CFD: Solution of Poiseuille Flow problem using FTCS, BTCS and Crank Nicolson Method

MASTER OF TECHNOLOGY

by

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Q. Solve the following non-dimensional partial differential equation for Couette Flow using finite difference method with specified boundary conditions for j=1 to j=101(M) grid size as shown in Fig. 1. Use pseudo-transient solution approach with initial condition of u-velocity as zero.

1. Explicit method: FTCS

2. Implicit method

a) BTCS: Gauss-Seidel iterative method

b) BTCS: Tridiagonal Matrix Algorithm

c) Crank-Nicolson: Tridiagonal Matrix Algorithm

Discretize the above PDE using the discretization schemes mentioned above with uniform grid of M. Take ReH =100 and Δt=5×10−3 for Explicit method and Δt=10−2 for Implicit method.

Submit results in terms of velocity profiles at different time* including steady state results as shown in Fig. 2, convergence history (ε vs t) for all schemes in a single plot and report on discretized algebraic equation of each discretization scheme, comparison study of number of time iterations and physical time taken to converge up to ϵ<10-6.

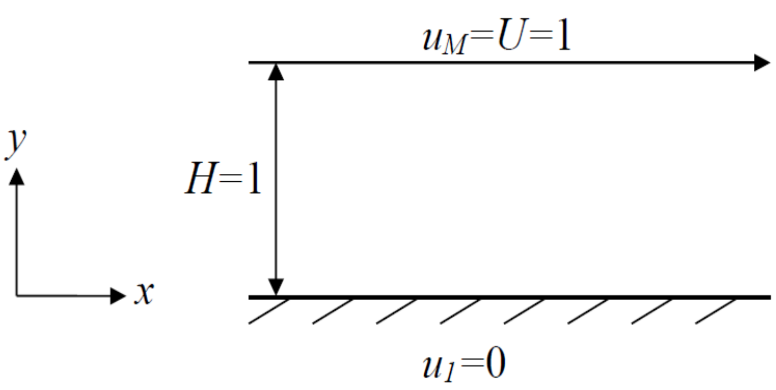


Fig. 1

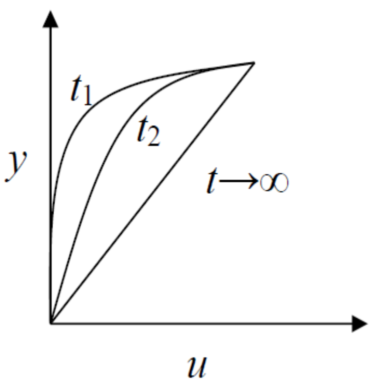


Fig. 2

Explicit method: FTCS

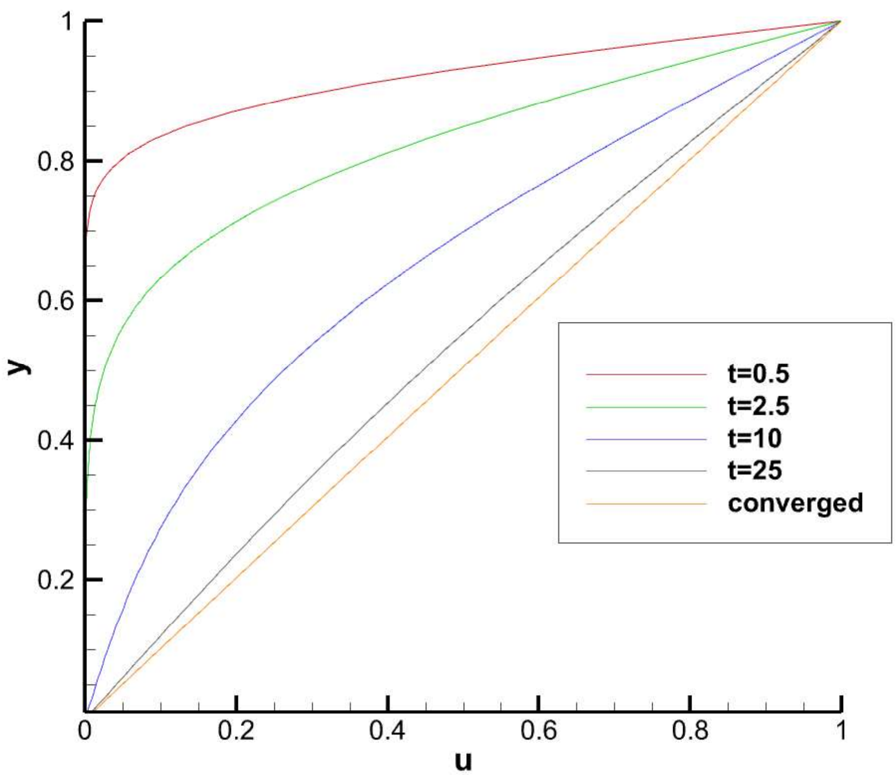
Discretization:

