

Table 1: Classification accuracies on training and validation data

(a) DATA SET 1			(b) DATA SET 2		
Model	training	validation	Model	training	validation
1	48.15	48.5	1	92.88	91.69
2	96.8	96.3	2	93.49	92.88
3	96.8	96.3	3	94.03	93.17
4	96.8	96.3	4	93.59	92.14
5	96.8	96.3	5	94.79	93.92

Table 2: Confusion matrix on test set -Model -5,DATA SET 1.

Class0	207 30.71	6 0.89	8 1.19	93.66 6.34
Class1	4 0.59	226 33.53	4 0.59	96.58 3.42
Class2	4 0.59	2 0.30	213 31.60	97.26 2.74
	96.28 3.72	96.58 3.42	94.67 5.33	95.84 4.16
	Class0	Class1	Class2	

Table 3: Confusion matrix on test set -Model -5,DATA SET 2.

Class0	197 29.22	4 0.59	7 0.10	94.71 5.29
Class1	6 0.89	237 35.16	1 0.59	97.13 2.87
Class2	12 1.78	8 1.18	202 29.97	91 9
	91.63 8.37	95.18 4.72	96.19 3.81	94.36 5.64
	Class0	Class1	Class2	

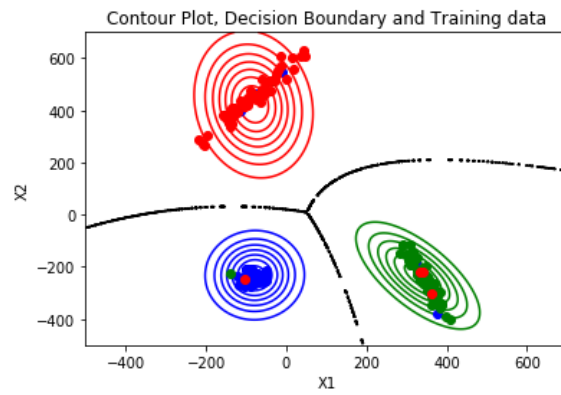
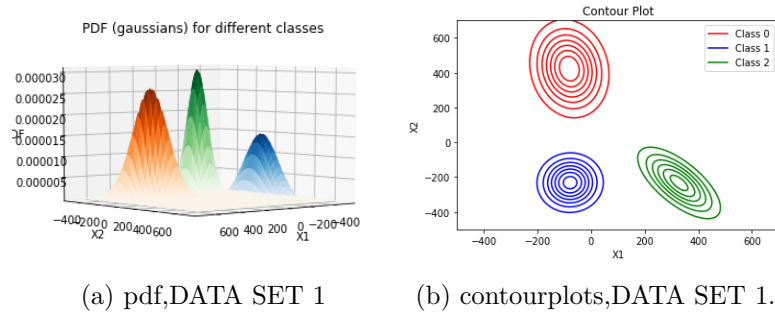


