MAE 8 - Spring 2022
Homework 7

Instructions: Follow the homework solution template. Put all answers in a MATLAB script named **hw7.m**. For this homework, you will need to submit multiple files. Create a zip archive named **hw7.zip**. The zip archive should include the following files: **hw7.m**, **terrain.mat**, **matB.mat**, **car.m** and **rocket.m**. Make sure the three figures are plotted on screen when the script is executed. Submit **hw7.zip** in CANVAS before 10 PM on 5/22/2020. Use double precision unless otherwise stated.

Problem 1: Consider the topography of a terrain stored in the file **terrain.mat**. Download the file from CANVAS and load it into MATLAB. The file contains the spatial coordinates of the terrain in vectors **x** and **y** and the altitude in a 2-dimensional matrix **altitude**. The coordinates and the altitude are given in the unit of meters.

(a - d) How many peaks (i.e. local maxima) are there on the terrain? Put the answer in **p1a**. Exclude the peaks on the boundaries of the terrain. Find the x- and y- coordinates and the altitude of the peaks and put the answers in **p1b**, **p1c** and **p1d**, respectively.

(e - g) Snow falls at elevations above 1,100 m. Find the x- and y-coordinates and the altitude of locations (points) on the terrain with snow cover and put the answers in **p1e**, **p1f** and **p1g**, respectively.

(h) Make figure 1 to include the following items:

- Use function **surf** to plot the terrain. Use **shading interp** to make the surface plot smooth. Set **view(3)** to put the plot in three-dimensional view.
- Use red filled circles with a marker size of 10 to identify the peaks.
- Use green filled circles with a marker size of 4 to identify the snow cover.
- Extra credit (5 points): A ball is initially released on the terrain at the coordinates (x = 8 km, y = -8 km). Assume that the ball follows a path with steepest slope. Use magenta solid line with a line width of 4 to mark the descent of the ball.

Be sure to label axes with correct units, provide a title and include a legend box. Set **p1h = 'See figure 1'**. If attempt the extra credit, set **p1i = 'See ball trajectory'**.

Problem 2:

Download the file **matB.mat** from CANVAS. The file contains a two-dimensional matrix **matB**. Use **break** or **continue** statements to perform the following exercises with **matB**.

(a) Sum all elements that are above the diagonal. Put the answer in **p2a**.

(b) Find the product of all elements that are below the diagonal. Put the answer in **p2b**.

(c) Sum all elements in **matB**. Exclude the elements whose column index is twice as large as their row index. Put the answer in **p2c**.

(d) Find the product of all elements in **matB**. Exclude the elements whose value is larger than their row index. Put the answer in **p2d**.

Problem 3: A car, initially at rest, accelerates in x direction with the following expression:

$$a(t) = 5\, sech^2(\frac{t}{20})\, tanh(\frac{t}{20}),$$

where **a** is acceleration and t is time. In this exercise, you are to explore the motion of the car numerically using Euler method. The motion is described by the following differential equations:

$$\begin{aligned} \frac{du}{dt} &= 5\, sech^2(\frac{t}{20})\, tanh(\frac{t}{20}), \\ \frac{dx}{dt} &= u, \end{aligned}$$

where **u** is velocity. Using Euler method and capital letters to denote the approximation, the equations can be approximated by

$$\begin{aligned} U_{n+1} &= U_n + 5\, sech^2(\frac{T_n}{20})\, tanh(\frac{T_n}{20})\Delta t, \\ X_{n+1} &= X_n + U_n \Delta t, \end{aligned}$$

where subscript n denotes variables at current time, subscript n+1 denotes variables at a time that is Δt ahead.

Write function **car.m** to numerically solve for the motion of the car. The function should have the following header: **function [T, X, U] = car(Tf, dt)**. The inputs are total traveling time **Tf** and the time step **dt**. The outputs vectors **T**, **X** and **U** are the time, distance and velocity of the car, respectively. Give the function a description.

(a) Set **p3a=evalc('help car')**.

