END TERM EXAMINATION

FIRST SEMESTER [BCA] DECEMBER 2016

Paper Code: BCA-101

Subject: Mathematics-I

Time: 3 Hours

Maximum Marks: 75

Note: Attempt any five questions including Q no. 1 which is compulsory. Select one question from each unit.

- (a) Prove that every square matrix is uniquely expressible as the sum of a Q1 symmetric matrix and a skew symmetric matrix. (5)
 - (b) For what value of x, the matrix

(5)

$$A = \begin{bmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{bmatrix}$$
 is singular.

(c) Using properties without expanding prove that:

(5)

$$\begin{vmatrix} x + y & y + z & z + x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0.$$

- (d) Show that $f(x) = \begin{cases} 2x 1; & x < 2 \\ 3; & x = 2 \\ x + 1; & x > 2 \end{cases}$ is continuous at x = 2. (5)
- (e) Show that function $f(x) = \sin x(1 + \cos x)$ is maximum when $x = \frac{\pi}{3}$. (5)

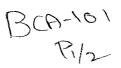
UNIT-I

(a) If the matrix is orthogonal, then find the values of a, b and c where O2matrix is

$$A = \begin{bmatrix} a & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}. \tag{6.5}$$

- (b) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} -2 & 2 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Also, find A⁻¹.
- (a) Find the eigen values and eigen vectors of the matrix Q3 $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$ (6)
 - (b) Examine the following system of vectors for linearly dependence. If dependent, find the relation between them (6.5) $X_1 = (1, -1, 1); X_2 = (2, 1, 1); X_3 = (3,0,2).$

P.T.O.





Unit-II

- (6)Evaluate $\lim_{x\to 1} \left([x] + \frac{|x-1|}{x-1} + 2 \right)$. Q4.
 - For what choice of 'a' and 'b' is the function continuous $\forall x \in R$ b) $f(x) = \begin{cases} ax^2 + b, & x < 2 \\ 2 & x = 2 \\ 2ax + b, & x > 2 \end{cases}$ (6.5)
- For what value of λ' does the $\lim_{x\to 1} f(x)$ exists, where f is defined by the rule $f(x) = \begin{cases} 2\lambda x + 3 & \text{if } x < 1 \\ 1 - \lambda x^2 & \text{if } x > 1 \end{cases}$ Q5. a)
 - Discuss the nature of discontinuity at x=0 of $f(x) = \begin{cases} \frac{\sin[x]}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$.(6) b)

- a) Q6. $-2x^2 + 2y^2 - 1 = 0$.
 - (6)If $x^y + y^x = a^b$, find $\frac{dy}{dx}$.
- (6.5)If $y = \sin^{-1} x$ then show that Q7. a) i) $(1 - x^2)y_2 - xy_1 = 0$. ii) $(1-x^2)y_{n+2} - (2n+1) xy_{n+1} - n^2y_n = 0$.
 - maxima/minima for function given the b) (6) $f(x) = \frac{(x-1)(x-6)}{(x-10)}, x \neq 10.$

Unit-IV

- Evaluate a) Q8. (ii) $\int_0^2 \frac{5x}{x^2+1} dx$. $\int \log(1+x)dx$
 - evaluate Obtain the reduction formula for $\int \tan^n x \ dx$. Also b) (6.5) $\int_{0}^{\pi/4} \tan^{n} x \ dx.$
- Show that Q9. a)

Show that
$$\int_{0}^{\pi/2} \sin^{p} \theta \cos^{q} \theta \ d\theta = \frac{\sqrt{\frac{p+1}{2}} \sqrt{\frac{q+1}{2}}}{2\sqrt{\frac{p+q+2}{2}}}, \ p,q > -1.$$
(6.5)

(6)

(6)Evaluate $\int_0^1 \frac{xe^x}{(x+1)^2} dx$. b)

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