Figure 2: Decision boundary and Decision surface,DATA SET 1

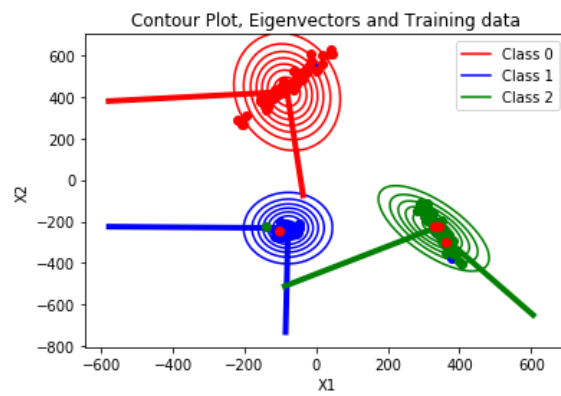


Figure 3: eigenvectors,DATA SET 1

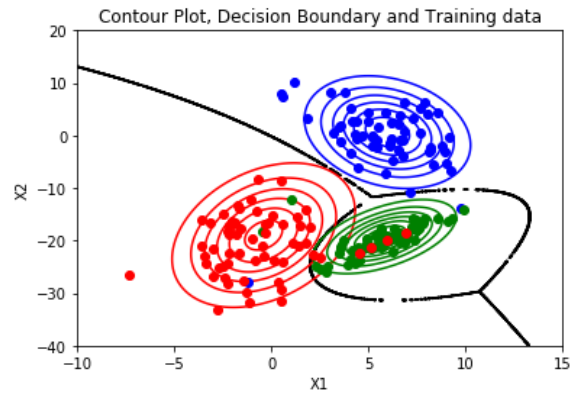
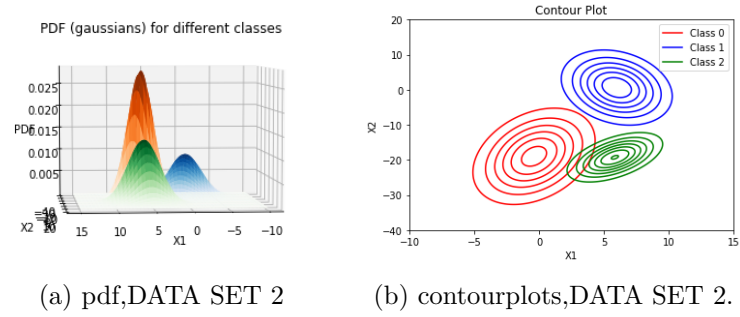


Figure 5: Decision boundary and Decision surface,DATA SET 2

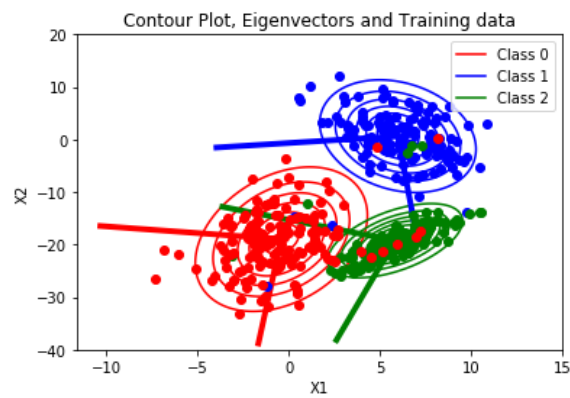


Figure 6: eigenvectors,DATA SET 2

2

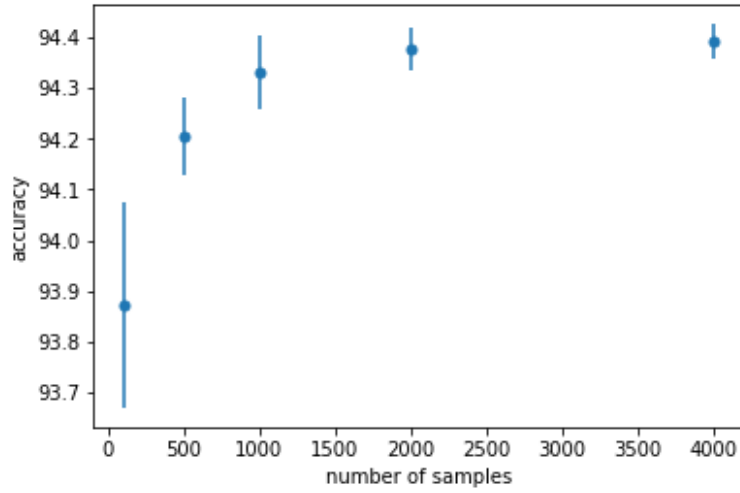


Figure 7: Averaged over 20 replicas varying data size vs accuracy

As training sample size increases the standard error in accuracy decreases  
 Number of training samples needed to get 85 percent is 20

