Exercise 5.1

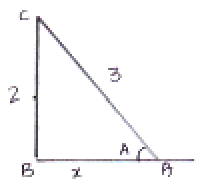
1. In each of the following one of the six trigonometric ratios is given. Find the values of the other trigonometric ratios.

Sol:

(i) Sin A = $\frac{2}{3}$

We know that Sin θ = $\frac{opposite\ side}{hypotenuse}$

Let us Consider a right angled Δ^{le} ABC.



By applying Pythagorean theorem we get

$AC^2 = AB^2 + BC^2$

9 = $x^2 + 4$

x^2 = 9 - 4

x = $\sqrt{5}$

We know that cos = $\frac{adjacent\ side}{hypotenuse}$ and

tanθ = $\frac{opposite\ side}{adjacent\ side}$

So, cosθ = $\frac{\sqrt{5}}{3}$;

sec = $\frac{1}{cos\theta} = \frac{3}{\sqrt{5}}$

tanθ = $\frac{2}{\sqrt{5}}$;

cot = $\frac{1}{tan\theta}$ = $\frac{\sqrt{5}}{2}$

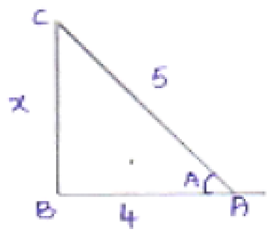
cosecθ = $\frac{1}{sin\theta} = \frac{3}{2}$

(ii)

Cos A = $\frac{4}{5}$

We know that cosθ = $\frac{adjacent\ side}{hypotenuse}$

Let us consider a right angled Δ^{le} ABC.



Let opposite side BC = x.

By applying pythagorn's theorem, we get

$AC^2 = AB^2 + BC^2$

25 = x + 16

x = 25 - 16 = 9

x = $\sqrt{9}$ = 3

We know that cosA = $\frac{4}{5}$

sinA = $\frac{opposite\ side}{hypotenuse} = \frac{3}{5}$

tanA = $\frac{opposite\ side}{adjacent\ side} = \frac{3}{4}$

cosecA = $\frac{1}{sinA} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$

secA = $\frac{1}{cosA} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$

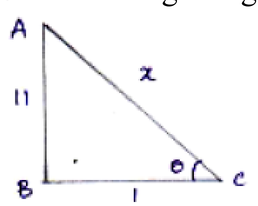
cotA = $\frac{1}{tanA} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

(iii)

tanθ = 11.

We know that tanθ = $\frac{opposite\ side}{adjacent\ side} = \frac{11}{1}$

Consider a right angled Δ^{le} ABC.



Let hypotenuse AC = x, by applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$x^2 = 11^2 + 1^2$

$x^2 = 121 + 1$

$x = \sqrt{122}$

We know that sinθ = $\frac{opposite\ side}{hypotenuse} = \frac{11}{\sqrt{122}}$

cosθ = $\frac{adjacent\ side}{hypotenuse} = \frac{1}{\sqrt{122}}$

cosecθ = $\frac{1}{sin\theta}$ = $\frac{\frac{1}{11}}{\sqrt{122}}$ = $\frac{\sqrt{122}}{11}$

secθ = $\frac{1}{cos\theta}$ = $\frac{1/1}{\sqrt{122}}$ = $\sqrt{122}$

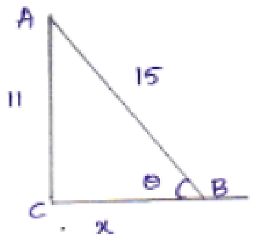
cotθ = $\frac{1}{tan\theta}$ = $\frac{1}{11}$ = $\frac{1}{11}$

(iv)

Sin θ = $\frac{11}{5}$

We know Sin θ = $\frac{opposite\ side}{hypotenuse}$ = $\frac{11}{15}$

Consider right angled Δ^{le} ACB.



Let x = $adjacent\ side$

By applying Pythagoras

$AB^2 = AC^2 + BC^2$

225 = 121+x^2

x^2 = 225 -121

x^2 = 104

x = $\sqrt{104}$

cos = $\frac{adjacent\ side}{hypotenuse}$ = $\sqrt{\frac{104}{15}}$

tan = $\frac{opposite\ side}{adjacent\ side}$ = $\frac{11}{\sqrt{104}}$

cosec$\theta = \frac{1}{sin\theta} = \frac{15}{11}$

sec = $\frac{1}{cos\theta} = \frac{15}{\sqrt{104}}$

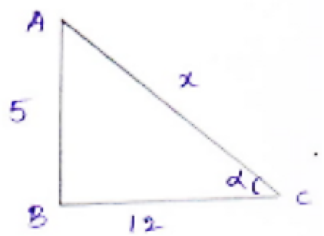
cot = $\frac{1}{tan\theta}$ = $\frac{\sqrt{104}}{11}$

(v)

tanα = $\frac{5}{12}$

We know that tanα= $\frac{opposite\ side}{adjacent\ side}$ = $\frac{5}{12}$

Now consider a right angled Δ^{le} ABC.



Let x = hypotenuse .By applying Pythagoras theorem

$AC^2 = \; AB^2 + BC^2$

$x^2 = 5^2 + 12^2$

$x^2 = 25 + 144 = 169$

$x = 13$

sinα $= \frac{opposite\ side}{hypotenuse} = \frac{5}{13}$

cosα $= \frac{adjacent\ side}{hypotenuse} = \frac{12}{13}$

cotα $= \frac{1}{tan\alpha} = \frac{12}{15}$

cosecα $= \frac{1}{sin\alpha} = \frac{1/5}{13} = \frac{13}{5}$

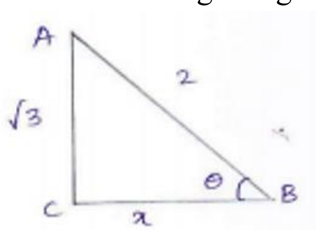
secα $= \frac{1}{cos\alpha} = \frac{\frac{1}{12}}{13} = \frac{13}{12}$.

(vi)

Sin $\theta = \frac{\sqrt{3}}{2}$

We know Sin $\theta = \frac{opposite\ side}{hypotenuse} = \frac{\sqrt{3}}{2}$

Now consider right angled Δ^{le} ABC.



