CFD: Solution of Lid Driven Cavity Flow problem using Stream-function Vorticity equation

MASTER OF TECHNOLOGY

by

Devendra Chaudhari

(Roll No. 234103306)

DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

GUWAHATI- 781039, INDIA

Problem Statement:

Solve the following partial differential equation using the finite difference method with the specified boundary conditions for the geometry with 100×100 grid size as shown in the figure.

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial x^2}=-\omega$$

$$u\frac{\partial\omega}{\partial x}+v\frac{\partial\omega}{\partial y}=\frac{1}{Re}\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial x^2}\right)$$

$$u=\frac{\partial\psi}{\partial y}\qquad v=-\frac{\partial\psi}{\partial x}$$

Convergence Criteria: Find the maximum error of stream function and vorticity and reduce that maximum error to 10-6. Apply the finite difference discretization to replace all derivatives with the corresponding central difference expressions with uniform grid and *write the discretized equations of the governing equations and boundary conditions of stream function & vorticity in the **report***. Write the code in such a way so that you can input the values of Re. Submit the results and discussion for **Re=100 and 400** in terms of streamlines, velocity vectors, *u* velocity along vertical centreline and *v* velocity along horizontal centreline.

Boundary conditions:

Left boundary:

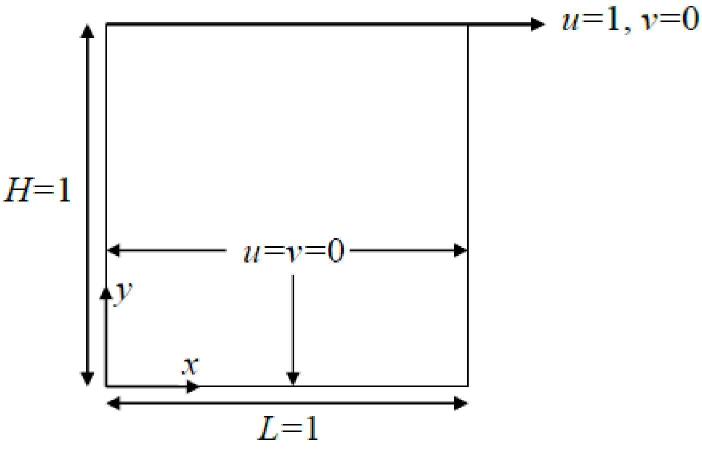


Figure: Flow inside a lid-driven cavity

$u = 0 \quad v = 0 \quad \psi = 0$

Bottom boundary:

$u = 0 \quad v = 0 \quad \psi = 0$

Top boundary:

$u = 1 \quad v = 0 \quad \psi = 0$

Right boundary:

$u = 0 \quad v = 0 \quad \psi = 0$

Vorticity boundary conditions:

Left boundary:

$$\omega_{i,j} = \frac{-2}{\Delta x^2}\left(\psi_{i+1,j} - \psi_{i,j}\right)$$

Bottom boundary:

$$\omega_{i,j} = \frac{-2}{\Delta y^2}\left(\psi_{i,j+1} - \psi_{i,j}\right)$$

Right boundary:

$$\omega_{i,j} = \frac{-2}{\Delta x^2}\left(\psi_{m-1,j} - \psi_{m,j}\right)$$

Top Boundary

$$\omega_{i,j} = \frac{-2}{\Delta y^2}\left(\psi_{i,n-1} - \psi_{i,n} + U\Delta y\right)$$

Solution of stream function:

$$\psi_{i,j} = \frac{1}{2(1+\beta^2)}\left[\Delta x^2 \omega_{i,j} + \beta^2\left(\psi_{i,j+1} + \psi_{i,j-1}\right) + \psi_{i+1,j} + \psi_{i-1,j}\right]$$

