SUPERSONIC NOZZLE

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INTRODUCTION

- **Name**: Devin Moltz
- **Education**: Undergraduate Mechanical Engineering (graduated)
  - Aerospace Specialization

PROJECT DESCRIPTION

- Design and build a supersonic nozzle from scratch that can intake airflow from a consumer-grade air compressor and accelerate the flow to Mach 2.

- Prepare for the real-world application of nozzles in propulsion.

- Research, formulation, calculations, and simulations were conducted using resources such as Rocket Propulsion Elements, 9th Edition, along with Python, Free CAD, and ANSYS.

GITHUB, LINKEDIN AND YOUTUBE

- https://github.com/Devinm345?tab=repositories

- www.linkedin.com/in/devin-moltz

- https://www.youtube.com/@DevMonsterZap/streams

APPROACH PT.1

- Review Chapters 2 and 3, with a particular focus on Sections 2.3, 3.2, 3.3, and 3.4 from Rocket Propulsion Elements, Ninth Edition by George P. Sutton and Oscar Biblarz.
  - Section 2.3
    - Establishes the concept of effective exhaust velocity and introduces notation for different sections of the nozzle.
  - Section 3.2
    - Introduces key thermodynamic principles, including the Ideal Gas Law and isentropic flow equations.
  - Section 3.3
    - Explores isentropic flow in greater detail, focusing on the expansion area ratio and the conditions for optimum expansion.
  - Section 3.4
    - Analyzes and determines the optimal nozzle shape for specific cases.

APPROACH PT.1

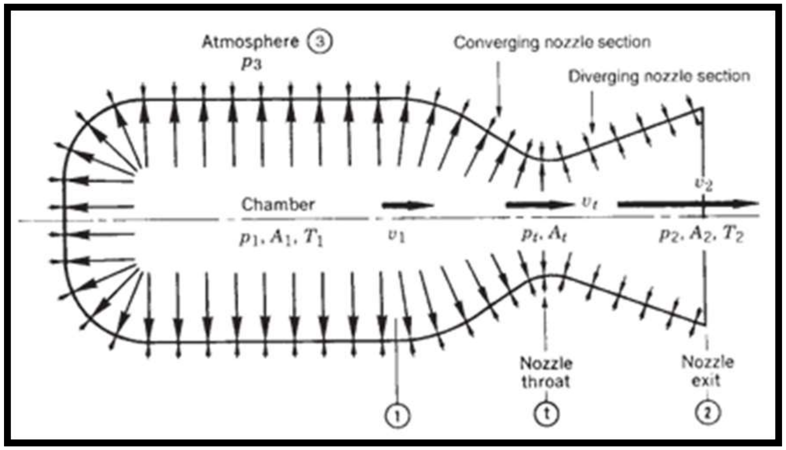
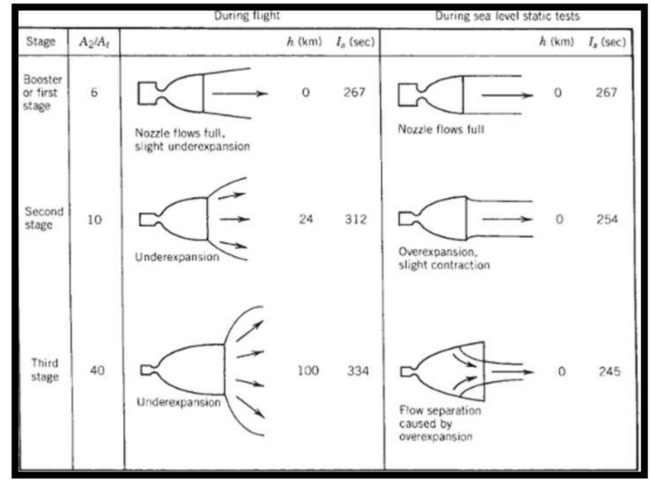


Figure 2-10

- Identifies the locations of each section within the nozzle and provides clear labels for the associated notation.

