

ARJUNA 2.0

-JEE 2024 Exam-

(14 JULY 2023)

MATHEMATICS

CHAPTER-02

SET THEORY

Lecture - 01



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Today's Targets



Introduction to Sets



Representation of Sets



Question Practice

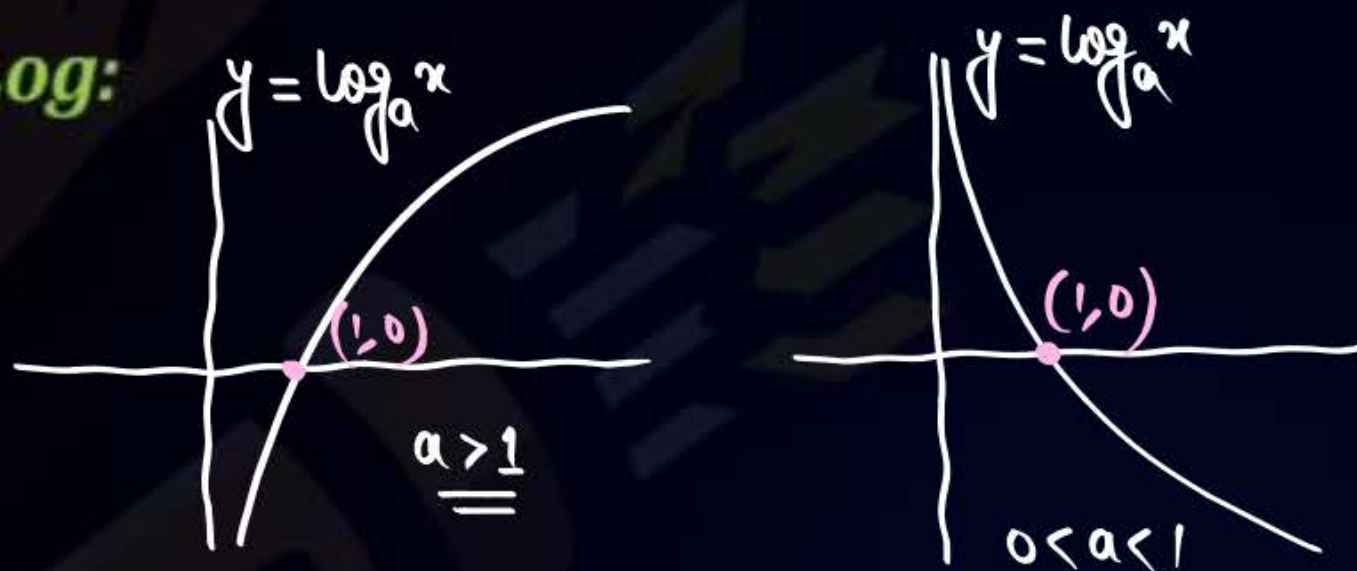




LAST CLASS



Graph of Log:



Log-Inequality:

Point-01 : If base $> 1 \Rightarrow \log$ / \log_a to sign of inequality will remain same.

Point-02 : If $0 < \text{base} < 1 \Rightarrow \log_a$ to sign of inequality will reverse.

Point-03 : Base \equiv variable \Rightarrow Make cases according to point - 01 & 02. ✓

Point-04 : Before final answer take intersection with Domain of Log.

Note : If number & base lies on same side of unity^① (both greater than 1 or both less than 1), then value of log is +ve.

If number & base lies on opposite side of unity, then value of log is -ve.