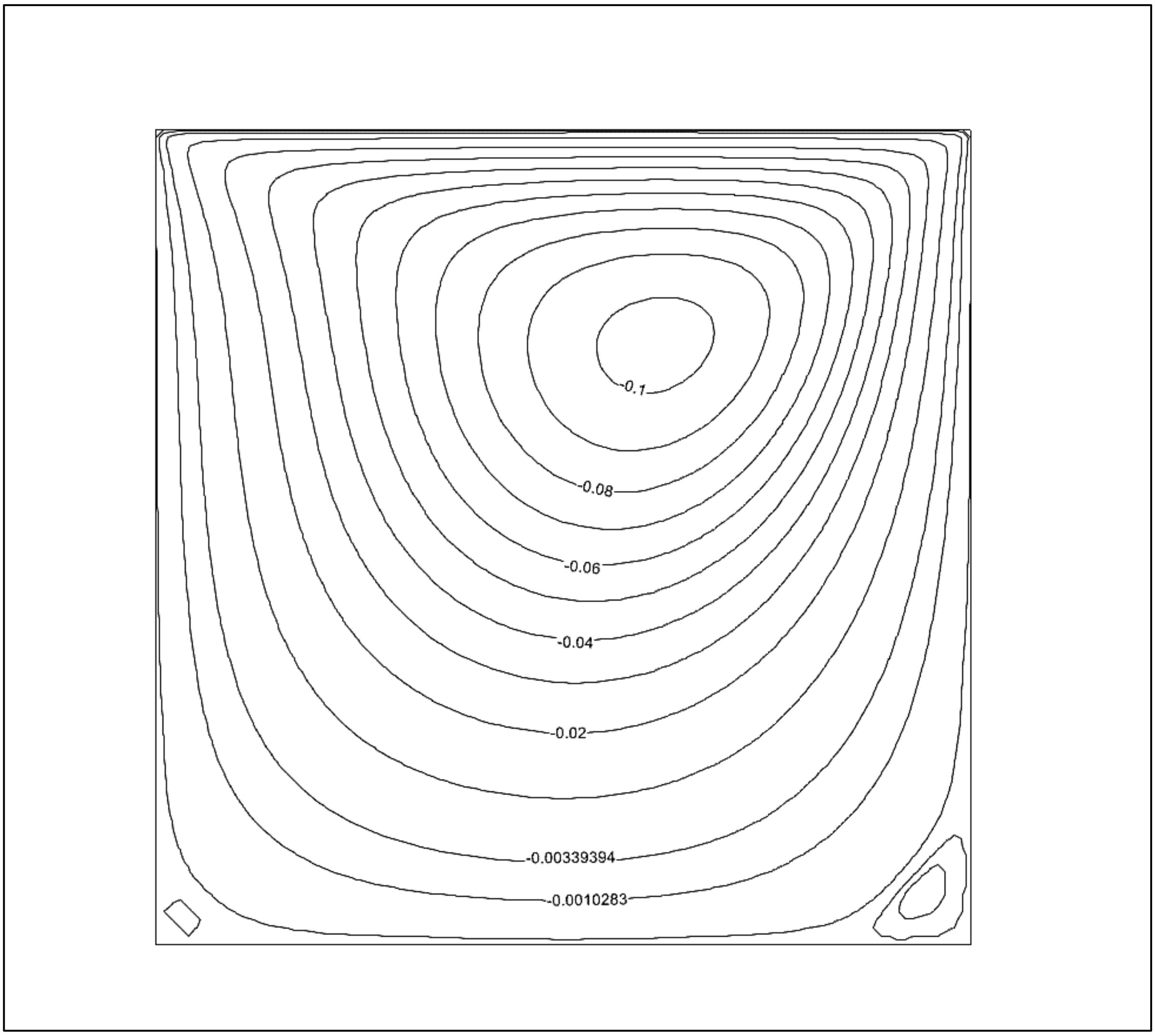
Solution of vorticity:

$$\begin{aligned}\omega_{i,j} = \frac{1}{2(1+\beta^2)}\Bigg[&\left\{1 - \left(\psi_{i,j+1} - \psi_{i,j-1}\right)\frac{\beta * Re}{4}\right\}\omega_{i+1,j} \\ &+ \left\{1 + \left(\psi_{i,j+1} - \psi_{i,j-1}\right)\frac{\beta * Re}{4}\right\}\omega_{i-1,j} \\ &+ \left\{1 + \left(\psi_{i+1,j} - \psi_{i-1,j}\right)\frac{Re}{4\beta}\right\}\beta^2\omega_{i,j+1} \\ &+ \left\{1 - \left(\psi_{i+1,j} - \psi_{i-1,j}\right)\frac{Re}{4\beta}\right\}\beta^2\omega_{i,j-1}\Bigg]\end{aligned}$$

