Brushless DC Motor Electromagnetic Torque Estimation with Single-Phase Current Sensing

2 authors, including:

Zahurin Samad
Universiti Sains Malaysia
108 PUBLICATIONS **496** CITATIONS

Some of the authors of this publication are also working on these related projects:

Contractile water jet thruster View project

Flash less cold forging to manufacture AUV propeller View project

Brushless DC Motor Electromagnetic Torque Estimation with Single-Phase Current Sensing

Chin-Long Cham[†] and Zahurin Bin Samad*

Abstract – The purpose of this paper was to find an effective method for measuring electromagnetic torque produced by a brushless DC motor with single-phase current sensing in real-time. A torque equation is derived from the theory of brushless DC motor. This equation is then validated experimentally with a motor dynamometer. A computer algorithm is also proposed to implement the electromagnetic torque estimation equation in real-time. Electromagnetic torque is a linear function of phase current. Estimating the electromagnetic torque in real-time using single-phase current is not appropriate with existing equations, however, because of the rectangular alternating-pulse nature of the excitation current. With some mathematical manipulation to the existing equations, the equation derived in this paper overcame this limitation. The equation developed is simple and so it is computationally efficient, and it takes only motor torque constant and single-phase current to evaluate the electromagnetic torque; no other parameters such as winding resistances, inductances are needed. The equation derived is limited to the three-phase brushless DC motor. It can, however, easily be extended to the multi-phase brushless DC motor with the technique described in this paper.

Keywords: Brushless DC motor, Electromagnetic torque, Algorithm, Mathematical methods

1. Introduction

The brushless DC motor is rapidly becoming more popular than the brushed DC motor. It needs less maintenance and has a longer life span since it contains no wearable brushes, and has flat speed-torque characteristics, higher efficiency, and better thermal handling as windings on the stator allow effective heat dissipation and better dynamic response with lower rotor inertia. The brushless DC motor is more expensive, however, and has a more complex controller design [1]. It also has safety issues, demagnetisation problems, and a high risk of inverter failure [2]. The differences between brushed and brushless DC motors are that the latter has its windings built on the stator and permanent magnets on the rotor and commutation is done electronically instead of via brushes. There are two kinds of brushless motors; namely, brushless AC motors and brushless DC motors. The difference between the brushless AC motor (also known as the permanent magnet synchronous motor) and the brushless DC motor depend on the current waveform supplied; the brushless AC motor is fed by sinusoidal current whereas the brushless DC motor is fed by rectangular current [3].

Research has been conducted to measure torque ripple in a brushless DC motor [4, 5]. Papers that describe the calculation of electromagnetic torque produced by a

brushless DC motor using the information of phase current alone are, however uncommon. Motor torque can be measured directly by a torque sensor, which is expensive and sometimes may be too bulky to be implemented in certain applications. Therefore, electromagnetic torque estimations with measurable parameters such as back EMF, rotor speed and phase current are highly desirable. The electromagnetic torque of brushless DC motor can be estimated by measuring phase currents [6]. It was claimed by [6] that at least two current sensors were required to estimate electromagnetic torque. Also, DC bus current can also be used to estimate the output torque as shown by [7]. Instantaneous torque was calculated by multiplying the torque constant with DC link current with the aid of line-to-line back EMF shape function in [8, 9]. Reference [10] derived an analytical equation from classical synchronous motor theory which correlated electromagnetic torque with motor speed. The equation required information such as back EMF constant, phase resistance and inductance, motor speed and DC source voltage to describe electromagnetic torque. Reference [11] estimated instantaneous electromagnetic torque with back EMF and rotor speed whereby back EMF was estimated with a sliding-mode observer and rotor speed was estimated with a simplified extended Kalman filter.

