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Un schéma à variation totale décroissante pour les équations de Saint-Venant

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ABSTRACT

A high-resolution algorithm is proposed for the solution of the two-dimensional shallow-water equations by adopting the numerical flux for the generalized formulation of the TVD Lax-Wendroff scheme developed by Sweby and Davis. To obtain high resolution near the discontinuities, we apply the ACM (artificial compression method) technique to the proposed scheme. Comparison of corresponding results is exposed. A variety of hydraulic test cases show that the coupled method (TVD-ACM) is quite robust and accurate.

RÉSUMÉ

Un algorithme à haute résolution est proposé pour résoudre les équations de Saint-Venant à deux dimensions en adoptant le flux numérique pour la formulation généralisée du schéma TVD de Lax-Wendroff développé par Sweby et Davis. Pour obtenir une haute résolution au voisinage des discontinuités, la technique de (compression artificielle) (ACM) a été appliquée au schéma proposé. L'article présente une comparaison des résultats obtenus. Des cas tests variés montrent que cette méthode couplée (TVD-ACM) est robuste et précise.

1 Introduction

Shallow water equations (SWE) models have been developed for the purpose of determining circulation patterns which are of primary interest in some models to predict the cause and effects of toxic substances for environmental applications.

The general approach, to resolve SWE, based upon the splitting methodology (advection-diffusion-propagation steps) [Benqué et al. 1982], seems to be inadequate. In fact, the interesting mathematical features of this system, hyperbolicity and conservation, were completely spoiled. An appropriate method which take these characteristics into account must be robust and efficient for the correct resolution of flow features such as shocks. Recently, some works in this direction, have been done [Alcrudo and Garcia-Navarro 1993 ; Ambrosi 1995 ; Louaked and Hanich 1995], and some of these methods are generally complicated and expensive owing to the use of approximate Riemann solver, which decomposes the fluctuation in each cell into waves, which are then distributed in an upwind sense.

The aim of this work is to present the extension of the Davis second-order symmetrical TVD (Total Variation Diminishing) scheme to SWE, characterized by it simplicity in coding and its ability to incorporate the ideas of a wide variety of schemes distinguished by their robustness and their usefulness in computing flows with very complicated shock structures for the compressible Euler equations [Collela and Woodward 1984 ; Harten 1984 ; Roe 1984 ; Yee 1987].

Harten [1984] developed the concept of TVD schemes and derived a set of sufficient conditions which is very useful in checking or constructing second order TVD schemes.

As a sequel of these works, Davis [1984] showed that it is possible to formulate the classical Lax-Wendroff scheme in TVD form. This is accomplished by appending to the scheme a non-linear

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term which applies precisely the correct amount of artificial viscosity needed at each mesh point to limit overshoots and undershoots. The TVD resulting scheme not only has the ability to damp oscillations, but also to highly resolve shocks and contact discontinuities and contains no terms depending on adjustable parameters.

The technique extended to SWE leads to a hybrid method that uses the second order flux in smooth regions but involves some limitation based on the gradient of the solutions so that in the vicinity of discontinuities it reduces to the monotone upwind method.

The control of switching between upwind and the second order scheme is obtained through the use of special non-linear functions, the limiters, which satisfy criteria deduced for a scalar one-dimensional conservation law.

To have a good resolution with fewer grids, the concept of the artificial compression method (ACM) [Harten 1978] is used. The ACM is a technique which modifies standard finite difference schemes which prevents the smearing of contact discontinuities and improves the resolution of shocks. The accurate TVD-ACM second order scheme achieves high resolution while preserving the robustness of the original non oscillatory scheme.

Numerical results obtained using the scheme constructed in this paper indicated good shocks and contact discontinuity transitions without any oscillations and high accuracy in smooth regions.

2 Governing equations

The shallow water equations present an average model which can be obtained from the Navier-Stokes equations by depth integration supposing hydrostatic pressure only and neglecting all dynamic effects in the vertical direction.

The governing equations, written in conservative form, are

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{E}}{\partial x} + \frac{\partial \boldsymbol{F}}{\partial y} = \boldsymbol{S} \tag{1}$$

With

$$\boldsymbol{U} = \begin{pmatrix} h \\ q_1 \\ q_2 \end{pmatrix}, \boldsymbol{E} = \begin{pmatrix} q_1 \\ \dfrac{q_1^2}{h} + \dfrac{gh^2}{2} \\ \dfrac{q_1 q_2}{h} \end{pmatrix}, \boldsymbol{F} = \begin{pmatrix} q_2 \\ \dfrac{q_1 q_2}{h} \\ \dfrac{q_2^2}{h} + \dfrac{gh^2}{2} \end{pmatrix}$$

Here h, $q_1 = uh$ and $q_2 = vh$ are respectively the total height above the bottom of the channel (figure 1) and $q = (q_1, q_2)$ is the unit-width discharge, (u, v) is the velocity field and g is the gravitational acceleration.

The vector S contains a number of physical effects such as the effect of bottom slop and friction. In this present study, only the numerical treatment of the homogenous system will be described.