$$\log_2 4 = +2$$

$$\log_3 27 = +3$$

$$\log_2 5 = 2.32$$

$$\log_{\frac{1}{2}} \frac{1}{4} = +2$$

$$\log_{\frac{1}{3}} \frac{1}{27} = +3$$

$$\log_{\frac{1}{2}} \frac{1}{5} = 2.32$$

$$\log_{\frac{1}{2}} 4 = -2$$

$$\log_3 \frac{1}{27} = -3$$

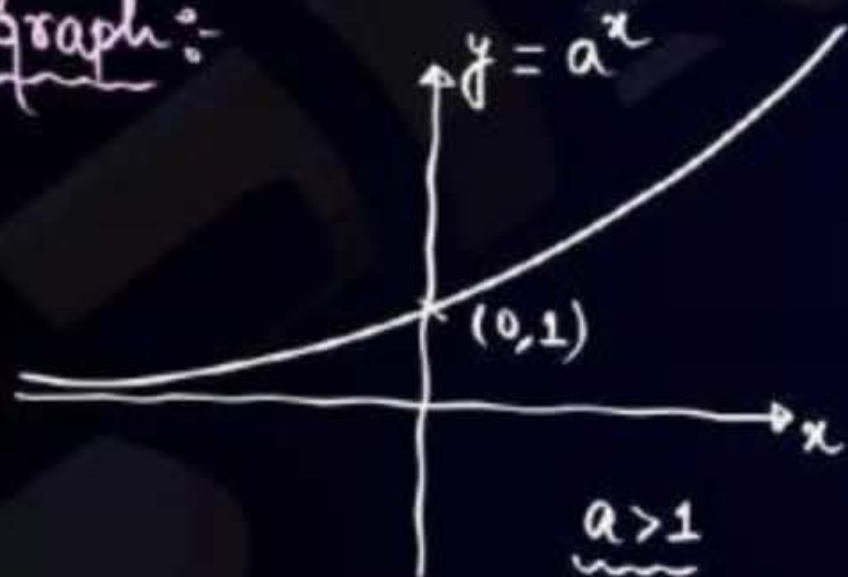
"exponential function":-

$$a=1 \Rightarrow y = (1)^x = 1 \Rightarrow \text{horizontal line at } y=1$$

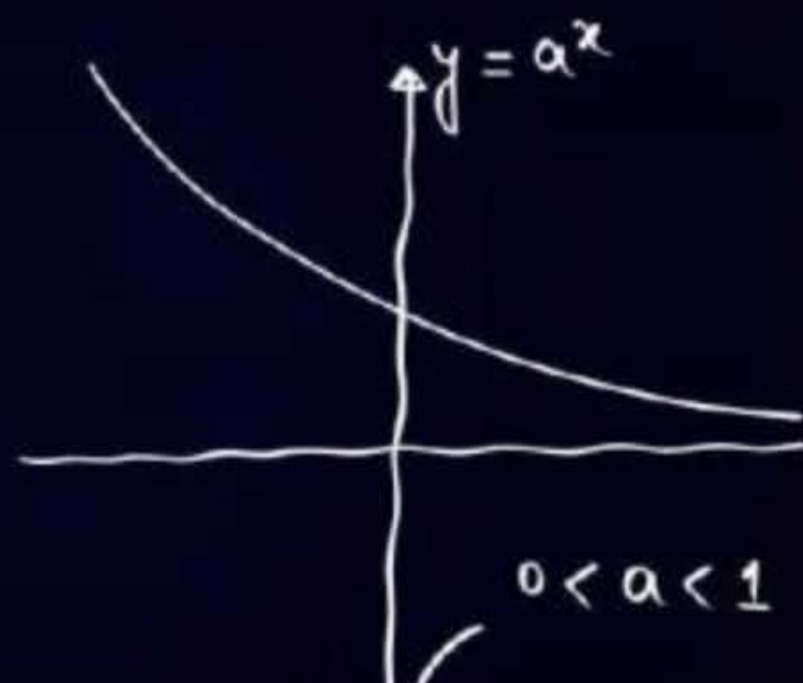
$$y = a^x, (a > 0), (a \neq 1)$$

ex: $y = 2^x, 3^x, 5^x, \dots, (\frac{1}{2})^x, (\frac{1}{3})^x, \dots$

Graph:-



ex: $y = 2^x, 3^x, 4^x, \dots$



ex: $y = (\frac{1}{2})^x, (\frac{1}{3})^x, (\frac{1}{4})^x, \dots$

Question



Solve following inequality :

(iii)

$$\log_{0.5} \left(\log_8 \left(\frac{x^2 - 2x}{x - 3} \right) \right) < 0$$

$$\log_{\frac{1}{2}} \left(\log_8 \left(\frac{x^2 - 2x}{x - 3} \right) \right) < 0$$

$$\log_8 \left(\frac{x^2 - 2x}{x - 3} \right) > \left(\frac{1}{2} \right)^0 = 1$$

