Question 1: [Total Marks: 100%]

a) Table 1 below is the launch velocity profile of a space vehicle.

(i) Using MATLAB, obtain mathematical expressions that respectively model the space vehicle's launch velocity, acceleration and distance-travelled profiles over the time period given in Table 1. **[30%]**

(ii) Determine the maximum and minimum acceleration, in ms^{-2} , of the vehicle over this period, as well as the distances from launch that they respectively occur. **[20%]**

Table 1: Launch velocity profile of a space vehicle following its launch

Event	Time(s)	Velocity (m/s)
Lift-off	0	0
Commence roll manoeuvre	10	56.40
End roll manoeuvre	15	97.23
Throttle to 89%	20	136.25
Throttle to 67%	32	226.16
Throttle to 104%	59	403.86
Maximum dynamic pressure	62	440.44
Solid rocket booster separation	125	1265.23

b) Use the following steps to evaluate an integral using pattern recognition:

(i) Use a computer algebra system, such as MATLAB, to evaluate the following integrals:

$$\int \ln x \, dx \quad \int x \ln x \, dx \quad \int x^2 \ln x \, dx$$
$$\int x^3 \ln x \, dx \quad \int x^7 \ln x \, dx$$

Submit your MATLAB code along with the answers you have obtained. **[20%]**

(ii) Based on the pattern of your responses in part (i) or otherwise, deduce the integral of

$$\int x^n \ln x \, dx$$

[10%]

(iii) Use integration by parts to validate the integral expression that you deduced in part (b). For what values of n is it valid? **[20%]**