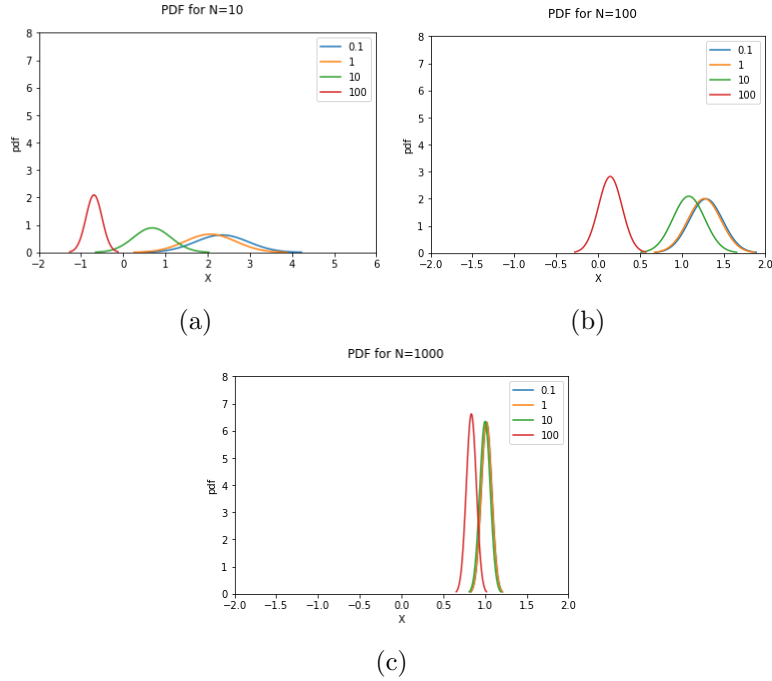
3

Table 4: Classification accuracies on training data

(a) DATA SET 3			(b) DATA SET 4		
features	accuracy	error	features	accuracy	error
1	83.50	16.50	1	76.67	23.33
2	87.03	12.97	2	80.50	19.50
3	31.03	68.97	3	83.77	16.23

We can see a drop in accuracy when we considered all the three features of data set 3 As the number of features increases to train the model,we need more training data but here we are using a fixed training data set,so it is possible that error increases with more number of features However in case of data set 4 accuracy increases as we consider all the three features.

4

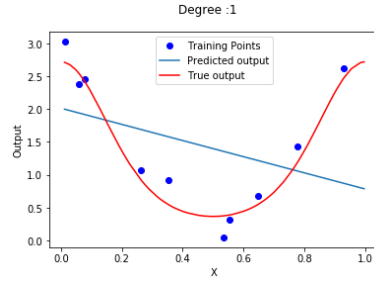


With each additional observation the uncertainty about the true value of mean decreases, In our plot we see the shift of mean and decrease in variance

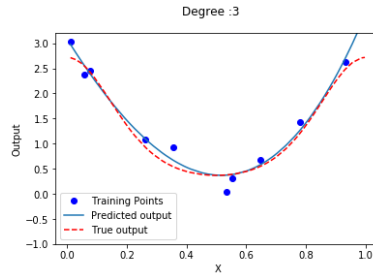
As the ratio of variances increases the mean shifts toward prior mean and updated variance decreases

5

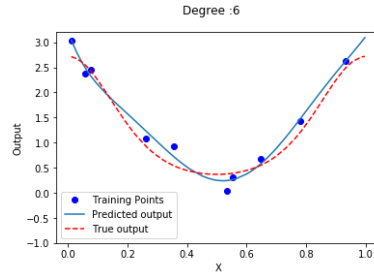
## Normal regression



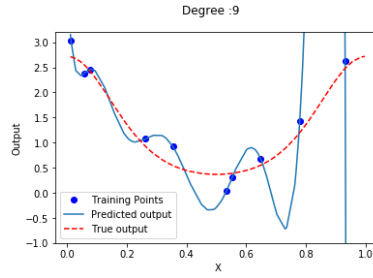
(a) degree 1.



(b) degree 3.



(c) degree 6.



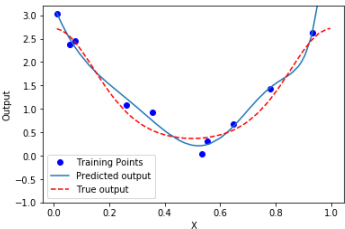
(d) degree 9.