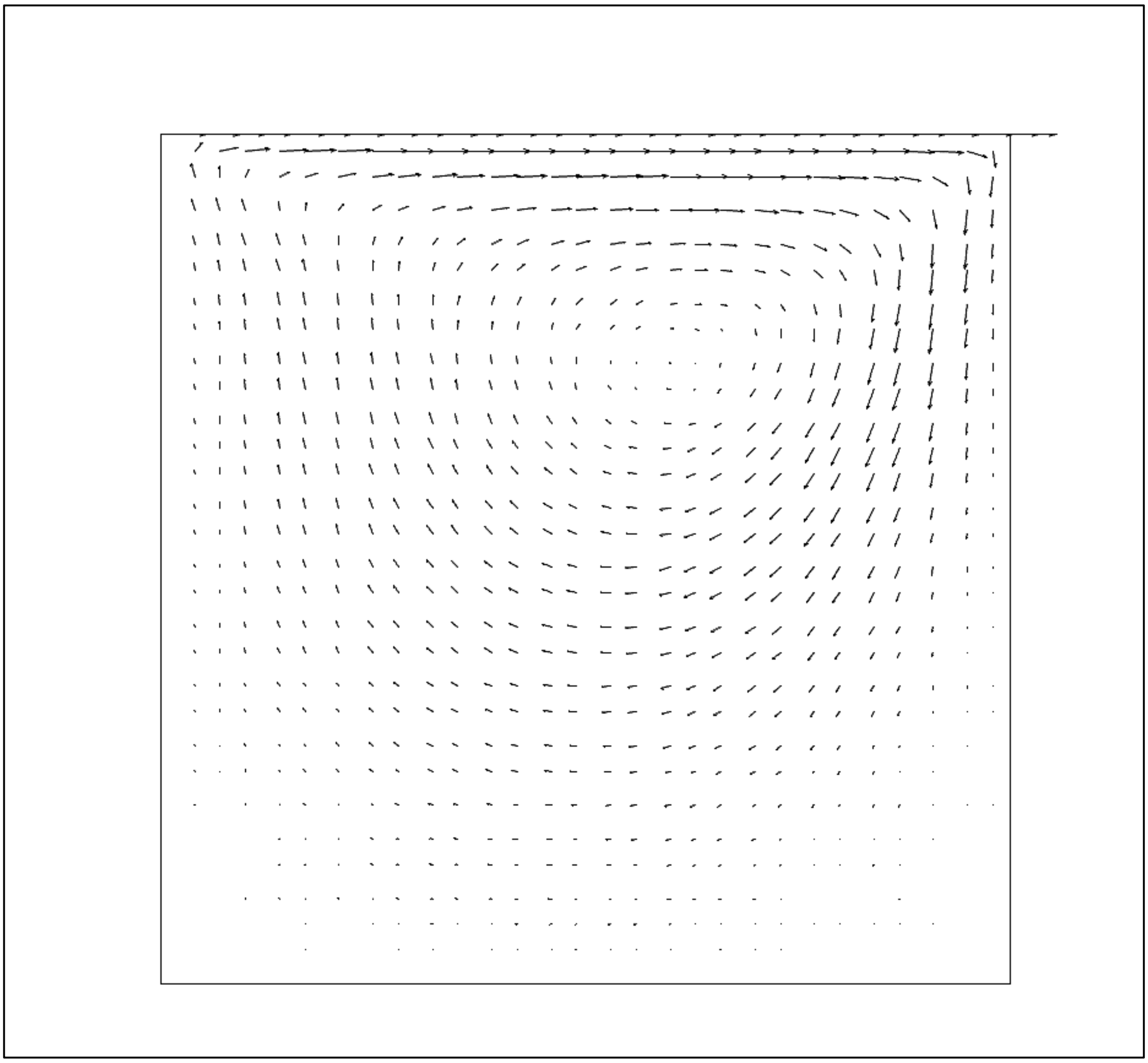
Results:

Reynolds number=100

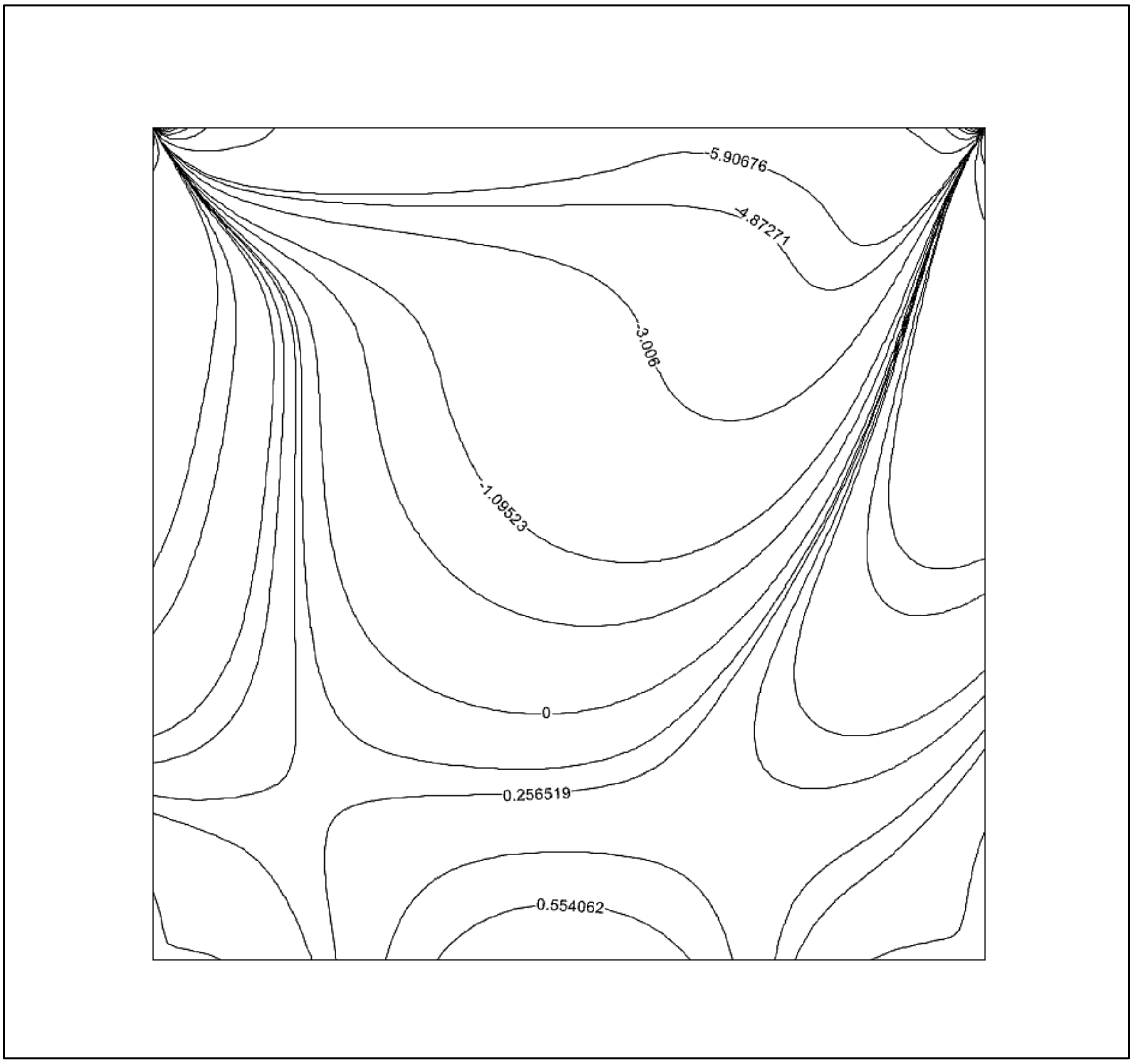
1. Streamlines:



2. Velocity vectors:

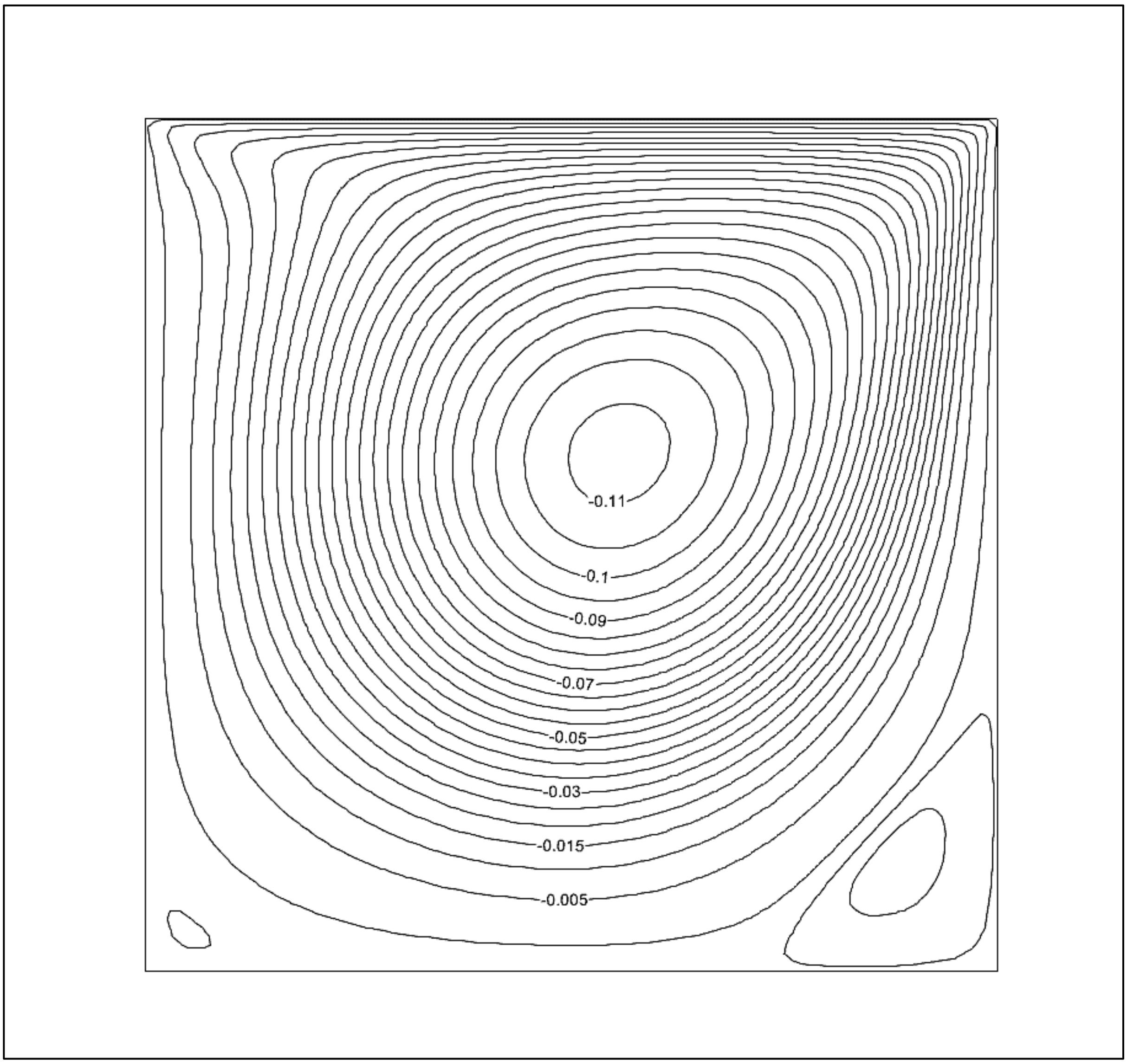