This paper describes how electromagnetic torque produced by a three-phase brushless DC motor can be calculated through single-phase current sensing. An equation has been derived to calculate electromagnetic torque from the theory of the brushless DC motor. In conjunction, an algorithm has been implemented in an

† Corresponding Author: School of Mechanical Engineering, Universiti Sains Malaysia, Malaysia. (cham1808@yahoo.com.my)
* School of Mechanical Engineering, Universiti Sains Malaysia, Malaysia. (zahurin@eng.usm.my)

Arduino microcontroller in order to carry out the electromagnetic torque measurement in real-time. Motor dynamometer testing has been conducted to verify the capability of the proposed equation and the measurement algorithm.

It is interesting to note that the abc (phase variable) reference frame of a permanent magnet synchronous motor is usually transformed to the dq0 (space vector) reference frame to simplify the analysis of a three-phase circuit fed by sinusoidal wave (hence sinusoidal back EMF). dq0 transformation is not appropriate and has not been implemented in this paper because the brushless DC motor is fed by rectangular wave and has a trapezoidal back EMF.

2. Mathematics of the Brushless DC Motor

A schematic diagram of a brushless DC motor drive (wye-connected) is shown in Fig. 1. The equations shown in the latter part of this paper are based on this system. Wye-connection is chosen in this study because it is applied in most applications thanks to its simpler commutation logic, which is less prone to bridge-shorting [12].

A few assumptions are made in order to reduce the mathematical complexity of the brushless DC motor drive. They are: (1) symmetrical three-phase winding, (2) no magnetic saturation, (3) no hysteresis and eddy current losses, (4) uniform air-gap, (5) mutual inductance is ignored, and (6) armature reaction is ignored.

Mathematical model of armature winding is

$$V_a = i_a R + L\frac{di_a}{dt} + e_a \tag{1}$$

$$V_b = i_b R + L\frac{di_b}{dt} + e_b \tag{2}$$

$$V_c = i_c R + L\frac{di_c}{dt} + e_c \tag{3}$$

where

V_a, V_b, V_c - terminal voltages of phase a, b, and c [V]
i_a, i_b, i_c - stator current of phase a, b, and c [A]
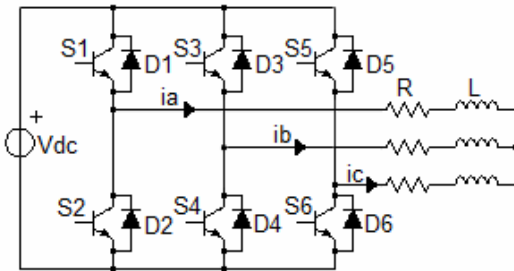e_a, e_b, e_c - back EMF of phase a, b, and c [V]

Fig. 1. Schematic of brushless DC motor drive.

L - per phase armature self-inductance [H]
R - per phase armature resistance [Ω]

The back EMF are displaced by 120 electrical degrees from one phase to another, and they can be expressed as

$$e_a = K_e f(\theta_e)\omega_m \tag{4}$$

$$e_b = K_e f(\theta_e - 2\pi/3)\omega_m \tag{5}$$

$$e_c = K_e f(\theta_e + 2\pi/3)\omega_m \tag{6}$$

where

ω_m - mechanical rotor speed [$rad.s^{-1}$]
K_e - back emf constant [$V/rad.s^{-1}$]
$f(\theta_e)$ - trapezoidal function
θ_e - electrical angle of rotor [rad]

Subtracting (2) from (1) and (3) from (2) yield

$$V_{ab} = R(i_a - i_b) + L(\frac{di_a}{dt} - \frac{di_b}{dt}) + (e_a - e_b) \tag{7}$$

$$V_{bc} = R(i_b - i_c) + L(\frac{di_b}{dt} - \frac{di_c}{dt}) + (e_b - e_c) \tag{8}$$

For a wye-connected three-phase winding, the total phase current is zero according to Kirchoff's Current Law, thus

$$i_a + i_b + i_c = 0 \tag{9}$$

Substituting i_c from (9) to (8) gives

$$V_{bc} = R(i_a + 2i_b) + L(\frac{di_a}{dt} + 2\frac{di_b}{dt}) + (e_b - e_c) \tag{10}$$

