Spring 2022 Project:
Dynamics of Bungee Jumping

https://www.booking.com/articles/best-places-in-the-world-to-bungee-jump.html

Objectives

- A person jumps from a cliff while attaching to a bungee cord
- How safe is the jump? How does the safety depend on the weight of the person, the spring constant and length of the bungee cord?
- Approach: modeling with spring - mass system in 3D
- Physics: pendulum, spring-mass, air frictional drag
- Three studies with 18 numerical experiments
  - Mass: Exp. 1 through 6
  - Spring constant: Exp. 7 through 12
  - Cord length: Exp. 13 through 18

Governing Equations

$$
\begin{aligned}
\frac{\partial U}{\partial t} &= -\frac{k}{m}\left(\frac{r-l}{r}\right)X - \frac{C_d \rho A V_{mag}}{2m}U, \\
\frac{\partial V}{\partial t} &= -\frac{k}{m}\left(\frac{r-l}{r}\right)Y - \frac{C_d \rho A V_{mag}}{2m}V, \\
\frac{\partial W}{\partial t} &= -\frac{k}{m}\left(\frac{r-l}{r}\right)Z - \frac{C_d \rho A V_{mag}}{2m}W - g, \\
\frac{\partial X}{\partial t} &= U, \\
\frac{\partial Y}{\partial t} &= V, \\
\frac{\partial Z}{\partial t} &= W,
\end{aligned}
$$

$$
\begin{aligned}
r &= \sqrt{X^2 + Y^2 + Z^2}, \\
V_{mag} &= \sqrt{U^2 + V^2 + W^2}.
\end{aligned}
$$

- Equation of motion includes elastic force in the cord, weight and air frictional drag.
- X, Y, Z: position of the persion in Cartesian coordinate
- U, V, W; velocity components in the x, y, z directions, respectively.

Physical Parameters

- m: Mass of the person in kilogram, to be varied in experiments
- k: Spring constant in Newton per meter, to be varied in experiments
- l: Length of the chord in meter, to be varied in experiments
- $C_d = 0.1$: Coefficient of drag due to air friction acted on the person
- $A = \pi$: Projected area of the person in flight in squared meter
- $\rho_a = 1.2$: Air density in kilogram per cubic meter
- $g = 9.81$: Gravity in meter per squared second

- $V_{mag} = \sqrt{U^2 + V^2 + W^2}$: Speed of the person in meter per second
- $Acc = dV_{mag}/dt$: Acceleration in meter per squared second
- $KE = 0.5mV_{mag}^2$: Kinetic energy in Joule