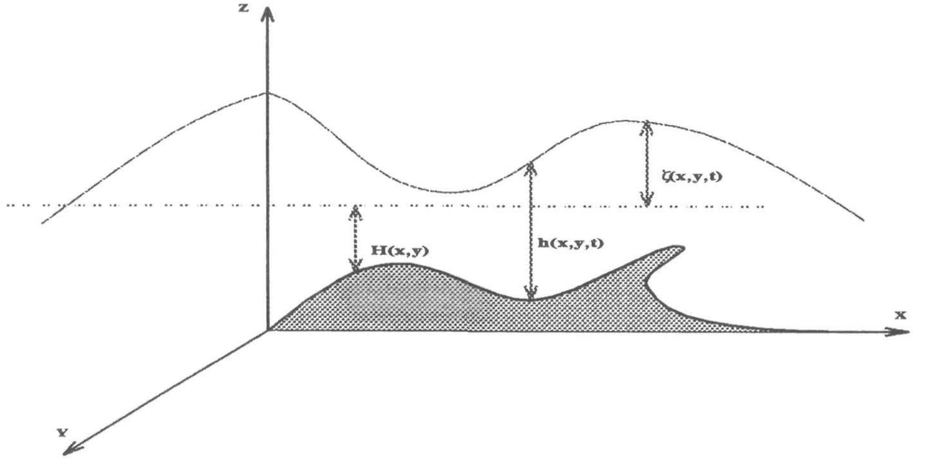

Fig. 1. Vertical section of the physical domain.

3 TVD finite difference scheme

3.1 *Shallow water and wave equation in one dimension*

The one dimensional homogenous SWE is as follows

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{E}}{\partial x} = 0 \tag{2}$$

Derivative of the flux **E** can be written in quasilinear forms as $A\dfrac{\partial \boldsymbol{U}}{\partial x}$ and a matrice A can be diagonalized to

$$\begin{aligned} \Lambda &= R^{-1}AR \\ A &= \begin{pmatrix} 0 & 1 \\ -u^2 + c^2 & 2u \end{pmatrix}; \quad \Lambda = \begin{pmatrix} u+c & 0 \\ 0 & u-c \end{pmatrix}. \end{aligned} \tag{3}$$

Then the equation can be rewritten as:

$$R^{-1}\frac{\partial \boldsymbol{U}}{\partial t} + \Lambda R^{-1}\frac{\partial \boldsymbol{U}}{\partial x} = 0 \tag{4}$$

For frozen Jacobien, using Roe's average, equation (4) becomes the decoupled two wave equations as

$$\frac{\partial \boldsymbol{V}}{\partial t} + \Lambda\frac{\partial \boldsymbol{V}}{\partial x} = 0, \tag{5}$$

where the wave speed $c = \sqrt{gh}$ and $V = R^{-1}U$.

3.2 *Wave equation*

Consider the scalar wave equation:

$$\frac{\partial v}{\partial t} + a\frac{\partial v}{\partial x} = 0 \qquad a > 0. \tag{6}$$

Let v_j^n be the numerical solution of (6) at $x = j\Delta x$, $t = n\Delta t$ with Δx the spatial mesh size and Δt the time step.

Following Sweby [1984] the Lax-Wendroff method is written to look like the upwind part

$$v_j^{n+1} = v_j^n - \alpha\Delta v^n_{j-\frac{1}{2}}, \tag{7}$$

with the additional anti-diffusive flux term

$$-\nabla\left[\frac{1}{2}(1-\alpha)\alpha\Delta v^n_{j+\frac{1}{2}}\right], \tag{8}$$

where $\alpha = \dfrac{a\Delta t}{\Delta x}$, $\Delta v^n_{j+\frac{1}{2}} = v^n_{j+1} - v^n_j$ and $\Delta v^n_{j-\frac{1}{2}} = v^n_j - v^n_{j-1}$.

Since the first order scheme does not produce spurious oscillations at discontinuities, a limited amount of anti-diffusive flux is added

$$-\nabla\left[\phi\frac{1}{2}(1-\alpha)\alpha\Delta v^n_{j+\frac{1}{2}}\right]. \tag{9}$$

A particular form of the resulting scheme is given by the incremental form

$$v_j^{n+1} = v_j^n - \alpha\left\{1 + \frac{1}{2}(1-\alpha)\left[\frac{\phi(r_j^+)}{r_j^+} - \phi(r^+_{j-1})\right]\right\}\Delta v^n_{j-\frac{1}{2}}, \tag{10}$$

where $r_j^+ = \dfrac{\Delta v^n_{j-\frac{1}{2}}}{\Delta v^n_{j+\frac{1}{2}}}$.

The flux limiter ϕ is defined under Harten's TVD inequalities. By the use of Davis' approach, Sweby's scheme is recovered by adding terms to the Lax-Wendroff method to obtain, a five point, symmetrical TVD scheme. A suitable term is

$$\begin{aligned}&[K^+(r_j^+) + K^-(r^-_{j+1})]\Delta v^n_{j+\frac{1}{2}}\\ &-[K^+(r^+_{j-1}) + K^-(r_j^-)]\Delta v^n_{j-\frac{1}{2}},\end{aligned} \tag{11}$$

where $K^{\pm}(r_j^{\pm}) = \dfrac{\alpha}{2}(1-\alpha)[1-\phi(r_j^{\pm})]$, $r_j^- = \dfrac{\Delta v^n_{j+\frac{1}{2}}}{\Delta v^n_{j-\frac{1}{2}}}$.