Let x = $adjacent\ side$

By applying Pythagoras

$AB^2 = AC^2 + BC^2$

4 = 3+x^2

x^2 = 4 − 3

x^2 = 1

x = 1

cos $= \frac{adjacent\ side}{hypotenuse} = \frac{1}{2}$

tan $= \frac{opposite\ side}{adjacent\ side} = \frac{\sqrt{3}}{1} = \sqrt{3}$

cosec$\theta = \frac{1}{sin\theta} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$

sec $= \frac{1}{cos\theta} = \frac{1}{\frac{1}{2}} = 2$

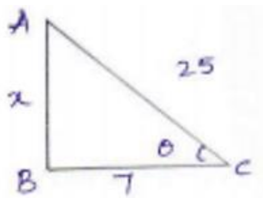
cot $= \frac{1}{tan\theta} = \frac{1}{\sqrt{3}}$

(vii)

Cos$\theta = \frac{7}{25}$.

We know that cos$\theta = \frac{adjacent\ side}{hypotenuse}$

Now consider a right angled Δ^{le} ABC,



Let x be the opposite side.

By applying pythagorn's theorem

$AC^2 = AB^2 + BC^2$

$(25)^2 = x^2 + 7^2$

625 - 49 = x^2

$576 = \sqrt{576} = 24$

sin$\theta = \frac{opposite\ side}{hypotenuse} = \frac{24}{25}$

tan$\theta = \frac{opposite\ side}{adjacent\ side} = \frac{24}{7}$

cosec$\theta = \frac{1}{sin\theta} = \frac{1}{\frac{3}{5}} = \frac{25}{24}$

sec$\theta = \frac{1}{cos\theta} = \frac{1}{\frac{4}{5}} = \frac{25}{7}$

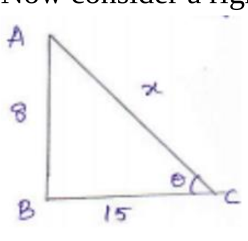
cot$\theta = \frac{1}{tan\theta} = \frac{1}{\frac{3}{4}} = \frac{7}{24}$

(viii)

tan$\theta = \frac{8}{15}$

We know that tan$\theta = \frac{opposite\ side}{adjacent\ side} = \frac{8}{15}$

Now consider a right angled Δ^{le} ABC.



By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$x^2 = 8^2 + 15^2$

$x^2 = 225 + 64 = 289$

$x = \sqrt{289} = 17$

$\sin\theta = \frac{opposite\ side}{hypotenuse} = \frac{8}{17}$

$\cos\theta = \frac{adjacent\ side}{hypotenuse} = \frac{15}{17}$

$\tan\theta = \frac{opposite\ side}{adjacent\ side} = \frac{8}{15}$

$\cot\theta = \frac{1}{tan\theta} = \frac{1}{\frac{8}{15}} = \frac{15}{8}$

$\text{cosec}\theta = \frac{1}{sin\theta} = \frac{\frac{1}{8}}{17} = \frac{17}{8}$

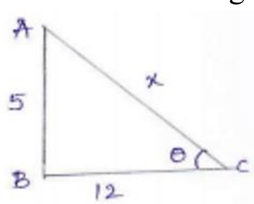
$\sec\theta = \frac{1}{cos\theta} = \frac{\frac{1}{15}}{17} = \frac{17}{15}$

(ix)

$\cot\theta = \frac{12}{5}$

$\cot\alpha = \frac{adjacent\ side}{opposite\ side} = \frac{12}{5}$

Now consider a right angled Δ^{le} ABC,



By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$x^2 = 25 + 144$

$x^2 = 169 = \sqrt{169}$

$x = 13$

$\tan\theta = \frac{1}{cot\theta} = \frac{\frac{1}{12}}{5} = \frac{5}{12}$

$\sin\theta = \frac{opposite\ side}{hypotenuse} = \frac{5}{13}$

$\cos\theta = \frac{adjacent\ side}{hypotenuse} = \frac{12}{13}$

$\text{cosec}\theta = \frac{1}{sin\theta} = \frac{1}{5/13} = \frac{13}{5}$

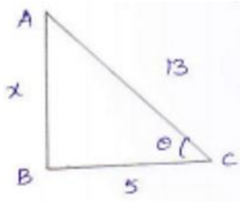
$\sec\theta = \frac{1}{cos\theta} = \frac{1}{12/13} = \frac{13}{12}$

(x)

$\sec\theta = \frac{13}{5}$

secθ = $\frac{hypotenuse}{adjacent\ side}$ $= \frac{13}{5}$

Now consider a right angled Δ^{le} ABC,



By applying Pythagoras theorem

$AC^2 = \ AB^2 + BC^2$

$169 \ = \ x^2 + 25$

$x^2 = 169 \ - 25 = 144$

$x = 12$

cosθ $= \frac{1}{sec\theta} = \frac{1}{\frac{13}{5}} = \frac{5}{13}$

tanθ $= \frac{opposite\ side}{adjacent\ side} = \frac{12}{5}$

sinθ $= \frac{opposite\ side}{hypotenuse} = \frac{12}{13}$

cosecθ $= \frac{1}{sin\theta} = \frac{1}{12/13} = \frac{13}{12}$

secθ $= \frac{1}{cos\theta} = \frac{1}{5/13} = \frac{13}{5}$

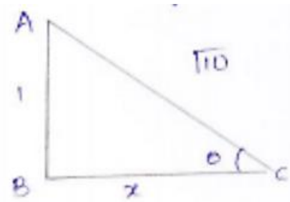
cotθ $= \frac{1}{tan\theta} = \frac{1}{12/5} = \frac{5}{12}$

(xi)

cosecθ = $\sqrt{10}$

cosecθ = $\frac{hypotenuse}{opposite\ side}$ $= \sqrt{10}$

consider a right angled Δ^{le} ABC, we get



Let x be the adjacent side.

