Mechanical properties of fluid

Static fluid

Fluid :- Substance that can flow.
e.g → Liquid & gas.

- Relative density or Specific gravity = $\frac{\rho_{object}}{\rho_{water}}$ = unitless

⇒ $\rho_{water} = 1\ gm/cm^3$
$\rho_{oil} = 0.8\ gm/cm^3$
$\rho_{Hg} = 13.6\ gm/cm^3$
$\rho_{milk} = 1.04\ gm/cm^3$
$\rho_{ice} = 0.9\ gm/cm^3$

$\boxed{\text{density} = \frac{\text{Mass}}{\text{Volume}}}$

Density of Mixture of two liquids

$$\boxed{\rho_{mix} = \frac{\text{Total Mass}}{\text{Total volume}} = \frac{(M_1+M_2)}{(V_1+V_2)}}$$

$\boxed{\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1+\rho_2}}$ (M = same)

$\boxed{\rho_{mix} = \frac{\rho_1+\rho_2}{2}}$ (V = same)

$$\boxed{\rho_1 < \rho_{mix} < \rho_2}$$

- Pressure (P) :-

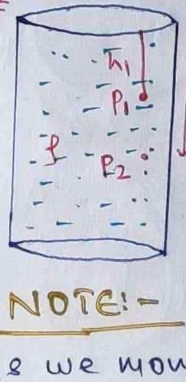
$\boxed{P = \frac{F}{A},\ \text{scaler},\ N/m^2,\ [ML^{-1}T^{-2}]}$

Atmospheric pressure = 1 Atm = $1.05 \times 10^5\ N/m^2$

★ Variation of 'P' with depth :-

$\boxed{\Delta P = \rho g h}$

#



$\boxed{P_2 - P_1 = \rho g h}$

$\boxed{P_2 - P_1 = \rho g (h_2 - h_1)}$

★ NOTE :-

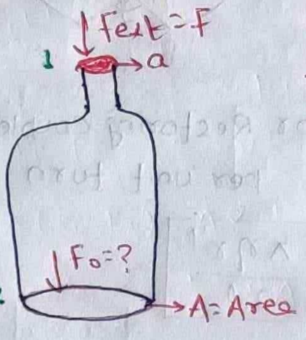
- As we move upward from ground, Atmospheric pressure decrease.
- In open container, P (Just inside liq.) = 1 ATM
- In closed container, P (just inside liq) = 0
- In Air molecules, Pressure applies same in all direction.
- Liquid Pressure is same at all horizon. position.

Pascal's Law :-

↳ Extra pressure applied at any point of closed fluid transmits undiminished to every point.

Pascal चाचा → Static fluid me pressure balance krte hai.

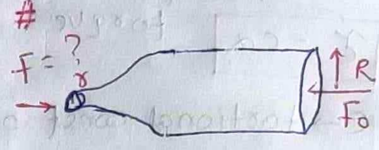
#



$P_1 = P_2$

$\boxed{\frac{F}{a} = \frac{F_0}{A}}$

#



$P_1 = P_2$

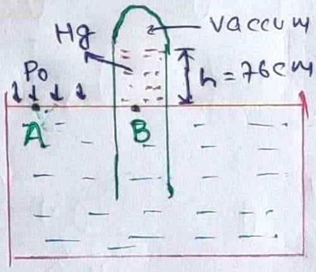
$\boxed{F = \frac{r^2}{R^2} F_0}$

★ Moving container 'P' calculation :-
(Bs "geff" le lena hai)

(i) Lift up :- $P = \rho(g+a)h$
(ii) Lift down :- $P = \rho(g-a)h$
(iii) free fall :- $P = \rho(g-g)h = 0$

Barometer :- to measure atmos. pressure.

$P_A = P_B$
$P_0 = \rho g h$
$P_0 = 1.01 \times 10^5\ N/m^2 = 1\ ATM$



Rawlal the garib आदमी

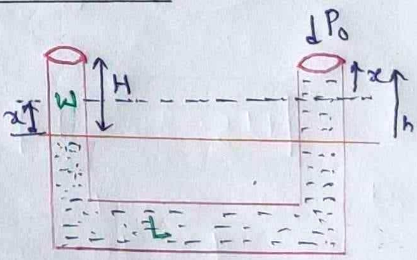
water Barometre $\boxed{h = 10.1m}$

⇒ Question on U-Tube :-

$P_A = P_B$

$P_w g H = P_L g (2x)$

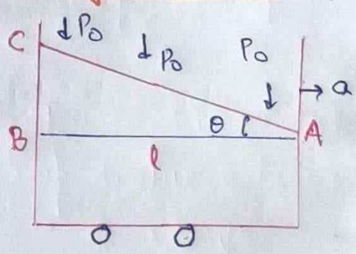
→ Ek aisa line Select karo jiske about 'P' same hona chahiye.



