ENGR–3546: SYSTEM MODELING AND SIMULATION
LAB NO. 5

SOLAR PANEL ELEVATION ANGLE CONTROL

Due date : December 7th, 2017 at 11 pm

Note : Complete the lab in groups of 2. Each group must submit only one report.

1 Lab objectives

— Develop the dynamic model an electromechanical system.
— Linearize a nonlinear model about a nominal operating point.
— Compare the response of a system based on linear and nonlinear models.
— Observe the effect of the nominal operating point used for linearization purposes on the validity of the resulting linear model.

2 Reference documents

— Simulink user guide : `http://www.mathworks.com/help/pdf_doc/simulink/sl_gs.pdf`
— MathWorks Matlab website : `http://www.mathworks.com/products/matlab/`
— MathWorks Simulink website : `http://www.mathworks.com/products/simulink/`

3 Description of the system and the design scenario

In order to supply electricity to a pump used to irrigate a secluded potato field, a farmer on Prince Edward Island [1] uses batteries that are recharged using a solar panel. With the goal of optimizing the solar energy produced by the panel, its orientation with respect to the sun (i.e., azimuth and elevation angles) can be controlled using two shunt DC motors (one for each angle). You have been contacted by the farmer to help in the design of a wormset-based transmission for the motor that is used to control the elevation angle of the solar panel.

An abstraction of the system used to control the solar panel's elevation angle is shown in Figure 1. The solar panel assembly, assumed to be a homogeneous and rigid cuboid, has mass m, length L, width w and thickness d (values are provided in Table 1). The panel is attached along one of its edges to a shaft supported by two bearings. The thickness of the solar panel assembly is considered negligible such that, from the point of view of its mass moment of inertia about the shaft

1. Home of the best potatoes worldwide (apologies to Don Poulin's in Azilda) !

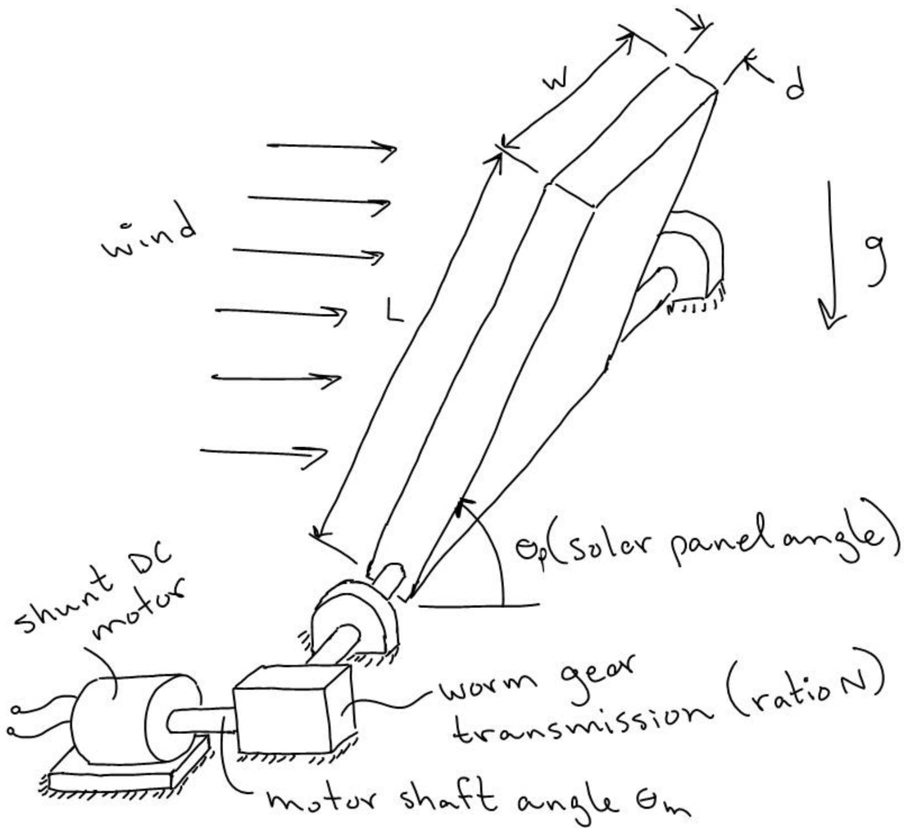

FIGURE 1 – Solar panel elevation angle actuation system.