3.3 *SWE equation*

The above scheme is applied to the uncoupled set of equations (5) to each scalar equation in turn, that is

$$\begin{aligned} V_j^{n+1} &= V_j^n - \frac{\alpha}{2}[V_{j+1}^n - V_{j-1}^n] + \frac{\alpha^2}{2}[V_{j+1}^n - 2V_j^n + V_{j-1}^n] \\ &\quad + [K^+(r_j^+) + K^-(r_{j+1}^-)]\Delta V^n_{j+\frac{1}{2}} \\ &\quad - [K^+(r_{j-1}^+) + K^-(r_j^-)]\Delta V^n_{j-\frac{1}{2}}, \end{aligned} \tag{12}$$

multiplied (12) by R^{-1} to obtain an equation in terms of the original dependent variables.

$$\begin{aligned} U_j^{n+1} &= U_j^n - A\frac{\Delta t}{2\Delta x}[U_{j+1}^n - U_{j-1}^n] + A^2\frac{\Delta t^2}{2\Delta x^2}[U_{j+1}^n - 2U_j^n + U_{j-1}^n] \\ &\quad + R[K^+(r_j^+) + K^-(r_{j+1}^-)]R^{-1}\Delta U^n_{j+\frac{1}{2}} \\ &\quad - R[K(r_{j-1}^+) + K^-(r_j^-)]R^{-1}\Delta U^n_{j-\frac{1}{2}}. \end{aligned} \tag{13}$$

The requirement that R and R^{-1} be known is removed by approximating the diagonal matrix $K^{\pm}$ by scalar matrices

$$K^{\pm}(r^{\pm}) \cong \bar{K}^{\pm}(r^{\pm})I, \tag{14}$$

where I is the unit matrix 2×2 and $r^{\pm}$ are choosen to be scalar functions of U

$$r_j^+ = \frac{\left(\Delta U^n_{j-\frac{1}{2}}, \Delta U^n_{j+\frac{1}{2}}\right)}{\left(\Delta U^n_{j+\frac{1}{2}}, \Delta U^n_{j+\frac{1}{2}}\right)}, \tag{15}$$

$$r_j^- = \frac{\left(\Delta U^n_{j-\frac{1}{2}}, \Delta U^n_{j+\frac{1}{2}}\right)}{\left(\Delta U^n_{j-\frac{1}{2}}, \Delta U^n_{j-\frac{1}{2}}\right)}, \tag{16}$$

in which (.,.) denotes the inner product on $\mathbb{R}^2$.

The resulting scheme does not depend explicitly on transformation (3), so it can be used without modification for non-linear problems. This is done by using the technique proposed by Roe for constructing a suitable coefficient matrix to define a modified, linear equation but equivalent to the system of the conservation law. The numerical method takes the form

$$\begin{aligned} U_j^{n+1} &= U_j^n - A\frac{\Delta t}{2\Delta x}[U_{j+1}^n - U_{j-1}^n] + A^2\frac{\Delta t^2}{2\Delta x^2}[U_{j+1}^n - 2U_j^n + U_{j-1}^n] \\ &+ [\bar{K}^+(r_j^+) + \bar{K}^-(r_{j+1}^-)]\Delta U_{j+\frac{1}{2}}^n \\ &- [\bar{K}^+(r_{j-1}^+) + \bar{K}^-(r_j^-)]\Delta U_{j-\frac{1}{2}}^n. \end{aligned} \tag{17}$$

The typical limiter function based on the ratio of consecutive gradients, and which guarantees second order accuracy and TVD property, is as follows

$$\phi(r) = minmod(2r, 1), \tag{18}$$

where the function minmod is defined by

$$minmod(a, b) = \mathrm{sgn}(a) max[0, min(|a|, b\,\mathrm{sgn}(a))]$$

Note that this limiter possesses the property

$$\frac{\phi(r)}{r} = \phi\left(\frac{1}{r}\right),$$

which guarantees that forward and backward gradients are treated in the same manner and also preserve the symmetry properties of the exact solution.

3.4 *ACM (Artificial Compression Method)*

The structure of a finite difference scheme generates a discrete shock by the continuous transition connecting the states on both sides of the jump discontinuity. When this discontinuity is an admissible shock (Rankine-Hugoniot relation and Oleinik's entropy condition [Oleinik 1963], are satisfied), the transition occurs with a finite number of mesh points. However, in presence of a contact discontinuity, the width transition is typically unbounded in time.

The essence of the ACM is to solve a modified equation

$$\frac{\partial U}{\partial t} + \frac{\partial (E+G)}{\partial x} = 0, \tag{19}$$

rather than the original equation (6)

$$\frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} = 0. \tag{20}$$

here $G(U,t)$ is any function which has the following properties [Harten 1977 & 1978]:

- $G(U, t) = 0\, for\ U \notin [U_L(t), U_R(t)]$
- $G(U, t)\cdot\mathrm{sgn}[U_R - U_L] > 0\, for\ U \in [U_L(t), U_R(t)]$

$G(U,t)$ is called an artificial compression flux (ACF). With either a shock or contact discontinuity (U_L, U_R, S) propagating with speed S, across which the value of U jump from U_L to U_R.
The original conservation equation (20) and the modified equation (19) have the same solution [Harten 1977]. However when the finite difference scheme is applied to both equations, the width of the viscous profile [Jennigs 1974] of the solution of the modified equation is kept reduced and one obtains a better resolution of shocks and contact discontinuities. The construction of a split operator C_Δ which compresses a TVD solution (17) at any given time level is given in Harten [1978]

$$C_\Delta = U_j - \frac{\hat{\lambda}}{2}\left(\theta_{j+\frac{1}{2}} G_{j+\frac{1}{2}} - \theta_{j-\frac{1}{2}} G_{j-\frac{1}{2}}\right) \tag{21}$$

here the vector $U_j = (U_{1,j}, U_{2,j})$ is the solution of equation (2) obtained by the scheme (17) and $G_{m,j+\frac{1}{2}}$ is the *mth* component of the vector $G_{j+\frac{1}{2}}$ given by