Horizontal Accelerating container

$\boxed{\tan\theta = \frac{a}{g} = \frac{H}{l}}$

$\boxed{\sin\theta = \frac{a}{\sqrt{a^2+g^2}}}$



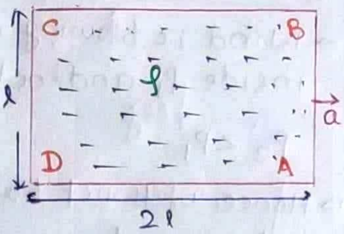
#

$\boxed{P_D > P_C > P_A > P_B}$

Max. (at P_D) Min (at P_B)

$P_A - P_B = \rho g l$

$P_C - P_B = \rho g 2l$

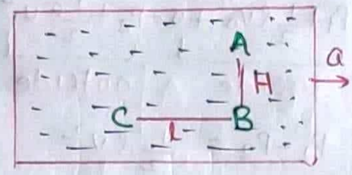


#

$P_B - P_A = \rho g H$

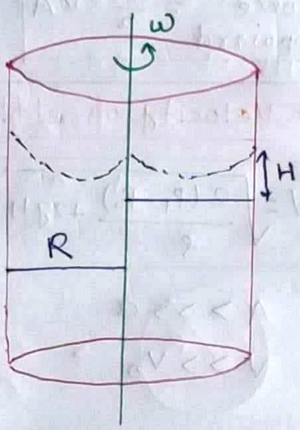
$P_C - P_B = \rho a l$

$\boxed{P_C - P_A = \rho(gH + al)}$



A vessel is rotated about vertical axis. Find rise in water 'H'.

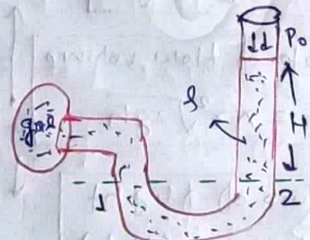
$$\boxed{H = \frac{R^2\omega^2}{2g}}$$



Manometer :- to measure gas pressure.

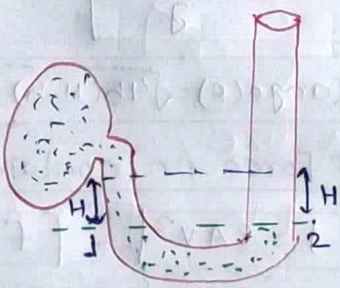
Case-I

$$\boxed{\begin{gathered} P_1 = P_2 \\ P_{gas} = P_0 + \rho g H \end{gathered}}$$



Case-II

$$\boxed{\begin{gathered} P_1 = P_2 \\ P_{gas} = P_0 - \rho g H \end{gathered}}$$



Archimedes Principle

↳ pressure diffⁿ in a fluid.

$F_{Buoyant\ force}$ = [weight of displaced liq.]

upthrust

↳ doesn't depend on area & Height of obj.

* volume of displaced liquid = volume of solid in liq.

$$\boxed{F_{Net} = \rho V g}$$

Apparent Weight [σ = object, ρ = liquid]

(i) $\sigma = \rho$ (float & completely submerged)

$N = 0$, objects remain where it's placed.

$$\boxed{V_{total} = V_{In}}$$

(ii) $\sigma < \rho$ (floating with partially submerged)

$$\boxed{\frac{V_{in}}{V_{Total}} = \frac{\sigma}{\rho}} \ \frac{\text{chota}}{\text{bada}}$$

(iii) $\sigma > \rho$ (completely sink)

$$\boxed{N = mg\left[1 - \frac{\rho}{\sigma}\right]} \ \frac{\text{chota}}{\text{bada}}$$

