(1)

Non-dimensional numbers

Let's manipulate LHS & RHS to arrive at a N-D representation

$$\frac{h_x x}{K} = \boxed{Nu_x = 0.332\, Pr^{1/3}\, Re_x^{1/2}} \qquad (Pr > 0.6)$$

Local Nusselt Number: $\dfrac{\text{convection in fluid}}{\text{conduction in fluid}} = \dfrac{L/K_f}{1/h} = \dfrac{\text{cond. resistance in fluid}}{\text{conv. resistance}}$

(is it Biot no.?)

⇓

Advantage of this representation is that, to test this relation, we only need to vary Pr & Re_x (~10 x 10 data points)

$Pr \to \dfrac{\nu}{\alpha}$ $\qquad Re_x \to \dfrac{U_\infty x}{\nu}$

⇓

In experiments, this means either

1. vary α and vary U_∞ ✓

or

2. vary α and vary ν ✗ (two N-D in RHS vary)

or

3. vary α and vary x ✗ (LHS ND also varies)

option 1 is the most convinient route to test the above numerical result numerically.

Most exp. use $Nu = \text{Const} \times Re_L^m\, Pr^n$ & calculate these unknowns.

Let us return to understanding results from analysis of T.B.L

↓

Further numerical results reveal that

$$\boxed{\delta_t = \frac{\delta}{P_r^{1/3}} = \frac{4.91x}{Re_x^{1/2}\,P_r^{1/3}}}$$

All of these are only valid for laminar regime

↓

(low to moderate Re_x)

& $P_r > 0.6$

* Average H.T coefficient

We generally use average: $\bar{h} = \frac{1}{L}\int_0^L h(x)\,dx$

$$\overline{Nu} = \frac{\bar{h}L}{K_f} = f(Re_L, P_r)$$

Laminar Flow over plate : $Nu_x = 0.332\,P_r^{1/3}\,Re_x^{1/2}$

$$\overline{Nu} = 0.664\,P_r^{1/3}\,Re_L^{1/2}$$

↓

$Re < 10^5$

$P_r > 0.6$

"Airfoil"

Q. You are performing H.T experiment on a cylinder of $L = 0.15\,m$

$\rightarrow 100\ m/s$ $T_\infty = 37^\circ C$ $T_s = 325^\circ C$ $(R_{cyl} \gg \delta_v \gg \delta_t)$

$\dot{Q} = 1600\ J$ is lost every second.

As a lazy experimentalist, how will you calculate $\dot{Q}$ for

$L = 0.30\,m$ $\rightarrow 50\ m/s$ $T_\infty = 37^\circ C$ $T_s = 425^\circ C$

$\dot{Q} = ?$

Ans. $\dot{Q} = ? = \bar{h} A_s (T_s - T_\infty)$

Expt 1. $\bar{h} A_s = \dfrac{1600}{325-37} = 5.56\ W/K$

$\Downarrow$

Then using this $\bar{h} A_s$ for Exp 2. will give wrong result because $\bar{h}$ is different.

$\Downarrow$

Do we know h & U_∞ relation? $h_x = 0.332\, P_r^{1/3}\, K_f \sqrt{\dfrac{U_\infty \rho}{\mu x}}$

$\downarrow$ average

$\bar{h} = 0.664\, P_r^{1/3}\, K_f \sqrt{\dfrac{U_\infty \rho}{\mu L}}$?

Exp 1. [5.56 ? ? ?

Exp 2. [? $\bar{h}_2 = 0.664\, P_r^{1/3}\, K_f \sqrt{\dfrac{U_{\infty_2} \rho}{M L_2}}$

Unknown But same as Exp. 1

$\Downarrow$

$$0.664 \left(P_r^{1/3} K_f \sqrt{\frac{\rho}{M}} \right) = \bar{h}_1 \sqrt{\frac{L_1}{U_{\infty_1}}} = \bar{h}_2 \sqrt{\frac{L_2}{U_{\infty_2}}}$$

$\Downarrow$

$$\bar{h}_2 = \bar{h}_1 \times \sqrt{\frac{L_1}{L_2} \times \frac{U_{\infty 2}}{U_{\infty 1}}}$$

?