TABLE 1 – Properties of the solar panel assembly.

Mass :	$m = 20$ kg
Length :	$L = 2$ m
Width :	$w = 2$ m
Thickness :	$d = 10$ cm

axis, it may be modeled as a long slender bar of length L. The angle measured from the horizontal to the solar panel is denoted by θ_p.

As its elevation angle is being changed, the solar panel will be subjected to a torque τ_p applied to its shaft by the motor through the wormset transmission, its own weight (use $g = 9.81$ m/s^2) as well as a drag force due to the wind. The wind is assumed to be blowing at a speed $\mathcal{V}_{\text{wind}}$ in a horizontal direction perpendicular to the axis of the solar panel's shaft. The drag force, distributed over the area of the solar panel, can be represented simply as a force F_D acting through the panel's centre of mass such that

$$F_D = \frac{1}{2}\rho C_d \mathcal{V}_{\text{wind}}^2 A_\perp \tag{1}$$

where values for the atmospheric air mass density ρ, the average drag coefficient C_d and the design wind speed are provided in Table 2. As observed in Eq. (1), the drag force is proportional to the

Table 2 – Parameters determining the wind drag force.

Mass density of atmospheric air at 15° C :	$\rho = 1.225$ kg/m^3
Average drag coefficient :	$C_d = 0.75$
Wind speed :	$\mathcal{V}_{\text{wind}} = 40$ km/h

Table 3 – Properties of the DC shunt motor.

Motor constant :	$k_m = 0.1$ N·m/A
Armature resistance :	$R_a = 0.75\ \Omega$

area of the solar panel that is perpendicular to the wind direction, i.e.,

$$A_\perp = A \sin\theta_p \tag{2}$$

where $A = Lw$.

The shunt DC motor is connected to a set of batteries providing a DC source voltage $v(t)$. Values for the motor constant k_m and armature resistance R_a are provided in Table 3 (note that the armature inductance is neglected for this analysis). The motor produces a torque τ_m that is related to the source voltage and the motor speed according to

$$\tau_m = \frac{k_m}{R_a}\left[v(t) - k_m\dot{\theta}_m\right] \tag{3}$$

where θ_m is the angular position of the motor's shaft.

The motor's shaft is connected to the solar panel's shaft through a wormset-based transmission. This transmission consists of a worm and a worm gear meshing together as illustrated in Figure 2. The gear ratio N provided by the wormset is defined such that

$$\tau_p = N\tau_m \tag{4}$$

and

$$\theta_m = N\theta_p \tag{5}$$

where τ_p is the torque applied to the solar panel's shaft by the worm gear. With typical gear ratios ranging from 1 : 1 to 360 : 1, one advantage of a wormset transmission is that it can be made to be non-backdrivable. This means that while applying a torque to the worm will turn the worm gear, one cannot turn the worm by applying a torque to the worm gear (due to friction). For the application being studied, this implies that the motor will not "feel" the torques generated by the weight of the solar panel assembly and the wind's drag force *when the system is at rest.* As such, electrical power is only consumed by the motor when the solar panel's elevation angle must be changed. It is very important to note, however, that the weight and wind drag force are felt by the motor when the system is in motion and they must be considered in the system model to be

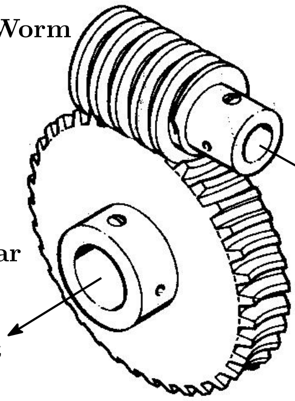

FIGURE 2 – Illustration of a typical wormset.

developed.