* object of density 'σ' is released then find accⁿ of object inside liquid. $(\sigma > \rho)$

$$\boxed{a = g\left[1 - \frac{\rho}{\sigma}\right]}$$

Buyant force on object with cavity

$V_{cavity} = V_{Total} - V_{matter}$

$N = mg - F_B$ $\boxed{N = \sigma V_M g - \rho V_T g}$

* $F_B = \rho_L V_T g$

* $V_{matter} = \frac{M}{\sigma}$

⇒ Rise / Fall of liquid :-

↳ When Ice placed on liquid will melt.

$$\boxed{\begin{gathered} \rho_L > \rho_0 = \text{Rise} \\ \rho_L < \rho_0 = \text{fall} \\ \rho_L = \rho_w = \text{same} \end{gathered}}$$

ρ_0 = density of water

ρ_L = surr. Liquid

Fluid dynamics

- Non-viscous
- Incompressible
- Friction less
- density constant
- Streamlined flow.

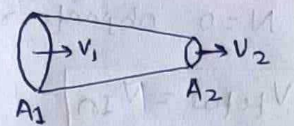
* Properties :-

- Tangent at any point → direcⁿ of flow
- No 2 streamline cross
- Velocity more where streamlines are close.

Equation of continuity

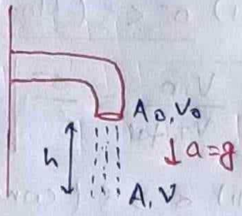
↳ Conservation of Mass

$\boxed{A_1V_1 = A_2V_2}$



$V > V_0$, $A < A_0$

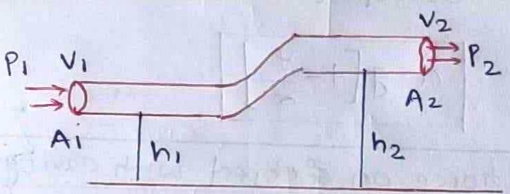
$$\boxed{V = \sqrt{\left(\frac{A_0^2}{A_0^2 - A^2}\right) 2gh}}$$



Rate of volume flow

$$\boxed{AV = AA_0\sqrt{\frac{2gh}{A_0^2 - A^2}}}$$

Bernoulli's Eqn :- (conservation of Energy)



$\boxed{A_1V_1 = A_2V_2}$

$$\boxed{P_1 + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g h_2}$$

$$\left[P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}\right]$$

divided by 'ρg' both side

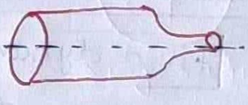
$$\left[\frac{P}{\rho g}\right] + \left[\frac{v^2}{2g}\right] + [h] = \text{constant}$$

pressure head, velocity head, gravitational

↳ Each head dimension equal to length.

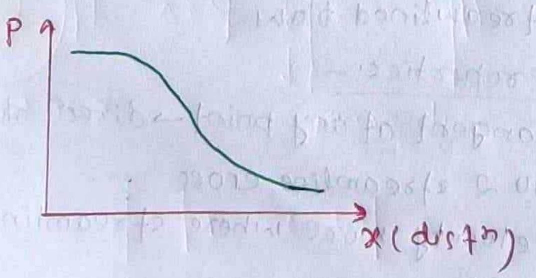
#

$P + \frac{1}{2}\rho v^2 = \text{cont}^n$



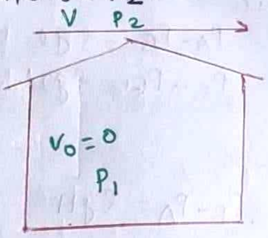
$AV = \text{const}^n$

↳ potential energy same on horizontal level.



⇒ Rawlas House

→ Wind is blowing outside then pressure inside P_1 and outside house P_2.



$P_2 < P_1$

↳ Hence up thrust force act on roof.

$$\underbrace{P_1 + \frac{1}{2}\rho V_0^2 + 0}_{\text{inside}} = \underbrace{P_2 + \frac{1}{2}\rho v^2}_{\text{outside}}$$

$P_1 - P_2 = \frac{1}{2}\rho v^2$

$A(P_1 - P_2) = A \times \frac{1}{2}\rho v^2$

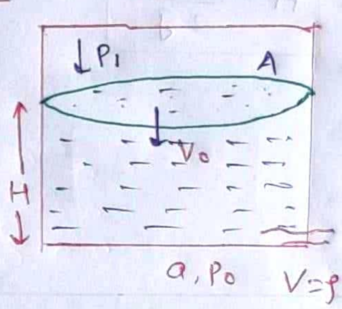
$\boxed{\text{Force upward} = \frac{1}{2}\rho v^2 A}$

