ChaP Test

Name : ______________________________

Batch Code : ________________ **Date of Practice:** ________________

Enroll. No. : ______________________________

Subject : Maths **Class : XII** **Q. P Code : 614080.0**

Definite Integral and Area – 3

Important Instructions

Attempt all the Questions of Section - I, Section - II & Section - III.

Section - I has Two Parts. Part - A and Part - C.

- Part - A has **10 Single choice** and **15 multiple choice** questions with one or more than one correct option.
- Part - C has **10 Integer type** questions.

Section - II has Three Parts. Part - A, Part - B and Part - C.

- Part - A has **2 Comprehension** type questions. Each comprehension describes an experiment, a situation or a problem. Three multiple choice questions will be asked based on this comprehension.
- Part - B has **2 Match the following** type questions and you will have to match entries in Column - I with the entries in Column - II.
- Part C has **10 Integer type** questions. The answer to each question is a single digit integer ranging from 0 to 9.

Section - III has Three Parts. Part - A, Part - B and Part - C.

- Part - A has **5 multiple choice** questions with one or more than one correct option and **1 Comprehension** type questions. Each comprehension describes an experiment, a situation or a problem. Three multiple choice questions will be asked based on this comprehension.
- Part - B has **1 Match the following** type questions and you will have to match entries in Column I with the entries in Column - II.
- Part - C has **10 Integer type** questions. The answer to each question is a single digit integer ranging from 0 to 9.

MARKING SCHEME :

Single choice: **+3** for correct answer**, 0** if not attempted and **–1** in all other cases.

Multiple choice: **+4** for correct answer, **0** if not attempted and **–2** in all other cases.

Comprehension: **+4** for correct answer, **0** if not attempted and **– 2** in all other cases.

Match the following: For each entry in Column I**, +2** for correct answer, **0** if not attempted and **– 1** in all other cases.

Integer type: **+4** for correct answer and **0** in all other cases.

All the best ....

SECTION I

PART A
Single Answer Questions

1. If $k \in N$ and $I_k = \int_{-2k\pi}^{2k\pi} |\sin x|[\sin x]\,dx$, (where [.] denotes the greatest integer function) then $\sum_{k=1}^{100} I_k$ equals to

(A) –10100 (B) –40400 (C) 20200 (D) None of these

2. If $f(x)$ is an integrable function in $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ and $I_1 = \int_{\pi/6}^{\pi/3} \sec^2\theta f(2\sin 2\theta)\,d\theta$ and $I_2 = \int_{\pi/6}^{\pi/3} \operatorname{cosec}^2\theta f(2\sin 2\theta)\,d\theta$, then

(A) $I_1 = 2I_2$ (B) $I_1 = 3I_2$ (C) $2I_1 = I_2$ (D) None of these

3. Area enclosed by the curve y=f9x) defined parametrically as $x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2}$ is equal to

(A) π *sq units* (B) $\pi/2$ *sq units* (C) $3\pi/2$ *sq units* (D) $3\pi/4$ *sq units*

4. The value of $\int_{-6}^{6} \max(|2-|x||, 4-|x|, 3)dx$ is

(A) 40 (B) 50 (C) 60 (D) 30

5. Area of the rectangle formed by asymptotes of the hyperbola $xy - 3y - 2x = 0$ and co-ordinate axes is

(A) 2 sq. units (B) 6 sq. units (C) 4 sq. units (D) none of these

6. A square ABCD is inscribed in a circle of radius 4. A point P moves inside the circle such that $d(P, AB) \le \min(d(P, BC), d(P, CD), d(P, DA))$ where $d(P, AB)$ is the distance of a point P from line AB. The area of region covered by moving point P is

(A) 4π (B) 8π (C) $8\pi - 16$ (D) none of these

7. Let $f(x)$ be a real valued function defined by $f(x) = x^2 + x^2\int_{-1}^{1} tf(t)dt + x^3\int_{-1}^{1} f(t)dt$ then the value of $\int_{-1}^{1} f(x)dx$ is equal to

(A) $\frac{10}{11}$ (B) $\frac{5}{11}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

8. The value of $\int_{0}^{2\pi} \frac{x\tan^3 x}{\tan^3 x + \cot^3 x}dx$ is equal to

(A) $\frac{\pi^2}{2}$ (B) $\frac{\pi^2}{4}$ (C) π^2 (D) $2\pi^2$

9. If $k = \int_0^1 \frac{e^t}{1+t}dt$, then $\int_0^1 e^t \ln(1+t)dt$ is equal to

(A) *k* (B) *2k* (C) *e. ln* 2 – *k* (D) None of these

10. The value of the definite integral $\int_{t+2\pi}^{t+\frac{5\pi}{2}} \left(\sin^{-1}(\cos x) + \cos^{-1}(\sin x)\right)dx$ is equal to

(A) $\frac{\pi^2}{2}$ (B) $\frac{\pi^2}{8}$ (C) $\frac{\pi^2}{4}$ (D) None of these

One or more than one correct option questions

11. Let $I=\int_{-\pi}^{199\pi}\sqrt{\frac{1-\cos 2x}{2}}dx$. Then

(A) $I=\int_{-\pi}^{199\pi}\sin x dx$
(B) $I=\int_{-\pi}^{199\pi}|\sin x|dx$
(C) $I=400$
(D) $I=198\int_{0}^{\pi}|\sin x|dx$

12. If the area of the region bounded by the curves $y=x^2+1$, $y=x$ and the pair of lines $x^2+y^2+2xy-4x-4y+3=0$ is k units, then the area of the region bounded by the curve $y=x^2+1, y=\sqrt{x-1}$ and the pair of lines $(x+y-2)(x+y-3)=0$, is

(A) k
(B) 2k
(C) $\frac{k}{2}$
(D) none of these

13. If $I_1=\int_0^1 2^{x^2}dx, I_2=\int_0^1 2^{x^3}dx, I_3=\int_1^2 2^{x^2}dx, I_4=\int_1^2 2^{x^3}dx$, then

(A) $I_1>I_2$
(B) $I_1<I_2$
(C) $I_3<I_4$
(D) $I_3=I_4$

14. Area bounded by the curves $y=|x|$ and $y=\sqrt{|x|}$ is

(A) $\frac{1}{6}$ sq. unit
(B) $\frac{1}{3}$ sq. unit
(C) $\frac{1}{4}$ sq. unit
(D) none of these

15. If $I_1=\int_0^1\frac{\tan^{-1}x}{x}dx$ and $I_2=\int_0^{\frac{\pi}{2}}\frac{x}{\sin x}dx$, then $\frac{I_1}{I_2}=\frac{a}{b}$, *then*

(A) a=1
(B) b=2
(C) a+b=3
(D) None

16. $\int_{-2}^{3}\left|1-x^2\right|dx=\frac{a}{b}$, then

(A) a=28
(B) b=3
(C) a=27
(D) b=2

17. If area bounded by $y=\log x, y=x$ and $x^2+y^2+2xy-k^2=0$ is 'a' sq. units, then area bounded by $y=e^x, y=\log x$ and $x^2+y^2+2xy-k^2=0$ will be

(A) a sq. units
(B) $\frac{a}{2}$ sq. units
(C) 2a sq. units
(D) none of these

18. Let $I=\int_0^{\sqrt{3}}\frac{\tan^{-1}\left(\frac{2x}{1-x^2}\right)}{1+x^2}dx$, then which of the following statements are correct

(A) I can be evaluated by the substituting $x=\tan\theta$ only

(B) $I=\int_0^{\sqrt{3}}\frac{2\tan^{-1}x}{1+x^2}dx$

(C) $I=\int_0^1\frac{2\tan^{-1}x}{1+x^2}dx+\int_1^{\sqrt{3}}\frac{\pi-2\tan^{-1}x}{1+x^2}dx$

(D) $I=\frac{7}{72}\pi^2$

19. Let A(K) be the area bounded by the curves $y = x^2 - 3$ and $y = kx + 2$

(A) The range of A(k) is $\left[\frac{10\sqrt{5}}{3}, \infty\right)$

(B) The range of A(k) is $\left[\frac{20\sqrt{5}}{3}, \infty\right)$

(C) If function $k \to A(k)$ is defined for $k \in [-2, \infty)$, then A (k) is many-one function.

(D) The value of k for which area is minimum is 1.

20. The area enclosed by the curves $x = a\sin^3 t$ and $y = a\cos^3 t$ is

(A) $12a^2\int_0^{\frac{\pi}{2}} \cos^4 t \sin^2 t\, dt$

(B) $12a\int_0^{\frac{\pi}{2}} \cos^2 t \sin^4 t\, dt$

(C) $2\int_{-a}^{a} \left(a^{2/3} - x^{2/3}\right)^{3/2} dx$

(D) $4\int_{-a}^{a} \left(a^{2/3} - x^{2/3}\right)^{3/2} dx$

21. Given that $\lim_{n\to\infty}\sum_{r=1}^{n} \frac{\log(n^2 + r^2) - 2\log n}{n} = \log 2 + \frac{\pi}{2} - 2$, then $\lim_{n\to\infty}\frac{1}{n^{2m}}\left[\left(n^2+1^2\right)^m\left(n^2+2^2\right)^m...\left(2n^2\right)^m\right]^{1/n}$ is equal to

(A) $2^m e^m(\pi/2 - 2)$

(B) $2^m e^{m(2-\pi/2)}$

(C) $e^{m(\pi/2-2)}$

(D) $e^{2m(\pi/2-2)}$

22. $\int_{-\frac{1}{2}}^{\frac{1}{2}}\left([x] + \ln\left(\frac{1+x}{1-x}\right)\right)dx =$

(A) $-\frac{1}{2}$

(B) 0

(C) 1

(D) $2\ln\frac{1}{2}$

23. $\int_{c^{-1}}^{c^2}\left|\frac{\ln x}{x}\right|dx =$

(A) $\frac{3}{2}$

(B) $\frac{5}{2}$

(C) 3

(D) 5

24. If a function $y = f(x)$ satisfying the conditions $f(x) + f(y) = f(x)\,f(y) + f(xy)$ where $f(1) = 0$ and $f'(1) = -2$ are the area bounded by $y = f(x)$ and $y = \left|\cos^{-1}(\cos x) - \sin^{-1}(\sin x)\right|$ from $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is *A*, then

(A) function $y = f(x)$ is $1 + x^2$

(B) $A = \frac{4\sqrt{2} - 3}{3}$ sq. units

(C) function $y = f(x)$ is $1 - x^2$

(D) $A = \frac{6 + 2\sqrt{2}}{3}$ sq. units

25. If $A\left(\frac{3}{\sqrt{2}}, \sqrt{2}\right)$, $B\left(\frac{-3}{\sqrt{2}}, \sqrt{2}\right)$, $C\left(\frac{-3}{\sqrt{2}}, -\sqrt{2}\right)$ and $D(3\cos\theta, 2\sin\theta)$ are four points, then value of θ for which area of quadrilateral *ABCD* is maximum is, where $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$

(A) maximum area is 10 sq. units

(B) $\frac{7\pi}{4}$

(C) $2\pi - \sin^{-1}\left(-\frac{3}{\sqrt{85}}\right)$

(D) maximum area is 12 sq. units

PART C
Integer Type

N1. Let f be a differentiable function satisfying the condition $f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}(y\neq 0, f(y)\neq 0)\ \forall\ x, y\in R$ and $f'(1)=2$. If the smaller area enclosed by $y=f(x), x^2+y^2=2$ is A, then find $[A]$, where $[.]$ represents the greatest integer function.

N2. Let $f(x)$ be a function which satisfy the equation $f(xy)=f(x)+f(y)$ for all $x>0, y>0$ such that $f'(1)=2$. Let *A* be the area of the region bounded by the curves $y=f(x)$, $y=\left|x^3-6x^2+11x-6\right|$ and $x=0$, then find value of $\frac{28}{17}A$.

N3. Let the function $f:[-4,4]\to[-1,1]$ be defined implicitly by the equation $x+5y-y^5=0$. If the area of triangle formed by tangent and normal to $f(x)$ at $x=0$ and the line $y=5$ is A, find $\frac{A}{13}$.

N4. Area of the region bounded by $[x]^2=[y]^2$, if $x\in[1,5]$, where [ ] denotes the greatest integer function, is: ……………………………

N5. Let a differentiable function $f(x)$ satisfies $f(x).f'(-x)=f(-x)\cdot f'(x)$ and $f(0)=1$. Find the value of $\int_{-2}^{2}\frac{dx}{1+f(x)}$.

N6. Find the number of points where $f(\theta)=\int_{-1}^{1}\frac{\sin\theta dx}{1-2x\cos\theta+x^2}$ is discontinuous where $\theta\in[0,2\pi]$.

N7. If $\int_{\sqrt{2}}^{x}\frac{dt}{t\sqrt{t^2-1}}=\frac{\pi}{12}$, then $x=$ ………………….

N8. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}[2\sin x]dx=-k\pi$, then $k=$ ....

N9. $\int_0^1\sqrt{\frac{1-x}{1+x}}dx=\frac{\pi}{2}-a$, then $a=$ ....

N10. $\int_0^1\frac{\log x}{\sqrt{1-x^2}}dx=k\frac{\pi}{a}\log b$, the k+a+b=…………………

SECTION II

PART A
Comprehension - I

Consider the function $f(x)$ and $g(x)$, both defined from $R \to R$ $f(x) = \frac{x^3}{2} + 1 - x\int_0^x g(t)dt$ and $g(x) = x - \int_0^1 f(t)\,dt$, then

1. Minimum value of $f(x)$ is:

(A) 0 (B) 1 (C) $\frac{3}{2}$ (D) Does not exist

2. The area bounded by $g(x)$ with co-ordinate axes is (in square units):

(A) $\frac{9}{4}$ (B) $\frac{9}{2}$ (C) $\frac{9}{8}$ (D) None of these

3. The number of points of intersection of $f(x)$ and $g(x)$ is/are:

(A) 0 (B) 1 (C) 2 (D) 3

Comprehension - 2

Let $f(x)$ be function defined on $[0,1]$ such that $f(1) = 0$ and for any $\alpha \in (0,1]$, $\int_0^a f(x)dx - \int_a^1 f(x)dx = 2f(a) + 3a + b$ where *b* is constant.

4. $b =$

(A) $\frac{3}{2e} - 3$ (B) $\frac{3}{2e} - \frac{3}{2}$ (C) $\frac{3}{2e} + 3$ (D) $\frac{3}{2e} + \frac{3}{2}$

5. The length of the subtangent of the curve $y = f(x)$ at $x = 1/2$ is:

(A) $\sqrt{e} - 1$ (B) $\frac{\sqrt{e} - 1}{2}$ (C) $\sqrt{e} + 1$ (D) $\frac{\sqrt{e} + 1}{2}$

6. $\int_0^1 f(x)dx =$

(A) $\frac{1}{e}$ (B) $\frac{1}{2e}$ (C) $\frac{3}{2e}$ (D) $\frac{2}{e}$

PART B
Matrix Match

M1. Match the following.

	COLUMN – I		COLUMN – II
(A)	If $f(x)=\int_0^{g(x)}\frac{dt}{\sqrt{1+t^3}}$, where $g(x)=\int_0^{\cos x}(1+\sin t^2)\,dt$, then value of $f'\left(\frac{\pi}{2}\right)$ is	(P)	- 2
(B)	If $f(x)$ is a non-zero differentiable function such that $\int_0^x f(t)\,dt=\{f(x)\}^2, \forall x\in R$, then $f(2)$ is equal to	(Q)	2
(C)	If $\int_a^b(2+x-x^2)dx$ is maximum, then $a+b$ is equal to	(R)	1
(D)	If $\lim_{x\to 0}\left(\frac{\sin 2x}{x^3}+a+\frac{b}{x^2}\right)=0$, then $3a+b$ has the value	(S)	-1
		(T)	0

M2. Match the following.

	COLUMN – I		COLUMN – II
(A)	The area bounded by the curve $y=x\,|x|$, $x-$axis and the ordinates $x=1, x=-1$	(P)	10/3 sq.units
(B)	The area of the region lying between the lines $x-y+2=0, x=0$ and the curve $x=\sqrt{y}$	(Q)	64/3 sq.units
(C)	The area enclosed between the curves $y^2=x$ and $y=|x|$	(R)	2/3 sq.units
(D)	The area bounded by parabola $y^2=x$, straight line $y=4$ and $y-$axis	(S)	1/6 sq.units
		(T)	2 sq units

PART C
Integer Type

N1. $\int_0^{\pi/2}\left(\frac{\theta}{\sin\theta}\right)^2 d\theta=\pi\log b$, then b=…………………

N2. Let $\lim_{n\to\infty} n^{\frac{1}{2}\left(1+\frac{1}{n}\right)}\cdot(1^1\cdot 2^2\cdot 3^3\cdot\ldots\ldots n^n)^{\frac{1}{n^2}}=e^{\frac{-p}{q}}$ where p and q are relative prime positive integers. Find the value of $|p+q|$………………..

N3. If the area enclosed by the curve $y=\sqrt{x}$ and $x=-\sqrt{y}$, the circle $x^2+y^2=2$ above the $x-$axis, is A then the value of $\frac{16}{\pi}A$ is………………

N4. The value of $'a'(a>0)$ for which the area bounded by the curves $y=\frac{x}{6}+\frac{1}{x^2}$, $y=0, x=a$ and $x=2a$ has the least value is………..

N5. Let $I_n = \int_{-1}^{1} |x| \left(1 + x + \frac{x^2}{2} + \frac{x^3}{3} + ... + \frac{x^{2n}}{2n}\right) dx$. If $\lim_{n\to\infty} I_n$ can be expressed as rational $\frac{p}{q}$ in its lowest form, then find the value of $\frac{pq(p+q)}{10}$. ………………….

N6. Area bounded by the relation $[2x] + [y] = 5,\ x, y > 0,$ is (where $[.]$ represents greatest integer function).

N7. If the area of the region $\{(x, y) : 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\}$ is A, then the value of $3A - 17$ is……………

N8. The area bounded by the curve $y^2 = 1 - x$ and the lines $y = \frac{|x|}{x}$, $x = -1$ and $x = \frac{1}{2}$ is $\frac{3}{\sqrt{a}} - \frac{11}{b}$, then a+b=………..

N9. Let $I = \int_0^{\pi} x^6 (\pi - x)^8 dx$, then $\frac{\pi^{15}}{({}^{15}C_9) I} =$ …………………

N10. If the area bounded by the curve $y = x^2 + 1$ and the tangents to it drawn from the origin is A, then the value of 3A is………………………

SECTION III

PART A

One or more than one correct option questions

1. The value of $\int_0^{n\pi + v} |\sin x|\, dx$ is

(A) $2n + 1 + \cos v$ (B) $2n + 1 - \cos v$ (C) $2n + 1$ (D) $2n + \cos v$

2. The points of intersection of $F_1(x) = \int_2^x (2t - 5) dt$ and $F_2(x) = \int_0^x 2t\, dt$, are

(A) $\left(\frac{6}{5}, \frac{36}{25}\right)$ (B) $\left(\frac{2}{3}, \frac{4}{9}\right)$ (C) $\left(\frac{1}{3}, \frac{1}{9}\right)$ (D) $\left(\frac{1}{5}, \frac{1}{25}\right)$

3. If $f(x) = \int_{x^2}^{x^2+1} e^{-t^2} dt$, then $f(x)$ increases in

(A) $(2, 2)$ (B) No value of (C) $(0, \infty)$ (D) $(-\infty, 0)$

4. Let $g(x) = \int_0^x f(t) dt$ where $\frac{1}{2} \le f(t) \le 1, t \in [0, 1]$ and $0 \le f(t) \le \frac{1}{2}$ for $t \in (1, 2]$, then

(A) $-\frac{3}{2} \le g(2) < \frac{1}{2}$ (B) $0 \le g(2) < 2$ (C) $\frac{3}{2} < g(2) \le \frac{5}{2}$ (D) $2 < g(2) < 4$

5. The function $L(x) = \int_1^x \frac{dt}{t}$ satisfies the equation

(A) $L(x + y) = L(x) + L(y)$ (B) $L\left(\frac{x}{y}\right) = L(x) + L(y)$ (C) $L(xy) = L(x) + L(y)$ (D) None of these

Comprehension - I

If the integral $I_n = \int_0^{\pi/4} \tan^n x\,dx$ is reduced to its lower integrals like $I_{n-1},\ I_{n-2}$ etc.,

6. The value of $n(I_{n-1} + I_{n+1})$ is
(A) 1 (B) 2 (C) $\pi/4$ (D) π

7. Then $I_2 + I_4, I_3 + I_5$ and $I_4 + I_6$ are in
(A) A.P. (B) G.P. (C) H.P. (D) None of these

8. The value of $\frac{I_3 + 2I_5}{I_1}$ is
(A) 1 (B) 2 (C) $\pi/4$ (D) π

PART B
Matrix Match

M1. Match the following.

COLUMN – I		COLUMN – II	
(A)	$\int_{-1}^{1} \frac{dx}{1+x^2}$ =…….	(P)	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
(B)	$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$ = ...	(Q)	$2\log\left(\frac{2}{3}\right)$
(C)	$\int_2^3 \frac{1}{1-x^2}dx$ = ...	(R)	$\pi/3$
(D)	$\int_1^2 \frac{1}{x\sqrt{x^2-1}}dx$ = ...	(S)	$\pi/2$
		(T)	π

PART C
Integer Type

N1. The integral value $\int_{-2}^{0}\left(x^3 + 3x^2 + 3x + 3 + (x+1)\cos(x+1)\right)\,dx$ is-------------------

N2. $\lim_{n\to\infty} \frac{1}{n}\sum_{r=1}^{2n} \frac{r}{\sqrt{n^2+r^2}}$ equals $-a+\sqrt{b}$, then a-b=……………

N3. If the ordinate $x = a$ divides the area bounded by the curve $y = \left(1 + \frac{8}{x^2}\right)$, $x-$axis and the ordinates $x = 2,\ x = 4$ into two equal parts, then $a = m\sqrt{n}$, then m^n **=…………………**

N4. If the area bounded by $y = ax^2$ and $x = ay^2$, $a > 0$, is 1, then $\frac{1}{a^2}$ = ...

N5. The area bounded by the curves $y = \sqrt{x},\ 2y + 3 = x$ and $x-$axis in the 1st quadrant is ---------------

N6. The area bounded by the curve $y=(x+1)^2, y=(x-1)^2$ and the line $y=\frac{1}{4}$ is 1/k, then k=………………..

N7. $4\int_0^{\pi/2} \cos^3 x \sin x\,dx$=…………

N8. $\int_{-1/2}^{1/2} \cos x \ln\left(\frac{1+x}{1-x}\right) dx$.=………………

N9. $\pi^2\int_0^{\pi} \frac{x\sin 2x \sin\left(\frac{\pi}{2}\cos x\right)dx}{2x-\pi}$ =………………

N10. Let f : R → R be a differentiable function, f(1) = 4 and f ′(1) = 2. Then evaluate $\sqrt{\lim_{x\to 1}\int_4^{f(x)} \frac{2t}{x-1}dt}$.

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