By applying pythagora's theorem

$AC^2 = \ AB^2 + BC^2$

$\left(\sqrt{10}\right)^2 = \ 1^2 + \ x^2$

$x^2 = 10 \ - 1 = 9$

$x = 3$

sinθ $= \frac{1}{cosec\theta} = \frac{1}{\sqrt{10}}$

cosθ $= \frac{adjacent\ side}{hypotenuse} = \frac{3}{\sqrt{10}}$

tanθ $= \frac{opposite\ side}{adjacent\ side} = \frac{1}{3}$

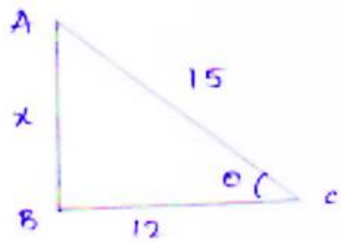
secθ $= \frac{1}{cos\theta} = \frac{\sqrt{10}}{3}$

cotθ $= \frac{1}{tan\theta} = \frac{1}{\frac{1}{3}} = 3.$

(xii)

Cosθ = $\frac{12}{5}$

cosθ = $\frac{adjacent\ side}{hypotenuse} = \frac{12}{15}$.



Let x be the opposite side.

By applying pythagorn's theorem

$AC^2 = AB^2 + BC^2$

$225 = x^2 + 144$

$225 - 144 = x^2$

$x^2 = 81$

$x = 9$

sinθ $= \frac{opposite\ side}{hypotenuse} = \frac{9}{15}$

tanθ $= \frac{opposite\ side}{adjacent\ side} = \frac{9}{12}$

cosecθ $= \frac{1}{sin\theta} = \frac{1}{\frac{9}{15}} = \frac{15}{9}$

secθ $= \frac{1}{cos\theta} = \frac{1}{\frac{12}{15}} = \frac{15}{12}$

cotθ $= \frac{1}{tan\theta} = \frac{1}{\frac{9}{12}} = \frac{12}{9}$

2. In a ΔABC, right angled at B, AB = 24 cm, BC = 7 cm. Determine

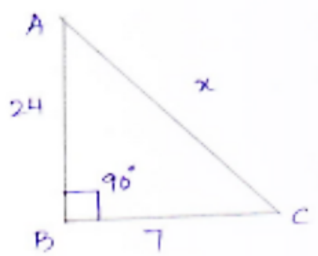
(i) Sin A, Cos A

(ii) Sin C, cos C

Sol:

ΔABC is right angled at B

AB = 24cm, BC = 7cm.

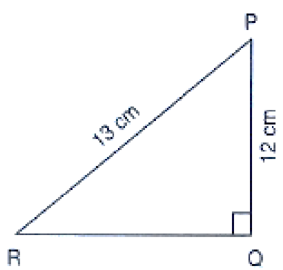


Let ‘x’ be the hypotenuse,
By applying Pythagoras
$AC^2 = AB^2 + BC^2$
$x^2 = 24^2 + 7^2$
$x^2 = 576 + 49$
$x^2 = 625$
$x = 25$

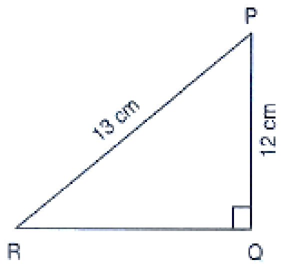
a. Sin A, Cos A
At ∠A, opposite side = 7
adjacent side = 24
hypotenuse = 25
sin A = $\frac{opposite\ side}{hypotenuse} = \frac{7}{25}$
cos A = $\frac{adjacent\ side}{hypotenuse} = \frac{24}{25}$

b. Sin C, Cos C
At ∠C, opposite side = 24
adjacent side = 7
hypotenuse = 25
sin C = $\frac{24}{25}$
cos C = $\frac{7}{25}$

3. In Fig below, Find tan P and cot R. Is tan P = cot R?

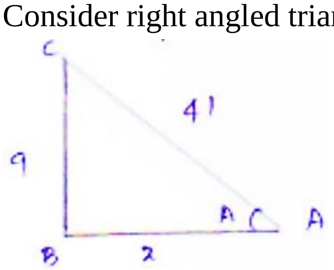


Sol:



Let x be the adjacent side.
By Pythagoras theorem
$PR^2 = PQ^2 + RQ^2$
$169 = x^2 + 144$
$x^2 = 25$
$x = 5$
At LP, opposite side = 5
Adjacent side = 12
Hypotenuse = 13
tan P = $\frac{\frac{1}{12}}{5} \Rightarrow \frac{5}{12}$
At LR, opposite side = 12
Adjacent side = 5
Hypotenuse = 13
cot R = $\frac{1}{\tan R} = \frac{1}{\frac{12}{5}} = \frac{5}{12}$
[∵ Tan R = $\frac{opposite\ side}{adjacent\ side}$]
∵ tan P = cot R

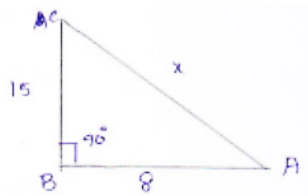
4. If sin A = $\frac{9}{41}$, $compute$ $\cos A$ and $\tan A$
Sol:
$\sin A = \frac{9}{41}$
Sin A = $\frac{opposite\ side}{adjacent\ side} = \frac{9}{41}$
Consider right angled triangle ABC,



Let x be the adjacent side
By applying Pythagorean

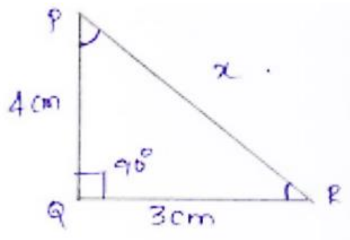
$AC^2 = AB^2 + BC^2$
$41^2 = 12^2 + 9^2$
$x^2 = 41^2 - 9^2$
$x = 40$
$\cos A = \frac{adjacent\ side}{hypotenuse} = \frac{40}{41}$
$\tan A = \frac{opposite\ side}{Hypotenuse\ side} = \frac{9}{40}$

5. Given 15 cot A = 8, find Sin A and sec A.
Sol:
15 cot A = 8, find Sin A and sec A
Cot A = $\frac{8}{15}$



Consider right angled triangle ABC,
Let x be the hypotenuse,
$AC^2 = AB^2 + BC^2$
$x^2 = (8)^2 + (15)^2$
$x^2 = 64 + 225$
$x^2 = 289$
$x = 17$
Sin A = $\frac{opposite\ side}{hypotenuse} = \frac{15}{17}$
Sec A = $\frac{1}{\cos A}$
$\cos A = \frac{adjacent\ side}{Hypotenuse} = \frac{8}{17}$
Sec A = $\frac{1}{\cos A} = \frac{1}{8/17} = \frac{17}{8}$

6. In ΔPQR, right angled at Q, PQ = 4 cm and RQ = 3 cm. Find the values of sin P, sin R, sec P and sec R.
Sol:
ΔPQR, right angled at Q.



