CSCI 301, Winter 2018
Math Exercises #7

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Problem 1. A $= \{a^i b^j \mid i \geq j \geq 0\}$

Proof. Suppose A is regular.

Let p = pumping length.
Let s be some string $\in A$ with $|s| \geq p$.

Suppose $s = a^{p-1}b^{p-1}$.
Therefore, s is in the language and $|s| > p$.

Suppose x, y, z exist.

Since $|x, y| \leq p$ and $|a| \leq p$, we know atleast that y MUST contain only b's.

Now, the pumping lemma states $xy^iz \in A$.
Yet, we can see that for $i > 1$,

i.e. $xyyz$, $xyyyz$ $xyyyyz$ ...

the resulting string CANNOT be in the language A because then we would have more b's than a's which would not satisfy the condition that $i \geq j \geq 0$. Therefore, this equation cannot be pumped and must NOT be regular.

□

Problem 2. $A = \{a^{4n+5}b^{3n+2} : n \geq 0\}$

Proof. Suppose A is regular.

Let p = pumping length.
Let s be some string ∈ A with $|s| > p$

Suppose $s = a^{4p+5}b^{3p+2}$.
Therefore, s is in the language A and $|s| \geq p$.

Suppose x, y, z exist.
Since, $|xy| \leq p$ we know that x and y must be all a's.

The pumping lemma states that for regular languages with x, y, z that xy^iz must also be in that language.
However, for any $i > 1$ this is not true for $s \in A$.

For example, consider $xyyz$

$$\text{then } s = a^{4p+5+1}b^{3p+2} = a^{4p+6}b^{3p+2} \notin A$$

In this case, any extra a's would result in a string that is not in our language, thus this expression cannot be pumped and must NOT be a regular language.

□

Problem 3. A $= \{a^ib^jc^k \mid i = j \text{ or } j = k\}$

Proof. Suppose A is regular.

Lets break this problem up into two cases and evaluate them individually.

Let $p =$ pumping length.
Let s be some string $\in$ A with $|s| \geq p$

Case 1: $[i = j]$

Suppose $s = a^pb^pc^k$.
Therefore, s is in the language and its length is greater than p.

Since, $|xy| \leq p$, we know x and y must consist of all a's.

We know by the pumping lemma for regular languages that $xy^iz \in A$
Consider one example of this, xyyz.

This results in $s = a^{p+1}b^pc^k$.

Now, $|a| \neq |b|$ which does not satisfy the condition for case 1 that $i = j$. We have a contradiction.

Case 2: $[j = k]$

Let $s = a^ib^pc^p$.
Therefore, s is in the language and $|s| \geq p$.

Suppose $i = 0$.
Then for $|xy| \leq p, x, y$ must be all b's.

According to the pumping lemma $xy^iz \in A$. Lets consider xy^2z.

This results in $s = a^ib^{p+1}c^p$ and now our condition is not met as $j \neq k$.
Furthermore, this contradiction arises for any value > 1. Since y cannot be pumped without such a contradiction, we have proven that this language is NOT regular.

□