3.16) Show that the collection of Turing-recognizable languages are closed under the operation of

a.) union

Suppose, X and Y be two Turing recognizable languages that have Turing machines M_x and M_y respectively.

Now the union of these languages is denoted by L_{xy} and Turing machine this language is M_{xy}.

"On input w:"

1.) Run x and y alternately on w step by step, if either accepts, accept. If both halt and reject, reject.

Suppose s be a word from L_{xy}. M_{xy} works for an input string s as shown.

- It executes M_x and M_y on s individually.
- If at least any one of M_x or M_y accepts s then M_{xy} also accepts after a finite number of states and reach to it's accepting state
- If both M_x and M_y reject and either of them do so looping then will loop.

Hence, it can be said that collection of Turing recognizable languages is closed under Union operation

b.) next page

b.) Concatenation

Suppose, X and Y be two Turing recognizable languages that have the Turing machine M_x and M_y respectively. Now the concatenation of these Languages is denoted by L_{xy} and the Turing machine recognizing this language is M_{xy}.

Let s be a word from L_{xy}. M_{xy} works for an input string s as shown:

- It divides each string of XY into s_1 and s_2 non-deterministically.
- If runs s_1 to M_x. If M_x halts and rejects, reject.
- If runs s_2 to M_y. If M_y accepts, accept. If M_y halts and rejects, reject.

Hence, it can be said that collection of Turing recognizable languages is closed under Concatenation operation.

C.) next page

C. Star

Suppose, X be a Turing recognizable language and the Turing machine is M_x.

- Now X^* is the language obtained from Star operation on X.
  The Turing machine for this language be M_{x*}.

M_{x*} works as follows:

- For an input string s of X, it non-deterministically divided the string into $s_1, s_2, \ldots, s_n$
- For each of those divided parts M_{x*} runs, Suppose, M_{x*} all divided parts then s is accepted by M_{x*} else s is rejected by M_{x*}

Hence, it can be said that collection of Turing recognizable languages is closed under Star operation.

d.) next page

d.) Intersection

Suppose, X, Y be two Turing recognizable languages that have Turing machines M_x and M_y respectively.

Now the intersection of these languages is denoted by L_{xy}, and the Turing machine recognizing this language is M_{xy}.

For an input string s from L_{xy}, M_{xy} works as shown:

Turing Machine M_x runs on s. if it accepts s then M_y runs on s. Else s is rejected.

Suppose, M_y accepts s then it is accepted by the Turing machine otherwise s is rejected.

Hence, it can be said that collection of Turing recognizable languages is closed under Intersection operation.

e.) next Page

e.) Homomorphism

Suppose, X a Turing recognizable language that have Turing machine M_x

To recognize h(x) the other Turing machine M_y is simulated in such a way that:

On input s, it will consider all strings w such that $h(w) = s$

The Turing Machine M_x will execute on input w by going through all strings in W.

If $h(w) = s$ start executing M_x on input w, using merging to interleave with other executions on M_x. Accept if any executions accept.

M_y will accept s if any of those executions of Mx accepts s. Else s will be rejected.

Hence, it can be said that collection of Turing recognizable languages is closed under homomorphism operation.