Practice Final Examination Math 100

Instructions:

1. Attempt every question
2. All work must be shown. No marks are given for answers alone.
3. No marks are given for guess and test methods.
4. You have 2h to complete this practice exam.
5. The maximum total score is 50.
6. No calculators allowed.

Question 1. (3 points) Show that:

$$\int_0^{-\frac{1}{2}} \ln\left(\frac{1+x}{1-x}\right)\, dx = \int_0^{\frac{1}{2}} \ln\left(\frac{1+x}{1-x}\right)\, dx$$

Question 2. (6 points) Use the definition of the definite integral to compute:

$$\int_0^1 (x^2 + x + 1)\, dx,$$

by using as sample points the right-end points of each subinterval of division of $[0,\ 1]$.

Make use of the following sums:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}.$$

Question 3. (8 points) Let

$$f(x) = \begin{cases} \frac{x^2}{1+x^2} & 0 \le x \le 1 \\ \frac{x^2}{2} & 1 < x \le 2 \end{cases}$$

Find the function

$$F(x) = \int_0^x f(t)\, dt$$

for $x \in [0,\ 2]$.

Question 4. (4 points) Evaluate the limit

$$\lim_{x \to +\infty} \left(\frac{2x+2}{2x+1}\right)^x.$$

Question 5. (9 points) Evaluate the following integrals:

$$\int \frac{1}{\sqrt{x(1-x)}}\,dx \qquad \text{and} \qquad \int_1^2 \frac{e^{1/x}}{x^2}\,dx$$

Question 6. (7 points) Find the dimensions of the isosceles triangle of largest area that can be inscribed in a circle of radius r.

Question 7. (7 points) For the function $f(x) = x^2 e^x$, make a sketch by analyzing f, f', f''. Be sure to identify and label clearly all intercepts, asymptotes, local extrema and inflection points. Provide charts of the first and second derivatives to support your answers.

Question 8. (6 points) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by:

$$2x + 3y = 6, \quad x - 4 = -(y-1)^2,$$

about $x = -5$.