L16: Unrecognizable Languages and Reductions

5.23) Show that A is decidable iff $A \leq_m 0^*1^*$

Decidability

Assume $B = 0^*1^*$. Thus it is required to prove that A is decidable iff $A \leq_m B$.

Solution can be divided into two parts.

1.) If A is decidable then $A \leq_m B$.
2.) If $A \leq_m B$ then A is decidable.

Part 1: If A decidable then $A \leq_m B$.

Proof:

First define a function f as follows:

$f(s) = 01$ if $s \in A$
$f(s) = 10$ otherwise.

Since A is decidable, decider can be used for A to compute f. Also, $s \in A$ iff $f(s) \in B$.

Hence, f is mapping reduction from A to B.

Part 2: If $A \leq_m B$, then A is decidable.

Proof:

Since $A \leq_m B$, there exist a function f, such that $w \in A$ iff $f(w) \in B$.

Now Consider Turing Machine M:

M = on input w

1.) Compute $f(w)$

2.) If $f(w)$ is in form of 0^*1^*, then accept, otherwise reject.

Now

$w \in A$
$\Leftrightarrow f(w)$ is of form 0^*1^*
$\Leftrightarrow M$ accepts w.

Thus M decides A.