Total No. of Questions :3] [2511]-F-001 [Seat No.:

10

Society for Computer Technology and Research's

PUNE INSTITUTE OF COMPUTER TECHNOLOGY

IN-SEM EXAMINATION OCT-2025

(FY) (B.Tech.) (BS&E) (SEM-I)

(F-001) (Linear Algebra and Calculus)

(2024 PATTERN)

Time : 1 Hour] [Max Marks : 20

Instructions to the candidates:

1. *Assume suitable data wherever required.*
2. *Neat diagrams must be drawn wherever necessary.*
3. *Use of non-programmable calculators is allowed.*
4. *[Marks], [Course Outcome] and [Bloom's Taxonomy Level] given at the end of each questions.*
5. ***Attempt Any Two Questions***

Q.1)

A) Solve the following system of linear equations, if consistent by reducing it to the row echelon form.

$$x + 2y - z = 3,$$
$$3x - y + 2z = 1,$$
$$2x - 2y + 3z = 2,$$
$$x - y + z = -1$$

[05] [CO1] [L2]

B) Examine the row vectors of the following matrix for linear dependence. If dependent, find the relation between them.

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 2 & 1 & 1 & 1 \\ 3 & -1 & 2 & -1 \\ 3 & 0 & 3 & 1 \end{bmatrix}$$

[05] [CO1] [L3]

Q.2)

A) Express each of the transformation $x_1 = 3y_1 + 5y_2$, $x_2 = -y_1 + 7y_2$ and $y_1 = z_1 + 3z_2$, $y_2 = 4z_1$ in the matrix form and find the composite transformation which expresses x_1, x_2 in terms of z_1, z_2 **[05] [CO1] [L2]**

B) If A and B are $n \times n$ matrices, and $AB = I$, where I is an identity matrix. Prove that rank of A is n. **[05] [CO1] [L3]**

Q.3)

A) Decode the cipher text message "UOUVNY" to plain text message using key matrix, $K = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$. **[05] [CO1] [L2]**

B) If A and B are orthogonal matrices, then show that both A^{-1} and $(AB)^{-1}$ are orthogonal matrices. [05] [CO1] [L3]

..... **End of question paper**.....

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IN-SEMESTER EXAMINATION
ACADEMIC YEAR 2025 - 26
SEMESTER – I

Program : F. Y. B. Tech. (Common) **Class: FY-1 to FY-13**

Date of Examination: 17/10/2025 **Max. Marks: 20**

Course: F001 - LINEAR ALGEBRA AND CALCULUS

Solution and Marking Scheme

COs	Course Outcomes
F001-1	**Solve** system of linear equations. **Examine** linear dependence of vectors. **Express** linear and orthogonal transformations in matrix form and discuss its nature.

★ **Note :** Any alternate solution/method should be given due credit.

Q1 A] Given non-homogeneous system is written in matrix form as follows

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

i.e. $A\,X = B$

Consider $(A|B) = \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{array}\right] \xrightarrow[\substack{R_3-2R_1 \\ R_4-R_1}]{R_2-3R_1}$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{array}\right] \xrightarrow{R_3-2R_4} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -7 & +5 & -8 \\ 0 & 0 & 1 & 4 \\ 0 & -3 & 2 & -4 \end{array}\right] \xrightarrow{R_2-2R_4} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -1 & 1 & 0 \\ 0 & -3 & 2 & -4 \\ 0 & 0 & 1 & 4 \end{array}\right] \xrightarrow{R_3-3R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 1 & 4 \end{array}\right] \xrightarrow{R_4+R_3} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R_3/(-1)]{R_2/(-1)} \left[\begin{array}{ccc|c} \boxed{1} & 2 & -1 & 3 \\ 0 & \boxed{1} & -1 & 0 \\ 0 & 0 & \boxed{1} & 4 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{ — (1M)}$$

This is row-echelon form.

$\rho(A|B) = \rho(A) = 3 =$ no. of variables $\Rightarrow$ system has unique soln. (1M)

$\therefore z = 4,\ y - z = 0 \Rightarrow y = 4$ & $x + 2y - z = 3 \Rightarrow x = -1$. $\{-1, 4, 4\}$. (1M)

B] Let $X_1 = (1,-1,2,0),\ X_2 = (2,1,1,1),\ X_3 = (3,-1,2,-1),\ X_4 = (3,0,3,1)$.

