CPSC 3750 Assignment 3

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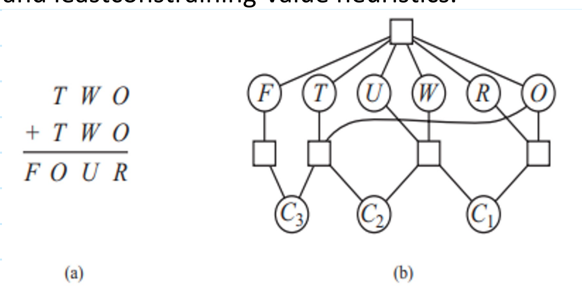
1) [10 points] Give precise formulation for the following as constraint satisfaction problems:
   - Rectilinear floor-planning: find non-overlapping places in a large rectangle for a number of smaller rectangles.

   Assuming an x, y gird for location points of the rectangles
   - Variables:
     - One set of points on the grid that mark the corners of each rectangle.
   - Domain:
     - The new rectangle fits within the confines of the larger rectangle before it.
     - The points match the shape consistent with a rectangle.
   - Constraints
     - No two rectangles overlap (sides overlapping).

2) [20 points] Solve the following cryptarithmetic problem by hand, using the strategy of backtracking with forward checking, most constrained variable, most constraining variable, and leastconstraining-value heuristics.



We need to assign a numeric value {Domain=0,1,2,3,4,5,6,7,8,9} to {F,0,U,R,T,W} so that it follows the outline given above.

- Starting with F as it is a carry and cannot be zero we can assign it a value of 1.
  - F is the most constrained variable as it only has one number it can logically be. This is because it cannot be zero, and cannot be greater than 1. Because the highest possible number would be from T=9, therefore F would be at max 1.
  - At the same time, 1 is the least constraining value, as T can be {5,6,7,8,or 9} in order to obtain an F value of 1.

- Next if we look at T, it will need to be something greater than 4 because we need a carry to equal F. It cannot be 5 because O + O will need to equal some other single digit value R. Therefore we can assign T to the next available number being 6.
  - T is the most constrained variable, because its possible selection is reduced greatly from F, but also from the restrictions of 0. Which means this variable has the most constraints placed on it then any other variable.

- From the last step we can deduce that 0 will then equal 2.

- With 0 being 2, we can now calculate R to be 4, because O + O = R.

- The next value to assign would be W. The remaining domain is {0,3,5,7,8,9}. But with the constraint that W + W = some value U. Which is not possible with the remaining domain. From here we need to backtrack.

- Because the value of R came from the value assigned to O, and O came from the value assigned to T we will need to back track all the way to the value of T.

- The next valid number in line for F would be 7. And would bring us to, F=1, T=7 with a {Domain=0,2,3,4,5,6,8,9} and {O,U,R,W} to be assigned.

- With T being 7, this will make the new value for O be 4.

- And O + O = R, therefore R = 8.

- Now back to W with the remaining domain of {0,2,3,5,6,9} for letters {W, U} since we know W + W = U we can then assign 3 to W, Which leaves us with a value of 6 for U.

- Therefore the final solution is F = 1, T = 7, O = 4, R = 8, W = 3, and U = 6.

3) Wumpus World - resolution by refutation. Prove $W_{1,3}$

KB {$Wumpus_{1,3} \Longleftrightarrow stench_{1,4} \wedge stench_{2,3} \wedge stench_{1,2}$ }

Convert KB to CNF: CNF(KB)

$W_{1,3} \Longleftrightarrow (S_{1,4} \wedge S_{1,2} \wedge S_{2,3})$

$(\neg W_{1,3} \vee (S_{1,4} \wedge S_{1,2} \wedge S_{2,3})) \wedge (W_{1,3} \vee \neg(S_{1,4} \wedge S_{1,2} \wedge S_{2,3}))$ (remove biconditional)