3. Vorticity:

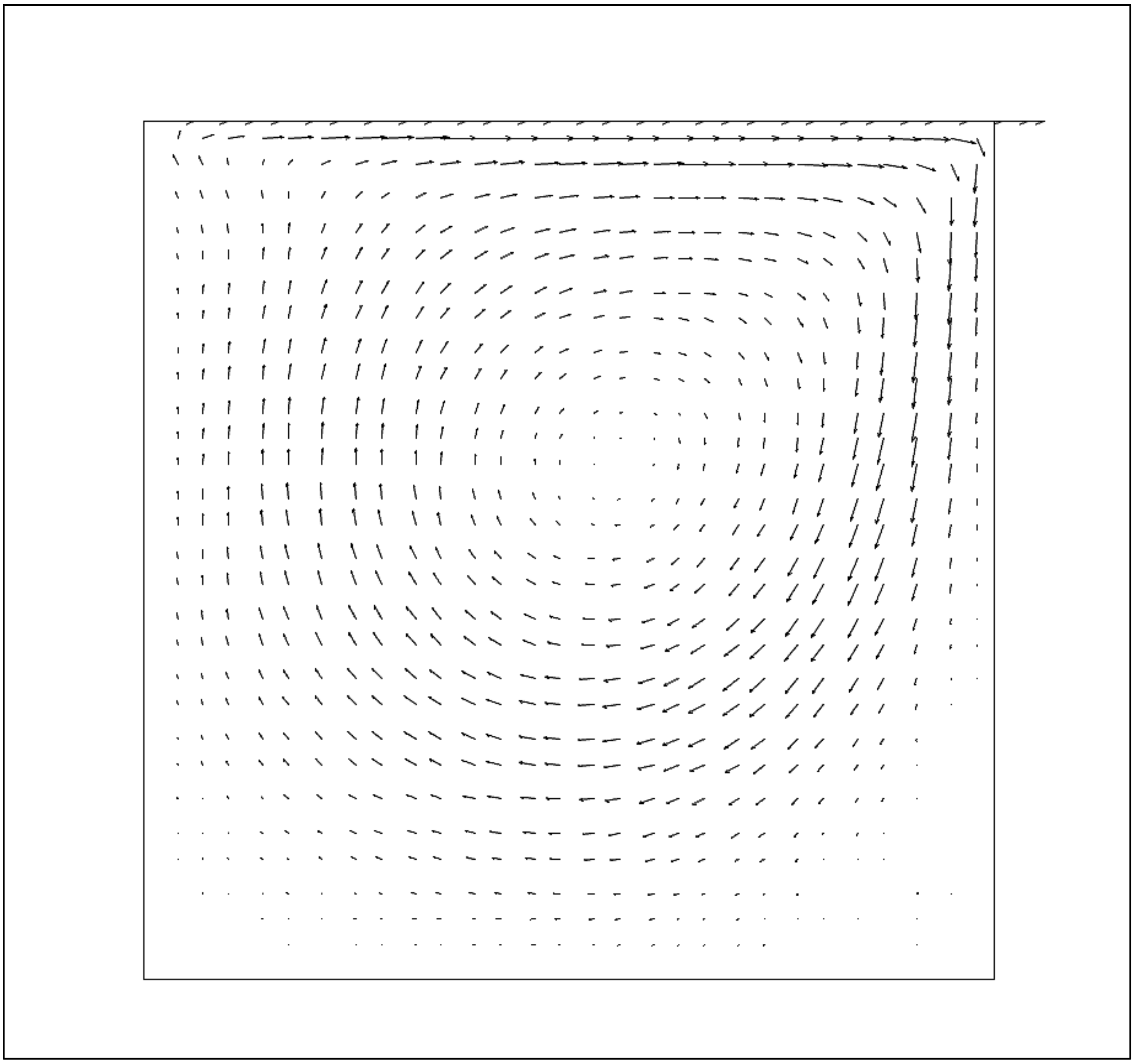


Reynold's number= 400