- Supports the project by defining key notation and illustrating the effects of nozzle geometry on flow behavior.



		During flight			During sea level static tests		
Stage	A_2/A_t		h (km)	I_s (sec)		h (km)	I_s (sec)
Booster or first stage	6	Nozzle flows full, slight underexpansion	0	267	Nozzle flows full	0	267
Second stage	10	Underexpansion	24	312	Overexpansion, slight contraction	0	254
Third stage	40	Underexpansion	100	334	Flow separation caused by overexpansion	0	245

Figure 3-10

- Illustrates the effects of varying expansion area ratios through a detailed visualization.

APPROACH PT.1

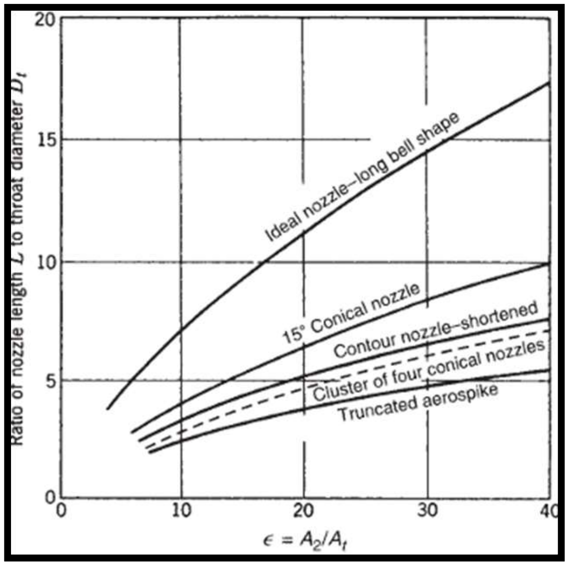


Figure 3-12

- Provides a reference for determining the expansion ratio and nozzle length.

- Serves as a guide for the final calculations of the project, providing key visual references.

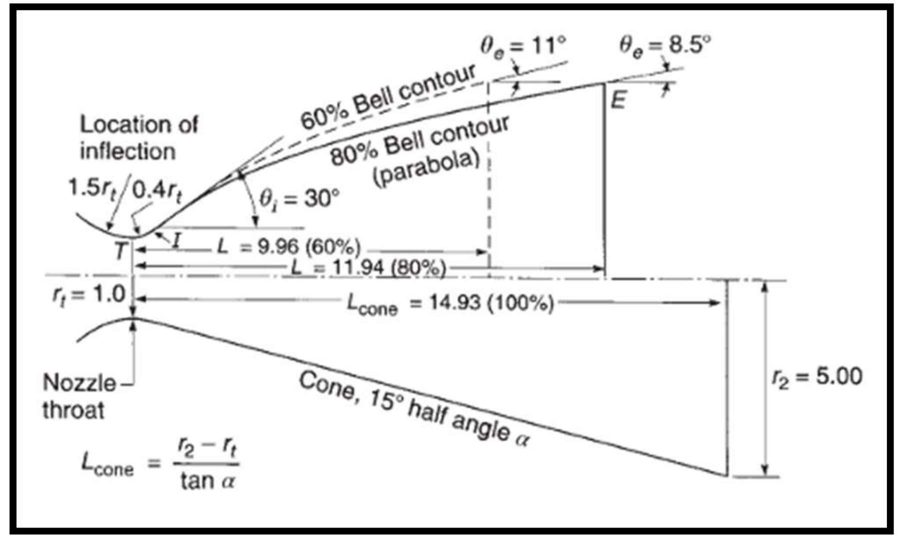


Figure 3-13

- Illustrates the accumulation of all the other figures into one reference that specifies throat and exit geometry as well as the overall length.