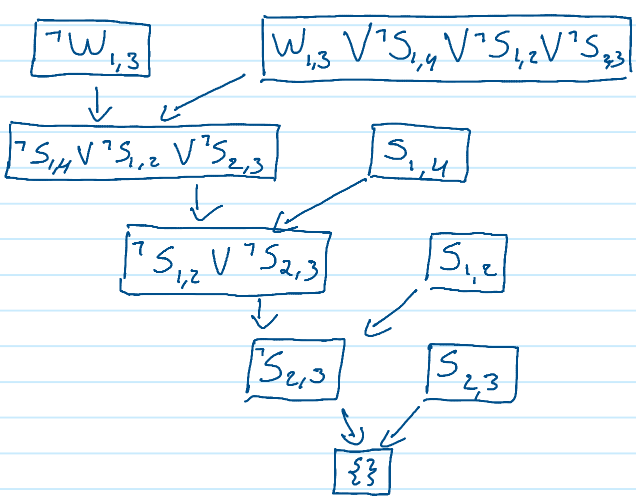
$(\neg W_{1,3} \vee (S_{1,4} \wedge S_{1,2} \wedge S_{2,3})) \wedge (W_{1,3} \vee \neg S_{1,4} \vee \neg S_{1,2} \vee \neg S_{2,3})$ (Move ¬ inwards, de Morgan's)

$(S_{1,4} \vee \neg W_{1,3}) \wedge (S_{1,2} \vee \neg W_{1,3}) \wedge (S_{2,3} \vee \neg W_{1,3}) \wedge (W_{1,3} \vee \neg S_{1,4} \vee \neg S_{1,2} \vee \neg S_{2,3})$ (distributive rule)

KB {$(S_{1,4} \vee \neg W_{1,3}), (S_{1,2} \vee \neg W_{1,3}), (S_{2,3} \vee \neg W_{1,3}), (W_{1,3} \vee \neg S_{1,4} \vee \neg S_{1,2} \vee \neg S_{2,3})\ S_{1,4}, S_{1,2}, S_{2,3}$}

Convert ¬ σ to CNF: CNF(¬σ)

KB {$\neg W_{1,3}, (S_{1,4} \vee \neg W_{1,3}), (S_{1,2} \vee \neg W_{1,3}), (S_{2,3} \vee \neg W_{1,3}), (W_{1,3} \vee \neg S_{1,4} \vee \neg S_{1,2} \vee \neg S_{2,3})\ S_{1,4}, S_{1,2}, S_{2,3}$}



KB

$\neg W_{1,3} \wedge W_{1,3} \vee \neg S_{1,4} \vee \neg S_{1,2} \vee \neg S_{2,3}$

$S_{1,4} \vee \neg W_{1,3}$

$S_{1,2} \vee \neg W_{1,3}$

$S_{2,3} \vee \neg W_{1,3}$

4) [10 points] Convert the following set of sentences to CNFs:

S1 : $A \Leftarrow\Rightarrow (B \vee E)$	$(A \Rightarrow (B \vee E)) \wedge ((B \vee E) \Rightarrow A)$ (remove biconditional) $(\neg A \vee B \vee E) \wedge (\neg(B \vee E) \vee A)$ (remove implication) $(\neg A \vee B \vee E) \wedge ((\neg B \wedge \neg E) \vee A)$ (Move ¬ inwards, de Morgan's) $(\neg A \vee B \vee E) \wedge (\neg B \vee A) \wedge (\neg E \vee A)$ (distributive rule)
S2 : $E \Rightarrow D$	$(\neg E \vee D)$ (remove implication)
S3 : $C \wedge F \Rightarrow \neg B$	$\neg(C \wedge F) \vee \neg B$ (remove implication) $(\neg C \vee \neg F) \vee \neg B$ (move ¬ inwards, de Morgan's) $(\neg C \vee \neg B) \vee (\neg F \vee \neg B)$ (distributive rule)
S4 : $E \Rightarrow B$	$(\neg E \vee B)$ (remove implication)
S5 : $B \Rightarrow F$	$(\neg B \vee F)$ (remove implication)
S6 : $B \Rightarrow C$	$(\neg B \vee C)$ (remove implication)

Use resolution to prove ¬B.
Goal: to find {} (clause w/ 0 literals)

- S3 says $(\neg C \vee \neg B) \vee (\neg F \vee \neg B)$
- S5 says $(\neg B \vee F)$

Put together resolve $R = (S2) \wedge (S5)$

$$R = (\neg C \vee \neg B) \vee (\neg F \vee \neg B) \wedge (\neg B \vee F)$$
$$R = (\neg F \vee \neg B) \wedge (\neg B \vee F)$$
$$R = \neg B$$