Euler-Cromer Method

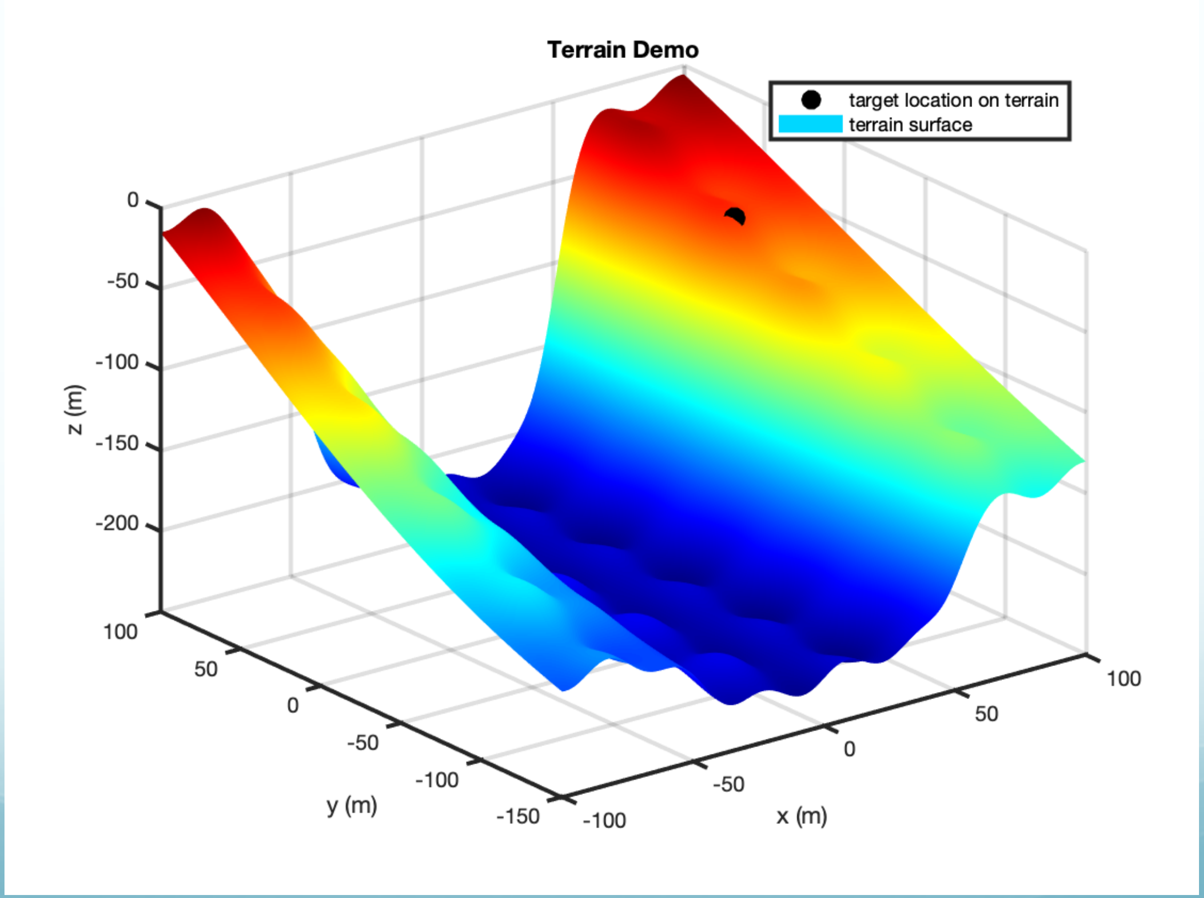
- Transform the differential equations into algebraic equations:

$$
\begin{aligned}
r_n &= \sqrt{X_n^2 + Y_n^2 + Z_n^2}, \\
V_{mag,n} &= \sqrt{U_n^2 + V_n^2 + W_n^2}, \\
U_{n+1} &= U_n - \left[\frac{k}{m}\left(\frac{r_n - l}{r_n}\right) X_n + \frac{C_d \rho A V_{mag,n}}{2m} U_n\right] \Delta t, \\
V_{n+1} &= V_n - \left[\frac{k}{m}\left(\frac{r_n - l}{r_n}\right) Y_n + \frac{C_d \rho A V_{mag,n}}{2m} V_n\right] \Delta t, \\
W_{n+1} &= W_n - \left[\frac{k}{m}\left(\frac{r_n - l}{r_n}\right) Z_n + \frac{C_d \rho A V_{mag,n}}{2m} W_n + g\right] \Delta t, \\
X_{n+1} &= X_n + U_{n+1} \Delta t, \\
Y_{n+1} &= Y_n + V_{n+1} \Delta t, \\
Z_{n+1} &= Z_n + W_{n+1} \Delta t, \\
T_{n+1} &= T_n + \Delta t,
\end{aligned}
$$

- March the governing equations forward in time with dt = 0.02 s **for a total of 120 s or until the person lands on terrain**.

