MAE 8 - Spring 2022
Homework 3

Instructions: Follow the homework solution template. Put all answers in a MATLAB script named **hw3.m**. Make sure the two figures are displayed on screen when your homework script is executed. Submit **hw3.m** in CANVAS before 10 PM on 04/22/2022. Use double precision unless otherwise stated.

Problem 1: Consider the linear system of equations below:

$$
\begin{aligned}
5x_1 - 2x_2 &= -2 & -2x_{10} + 5x_{11} - 2x_{12} &= 0 & -2x_{20} + 5x_{21} - 2x_{22} &= 0 \\
-2x_1 + 5x_2 - 2x_3 &= 0 & -2x_{11} + 5x_{12} - 2x_{13} &= 0 & -2x_{21} + 5x_{22} - 2x_{23} &= 0 \\
-2x_2 + 5x_3 - 2x_4 &= 0 & -2x_{12} + 5x_{13} - 2x_{14} &= 0 & -2x_{22} + 5x_{23} - 2x_{24} &= 0 \\
-2x_3 + 5x_4 - 2x_5 &= 0 & -2x_{13} + 5x_{14} - 2x_{15} &= 0 & -2x_{23} + 5x_{24} - 2x_{25} &= 0 \\
-2x_4 + 5x_5 - 2x_6 &= 0 & -2x_{14} + 5x_{15} - 2x_{16} &= 0 & -2x_{24} + 5x_{25} - 2x_{26} &= 0 \\
-2x_5 + 5x_6 - 2x_7 &= 0 & -2x_{15} + 5x_{16} - 2x_{17} &= 0 & -2x_{25} + 5x_{26} - 2x_{27} &= 0 \\
-2x_6 + 5x_7 - 2x_8 &= 0 & -2x_{16} + 5x_{17} - 2x_{18} &= 0 & -2x_{26} + 5x_{27} - 2x_{28} &= 0 \\
-2x_7 + 5x_8 - 2x_9 &= 0 & -2x_{17} + 5x_{18} - 2x_{19} &= 0 & -2x_{27} + 5x_{28} - 2x_{29} &= 0 \\
-2x_8 + 5x_9 - 2x_{10} &= 0 & -2x_{18} + 5x_{19} - 2x_{20} &= 0 & -2x_{28} + 5x_{29} - 2x_{30} &= 0 \\
-2x_9 + 5x_{10} - 2x_{11} &= 0 & -2x_{19} + 5x_{20} - 2x_{21} &= 0 & -2x_{29} + 5x_{30} &= 1
\end{aligned}
$$

Use blackslash (\) operator to solve the system of equations, $\mathbf{Ax} = \mathbf{b}$, for unknown **x's** and put the answer in **p1**. The coefficient matrix **A** has a dimension of 30 rows by 30 columns. All elements are zero except for those on the diagonal, subdiagonal and superdiagonal. To create **A**, first create a 30×30 matrix with all zero and then modify the non-zero elements using linear indexing. Another method is to explore function **diag**.

Problem 2: Use forward difference formula to estimate the derivative of the function $f(x) = tanh^4(0.5x)\, e^{-sin^2(x)}$ by performing the following tasks:

(a) Create a vector **x** that has values from 0 to 10 with a step of 0.1 (in radians). Set **p2a=x**.

(b) Compute **f** and put the answer in **p2b**.

(c) Use function **diff** to estimate the derivative f'. Put the answer in **p2c**.

(d) What is the value of f' at x = 5? Put the answer in **p2d**.

Problem 3: Use the trapezoid method discussed in class to estimate the following integral: $\int_{-5}^{5} g(z)\, dz$ where $g(z) = sech^2(z) sin^4(4z)$ by performing the following exercises.

(a) Create a vector **z** that has values from -5 to 5 with a step of 0.1 (in radians). Set **p3a=z**.

(b) Compute **g** and set **p3b=g**.

(c) Use function **sum** to approximate the integral. Put the answer in **p3c**.

