Data Science Hw 2

工科海洋四 B07505015 梁瑞翔

Problem 1 Type I and II errors

(1) α=P(Type I Error)=P(Reject H_0/H_0 is true)=$P(\sum_{i=1}^{5} Xi = 5$ / H_0:p=1/2)

=$\binom{5}{5}$(1/2)^5=0.03125

(2) 1-β=P(Reject H_0/H_a is true)=$P(\sum_{i=1}^{5} Xi = 5$ / H_a:p=3/4)

=$\binom{5}{5}$(3/4)^5=0.237

β=0.763

Problem 2 Hypothesis testing

State hypotheses: H_0: σ^2=0.81, H_a: σ^2>0.81

Compute test statistic: S^2=1.44, n=10, X^2=9*1.44/0.81=16

Critical region: The null hypothesis is rejected when X^2>16.919,

where X^2=(n-1)*s^2, with v=9 degrees of freedom

The X^2 statistic is not significant at the 0.05 level. There is insufficient evidence to claim that σ>0.9 year.

Problem 3 Markov Properties

(1) Pairwise Markov property: Any two non-adjacent variables are conditionally independent given all other variables: $X_u \perp\!\!\!\perp X_v \mid X_{V\setminus\{u,v\}}$

Local Markov property: A variable is conditionally independent of all other variables given its neighbors: $X_v \perp\!\!\!\perp X_{V\setminus \mathrm{N}'[v]} \mid X_{\mathrm{N}(v)}$, where N(v) is the set of neighbors of v, N'[v] = v∪ N(v) is th eclosed neighboourhood of v.

Global Markov property: Any two subsets of variables are conditionally independent given a separating subset: $X_A \perp\!\!\!\perp X_B \mid X_S$, where every path from a node in A to a node in B passes through S.

(2) Each variable is conditionally independent of its non-descendants given its parent variables: $X_v \perp\!\!\!\perp X_{V\,\setminus\,\mathrm{de}(v)} \mid X_{\mathrm{pa}(v)} \quad \text{for all } v \in V$, where de(v) denotes the set of descendants of v (thus V \ de(v) is the set of non-descendants of v)

(3) The Markov blanket of a node is the set of nodes consisting of its parents, its children, and any other parents of its children.

Problem 4 LDA

```
V =

    0.9913   -0.3714
   -0.1316    0.9285



D =

    8.6873         0
         0    0.0000
```

Optimal projection vectors and their corresponding eigenvalues:

$w_1=\begin{pmatrix} 0.9913 \\ -0.1316 \end{pmatrix}$

$w_2=\begin{pmatrix} 0.3714 \\ 0.9285 \end{pmatrix}$

$\lambda_1=8.6873$

$\lambda_2=0$

```
% code
X1=[5,3;3,5;3,4;4,5;4,7;5,6];
X2=[9,10;7,7;8,5;8,8;7,2;10,8];
Mu1=transpose(mean(X1));
Mu2= transpose(mean(X2));
S1=cov(X1);
S2=cov(X2);
Sw=S1+S2;
SB=(Mu1-Mu2)*transpose(Mu1-Mu2);
invSw=inv(Sw);
invSw_by_SB=invSw*SB;
% getting the projection vector
[V,D]=eig(invSw_by_SB)
```