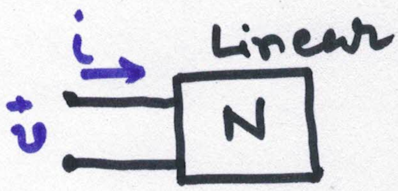
Impedance.



Complex freq. $= \sigma + j\omega$

voltage of the form e^{st}

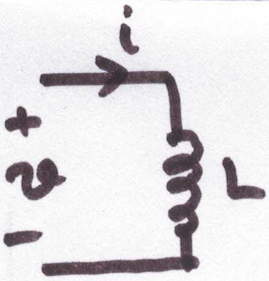
$$\frac{v}{i} = \text{Constant}$$

$Z(s) =$ Impedance

$$v = iR$$

$$\frac{v}{i} = R \quad [\text{independent of } s]$$

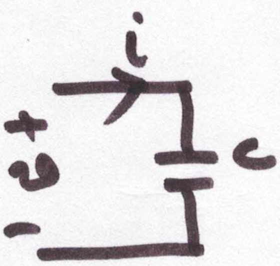
2



$$v = L\frac{di}{dt} \qquad i = e^{st}$$

$$v = sLe^{st}$$

$$\frac{v}{e^{st}} = sL \Rightarrow \frac{v}{i} = sL = Z_L(s)$$



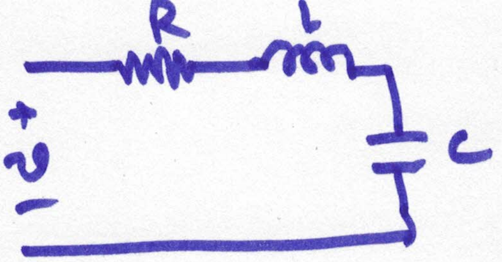
$$i = C\frac{dv}{dt} \; ; \; v = e^{st}$$

$$\frac{v}{i} = \frac{1}{sC} = Z_c(s)$$

$$\text{Admittance} = \frac{1}{\text{Impedance}}$$

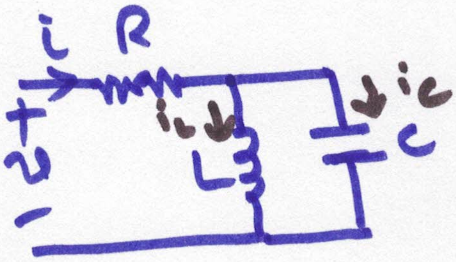
3

#1.



$$Z(s) = R + sL + \frac{1}{sC}.$$

#2.



$R + (L \parallel C)$

$$Z(s) = R + \frac{sL \cdot \frac{1}{sC}}{sL + \frac{1}{sC}}$$

$$= R + \frac{sL}{s^2LC + 1}$$

$\frac{d}{dt} \to s$

$\int dt \to \frac{1}{s}$

$v = 6e^{-2t}$

4

Phase.

Let F be a periodic signal, and T be it's period i.e. $F(t+T) = F(t)$ for all t. Then the phase of F at any argument t is

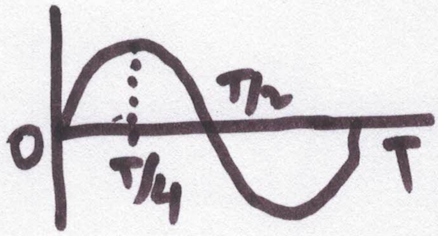
$$\phi(t) = 2\pi \left[\!\left[\frac{t - t_0}{T} \right]\!\right]$$

$[\![\cdot]\!] \to$ Represents the fractional part of a real no.

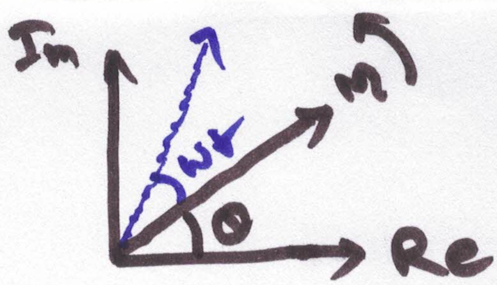
$t_0 \to$ begining of a cycle.