$$G_{m,j+\frac{1}{2}} = g_{m,j} + g_{m,j+1} - |g_{m,j+1} - g_{m,j}|\,\mathrm{sgn}(U_{m,j+1} - U_{m,j}), \tag{22}$$

where

$$g_{m,j} = \alpha_j(U_{m,j+1} - U_{m,j-1}),$$

and

$$\alpha_j = max\left\{0,\ min_{1 \le m \le M} \frac{min[|U_{m,j+1} - U_{m,j}|, (U_{m,j} - U_{m,j-1})\,\mathrm{sgn}(U_{m,j-1} - U_{m,j})]}{|U_{m,j+1} - U_{m,j}| + |U_{m,j} - U_{m,j-1}|}\right\},$$

the integer M being the number of the components of the vector solution U_j.
The automatic switch θ_j is defined by

$$\theta_{j+\frac{1}{2}} = max(\hat{\theta}_j, \hat{\theta}_{j+1}), \tag{23}$$

where

$$\hat{\theta}_j = \begin{cases} \left|\dfrac{\left|\Delta h_{j+\frac{1}{2}}\right| - \left|\Delta h_{j-\frac{1}{2}}\right|}{\left|\Delta h_{j+\frac{1}{2}}\right| + \left|\Delta h_{j-\frac{1}{2}}\right|}\right| & \text{for } \left|\Delta h_{j+\frac{1}{2}}\right| + \left|\Delta h_{j-\frac{1}{2}}\right| > \varepsilon \\ 0. & \text{else,} \end{cases} \tag{24}$$

here $\varepsilon > 0$ is some suitable chosen measure of insignificant variation in h.

The hybrid artificial compressor (21) is stable under $\hat{\lambda} \le 1$ and is completly independent of the choice of the scheme (17).

4 Two dimensional flow

The two dimensional problem is treated by the local one-dimensional decomposition of the limiting process. The Strang type operator splitting is used. Solution at the next time $(n+1)\Delta t$ is obtained as follow

$$U_{ij}^{n+1} = L_x\left(\frac{\Delta t}{2}\right) L_y\left(\frac{\Delta t}{2}\right) L_y\left(\frac{\Delta t}{2}\right) L_x\left(\frac{\Delta t}{2}\right) U_{ij}^n, \tag{25}$$

where

$$\begin{aligned} L_x U_{i,j}^n = U_{i,j}^* = {} & U_{i,j}^n - A\frac{\Delta t}{2\Delta x}[U_{i+1,j}^n - U_{i-1,j}^n] + A^2\frac{\Delta t^2}{2\Delta x^2}[U_{i+1,j}^n - 2U_{i,j}^n + U_{i-1,j}^n] \\ & + [\bar{K}^+(r_{i,j}^+) + \bar{K}^-(r_{i+1,j}^-)]\Delta U_{i+\frac{1}{2},j}^n \\ & - [\bar{K}^+(r_{i-1,j}^+) + \bar{K}^-(r_{i,j}^-)]\Delta U_{i-\frac{1}{2},j}^n, \end{aligned} \tag{26}$$

$$\begin{aligned} L_y U_{i,j}^* = {} & U_{i,j}^* - B\frac{\Delta t}{2\Delta y}[U_{i,j+1}^* - U_{i,j-1}^*] + B^2\frac{\Delta t^2}{2\Delta y^2}[U_{i,j+1}^* - 2U_{i,j}^* + U_{i,j-1}^*] \\ & + [\bar{K}^+(r_{i,j}^+) + \bar{K}^-(r_{i,j+1}^-)]\Delta U_{i,j+\frac{1}{2}}^* \\ & - [\bar{K}^+(r_{i,j-1}^+) + \bar{K}^-(r_{i,j}^-)]\Delta U_{i,j-\frac{1}{2}}^*, \end{aligned} \tag{27}$$

here the Jacobien matrices

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -u^2 + c^2 & 2u & 0 \\ -uv & v & u \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 1 \\ -uv & v & u \\ -u^2 + v^2 & 0 & 2v \end{bmatrix},$$

are evaluated at some state $(\tilde{u}, \tilde{v}, \tilde{c})$ known as "Roe's average" which are given by

$$\tilde{u} = \frac{u_{j+1}\sqrt{h_{j+1}} + u_{j-1}\sqrt{h_{j-1}}}{\sqrt{h_{j+1}} + \sqrt{h_j}}, \ \tilde{v} = \frac{v_{j+1}\sqrt{h_{j+1}} + v_{j-1}\sqrt{h_{j-1}}}{\sqrt{h_{j+1}} + \sqrt{h_j}}, \ \tilde{c} = \sqrt{\frac{g(h_{j+1} + h_{j-1})}{2}}.$$

5 Numerical tests

5.1 *Test 1: Dam break problem*

Consider a wide channel having a barrier placed across its width. Let h_1 and h_2 be respectively the height of the water upstream and downstream. At time $t = 0$ the barrier is suddenly removed. The flow consists of bore travelling downstream and a rarefaction wave travelling upstream. The analytical solution of this problem is given in Stoker [1957]:

$$h(x,t) = \begin{cases} h_1 & \text{if } \dfrac{x}{t} \le -\sqrt{gh_1} \\ \left(\dfrac{1}{9g}\right)\left[2\sqrt{gh_1} - \dfrac{x}{t}\right]^2 & \text{if } -\sqrt{gh_1} \le \dfrac{x}{t} \le [u_m - \sqrt{gh_m}] \\ h_m & \text{if } [u_m - \sqrt{gh_m}] \le \dfrac{x}{t} \le s \\ h_2 & \text{if } s \le \dfrac{x}{t} \le \infty, \end{cases} \tag{28}$$