Velocity of efflux

$$\boxed{V = \sqrt{\frac{2(P_1 - P_0)}{\rho} + 2gH}}$$

$A >>> a$

$V >> V_0$



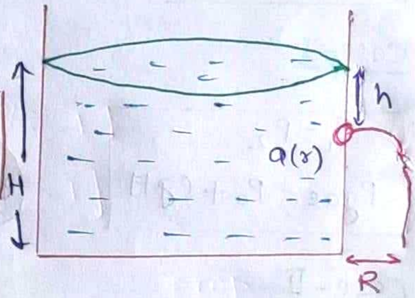
open container

$\boxed{\text{Velocity} = \sqrt{2gh}}$

$\boxed{\text{Rate of flow volume} = \pi r^2 \sqrt{2gh}}$

$\boxed{\text{Time of flight } (T) = \frac{2(H-h)}{g}}$

$\boxed{\text{Range}(R) = \sqrt{4h(H-h)}}$



$\boxed{R_{max} = H \text{ when } h = H/2}$

Force on container :-

$\boxed{F = \rho A V^2}$ $\boxed{F = 2\rho a g H}$

★ For Massless container, 'μ_{min}' to keep container at rest

$\mu mg = 2\rho a g h$

a = hole

A = container Area

$$\boxed{\mu_{min} = \frac{2a}{A}}$$

★ Hole at same distance from top and bottom or same distance from mid-point has same range.

Time taken to move liquid from Height H_2 to H_1:-

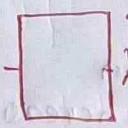
$$V_0 = \frac{a}{A}\sqrt{2gh}, \quad t = \frac{A}{a}\sqrt{\frac{2}{g}}\left(\sqrt{H_2} - \sqrt{H_1}\right)$$

↳ To reach at ground level

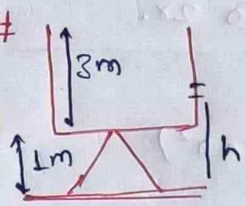
$$\boxed{t = \frac{A}{a}\sqrt{\frac{2H}{g}}}$$

↳ Ratio from $h \to \frac{h}{2}$ & $\frac{h}{2} \to$ bottom

$$\frac{t_1}{t_2} = \frac{\sqrt{2}-1}{1}$$



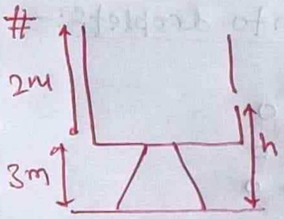
#



For Max range,

$$h = \frac{\text{total height}}{2} = \frac{1+3}{2} = 2m$$

#



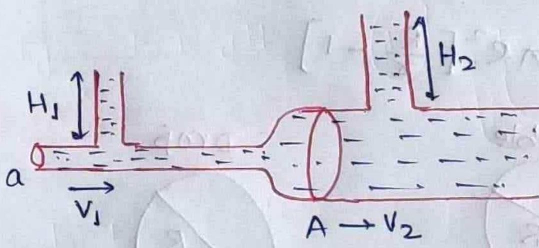
for Max range

$$h = \frac{3+2}{2} = 2.5$$

but $h = 3m$ i.e bottom of container.

Venturimetre:-

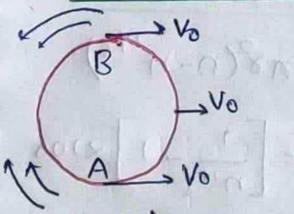
- Measure rate of volume flow.
- Based on Bernoulli's principle.

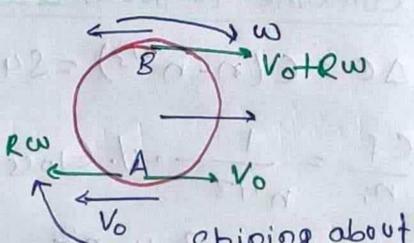


$aV_1 = AV_2 = V =$ Rate of volume flow

$$\boxed{V = \sqrt{\left(\frac{a^2A^2}{A^2 - a^2}\right)2g(h_2 - h_1)}}$$

Dynamic lift & Magnus Effect





$V_A = V_B = V_0 + V$

$V_A = V + V_0 - R\omega$

$V_B = V + V_0 + R\omega$

Viscocity:-

[viscous force betw two liquid layers]

$$\boxed{F = \eta A \frac{\Delta V}{l}} \qquad \boxed{\eta = \frac{\text{Shear stress}}{\text{Velocity grad}^n} = \frac{F/A}{\Delta V/l}}$$