$t = \frac{T}{4}$

$$\Phi(t) = 2\pi \left[\frac{T/4 - 0}{T} \right]$$



$$= 2\pi \left[\frac{1}{4} \right] = \frac{\pi}{2}.$$



$$\bar{M} = Me^{j\theta} = M\angle\theta$$

$$Me^{j(\omega t + \theta)}$$

* Phasor Let $v(t) = V_m \cos(\omega t + \theta)$

$$e^{st} = e^{(\sigma + j\omega)t} = e^{j\omega t}$$

$$= Re\left[V_m e^{j(\omega t + \theta)}\right]$$

$$= Re\left[\sqrt{2}\, V e^{j(\omega t + \theta)}\right]$$

rms voltage

Identifying quantity (Phasor) $= Re\left[\underbrace{(Ve^{j\theta})} (\sqrt{2} e^{j\omega t})\right]$

Representation of voltage $v(t)$

$$\bar{V} = Ve^{j\theta}$$

$$\bar{I} = I_0 e^{j\alpha}$$

$$i(t) = \mathrm{Re}\left[I_0 e^{j\alpha} \sqrt{2} e^{j\omega t}\right]$$

$$= \sqrt{2} I_0 \cos(\omega t + \alpha)$$

(i) $i_1 = 10\sqrt{2}\cos\omega t \rightarrow \bar{I}_1 = 10e^{j0} = 10\angle 0$

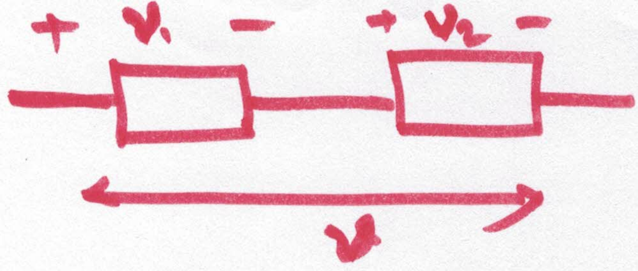
(ii) $i_2 = 10\cos\left(\omega t + \frac{\pi}{6}\right) \rightarrow \bar{I}_2 = \frac{10}{\sqrt{2}} e^{j\frac{\pi}{6}}$.

(iii) $v_1(t) = 20\sqrt{2}\sin\left(\omega t + \frac{\pi}{6}\right)$

$$= 20\sqrt{2}\cos\left(\omega t + \frac{\pi}{6} - \frac{\pi}{2}\right)$$

$$= 20\sqrt{2}\cos\left(\omega t - \frac{\pi}{3}\right)$$

$$\bar{V}_1 = 20 e^{-j\frac{\pi}{3}}.$$

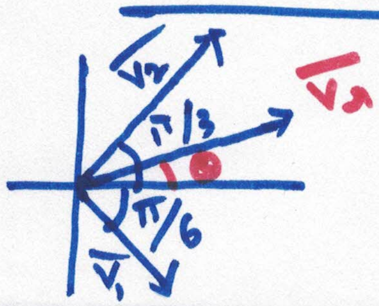


$$v_1 = 150\sqrt{2}\cos\left(100\pi t - \frac{\pi}{6}\right)$$

$$v_2 = 200\sqrt{2}\cos\left(100\pi t + \frac{\pi}{3}\right)$$

$$v = ?$$

① Graphical



$$\bar{V}_1 = 150\, e^{-j\frac{\pi}{6}}.$$

$$\bar{V}_2 = 200\, e^{j\frac{\pi}{3}}.$$

$$\bar{V}_r = V_r \angle \theta$$

$$v = \sqrt{2}\, V_r \cos(\omega t + \theta)$$

② Analytical

$$\bar{V}_1 = 150\, e^{-j\frac{\pi}{6}} = 130 - j75.$$

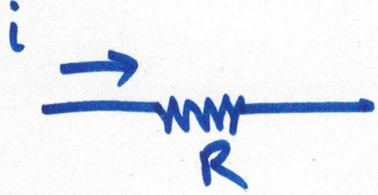
$$\bar{V}_2 = 200\, e^{j\frac{\pi}{3}} = 100 + j173$$