Ans. $\bar{\dot{Q}} = ? = \bar{h} A_s (T_s - T_\infty)$

Exp 1. $\bar{h} A_s = \dfrac{1600}{325-37} = 5.56 \ W/K$

$\Downarrow$

Then using this $\bar{h} A_s$ for Exp 2. will give wrong result because $\bar{h}$ is different.

$\Downarrow$

Do we know h & U_∞ relation? $h_x = 0.332 \ Pr^{1/3} K_f \sqrt{\dfrac{U_\infty \rho}{\mu x}}$

$\downarrow$ average

$$\bar{h} = 0.664 \ Pr^{1/3} K_f \sqrt{\frac{U_\infty \rho}{\mu L}}$$

Exp 1. [5.56 ? ? ? ?

Exp 2. [? $\bar{h}_2 = 0.664 \ Pr^{1/3} K_f \sqrt{\dfrac{U_{\infty_2} \rho}{\mu L_2}}$

Fluid prop. identical to Exp. 1

$\Downarrow$

$$0.664 \left(Pr^{1/3} K_f \sqrt{\frac{\rho}{\mu}} \right) = \bar{h}_1 \sqrt{\frac{L_1}{U_{\infty_1}}} = \bar{h}_2 \sqrt{\frac{L_2}{U_{\infty_2}}}$$

$\Downarrow$

$$\bar{h}_2 = \underset{?}{\bar{h}_1} \times \sqrt{\frac{L_1}{L_2} \times \frac{U_{\infty_2}}{U_{\infty_1}}}$$

$$\bar{h}_2 A_1 = \bar{h}_1 A_1 \times \sqrt{\frac{1}{4}} \Rightarrow \bar{h}_2 A_1 = \frac{\bar{h}_1 A_1}{2}$$

$\Downarrow$

$$\bar{h}_2 \underbrace{\frac{A_1}{A_2}}_{0.5} A_2 = \frac{\bar{h}_1 A_1}{2}$$

$\Downarrow$

$$\bar{h}_2 A_2 = \bar{h}_1 A_1$$

$\Downarrow$

$$\dot{Q}_2 = \bar{h}_2 A_2 (T_s^2 - T_\infty)$$

$$\bar{h}_2 A_1 = \bar{h}_1 \bar{A}_1 \times \sqrt{\frac{1}{4}} \Rightarrow \bar{h}_2 A_1 = \frac{\bar{h}_1 \bar{A}_1}{2}$$

$$\Downarrow$$

$$\bar{h}_2 \underbrace{\frac{A_1}{A_2}}_{0.5} A_2 = \frac{\bar{h}_1 \bar{A}_1}{2}$$

$$\Downarrow$$

$$\bar{h}_2 A_2 = \bar{h}_1 A_1$$

$$\Downarrow$$

$$\dot{Q}_2 = \bar{h}_2 A_2 (T_s^2 - T_\infty) \checkmark$$

Alternative: Work with Non-dim. relation

$$\bar{h} = 0.664\, Pr^{1/3} K_f \sqrt{\frac{\rho U_\infty}{\mu L}} \Rightarrow \overline{Nu} = 0.664\, Pr^{1/3} Re_L^{1/2}$$

Now, what is changed in Exp.: Re_L? No. $\left(\frac{U_\infty}{L}\right)_1 = \left(\frac{U_\infty}{L}\right)_2$

Pr? obviously not

$$\Downarrow$$

$$(\overline{Nu})_1 = (\overline{Nu})_2$$

$$\Downarrow$$

$$\frac{\bar{h}_1 L_1}{K_f} = \frac{\bar{h}_2 L_2}{K_f} \Rightarrow \bar{h}_2 = \frac{\bar{h}_1 L_1}{L_2}$$