SI → Poiseuilles , CGS → Poise

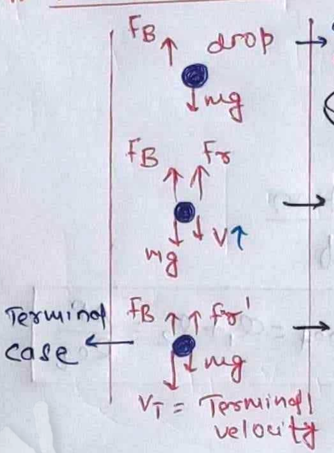
1 Poiseuille = 10 poise

Stoke's law:- [only for sphere]

↳ Viscous force b/w solid & liquid.

$$\boxed{F = 6\pi\eta r\vec{v}} \qquad \boxed{F \propto \sqrt{\text{Area}}}$$

Terminal velocity



at $t = 0$

$\boxed{F_{net} = mg - F_B}$ → acc^n downward

$\boxed{F_{net} = mg - F_B - F_r}$

downward acc^n ↓, V↑

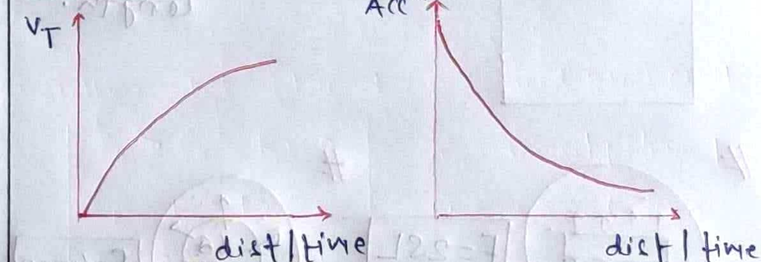
$F_{net} = mg - F_B - F_r' = 0$

$a = 0$ V = constant

$\to mg = F_B + F_r$

$$\boxed{V_T = \frac{2r^2}{9\eta}(\sigma - \rho)g}$$

Graph:-



- $V_T \propto r^2$:- Bigger rain drops, velocity is greater.

★ If 'n' drop collapse

$$\boxed{V_{T_f} = V_{T_i} \times n^{2/3}} \qquad \cancel{R = (nr)^3}$$

$$\boxed{R = n^{1/3}r}$$

Temperature dependence

- For liquid, Temp↑ → Viscocity↓
- For gas, Temp↑ → Viscocity↑

Poiseuille equation

↳ Rate of volume flow of viscous fluid

$Q = AV$

$$\boxed{Q = \frac{\pi \Delta P r^4}{8 \eta l}}$$

Fluid resistance → Fluid current

$$\boxed{R = \frac{8\eta l}{\pi r^2}}$$

ΔP = pressure diff.

r = radius of pipe

l = length of pipe

η = coefficient of viscosity

① Series combination

Q = same , ΔP = diff.

$R_{eq} = R_1 + R_2$

$$\boxed{Q_1 = Q_2 = \frac{P_1 - P_2}{R_{eq}}}$$

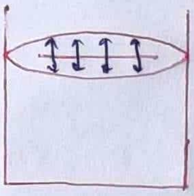
② Parallel combination

ΔP = same , Q, diff

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}} \quad \boxed{Q_{net} = Q_1 + Q_2 = \frac{\Delta P}{R_{eq}}}$$

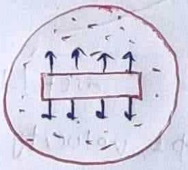
Surface tension

↳ Force per unit length parallel to contact surface.



$S = \frac{f}{l}$ = force per unit length.

