

$$\begin{cases} \frac{\partial h}{\partial t} + \frac{\partial(hv_x)}{\partial x} + \frac{\partial(hv_y)}{\partial y} = 0 \\ \frac{\partial(hv_x)}{\partial t} + \frac{\partial(h(v_x^2 - B_x^2) + \frac{1}{2}gh^2)}{\partial x} + \frac{\partial(h(v_xv_y - B_xB_y))}{\partial y} + \left(B_0B_x - hv_y\left(f_0 + \frac{\partial f}{\partial y}y\right)\right) = -h\nu\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right) \\ \frac{\partial(hv_y)}{\partial t} + \frac{\partial(h(v_xv_y - B_xB_y))}{\partial x} + \frac{\partial(h(v_y^2 - B_y^2) + \frac{1}{2}gh^2)}{\partial y} + \left(B_0B_y + hv_x\left(f_0 + \frac{\partial f}{\partial y}y\right)\right) = -h\nu\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}\right) \\ \frac{\partial(hB_x)}{\partial t} + \frac{\partial(h(v_yB_x - v_xB_y))}{\partial y} - B_0v_x = -h\nu\left(\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2}\right) \\ \frac{\partial(hB_y)}{\partial t} + \frac{\partial(h(v_xB_y - v_yB_x))}{\partial x} - B_0v_y = -h\nu\left(\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2}\right) \end{cases} \quad (1)$$

$$\frac{\partial H}{\partial t} + \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + F + \nu V = 0 \quad (2)$$

$$\begin{cases} H = \begin{pmatrix} h & hv_x & hv_y & B_x & B_y \end{pmatrix}^T \\ X = \begin{pmatrix} hv_x & h(v_x^2 - B_x^2) + \frac{1}{2}gh^2 & h(v_xv_y - B_xB_y) & 0 & h(v_xB_y - v_yB_x) \end{pmatrix}^T \\ Y = \begin{pmatrix} hv_y & h(v_xv_y - B_xB_y) & h(v_y^2 - B_y^2) + \frac{1}{2}gh^2 & h(v_yB_x - v_xB_y) & 0 \end{pmatrix}^T \\ F = \begin{pmatrix} 0 & B_0B_x - hv_y\left(f_0 + \frac{\partial f}{\partial y}y\right) & B_0B_y + hv_x\left(f_0 + \frac{\partial f}{\partial y}y\right) & -B_0v_x & -B_0v_y \end{pmatrix}^T \\ V = \begin{pmatrix} 0 & -h\left(\frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2}\right) & -h\left(\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2}\right) & -h\left(\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2}\right) & -h\left(\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2}\right) \end{pmatrix}^T \end{cases} \quad (3)$$

$$\begin{cases} H_{i-\frac{1}{2},j}^{n+\frac{1}{2}} = \frac{H_{i,j}^n + H_{i-2,j}^n + H_{i-1,j+1}^n + H_{i-1,j-1}^n}{4} - \frac{\Delta t}{2} \left( \frac{X_{i,j}^n - X_{i-2,j}^n}{\Delta x} + \frac{Y_{i-1,j+1}^n - Y_{i-1,j-1}^n}{\Delta y} + 2F_{i-1,j}^n + 2\nu V_{i-1,j}^n \right) \\ H_{i+\frac{1}{2},j}^{n+\frac{1}{2}} = \frac{H_{i+2,j}^n + H_{i,j}^n + H_{i+1,j+1}^n + H_{i+1,j-1}^n}{4} - \frac{\Delta t}{2} \left( \frac{X_{i+2,j}^n - X_{i,j}^n}{\Delta x} + \frac{Y_{i+1,j+1}^n - Y_{i+1,j-1}^n}{\Delta y} + 2F_{i+1,j}^n + 2\nu V_{i+1,j}^n \right) \\ H_{i,j-\frac{1}{2}}^{n+\frac{1}{2}} = \frac{H_{i+1,j-1}^n + H_{i-1,j-1}^n + H_{i,j}^n + H_{i,j-2}^n}{4} - \frac{\Delta t}{2} \left( \frac{X_{i+1,j-1}^n - X_{i-1,j-1}^n}{\Delta x} + \frac{Y_{i,j}^n - Y_{i,j-2}^n}{\Delta y} + 2F_{i,j-1}^n + 2\nu V_{i,j-1}^n \right) \\ H_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{H_{i+1,j+1}^n + H_{i-1,j+1}^n + H_{i,j+2}^n + H_{i,j}^n}{4} - \frac{\Delta t}{2} \left( \frac{X_{i+1,j+1}^n - X_{i-1,j+1}^n}{\Delta x} + \frac{Y_{i,j+2}^n - Y_{i,j}^n}{\Delta y} + 2F_{i,j+1}^n + 2\nu V_{i,j+1}^n \right) \end{cases} \quad (4)$$

$$H_{i,j}^{n+1} = H_{i,j}^n - \frac{\Delta t}{2} \left( \frac{X_{i+\frac{1}{2},j}^{n+\frac{1}{2}} - X_{i-\frac{1}{2},j}^{n+\frac{1}{2}}}{\Delta x} + \frac{Y_{i,j+\frac{1}{2}}^{n+\frac{1}{2}} - Y_{i,j-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta y} + 2F_{i,j}^n + 2\nu V_{i,j}^n \right) \quad (5)$$