APPROACH PT.2

- Key symbols:

a	m/s	Velocity of sound
c^*	m/s	Characteristic velocity
M		Mach number
γ		Specific heat ratio(Aero)
k		Specific heat ratio(Thermo)
v	m/s	Velocity
V	m^3/kg	Specific Volume
ϵ		Expansion area ratio
ρ	kg/m^3	Density
r	m	Radius

- Legend:

x_1	Subscript 1	Location 1 (chamber/inlet)
x^*	Exponent *	Characteristic flow notation
x_t	Subscript t	Location t (throat)
x_2	Subscript 2	Location 2 (outlet/exit)
x_3	Subscript 3	Location 3 (ambient)

APPROACH PT.2

- Equations used:

$c^* = v_2$ (2-15)	$T_e = T_1\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-1}$ (3-12)	$T^* = \frac{2T_1}{(\gamma+1)}$ (3-22)
		<u>Extra equations used</u>
$\dot{m} = \frac{Av}{V}$ (3-3)	$P_e = P_1\left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{-\gamma}{\gamma-1}}$ (3-13)	$r = \sqrt{\frac{A}{\pi}}$ (1)
$p_x V_x = RT_x$ (3-4)	$\frac{A_y}{A_x} = \frac{M_x}{M_y}\sqrt{\left\{\frac{1+\left[\frac{\gamma-1}{2}\right]M_y^2}{1+\left[\frac{\gamma-1}{2}\right]M_x^2}\right\}^{\frac{(\gamma+1)}{(\gamma-1)}}}$ (3-14)	$A^* = \frac{\dot{m}}{\left(\frac{P^*}{RT^*}\right)\sqrt{\gamma RT^*}}$ (2)
$a = \sqrt{\gamma RT}$ (3-1)		$P^* = \rho^* RT^*$ (3)
$M = \frac{v}{\sqrt{\gamma RT}}$ (3-11)		$\dot{m} = \rho^* A^* V^*$ (4)
	$P^* = P_1\left[\frac{2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}$ (3-20)	$L_{cone} = \frac{r_2 - r^*}{\tan(\alpha)}$ (5)

APPROACH PT.3

Initial conditions

- Declare the assumptions, initial inlet conditions, outlet conditions, ambient conditions and target Mach number.

Formulation of equations

- Manually document all the essential formulas required to calculate the throat-to-exit area ratio.

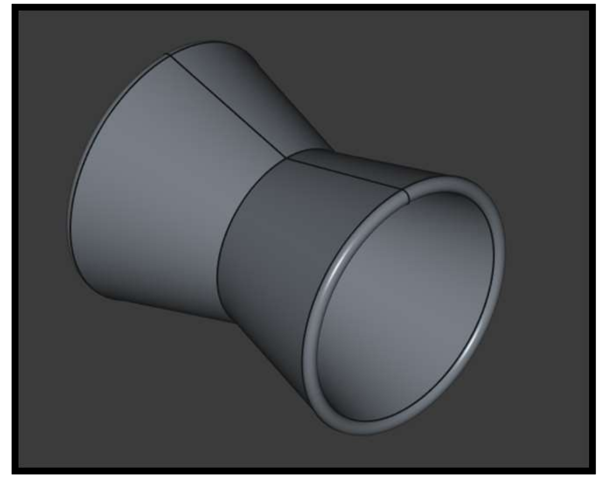
Automation with python

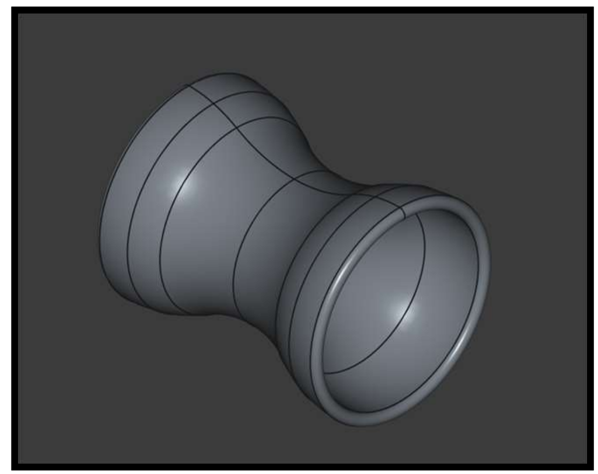
- In a logical sequence, specify the initial conditions and systematically apply the necessary formulations to compute the throat-to-exit area ratio.