$$\bar{V}_1 + \bar{V}_2 = 230 + j98$$

$$\bar{V} = 250 \angle 23.1$$

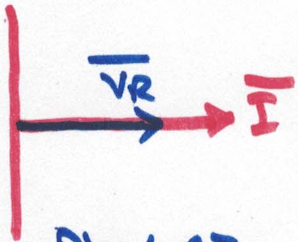
$$v = 250\sqrt{2}\cos(100\pi t + 23.1)$$

11



$$i = \sqrt{2}\, I \cos \omega t .$$

$$\bar{I} = I e^{j0} = I \angle 0$$



Phasor.

$$\bar{V}_R = \bar{I} R = V_R \angle 0$$

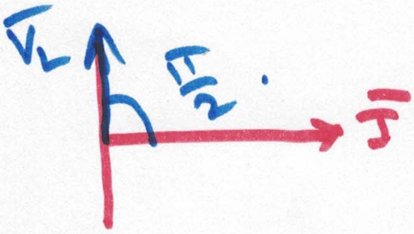
$$v_R = \sqrt{2}\, I R \cos \omega t$$

$$\frac{\bar{V}_R}{\bar{I}} = R = R \angle 0^\circ$$

12

$$i = \sqrt{2}\, I \cos \omega t .$$

$$\bar{I} = I \angle 0^\circ$$



$$v = L \frac{di}{dt}$$

$$= -\omega L \sqrt{2}\, I \sin \omega t$$

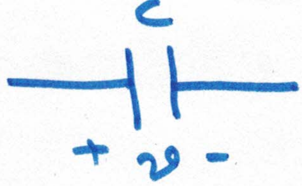
$$= \sqrt{2}\, \omega L I \cos \left(\omega t + \frac{\pi}{2}\right)$$

$$\bar{V}_L = \omega L I \angle \frac{\pi}{2} .$$

$$= V_L \angle \frac{\pi}{2} .$$

$$\frac{\bar{V}_L}{\bar{I}} = \omega L \angle \pi/2 = Z_L \text{ (Impedance)}$$

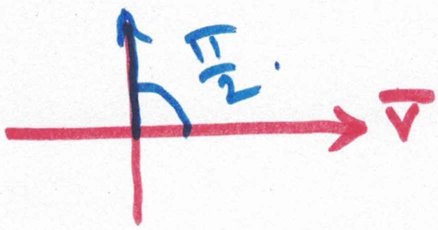
$$= j \omega L$$



$v = \sqrt{2} V \cos \omega t \rightarrow \bar{V} = V \angle 0^\circ$

$i = C \dfrac{dv}{dt}$

$= -\sqrt{2}\, \omega C V \sin \omega t$



$i = \sqrt{2}\, \omega C V \cos\left(\omega t + \dfrac{\pi}{2}\right)$

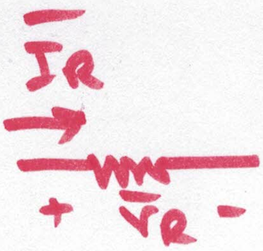
$\bar{I}_C = \omega C V \angle \pi/2$.

$$\frac{\bar{V}}{\bar{I}_C} = \frac{1}{\omega C \angle \pi/2} = \frac{1}{j\omega C}$$

$$= Z_C$$

(Not a Phasor)

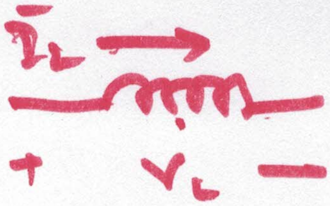
$\sqrt{2} K \cos(\omega t + \theta)$



$\bar{V}_R = R \bar{I}_R$

$\bar{I}_R = \dfrac{\bar{V}_R}{R}$

$= G \bar{V}_R$

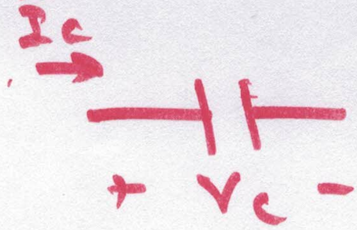


$\bar{V}_L = j\omega L \bar{I}_L$

$\bar{I}_L = \dfrac{\bar{V}_L}{j\omega L}$

$Z_L = j\omega L$

$Y_L = \dfrac{1}{j\omega L}$



$\bar{V}_C = \dfrac{1}{j\omega C} \bar{I}_C$

$\bar{I}_C = j\omega C \bar{V}_C$

$Z_C = \dfrac{1}{j\omega C}$

$Y_C = j\omega C$.

when impedance is purely imaginary

$$Z_L = j\omega L = jX_L$$

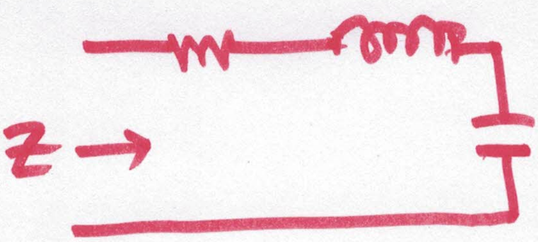
$$X_L = \omega L = \text{Reactance}.$$

$$Y_C = j\omega C = jB_C$$

$$B_C = \omega C = \text{Susceptance of } C.$$

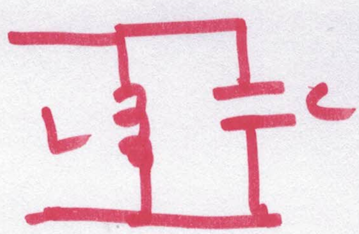
$$Z_C = j\left(-\frac{1}{\omega C}\right) = jX_C$$

$$X_C = -\frac{1}{\omega C} = \text{Reactance of } C$$



$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$= \underbrace{R}_{\text{Resistive part}} + j\underbrace{\left(\omega L - \frac{1}{\omega C}\right)}_{\text{Reactive part.}}$$



$$Z = \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{j\omega L}{1 - \omega^2 LC}$$

$$\omega^2 LC = 1$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$\boxed{\text{Resonance condition}}$