$$u(x,t) = \begin{cases} 0 & \text{if } \dfrac{x}{t} \le -\sqrt{gh_1} \\ \left(\dfrac{2}{3}\right)\left[\sqrt{gh_1} + \dfrac{x}{t}\right] & \text{if } -\sqrt{gh_1} \le \dfrac{x}{t} \le [u_m - \sqrt{gh_m}] \\ u_m & \text{if } [u_m - \sqrt{gh_m}] \le \dfrac{x}{t} \le s \\ 0 & \text{if } s \le \dfrac{x}{t} \le \infty, \end{cases} \tag{29}$$

where h_m and u_m are given in terms of velocity of propagation of the shock s by

$$h_m = \frac{1}{2}\left[\sqrt{1 + \frac{8s^2}{gh_2}} - 1\right]h_2, \tag{30}$$

$$u_m = s - \frac{gh_2}{4s}\left[1 + \sqrt{1 + \frac{8s^2}{gh_2}}\right], \tag{31}$$

and s is the positive real solution of the equation:

$$u_m + 2\sqrt{gh_m} - 2\sqrt{gh_1} = 0. \tag{32}$$

The initial conditions are defined as:

$$h_0 = \begin{cases} h_1 \text{ if } x < 0 \\ h_2 \text{ if } x > 0 \end{cases} \tag{33}$$

$$u_0(x) = 0$$

Different values of h_2 are tested to simulate waves propagating on wet and dry channel beds. Figures (2)–(5) show the computational results obtained with or without ACM together with the exact solutions, they are found to be valid for h_2 larger than 10^{-4}. Although for very small values of h_2, error is negligible except in a small neighbourhood of $x/t = 2\sqrt{gh}$ where strict hyperbolicity fails. We present in figure 6 and figure 7, the result obtained by Hou Zhang et al. [1992] for the same test. We note that our results are in better agreement with the analytical solution than that obtained by Hou Zhang et al.

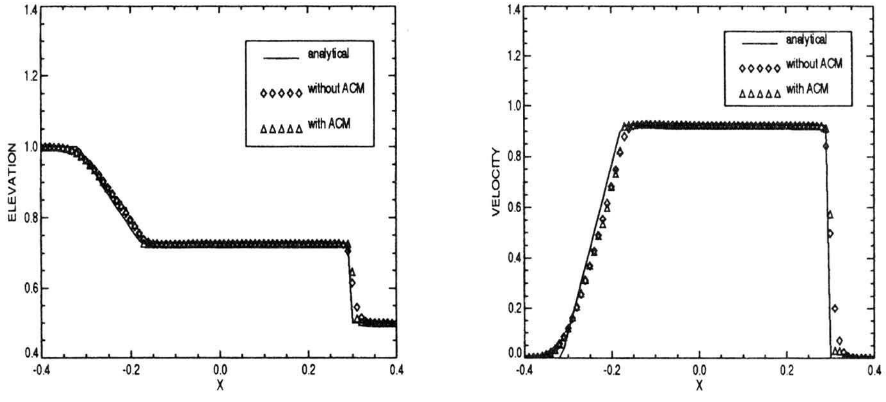

Fig. 2. Elevation and Velocity for $h_1 = 1.$ and $h_2 = .5$ at $t = 0.1$.

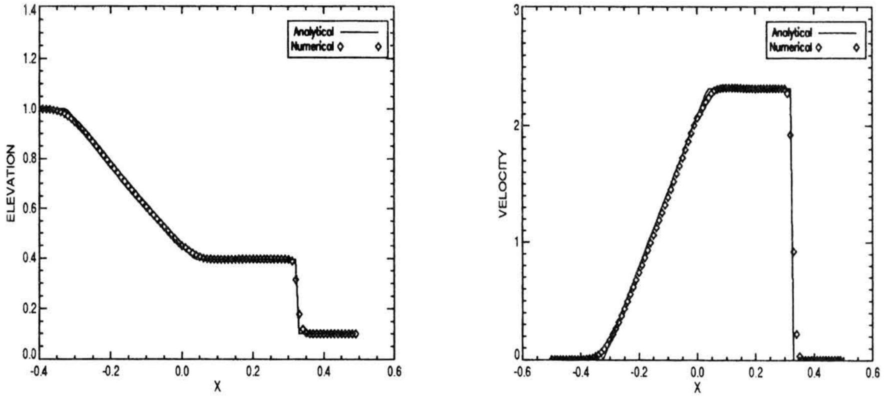

Fig. 3. Elevation and Velocity for $h_1 = 1.$ and $h_2 = .1$ at $t = 0.1$.

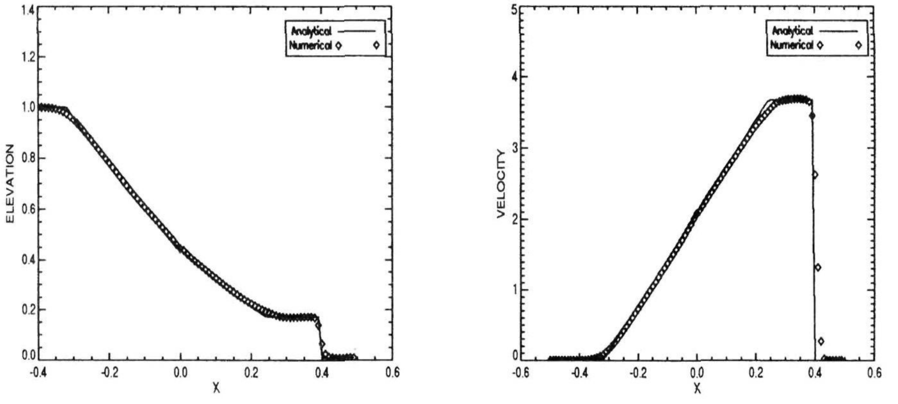

Fig. 4. Elevation and Velocity for $h_1 = 1.$ and $h_2 = .05$ at $t = 0.1$.