Degree	Train error	Valid error	Test error
1	0.9283008115778021	0.971499902634403	0.5649857203143979
3	0.16130934063028565	1.1021676649503447	1.0271045078692982
6	0.12138157352252413	1.1617315830580974	0.9754174639925292
9	0.0014145153022794915	7.619038944554978	59.045916792231274

(e)

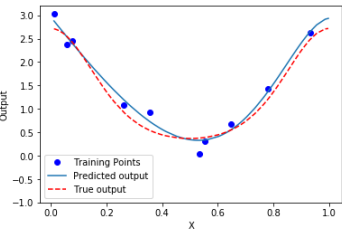
Regularisation degree 9

Degree is 9 and ridge regression parameter is  $\lambda=0.00000001$



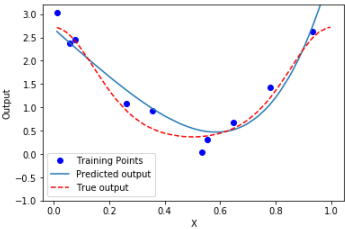
(a)

Degree is 9 and ridge regression parameter is  $\lambda=0.00100000$



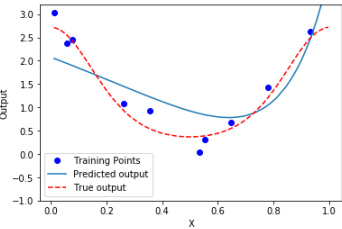
(b)

Degree is 9 and ridge regression parameter is  $\lambda=0.01000000$



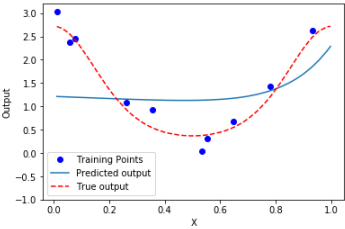
(c)

Degree is 9 and ridge regression parameter is  $\lambda=0.10000000$



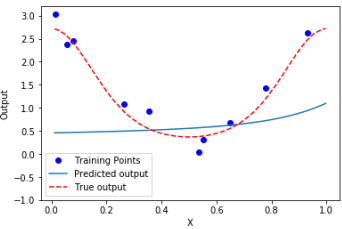
(d)

Degree is 9 and ridge regression parameter is  $\lambda=1.00000000$



(e)

Degree is 9 and ridge regression parameter is  $\lambda=10.00000000$



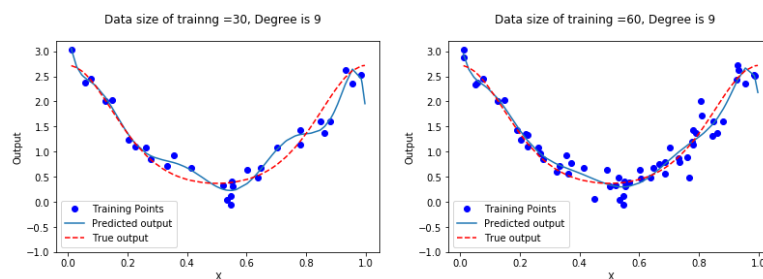
(f)

Ridge regression

Lambda	Train error	Valid error
$10^{-3}$	0.11058	1.07982
$10^{-3}$	0.143255	1.15079
0.01	0.263423	1.06158
0.1	0.53419	0.957797
1	0.950917	0.820512
10	1.35736	0.201919

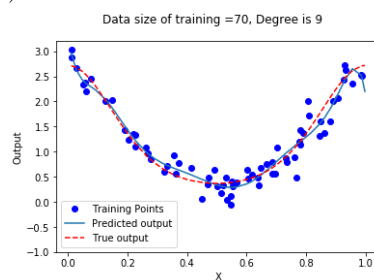
(g)

## Varying data size



(a)

(b)



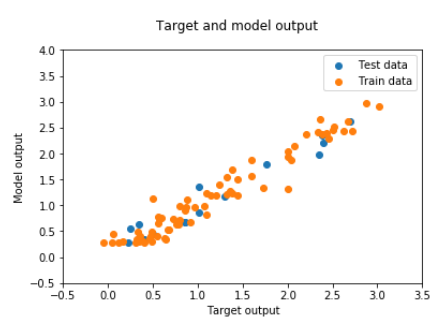
(c)