$$\log_8 \left(\frac{x^2 - 2x}{x - 3} \right) > 1$$

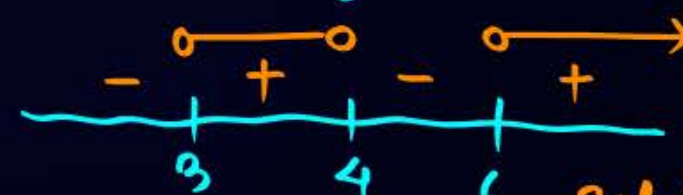
$$\frac{x^2 - 2x}{x - 3} > 8$$

$$\frac{x^2 - 2x - 8}{x - 3} > 0$$

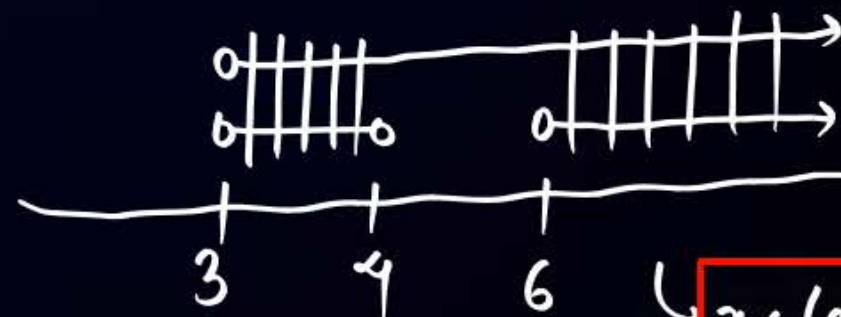
$$\frac{x^2 - 2x - 8x + 24}{x - 3} > 0$$

$$\frac{x^2 - 10x + 24}{x - 3} > 0$$

$$(x - 6)(x - 4) > 0$$



Solution.



$$x \in (3, 4) \cup (6, \infty)$$

def

defining :-

$$\log_8 \left(\frac{x^2 - 2x}{x - 3} \right) > 0 \cap \frac{x^2 - 2x}{x - 3} > 0$$

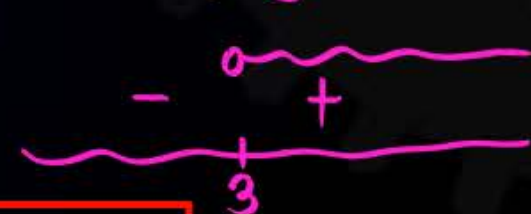
$$\frac{x^2 - 2x}{x - 3} > 1$$

$$\frac{x^2 - 2x}{x - 3} > 1$$

$$\frac{x^2 - 2x - x + 3}{x - 3} > 0$$

$$\frac{x^2 - 3x + 3}{x - 3} > 0$$

$$\frac{x^2 - 3x + 3}{x - 3} > 0$$



Solve : $\log_{\frac{1}{x}} \left(\frac{2(x-2)}{(x+1)(x-5)} \right) \geq 1$

Solve:

C-1: $\boxed{\frac{1}{x} > 1} \Rightarrow x \in (0, 1)$

$\frac{2(x-2)}{(x+1)(x-5)} \geq \frac{1}{x}$

$\frac{2(x-2)}{x^2-4x-5} - \frac{1}{x} \geq 0$

$\frac{2x^2 - 4x - x^2 + 4x + 5}{x(x^2-4x-5)} \geq 0$

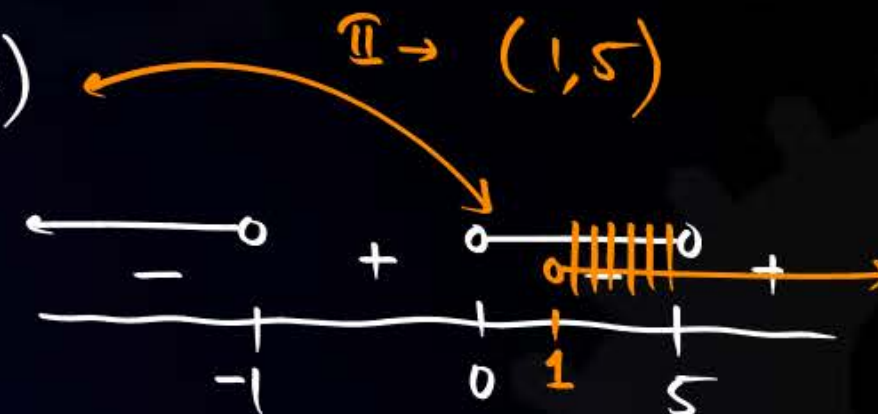
$\frac{x^2+5}{x(x+1)(x-5)} \geq 0$



C-II

$0 < \frac{1}{x} < 1 \Rightarrow x \in (1, \infty)$

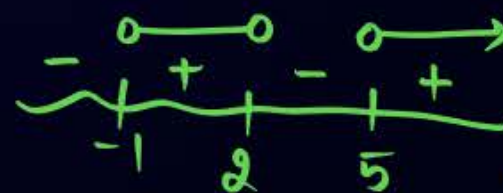
$\frac{x^2+5}{x(x+1)(x-5)} \leq 0$



Solution $x \in (1, 5)$

def:

$\frac{2(x-2)}{(x+1)(x-5)} > 0, \frac{1}{x} > 0, \frac{1}{x} \neq 1$



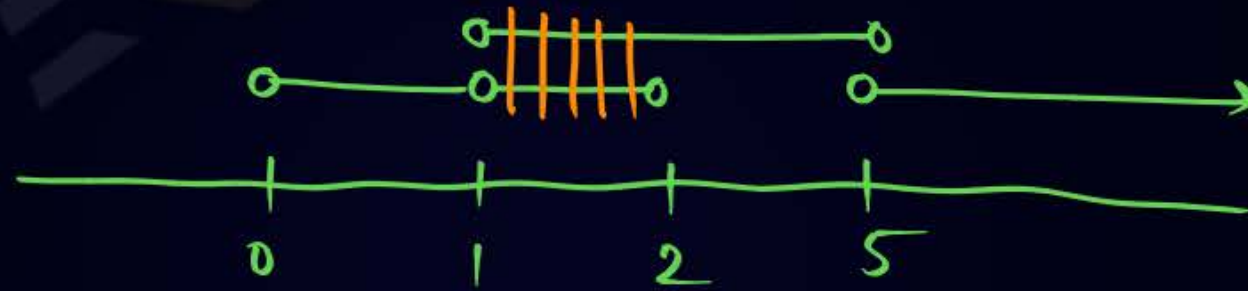
$x \in (0, 1) \cup (1, 2) \cup (5, \infty)$

Solⁿ:

$$x \in (1, 5)$$

Def:

$$x \in (0, 1) \cup (1, 2) \cup (5, \infty)$$



$$x \in (1, 2) \text{ for}$$

Question



Solve following exponential equations:

(i) $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

(ii) $4^x - 5 \cdot 2^x + 6 = 0$

$$4^x - \frac{3^x}{\sqrt{3}} = 3^x \cdot \sqrt{3} - \frac{4^x}{2}$$

$$4^x + \frac{4^x}{2} = 3^x \sqrt{3} + \frac{3^x}{\sqrt{3}}$$

$$4^x \left(1 + \frac{1}{2}\right) = 3^x \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$$

$$4^x \left(\frac{3}{2}\right) = 3^x \left(\frac{4}{\sqrt{3}}\right)$$

$$\log_2 3, 1.$$

$$\frac{4^x}{2 \times 4} = \frac{3^x}{3 \sqrt{3}}$$

$$\frac{2^{2x}}{2^3} = \frac{3^x}{3^{3/2}}$$

$$2^{2x-3} = 3^{x-\frac{3}{2}}$$

$$\left. \begin{array}{l} 2x-3=0 \\ x-\frac{3}{2}=0 \end{array} \right\} x = \frac{3}{2}$$

(ii) $4^x - 5 \cdot 2^x + 6 = 0$

$$2^x = t \Rightarrow 4^x = t^2$$

$$t^2 - 5t + 6 = 0$$

$$(t-3)(t-2) = 0$$

$$t = 3, +2$$

$$2^x = 3, \quad 2^x = 2$$

$$\log_2()$$

$$\log_2(2)^x = \log_2 3$$

$$x = \log_2 3$$

$$x = 1 \checkmark$$

Question



Solve : $4^{\sqrt{x^2-2}+x} - 5 \cdot 2^{\underline{x-1}+\sqrt{x^2-2}} = 6$

$\underline{\underline{EF}}$ # $\frac{x+\sqrt{x^2-2}}{2} = \frac{-\frac{3}{2}}{2} \xrightarrow{-ve} \Rightarrow \underline{\underline{n.p.}}$

