6 find a_0 for $f(x) = |x|$, $-\pi \le x \le \pi$

$\rightarrow$ $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x\,dx$

$= \frac{2}{\pi}\int_{0}^{\pi} x\,dx$

$= \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$

$= \frac{\pi^2}{\pi}$ $\boxed{a_0 = \pi}$

Q-7 write fourier series expansion for odd & even fun^n.

$\rightarrow$ for ~~odd~~ even $b_n = 0$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx\,dx$

for ~~even~~ odd $a_0 = a_n = 0$

$f(x) = \sum_{n=1}^{\infty} b_n \sin nx\,dx$

Q-8 find a_0 for given fun^n $f(x) = \cos\frac{x}{2}$, $0 < x < 2$

$P = 2l = 2$

$l = 1$

$a_0 = \frac{1}{l}\int_{0}^{c+2l} f(x)\,dx$

$= \int_{0}^{2} \cos\frac{x}{2}\,dx$

$= \left[\sin\frac{x}{2}\right]_0^{2} \cdot 2 = [\sin 1 - \sin 0]\,2 = 2(0.84)$

$\boxed{a_0 = 1.68}$

Q-9 find a_n for $f(x) = 3x^2$ $-\pi \le x \le \pi$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} 3x^2$

$= \frac{3 \times 2}{\pi}\int_{0}^{\pi} x^2$

$= \frac{6}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi} = \frac{2}{\pi}\pi^3 = 2\pi^2$ $\therefore \boxed{a_n = 2\pi^2}$

Q-10 write fourier series expansion for $f(x)$ in $0 < x < 2l$ & $-l < x < l$

$\rightarrow$ (1) for $0 < x < 2l$

$a_0 = \frac{1}{l}\int_{0}^{2l} f(x)\,dx$

$a_n = \frac{1}{l}\int_{0}^{2l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$

$b_n = \frac{1}{l}\int_{0}^{2l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$

(2) for $-L < x < L$

$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$

$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx$

$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx$

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Q-4

(1) $f(x) = e^x \quad 0<x<\pi$

$\rightarrow \quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)\, dx$

$a_0 = \frac{1}{\pi}\int_0^\pi e^x\, dx$

$= \frac{1}{\pi}[e^x]_0^\pi = \boxed{\frac{1}{\pi}[e^\pi - 1]}$

$a_n = \frac{1}{\pi}\int_0^\pi e^x \cos nx\, dx$

$= \frac{1}{\pi}\left[\frac{e^x}{n^2+1}(\cos nx + n\sin nx)\right]_0^\pi$

$= \frac{1}{\pi}\left(\frac{e^\pi}{1+n^2}[(-1)^n) - \frac{1}{1+n^2}(-1)\right)$

$\boxed{a_n = \frac{((-1)^n)\,(e^\pi)}{\pi(1+n^2)}}$

$b_n = \frac{1}{\pi}\int_0^\pi f(x)\sin nx\, dx$

$= \frac{1}{\pi}\int_0^\pi e^x \sin nx\, dx$

$= \frac{1}{\pi}\left[\frac{e^x}{1+n^2}(\sin nx - n\cos nx)\right]_0^\pi$

$= \frac{1}{\pi}\left[\frac{e^\pi}{1+n^2}(-n((-1)^n)] - \frac{1}{1+n^2}((-1)^n - 1)\right)$

$\boxed{b_n = \frac{(e^\pi)\, n\,(1-(-1)^n)}{\pi(1+n^2)}}$

(2) $f(x) = 3e^{2x}, \quad 0 < x < 2\pi$

$\rightarrow \quad f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)\, dx$

$a_0 = \frac{1}{\pi}\int_0^{2\pi} 3e^{2x}\, dx$

$= \frac{3}{\pi}\left[\frac{e^{2x}}{2}\right]_0^{2\pi} dx$

$\boxed{a_0 = \frac{3}{2\pi}(e^{4\pi} - 1)}$

$a_n = \frac{1}{\pi}\int_0^{2\pi} 3e^{2x}\cos nx\, dx$

$= \frac{3}{\pi}\left[\frac{e^{2x}}{n^2+4}(2\cos nx + n\sin nx)\right]_0^{2\pi}$

$= \frac{3}{\pi}\left(\frac{e^{4\pi}}{n^2+4}(2(1)) - \frac{1}{n^2+4}(2(1))\right)$