Data Size	Train error	Valid error
10	0.0014145153022794915	7.619038944554978
30	0.14359859648995235	0.9906333781862198
60	0.20996586692904454	0.9952349440997669
70	0.20056975079060063	0.9866803983214962

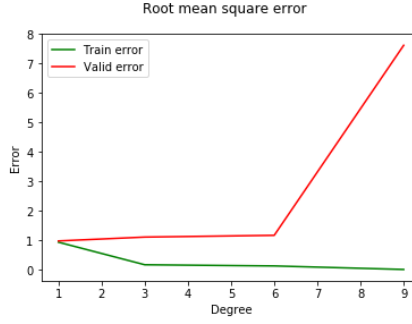
(d)



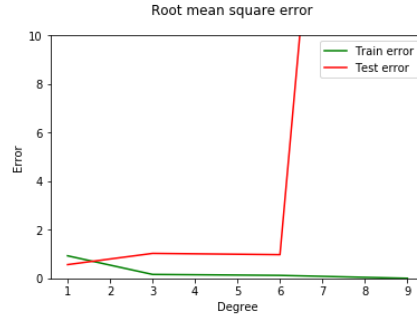
Training Data size 70, degree 9 is chosen as best model on basis of tables



(a) target-model



(b) RMS error-train vs valid



(c) RMS error-train vs test

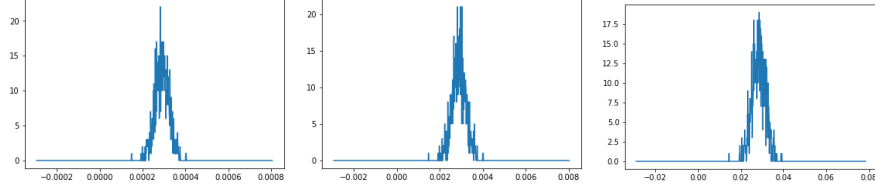
Coefficient of	1 degree	3 degree	6 degree	9 degree
$w^9$				-297983
$w^8$				1.13025e+06
$w^7$				-1.77808e+06
$w^6$			143.813	1.50374e+06
$w^5$			-477.462	-740726
$w^4$			584.435	215263
$w^3$		3.68456	-320.335	-35669.9
$w^2$		6.32403	86.0133	3091.45
$w^1$	-1.22698	-9.50108	-18.5657	-124.955
$w^0$	2.01277	3.08138	3.21685	4.19054

(a)

Coefficient of	$\ln(\lambda) = -18.42$	$\ln(\lambda) = -6.9077$	$\ln(\lambda) = -4.605$	$\ln(\lambda) = -2.3025$	$\ln(\lambda) = 0$	$\ln(\lambda) = 2.3025$
$w^9$	276.76	-2.35335	-0.462206	0.621865	0.200221	0.0470843
$w^8$	-290.3	-1.52781	-0.100236	0.685354	0.210766	0.0522241
$w^7$	-232.859	-0.440062	0.365758	0.745059	0.218586	0.0569209
$w^6$	147.792	0.881629	0.927632	0.786794	0.220697	0.0621078
$w^5$	258.223	2.30068	1.53208	0.780796	0.211638	0.0677987
$w^4$	-138.462	3.47236	2.01729	0.665298	0.18138	0.0739988
$w^3$	-46.8632	3.67343	1.95876	0.3107	0.11149	0.0808511
$w^2$	43.226	1.40446	0.284229	-0.553412	-0.0290924	0.0897927
$w^1$	-14.4715	-7.43556	-5.28043	-2.28264	-0.233213	0.116394
$w^0$	3.19854	2.96045	2.68583	2.07551	1.21495	0.456602

(b)

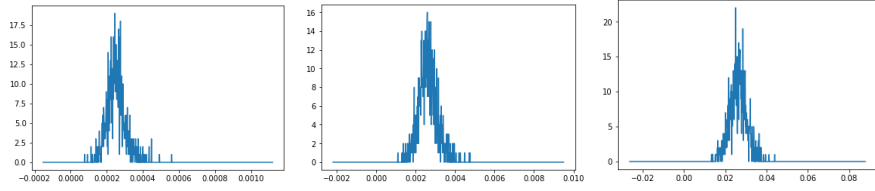
## Histograms- zoom to observing variance