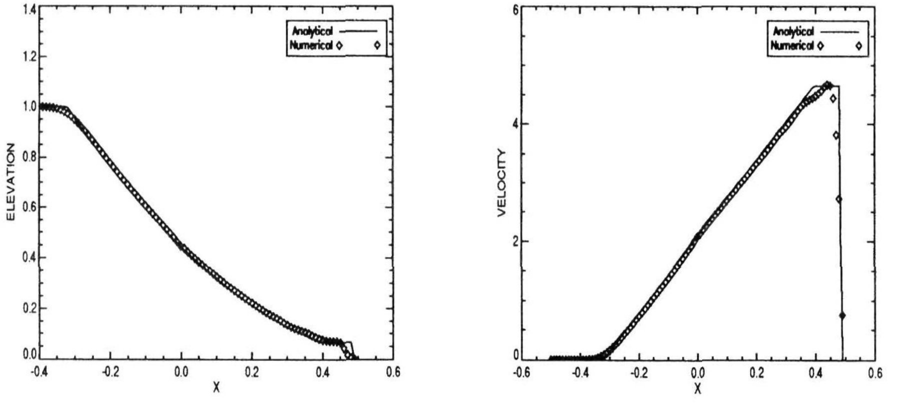

Fig. 5. Elevation and Velocity for $h_1 = 1.$ and $h_2 = .001$ at $t = 0.1$.

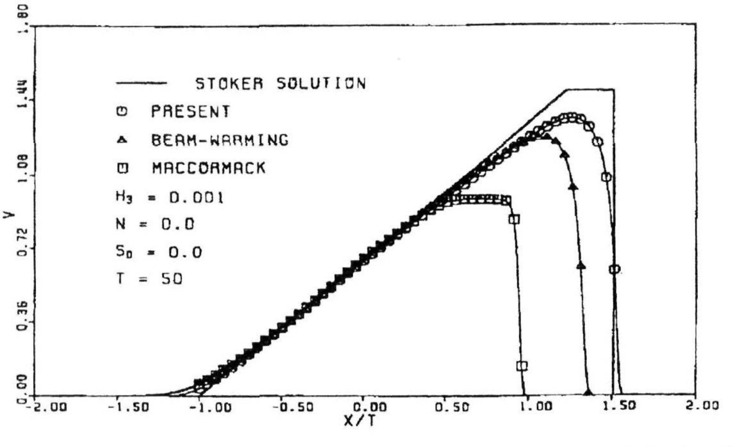

Fig. 6. Velocity results and comparaisons, obtained by Hou Zhang et al, for $h_2 = 0.001$.

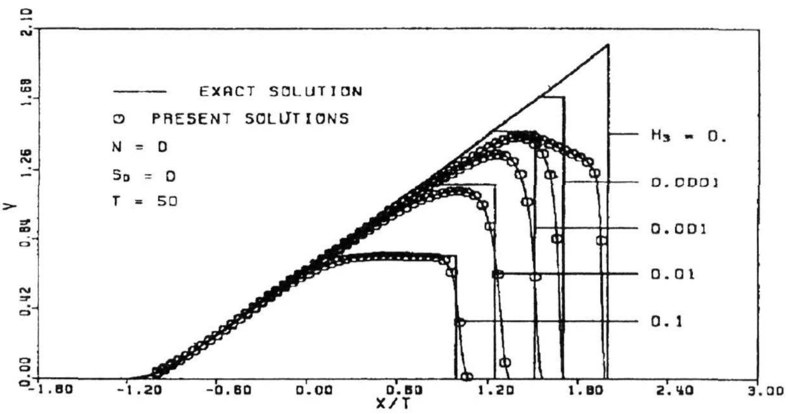

Fig. 7. Velocity results and comparaisons, obtained by Hou Zhang et al, for $h_2 = 0$.

Remark: For the dam break wave on dry bottom problem ($h_2 = 0.$) we have considered another formulation of the SWE:

$$\frac{\partial U}{\partial t} + \frac{\partial E}{\partial x} = 0 \tag{34}$$

with

$$U = \begin{pmatrix} h \\ uh \end{pmatrix}, E = \begin{pmatrix} uh \\ u^2 h + \frac{gh^2}{2} \end{pmatrix},$$

and h is the elevation and u the velocity.

These two equations are equivalent to those in conservative form with (h,Q) formulation. However, when shocks appear, this equivalency fails because the Ranking-Hugoniot condition is not verified. Consequently, in this context the numerical solution will not be correctly obtained. Our numerical simulations show that the discontinuities on the velocity and water level profiles can be spuriously produced, if the formulation (34) should be used.

5.2 *Test 2: Breaking of a circular dam*

To test the ability of the method to conserve high symmetries, the TVD-ACM scheme is applied to the test case computed by Alcrudo et al. [1993] for the breaking of dam of cylindrical geometry. The initial conditions consist in two regions of water separated by a cylindrical wall with radius equal to 11 m (Figure 8). The water depth inside the dam is 10 m, whilst outside the dam it is 1 m. The numerical results of the evolution of the flow between time $t = 0.1$ to $t = 0.75$, computed in 50×50 cell rectangular grid, are shown in figures (8)–(10).

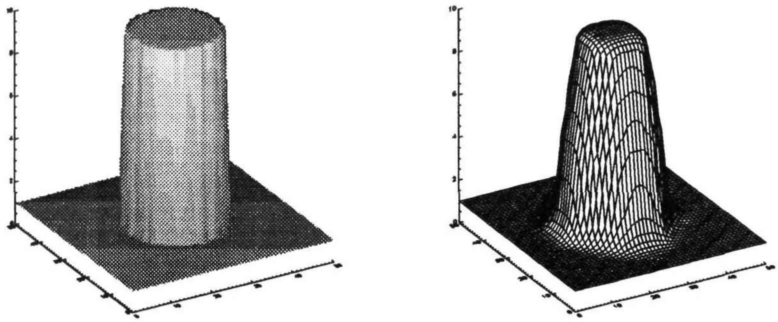

Fig. 8. Elevation at $t = 0$ and $t = 0.1$ for Breaking of circular dam.

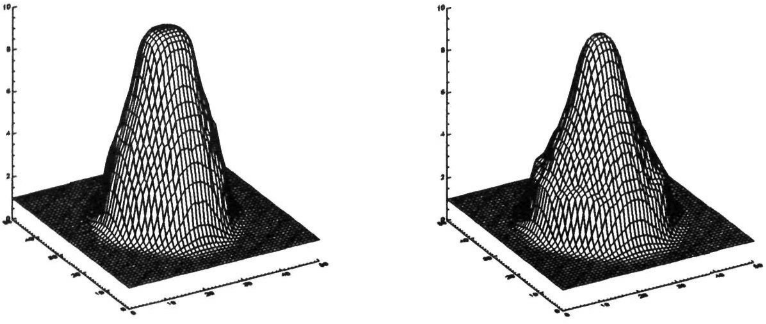

Fig. 9. Elevation at $t = 0.4$ and $t = 0.6$ for Breaking of circular dam.

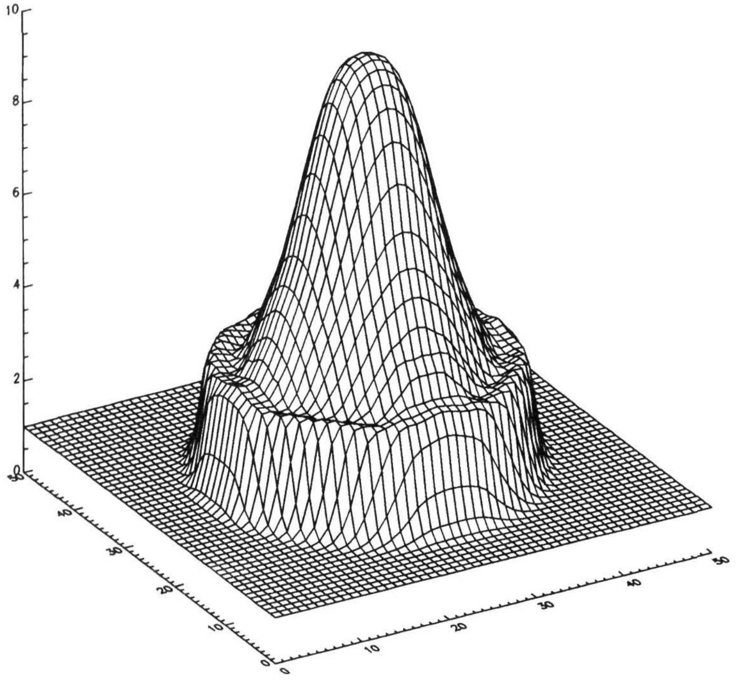

Fig. 10. Water elevation at $t = 0.75$.

5.3 *Test 3: Partial breach*

A partial breach or the opening of sluice gates test suggested by Alcrudo et al. [1993]; Ambrosi [1995] and Fennema et al. [1990] is widely used to check out the shock capturing ability of the method. In this calculation, the computational domain comprises of a 200 × 200 m channel with a nonsymmetrical breach. Figure (11) shows the basin geometry and parameters.
As initial conditions, the upstream and downstream water levels are imposed as follows $h_1 = 10\ m$ and $h_2 = 5\ m$ respectively. The solid boundary conditions are based on extrapolations of the interior-point variables towards the boundary (Space-time extrapolation). A uniform grid 40 × 40 is used, figures (12)-(13) represent the water surface at $t = 2.23$, $t = 4.3$, $t = 8.22$ and $t = 10.1$. At time $t =$ 6.29 the wave front has reached one side of the channel.