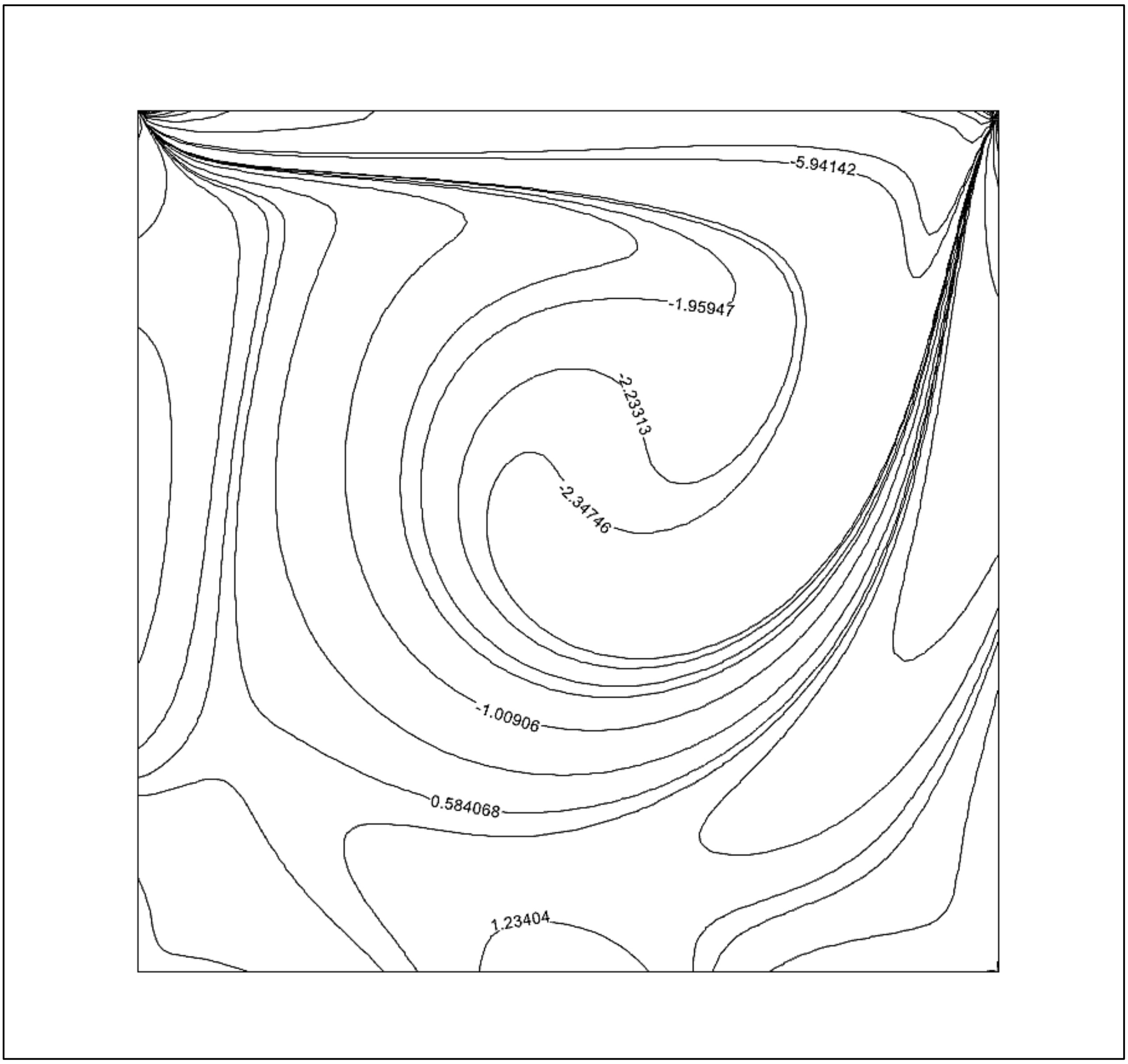
1. Streamlines:



