Complexity Theory

Introduce time, a clock into the model

$f: N \Rightarrow N$

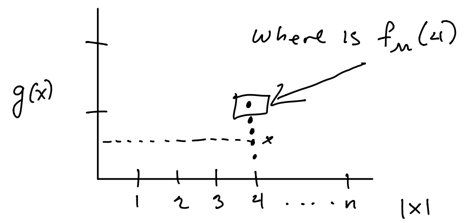
Non-neg integers

Given a Deterministic Turing Machine M which decides some language L, and input x

M takes a specific finite number of steps

$g(x) \equiv$ # of steps on input x

Define $f_M(n) = \max[g(x) \mid |x| = n]$



$f_M(n)$ is called the worst case # of steps of M, as a function of input length n.

Def: f and g are functions

$n \to R^+$

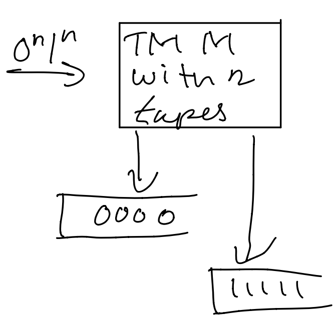
$f(n) = O(g(n))$

If there are positive integers c and n_0 such that $f(n) \leq c\, g(n)$ for all $n \geq n_0$

Consider the TM that decides the language $A = \{0^n1^n \mid n \geq 1\}$

M is the TM we studied to decide A, what is

$f_M(2n) = O(n^2)$

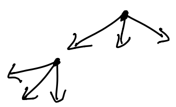


$f_M(n) = O(n)$

Worst case running-time of a DTM (Deterministic Turing Machine) M as a function of input length n.

$\delta(Q \times \Gamma) \rightarrow P(Q, \Gamma\{L, R\})$

A NTM (Nondeterministic Turing Machine) M decides L, for any input w.



M accepts input w if some Computation path accepts,

M reject if all path rejects.

The worst case running time of a NTM M as a function of the length of the input n is given by function $f_m(n) \equiv$ Max # of steps that M uses on any of the Computation paths on any input of size n

Consequence: If NTM M has worst case running time $O(n^2)$

P = Set of all languages which can be decided by a DTM whose worst case running time is $O(n^k)$ for some k

P means Polynomial.

NP = Set of all languages which can be decided by a NTM whose worst case running time is $O(n^k)$ for some k.

NP Means Nondeterministic polynomial