The electromagnetic torque produced by a brushless DC motor can be expressed as

$$T_e = \frac{e_a i_a + e_b i_b + e_c i_c}{\omega_m} \tag{11}$$

or

$$T_e = K_t[f(\theta_e)i_a + f(\theta_e - \frac{2\pi}{3})i_b + f(\theta_e + \frac{2\pi}{3})i_c] \tag{12}$$

where

T_e - electromagnetic torque [Nm]
K_t - torque constant [Nm/A]

Under steady-state operation, electromagnetic torque will be counter-balanced by load torque, inertia torque and friction torque. Therefore,

$$T_e = T_L + J\frac{d\omega_m}{dt} + \beta\omega_m \tag{13}$$

where

T_L - load torque [Nm]
J - inertia of the rotor and coupled shaft [kgm^2]
β - friction factor [$Nm.s.rad^{-1}$]

Eqs. (7, 10), and (13) can be transformed into Laplace domain with the Differentiation Theorem, which can then be expressed in the form of state-space equations giving a multiple-input-multiple-output (MIMO) correlation which is crucial in computer simulation. State space equations of the brushless DC motor are

$$\begin{bmatrix} i_a' \\ i_b' \\ \omega_m' \end{bmatrix} = \begin{bmatrix} -\dfrac{R}{L} & 0 & 0 \\ 0 & -\dfrac{R}{L} & 0 \\ 0 & 0 & -\dfrac{\beta}{J} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ \omega_m \end{bmatrix} + \begin{bmatrix} \dfrac{2}{3L} & \dfrac{1}{3L} & 0 \\ -\dfrac{1}{3L} & \dfrac{1}{3L} & 0 \\ 0 & 0 & \dfrac{1}{J} \end{bmatrix} \begin{bmatrix} V_{ab} - e_{ab} \\ V_{bc} - e_{bc} \\ T_e - T_L \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} i_a \\ i_b \\ \omega_m \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ \omega_m \end{bmatrix} \quad (15)$$

3. Calculation of Electromagnetic Torque

Theoretically, the back EMF of each phase of a brushless DC motor has a trapezoidal waveform and the current has a pulse-like waveform which alternates its direction every half wave, as shown in Fig. 2.
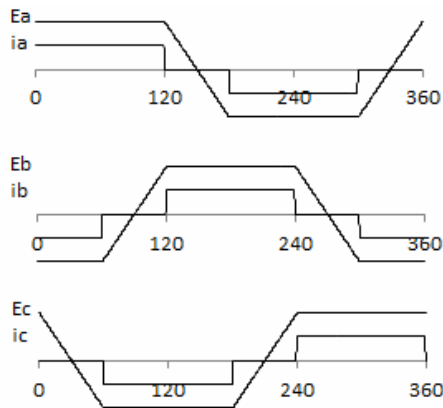
Fig. 2. Waveforms of back EMFs and currents of the three phases as a function of electrical angle (degree).

Table 1. Switching sequence (clockwise direction)

Electrical Angle (Deg.)	Conducting Switch		Commutating Diode
0 to 60	S1	S4	D6
60 to 120	S1	S6	D3
120 to 180	S3	S6	D2
180 to 240	S3	S2	D5
240 to 300	S5	S2	D4
300 to 360	S5	S4	D1

The waveforms showed in Fig. 2 result from the switching sequence shown in Table 1. The switching sequence shows the motor is rotating in the clockwise direction. Referring to Fig. 1, S1 to S6 are power switches (e.g. MOSFET, IGBT, etc.) and D1 to D6 are the associated diodes.

Assuming the windings of the three phases are symmetrical, the magnitudes of back EMFs and currents should be equal for the three phases. From Eq. (11) and Fig. 2 it can be seen that the electromagnetic torque developed by a brushless DC motor at any instant is

$$T_e = \frac{2e_p i_p}{\omega_m} \quad (16)$$

where

e_p - phase back EMF [V]
i_p - non-zero phase current [A]

Likewise, Eq. (16) can also be written as

$$T_e = \left| 2K_t i_p \right| \quad (17)$$

The absolute sign has to be implemented in Eq. (17) because multiplication of phase back EMF and phase current in Eq. (16) gives only a positive value. In other words, phase back EMF and phase current are always in phase for the motoring mode. Assuming the absolute in Eq. (17) is especially important in the real-time application, as lack of the absolute operation will give negative torque value because of the alternating nature of phase current as can be seen in Fig. 2.

Eq. (17) is theoretically correct, but it is not directly implementable in the real-time application because a complete single phase current waveform is composed of 240 electrical degrees of non-zero ampere and 120 electrical degrees of zero ampere. These zeroes' current moment will give an incorrect result, as shown in Fig. 3.

In order to get rid of the error introduced by Eq. (17) and to obtain a torque value as in the actual condition, some post-processing is required. The suggested post-processing is expressed as

$$T_e = \left| \left| 2K_t i_p \right| - 0.75\alpha \right| + 0.75\alpha \quad (18)$$
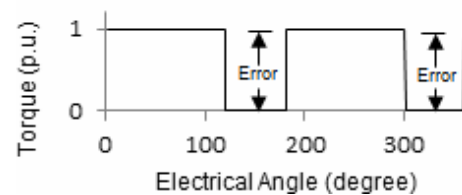
Fig. 3. Per-unit electromagnetic torque derived from Eq. (17).

in which α is an average value of electromagnetic torque over a number of sampling data. The constant 0.75 is derived from the fact that the alternating square pulse (phase current) is composed of 2/3 high pulse and 1/3 low pulse; only the average magnitude of high pulse is of interest and it has to be half of the magnitude of the high pulse, so the constant 0.75 is obtained by dividing 0.5 by 2/3. The derivation of the constant 0.75 is illustrated in Fig. 4.

The idea behind Eq. (18) is to find an average value produced by Eq. (17) and invert the low side of the calculated electromagnetic torque to the high side about that average level. Without violating the theory of the brushless DC motor, Eq. (18) gives a precise estimation of the electromagnetic torque produced by a brushless DC motor using information on the single-phase current alone.

Eq. (18) is theoretically verified by comparing the result generated by Eq. (18) and the result generated by Eq. (11). The results are shown in Figs. 5(a) and (b).

Figs. 5(a) and (b) show identical results and hence Eq. (18) is able to calculate electromagnetic torque correctly by theory.
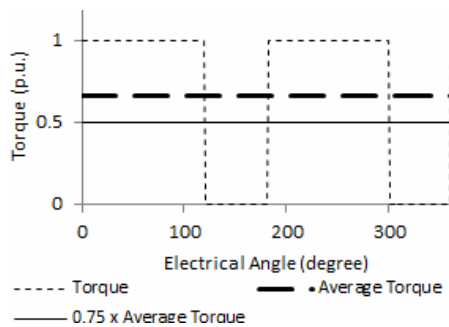

Fig. 4. Per-unit torque, average torque (α), and 0.75 of average torque (0.75 α) of the brushless DC motor.
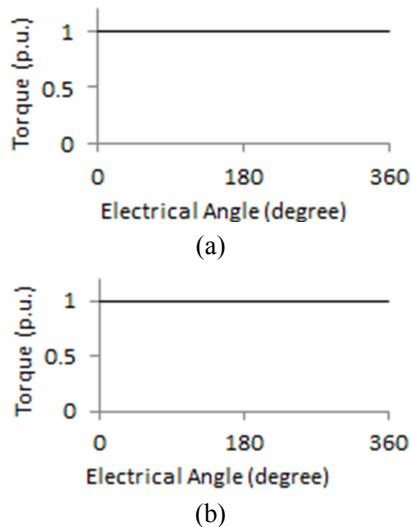

(a)

(b)

Fig. 5. (a) Calculation of torque with Eq. (18); (b) Calculation of torque with Eq. (11).

4. Experimental Verification of Torque Estimation Equation

Section 3 theoretically shows how Eq. (18) correctly calculates the electromagnetic torque produced by a brushless DC motor. In order to verify the validity of Eq. (18), laboratory experiments need to be conducted.

A brushless DC motor of 1.2 kW is tested with a motor dynamometer to validate Eq. (18). Measurement shows that the corresponding brushless DC motor has a back EMF constant and hence a torque constant of 0.07 Nm/A (V/rad.s^{-1}). Measurement of motor parameters like phase resistance, inductance, and rotor inertia is not needed because these parameters are irrelevant in the evaluation of electromagnetic torque. The motor dynamometer test set-up is shown in Fig. 6. For better interpretation, it is expressed as a schematic diagram in Fig. 7.

Eq. (18) is implemented with an Arduino microcontroller (a low cost open source microcontroller) in an algorithmic way to estimate the instantaneous electromagnetic torque. The algorithm used for instantaneous electromagnetic torque estimation is shown in Fig. 8.

The symbol i shown in Fig. 8 indicates the current number of iterations. The symbol n indicates the total number of iterations which also represents the number of sampling data used in the calculation of α. The average value (α) is expressed as:

$$\alpha = \frac{\sum \text{Instantane ous Torque}}{n} \quad (19)$$

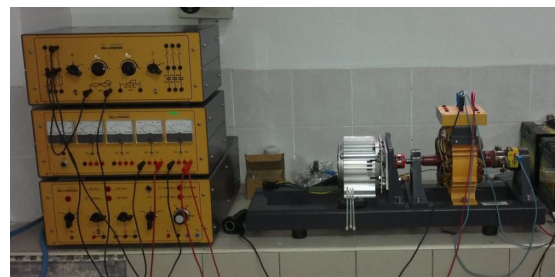The value of n should be carefully selected so that it is large enough to smooth out the measurements and small

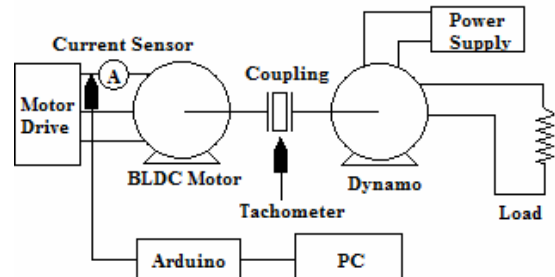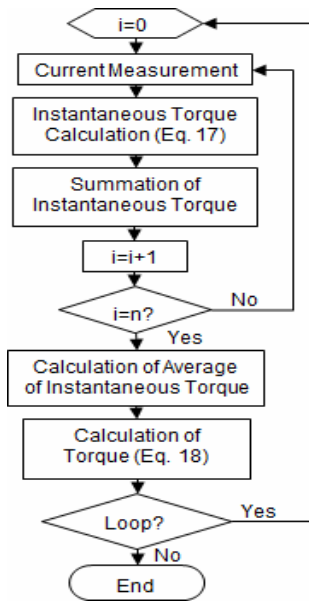**Fig. 6.** Experimental set-up.

Fig. 7. Schematic diagram of the experimental setup.

Fig. 8. Algorithm implemented in the Arduino micro-controller for torque estimation.

enough to be utilised for real-time application.

The loop decision block (Loop?) is added for the sake of completeness of the algorithm. It indicates the termination criteria of a measurement. For example, the algorithm can be designed in such a way that it will acquire 10 readings and then exit; then the loop decision block will be "No" upon 10 cycles of looping. It has no meaning in the determination of electromagnetic torque.

With the algorithm shown in Fig. 8, the torque is estimated at a motor speed of 3150 RPM with a load of 0.30kW and is shown in Fig. 9. In the same figure, the actual torque generated by the brushless DC motor is superimposed. Since the brushless DC motor has 89% efficiency at 3150rpm, the electrical power produced by the motor is 0.33kW. The term efficiency has taken the load torque imposed by the dynamometer, motor friction and inertia into account as described by Eq. (13).

The brushless DC motor has been tested under various speeds and loads with the motor dynamometer; results obtained for a motor speed of 1960 rpm with a load of 0.14kW and speed of 1550rpm with a load of 0.094kW are shown in Figs. 10 and 11 respectively.

5. Discussion of Experimental Results

There are several types of dynamometer; for example, engine dynamometer, motor dynamometer and chassis dynamometer. The motor dynamometer is commonly used to measure the force, torque, or power of an electric motor, and it is used in this research. It is observed from Figs. 9, 10 and 11 that the electromagnetic torque evaluated with Eq. (18) is comparable to the actual torque developed to counterbalance load torque imposed by the dynamometer.
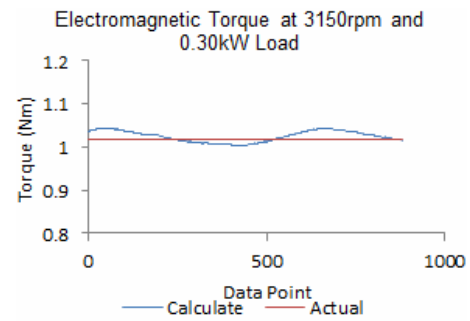
Fig. 9. The electromagnetic torque produced and estimated (Eq. 18) at 3150 rpm with 0.30kW loading.
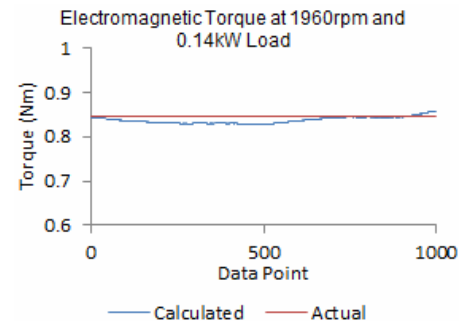
Fig. 10. The electromagnetic torque produced and estimated at 1960 rpm with 0.14kW loading.
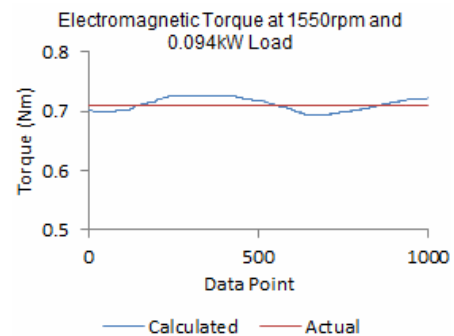
Fig. 11. The electromagnetic torque produced and estimated at 1550 rpm with 0.094kW loading.

Therefore, Eq. (18) can estimate the electromagnetic torque accurately with single-phase current measurement alone, which is cost-effective and computationally efficient.

Theoretically, electromagnetic torque developed should be of a constant value. Measurement showed, however, that there was torque fluctuation about a mean value, which can be seen from Fig. 9, 10 and 11. This fluctuation is known as ripples in the context of the brushless DC motor. Torque ripple is unavoidable because the phase flux linkage and current in stator windings are not sinusoidal [4]. The greatest torque fluctuation is found to be 0.025Nm in Fig. 9, 0.033Nm in Fig. 10, and 0.032Nm in Fig. 11, which correspond to 2.5%, 3.9% and 4.5% of measurement deviation respectively. The average of the estimated value showed that the actual and the calculated electromagnetic

torque were identical in each test.

Care should be taken while implementing Eq. (18) because it has the following limitations

1. Applicable to three-phase brushless DC motor only.
2. Identical back EMF spatial distribution with 120 electrical degrees offset among the three phases.
3. Half-wave symmetry and zero average back EMF and excitation current.
4. Constant and identical phase resistance and inductance with negligible mutual inductance among motor phases.
5. Excitation currents are in phase with their respective phase back EMF.

Limitation 1 can be extended to the multi-phase brushless DC motor by using the technique discussed in section 3 as long as condition 4 and condition 5 are fulfilled. In that case, condition 2 and condition 3 must be adjusted accordingly.

6. Conclusions

There are several ways in which electromagnetic torque can be evaluated, and the simplest solution is through torque sensor measurement. A torque sensor is very expensive, however, and sometimes it may be too bulky for certain applications. Therefore, torque estimation with measurable parameters such as back EMF, currents and rotor speed is highly desirable. This paper shows how the electromagnetic torque of a brushless DC motor can be evaluated with single-phase current measurement alone and with proper mathematical manipulation of the theory of the brushless DC motor. A flow chart has been proposed to implement the electromagnetic torque estimation equation in an algorithmic way. This algorithm can be used as it is or it can be plugged into any algorithm for estimation of brushless DC motor electromagnetic torque. The measurement technique suggested in this paper is cost effective because only one current sensor is needed, it is measurement-friendly with minimum intrusion in the system of interest compared with the torque sensor, and it is computationally effective since no complex mathematics are involved. Furthermore, only the motor torque constant is required to evaluate the electromagnetic torque; no other parameters such as winding resistances, inductances, etc. are needed. The equation and the algorithm suggested have been verified experimentally and show a high degree of accuracy in evaluating electromagnetic torque in real-time.

References

[1] P. Yedamale, "Brushless DC (BLDC) Motor Fundamentals," Microchip Technology Inc., Arizona, 2003.

[2] M. Ehsani, , Y. Gao, A. Emadi, "Permanent Magnetic BLDC Motor Drives," in Modern Electric, Hybrid Electric, and Fuel Cell Vehicles Fundamentals, Theory, and Design, 2nd ed., New York: CRC Press, 2010, pp. 200-216.

[3] N. Hashemnia, "Comparative study of using different motors in the electric vehicles," in 18th International Conference on Electrical Machines ICEM 2008, 2008, pp. 1-5.

[4] D. C. Hanselman, "Minimum torque ripple, maximum efficiency excitation of brushless permanent magnet motor," IEEE transaction on industrial electronics, vol. 41, pp. 292-300, Jun. 1994.

[5] L. Sun, H. Gao, Q. Song, J. Nei, "Measurement of torque ripple in pm brushless motors," in Industry Applications Conference, 2002. 37th IAS Annual Meeting, 2002, pp. 2567-2571.

[6] C. L. Chu, M. C. Tsai, H. Y. Chen, "Torque control of brushless dc motors applied to electric vehicles," in IEEE International Electric Machines and Drives Conference, 2001, pp. 82-87.

[7] H. Tan, S. L. Ho, "A novel single current sensor technique suitable for bldcm drives," in proceedings of the IEEE 1999 International Conference on Power Electronics and Drive Systems, 1999, pp. 133-138.

[8] J. C. Fang, X. X. Zhou, G. Liu, "Instantaneous torque control of small inductance brushless DC motor," IEEE Transactions on Power Electronics, Vol. 27, pp. 4952-4964, Dec. 2012.

[9] U. Y. Huh, J. H. Lee, T. G. Lee, "A torque control strategy of brushless DC motor with low resolution encoder," in Proc. International Conference on Power Electronics and Drive Systems, 1995, pp. 496-501.

[10] M. Markovic, A. Holder, Y. Perriard, "An analytical determination of the torque speed and efficiency speed characteristics of a bldc motor," in IEEE 2009 Energy Conversion Congress and Exposition, 2009, pp. 168-172.

[11] Y. Liu, Z. Q. Zhu, D. Howe, "Instantaneous torque estimation in sensorless direct-torque-controlled brushless DC motors," IEEE Transactions on Industry Applications, Vol. 42, pp. 1275-1283, Sept. 2006.

[12] P. D. Gipper, "Accurate torque-speed performance prediction for brushless DC motors," in Proc. NAECON, 1988, pp. 1092-1097.

Chin-Long Cham received B.Eng degree in mechanical engineering from University Malaya in 2011. He is currently working toward the Ph.D degree at Universiti Sains Malaysia, School of Mechanical Engineering. His research interests are control system and hybrid electric system.

Zahurin Bin Samad received B.Sc degree in aerospace engineering from University College of Swansea in 1988, M.Sc degree in mechanical engineering from University of Wales in 1992 and Ph.D degree in mechanical engineering from University of Leeds in 1995. He is currently an Associate Professor in School of Mechanical Engineering at Universiti Sains Malaysia. His fields of specialization are manufacturing automation, computer numerical control, machine tools, computer aided manufacturing, robotics, machine vision, control system artificial intelligence and forging process.