Final Project Review Report

Monte Carlo Simulation for Financial Risk Analysis

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1. Introduction

Financial risk arises from uncertainty in market behavior, asset price movements, interest rates, and volatility. Accurately measuring this risk is essential for investors, financial institutions, and risk managers. Traditional risk assessment methods often rely on fixed assumptions and single-point estimates, which may underestimate extreme outcomes.

This project focuses on the application of **Monte Carlo simulation** as a stochastic modeling technique to analyze financial risk. Monte Carlo methods allow for the generation of thousands of possible future scenarios, offering a probabilistic view of potential gains and losses. The primary objective of this project is to evaluate how Monte Carlo simulation can be used to estimate future asset values and calculate key risk measures such as **Value at Risk (VaR)**.

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2. Theoretical Background

2.1 Financial Risk

Financial risk refers to the possibility of losing money due to unfavorable market movements. Common types include:

- **Market risk** (price fluctuations)
- **Credit risk** (default risk)
- **Liquidity risk** (inability to sell assets quickly)
- **Operational risk**

This project concentrates on **market risk**, specifically the uncertainty in asset prices.

2.2 Monte Carlo Simulation

Monte Carlo simulation is a numerical method that uses random sampling to approximate the behavior of complex systems. In finance, it is widely used because asset prices evolve randomly and cannot be predicted with certainty.

The method involves:

- Defining a mathematical model
- Generating random variables
- Repeating simulations many times
- Analyzing the distribution of outcomes

The law of large numbers ensures that increasing the number of simulations improves result accuracy.

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3. Mathematical Model

3.1 Asset Price Dynamics

Asset prices are assumed to follow **Geometric Brownian Motion (GBM)**:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

Where:

- S_t = asset price at time t
- μ = expected return (drift)
- σ = volatility
- dW_t = Wiener process (random shock)

The discrete-time approximation used in simulations is:

$$S_{t+1} = S_t \cdot e^{\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma \sqrt{\Delta t} Z}$$

Where $Z \sim N(0,1)$.

3.2 Model Assumptions

- Returns are normally distributed
- Volatility and drift are constant
- Markets are frictionless
- No arbitrage opportunities

While these assumptions simplify modeling, they may not always hold in real markets.

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4. Data and Parameter Estimation

Historical price data is used to estimate:

- **Mean return (μ\muμ)** using logarithmic returns
- **Volatility (σ\sigmaσ)** using standard deviation

Log returns are calculated as:

rt=ln(StSt−1)r_t = \ln\left(\frac{S_t}{S_{t-1}}\right)rt=ln(St−1St)

These parameters serve as inputs for the simulation.

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5. Simulation Methodology

5.1 Simulation Steps

1. Select initial asset price S0S_0S0
2. Estimate μ\muμ and σ\sigmaσ from historical data
3. Generate random values from a standard normal distribution
4. Simulate asset price paths over a chosen time horizon
5. Repeat the process for a large number of iterations (e.g., 10,000)
6. Record final asset values and returns

5.2 Number of Simulations

A higher number of simulations reduces sampling error. In this project, thousands of simulations were conducted to ensure stability and reliability of results.

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6. Risk Measurement Techniques

6.1 Value at Risk (VaR)

Value at Risk measures the maximum expected loss over a specific period at a given confidence level.

For example:

- **95% VaR** indicates that there is only a 5% chance the loss will exceed this value.

Monte Carlo VaR is calculated directly from the simulated loss distribution:

VaRα=Percentile(1−α)VaR_{\alpha} = \text{Percentile}_{(1-\alpha)}VaRα=Percentile(1−α)

6.2 Interpretation of Results

- A higher VaR indicates greater downside risk
- VaR increases with volatility and time horizon
- Monte Carlo VaR captures non-linear risk better than variance-based methods

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7. Results and Visualization

7.1 Simulated Price Paths

Multiple price paths demonstrate how asset prices can evolve differently even with identical starting conditions.

7.2 Distribution of Final Prices

Histograms of final asset prices show:

- Right-skewed distributions
- Presence of tail risks
- Large dispersion under high volatility

7.3 Loss Distribution

The loss distribution provides direct insight into worst-case scenarios and tail behavior.

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8. Sensitivity Analysis

The simulation results were tested under varying conditions:

- Increased volatility → higher VaR
- Longer investment horizon → greater uncertainty
- Lower expected returns → higher probability of losses

This analysis highlights the sensitivity of financial risk to key parameters.

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9. Advantages of Monte Carlo Simulation

- Models uncertainty realistically
- Handles complex, non-linear payoffs
- Useful for portfolio risk and derivatives pricing
- Allows scenario and stress testing

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10. Limitations

- Strong dependence on assumptions
- Normal distribution may underestimate extreme events

- Computationally demanding
- Historical data may not predict future behavior

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11. Conclusion

This project demonstrates that Monte Carlo simulation is a robust and flexible approach to financial risk analysis. By generating a full probability distribution of outcomes, it provides deeper insight into potential losses than traditional methods. Despite limitations, Monte Carlo simulation remains a vital tool in modern risk management.

Future enhancements could include:

- Fat-tailed distributions
- Stochastic volatility models
- Multi-asset portfolio simulations

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12. References

- Hull, J. C. (2018). *Options, Futures, and Other Derivatives*
- Glasserman, P. (2004). *Monte Carlo Methods in Financial Engineering*
- Jorion, P. (2007). *Value at Risk*