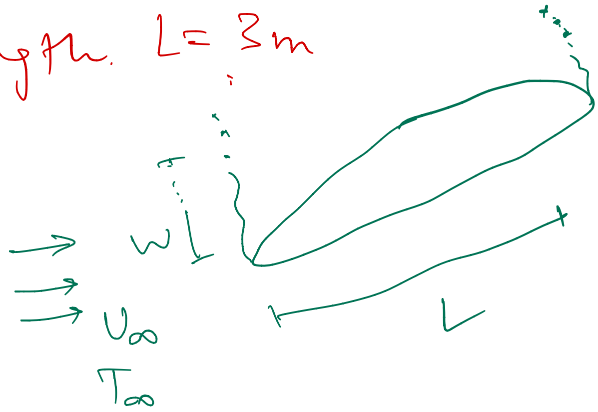
$$\dot{Q}_2 = \bar{h}_2 A_2 (T_{s_2} - T_\infty) \times \frac{A_1}{A_1}$$

$$\dot{Q}_2 = [\bar{h}_1 A_1] \frac{L_1}{L_2} \times \frac{A_2}{A_1} (T_{s_2} - T_\infty) = [\bar{h}_1 A_1] (T_{s_2} - T_\infty)$$

Q. A large object of char. length $L = 3\,m$ & width $= 2\,m$.

$U_\infty = 7\,m/s$

$T_\infty = 20^\circ c$

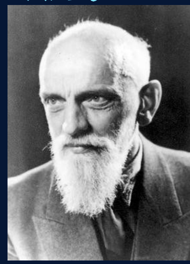


Total drag force on plate is 1 N. What is $\bar{h}$?

What's the relation ?

Ans

Blasius



* Mom. & Heat Transfer Analogies

Consider the following Non-dim. G.E^s

prime denotes Non-dim.

Mom$_x$ Transfer : $$u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = \frac{1}{Re_L}\frac{L^2}{\delta_v^2}\frac{\partial^2 u'}{\partial y'^2}$$

$\Downarrow$ using same char. scales as before (only $y_{ch} = \delta_t$ instead of δ_v)

Energy Transfer : $$u'\frac{\partial \theta'}{\partial x'} + v'\frac{\partial \theta'}{\partial y'} = \frac{1}{Re_L Pr}\frac{L^2}{\delta_t^2}\frac{\partial^2 \theta'}{\partial y'^2}$$

How? H.W

You'll get : $\frac{\alpha}{U_\infty L}$

$\Downarrow$

If $Pr = 1$

$\Downarrow$

$\delta_t = \delta_v$ & The governing Eq^n (and B.C) are identical for mom. & Energy transfer

$\Downarrow$

$$\left.\frac{\partial u'}{\partial y'}\right|_{y'=0} = \left.\frac{\partial \theta'}{\partial y'}\right|_{y'=0}$$

$\Downarrow$

We found earlier:

$$T_w = M\left.\frac{du}{dy}\right|_{y'=0} = C_f\left(\frac{\rho U_\infty^2}{2}\right)$$

C_f: skin friction coeff.

$0.664\, Re_x^{-1/2}$

Dim. less.

$$\frac{du'}{dy'} = \frac{C_f}{2}\frac{\rho U_\infty \delta}{M}$$

$\Downarrow$

$$\left.\frac{\partial \theta'}{\partial y'}\right|_{y'=0} = 0.332\, Pr^{1/3}\sqrt{\frac{U_\infty \rho}{M x}}$$

Non-dim.

$$Nu_x = \frac{h x}{K_f} = \left[\frac{-K_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}\right]\frac{x}{K_f} = +\left(\left.\frac{\partial \theta'}{\partial y'}\right|_{y'=0}\right)\frac{x}{\delta}$$

$\Downarrow$

$$\left.\frac{\partial \theta'}{\partial y'}\right|_{y'=0} = Nu_x\left(\frac{\delta}{x}\right)$$

$$Nu_x = 0.332\, Pr^{1/3}\, Re^{1/2}$$

$$\Downarrow$$

$$\frac{C_f}{2}\left(\frac{\rho U_\infty}{\mu}\right)\delta = Nu_x\left(\frac{\delta}{x}\right)$$

$$\Downarrow$$

$$\boxed{C_f \frac{Re_x}{2} = Nu_x} \rightarrow$$

Analogy-I :

Reynolds Analogy
(for $Pr \simeq 1$)

Advantages :

⊕ No need to conduct exp. to vary thermal properties.
Only the knowledge of C_f is enough

Disadvantages :

