L20: P, NP, and Polynomial-Time Reduction

Problem 7.9 from the Book

7.9) A triangle in an undirected graph is a 3-clique. Show that TRIANGLE ∈ P where TRIANGLE = {<G> | G contains a triangle}

Class P: P is a class of languages that are decidable in polynomial time on a deterministic single tape Turing-machine.

Specified language

A triangle is an undirected graph is a 3-clique.

TRIANGLE = {<G> | G contains a triangle}

Now we have to show that TRIANGLE ∈ P

Let A be the Turing Machine that decides TRIANGLE is Polynomial time

A can be described as follows:

A = "on input $G\langle V, E\rangle$:

V denotes set of vertices of the graph G.
E denotes set of edges of the graph G.

For $u, v, w \in V$ and $u < v < w$, we enumerate all triples (u, v, w).

Check whether all three edges $(u, v), (v, w), (w, u)$ exist in E or not. If exist then accept.

Otherwise reject

Enumeration of all triple require $O(|V|^3)$ time

Checking whether all three edges belong to E take $O(|E|)$ time

Overall time is $O(|V|^3 |E|)$ which is polynomial in the length of the inplet

Therefore TRIANGLE ∈ P.