In the actual implementation of the proposed system, the source voltage provided to the DC motor would need to be varied based on an adequate control algorithm to allow the elevation angle of the solar panel to be at the optimal value for energy harvesting. However, for the purpose of the lab exercises described herein, the source voltage will be applied as a simple step input, i.e., $v(t) = V u_s(t)$, with $V = 20$ V. This will suffice to allow for an understanding of the level of approximation of linearized dynamic models as well as a preliminary design and validation of the proposed system.

4 Procedure

1. Draw a proper free body diagram (FBD) of the solar panel assembly showing all forces, torques and inertial effects. Using the FBD, find the dynamic model of the solar panel assembly with the torque applied to the solar panel's shaft by the worm gear (i.e., τ_p) as the input and the angular position of the panel (i.e., θ_p) as the output. Include the FBD as well as all mathematical developments leading to the dynamic model in your report. **Do not substitute numerical parameter values at this time.**

2. Starting with the dynamic model obtained in the previous step, substitute Eqs. (1) through (5) to obtain the dynamic model for the entire system (i.e., solar panel, motor, transmission, environment) with the source voltage $v(t)$ as the input variable and the solar panel angle θ_p as the output. The dynamic model will correspond to a nonlinear differential equation of motion. In your report, express the equation of motion in the following form :

$$B_1\ddot{\theta}_p + B_2\dot{\theta}_p + B_3\cos\theta_p + B_4\sin^2\theta_p = B_5 v(t) \tag{6}$$

   and clearly define the B_i coefficients ($i = 1, 2, \ldots, 5$) while showing intermediate results as appropriate. **Do not substitute numerical parameter values at this time.**

3. Linearize the dynamic model about a nominal operating point based on the following procedure :
   (a) Determine the nominal operating point about which to linearize the nonlinear model :
      i. Represent the nominal value of the output as $\overline{\theta}_p$. This value will eventually be chosen by you during the lab (keep it in symbolic form for now).
      ii. Using Eq. (6), solve for the corresponding nominal value of the input $\overline{v}$ by substituting $v = \overline{v}$ and $\theta_p = \overline{\theta}_p$ and setting all time derivatives to zero. Show all of your work and report on the expression found for $\overline{v}$ as a function of $\overline{\theta}_p$ in your report.
   (b) Using the Taylor series expansion approach, linearize the nonlinear terms appearing in Eq. (6), i.e., $\cos(\theta_p)$ and $\sin^2(\theta_p)$, about the nominal operating point defined by $\overline{\theta}_p$. In your report, clearly show the linearized versions of each nonlinear term as well as all mathematical developments used. Each expression should be a function of $\overline{\theta}_p$ and $\Delta\theta_p = \theta_p - \overline{\theta}_p$, i.e., $\cos(\theta_p) \approx f_1(\overline{\theta}_p, \Delta\theta_p)$ and $\sin^2(\theta_p) \approx f_2(\overline{\theta}_p, \Delta\theta_p)$.
   (c) Make the following substitutions in Eq. (6) :
      — The linearized versions of the nonlinear terms (obtained in the previous step).
      — $\ddot{\theta}_p = \Delta\ddot{\theta}_p$
      — $\dot{\theta}_p = \Delta\dot{\theta}_p$
      — $v = \overline{v} + \Delta v$

      and cancel terms as appropriate based on the previously determined nominal operating point.
   (d) Express the linearized dynamic model of the system in the following form :

$$C_1\Delta\ddot{\theta}_p + C_2\Delta\dot{\theta}_p + C_3\Delta\theta_p = C_4\Delta v \tag{7}$$

      Provide this model in your report while clearly defining the values of coefficients C_j ($j = 1, 2, \ldots, 4$) as functions of the system's parameters as well as the B_i coefficients as appropriate. **Do not substitute numerical parameter values at this time.**

4. Observe the change in the term representing the effect of the wind drag force on the system as the dynamic model is linearized (i.e., compare this term in the nonlinear and linear models). What special behaviour would manifest itself if the model linearization were performed around a nominal operating point defined by $\overline{\theta}_p = 0$? Provide a brief discussion of the situation in your report.