⊖ Only works for $Pr \simeq 1$

⊖ Implicit assumption of Const. U_∞

⊖ Laminar B.L

Chilton-Colburn Analogy-II

Arbitrary Pr.

$$C_{f,x} = 0.664\, Re_x^{-1/2} \quad \& \quad Nu_x = 0.332\, Pr^{1/3}\, Re_x^{1/2}$$

$\Downarrow$ Divide $\Downarrow$

$$\boxed{C_{f,x}\frac{Re_x}{2} = Nu_x\, Pr^{-1/3}}$$

Or use averaged versions:

$$\bar{C}_f = 1.32\, Re_L^{-1/2}, \quad \overline{Nu} = 0.664\, Pr^{1/3} Re_L^{1/2} \Rightarrow \boxed{\frac{\bar{C}_f Re_L}{2} = \overline{Nu}\, Pr^{-1/3}}$$

$\bar{h}L/k$

Advantages

(+) Pr is arbitrary (Pr > 0.6 though)

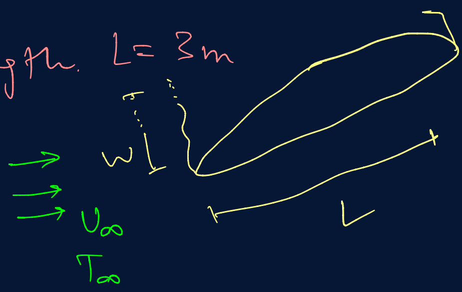
(+) Surprisingly, works for Turbulent flows

Disadvantages

(−) U_∞ still needs to const. so that B-L Eqⁿ (that it's based on) are valid

Let's Return to our example:

Q. A large object of char. length $L = 3\,m$ & width $= 2\,m$.



$U_\infty = 7\,m/s$

$T_\infty = 20^\circ C$

Total drag force on plate is 1 N. What is $\bar{h}$?

Ans

Allen Colburn

We can now solve this by Colburn's blessing:

["Give me info. on mom. transfer, I'll give you info. on heat transfer"]

$$\left[\frac{\overline{C_f}\, Re_L}{2} = \overline{Nu}\, Pr^{-1/3}\right]$$

Recall: $\tau_w = C_f \dfrac{\rho U_\infty^2}{2}$ → Units? $\dfrac{kg}{m\text{-}s^2} = \dfrac{N}{m^2}$

Here, $F_{total} = \tau_w A_s = \overline{C_f}\, \dfrac{\rho U_\infty^2}{2} A_s$

$$\left(\frac{F_{total}}{\rho U_\infty^2 A_s}\right) \times Re_L = \underbrace{\overline{Nu}}_{\text{Find } \bar{h}}\, Pr^{-1/3}$$

When you finally understand the reason for Analogies & Non-dimensional numbers

Mom. Transfer Correlation for Turbulent flows

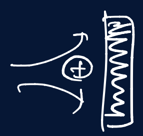
Laminar Flow:

$F_{Drag} = C_D \frac{\rho U_\infty^2}{2} A$; $C_D \downarrow C_{friction}$

Drag coefficient = Friction coeff

Planes $\downarrow A$ while taking off
$\uparrow A$ while landing

For Blunt shapes :



$\ominus$ low Pressure Region $\Rightarrow$ Additional Pressure drag

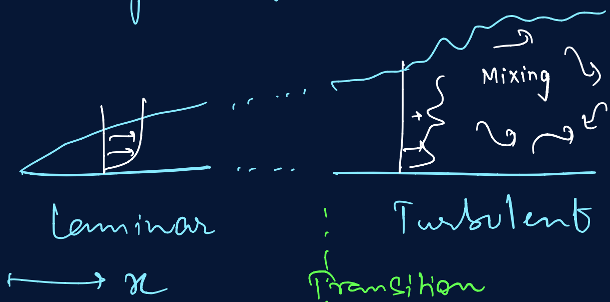
From Blasius numerical sol.^{n}

$$\delta_v \approx \frac{5x}{Re_x^{1/2}} , \qquad C_{f,x} = 0.664\, Re_x^{-1/2}$$

$$\Downarrow$$

$$\overline{C_f} = 1.33\, Re_L^{-1/2} \rightarrow \left(\frac{1}{L}\int_0^L C_{f,x}\,dx\right)$$

