ECE 310 Fall 2023

Lecture 1

Introduction to digital signals

Corey Snyder

Learning Objectives

After this lecture, you should be able to:

- Describe what defines a signal.
- Explain the difference between continuous-time and discrete-time signals.
- Understand the advantages of working with digital signals over analog signals.

1 What is a signal?

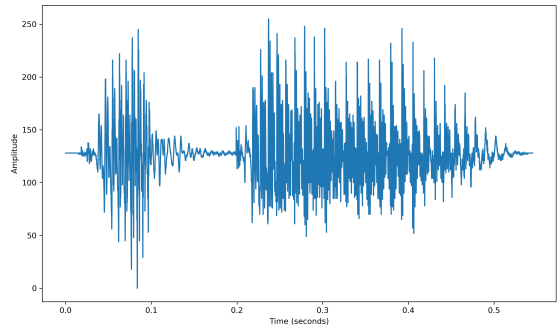
We may define a signal as any measurable quantity that varies over time, space, or any other variable that orders or organizes these measurements. With respect to time, we often talk about audio signals where we have the amount of acoustic vibration as a function of time. These acoustic vibrations convey the information we perceive as speech, music, or any other sound (Fig. 1a). We may also measure a non-physical quantity against time like stock prices or financial data (Fig. 1b).

The most common signals we observe in space are images where the intensity and distribution of electromagnetic wavelengths convey information. The organization for images is described by locations in space which could be (x, y, z) coordinates in a physical scene or (x, y) coordinates in a 2D image. The example image in Fig. 1c is an RGB (red, green, blue) image where at each integer-valued (x, y) coordinate, we have three color values to describe a single pixel amongst thousands of other measured values. We could enrich these visual signals further into videos that have sequences of images such that our information is described both spatially and temporally! Lastly, we can use a more abstract ordering for our signal using a graph described by a collection of edges and vertices. Each vertex can contain its own signal while the edges encode arbitrary relationships between vertices. For example, a transportation network like vehicle roadways is naturally described as a graph signal. We can measure quantities like number of vehicles, number of pedestrians, or weather at each intersection over time while adjacent intersections are connected by edges (Fig. 1d). Graph signals and the field of graph signal processing is beyond the scope of this course, but an emerging modern signal processing discipline worth mentioning!

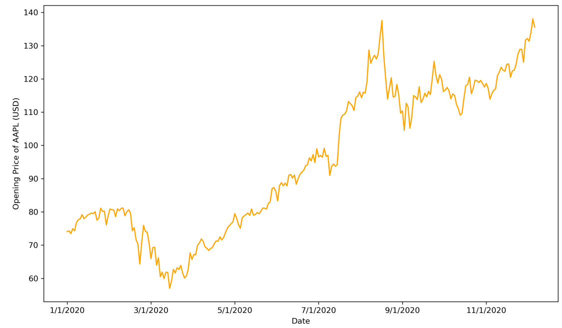
2 Continuous-time vs. discrete-time signals

In the previous section, we described various types of signals that vary with time or space. How we measure over time and space and how we measure these quantities is the key topic of this section. For conciseness, let's focus on audio signals for now.

A *continuous-time* signal is a signal that is (as the name suggests) observed continuously over time at infinitely and uncountably-many points in any interval. This means the signal or phenomenon is experienced for *any* possible value of time, e.g. t=0, 1, 3.10, 3.14159, etc. Conversely, a *discrete-time* signal is measured over regular intervals in time. We say that the time domain is *discretized* and thus we only observe the signal at integer values, e.g. $n = -1, 0, 1$, etc. Returning to audio signals, we as humans experience speech

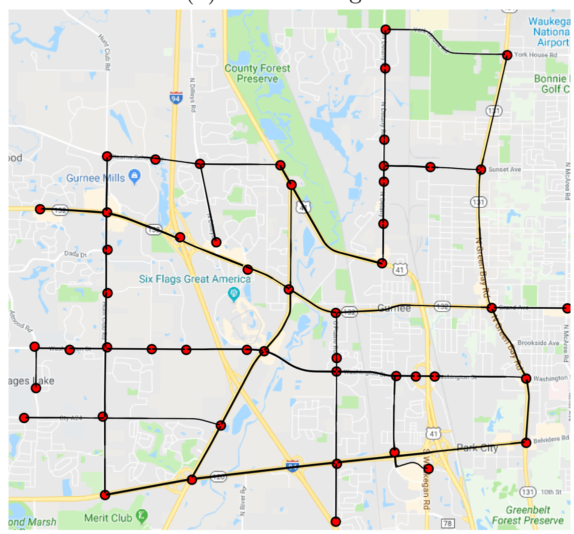


(a) "Goodbye" audio signal



(b) Financial signal

(c) Image signal

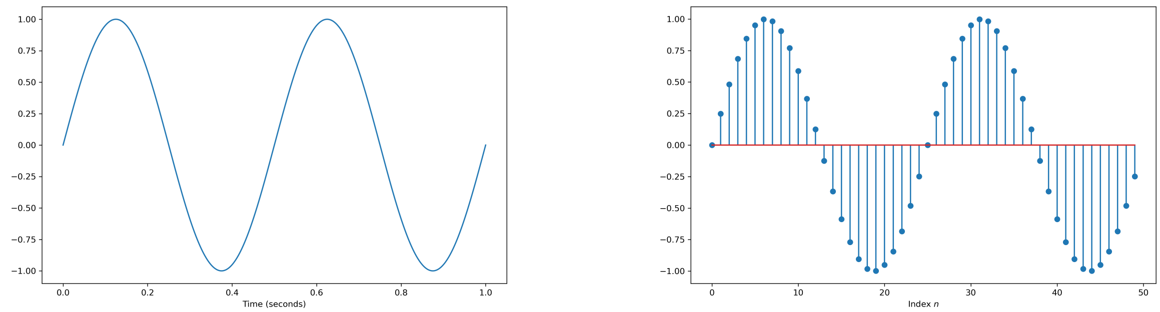


(d) Graph signal

Figure 1: Various example signals for different types of audio, visual, spatial, and non-physical data sources.

or music as continuous-time signals when acoustic vibrations reach our ears. However, when we record audio, we *sample* at some *sampling rate* that describes how often we capture a measurement of the audio. For example, music is often recorded at 44.1 kHz. This means every second 44,100 samples of the audio are collected. Alternatively, we receive a new sample every $\approx 2.2675 \times 10^{-5}$ seconds. In more standard signal processing terms, we would say our *sampling period* is $T = 2.2675 \times 10^{-5}$. Figure 2 depicts continuous and discrete-time representations of a given sinusoid. For a signal x, we typically denote it as being a continuous-time signal using parentheses and the variable $t \in \mathbb{R}$ for time: $x(t)$. Here, the symbol $\mathbb{R}$ denotes the continuous real line. A discrete-time signal is then identified using brackets and the variable $n \in \mathbb{Z}$ for our index, $x[n]$, where $\mathbb{Z}$ denotes the set of integers. With a given sampling period T, we can ideally sample a continuous-time signal to obtain a discrete-time signal: $x[n] = x(nT)$.

Next, we can differentiate between how we measure the actual quantity of the signal. Similar to continuous-time signals, we can also have *continuous-valued* signals that measure any possible continuous value for an observed quantity. We often refer to continuous-time, continuous-valued signals as *analog* signals. Conversely, we have *discrete-valued* signals that *quantize* measurements to a pre-defined set of values. We typically refer to discrete-time, discrete-valued signals as *digital* signals. Our digital computing systems operate with some fixed number of bits for computation, thus we cannot sample every possible continuous value and must quantize within some known range. In this course, we will treat the values of digital signals as being effectively continuous since standard floating point representations of numbers achieve very high precision for our purposes; however, we know that the signals we handle in computer code do not take on continuous values. We will talk much more about analog-to-digital, digital-to-analog conversion, and sampling later in the course.



(a) Continuous-time signal

(b) Discrete-time signal

Figure 2: Comparison of continuous-time and discrete-time representations of a sinusoidal signal.

3 Why use digital signals?

Finally, as we begin this course on digital signal processing, it is important we take a step back and ask "why work with digital signals instead of analog signals?". This may feel like an unnecessary question living in a world filled with digital devices in our pockets, on our wrists, and in front of you as you read these notes. But let's list some reasons to make the practical considerations clear:

1. **We can perform complex mathematical operations easier.** There are many different mathematical operations that are not only easier in digital electronics, but arguably only possible digitally (without incredible effort). Consider for example: multiplication, exponentiation, matrix operations, square roots, and many more.

2. **Digital electronics are more reliable.** Analog electronics are susceptible to changes due to environmental conditions and component aging. Digital electronics are highly repeatable, less prone to mistakes, and can easily implement error-correcting schemes for when mistakes are made.

3. **It is expensive to store analog signals.** Consider analog and digital audio storage, for example. One 12-inch vinyl record can hold a single album while a computer chip the size of your thumbnail can store tens of thousands of songs in your phone. Moreover, this same storage medium can also hold images, text data, memes, and any other information!

4. **Digital signal processing provides greater design flexibility.** Altering the design of an analog system requires ordering new physical parts, potentially allocating more space on a PCB, and running new simulations for any non-idealities of your system. Conversely, we only need to tweak some numbers or code to provide large, meaningful changes to our digital systems.

The above list is by no means exhaustive, nor do we mean to indicate analog signals are not important! We need analog signals for many applications ranging from power electronics to acoustics to radio frequency (RF) communication and beyond. Furthermore, *mixed signal* systems that blend digital and analog signals are prevalent. This course will focus on the fundamental techniques to represent and analyze discrete-time signals and to design and implement digital systems. The goal of this course is to demonstrate how digital signal processing provides a mathematical framework that supports diverse applications in audio engineering, image and video processing, data science, communications, machine learning, and much more.