Landing on Terrain

- File **terrain.mat** includes the x-y coordinate and the altitude of the terrain.
- See **terrain_demo.m** for illustration of **surf** and **interp2**



Files to download

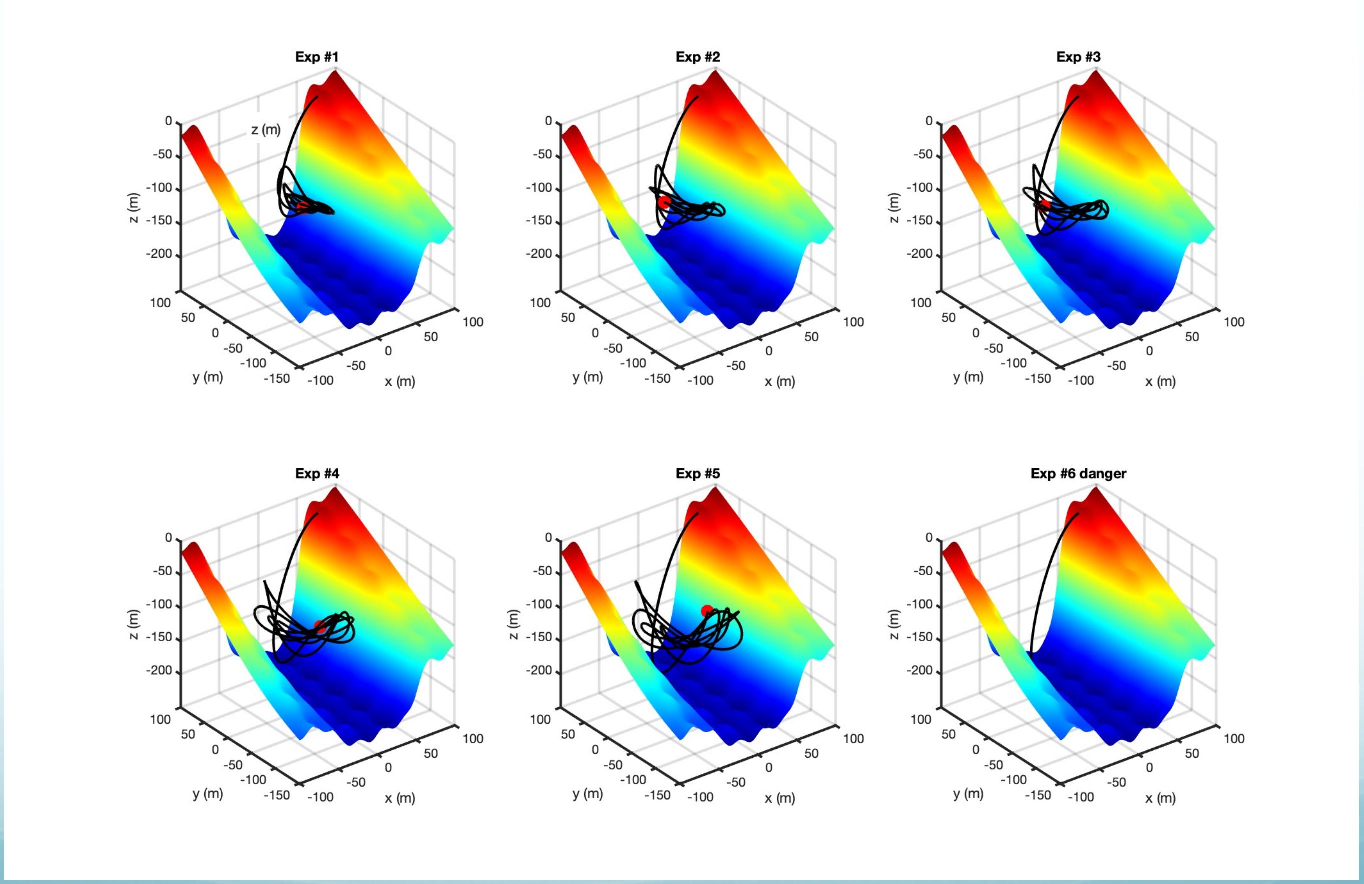
- File **bungee_data.txt:** includes parameters for the eighteen experiments
- File **terrain.mat:** includes terrain topography data.
- File **terrain_demo.m:** includes demo of surf and interp2.

Files to write

- Write **bungee.m** to solve for positions and velocities along the trajectories of the jump (T, X, Y, Z, U, V, W).
- Write **read_input.m** to read simulation parameters from the text file: **bungee_data.txt.** Use functions **importdata.**
- All project results are to be put in a script named **project.m**. Follow instructions on the posted project description.