(b, c, d) Use function **car** to get the time, distance, and velocity of the car for $T_f = 60$ s. Put the answers to **p3b**, **p3c**, and **p3d**, respectively. Use 10-second time step.

(e, f, g) Repeat the step above with 1-second time step. Put the time, distance, and velocity into **p3e**, **p3f**, and **p3g**, respectively.

(h) Create **figure 2**. Use function **subplot** to include 2 panels with one on top of the other. Plot distance versus time in the top panel. The top panel should include 2 curves: the 10-second time step in parts (b, c) and the 1-second time step in parts (e, f). Plot velocity versus time in the bottom panel. The bottom panel should also include 2 curves: the 10-second time step in parts (b, d) and the 1-second time step in parts (e, g). Use solid lines with different filled symbols for the curves. Set **p3h='See figure 2'**.

Problem 4: A model rocket (with mass **m** = 10 kg) initially at rest on the ground (Zo = 0) is launched upward. The motion of the rocket is described by the following differential equations:

$$\begin{aligned} \frac{\partial w}{\partial t} &= -g + \frac{Th}{m}, \\ \frac{\partial z}{\partial t} &= w, \end{aligned}$$

where **t** is time, **z** is altitude, **w** is velocity, and Th is the upward thrust due to the propulsion of the rocket and **g** is gravity. In this exercise, the upward thrust **Th** varies with time as follows:

$$\begin{aligned} Th(t) &= 670N \quad for\, 0 \leq t < 2(s), \\ Th(t) &= 1360N \quad for\, 2 \leq t < 4(s), \\ Th(t) &= 0N \qquad for\, t \geq 4(s), \end{aligned}$$

and the gravity **g** varies with altitude **z** as follows:

$$\begin{aligned} g(z) &= 9.81 \left[1 - \left(\frac{z}{10000} \right)^3 \right] (m/s^2) \;\; for\, 0 \leq z < 10,000\, (m), \\ g &= 0 \; (m/s^2)\, for\, z \geq 10,000\, (m), \end{aligned}$$

Using Euler-Cromer method, the differential equations can be approximated by

$$\begin{aligned} W_{n+1} &= W_n + \left(-g_n + \frac{Th_n}{m} \right) \Delta t, \\ Z_{n+1} &= Z_n + W_{n+1} \Delta t, \end{aligned}$$

where Δt is the time step and capital letters are used to denote approximated quantities.

In this exercise, you will create a file **rocket.m** to include 3 functions. The primary function should be named **rocket** and two subfunctions should be named **gravity** and **thrust**.

The **rocket** function solves the governing equations for the projectile motion of the rocket. It should have the following header: **function [T, Z, W] = rocket(Tf, dt)** where the input **Tf** is the duration of the flight of the rocket and **dt** is the time step. The output vectors **T**, **Z** and **W** are the time, altitude and velocity of the rocket, respectively.

The **thrust** subfunction computes the thrust at a given time during the flight. It should have the following header: **function [Th] = thrust(t)** where the input **t** is the time and the output **Th** is the upward thrust given above. Both input and output are single numbers.

The **gravity** subfunction computes the value of gravity at a given time during the flight. It should have the following header: **function [g] = gravity(z)** where the input **z** is the altitude of the rocket and the output **g** is the altitude-dependent gravity given above. Both input and output are single numbers.

Give all functions a description.

(a) Set **p4a = evalc('help rocket')**.

(b) Set **p4b = evalc('help gravity')**.

(c) Set **p4c = evalc('help thrust')**.

(d) Set **p4d = evalc('help rocket>gravity')**.

(e) Set **p4e = evalc('help rocket>thrust')**.

(f,g) Compute the altitude and velocity of the rocket after 120 s using $\Delta t = 0.1s$. Put the answers in **p4f** and **p4g**, respectively. The answers should be single numbers, not vectors.

(h) Create **figure 3**. Use function **subplot** to include 2 panels with one on top of the other. The top panel shows how the altitude of the rocket changes with time during the 120-second flight. The bottom panel shows velocity versus time. Set **p4h = 'See figure 3'**.