$$\frac{u_i{}^{n+1} - u_i{}^n}{\Delta t} = \frac{1}{Re_h}\frac{u_{i+1}{}^n - 2u_i{}^n + u_{i-1}{}^n}{\Delta y^2}$$

$$u_i{}^{n+1} = u_i{}^n + \gamma(u_{i+1}{}^n - 2u_i{}^n + u_{i-1}{}^n$$

$$u_i{}^{n+1} = u_i{}^n(1 - 2\gamma) + \gamma u_{i+1}{}^n + \gamma u_{i-1}{}^n$$

Velocity profile:



BTCS:

Discretization:

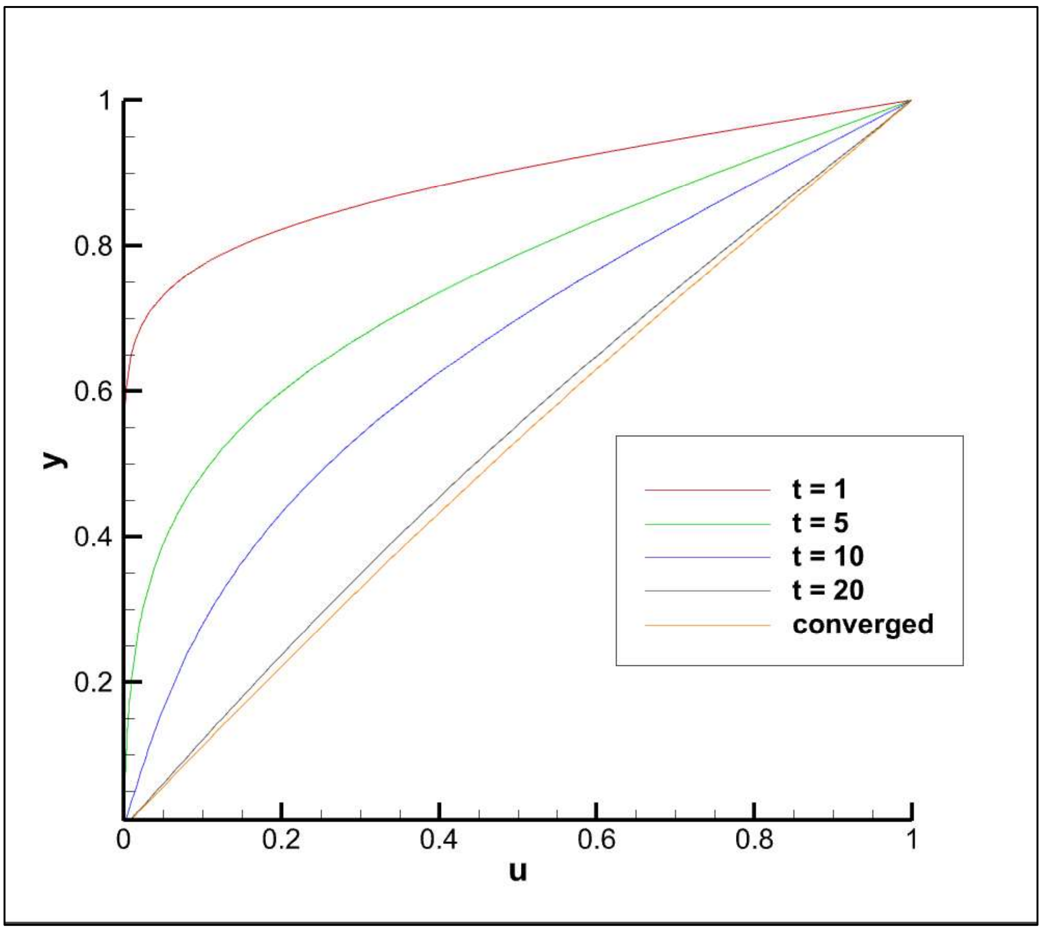
$$\boldsymbol{\frac{u_i{}^{n+1} - u_i{}^{n}}{\Delta t} = \frac{1}{Re_h}\frac{u_{i+1}{}^{n+1} - 2u_i{}^{n+1} + u_{i-1}{}^{n+1}}{\Delta y^2}}$$