5. Download the following files from D2L to your working folder :
   — `parameters.m`
   — `nonlinear_model.m`
   — `solar_panel_simulation.mdl`

6. Open the `parameters.m` file and enter the parameter values using the units listed as well as the required expressions to compute I, the B_i and C_j coefficients and $\overline{v}$. For the moment, do not specify the wormset ratio N or the nominal solar panel angle $\overline{\theta}_p$.

NOTE : For the next sequence of steps use $N = 102.5$. You will not be using the linearized model just yet so $\overline{\theta}_p$ has no impact on the results you will obtain (you can use $\overline{\theta}_p = 0$).

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7. Enter the values of N and $\overline{\theta}_p$ in the `parameters.m` file. Run the script. All parameters and computed coefficients should now be stored in your MATLAB workspace.

8. Open the `solar_panel_simulation.mdl` SIMULINK model file. This file contains a SIMULINK program of the solar panel system's nonlinear model. Explore the `Solar panel tilt mechanism: Nonlinear model` subsystem to understand how a nonlinear model may be simulated using MATLAB/SIMULINK. Open the `nonlinear_model.m` file and explore its contents as well.

9. In `Simulation` $\rightarrow$ `Configuration parameters`, verify that `Max step size` is set to `0.01`. Set `Stop time` to `30`. Verify that `output_switch` is in the upper position.

10. Open the `v_step` step input block and modify it to match the desired input for the system.

11. Run the simulation. Open the `theta v. time` scope and observe the results. In your report, explain the system's curious behaviour in the $10 \leq t \leq 20$ seconds time range (approx.). Base your answer on your understanding of the mechanical aspects of the system including the forces that are applied to the solar panel assembly. You are not required to include the plot in your report.

NOTE : For the next sequence of steps use $N = 85$.

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12. Prepare a block diagram for the linearized dynamic model with Δv as the input and $\Delta\theta$ as the output. Include a sketch of your diagram in your report along with any supporting mathematical equations. *Hint : Be wary of unit consistency.*

13. In the `solar_panel_simulation.mdl` file, open the `Solar panel tilt mechanism: Linear model` subsystem. Delete the direct link between the subsystem's input and output (i.e., `v` and `theta`) and replace it with your block diagram (obtained above).

    Important : Keep in mind that the subsystem's input and output are v and θ, respectively, while for the block diagram these are Δv and $\Delta\theta$. You will need to add additional summation blocks to the SIMULINK diagram to convert the inputs and outputs as required using $\overline{v}$ and $\overline{\theta}_p$.

14. Simulate the system for the following values of $\overline{\theta}_p$:

$$\overline{\theta}_p = 0°,\ 10°,\ 20°,\ 30°,\ 40° \text{ and } 50° \tag{8}$$

    In each case, you must update the value of `theta_bar` in the `parameters.m` file. Moreover, you must save the simulation data (i.e., θ_p vs t) obtained using both the nonlinear and linear models for each value of $\overline{\theta}_p$. The `solar_panel_simulation.mdl` file is already setup to export the data to a set of `*.mat` files (one for each simulation). Switch `output_switch` to the lower position to export the data from both models. Also, you will want to change the file name in the `To File` block for each simulation. For example, use `panel_rotation_0`, `panel_rotation_10`, etc.

15. In a MATLAB script named `plotting.m`, plot the solar panel angle as a function of time for the different values of $\overline{\theta}_p$ used above. You may use either the `load` or `uiload` commands to access the appropriate data from your prior simulations (i.e., `panel_rotation_#.mat` data files). Use the `subplot` command to create six plots on the same MATLAB figure (i.e., one for each value of $\overline{\theta}_p$). On each plot, include $\theta_p(t)$ obtained from the nonlinear model as well as from the linear model (with the corresponding $\overline{\theta}_p$). This will allow for a direct comparison of the two models. Add axis labels, a title and a legend to each plot. Include a printout of your plot in your report.

16. While observing the plots you just created, address the following in your report :

    (a) Discuss the sensitivity of the linearized model's accuracy to variations of $\overline{\theta}_p$ while considering the transient portion of the response (i.e., the significance of variations of $\overline{\theta}_p$ on the similarity between the nonlinear and linear models during the transient portion of the response).