Simulate using ANSYS

- Convert the geometry to a 2D representation, generate the computational mesh, then apply the appropriate boundary conditions, and proceed with the simulation.

- Repeat previous steps for other geometries.





CALCULATIONS PT.1

Throat conditions

eq 3-11

$\dot{m} = \rho^* A^* V^* \qquad V^* = a = \sqrt{\gamma R T}$

$A^* = A_t \qquad P^* = \rho^* R T^* \Rightarrow \rho^* = \frac{P^*}{R T^*}$ eq 3-22

3-24 $A^* = \frac{\dot{m}}{\rho^* V^*} = \frac{\dot{m}}{\left(\frac{P^*}{RT^*}\right)\cdot\left(\sqrt{\gamma R T^*}\right)} \qquad T^* = \frac{2T_1}{\gamma+1}$

$= \frac{\dot{m} R T^*}{P^* \sqrt{\gamma R T^*}} = \frac{\dot{m}}{P^*} \sqrt{\frac{RT^*}{\gamma}} \qquad P^* = P_1 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$

$\sqrt{RT} = (RT)^{1/2}$

```python
# Solving for throat conditions
# V* = a
T_star = T1 * ((2)/(gamma + 1))
P_star = P1 * ((2)/(gamma + 1))**(gamma/(gamma - 1))
a = np.sqrt(gamma * R * T_star) # Local speed at the throat
A_star = ((m_dot)/((P_star / (R * T_star)) * a))
print(f'Throat area = {A_star:.5g} m^2\nThroat temp = {T_star
```

```
Throat area = 0.0011472 m^2
Throat temp = 248.33 K
Throat Pressure = 327.81 kPa
```

- The throat area was determined using equations 3-1, 3-11, 3-22, 3-24, along with variations of the mass conservation law and the ideal gas law.
- Once the correct formulas are identified, they are implemented in Python to facilitate efficient recalculations in case of errors.
- Since Inlet conditions and outlet Mach number is known, calculations begin at the throat
- In reference to the legend, the characteristic flow location in this case is located at the throat, there for $v^* = a$.

CALCULATIONS PT.2

exit conditions

$$P_e = P_1\left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-\frac{\gamma}{\gamma-1}} \quad \text{eq } 3\text{-}13$$

let: $M_e = 2$ & $M_t = 1$

$$T_e = T_1\left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-1} \quad \text{eq } 3\text{-}12$$

$$\frac{A_e}{A_t} = \frac{1}{M_e}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma - 1}{2} M_e^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad \text{eq } 3\text{-}14$$

```
# Solving For exit conditions
P2 = P1 * (1 + ((gamma - 1) / 2) * M2**2)**(-gamma/(gamma-1))
A2 = (A_star / M2) * ((2/(gamma + 1)) * (1 + ((gamma - 1) / 2) * M2**2)**((gamma + 1)/(2 * (gamma - 1))))
print(f'Exit area = {A2:.6g} m^2\nExit pressure = {P2:.6g} kPa')
```

```
Exit area = 0.0027876 m^2
Exit pressure = 79.3063 kPa
```

- Following the same approach as the previous calculations, the formulations were initially outlined by hand and then translated into Python for rapid prototyping.
- After calculating the throat area, temperature, and pressure, the exit area and outlet pressure are then determined for later comparison to figures 3-10 and 3-12
- The equations utilized include: 3-13 and 3-14
- With the area ratio and exit pressure determined, the outlet conditions are defined and nearly ready for simulation.

