MAD 4401 Numerical Analysis

Homework 2

Sep. 08, 2022 by Dr. Lei-Hsin Kuo

This assignment is due by 11:59 pm on Thursday, Sep. 15.

1. Use algebraic manipulation to show that each of the following functions has a fixed point p precisely when $f(p) = 0$, where $f(x) = x^4 + 2x^2 - x - 3$.

   a.) $g_2(x) = \left(\dfrac{x + 3 - x^4}{2}\right)^{\frac{1}{2}}$

   b.) $g_3(x) = \left(\dfrac{x + 3}{x^2 + 2}\right)^{\frac{1}{2}}$

   (Section 2.2, Exercise 1)

2. a.) Perform four iterations of fixed-point iteration, if possible, on each of the functions defined in Exercise 1 with starting guess $p_0 = 1$.

   b.) Which function do you think gives the best approximation to the solution? Why?

   (Section 2.2, Exercise 2)

3. a.) Use Theorem 2.3 (fixed-point theorem) to show that

   $$g(x) = \frac{1}{10}\left(\frac{5}{x^2} + 2x + 9\right)$$

   has a unique fixed point on $[1, 3]$.

   b.) Use Corollary 2.5 (error bound for fixed-point iteration) to estimate the number of iterations required to find an approximation to the fixed point accurate to within 10^{-5} using fixed-point iteration with any starting guess p_0 in the interval $[1, 3]$.

   c.) Used fixed-point iteration starting with $p_0 = 2$ to find an approximation to the fixed point accurate to within 10^{-5}. Since we don't know the fixed point you can stop when $|p_n - p_{n-1}| < 10^{-5}$ (don't worry, this will not take the amount of iterations predicted in part b).