    (b) The sensitivity of the linearized model's accuracy to variations of $\overline{\theta}_p$ while considering the steady-state portion of the response (i.e., the significance of variations of $\overline{\theta}_p$ on the similarity between the nonlinear and linear models during the steady-state portion of the response).

(c) Is the impact of changing $\overline{\theta}_p$ on the accuracy of the linear model more significant with regards to the steady-state or transient portions of the response ? Explain your answer based on the form of the linearized model. *Hint :* Observe which of the C_j coefficients ($j = 1, 2, 3, 4$) have been most impacted by the linearization.

(d) What nominal operating point (i.e., value of $\overline{\theta}_p$) appears as the most appropriate in this scenario ? What special relationship does it have with the steady-state response of the system ?

17. How would you evaluate the overall mechanical design of the solar panel tilt control system shown in Figure 1 ? What modifications would you make to the system to simplify its operation and allow for the use of smaller motors ?

5 Deliverables

The following deliverables must be submitted by the stated due date. Your lab will only be considered to be complete once all deliverables have been received :

- — A report (one report for each lab group) submitted in PDF format.
- — Copies of all MATLAB/SIMULINK files used in completing the lab (original files, not compressed).
- — A compressed (i.e., `*.zip`) file containing all of the aforementioned files (report, Matlab files, Simulink model files, etc.).

Note : Include your group number on your report's title page (you will know your group number once you enroll in a group through D2L).

The printed report should include a title page and should address all the required work identified in Section 4, e.g., :

- — Calculations
- — Plots (printed using MATLAB)
- — Numerical results
- — Analyses
- — Discussions
- — Printout of all SIMULINK diagrams.

Organize your report based on the numbering scheme used in this document (e.g., Question 1(b), 2(e), etc.). When presenting mathematical developments, ensure you define any new variables you choose to introduce (variables that were already defined in the lab document do not need to be defined in your report). Answer all questions concisely and clearly. Use sketches where appropriate.

When preparing plots using MATLAB, include the following :

- — Descriptive title
- — Axes labels
- — Legend (when the plot contains multiple results)
- — Grid lines

Moreover, use logical axis limits (both for viewing and possibly for comparison with other similar plots) and don't forget to distinguish multiple curves on the same plot by line type (solid, dashed, dotted, etc.) rather than only by colour.

IMPORTANT :

- — Use logical filenames for your MATLAB script files (i.e., `*.m` files).
- — Add comments to your code to clearly identify what each section of code is doing.

LAB NO. 5 - MARKING SCHEME

Calculation of overall mark:
- Each criteria is assigned a score (0 to 4).
- Overall mark obtained as of [criteria weight] x [criteria score]/4

Weights	
10	**Presentation and organization of document:** - Spelling, grammar - Organization of report into logical sections, etc. - Definition of all variables used
5	**Presentation of Matlab code and Simulink diagrams:** - Use of logical filenames - Adequate and appropriate commenting of code - Logical and "visually pleasing" arrangements of Simulink blocks and signal lines.
15	**Development of the nonlinear model:** - FBD of the solar panel assembly - Dynamic model of the solar panel assembly (input tau_p, output theta) - Development of the nonlinear model (coefficients B1 through B5) - Proper documentation of all mathematical developments
15	**Linearization:** - Identification of the nominal operating configuration (E_bar given theta_bar) - Linearization of cos(theta) and sin(theta)^2 about theta_bar - Nonlinear model (i.e., coefficients C1 through C4)
20	**Programming of the linearized model in MATLAB/Simulink:** - Sketch of the block diagram (e.g., by hand) - Completion of the parameters.m file - Programming of the block diagram in Simulink
20	**Discussions:** - Quality of observations from an engineering standpoint - Demonstrated insight into the functioning of the system and the model - Addressing of all the discussion points (i.e., #4, #11, #16, #17)
15	**Plots:** - Programming of the "plotting.m" file - Titles and axis labels - Distinguishing of curves and legend - Use of appropriate axes scales - Use of grid lines - Etc.
100	**TOTAL**