For long-enough surfaces :



happens earlier for Rougher surfaces

for x such that $Re_x > 10^5$
Turbulent Reynolds

Turbulence:

What do you expect? $\delta_v \propto x^?$ $\Rightarrow$ Increased

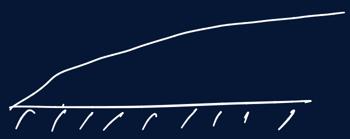
$$\delta_t \approx ? = \frac{0.38x}{Re_x^{1/5}} , \qquad C_{f,x} = 0.059\, Re_x^{-1/5}$$

$$\downarrow$$

$$\overline{C_f} = 0.074\, Re_L^{1/5}$$

Turbulent $\overline{C_f}$ $\uparrow^{es}$ rapidly with roughness ($C_{f\,laminar}$, not much)

if

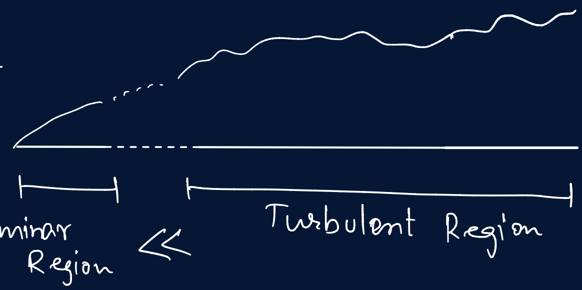


$Re = 10^3$

$\Downarrow$

Use Laminar Correlations

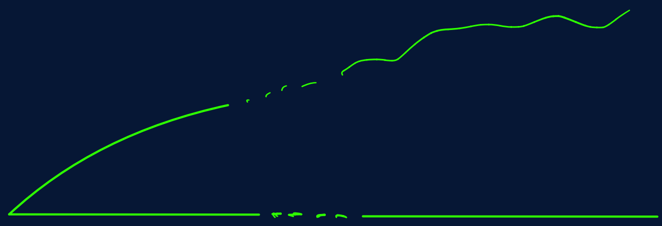
if



$\Downarrow$

Use Turbulent Correlations

Q. What if



Comparable regions?

$\Downarrow$

What to use?

* H.T correlations for Turbulent flows

Laminar:

From Blasius numerical solⁿ

$$\delta_t = \frac{\delta_v}{Pr^{1/3}}$$

$$Nu_x = \frac{h_x x}{K_f} = 0.332\, Re_x^{1/2}\, Pr^{1/3} \qquad (Pr > 0.6)$$

$$\downarrow$$

$$\overline{Nu} = 0.664\, Re_x^{0.5}\, Pr^{1/3}$$

Turbulent: $Re > 10^5$

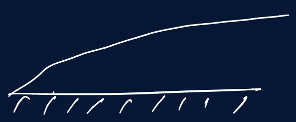
$$Nu_x = 0.0296\, Re_x^{0.8}\, Pr^{1/3} \qquad (Pr > 0.6)$$

$$\downarrow$$

$$\overline{Nu} = 0.037\, Re_L^{0.8}\, Pr^{1/3}$$

Similarly, like before, use correlations wisely!

if

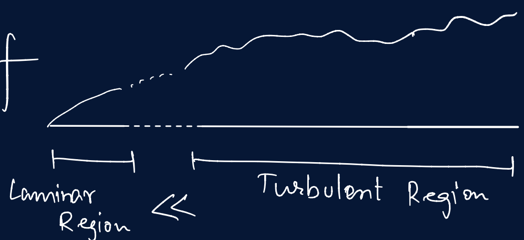


$Re = 10^3$

$\Downarrow$

Use Laminar Correlations

if



$\Downarrow$

Use Turbulent Correlations

* Spl. Case! Uniform heat flux (Newmann BC for θ instead of Dirichlet)

Laminar: $Nu_x = 0.453\, Re_x^{0.5}\, Pr^{1/3}$

Turbulent: $Nu_x = 0.031\, Re_x^{0.8}\, Pr^{1/3}$

$Pr > 0.6$

Slightly increased Nu_x from isothermal case?