L14: More Diagonalization; Proof that Turing Machines are Countable

4.11) Let $INFINITE_{PDA} = \{\langle M \rangle \mid M$ is a PDA and $L(M)$ is an infinite language$\}$

To decide the InfinitePDA Convert the given PDA M into a CFG G.

Now convert into an equivalent grammar G' in Chompsky Normal Form (CNF).

Generate a graph A1: each variable is a vertex, and for a rule or production.

$T \rightarrow UV$ add edges (T, U) from T to U. and (T, V) from T to V.

Apply DFS or BFS from the start state to see if A1 has a directed cycle. if it does, accept. Otherwise reject.

If $\langle M \rangle \in INFINITE_{PDA}$, then it has a string of length greater than pumping length of $L(M)$.

Following the proof of pumping lemma, it means there is a variable V which can derive sVt for some strings s. t, since it is in CNF it must be non-empty.

This implies that the graph A1 has a cycle involving the variable V

Assume their is a circle in A1. Then it is clear that for some variable V, a derivation $V \rightarrow^* sVt$ must exist and s and t must be none empty, since G' is in CNF.

Since there is a scope for finding a cycle from s. There is rule $S \rightarrow^* aVb$

Thus, $S \rightarrow^* a v^i V v^i b$, for all $i \geq 0$ and so $L(M)$ is infinite.

$\therefore$ $INFINITE_{PDA}$ is Decidable.