COSC 600
Class Exercises #3

1. For each function f(n) and time t in the following table, determine the largest size n of a problem that can be solved in time t, assuming that the algorithm to solve the problem takes f(n) milliseconds (ms).

	1 second	1 minute	1 hour	1 day	1 month	1 year
log n						
$\sqrt{n}$						
n						
n^2						
n^3						
2^n						
n!						

2. Indicate, for each pair of expressions (A, B) in the table below, whether A is O, Ω, Θ of B.

   Assume that k ≥ 1, and c > 1 are constants.

   Your answer should be in the form of the table with “yes” or “no” written in each box.

A	B	O	Ω	Θ	o
n^k	c^n				
$\sqrt{n}$	n				
$logn^5$	n^2				

3. Let f(n) and g(n) be asymptotically **positive** functions. Is it true or false each of the following conjectures?

   a. f(n) = O(g(n)) implies g(n) = O(f(n)).

   b. f(n) + g(n) = Θ (max(f(n), g(n))).

   c. f(n) = O(g(n)) implies log(f(n)) = O(log(g(n))), where log(g(n)) ≥ 1 and f(n) ≥ 1 for all sufficiently large n.

   d. f(n) = O((f(n))^2) where f(n) is an increasing function and f(n) >= 1.

   e. F(n) = O(g(n)) implies g(n) = Ω(f(n).

4. Rank the following functions by order of growth where log n is $\log_2 n$

$(\sqrt{n}\quad)$, n^3 , $n!$, $(\log n)^5$, 1, n , $\sqrt{\log n}$, $2^{\log n}$, 2^n, $nlogn$, $n + logn$

5. Give a tight asymptotic bounds for the following recurrence.

   a. $T(n) = 2T(n/4) + 1$

   b. $T(n) = 2T(n/4) + \sqrt{n}$

   c. $T(n) = 2T(n/4) + n$

   d. $T(n) = 2T(n/4) + n^2$

   e. $T(n) = 2T(n/2) + n^4$

   f. $T(n) = T(n-2) + n^2$

   g. $T(n) = 2T(n/2) + 1$

6. Suppose $T_1(N) = O(f(N))$ and $T_2(N) = O(f(N))$.

   a. Is $T_1(N) + T_2(N) = O(f(N)$ true?

   b. Is $T_1(N) – T_2(N) = o(f(N)$ true?

   c. Is $T_1(N) / T_2(N) = O(1)$ true?

7. In a recent court case, a judge cited a city for contempt and ordered a fine of \$1 for the first day. Each subsequent day, until the city followed the judge's order, the fine was squared (that is, the fine progressed as follows: \$1, \$2, \$4, \$16, \$256, \$65,536, …..).

   a. What would be the fine on day N?

   b. How many days would it take the fine to reach D dollars? (A Big-Oh answer will do.)

8. An algorithm takes 0.1 second for input size 50. How long will it take for input size 800 if the running time is the following (assume low-order terms are negligible):

   a. linear
   b. O(logN)
   c. quadratic
   d. cubic

9. An algorithm takes 1 second for input size 100. How large a problem can be solved in 8 seconds if the running time is the following (assume low-order terms are negligible):

a. linear
b. O(logN)
c. quadratic
d. cubic

10. For each of the following program fragments,
    a. What is run time function, T(n)?
    b. Give an analysis of the running time in Θ notation.

1)
```
sum = 0;
for ( i = 0; i < n; i++)
    sum++;
```
2)
```
sum = 0;
for ( i = 0; i< n; i++)
        for( k = 0; k < i; k++)
                sum++;
```
3)
```
sum = 0;
for ( i = 0; i< 6*n; i++)
    for ( j = 0; j < i; j++)
            sum++;
```
4)
```
sum = 0;
for( i = 1; i <= n; i++)
    for( j = 1; j <=  i; j++)
            for(k = 1; k <=  j; k++)
                    sum++;
```
5)
```
sum = 0;
for( i = 1; i <= n, i++)
    sum++;
for( j = 1; j<=n * n; j++)
    sum ++;
```