1.29) Use the pumping lemma to show that the following languages are not regular

a.) $A_1 = \{ 0^n 1^n 2^n \mid n \geq 0 \}$

Consider A_1 is a regular language.

Let p be the pumping length given by the pumping lemma.
Choose s to be the string $0^p 1^p 2^p$. Because s is a member of A_1, and $|s| > p$, the pumping lemma guarantees that s can be split into three pieces, $s = xyz$, where for any $i \geq 0$ the string xy^iz is in A_1. Now consider the two possibilities:

The string y consists only of 0's, only of 1's, or only of 2's. In these cases the string $xyyz$ will not have equal numbers of 0's, 1s, and 2's. Hence, $xyyz$ is not a member of A_1, this is a contradiction.

The string y consists of more than 1 kind of symbol. In this case, $xyyz$ will have the 0s, 1s, or 2s out of order. Hence $xyyz$ is not a member of A_2, this is a contradiction.

Either way we arrive at a contradiction. Therefore, A_1 is not regular.

b) next page

b.) $A_2 = \{www \mid w \in \{a,b\}^*\}$

Consider A_2 is a regular language.

Let p be pumping length given by the pumping lemma.

Let $s = a^p b a^p b a^p b \in A_2$.

By the pumping lemma, this string can be divided into three pieces xyz such that $|xy| \le p$, $|y| > 0$, and $xy^iz \in A_2$, $\forall i \geq a$

So $s = a^p b a^p b a^p b = xyz$

Let aabaabaab be the string that belongs to A_2. The pumping length of the string is 2. To satisfy the conditions of the pumping lemma, $x = a$, $y = a$, $z = baabaab$.

$s = aabaabaab = \underset{x}{\underline{a}}\ \underset{y}{\underline{a}}\ \underset{z}{\underline{baabaab}}$

Pump the middle part such that xy^iz $(i \geq 0)$. For $i = 2$, the y becomes aa. The string after pumping is aaabaabaab.

$s = (a)(a)^i(baabaab) = \underset{x}{\underline{a}}\ \underset{y}{\underline{aa}}\ \underset{z}{\underline{baabaab}}$ (when $i = 2$)

The string aaabaabaab $\notin A_2$. It is a contradiction. So, the pumping lemma is violated.

Therefore A_2 is not a regular language.

c.) next page

c.) $A_3 = \{a^{2^n} \mid n \geq 0\}$ (Here, a^{2^n} means a string of 2^n a's.)

Consider that A_3 is regular. Let p be the pumping length given by the pumping lemma. Let s be the string a^{2^p}. Because s is a member of A_3 and $|s| > p$, the pumping lemma guarantees that s can be split into three pieces, $s = xyz$, satisfying the three conditions of the pumping lemma.

The third condition tells us that $|xy| \leq p$. Furthermore $p < 2^p$ and so $|y| < 2^p$. Therefore, $|xyyz| = |xyz| + |y| < 2^p + 2^p = 2^{p+1}$. The second condition requires $|y| > 0$ so $2^p < |xyyz| < 2^{p+1}$. The length of $xyyz$ cannot be a power of 2. Hence $xyyz$ is not a member of A_3, this is a contradiction. Therefore, A_3 is not regular.