Consider the linear combination of given vectors — (1M)

$$c_1X_1 + c_2X_2 + c_3X_3 + c_4X_4 = 0$$

$\therefore c_1(1,-1,2,0) + c_2(2,1,1,1) + c_3(3,-1,2,-1) + c_4(3,0,3,1) = 0$

$\therefore c_1 + 2c_2 + 3c_3 + 3c_4 = 0$
$-c_1 + c_2 - c_3 = 0$
$2c_1 + c_2 + 2c_3 + 3c_4 = 0$
$c_2 - c_3 + c_4 = 0$

This homogeneous system can be written in matrix form as follows.

$$\begin{bmatrix} 1 & 2 & 3 & 3 \\ -1 & 1 & -1 & 0 \\ 2 & 1 & 2 & 3 \\ 0 & 1 & -1 & 1 \end{bmatrix} \xrightarrow[R_3-2R_1]{R_2+R_1} \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 3 & 2 & 3 \\ 0 & -3 & -4 & -3 \\ 0 & 1 & -1 & 1 \end{bmatrix} \xrightarrow{R_3+R_2} \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 3 & 2 & 3 \\ 0 & 0 & -2 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}.$$

$$\xrightarrow[R_3)(-2)]{R_2-3R_4} \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix} \xrightarrow{R_2)(5)} \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix} \xrightarrow{R_3-R_2} \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow{R_{24}} \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{R_{34}} \begin{bmatrix} 1 & 2 & 3 & 3 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 3 <$ no. of variables

$\Rightarrow$ system has infinitely many soln

$\therefore X_1, X_2, X_3, X_4$ are linearly dependent. — (3M)

Let $c_4 = t$, $c_3 = 0$, $c_2 - c_3 + c_4 = 0 \Rightarrow c_2 = -t$,

$c_1 + 2c_2 + 3c_3 + 3c_4 = 0 \Rightarrow c_1 = -t$

Put in linear combination of vectors, $-tX_1 - tX_2 + tX_4 = 0$.

$\therefore \boxed{X_1 + X_2 = X_4}$. — (1M)

Q2 A] Given $x_1 = 3y_1 + 5y_2$, $x_2 = -y_1 + 7y_2$.

This linear transformation is expressed in matrix form as follows.

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

i.e. $X = AY$

Given $y_1 = z_1 + 3z_2$, $y_2 = 4z_1$ This L.T. is expressed in matrix form as follows $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ i.e. $Y = BZ$.

$\therefore$ Composite transformation is $X = AY$ — (2M)

$= A(BZ) = (AB)Z$.

Consider $AB = \begin{bmatrix} 3 & 5 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 23 & 9 \\ 27 & -3 \end{bmatrix}$ — (2M)

$\therefore$ Composite transformation is

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 23 & 9 \\ 27 & -3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \quad \text{i.e.} \quad \boxed{\begin{aligned} x_1 &= 23z_1 + 9z_2, \\ x_2 &= 27z_1 - 3z_2 \end{aligned}}$$

— (1M)

Q 2 B] Given A & B are $n \times n$ matrices & $AB = I$.

$\Rightarrow \rho(AB) = \rho(I) = n$. — (2M)

We know, $\rho(AB) \leq \rho(A)$ — (1M)

$\therefore n \leq \rho(A)$

$\Rightarrow \rho(A) = n$ — (2M)

Q3) A) "UOUVNY"

21 15 21 22 14 25

It is cut into pieces each having 2 letters.

Let $A = \begin{bmatrix} 21 & 21 & 14 \\ 15 & 22 & 25 \end{bmatrix}_{2\times3}$ Given key matrix $K = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ — (1M)

$\therefore K^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$. — (1M)

Pre-multiplying by K^{-1} to matrix A,

$\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 21 & 21 & 14 \\ 15 & 22 & 25 \end{bmatrix} = \begin{bmatrix} 27 & 20 & 3 \\ -6 & 1 & 11 \end{bmatrix}$ — (2M)

$27 \equiv 1 \pmod{26}$, $-6 \equiv 20 \pmod{26}$ $= \begin{bmatrix} 1 & 20 & 3 \\ 20 & 1 & 11 \end{bmatrix}$

$\therefore$ Plain text is $\boxed{\text{"ATTACK"}}$ — (1M)

Q3) B) Given A & B orthogonal matrices, we have

$AA' = I$, $BB' = I$ — ① — (1M)

Also $A^{-1} = A'$, $B^{-1} = B'$. — ②

Consider $AA' = I$ ... given ①

$(AA')^{-1} = (I)^{-1} \Rightarrow (A')^{-1} A^{-1} = I \Rightarrow (A^{-1})'(A^{-1}) = I$

$\Rightarrow A^{-1}$ is an orthogonal matrix. — (2M)

T.S.T. $(AB)^{-1}$ is orthogonal we need t.s.t. $[(AB)^{-1}]'(AB)^{-1} = I$

LHS $[(AB)^{-1}]'[(AB)^{-1}]$

$= [B^{-1}A^{-1}]'[B^{-1}A^{-1}]$ ... $\because (AB)^{-1} = B^{-1}A^{-1}$

$\therefore (A^{-1})'(B^{-1})'(B^{-1})(A^{-1})$ ... from $(AB)' = B'A'$

$\therefore (A')'(B')'(B^{-1})(A^{-1})$ ... from ②

i.e. $A(BB^{-1})A^{-1} = I = RHS$

$\therefore (AB)^{-1}$ is an orthogonal matrix. — (2M)