$4^{\sqrt{x^2-2}+x} - 5 \cdot \frac{2^{x+\sqrt{x^2-2}}}{2} = 6$

$2^{x+\sqrt{x^2-2}} = t$

$(2^2)^{\sqrt{x^2-2}+x} = (2^{\sqrt{x^2-2}+x})^2 = t^2$

$t^2 - \frac{5}{2}t = 6$

$2t^2 - 5t = 12$

$2t^2 - 5t - 12 = 0$

$2t^2 - 8t + 3t - 12 = 0$

$2t(t-4) + 3(t-4) = 0$

$(2t+3)(t-4) = 0$

$t = -\frac{3}{2}, 4$
 $\frac{2}{x}$

$2^{x+\sqrt{x^2-2}} = 2^2$

$x + \sqrt{x^2-2} = 2$

$\sqrt{x^2-2} = 2-x$

sq. $x^2 - 2 = 4 + x^2 - 4x$
 $4x = 6$

$x = \frac{3}{2}$
 $\frac{3}{2}$

Question



Solution set of exponential inequality $4^x - 3(2)^x - 4 < 0$

A $(-\infty, -2)$

B $(2, \infty)$

C $(-\infty, 2]$

D $(-\infty, 2)$

$$\begin{aligned} 2^x &= t \\ t^2 - 3t - 4 &< 0 \\ t^2 - 4t + t - 4 &< 0 \\ (t-4)(t+1) &< 0 \\ (2^x-4)(\overbrace{2^x+1}^{+ve}) &< 0 \end{aligned}$$

$$2^x = 4 \Rightarrow x = 2$$

$$2^x + 1 = 0 \rightarrow 2^x = -1 \rightarrow \text{n.p.}$$

Question



Solve following inequality :

(i) $x^2 5^x - 5^{2+x} < 0$

(ii) $\frac{2^{x-1}-1}{2^{x+1}+1} < 2$

new

* NOTE:

$a^b = 1$

$a = 1$
 $b \equiv \text{kuch bhi}$
 $b \in \mathbb{R}$

include

$a = 1$
 $b = 1$

$b = 0$
 $a \equiv \text{kuch bhi except zero}$
 or
 $a \neq 0$

$a = -1$
 &
 $b \equiv \text{aisi value}$
 jo dHS ko 1 bnade.



IMPORTANT POINT TO NOTE



JEE wale puchte hain kyunki ismein cases sochne pdte hai

$$(v_1)^{v_2} = 1$$

Case-I :

$$v_1 = 1 \text{ \& } v_2 \in R$$

Solve \rightarrow
$v_1 = 1$

Case-II :

$$v_1 = -1 \text{ \& }$$

$v_2 =$ aisa value jo
LHS ko '1' bna de

Solve : $v_1 = -1$
& check your ans

Case-III :

$$v_2 = 0 \text{ \& } v_1 \neq 0$$

$v_2 = 0 \rightarrow$
Solve
 \Downarrow
Check your ans

The sum of all real values of x satisfying

$$(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1 \text{ is}$$

A 5

B 3

C -4

D 6

C-I: $v_1 = 1$
 $x^2 - 5x + 5 = 1$
 \Downarrow
 $x^2 - 5x + 4 = 0$
 \Downarrow
 $(x-1)(x-4) = 0$
 $\hookrightarrow x = 1, 4$

C-III: $v_2 = 0$
 $x^2 + 4x - 60 = 0$
 $x^2 + 10x - 6x - 60 = 0$
 $(x+10)(x-6) = 0$
 $\hookrightarrow x = 6, -10$

Check: $(36 - 30 + 5)^0 = 11^0 = 1 \checkmark$
 $(100 + 50 + 5)^0 = 155^0 = 1 \checkmark$

fnf $\Rightarrow x = 1, 4, 6, -10, 2$
 Sum = 3

C-II: $v_1 = -1$
 $x^2 - 5x + 5 = -1$
 $x^2 - 5x + 6 = 0$
 $\hookrightarrow (x-3)(x-2) = 0$
 $\hookrightarrow x = 2, 3$

Check:-
 $x=2 \Rightarrow (-1)^{-48} = \frac{1}{(-1)^{48}} = 1$
 $x=3 \Rightarrow (-1)^{-39} = \frac{1}{(-1)^{39}} = -1 \neq 1$
 \downarrow
 Reject.



IMPORTANT POINT TO NOTE



Memorise

#

$$x + \frac{1}{x} \geq 2, \quad x > 0$$

$$x + \frac{1}{x} \leq -2, \quad \underline{\underline{x < 0}}$$

$$x + \frac{1}{x} \in \underbrace{(-\infty, -2]} \cup \underbrace{[2, \infty)}$$

\downarrow \downarrow
 $x \rightarrow \text{(-ve)}$ $x \rightarrow \text{(+ve)}$

Solve:-
ex:- $\log \left(x + \frac{1}{x} \right) \geq 1$
 \rightarrow 2 se bada.
 $x \geq \left(x + \frac{1}{x} \right)$
 $0 \geq \frac{1}{x}$
 \downarrow
 $\frac{-}{+}$
 0