$\boxed{a_n = \frac{6}{\pi}\left[\frac{e^{4\pi} - 1}{n^2+4}\right]}$

$b_n = \frac{1}{\pi}\int_0^{2\pi} 3e^{2x}\sin nx\, dx$

$= \frac{3}{\pi}\left\{\frac{e^{2x}}{4+n^2}(2\sin nx - n\cos nx)\right\}_0^{2\pi}$

$= \frac{3}{\pi}\left(\frac{e^{4\pi}}{4+n^2}(-n(1)) - \frac{1}{4+n^2}(-n(1))\right)$

$\boxed{b_n = -\frac{3}{\pi}\left(\frac{(e^{4\pi} - 1)\, n}{n^2+4}\right)}$

Q.5

Q.3 $f(x) = e^{ax} \quad 0 < x < 2\pi$

$\rightarrow a_0 = \frac{1}{\pi} \int_c^{c+2\pi} f(x)\, dx$

$= \frac{1}{\pi} \int_0^{2\pi} e^{ax}\, dx$

$= \frac{1}{\pi} \left[\frac{e^{ax}}{a} \right]_0^{2\pi} dx$

$\boxed{a_0 = \frac{1}{\pi a} \left[e^{2ax} - 1 \right]}$

$a_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \cos nx\, dx$

$= \frac{1}{\pi} \int_0^{2\pi} e^{ax} \cos nx\, dx$

$= \frac{1}{\pi} \left[\frac{e^{ax}}{n^2+a^2} \; a\cos nx + n\sin nx \right]_0^{2\pi}$

$= \frac{1}{\pi} \left[\frac{e^{2a\pi}}{n^2+a^2} \; a(1) - \frac{1}{n^2+a^2} a(1) \right]$

$\boxed{a_n = \frac{a}{\pi(n^2+a^2)} (e^{2a\pi} - 1)}$

$b_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \sin nx\, dx$

$= \frac{1}{\pi} \int_0^{2\pi} e^{ax} \sin nx\, dx$

$= \frac{1}{\pi} \left[\frac{e^{ax}}{a^2+n^2} \; a\sin nx - n\cos nx \right]_0^{2\pi}$

$= \frac{1}{\pi} \left(\frac{e^{2a\pi}}{n^2+a^2} (-n) - \frac{1}{n^2+a^2} (-n) \right]$

$\boxed{b_n = \frac{-n}{\pi(n^2+a^2)} (e^{2a\pi} - 1)}$

Q-4 $f(x) = \frac{1}{2}(\pi - x) \quad , 0 < x < 2\pi$

$a_0 = \frac{1}{\pi} \int_c^{c+2\pi} \frac{1}{2}(\pi - x)$

$= \frac{1}{2\pi} \left[\pi x - \frac{x^2}{2} \right]_0^{2\pi}$

$= \frac{1}{2\pi} \left(2\pi^2 - \frac{4\pi^2}{2} \right)$

$\boxed{a_0 = 0}$

$a_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \cos nx\, dx$

$= \frac{1}{2\pi} \int_0^{2\pi} (\pi - x) \cos nx\, dx$

$= \frac{1}{2\pi} \left[(\pi - x) \frac{\sin nx}{n} - (-1) \left(\frac{-\cos nx}{n^2} \right) \right]_0^{2\pi}$

$= \frac{1}{2\pi} \left[- \frac{\cos nx}{n^2} \right]_0^{2\pi}$

$= \frac{-1}{2\pi n^2} (1 - 1)$

$\boxed{a_n = 0}$

$b_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \sin nx\, dx$

$= \frac{1}{2\pi} \int_0^{2\pi} (\pi - x) \sin nx\, dx$

$= \frac{1}{2\pi} \left[(\pi - x) \left(-\frac{\cos nx}{n} \right) - (-1) \left(-\frac{\sin nx}{n^2} \right) \right]_0$

$= \frac{1}{2\pi n} \left((-\pi)(-(1)) - \pi(-1) \right)$

$= \frac{1}{2\pi n} (\pi + \pi)$

$= \frac{1}{2\pi n} (2\pi)$

$b_n = \frac{1}{}$

Q.5 $f(x) = x^3, \ -\pi < x < \pi$

$f(-x) = -x^3 = -f(x)$

$a_0 = a_n = 0$

$b_n = \frac{2}{\pi} \int_0^\pi x^3 \sin nx \, dx$

$= \frac{2}{\pi} \left[x^3 \left(-\frac{\cos nx}{n} \right) - 3x^2 \left(-\frac{\sin nx}{n^2} \right) + 6x \left(\frac{\cos nx}{n^3} \right) \mp 6 \left(\frac{\sin nx}{n^4} \right) \right]_0^\pi$

$= \frac{2}{\pi} \left(-\frac{\pi^3 (-1)^n}{n} + 6\pi \frac{(-1)^n}{n^3} \right]$

$= -\frac{2\pi^2 (-1)^n}{n} + \frac{6 (-1)^n}{n^3} = \boxed{\frac{2(-1)^n}{n} \left(-\pi^2 + \frac{6}{n^2} \right)}$

Q-6 $f(x) = x^2, \ -2 \le x \le 2$

$L = 2$

$f(-x) = x^2 = f(x) \quad \therefore f(x)$ is even funⁿ.

$\boxed{\therefore b_n = 0}$

$a_0 = \frac{1}{l} \int_{-l}^{l} f(x) \, dx$

$= \frac{1}{2} \int_{-2}^{2} x^2 dx$

$= \frac{2}{2} \left[\frac{x^3}{3} \right]_0^2$

$= \frac{2^3}{3} = \boxed{\frac{8}{3}}$

$a_n = \frac{1}{l} \int_{-l}^{l} f(x) \cos nx \, dx$

$= \frac{1}{2} \int_{-2}^{2} x^2 \cos \frac{n\pi x}{2} dx$

$= \frac{2}{2} \left[x^2 \sin \frac{n\pi x}{n} - 2x \left(-\frac{\cos n\pi x}{n^2} \right) + 2 \left(-\frac{\sin n\pi x}{n^3} \right) \right]_0^2$

$-2x \left(-\cos \frac{n\pi x}{2} \right) \Big/ \left(\frac{n\pi}{2} \right)^2$

$= \frac{2(2) \cdot (-1)^n}{n^2 \frac{\pi^2}{4}}$

$\boxed{a_n = \frac{8(-1)^n}{n^2}}$

Q-7 $f(x) = x + |x|, \ -\pi < x < \pi$

$\rightarrow a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} x + |x|$

$= \frac{1}{\pi} \left(\int_{-\pi}^{\pi} x + \int_{-\pi}^{\pi} |x| \right)$

$= \frac{1}{\pi} \left(0 + 2 \left[\frac{x^2}{2} \right]_0^\pi \right)$

$= \frac{2}{\pi} \left[\frac{\pi^2}{2} \right] \qquad \boxed{a_0 = \pi}$

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (x + |x|) \cos nx \, dx$

$= \frac{1}{\pi} \left[\int_{-\pi}^{\pi} x \cos nx + \int_{-\pi}^{\pi} |x| \cos nx \, dx \right]$

$= \frac{1}{\pi} \left[0 + 2 \int_0^\pi x \cos nx \, dx \right)$

$= \frac{2}{\pi} \left[x \frac{\sin nx}{n} - 1 \left(-\frac{\cos nx}{n^2} \right) \right]_0^\pi$

$\boxed{a_n = \frac{2}{\pi n^2} \left((-1)^n - 1 \right)}$

$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (x + |x|) \sin nx \, dx$

$= \frac{1}{\pi} \left(\int_{-\pi}^{\pi} x \sin nx + \int_{-\pi}^{\pi} |x| \sin nx \, dx \right)$

$= \frac{1}{\pi} \left(2 \int_0^\pi x \sin nx + 0 \right)$

$= \frac{2}{\pi} \left[x \left(-\frac{\cos nx}{n} \right) - (1) \left(-\frac{\sin nx}{n^2} \right) \right]_0^\pi$

$= \frac{2}{\pi} \left(\frac{\pi}{n} (-(-1)^n \right)$

$\boxed{b_n = -\frac{2}{n} (-1)^n}$

8) $f(x) = x + x^2$ in $(-1, 1)$

$\Rightarrow a_0 = \frac{1}{1} \int_{-1}^{1} (x + x^2)\, dx$

$= \frac{1}{1} \left(0 + 2 \left[\frac{x^3}{3} \right]_0^1 dx \right)$

$= \frac{2}{3 \cdot 1} (1 - 0) \Rightarrow \boxed{a_0 = \frac{2}{3}}$

$a_n = \frac{1}{1} \int_{-1}^{1} f(x) \cos(n\pi x)\, dx$

$= 1 \int_{-1}^{1} (x + x^2) \cos n\pi x\, dx$

$= \left[0 + 2 \int_0^1 x^2 \cos n\pi x\, dx \right)$

$= 2 \left[x^2 \frac{\sin n\pi x}{n\pi} - 2x \frac{(-\cos n\pi x)}{(n\pi)^2} + 2 \frac{(-\sin n\pi x)}{(n\pi)^3} \right]_0^1$

$= 2 \left(\frac{2(1)(\cos n\pi)}{n^2 \pi^2} \right)$

$\boxed{a_n = \frac{4}{n^2 \pi^2} (-1)^n}$

$b_n = \frac{1}{1} \int_{-1}^{1} f(x) \sin n\pi x\, dx$

$= 1 \int_{-1}^{1} (x + x^2) \sin n\pi x\, dx$

$= 2 \int_0^1 x \sin n\pi x + 0$

$= 2 \left[x \frac{(-\cos n\pi x)}{n\pi} - 1 \left(\frac{-\sin n\pi x}{n^2 \pi^2} \right) \right]_0^1$

$= 2 \left[1 \left(\frac{-\cos n\pi}{n\pi} \right) \right]$

$\boxed{b_n = \frac{-2(-1)^n}{n\pi}}$

9) $f(x) = \begin{cases} -1, & -1 < x < 0 \\ 1, & 0 < x < 1 \end{cases}$

$a_0 = \frac{1}{\pi} \int_{-1}^{1} f(x)\, dx$

$= \frac{1}{\pi} \left[\int_{-1}^{0} f(x)\, dx + \int_0^1 f(x) dx \right]$

$= \frac{1}{\pi} \left(\int_{-1}^{0} -1\, dx + \int_0^1 1\, dx \right)$

$= \frac{1}{\pi} \left(-[x]_{-1}^{0} + (x]_0^1 \right]$

$= \frac{1}{\pi} \left[-(0 - (-1)) + 1 \right]$

$= \frac{1}{\pi} (-1 + 1) \Rightarrow \boxed{a_0 = 0}$

$a_n = \frac{1}{\pi} \int_{-1}^{1} f(x) \cos nx\, dx$

$= \frac{1}{\pi} \left[\int_{-1}^{0} f(x) \cos nx + \int_0^1 f(x) \cos nx\, dx \right)$

$= \frac{1}{\pi} \left[-\int_{-1}^{0} \cos nx + \int_0^1 \cos nx\, dx \right)$

$= \frac{1}{\pi} \left[-\left[\frac{\sin nx}{n} \right]_{-1}^{0} + \left[\frac{\sin nx}{n} \right]_0^1 \right)$

$= \frac{1}{n} \left[-\left[0 - \frac{\sin n}{n} + \frac{\sin n}{n} - 0 \right] \right.$

$= \frac{1}{n} (-1 + 1)$ $\quad \boxed{a_n = 0}$

$b_n = \frac{1}{\pi} \int_{-1}^{1} f(x) \sin nx\, dx$

$= \frac{1}{\pi} \left[\int_{-1}^{0} f(x) \sin nx + \int_0^1 f(x) \sin nx\, dx \right]$

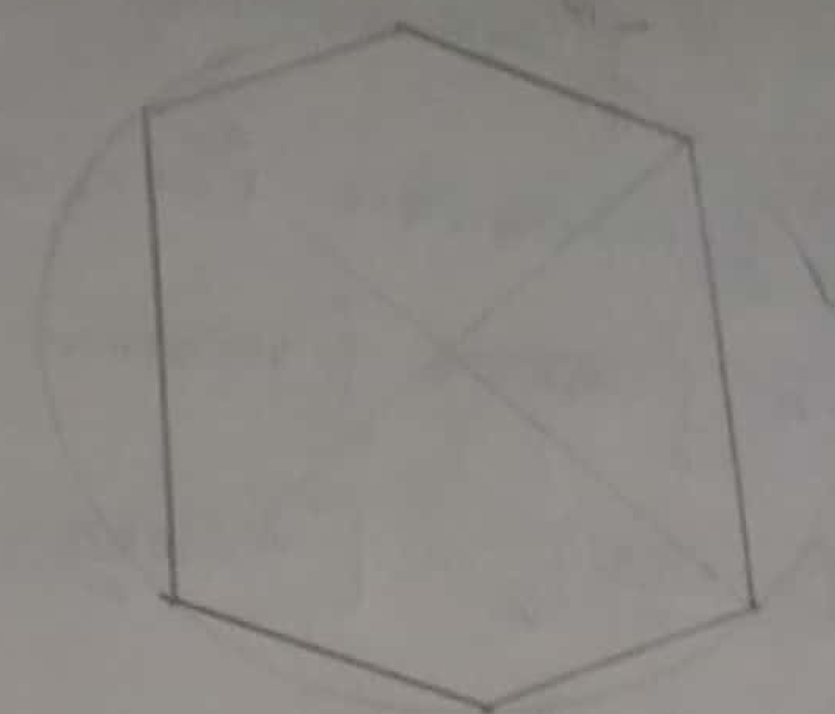
$= \frac{1}{\pi} \left[-1 \int_{-1}^{0} \sin nx + \int_0^1 \sin nx\, dx \right]$

$= \frac{1}{\pi} \left[-1 \left(\frac{-\cos nx}{n} \right)_{-1}^{0} + \left(\frac{-\cos nx}{n} \right]_0^1 \right]$

$= \frac{1}{\pi} \left(\frac{1 - \cos n}{n} + - \frac{\cos n}{n} + \frac{1}{n} \right)$

$\boxed{b_n = \frac{2}{n\pi} (1 - \cos n)}$

) $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$



$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)dx + \int_{0}^{\pi} f(x)dx\right]$

$= \frac{1}{\pi}\left[-k\,[x]_{-\pi}^{0} + k\,[x]_0^{\pi}\right]$

$= \frac{1}{\pi}\left[-k[0+\pi] + k[\pi-0]\right]$

$= \frac{1}{\pi}[-k\pi + k\pi] \Rightarrow \boxed{a_0 = 0}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$

$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cos nx\,dx + \int_0^{\pi} f(x)\cos nx\,dx\right]$

$= \frac{1}{\pi}\left[-k\int_{-\pi}^{0}\cos nx\,dx + k\int_0^{\pi}\cos nx\,dx\right]$

$= \frac{1}{\pi}\left[-k\left(\frac{\sin nx}{n}\right)_{-\pi}^{0} + k\left(\frac{\sin nx}{n}\right)_0^{\pi}\right]$

$= \frac{1}{\pi}\left[-k(0) + k(0)\right]$

$\boxed{a_n = 0}$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\sin nx + \int_0^{\pi} f(x)\sin nx\,dx\right]$

$= \frac{1}{\pi}\left[-k\int_{-\pi}^{0}\sin nx + k\int_0^{\pi}\sin nx\,dx\right]$

$= \frac{1}{\pi}\left(-k\left[-\frac{\cos nx}{n}\right]_{-\pi}^{0} + k\left[-\frac{\cos nx}{n}\right]_0^{\pi}\right)$

$= \frac{1}{\pi}\left[\frac{k}{n}(1-(-1)^n + \frac{k}{n}(1-(-1)^n\right]$

$= \frac{1}{\pi}\left[k\,(\cancel{1-(-1)^n} + \cancel{1-(-1)^n})\right)$

$= \frac{1}{\pi}(0)$ $\quad \boxed{\cancel{b_n = 0}}$ $\quad \boxed{b_n = \frac{2k}{\pi}(1-(-1)^n)}$

(ii) $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$

$\rightarrow$

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)dx + \int_0^{\pi} f(x)\,dx\right]$

$= \frac{1}{\pi}\left(-\pi\int_{-\pi}^{0} dx + \int_0^{\pi} x\,dx\right)$

$= \frac{1}{\pi}\left(-\pi\,[x]_{-\pi}^{0} + \left[\frac{x^2}{2}\right]_0^{\pi}\right)$

$= \frac{1}{\pi}\left(-\pi^2 + \frac{\pi^2}{2}\right]$

$= \pi\left(-1 + \frac{1}{2}\right)$

$= -\frac{\pi}{2}$

$\boxed{\therefore\ a_0 = -\frac{\pi}{2}}$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$

$= \frac{1}{\pi}\left[\int_{-\pi}^{0} f(x)\cos nx + \int_{0}^{\pi} f(x)\cos nx\right]$

$= \frac{1}{\pi}\left[-\pi\int_{-\pi}^{0}\cos nx + \int_{0}^{\pi} x\cos nx\right]$

$= \frac{1}{\pi}\left[-\pi\left[\frac{\sin nx}{n}\right]_{-\pi}^{0} + \left[x\frac{\sin nx}{n} - \left(-\frac{\cos nx}{n^2}\right)\right]_0^{\pi}\right]$

$= \frac{1}{\pi}\left[0 + \frac{(-1)^n - 1}{n^2}\right)$

$\boxed{a_n = \frac{(-1)^n - 1}{n^2\pi}}$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

$= \frac{1}{\pi}\left(\int_{-\pi}^{0} f(x)\sin nx + \int_0^{\pi} f(x)\sin nx\,dx\right)$

$= \frac{1}{\pi}\left[-\pi\int_{-\pi}^{0}\sin nx + \int_0^{\pi} x\sin nx\,dx\right]$

$= \frac{1}{\pi}\left[-\pi\left(-\frac{\cos nx}{n}\right)_{-\pi}^{0} + \left(x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right)_0\right.$

$= \frac{1}{\pi}\left(\frac{\pi}{n}(1-(-1)^n + \frac{\pi}{n}(-(-1)^n)\right)$

$\boxed{b_n = \frac{1 - 2(-1)^n}{n}}$

12) $P = 3 \quad f(x) = 2x - x^2$ range $(0,3)$

$\rightarrow \quad P = 2l = 3$

$l = 3/2$

$a_0 = \frac{1}{l}\int_0^{2l} f(x)\,dx$

$= \frac{2}{3}\int_0^3 2x - x^2\,dx$

$= \frac{2}{3}\left(\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^3$

$= \frac{2}{3}(3^2 - 3^2)$

$\boxed{a_0 = 0}$

$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}\,dx$

$= \frac{2}{3}\int_0^3 (2x - x^2)\cos\frac{n\pi x}{2/3}\,dx$

$= \frac{2}{3}\left[2\left[\frac{x(\sin n\pi x/2/3)}{2n\pi/2/3} - 1\left(\frac{-\cos n\pi x\cdot 3/2}{\left(\frac{2n\pi}{2/3}\right)^2}\right)\right]\right.$

$-\left(\frac{x^2\sin n\pi x\,3/2}{3n\pi/2} - 2x\left(\frac{-\cos 3n\pi x/2}{\left(\frac{3n\pi}{2/3}\right)^2}\right) + 2\left(\frac{-\sin 3n\pi x/2}{\left(\frac{3n\pi}{2/3}\right)^3}\right)\right)_0^3$

$= \frac{2}{3}\left[\frac{6\times 2((-1)^n - 1)}{\frac{9n^2\pi^2}{(2/3)^2}} + 4\frac{\cos(-1(-1)^n)}{\frac{9n^2\pi^2}{(2/3)^2}}\right)$

~~$= \frac{2}{3}\left[\frac{-8 + 16}{9n^2\pi^2}\right($~~

$= \frac{2}{3}\left[\frac{2(9)}{4\pi^2 n^2} - \frac{6(1)(9)}{4\pi^2 n^2} - \frac{2(3)\times 9}{2\times 4\, n^2\pi^2}\right)$

$= \boxed{a_n = \frac{-9}{n^2\pi^2}}$

$b_n = \frac{2}{3}\int_0^3 (2x - x^2)\sin\frac{2\pi n x}{3/2}\,dx$

$= \frac{2}{3}\left[\int_0^3 2x\sin\frac{2\pi n x}{3} - \int_0^3 x^2\sin\frac{2\pi n x}{3}\,dx\right]$

$= \frac{2}{3}\left(2\left[\frac{x\left(-\cos\frac{2\pi n x}{3}\right)}{\frac{2\pi n}{3}} - 1\left(\frac{-\sin\frac{2\pi n x}{3}}{\left(\frac{2\pi n}{3}\right)^2}\right)\right]_0^3\right.$

$-\left(x^2\left(\frac{-\cos\frac{2\pi n x}{3}}{\left(\frac{2\pi n}{3}\right)}\right) - 2x\left(\frac{-\sin\frac{2\pi n x}{3}}{\left(\frac{2\pi n}{3}\right)^2}\right) + 2\left(\frac{\cos\frac{2\pi n x}{3}}{\left(\frac{2\pi n}{3}\right)^3}\right)\right)_0$

$= \frac{2}{3}\left(\frac{6(-1)\times 3}{2\pi n} + \frac{9\times 3}{2\pi n} - \frac{18}{4n^2\pi^2} + \frac{18}{4n^2\pi^2}\right)$

$= \frac{2}{3}\left(\frac{-18 + 27}{2n\pi}\right)$

$= \frac{2}{3}\left(\frac{9}{2n\pi}\right)$

$\boxed{b_n = \frac{3}{n\pi}}$

13) Obtain cosine series for $f(x) = e^{x}$, $-l < x < l$

→ $a_0 = \frac{2}{l} \int_0^l f(x)\,dx$

$= \frac{2}{l}\left[e^{x}\right]_0^l = \boxed{\frac{2}{l}\left[e^{l}-1\right]}$

$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$

$= \frac{2}{l}\left[\int e^{x}\cos\frac{n\pi x}{l}\,dx\right)$

$= \frac{2}{l}\left[\frac{e^{x}}{1+\left(\frac{n\pi}{l}\right)^2}\left(\cos\frac{n\pi x}{l} + \frac{n\pi}{l}\sin\frac{n\pi x}{l}\right]\right]_0^l\Big)$

$= \frac{2}{l}\left[\frac{l^2}{l^2+n^2\pi^2}\left[e^{l}(-1)^{n}-1\right]\right.$

$= \cancel{\frac{2\,l\,e^{x}}{l^2+n^2\pi^2}}$ $\boxed{a_n = \frac{2l\left(e^{l}(-1)^n-1\right)}{l^2+n^2\pi^2}}$

14) obtain sine series for $f(x) = 2x$, $-1 \le x \le 1$

→ Here $f(x) = 2x$

$f(-x) = -f(x)$ $\therefore f(x)$ is odd funⁿ

$\therefore a_0 = a_n = 0$

$b_n = \frac{2}{l}\int_0^l f(x)\sin\left(\frac{n\pi x}{l}\right)dx$

$= \frac{2}{1}\int_0^1 2x\sin\left(\frac{n\pi x}{1}\right)dx$

$= 4\left[x\left(\frac{-\cos n\pi x}{n\pi}\right) - 1\left(\frac{-\sin n\pi x}{(n\pi)^2}\right)\right]_0^1$

$= 4\left(\frac{-(-1)^n}{n\pi}\right) = \boxed{\frac{-4(-1)^n}{n\pi}}$

15) find cosine integral of $f(x) = e^{-kx}$ $x > 0$ $k > 0$

16) sine integral of $f(x) = e^{-kx}$ $x > 0$ $k > 0$

15) → Fourier cosine integral

$B(\lambda) = 0$

$f(x) = \int_0^\infty A(\lambda)\cos\lambda x\,d\lambda$

$A(\lambda) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(x)\cos\lambda x\,dx$

$= \frac{2}{\pi}\int_0^\infty f(x)\cos\lambda x\,dx.$

$$= \frac{2}{\pi}\left[\int_0^\infty e^{-kx}\cos\lambda x\, dx\right]$$

$$= \frac{2}{\pi}\left[\frac{e^{-kx}}{k^2+\lambda^2}\left(-k\cos\lambda x + \lambda\sin\lambda x\right)\right]_0^\infty$$

$$= \frac{2}{\pi(k^2+\lambda^2)}\left[e^{-\infty}\left(-k\cos\infty + \lambda\sin\infty\right) - e^0\left(-k\cos 0 + \lambda\sin 0\right)\right]$$

$$= \frac{2}{\pi(k^2+\lambda^2)}\left[0 - 1(-k)\right] = \frac{2k}{\pi(k^2+\lambda^2)}$$

$$\boxed{A(\lambda) = \frac{2k}{\pi(k^2+\lambda^2)}}$$

Fourier sine integral

$$A(\lambda) = 0$$

$$f(x) = \int_0^\infty B(\lambda)\sin\lambda x\, d\lambda$$

$$B(\lambda) = \frac{1}{\pi}\int_{-\infty}^\infty f(x)\sin\lambda x\, dx$$

$$= \frac{2}{\pi}\int_0^\infty f(x)\sin\lambda x\, dx$$

$$= \frac{2}{\pi}\int_0^\infty e^{-kx}\sin\lambda x\, dx$$

$$= \frac{2}{\pi}\left[\frac{e^{-kx}}{k^2+\lambda^2}\left[-k\sin\lambda x - \lambda\cos\lambda x\right]\right]_0^\infty$$

$$= \frac{2}{\pi(k^2+\lambda^2)}\left[0 + \lambda\right] =$$

$$\boxed{B(\lambda) = \frac{2\lambda}{\pi(k^2+\lambda^2)}}$$

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