

L12: Universal Turing Machines; The Halting Problem is Recognizable but Not Decidable

3.22) Let A be the language containing only the single string S , where

$$S = \begin{cases} 0 & \text{If life never will be found on Mars.} \\ 1 & \text{If life will be found on Mars.} \end{cases}$$

Is A decidable? why or why not? For the purpose of this problem, assume that the question of whether life will be found on Mars has an unambiguous YES or NO answer.

Decidable language: A language is decidable if and only if some Turing machine decides it.

We call a Turing machine a decider if all branches halt on all inputs.

The given data is, A is a language containing only the single string S , where

$$S = \begin{cases} 0 & \text{If life never will be found on Mars.} \\ 1 & \text{If life will be found on Mars.} \end{cases}$$

So, the language A may contain either 0 or 1 but not both.

Thus $A = \{0\}$ (or) $A = \{1\}$

In both cases A are finite and S is fixed, so we know the same finite language is always decidable.

We are not able to determine whether

$$A = \{0\} \text{ or } A = \{1\}.$$

So we will not be able to describe the decider for A .

In this case, we need to give two Turing machines for both the cases. So, definitely one of them will be the decider of A .