Task 1: Figure 1

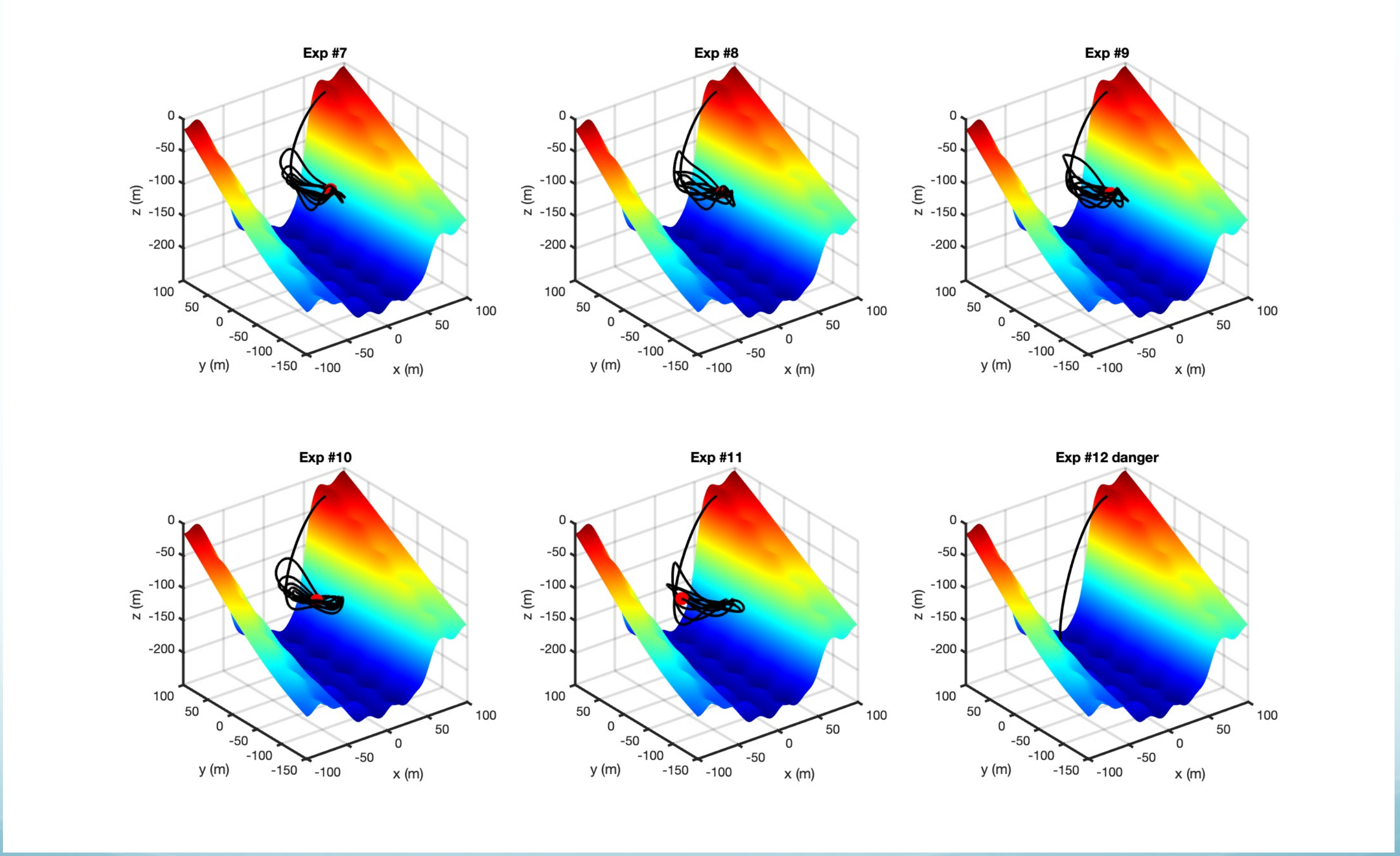
- Create **figure 1** to show the result of the mass study (Exp. #1 through 6). In each panel, show the trajectory, final position and the terrain. Use functions **plot3** and **surf**.