Impedance $\longrightarrow \infty$

KVL

$$\sum_i v_i = 0$$

$$\mathrm{Re} \sum_i \sqrt{2} V_i e^{j\theta} e^{j\omega t} = 0$$

$$\sum_i \bar{V}_i = 0$$

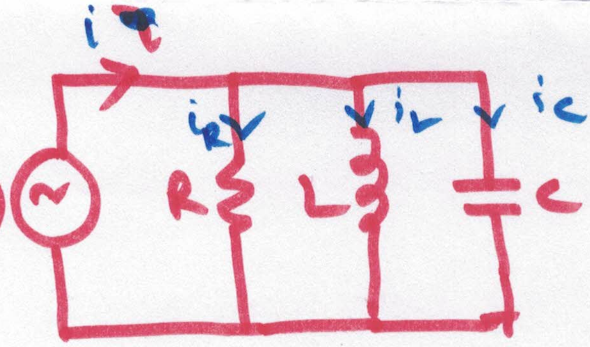
KCL

$$\sum_j i_j = 0$$

$$\downarrow$$

$$\sum_j \bar{I}_j = 0$$

$230\sqrt{2} \cos(100\pi t)$



$$\downarrow$$

$230 \angle 0$

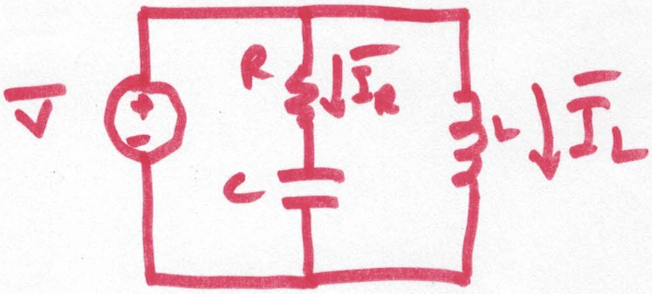
$$\bar{I}_R = \frac{230}{R} \angle 0$$

$$\bar{I}_L = \frac{230}{\omega L} \angle -\frac{\pi}{2}.$$

$$\bar{I}_C = \frac{230}{\frac{1}{\omega C}} \angle \frac{\pi}{2}.$$

$$\bar{I} = \bar{I}_R + \bar{I}_L + \bar{I}_C$$

$i =$



$R = 2\,\Omega$, $L = 0.2\,H$

$i_R = 10\sqrt{2}\cos(10t + 45)$

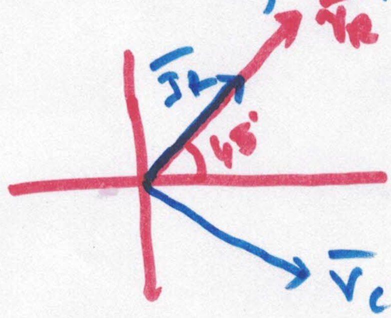
(i) Show $\bar{I}_R$ & $\bar{V}_R$ in a phasor diagram.

(ii) Show the direction of $\bar{V}_C$

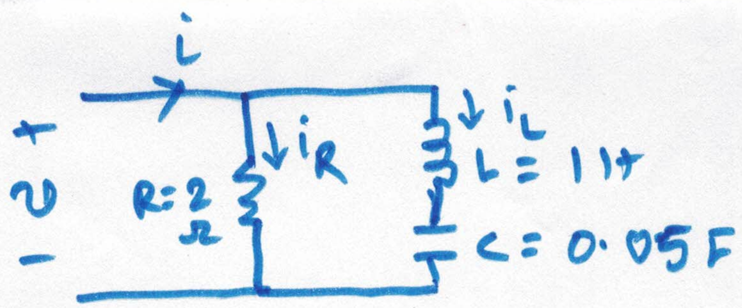
(iii) If the voltage phasor $\bar{V} = V\angle 0°$, $v(t) =$

$$\bar{I}_R = 10\angle 45°$$

$$\bar{V}_R = 20\angle 45°$$



$$\bar{V}_C = \frac{\bar{I}_R}{j\omega C} = \frac{I_R}{\omega C}\angle -\frac{\pi}{2}.$$

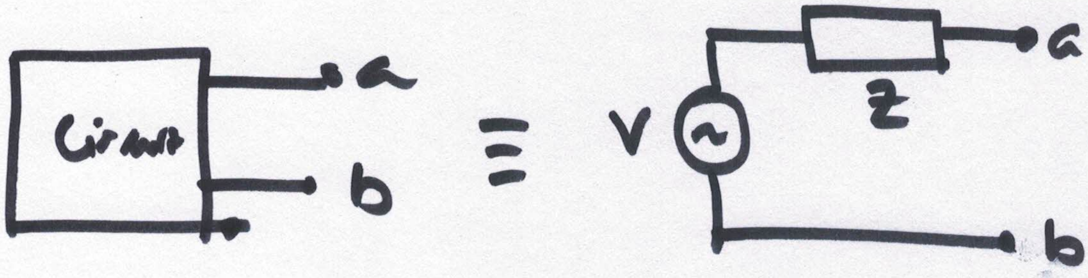


$v(t) = 5\sqrt{2}\cos 5t$

$v_C = ?$

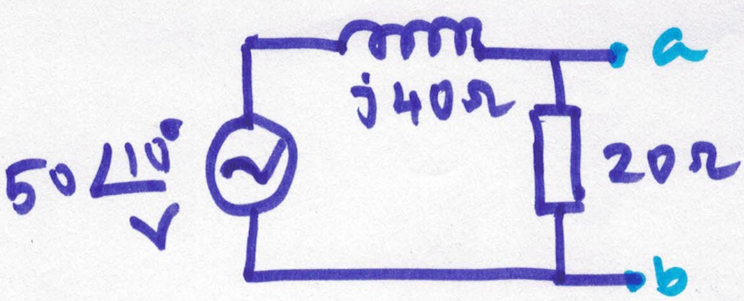
Complete phasor Diagram.

Thevenin & Norton:-

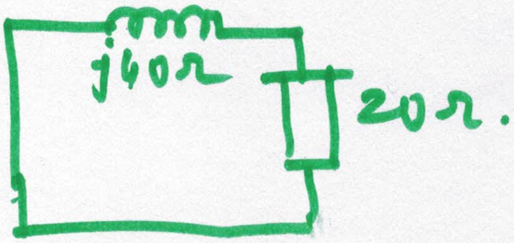


(i) Find V_{oc}

(ii) Find I_{sc}. $\Rightarrow Z = \dfrac{V_{oc}}{I_{sc}}$



By killing source:-



$$Z = Z_{ab} = \frac{j40 \cdot 20}{20 + j40}$$

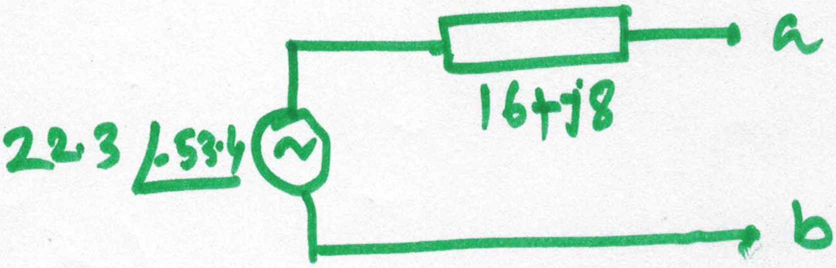
$$= 16 + j8$$

$$V_{oc} = V_{ab} = \frac{20}{20 + j40} 50\angle 10^\circ$$

(3)

$$V_{ab} = \frac{1}{\sqrt{5}\angle 63.4^\circ} \; 50\angle 10^\circ$$

$$V_{ab} = 22.3\angle -53.4$$

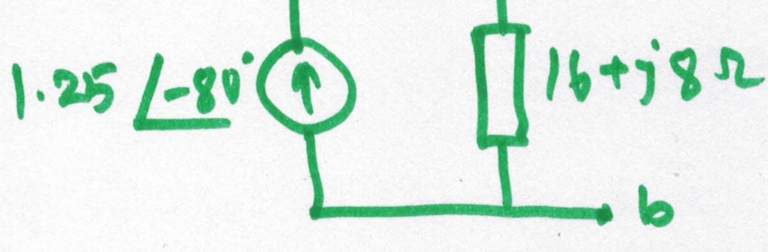


Thevenin Equivalent

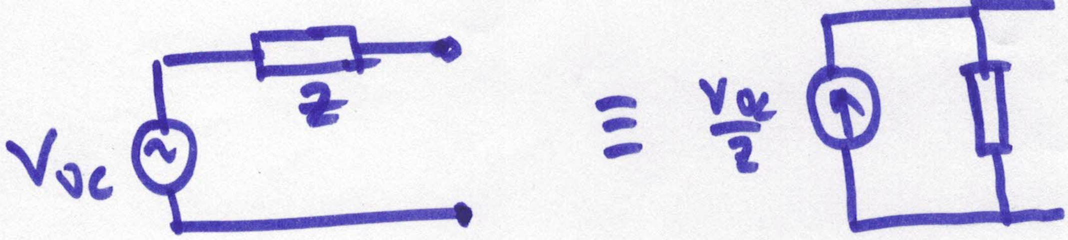
(4)

$$I_{sc} = \frac{50\angle 10}{j40} = \frac{50\angle 10}{40\angle 90^\circ}$$

$$= 1.25\angle -80^\circ$$



Norton's Equivalent



$$\frac{V_{oc}}{Z} = \frac{22.3\angle -53.4}{16 + j8}$$

$$= \frac{22.3\angle -53.4}{17.89\angle 26.6}$$

$$= 1.25\angle -80^\circ$$

(By Source Transformation)