Problem 4: Create 2-dimensional matrix **matA** using the following commands: **matA = 1:100; matA = abs(fix(100*cos(matA))); matA = reshape(matA,10,10)**. Perform the following exercises with **matA**:

a) Set **p4a** to be equal to **matA**. Use logical indexing to replace the maximum values of each column in matrix **p4a** with **-1**.

b) Set **p4b** to be equal to **matA**. Use logical indexing to replace the maximum values of the entire matrix **p4b** with **-2**.

c) Use function **isprime** to check whether any element in **matA** is a prime number. Put the answer in a logical number **p4c**.

d) Identify the elements in **matA** that are the prime numbers. Use linear indexing to report the answer in a column vector **p4d**.

Problem 5: The built-in function **clock** returns a row vector that contains 6 elements: the first three are the current date (year, month, day) and the last three represent the current time in hours (24 hour clock), minutes, and seconds. The seconds is a real number, but all others are integers. Use function **sprintf** to accomplish the following formatting exercises.

a) Get the current date and time and store them in **p5a**. The current date and time should be the date and time when your homework script is executed while being graded.

b) Using the format 'YYYY:MM:DD', write the current date to string **p5b**. Here, YYYY, MM, and DD correspond to 4-digit year, 2-digit month, and 2-digit day, respectively.

c) Using the format 'HH:MM:SS.SSSS', write the current time to string **p5c**. Here, HH, MM, and SS.SSSS correspond to 2-digit hour, 2-digit minute and 7-character second (2 digits before the decimal point and 4 digits after the decimal points), respectively.

d) Remove the last 5 characters from the string in part (c) so that the format is now 'HH:MM:SS'. Put the answer into string **p5d**.

e) Combine the strings in part (b) and part(d) together separated by a single space. Put the answer in string **p5e**.

Problem 6: Consider the following two-dimensional parametric curve:

$$\begin{aligned} x &= 16\, sin^3(\theta) \\ y &= 13\, cos(\theta) - 5\, cos(2\theta) - 2\, cos(3\theta) - cos(4\theta); \end{aligned}$$

for $1 \le \theta \le 360°$.

(a) Create a vector **theta** to include values from 1 to 360 with a consecutive difference of 1. Use the expressions above to obtain the values for vectors **x** and **y**. Create **figure 1** and use function **plot** with the vectors **x** and **y** to plot the curve. The figure needs to include the following items:

- Use red solid line with a line width of 5 to mark the curve.
- Use solid cyan diamond symbol to mark the last data point on the curve. Use a marker size of 30 for the symbol.

- Remember to provide axis labels, legend and figure title.

Set **p6a = 'See figure 1'**.

(b) Estimate the arc length of the curve. Approximate the arc length with straight lines between consecutive points. Put the answer in **p6b**.

Problem 7: Consider the following three-dimensional parametric curve:

$$\begin{aligned} x &= [1 + 0.25\,cos(50\theta)]\,cos(\theta) \\ y &= [1 + 0.25\,cos(50\theta)]\,sin(\theta) \\ z &= \frac{\pi\theta}{180} + 2sin(50\theta) \end{aligned}$$

for $0 \leq \theta \leq 1200°$.

(a) Create a vector **theta** to include values from 0 to 1200 with a consecutive difference of 0.5. Use the expressions above to obtain the values for vectors **x**, **y** and **z**. Estimate the arc length of the three-dimensional curve. Approximate the arc length with straight lines between consecutive points. Put the answer in **p7a**.

(b) Identify the data point at which the distance along the curve from the first data point is equal to 500. Put the location of the data point (x, y, and z coordinates) into a 3-element row vector **p7b**.

(c) Create **figure 2** and use function **plot3** with the vectors **x**, **y** and **z** to plot the curve. The figure needs to include the following items:

- Use magenta solid line with a line width of 0.5 to mark the curve.
- Use solid black circle symbol to mark the first data point on the curve. Use a marker size of 10 for the symbol.
- Use solid red triangle symbol to mark the last data point on the curve. Use a marker size of 10 for the symbol.
- Use solid blue square symbol to mark the data point found in part (b). Also use a marker size of 10 for the symbol.
- Remember to provide axis labels, legend and figure title.
- Include **view(3)** to put the figure in three-dimensional view.

Set **p7c = 'See figure 2'**.