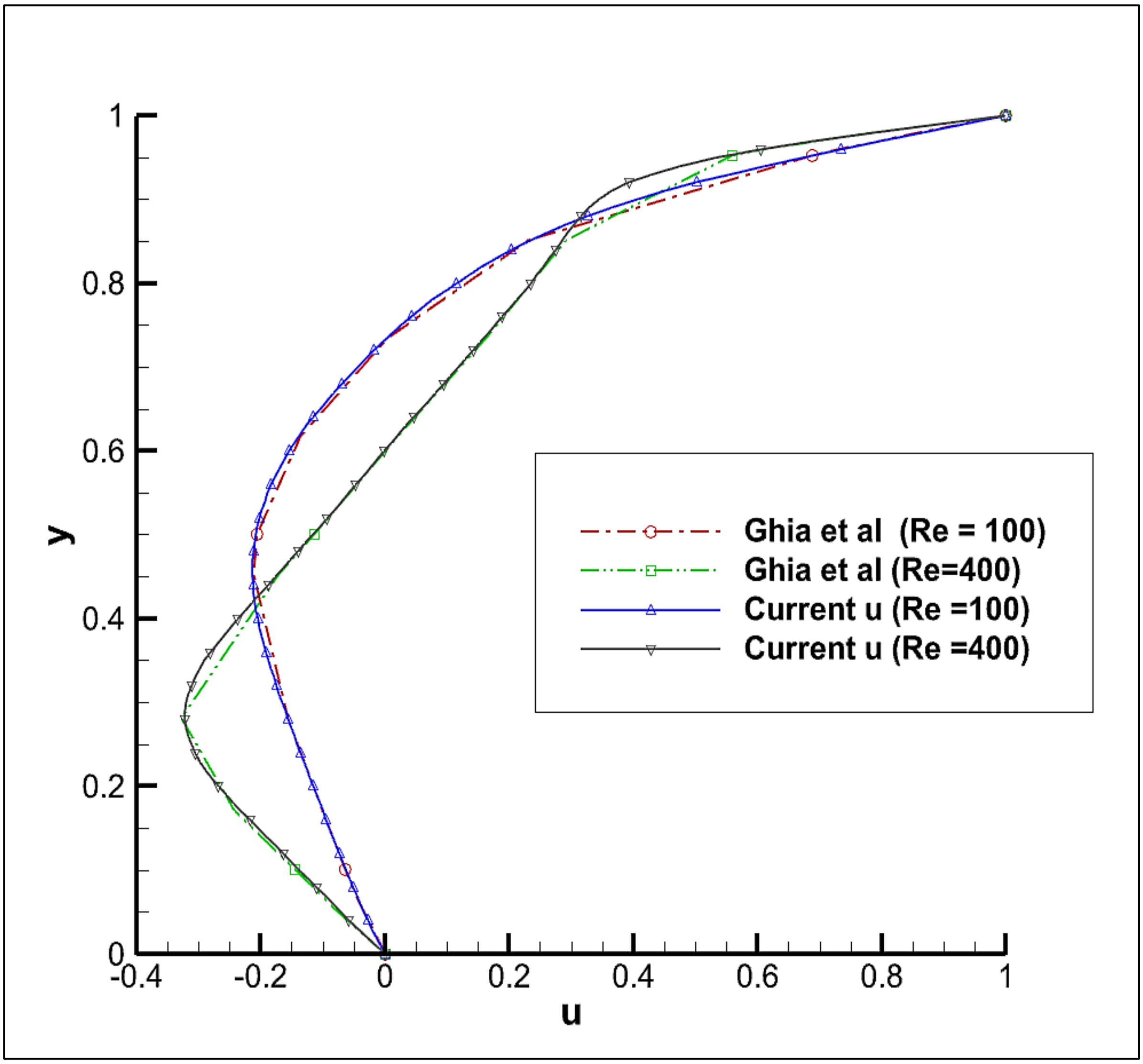
2. Velocity vectors:



3. Vorticity:



Comparison of u velocity at Re=100 and Re=400 along centreline with standard data:



Comparison of ‘v’ velocity at Re=100 and Re=400 along centreline with standard data: