$A_{TM} = \{<M, w> \mid M \text{ accepts } w\}$

we reduced A_{TM} to

$Halt_{TM} = \{<M, w> \mid M \text{ halts on } w\}$

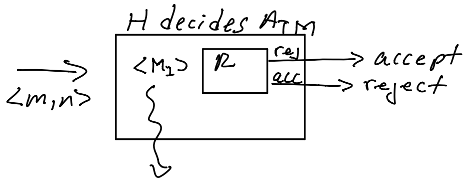
$A\,circ = \{<M> \mid M$ accepts w iff M accepts w^c, i.e. w rotated by one Position $\}$

$A\,rev = \{<M> \mid M$ accepts w iff M accepts $w^R\}$

$E_{TM} = \{<M> \mid M \text{ is a TM and } L(M) = \emptyset\}$

Theorem: E_{TM} is undecidable

Pf: Suppose R is a TM that decides E_{TM}. Reduce A_{TM} to E_{TM} i.e. we will use the supposed existence of R to describe a TM H that decides A_{TM}



creates on its tape the description of a TM M_1 which in fact is identical to M except that it first checks if the input to M is w. If the input is not w, then M_1 rejects, but if input is w, then H uses R to decide if $L(M_1) = \emptyset$

By construction, $L(M_1)$ is either $\emptyset$ or $\{w\}$

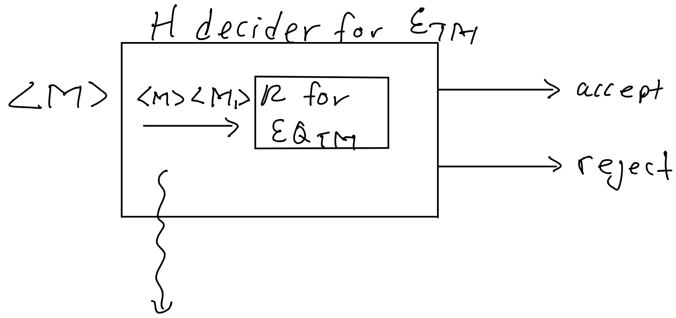
$L(M_1) = \emptyset$ iff M rejects w.

$L(M_1) \neq \emptyset$ iff M accepts w.

$EQ_{TM} = \{<M_1>, <M_2> \mid L(M_1) = L(M_2)\}$

Pf: Reduce E_{TM} to EQ_{TM}.

Suppose EQ_{TM} is decidable, so there is a TM R that decides EQ_{TM}. Want to use R to decide E_{TM}.



H creates a description of a TM M_1 that rejects every input. $L(M_1) = \emptyset$

Then inputs $\langle M \rangle \langle M_1 \rangle$ to R, H accepts if R accepts, rejects if R rejects.

So H is a decider for E_{TM}, but E_{TM} is undecidable, so R cannot exist

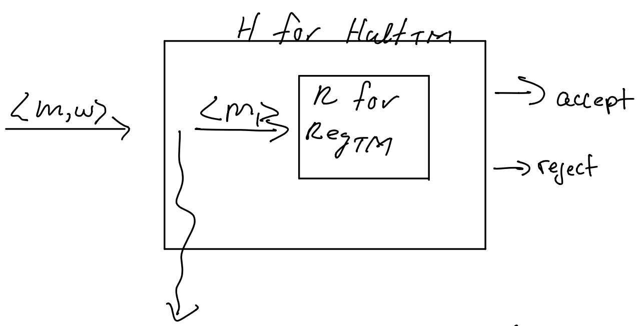
So EQ_{TM} is undecidable

$Regular_{TM} = \{\langle M \rangle \mid L(M) \text{ is regular}\}$

$Halt_{TM} = \{\langle m, w \rangle \mid M \text{ halts on } w\}$
Is undecidable.

Theorem: $Regular_{TM}$ is undecidable

proof: Reduce $Halt_{TM}$ to $Regular_{TM}$



want to describe how H creates description of TM M_1 so that R decide whether or not $L(M_1)$ is regular, that result can be used to decide whether or not M halts on input w.

M_1 simulates M on input w. if the simulation halts then M_1 takes in it's input w_1. M_1 accepts iff it's input w_1 is of the form $0^n 1^n \mid n \geq 1$, else M_1 rejects

What can M_1 accept?

M_1 migut accept $\emptyset$, iff M does not halt on w

M_1 might accept strings of the form $\{0^n 1^n \mid n \geq 1\}$ iff
M does halt on w

$L(M_1) = \emptyset \longrightarrow$ regular langung iff M doesn't halt on w

$L(M_1) = \{0^n 1^n \mid n \geq 1\} \Rightarrow$ non-regular language iff M haltson w.

function $f(x)$

$$f(x) = \frac{x^2}{3}$$

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x, x' TM takes in x and leaves $f(x)$ on its tape.

TM M compute $f(x)$ if M takes in x and writes $f(x)$.

Question: Is there a function f. s.t there is no TM which can Compute $f(x)$?