MAD 4401 Numerical Analysis **Name (Print):** ______________________
Fall 2022
Sample Test 1
09/22/22
Time Limit: 75 Minutes Instructor ______________________

This exam contains 2 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, and graphing calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	30	
3	10	
4	20	
5	20	
Total:	100	

1. (20 points) Consider the function

$$f(x) = x - \cos(x).$$

   a [10 points] Starting with the interval $\left[0, \frac{\pi}{2}\right]$, carry out 3 iterations of bisection method to produce p_3.

   b [10 points] Use the error bound (from THM 2.1) for bisection method to determine the number of iterations necessary to ensure an approximation with accuracy 10^{-6}.

2. (30 points) Consider the function $g(x) = \frac{1}{2}\left(10 - x^2\right)^{1/2}$.

   a [20 points] Show $g(x)$ has a unique fixed-point on $[1, 2]$.

   b [10 points] How many iteration are needed to ensure fixed-point iteration converges to the fixed-point, p, for any $p_0 \in [1, 2]$ with accuracy 10^{-6}.

3. (10 points) Consider the function

$$f(x) = x - \cos(x),$$

   starting with the initial guess $p_0 = 0$, carry out 2 iterations of Newton's method to produce p_2.

4. (20 points) Consider the function

$$f(x) = (x + 1)(x - 2)^2.$$

   a.) [10 points] For the root on the interval $[-3, 0]$, do you expect linear or quadratic convergence of Newton's method? Why?

   b.) [10 points] For the root on the interval $[1, 3]$, do you expect linear or quadratic convergence of Newton's method? Why?

5. (20 points) Consider the function $f(x) = \sin(x)$.

   a [10 points] Find the third Taylor polynomial approximation $p_3(x)$ of $f(x)$ at $x_0 = \frac{\pi}{6}$.

   b [10 points] Show that if $0 < h < 1$, then the error associated to the approximating $f\left(\frac{\pi}{6} + h\right)$ by $p_3\left(\frac{\pi}{6} + h\right)$ is of order $O\left(h^4\right)$.