(The figure is for illustrative purpose)

Task 1: Figure 2

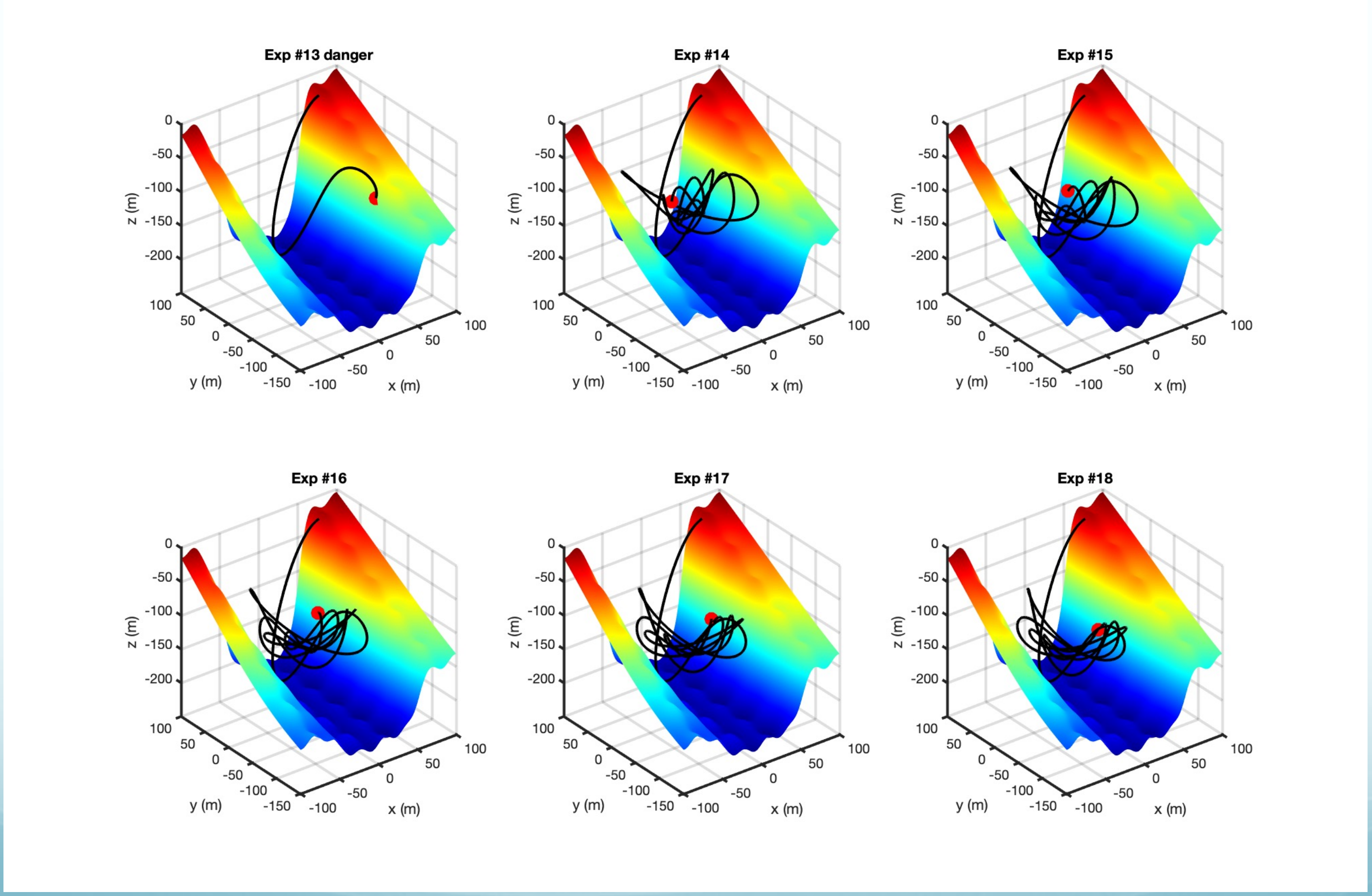
- Create **figure 2** to show the result of the spring constant study (Exp. #7 through 12). In each panel, show the trajectory, final position and the terrain.



(The figure is for illustrative purpose)

Task 1: Figure 3

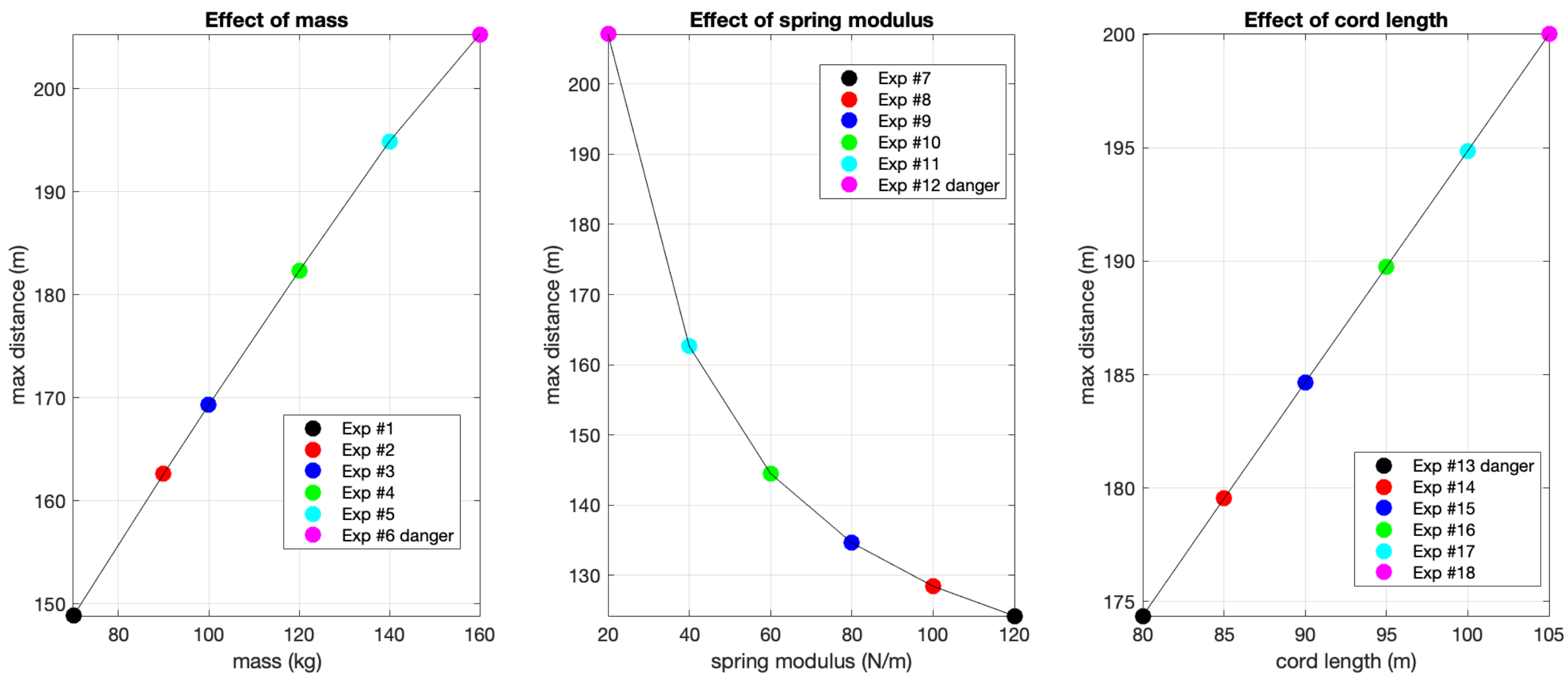
- Create **figure 3** to show the result of the <u>cord length study</u> (Exp. #13 through 18). In each panel, show the trajectory, final position and the terrain.



(The figure is for illustrative purpose)

Task 1: Figure 4

- Create **figure 4** to show how the maximum distance from the origin varies with mass, spring constant and cord length.



(The figure is for illustrative purpose)

Task 2: exp_res

- Create a data structure named **exp_res** which has 18 elements for the 18 experiments. The structure has the following fields:
  - **number**: experiment number
  - **max_speed**: maximum speed
  - **max_acceleration**: maximum acceleration
  - **integrated_KE**: time integrated kinetic energy
  - **travel_distance**: total travel distance along the trajectories

Task 3: report.txt

- Use function **fprintf** to create a text file named **report.txt** which includes the following:
  - Your name on the first line
  - Your PID on the second line
- The string**: 'exp number, max speed (m/s), max acc (m/s^2), int KE (J s), travel dist (m)'** on the third line
- Corresponding values of experiment number, maximum speed, maximum acceleration, integrated kinetic energy and travel distance for each of the eighteen experiment on the next eighteen lines.

Task 3: report.txt

report.txt

```
AAAAA AAAAA
A00000000
exp number, max speed (m/s), max acc (m/s^2), int KE (J s), travel dist (m)
         1    3.3400133e+01    8.5659617e+00    6.3710999e+05    1.2175989e+03
         2    3.5375753e+01    8.5268220e+00    1.0218838e+06    1.4007849e+03
         3    3.6205040e+01    8.5171991e+00    1.2491785e+06    1.4911453e+03
         4    3.7646468e+01    8.5078538e+00    1.7488854e+06    1.6413862e+03
         5    3.8879649e+01    8.5054426e+00    2.3361048e+06    1.7879980e+03
         6    3.9964793e+01    8.5066513e+00    5.5636083e+05    2.2011327e+02
         7    3.6323610e+01    1.0881769e+01    9.7301189e+05    1.3674191e+03
         8    3.5941136e+01    1.0337001e+01    9.8238327e+05    1.3825792e+03
         9    3.5290119e+01    9.7736006e+00    9.9147572e+05    1.3898828e+03
        10    3.4271379e+01    9.1786900e+00    1.0010814e+06    1.3937952e+03
        11    3.5375753e+01    8.5268220e+00    1.0218838e+06    1.4007849e+03
        12    3.8123099e+01    7.7732627e+00    3.0067690e+05    2.2087815e+02
        13    3.8382117e+01    9.2819438e+00    8.3344957e+05    4.4395412e+02
        14    3.8446325e+01    8.6226987e+00    2.2038868e+06    1.7466001e+03
        15    3.8549172e+01    8.0360805e+00    2.2360883e+06    1.7548282e+03
        16    3.8691938e+01    7.9882338e+00    2.2812231e+06    1.7692110e+03
        17    3.8879649e+01    8.5054426e+00    2.3361048e+06    1.7879980e+03
        18    3.9119560e+01    9.0364252e+00    2.3964844e+06    1.8058769e+03
```

(The figure is for illustrative purpose)

Project Submission

- All files related to project must be included in an archive named **project.zip**
- Submit the archive in CANVAS before 10 PM on Sunday June 5th 2022. The zip file must contain the following:

1. File **project.m** to include answers to all the tasks.
2. Files **read_input.m, bungee_data.txt** and **terrain.mat**
3. The **project.m** must plot the four figures and generate **report.txt**.
4. Any other scripts or functions that you have written so that your **project.m** will be executed properly.

Suggested Schedule

- Start with one trajectory: simulate, make figures, and analyze.
- Function **read_input.m** and **bungee.m** should be operational by the end of week 9.
- Get results of all 18 trajectories, plot the three figures, construct structure **exp_res**, and create **report.txt** during week 10.
- The project must be submitted in CANVAS before deadline. Late submission will be deducted 20% of the total project grade per day (24 hours).
- <u>Any syntax error will result in zero credit</u>.
- <u>Total run time must be less than 120 s.</u>