$$\boldsymbol{u_i{}^{n+1} = u_i{}^{n} + \gamma(u_{i+1}{}^{n+1} - 2u_i{}^{n+1} + u_{i-1}{}^{n+1}}$$

$$\boldsymbol{-u_i{}^{n+1}(1 + 2\gamma) + \gamma u_{i+1}{}^{n+1} + \gamma u_{i-1}{}^{n+1} = -u_i{}^{n}}$$

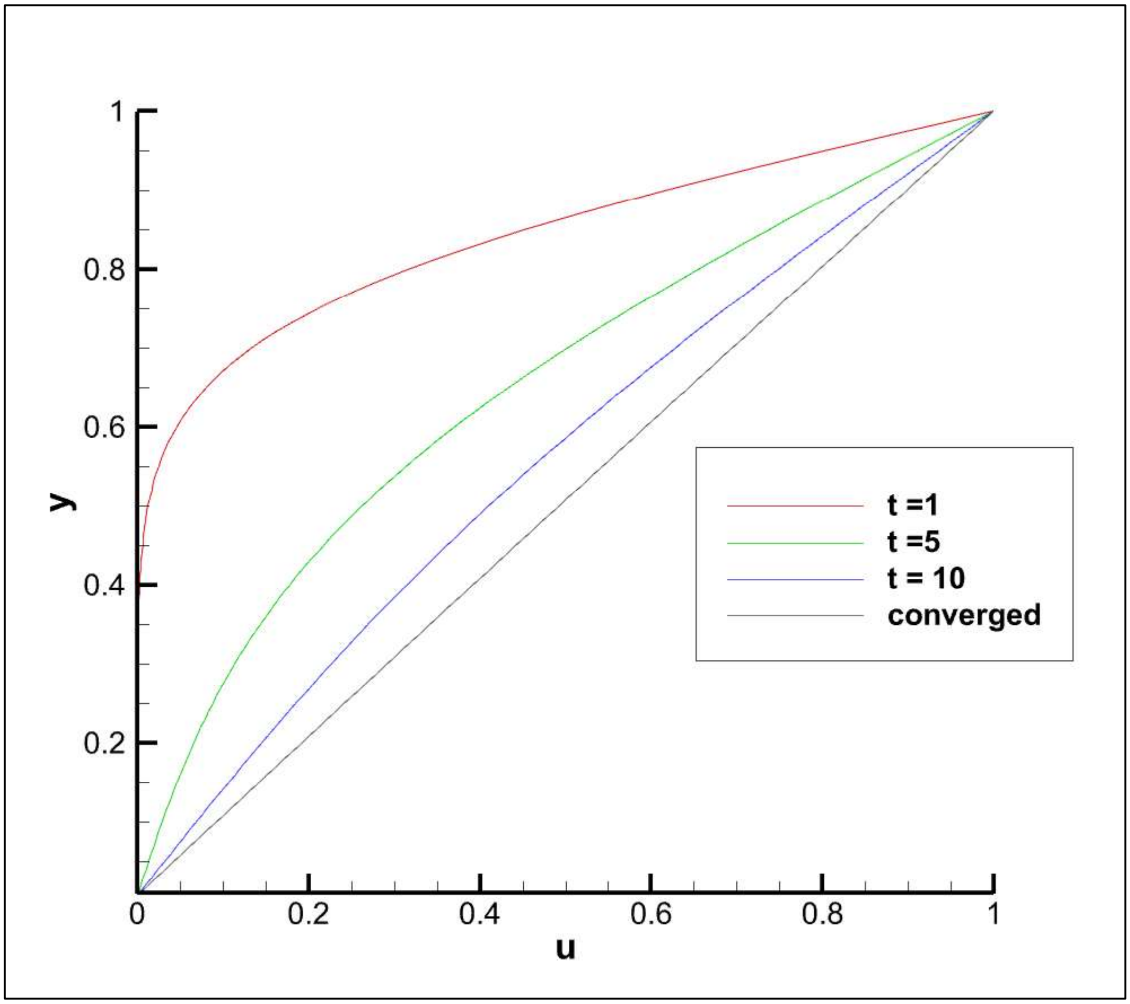
BTCS: Gauss-Seidel iterative method

Velocity profile:



BTCS: Tridiagonal Matrix Algorithm

Velocity profile:



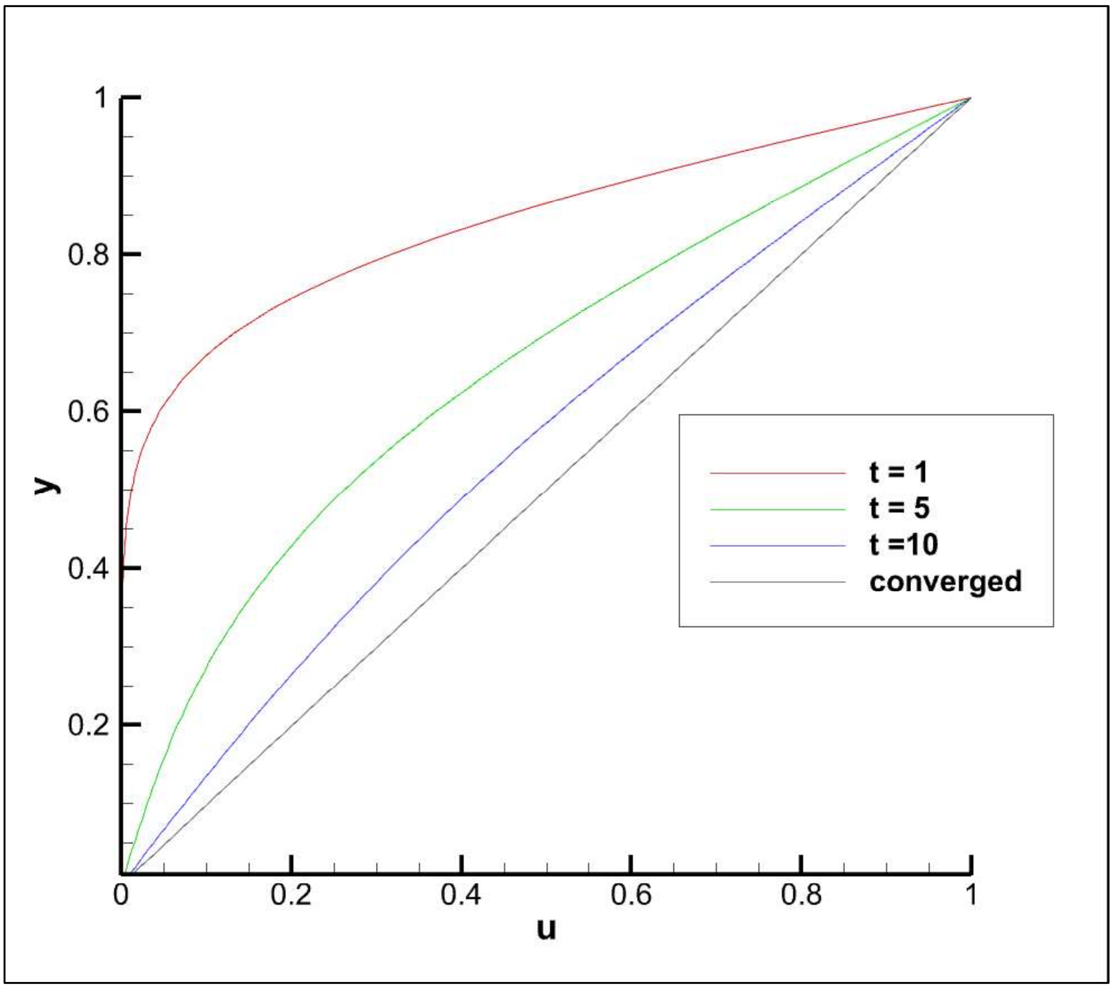
Crank-Nicolson: Tridiagonal Matrix Algorithm