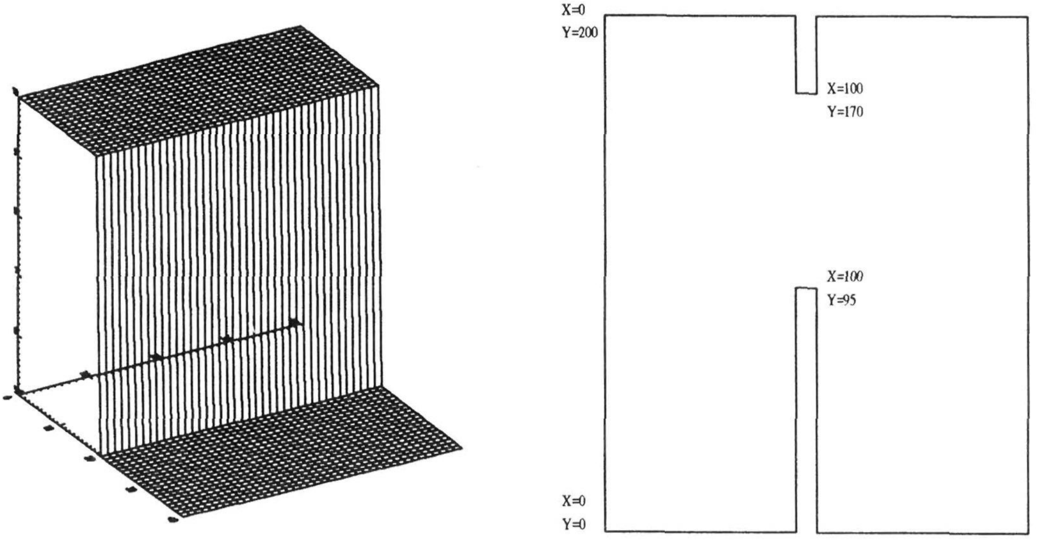

Fig. 11. Initial water elevation and basin geometry for partial breach.

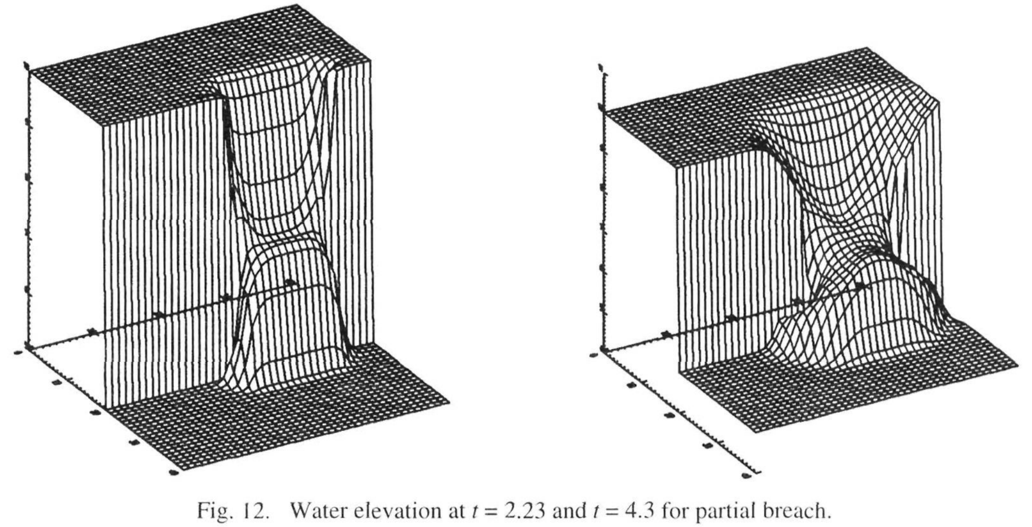

Fig. 12. Water elevation at $t = 2.23$ and $t = 4.3$ for partial breach.

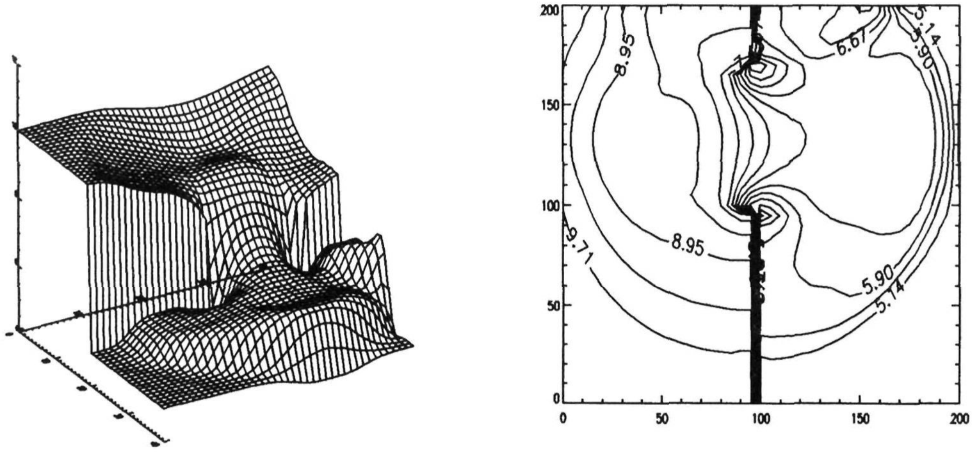

Fig. 13. Water elevation and contour plot of the depth at $t = 10.1$ for partial breach.

6 Conclusion

The high resolution scheme for the shallow water equations with rapidly varying flows is presented. Computational results for the one dimensional flows indicate that the method based on the TVD concept in conjuction with the ACM technique, provides a robust tool. However, in the case of the dry channel beds or when the downstream elevation takes very small values, error is negligible except in a small neighbourhood of $x/t = 2\sqrt{gh}$. This error is due to the change of the mathematical behavior: The coupling of equations, in the shallow water system, becomes strongly nonlinear, the strict hyperbolicity fails and the use of characteristic variables ceases to be correct.
The solutions show the high reliability and efficiency of the present scheme in solving 2-D free-surface flow problems.

Notations

E, F	Vectors of fluxes
S	Vector of source terms
U	Vector of conserved variables
u, v	Velocity components
h	The flow depth
h_1	Upstream water level
h_2	Downstream water level
x, y	Spacial coordinates
t	Time
g	Gravitational acceleration
q	Unit-width discharge
Δx	Spacial mesh
Δt	Time step
ϕ	Flux limiter
α	Courant number

G Artificial Compression Flux (ACF)

C_Δ Split operator

θ_j Automatic switch

L_x, L_y Split operators

A, B Jacobien matrices

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