

Lecture 7: Context-Free Grammars

2.4) Given context-free grammars that generate the following languages. In all parts, the alphabet Σ is $\{0,1\}$.

a.) $\{w \mid w \text{ contains at least three 1's}\}$

$$S \rightarrow R1R1R1R$$

$$R \rightarrow 0R \mid 1R \mid \epsilon$$

b.) $\{w \mid w \text{ starts and ends with the same symbol}\}$

$$S \rightarrow 0P0 \mid 1P1 \mid 0 \mid 1$$

$$P \rightarrow 0P \mid 1P \mid \epsilon$$

c.) $\{w \mid \text{the length of } w \text{ is odd}\}$

$$S \rightarrow 0 \mid 1 \mid 00S \mid 01S \mid 10S \mid 11S$$

$$S \rightarrow 0 \mid 1 \mid \overset{(\text{or})}{0S0 \mid 0S1 \mid 1S0 \mid 1S1}$$

d.) $\{w \mid \text{The length of } w \text{ is odd and its middle symbol is a 0}\}$

$$S \rightarrow 0 \mid 0S0 \mid 0S1 \mid 1S0 \mid 1S1$$

e.) $\{w \mid w = w^R, \text{ that is, } w \text{ is a Palindrome}\}$

$$S \rightarrow 0 \mid 1 \mid 0S0 \mid 1S1 \mid \epsilon$$

f.) The empty set ($\{\}$)

$$S \rightarrow S$$