(a) deg-1 lamda-0.001

(b) deg-1 lamda-0.01

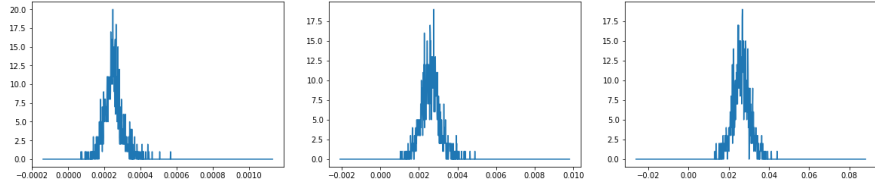
(c) deg-1 lamda-0.1



(a) deg-5 lamda-0.001

(b) deg-5 lamda-0.01

(c) deg-5 lamda-0.1



(a) deg-9 lamda-0.001

(b) deg-9 lamda-0.01

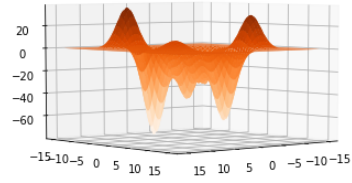
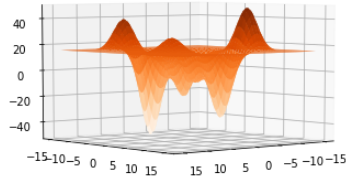
(c) deg-9 lamda-0.1

For a given degree with increase in lambda value the values of empirical risk increase from the order 0.0001 to 0.001 to 0.1 and also the variance in the empirical risk increases from 0.0002 to 0.002 to 0.02

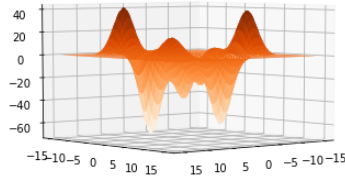
For a given degree as lambda increases there is a slight decrease in bias  
At a given lambda as the degree increases bias decreases, In linear model as the bias is high variance is very low

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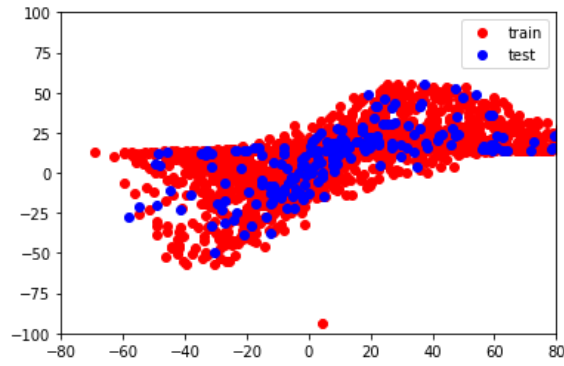
Best model training size 2000,  $K=10$ , variance=3



(a) training data size=100 , $K$  =10 ,variance=3 (b) training data size=1000 , $K$  =10 ,variance=3



(c) training data size=2000 , $K$  =10 ,variance=3



(d) training data size=2000 , $K$  =10 ,variance=3

2.0718083205670914	1.8218922536764157	1.9417863524157424
1.3028668210423784	1.8058381465793523	1.9152217990443392
1.1013914121084512	1.8050728831527947	1.9195257113518152

(a) training data size increases from top to down vs error rms      (b) training data size increases from top to down vs error rms      (c) training data size increases from top to down vs error rms

2.071808320693018	1.8218922537293674	1.9417863525257484
2.0719341732976417	1.8219451950547019	1.9418962472991146
2.0730602417323314	1.822420636990759	1.9428754331631657
2.0836942962794005	1.8270210376804983	1.951815859570481
2.151781958186159	1.8574117182975096	2.0024094284687854
2.238533899718736	1.8957425828059584	2.0597486884736815

(a) lamda increases from top to down,train error rms      (b) lamda increases from top to down,valid error rms      (c) lamda increases from top to down,test error rms