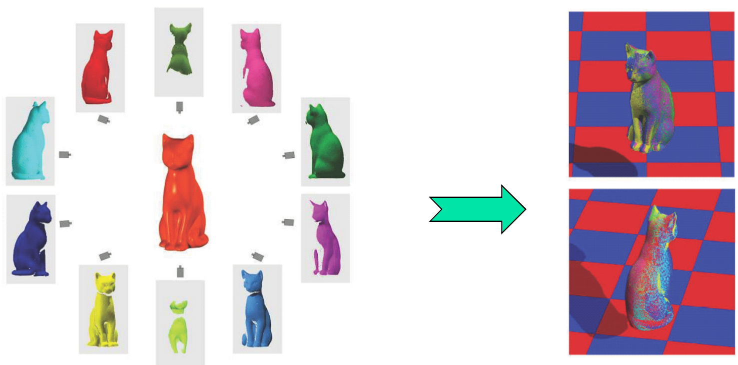
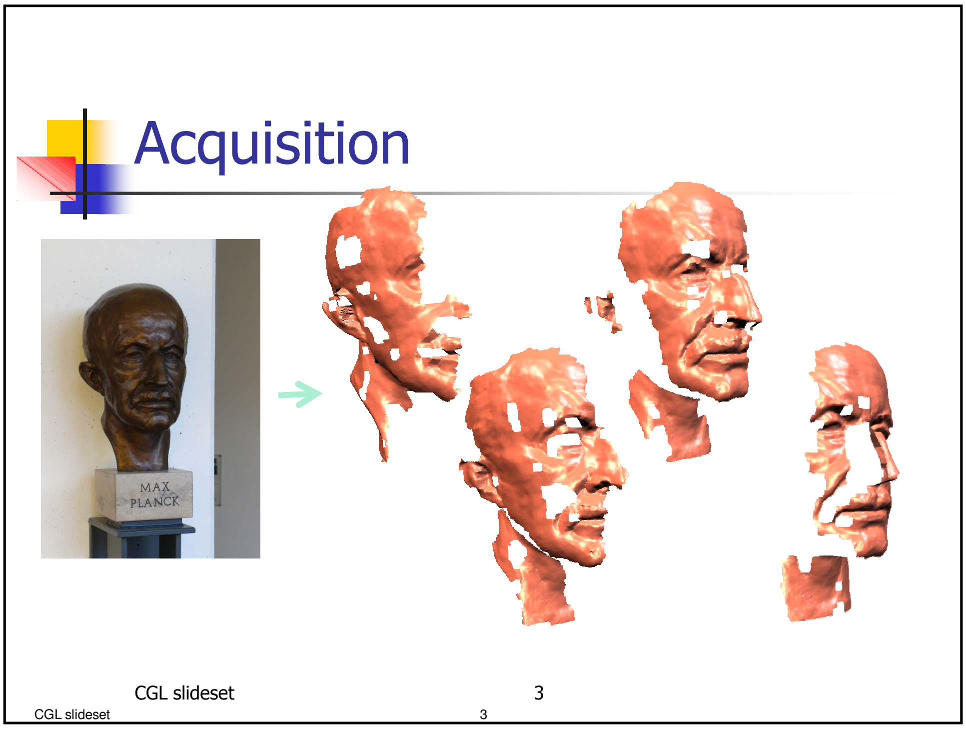
Iterative Closest Point

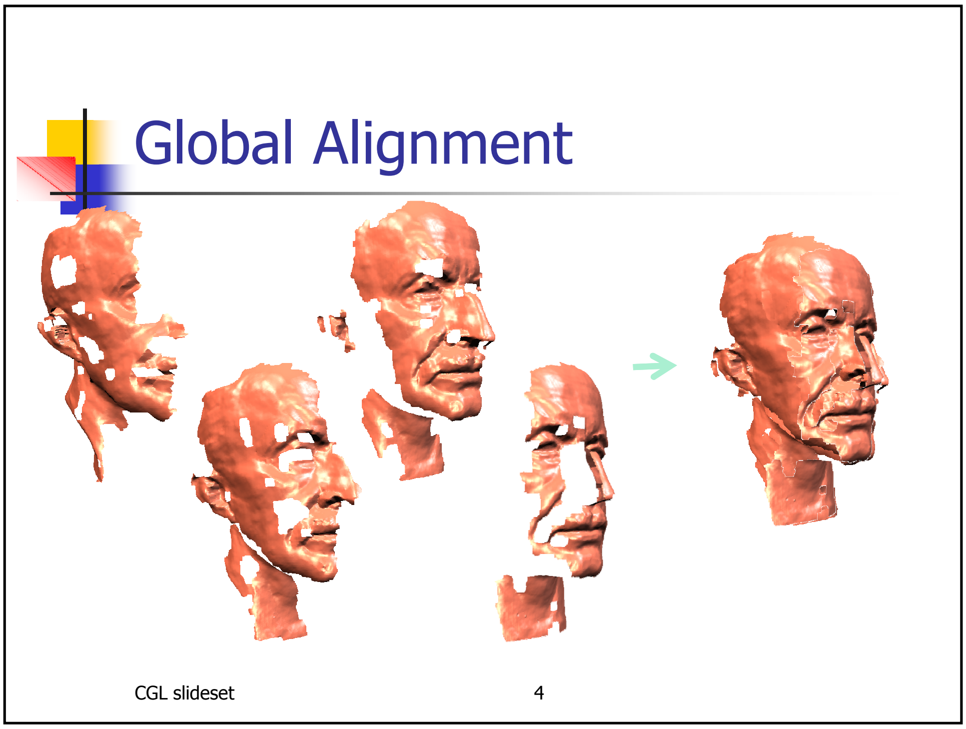
Motivation

- Align partially overlapping meshes

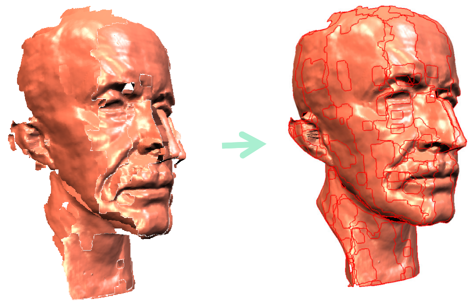


Images from: "Geometry and convergence analysis of algorithms for registration of 3D shapes" by Pottman


Acquisition
MAX PLANCK
CGL slideset 3
CGL slideset 3


Global Alignment
CGL slideset 4

Refinement by ICP



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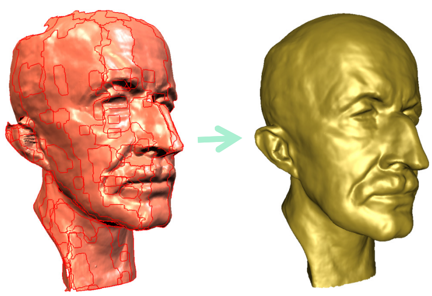
Reconstruction



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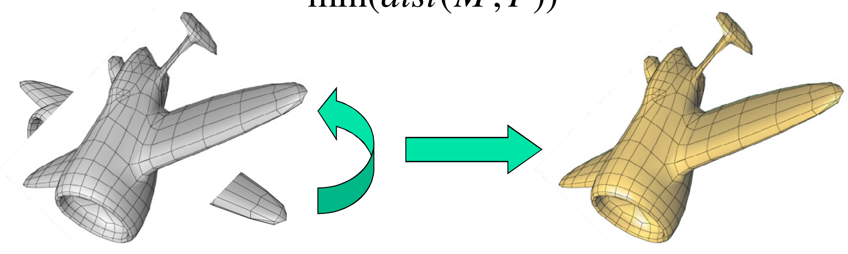
The Problem

- Input: Meshes *M,P*
- Output: Rotation *R* , translation *T*, s.t.

$$\tilde{M} = R * M + T$$
$$\min(dist(\tilde{M}, P))$$

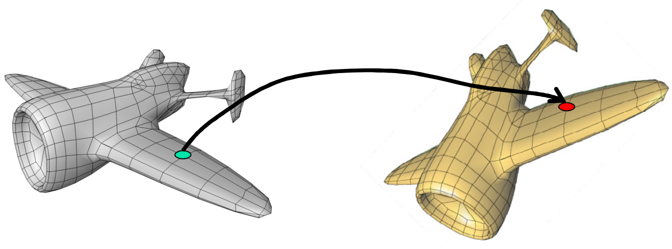


The Challenges

- Should support partial matching
- Should be robust to noise
- Should be efficient

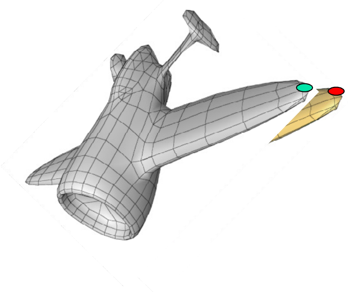
ICP Insight 1

- If correspondance is known, easy to find transformation



ICP Insight 2

- If transformation is known, easy to find correspondance (closest point)

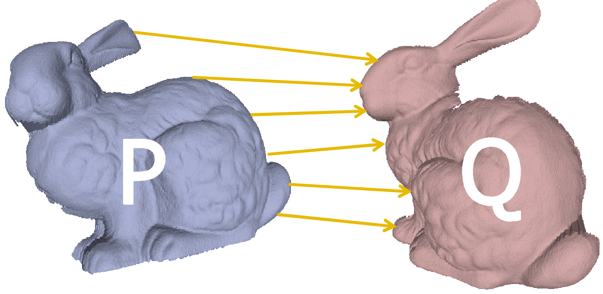


ICP Algorithm

- Start from initial guess
- Iterate
  - For each point on *M*, find closest point on *P*
  - Find best transform for this correspondance
  - Transform *M*

ICP Algorithm

- Step 1:
  Find some pairs of closest points (p_i, q_i)



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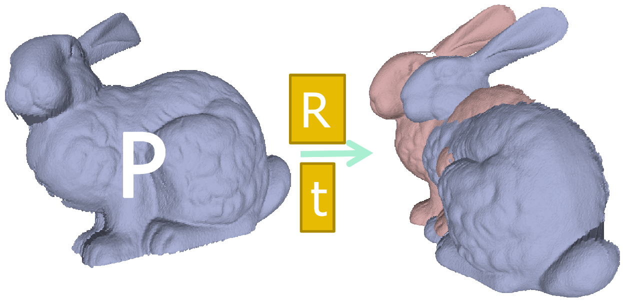
ICP Algorithm

- Step 2:
  Estimate rotation R and translation t



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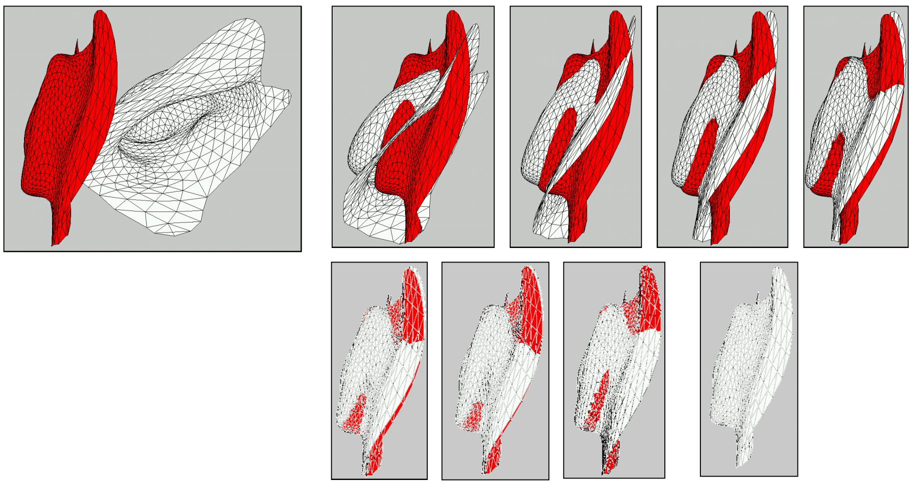
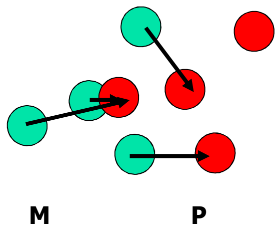
Example



Find Closest Point

- For each point in *M*
  - Choose closest point (Euclidean) from *P*



  - Minimizes $\frac{1}{|M|}\sum_{v\in M}\left\| v - match_p(v)\right\|_2^2$

Find Best Transform

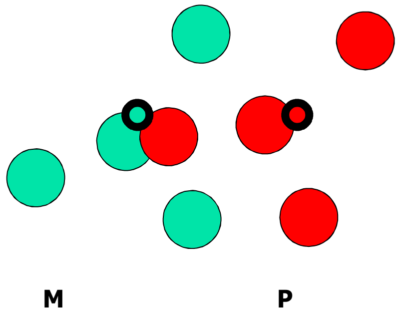
- Find R and T that minimize

$$\frac{1}{|M|}\sum_{v\in M}\left\| match_p(v) - (R * v + T)\right\|_2^2$$

- R – 3D rotation
- T – 3D translation

Find Best Transform

- Translation part – from centroids



$$T = avg(P) - R * avg(M)$$

Rotation Estimation

- Estimate Rotation
  - Approximate by general affine matrix

$$\min_{\mathbf{R}} \sum_i \|\hat{\mathbf{p}}_i - \mathbf{R}\hat{\mathbf{q}}_i\|^2 \longrightarrow \min_{\mathbf{A}} \sum_i \|\hat{\mathbf{p}}_i - \mathbf{A}\hat{\mathbf{q}}_i\|^2$$

Unitary Matrix

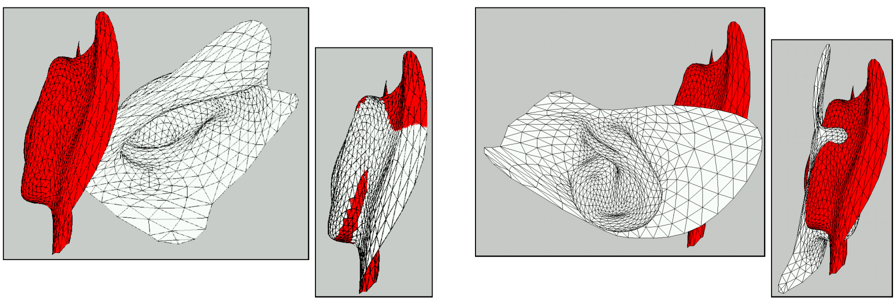
General Affine Matrix

Rotation Estimation - SVD

- A general affine matrix A induces
  - Rotation
  - Shear
  - Scaling
- Singular value decomposition: $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$
  - Exists for all matrices $\mathbf{R} = \mathbf{U}\mathbf{V}^T$
  - U,V – orthogonal
  - Σ – diagonal
  - Rotation + shear (and scaling) + rotation

Converges?

- Errors decrease monotonically
- Converges to local minimum
- Good initial guess → Converges to global minimum



Extensions

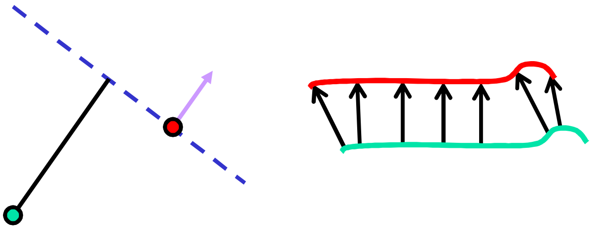
- Speed up correspondance
  - Use spatial subdivision
- Select only sample of points
- Different error metrics
- Change point matching
- Reject outliers

Points Sampling

- All points
- Uniform sampling
- Random sampling
- Uniform normal distribution

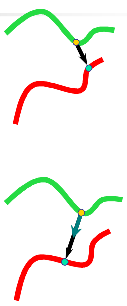
Error Metrics

- Point-to-plane distance instead of point to point



Point Matching

- Standard – closest point
  - Slow
- Normal shooting
  - Bad for noisy meshes



- Consider only compatible points
  - Same curvature, normals, colors

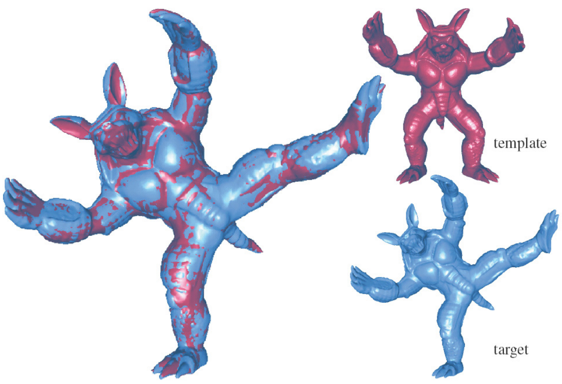
More Extensions

- Can be done in real time
  - Interactive scanning & registration

Movie from: "Efficient Variants of the ICP Algorithm" by Rusinkiewicz et al.

More Extensions

- Non rigid deformations



Images from: "Generalized Surface Flows for Mesh Processing" by Eckstein et al