BCA

(Please write your Exam Roll No.) Exam Roll No. ....

END TERM EXAMINATION

FIRST SEMESTER [BCA] DECEMBER 2016

Paper Code: BCA-101	Subject: Mathematics-I
Time: 3 Hours	Maximum Marks: 75

Note: Attempt any five questions including Q no.1 which is compulsory. Select one question from each unit.

Q1 (a) Prove that every square matrix is uniquely expressible as the sum of a symmetric matrix and a skew symmetric matrix. (5)

(b) For what value of x, the matrix (5)

$$A=\begin{bmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{bmatrix} \text{ is singular.}$$

(c) Using properties without expanding prove that: (5)

$$\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0.$$

(d) Show that $f(x)=\begin{cases} 2x-1; & x<2 \\ 3; & x=2 \\ x+1; & x>2 \end{cases}$ is continuous at x = 2. (5)

(e) Show that function $f(x)=\sin x(1+\cos x)$ is maximum when $x=\frac{\pi}{3}$. (5)

UNIT-I

Q2 (a) If the matrix is orthogonal, then find the values of a, b and c where matrix is

$$A=\begin{bmatrix} a & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}.$$ (6.5)

(b) Verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix} -2 & 2 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Also, find A^{-1}. (6)

Q3 (a) Find the eigen values and eigen vectors of the matrix

$$A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$ (6)

(b) Examine the following system of vectors for linearly dependence. If dependent, find the relation between them

X_1 = (1, -1, 1); X_2 = (2, 1, 1); X_3 = (3,0,2). (6.5)

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Unit-II

Q4. a) Evaluate $\lim_{x \to 1}\left([x]+\frac{|x-1|}{x-1}+2\right)$. **(6)**

b) For what choice of 'a' and 'b' is the function continuous $\forall$ x $\in$ R **(6.5)**

$$f(x)=\begin{cases} ax^2+b, & x<2 \\ 2 & x=2 \\ 2ax+b, & x>2 \end{cases}$$

Q5. a) For what value of 'λ' does the $\lim_{x \to 1}$ f(x) exists, where f is defined by the rule $f(x)=\begin{cases} 2\lambda x+3 & \text{if } x<1 \\ 1-\lambda x2 & \text{if } x>1 \end{cases}$. **(6.5)**

b) Discuss the nature of discontinuity at x=0 of $f(x)=\begin{cases} \frac{\sin[x]}{[x]}, & x \neq 0 \\ 0, & x=0 \end{cases}$ **(6)**

Unit-III

Q6. a) Find all the asymptotes of $y^4 - 2xy^3 + 2x^3y - x^4 - 3x^3 + 3x^2y + 3xy^2 - 3y^3 - 2x^2 + 2y^2 - 1 = 0$. **(6.5)**

b) If $x^y + y^x = a^b$, find $\frac{dy}{dx}$. **(6)**

Q7. a) If $y = \sin^{-1} x$ then show that **(6.5)**
i) $(1-x^2)y_2 - xy_1 = 0$.
ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$.

b) Examine the given function for maxima/minima $f(x)=\frac{(x-1)(x-6)}{(x-10)}, x \neq 10$. **(6)**

Unit-IV

Q8. a) Evaluate **(6)**
i) $\int \log(1+x)dx$ (ii) $\int_0^2 \frac{5x}{x^2+1}dx$.

b) Obtain the reduction formula for $\int \tan^n x\, dx$. Also evaluate $\int_0^{\pi/4} \tan^n x\, dx$. **(6.5)**

Q9. a) Show that **(6.5)**

$$\int_0^{\pi/2} \sin^p\theta\cos^q\theta\, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}, \quad p,q > -1.$$

b) Evaluate $\int_0^1 \frac{xe^x}{(x+1)^2}dx$. **(6)**

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