Discretization:

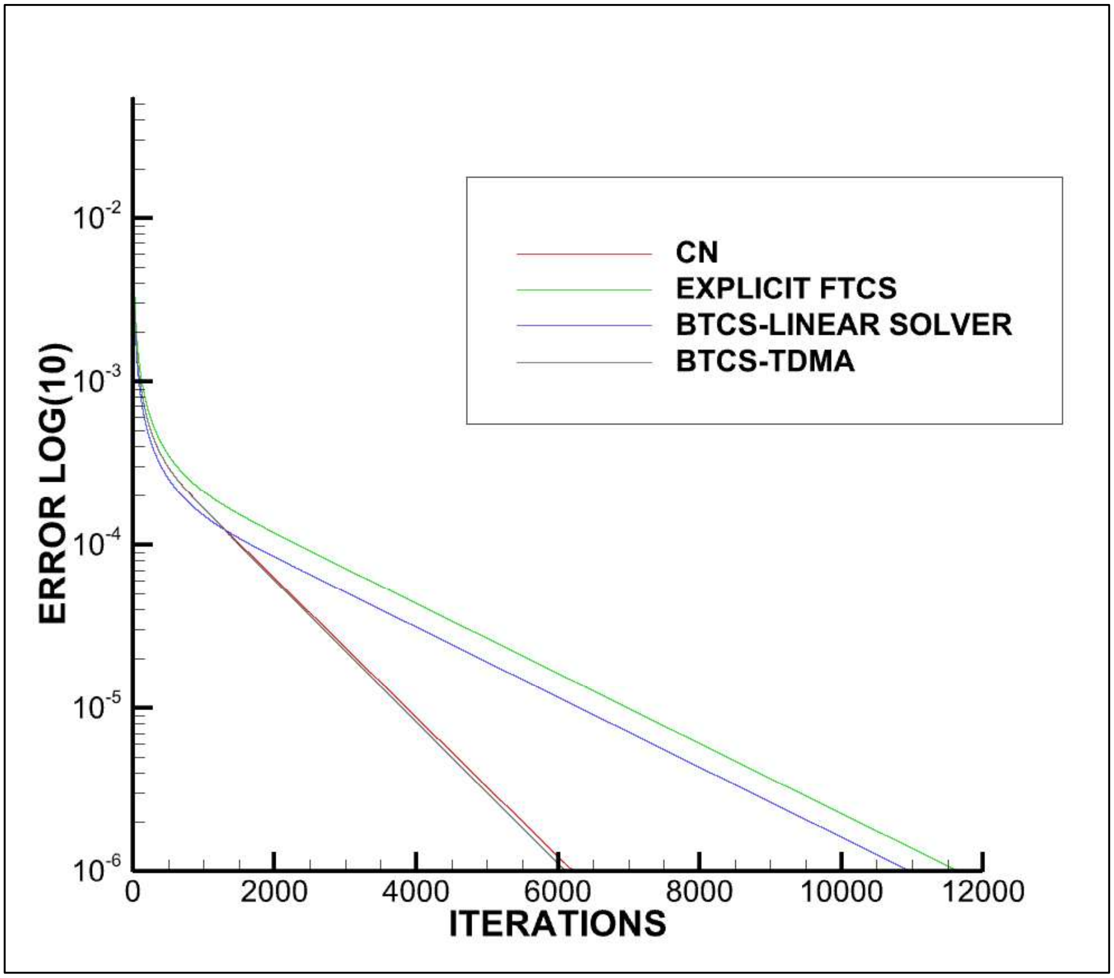
$$u_i^{n+1} - u_i^{n} = \frac{\gamma}{2}\left[u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1} + u_{i+1}^{n} - 2u_i^{n} + u_{i-1}^{n+1}\right]$$

$$u_i^{n+1}(1+\gamma) - \frac{\gamma}{2}u_{i+1}^{n+1} - \frac{\gamma}{2}u_{i-1}^{n+1} = u_i^{n}(1-\gamma) + \frac{\gamma}{2}u_{i+1}^{n} + \frac{\gamma}{2}u_{i-1}^{n}$$

Velocity profile:



Error vs Iterations:



Time vs Error: