Assignment

January 6, 2024

Questions

1. Form the differential equation representing the family of curves $y = \frac{A}{x} + 5$, by eliminating the arbitrary constant A.

2. If A is a square matrix of order 3, with $|A| = 9$, then write the value of $\left|2.\ \text{adj}A\right|$.

3. Find the acute angle between the planes

$$\vec{r}.\ \left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 1$$

   and

$$\vec{r}.\ \left(3\hat{i} - 6\hat{j} + 2\hat{k}\right) = 0.$$

4. Find the length of the intercept, cut off by the plane $2x + y - z = 5$ on the x-axis.

5. If $y = \log(\cos e^x)$, then find $\frac{dy}{dx}$.

6. Find :

$$\int_{-\frac{\pi}{4}}^{0} \frac{1 + \tan x}{1 - \tan x}\,dx$$

7. Let $*$ be an operation defined as $*$: $\mathbf{R} \times \mathbf{R} \to \mathbf{R}$ such that $a * b = 2a + b$, a, $b \in \mathbf{R}$. Check if * is a binary operation. If yes, find if it is associative too.

8. X and Y are two points with position vectors $3\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ respectively. Write the position vector of a point Z which divides the line segment XY in the ratio 2 : 1 externally.

9. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ be two vector. Show that vector $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$ are perpendicular to each other.

10. If A and B are symmertic matrices, such that AB and BA are both defined, then prove that $AB - BA$ is a skew matrix.

11. 12 cards numbered 1 to 12 (one number on one card), are placed in a box and mixed up thoroughly. Then a card is drawn at random from the box. If it is known that the number on the drawn card is greater than 5, find the probability that the card bears an odd number.

12. Out of 8 outstanding students of a school, in which there are 3 boys and 5 girls, a team of 4 students is to be selected for a quiz competition. Find the probability that 2 boys and 2 girls are selected.

13. In a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing ?

14. Solve the following differential equation :

$$\frac{dy}{dx} + y = \cos x - \sin x.$$

15. Find :

$$\int x.\ \tan^{-1} x\, dx.$$

16. Find :

$$\int \frac{dx}{\sqrt{5 - 4x - 2x^2}}.$$

17. Using properties of determinants, find the value of x for which

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0.$$

18. Solve the differential equation $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

19. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$, given that $y = 1$ when $x = 0$.

20. Let $A = R - \{2\}$ and $B = R\{1\}$. If $f : A \rightarrow B$ is a function defined by $f(x) = \frac{x-1}{x-2}$, show that f is one-one and onto. Hence, find f^{-1}.

21. Show that the relation S in the set $A = \{x \in Z : 0 \leq x \leq 12\}$ given by $S = \{(a, \ b) : a, \ b \in Z, \ |a-b|\}$ is divisible by 3 is an equivalence relation.

22. Integrate the function

$$\frac{\cos(x+a)}{\sin(x+b)}$$

w. r. t. x.

23. If $x = \sin t$, $y = \sin pt$, prove that

$$\left(1-x^2\right)\frac{d^2x}{dx^2} - x\frac{dy}{dx} + p^2y = 0.$$

24. An insurance company insured 3000 cyclists, 6000 scooter drivers and 9000 car drivers. The probability of an accident involving a cyclist, a scooter driver and a car driver are 0.3, 0.05 and 0.02 respectively. One of the insured persons meets with an accident. What is the probability that he is a cyclist ?

25. If $y = \csc\left(\cot \sqrt{x}\right)$, then find $\frac{dy}{dx}$.

26. Write the integrating factor of the differential equation

$$\left(\tan^{-1} y - x\right) dy = \left(1 + y^2\right) dx.$$

27. Let $* : N \times N \to N$ be an operation defined as $a * b = a + ab, \quad \forall a, \quad b \in N$. Check if * is a binary operation. If yes, find if it is associative too.

28. Solve the following equation :

$$\frac{dx}{dy} + x = \left(\tan y + \sec^2 y\right).$$