Let x be the hypotenuse

By applying Pythagoras

$PR^2 = PQ^2 + QR^2$

$x^2 = 4^2 + 3^2$

$x^2 = 16 + 9$

$\therefore x = \sqrt{25} = 5$

Find $\sin P$, $\sin R$, $\sec P$, $\sec R$

At LP, opposite side = 3 cm

Adjacent side = 4 cm

Hypotenuse = 5

$\sin P = \frac{opposite\ side}{Hypotenuse} = \frac{3}{5}$

$\sec P = \frac{Hypotenuse}{adjacent\ side} = \frac{5}{4}$

At LK, opposite side = 4 cm

Adjacent side = 3 cm

Hypotenuse = 5 cm

Sin R = $\frac{4}{5}$

Sec R = $\frac{5}{3}$

7. If cot $\theta = \frac{7}{8}, evaluate$:

(i) $\frac{(1+\sin\theta)\,(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) Cot$^2\theta$

Sol:

Cot $\theta = \frac{7}{8}$

(i) $\frac{(1+\sin\theta)\,(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\sin^2\theta}{1-\cos^2\theta}$ [$\because$ (a + b) (a – b) = $a^2 - b^2$] a = 1, b = sin θ

We know that $Sin^2\theta + \cos^2\theta = 1$

$1 - \sin^2\theta = \cos^2\theta = \cos^2\theta$

$1 - \cos^2\theta = \sin^2\theta$

$= \frac{\cos^2\theta}{\sin^2\theta}$

$= \cot^2 \theta$

$= (\cot\ \theta)^2 = \left[\frac{7}{8}\right]^2$

$= \frac{49}{64}$

(ii) $\cot^2 \theta$

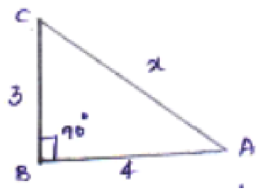
$\Rightarrow (\cot \theta)^2 = \left[\frac{7}{8}\right]^2$

$= \frac{49}{64}$

8. If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Sol:

3 cot A = 4, check $= \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$



Cot A = $\frac{adjacent\ side}{opposite\ side} = \frac{4}{3}$

Let x be the hypotenuse

By Applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$x^2 = 4^2 + 3^2$

$x^2 = 25$

$x = 5$

Tan A = $\frac{1}{\cos^2 A} = \frac{3}{4}$

Cos A = $\frac{adjacent\ side}{hypotenuse} = \frac{4}{5}$

Sin A = $\frac{3}{5}$

LHS = $\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}} = \frac{7}{25}$

RHS $\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16-9}{25}$

$= \frac{7}{251}$

9. If $\tan \theta = \frac{a}{b}$, find the value of $\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$

Sol:

Tan $\theta = \frac{a}{b}$ find $\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$ ….(i)

Divide equation (i) with $\cos\theta$, we get

$\Rightarrow \frac{\frac{\cos\theta+\sin\theta}{\cos\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$\Rightarrow \frac{1+\frac{\sin\theta}{\cos\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta}$

$= \frac{1+\frac{a}{b}}{1-\frac{a}{b}}$

$= \frac{b+a}{b-a}$

10. If 3 tan θ = 4, $find\ the\ value\ of\ \frac{4\cos\theta-\sin\theta}{2\cos\theta+\sin\theta}$

Sol:

$3\tan\theta = 4$ find $\frac{4\cos\theta-\sin\theta}{2\cos\theta+\sin\theta}$ ...(i)

Tan $\theta = \frac{4}{3}$

Dividing equation (i) with $\cos\theta$ we get

$= \frac{\frac{4\cos\theta-\sin\theta}{\cos\theta}}{\frac{2\cos\theta+\sin\theta}{\cos\theta}} = \frac{4-\tan\theta}{2+\tan\theta}\left[\because \frac{\sin\theta}{\cos\theta} = \tan\theta\right]$

$= \frac{4-\tan\theta}{2+\tan\theta}$ $[\because \frac{\sin\theta}{\cos\theta} = \tan\theta]$

$= \frac{4-\frac{4}{1}}{2+\frac{4}{5}}$

$= \frac{12-4}{6+4}$

$= \frac{8}{10}$

$= \frac{4}{5}$

11. If 3 cot θ = 2, find the value of $= \frac{4\sin\theta-3\cos\theta}{2\sin\theta+6\sin\theta}$

Sol:

3 cot θ = 2 find $\frac{4\sin\theta-3\cos\theta}{2\sin\theta+6\cos\theta}$ … (i)

Cot $\theta = \frac{2}{3}$

$= \frac{\frac{4\sin\theta-3\cos\theta}{\sin\theta}}{\frac{2\sin\theta+6\cos\theta}{\sin\theta}}$

$= \frac{4-3\cot\theta}{2+6\cot\theta}$

$= \frac{4-3\times\frac{2}{3}}{2+6\times\frac{2}{3}}$

$= \frac{4+2}{2+4} = \frac{2}{6}$

$= \frac{1}{3}$

12. If $\tan\theta = \frac{a}{b}$, prove that $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2-b^2}{a^2+b^2}$

Sol:

Tanθ = $\frac{a}{b}$. PT $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2-b^2}{a^2+b^2}$

Let $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$ ...(i)

Divide both Nr and Dr with cos θ of (a)

$= \frac{\frac{a\sin\theta - b\cos\theta}{\cos\theta}}{\frac{a\sin\theta + b\cos\theta}{\cos\theta}}$

$= \frac{a\tan\theta - b}{a\tan\theta + b}$

$= \frac{a\times\left(\frac{a}{b}\right) - b}{a\times\left(\frac{a}{b}\right) + b}$

$= \frac{a^2-b^2}{a^2+b^2}$

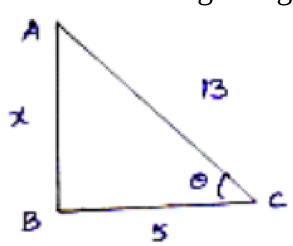
13. If sec θ = $\frac{13}{5}$, show that $\frac{2\cos\theta - 3\cos\theta}{4\sin\theta - 9cos\theta}$ = 3