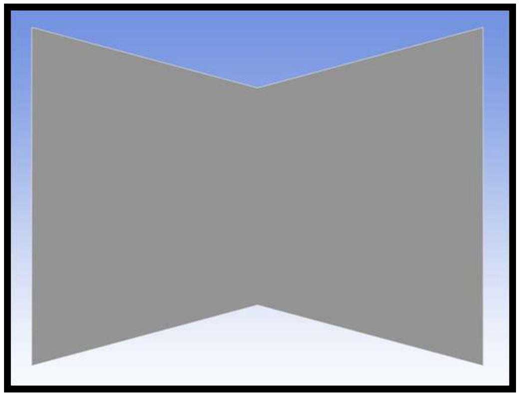
CALCULATIONS PT.3

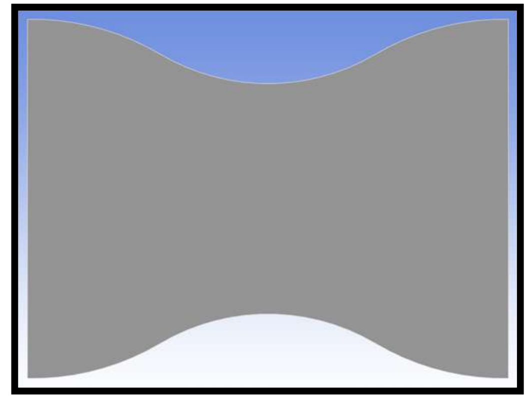
```
r2 = np.sqrt(A2/np.pi)
r_star = np.sqrt(A_star/np.pi)
L_cone = ((r2 - r_star)/(np.abs(np.tan(alpha*(np.pi/180)))))
print(f'Throat radius {r_star * 1000:.5g} mm\nExit radius {r2

Throat radius 19.109 mm
Exit radius 29.788 mm
Length of divereging section 39.854 mm
```

- The final geometric calculations were performed using Python.
- The radii were determined based on the area of the circular exits, along with an unreferenced equation (1) from the propulsion textbook to calculate the length of the diverging section.

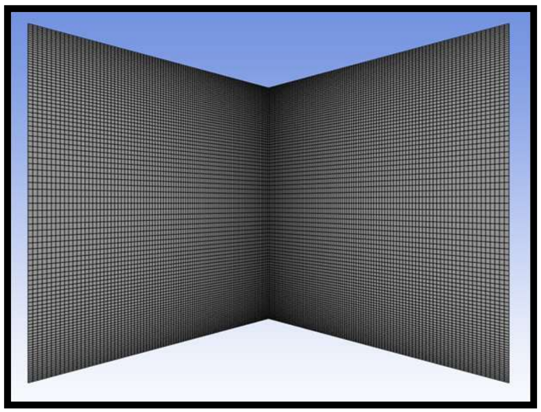
SIMULATIONS PT.1



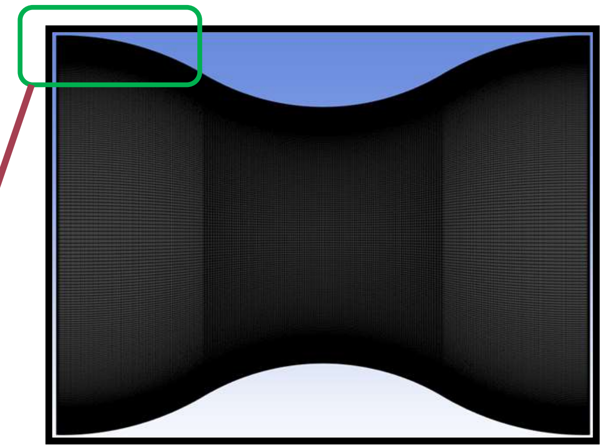


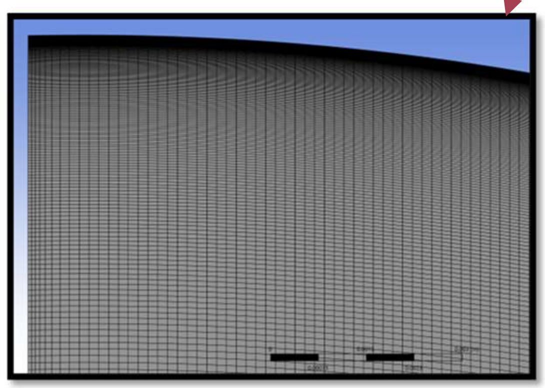
- Geometry:
  - Both nozzles have the same overall length and throat to exit area ratios.
- Inlet and outlet conditions
  - Inlet conditions of 620.528 kPa with 2.7 SCFM at average ambient temp ~298K
  - Outlet conditions of calculated pressure 79.306 kPa and ambient temp ~298K

SIMULATIONS PT.2



- Meshing:
  - A structured mesh aligned with the flow was obtained using face meshing in ANSYS.
  - Edge sizing with the number of divisions type allowed for subdividing the mesh with different biases.

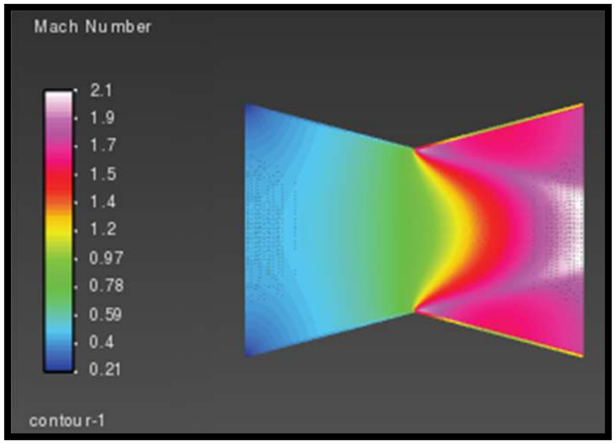




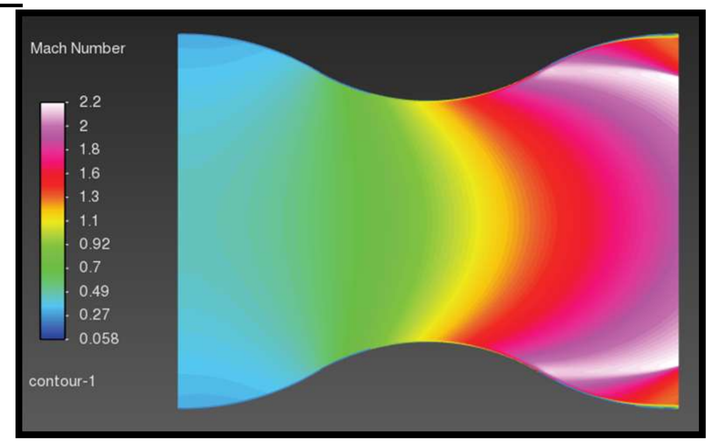
- Mesh elements: 20,000
- Vertical edge sizing bias: 5.0
- Wall y+ > 5
- Lower fidelity mesh for linear walls

- Mesh elements: 302,500
- Vertical edge sizing bias: 150
- Wall y+ ~1.6
- Higher fidelity mesh to capture curvature

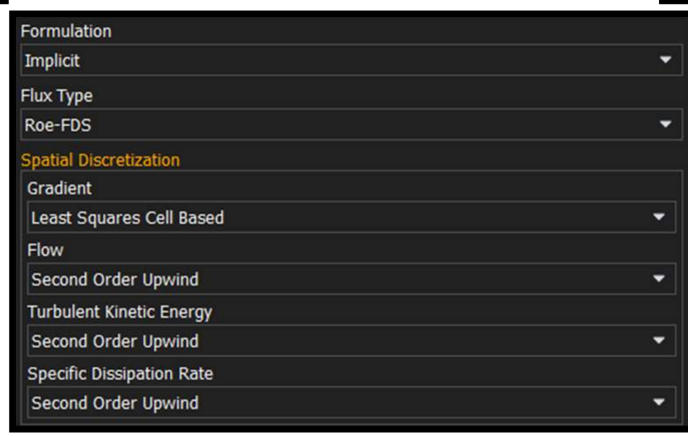
SIMULATIONS PT.3



- Viscous model: SST k-ω
- Material: Air
  - Density: ideal-gas
  - Viscosity: Sutherland
- Convergence criteria 1e-6



- Converged
- Achieved target exit Mach flow
- Ideally expanded flow



- Steady state convergence
- Achieved target exit Mach flow
- Over expanded flow

CHALLENGES AND SOLUTIONS

1. Simulation software:

   a. The original software to be used was Open FOAM. After extended trial and error, it was discovered that Open FOAM would not interface with the Free CAD as intended. Switched to ANSYS for CFD simulation.

2. Meshing errors:

   a. When meshing the smooth nozzle, the simulation residuals would indicate instability, which could be caused by many variables.

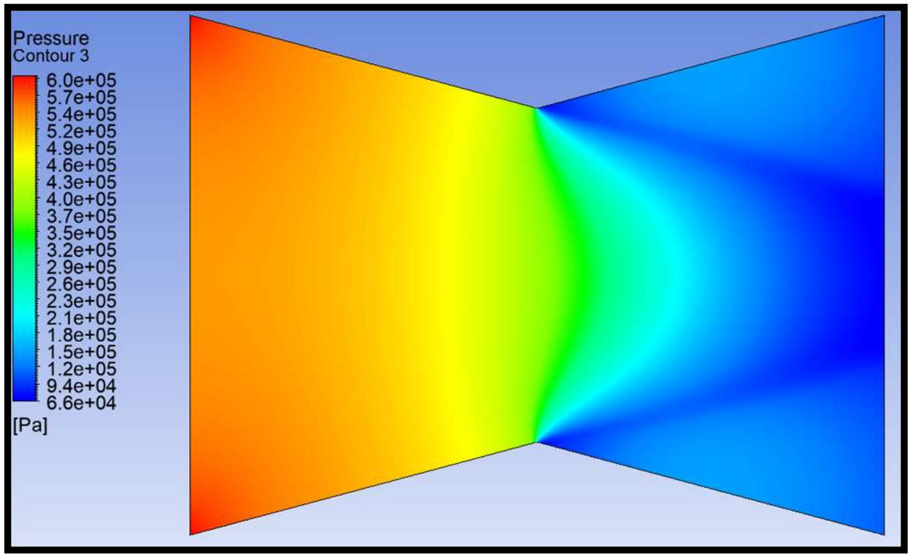
      1) The solution to this was to obtain a smaller wall y+, this was achieved through a high edge sizing bias.

SKILL AREAS DEVELOPED

1. Propulsion
   a. Nozzle expansion with respect to different geometries.
      1) Specifically, equation 3-14 and figure 3-13
   b. Nozzle geometry derivation from inlet and outlet conditions.
      1) Specified outlet Mach number, inlet mass flowrate, pressure and temperature
2. ANSYS
   a. 2D Supersonic nozzle simulations.
      1) Flux types: AUSM and Roe-FDS
      2) Viscous models: SST k-ω, Realizable k-ϵ and Reynolds Stress

CONCLUSIONS

- The mesh is equally important to the inlet and outlet boundary conditions.
- Mesh refinement at the edge and throat appears to be critical for achieving higher accuracy in the parabolic nozzle.
- In comparison of the two nozzles, the conical nozzle kept the intended expansion even though both nozzles have the same overall dimensions.



THANK YOU

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