Solⁿ $\rightarrow x \in (-\infty, 0)$

def $x > 0, x + \frac{1}{x} > 0, x + \frac{1}{x} \neq 1$

$\frac{0}{0}$
 ϕ

\downarrow
 $x > 0$
 \checkmark
 $\rightarrow x \in (0, \infty)$

BASIC MATHEMATICS

- ✓ ☒ Video Lecture
- ✓ ☒ Notes
- ✓ ☒ Class-Room example khud se solve
- ✓ ☒ DPP's
- ✓ ☒ Short Notes
- ✓ ☒ ☒ ☒ ☒ Revision

SET THEORY

- # Most easy chapter
- # Mains - Ques. ✓
- # Advanced → use hoga.
↳ Ques. ✓

Set

group of "well defined" objects / things / numbers.

$$\# A = \{1, 2, 3, 4, 5\}$$

elements / members.

$$\# 5 \in A$$

$$\# 6 \notin A$$

belongs to.

$$\# X = \{a, b, c\}$$

$$a \in X$$

$$b \in X$$

$$c \in X$$



DEFINITION OF SET



A set is well-defined **collection of distinct objects**. Sets are usually denoted by capital letters **A, B, C, X, Y, Z**.

Note:

The elements of the set are denoted by small letters. If x is an element of a set A , we write $x \in A$ (read as 'x belongs to A')

If x is not an element of A , then we write $x \notin A$ (read as 'x does not belongs to A').

For example : If $A = \{1, 2, 3, 4, 5\}$, then $3 \in A$, $6 \notin A$.

Which of following statements is well defined :

- ✓ (i) Collection of even number. → well def ✓
- ✓ (ii) Collection of Best Teacher. → not well defined.
- ✓ (iii) Collection of Most Talented writers of India. → not well defined
- ✓ (iv) Collection of all roots of equation : $(x^2 - 5x + 6 = 0)$ → well defined
- ✓ (v) Collection of all the vowels in English alphabets.

well defined^{*}
A = {a, e, i, o, u}

$x = 2, 3$
A = {2, 3}

Note :

Cardinal numbers of a set

Or

Order of a set

Or

Number of elements in a set

$n(A)$ = Number of elements in set (A)

$A = \{1, 2, 17\}$
 cardinal no. = 3.
 order of set A = 3.
 ** $n(A) = 3$.



1. Roster form or Tabular Form :

Structure : $A = \{ , , , , \dots \}$

elements

↓ ↓ ↓

$$A = \{ -, -, -, \dots \}$$

Notes :

- (i) Repetition of elements (numbers) is not allowed.*
- (ii) Order of elements (numbers) is not important.

Wrong

$$A = \{ 1, 1, 2, 2, 3 \}$$

↘

$$A = \{ 1, 2, 3 \}$$

OR

$$A = \{ 3, 1, 2 \}$$

$$A = \{ 2, 1, 3 \}$$

⋮

Ex. : Form a set of all the “vowels”

$$\hookrightarrow X = \{a, e, i, o, u\} \Rightarrow n(X) = 5$$

Ex. : Form a set of first 5 whole numbers.

$$\hookrightarrow A = \{4, 1, 3, 0, 2\} \Rightarrow n(A) = 5$$

Ex. : Form a set of roots of equation : $(x^2 - 23x + 120 = 0)$

$$\hookrightarrow x^2 - 18x - 5x + 120$$

$$(x - 8)(x - 15) = 0$$

$$\hookrightarrow x = 15, 8$$

$$\hookrightarrow A = \{15, 8\} \Rightarrow n(A) = 2$$

Question



If set X denotes set of all the letters of word **HONOLULU**, then cardinal number of set X is 5.

H, O, N, O, L, U, L, U.

$$X = \{h, o, n, l, u\}$$

OR

$$X = \{H, O, N, L, U\}$$

$\rightarrow n(X) = 5.$

Acidic Question



Solve : $\log_{\pi/4} \log_2 \log_{1/3}(x) \geq 0$

Ans. $x \in \left[\frac{1}{9}, \frac{1}{3}\right)$

HW

Simplify following :

$$\sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 =$$

HW

DAILY HOME WORK

All DPP's till date & Re-attempt all the Ques. of Today's Lecture.

Module:

Chapter-01: Basic Mathematics.

Exercise-04: Complete (Solve it Partwise)

#futureITians

THANK YOU