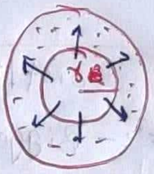
#



$\boxed{F = 2SL}$

Rod

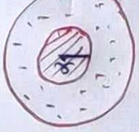
#



$\boxed{f = 4\pi r S}$

ring radius 'r'

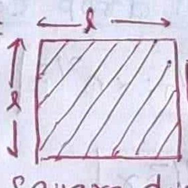
#



$\boxed{f = 2\pi r S}$

Disc radius 'r'

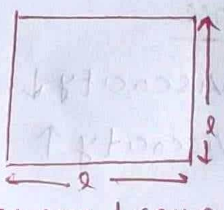
#



$\boxed{f = S(4l)}$

Square disc

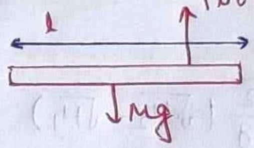
#



$\boxed{f = S(8L)}$

Square frame

#



$F_{ext} = F_{ST} + Mg$

$= 2Sl + Mg$

$\boxed{\text{Surface tension} \propto \frac{1}{\text{Temp}}}$

Surface Energy

$\boxed{E = S \cdot A}$

↓

Surface tension

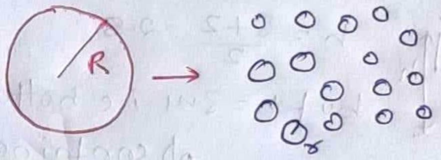
Work done in increasing radius of water drop from 'r' to '2r'.

$\text{work} = \Delta E = S(\Delta A)$

$= S(4\pi(2r)^2 - 4\pi r^2)$

$= 12\pi S r^2$.

Spilitting of drops into droplets:–



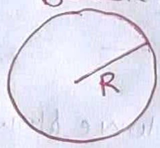
$R = n^{1/3} r$

$\Delta A = 4\pi (n r^2 - R^2]$

$\Delta A = 4\pi R^2 [n^{1/3} - 1]$

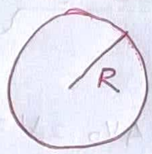
$\Delta A = 4\pi R^2 [\frac{R}{r} - 1]$

⇒ Bubble



$A = 4\pi R^2 \times 2$

Drop



$A = 4\pi R^2$

Film :– Take Area double

Energy released when droplets combine to form a drop:–

$\Delta E = E(n - n^{2/3}) = S4\pi r^2 (n - n^{2/3})$

$\frac{E_f}{n_i} = \frac{1}{n^{1/3}}$, $\% E_{loss} = \left[\frac{1}{n^{1/3}} - 1\right] \times 100$

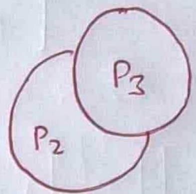
$\Delta E = n 4\pi r^2 - 4\pi R^2$

$\Delta E = 3VT\left[\frac{1}{r} - \frac{1}{R}\right]$

$$Work = 4\pi S R^3 \left[\frac{1}{R} - \frac{1}{r}\right]$$

$$\boxed{R = n^{1/3} r \quad \frac{R}{r} = n^{1/3} \quad n = \frac{R^3}{r^3}}$$

Pressure is always high in concave side.



$P_3 > P_2 > P_1$

★ Radius of Interface

$$\boxed{\frac{1}{R} = \frac{1}{R_1} - \frac{1}{R_2}}$$

Values of Excess pressure

DROP $\Delta P = P_{in} - P_{out} = \frac{2S}{R}$

$P_{in} = P_{out} + \frac{2S}{R}$

Bubble $\Delta P = P_{in} - P_{out} = \frac{4S}{R}$

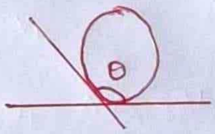
$P_{in} = P_{out} + \frac{4S}{R}$

Radius of coalesce

↳ Two drops of radius r_1 & r_2 coalesce under isothermal condition

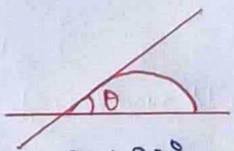
$$\boxed{R = \sqrt{R_1^2 + r_2^2}}$$

Angle of contact



$\theta > 90°$

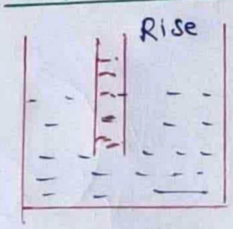
Cohesion > Adhes



$\theta < 90°$

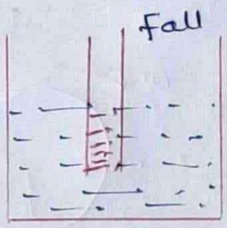
Cohe < Adhes

Capillary tube



$\theta < 90°$

Concave Meniscus



$\theta > 90°$

Convex Meniscus

Height of liq. rising in CT

$$\boxed{h = \frac{2S\cos\theta}{\rho r g}}$$