Sol:

$Sec\ \theta = \frac{13}{5}$

Sec θ = $\frac{Hypotenuse}{adjacent\ side} = \frac{13}{5}$

Now consider right angled triangle ABC



By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$169 = x^2 + 25$

$x^2 = 169 - 25 = 144$

$x = 12$

$cos\theta = \frac{1}{\sec\theta} = \frac{1}{13} = \frac{5}{3}$

$\tan\theta = \frac{opposite\ side}{adjacent\ side} = \frac{12}{13}$

$\sin\theta = \frac{opposite\ side}{hypotenuse} = \frac{12}{13}$

Cosec θ = $\frac{1}{\sin\theta} = \frac{1}{12/13} = \frac{13}{12}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{5/13} = \frac{13}{5}$

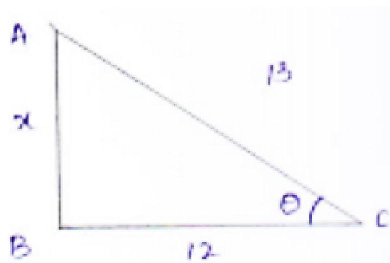
Cot $\theta = \frac{1}{\tan\theta} = \frac{1}{12/5} = \frac{5}{12}$

14. If cos θ = $\frac{12}{13}$, show that sin θ (1 – tan θ) = $\frac{35}{156}$

Sol:

Cos θ = $\frac{12}{3}$ S.T Sin θ $(1 - \tan\theta) = \frac{35}{156}$

Cos θ = $\frac{adjacent\ side}{hypotenuse} = \frac{12}{13}$



Let x be the opposite side

By applying Pythagoras

$AC^2 = AB^2 + BC^2$

169 = $x^2 + 144$

x = 25

x = 5

$\sin\theta = \frac{AB}{AC} = \frac{5}{3}$

Tan θ = $\frac{AB}{BC} = \frac{5}{12}$

$\sin\theta\,(1 - \tan\theta) = \frac{5}{13}\left(1 - \frac{5}{12}\right)$

= $\frac{5}{13}\left[\frac{7}{12}\right] = \frac{35}{156}$

15. If cot θ = $\frac{1}{\sqrt{3}}$, show that $\frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

Sol:

$\cot\theta = \frac{1}{\sqrt{3}} \frac{1-\cos^2\theta}{2-\sin^2\theta} = \frac{3}{5}$

Cot θ = $\frac{adjacent\ side}{opposite\ side} = \frac{1}{\sqrt{3}}$

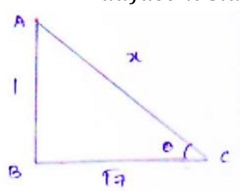
Let x be the hypotenuse

By applying Pythagoras

$AC^2 = AB^2 + BC^2$

$x^2 = (\sqrt{3})^2 + 1$
$x^2 = 3 + 1$
$x^2 = 3 + 1 \Rightarrow x = 2$
Cos $\theta = \frac{BC}{AC} = -\frac{1}{2}$
Sin $\theta = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$
$\frac{1-\cos^2\theta}{2-\sin^2\theta} \Rightarrow \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$
$\Rightarrow \frac{1-\frac{1}{4}}{2-\frac{3}{4}} \Rightarrow \frac{\frac{3}{4}}{\frac{5}{4}}$
$= \frac{3}{5}$

16. If tan $\theta = \frac{1}{\sqrt{7}}$ $\frac{cosec^2\theta-\sec^2\theta}{cosec^2\theta+\sec^2\theta} = \frac{3}{4}$
Sol:
Tan$\theta = \frac{1}{\sqrt{7}}$ $\frac{cosec^2\theta-\sec^2\theta}{cosec^2\theta+\sec^2\theta} = \frac{3}{4}$
Tan $\theta = \frac{opposite\ side}{adjacent\ side}$



Let 'x' be the hypotenuse
By applying Pythagoras
$AC^2 = AB^2 + BC^2$
$x^2 = 1^2 + (\sqrt{7})^2$
$x^2 = 1 + 7 = 8$
$x = 2\sqrt{2}$
Cosec $\theta = \frac{AC}{AB} = 2\sqrt{2}$
Sec $\theta = \frac{AC}{BC} = \frac{2\sqrt{2}}{\sqrt{7}}$
Substitute, cosec θ, sec θ in equation
$\Rightarrow \frac{(2\sqrt{2})^2-\left(2\sqrt{\frac{2}{7}}\right)^2}{(2\sqrt{2})^2+\left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$
$\frac{8-4\times\frac{2}{7}}{8+4\times\frac{2}{7}}$

$\Rightarrow \frac{8-\frac{8}{7}}{8+\frac{8}{7}}$

$= \frac{\frac{56-8}{7}}{\frac{56+8}{7}}$

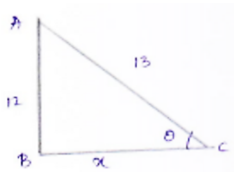
$= \frac{48}{64}$

$= \frac{3}{4}$

$L.H.S = R.H.S$

17. If Sin $\theta = \frac{12}{13}$ find $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{tan^2\theta}$

Sol:



Let x be the adjacent side

By applying Pythagoras

$AC^2 = AB^2 + BC^2$

$169 = 144 + x$

$x^2 = 25$

$x = 5$

Cos θ $= \frac{BC}{AC} = \frac{5}{13}$

Tan θ $= \frac{AB}{BC} = \frac{12}{5}$

$\Rightarrow \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)}{\alpha \times \frac{2}{13} \times \frac{5}{13}} \times \frac{1}{\left[\frac{12}{5}\right]^2}$

$\Rightarrow \frac{\frac{144-25}{169}}{\frac{24\times5}{169}} \times \frac{25}{144}$

$\Rightarrow \frac{\frac{119}{169}}{\frac{120}{169}} \times \frac{25}{144} = \frac{129}{120} \times \frac{25}{144} = \frac{595}{3456}$

18. If sec $\theta = \frac{5}{4}$, find the value of $\frac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta}$

Sol:

Not given

19. If cos $\theta = \frac{5}{13}$, find the value of $\frac{\sin^2\theta - \cos^2\theta}{2sin\,\theta\cos\theta} = \frac{3}{5}$

Sol:

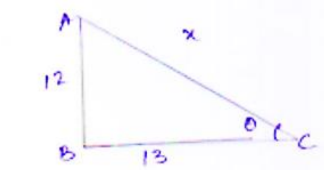
Not given

20. Tan $\theta = \frac{12}{13}$ Find $\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

Sol:

Tan $\theta = \frac{opposite\ side}{adjacent\ side}$

Let x be, the hypotenuse



By Pythagoras we get

$AC^2 = AB^2 + BC^2$

$x^2 = 144 + 169$

$x = \sqrt{313}$

Sin $\theta = \frac{AB}{AC} = \frac{12}{\sqrt{313}}$

Cos $\theta = \frac{BC}{AC} = \frac{13}{\sqrt{313}}$

Substitute, Sin θ, cos θ in equation we get

$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} \Rightarrow \frac{2\times\frac{12}{\sqrt{313}}\times\frac{13}{\sqrt{313}}}{\frac{169}{313}-\frac{144}{313}}$

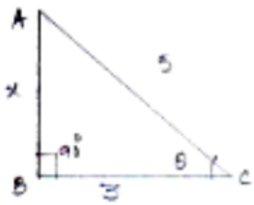
$= \frac{\frac{312}{313}}{\frac{25}{313}} = \frac{312}{25}$

21. If cos $\theta = \frac{3}{5}$, find the value of $\frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta}$

Sol:

Cos $\theta = \frac{3}{5}$ find value of $\frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta}$

We know that cos $\theta = \frac{adjacent\ side}{hypotenuse}$



Let us consider right angled Δle ABC

Let x be the opposite side, By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$25 = x^2 + 9$

$x^2 = 16 \Rightarrow x = 4$

Sin $\theta = \frac{AB}{AC} = \frac{4}{5}$

Tan $\theta = \frac{AB}{BC} = \frac{4}{3}$

Substitute sin θ, tan θ in equation we get

$\frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta} = \frac{\frac{4}{5}-\frac{3}{4}}{2\times\frac{4}{3}}$

$= \frac{\frac{16-15}{20}}{\frac{8}{3}} = \frac{\frac{1}{20}}{\frac{8}{3}}$

$= \frac{1}{20}\times\frac{3}{8} = \frac{3}{160}$

22. If sin $\theta = \frac{3}{5}, evaluate\ \frac{\cos\theta - \frac{1}{\tan\theta}}{2\cot\theta}$

Sol:

Not given

23. If sec A = $\frac{5}{4}$, verify that $\frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A} = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Sol:

Not given

24. If sin $\theta = \frac{3}{4}$, prove that $\sqrt{\frac{cosec^2\theta - \cot^2\theta}{\sec^2\theta - 1}} = \frac{\sqrt{7}}{3}$

Sol:

Not given

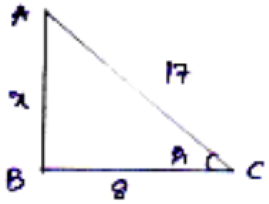
25. If sec A = $\frac{17}{8}$, verify that $\frac{3-4\sin^2 A}{4\cos^2 A - 3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$

Sol:

Sec A = $\frac{17}{8}$ verify that $\frac{3-4\sin^2 A}{4\cos^2 A - 3} = \frac{3-\tan^2 A}{1-3\tan^2 A}$

We know sec A = $\frac{hypotenuse}{adjacent\ side}$

Consider right angled triangle ABC



Let x be the adjacent side

By applying Pythagoras we get

$AC^2 = AB^2 + BC^2$

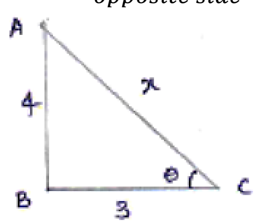
$(17)^2 = x^2 + 64$

$x^2 = 289 - 64$
$x^2 = 225 \Rightarrow x = 15$
Sin A = $\frac{AB}{BC} = \frac{15}{17}$
Cos A = $\frac{BC}{AC} = \frac{8}{17}$
Tan A = $\frac{AB}{BC} = \frac{15}{8}$
L.H.S = $\frac{3-4\sin^2 A}{4\cos^2 A-3} = \frac{3-4\times\left(\frac{15}{17}\right)^2}{4\times\left(\frac{8}{17}\right)^2-3} = \frac{3-4\times\frac{225}{289}}{4\times\frac{64}{289}-3} = \frac{867-900}{256-867} = \frac{-33}{-611} = \frac{33}{611}$
R.H.S = $\frac{3-\tan^2 A}{1-3\tan^2 A} = \frac{3-\left(\frac{15}{8}\right)^2}{1-3\times\left(\frac{15}{8}\right)^2} = \frac{3-\frac{225}{64}}{1-3\times\frac{225}{64}} = \frac{\frac{-33}{64}}{\frac{-611}{64}} = \frac{-33}{-611} = \frac{33}{611}$
∴ LHS = RHS

26. If cot $\theta = \frac{3}{4}$, prove that $\sqrt{\frac{\sec\theta - cosec\,\theta}{\sec\theta + cosec\,\theta}} = \frac{1}{\sqrt{7}}$

Sol:

Cot $\theta = \frac{3}{4}$ P.T $\sqrt{\frac{\sec\theta - cosec\,\theta}{\sec\theta + cosec\,\theta}} = \frac{1}{\sqrt{7}}$
Cot $\theta = \frac{adjacent\ side}{opposite\ side}$

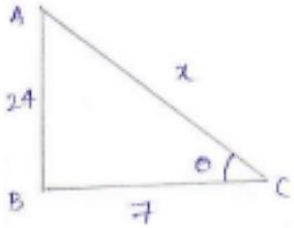


Let x be the hypotenuse by applying Pythagoras theorem.
$AC^2 = AB^2 + BC^2$
$x^2 = 16 + 9$
$x^2 = 25 \Rightarrow x = 5$
$\sec\theta = \frac{AC}{BC} = \frac{5}{3}$
Cosec $\theta = \frac{AC}{AB} = \frac{5}{4}$
On substituting in equation we get
$\sqrt{\frac{\sec\theta - cosec\,\theta}{\sec\theta + cosec\,\theta}} = \sqrt{\frac{\frac{5}{3}-\frac{5}{4}}{\frac{5}{3}+\frac{5}{4}}}$
$= \sqrt{\frac{\frac{20-15}{12}}{\frac{20+15}{12}}} = \sqrt{\frac{5}{35}} = \frac{1}{\sqrt{7}}$

27. If tan $\theta = \frac{24}{7}$, find that sin θ + cos θ

Sol:

Tan θ = $\frac{24}{7}$ $find\ \sin\theta + \cos\theta$



Let x $-$ 1 be the hypotenuse By applying Pythagoras theorem we get

$AC^2 = AB^2 + BC^2$

$x^2 = (24)^2 + (7)^2$

$x^2 = 576 + 49 = 62.5$

x = 25

sin θ = $\frac{AB}{AC} = \frac{24}{25}$

cos θ = $\frac{BC}{AC} = \frac{7}{25}$

sin θ + cos θ = $\frac{24}{25} + \frac{7}{25}$

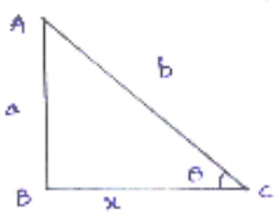
= $\frac{31}{25}$

28. If sin θ = $\frac{a}{b}$, find sec θ + tan θ in terms of a and b.

Sol:

Sin θ = $\frac{a}{b}$ find sec θ + tan θ

We know sin θ = $\frac{opposite\ side}{hypotenuse}$



Let x be the adjacent side

By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$b^2 = a^2 + x^2$

$x^2 = b^2 - a^2$

$x = \sqrt{b^2 - a^2}$

sec $\theta = \frac{AC}{BC} = \frac{b}{\sqrt{b^2-a^2}}$

Tan θ = $\frac{AB}{BC} = \frac{a}{\sqrt{b^2-a^2}}$

Sec θ + tan θ = $\frac{b}{\sqrt{b^2-a^2}} + \frac{a}{\sqrt{b^2-a^2}}$

$= \frac{b+a}{\sqrt{b^2-a^2}} = \frac{b+a}{\sqrt{(b+a)(b-a)}} = \frac{b+a}{\sqrt{b+a}} - \frac{1}{\sqrt{b-a}} = \sqrt{\frac{b+a}{b-a}}$

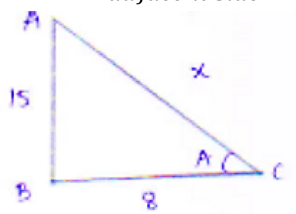
29. If 8 tan A = 15, find sin A – cos A.

Sol:

8 tan A = 15 find. Sin A – cos A

Tan A = $\frac{15}{8}$

Tan A = $\frac{opposite\ side}{adjacent\ side}$



Let x be the hypotenuse By applying theorem.

$AC^2 = AB^2 + BC^2$

$x^2 = 15^2 + 8^2$

$x^2 = 225 + 64$

$x^2 = 289 \Rightarrow$ x = 17

$Sin\ A = \frac{AB}{AC} = \frac{15}{17}$

Sin A – cos A = $\frac{15}{17} - \frac{8}{17}$

$= \frac{7}{17}$

30. If 3cos θ – 4sin θ = 2cos θ + sinθ Find tan θ

Sol:

3 cos θ – 2 cos θ = 4 sin θ + sin θ find tan θ

3 cos θ – 2 cos θ = sin θ + 4 sin θ

Cos θ = 5 sin θ

Dividing both side by use we get

$\frac{\cos\theta}{\cos\theta} = \frac{5\sin\theta}{\cos\theta}$

1 = 5 tan θ

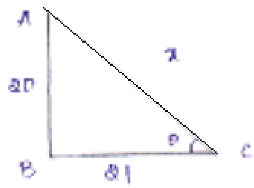
$\Rightarrow \tan\theta = 1$

31. If tan $\theta = \frac{20}{21}$, show that $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{3}{7}$

Sol:

Tan $\theta = \frac{20}{21}$ S.T $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta} = \frac{3}{7}$

Tan θ = $\frac{opposite\ side}{efficient\ side} = \frac{20}{21}$



Let x be the hypotenuse By applying Pythagoras we get

$AC^2 + AB^2 + BC^2$

$x^2 = (20)^2 + (21)^2$

$x^2 = 400 + 441$

$x^2 = 841 \Rightarrow$ x = 29

$Sin\ \theta = \frac{AB}{AC} = \frac{20}{29}$

Cos θ = $\frac{BC}{AC} = \frac{21}{29}$

Substitute sin θ, cos θ in equation we get

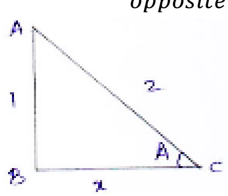
$\Rightarrow \frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta}$

$\Rightarrow \frac{1-\frac{20}{29}+\frac{21}{29}}{1+\frac{20}{29}+\frac{21}{29}} = \frac{\frac{29-20+21}{29}}{\frac{29+20+21}{29}} = \frac{30}{70} = \frac{3}{7}$

32. If Cosec A = 2 find $\frac{1}{Tan\ A} + \frac{\sin A}{1+\cos A}$

Sol:

Cosec A = $\frac{hypotenuse}{opposite\ side} = \frac{2}{1}$



Let x be the adjacent side

By applying Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$4 = 1 + x^2$

$x^2 = 3 \Rightarrow x = \sqrt{3}$

Sin A = $\frac{1}{cosec\ A} = \frac{1}{2}$

Tan A = $\frac{AB}{BC} = \frac{1}{\sqrt{3}}$

Cos A = $\frac{BC}{AC} = \frac{\sqrt{3}}{2}$

Substitute in equation we get

$\frac{1}{TanA} + \frac{\sin A}{1+\cos A} = \frac{1}{\frac{1}{\sqrt{3}}} + \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$

$= \sqrt{3} + \frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}} = \sqrt{3} + \frac{1}{2+\sqrt{3}} = \frac{2\sqrt{3}+3+1}{2+\sqrt{3}} = \frac{2\sqrt{3}+4}{2+\sqrt{3}} = \frac{2(2+\sqrt{3})}{2+\sqrt{3}} = 2$

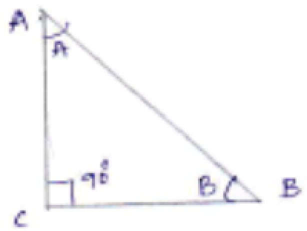
33. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Sol:

∠A and ∠B are acute angles.

Cos A = cos B S.T ∠A = ∠B

Let us consider right angled triangle ACB.



We have cos A = $\frac{adjacent\ side}{Hypotenuse}$

= $\frac{AC}{AB}$

Cos B = $\frac{BC}{AB}$

$Cos\ A = \cos B$

$\frac{AC}{AB} = \frac{BC}{AB}$

AC = BC

∠A = ∠B

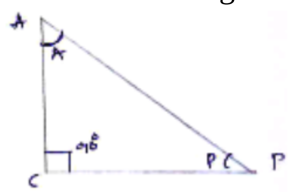
34. If ∠A and ∠P are acute angles such that tan A = tan P, then show that ∠A = ∠P.

Sol:

A and P are acute angle tan A = tan P

S. T. ∠A = ∠P

Let us consider right angled triangle ACP,



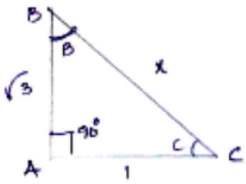
We know tan θ = $\frac{opposite\ side}{adjacent\ side}$

Tan A = $\frac{PC}{AC}$

Tan A = $\frac{AC}{PC}$

Tan A = $\frac{AC}{PC}$
Tan = tan P
$\frac{DC}{AC} = \frac{AC}{PC}$
$(PC)^2 = (AC)^2$
$PC = AC$ [∵ Angle opposite to equal sides are equal]
$\angle P = \angle A$

35. In a ΔABC, right angled at A, if $\tan C = \sqrt{3}$, find the value of $\sin B \cos C + \cos B \sin C$.
Sol:
In a Δle ABC right angled at A tan C = $\sqrt{3}$
Find sin B cos C + cos B sin C



Tan c = $\sqrt{3}$
Tan C = $\frac{opposite\ side}{adjacent\ side}$
Let x be the hypotenuse By applying Pythagoras we get
$BC^2 = BA^2 + AC^2$
$x^2 = \left(\sqrt{3}\right)^2 + 1^2$
$x^2 = \Delta \Rightarrow x = 2$
At ∠B, $\sin B = \frac{AC}{BC} = \frac{1}{2}$
$\text{Cos B} = \frac{\sqrt{3}}{2}$
At ∠C, $\sin = \frac{\sqrt{3}}{2}$
$\text{Cos c} = \frac{1}{2}$
On substitution we get
$\Rightarrow \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$
$\Rightarrow \frac{1}{4} + \frac{(\sqrt{3})}{4} \times \left(\sqrt{3}\right) = \frac{\sqrt{3}\times\sqrt{3}+1}{4} = \frac{3+1}{4} = \frac{4}{4} = 1$

36. State whether the following are true or false. Justify your answer.
(i) The value of tan A is always less than 1.
(ii) Sec A = $\frac{12}{5}$ for some value of angle A.
(iii) Cos A is the abbreviation used for the cosecant of angle A.
(iv) Sin θ = $\frac{4}{3}$ for some angle θ.

Sol:

(a) Tan A ∠1

Value of tan A at 45° i.e., tan 45 = 1

As value of A increases to 90°

Tan A becomes infinite

So given statement is false.

(b) Sec A = $\frac{12}{5}$ for some value of angle of

M-I

Sec A = 2.4

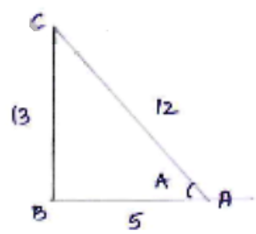
Sec A > 1

So given statement is True

M-II

For sec A = $\frac{12}{5}$

For sec A = $\frac{12}{5}$ we get adjacent side = 13



We get a right angle Δle

Subtending 9i at B.

So, given statement is true

(c) Cos A is the abbreviation used for cosecant of angle A.

The given statement is false. ∴ Cos A is abbreviation used for cos of angle A but not for cosecant of angle A.

(d) Cot A is the product of cot A and A

Given statement is false

∵ cot A is co-tangent of angle A and co-tangent of angle A = $\frac{adjacent\ side}{opposite\ side}$

(e) Sin θ = $\frac{4}{3}$ for some angle θ

Given statement is false

Since value of sin θ is less than (or) equal to one. Here value of sin θ exceeds one, so given statement is false.

Exercise 5.2

Evaluate each of the following (1 – 19):

1. $\sin 45° \sin 30° + \cos 45° \cos 30°$

Sol:

$\sin 45° \sin 30° + \cos 45° \cos 30° \dots.(i)$

We know that by trigonometric ratios we have,

Sin 45° $= \frac{1}{\sqrt{2}}$ $\sin 30° = \frac{1}{2}$

Cos 45° $= \frac{1}{\sqrt{2}}$ $\cos 30° = \frac{\sqrt{3}}{2}$

Substituting the values in (i) we get

$\frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

2. Sin 60° cos 30° + cos 60° sin 30°

Sol:

Sin 60° cos 30° + cos 60° sin 30° …(i)

By trigonometric ratios we have,

Sin 60° = $\frac{\sqrt{3}}{2}$ sin 30° = $\frac{1}{2}$

Cos 30° = $\frac{\sqrt{3}}{2}$ cos 60° = $\frac{1}{2}$

Substituting above values in (i), we get

$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

3. Cos 60° cos 45° - sin 60° · sin 45°

Sol:

Cos 60° cos 45° - sin 60° · sin 45° …(i)

By trigonometric ratios we know that,

Cos 60° = $\frac{1}{2}$ cos 45° = $\frac{1}{\sqrt{2}}$

Sin 60° = $\frac{\sqrt{3}}{2}$ sin 45° = $\frac{1}{\sqrt{2}}$

By substituting above value in (i), we get

$\frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \Rightarrow \frac{1-\sqrt{3}}{2\sqrt{2}}$

4. $Sin^2\,30° + \sin^2 45° + \sin^2 60° + \sin^2 90°$

Sol:

$Sin^2\,30° + \sin^2 45° + \sin^2 60° + \sin^2 90°$ …(i)

By trigonometric ratios we have

Sin 30° = $\frac{1}{2}$ sin 45° = $\frac{1}{\sqrt{2}}$

Sin 60° = $\frac{\sqrt{3}}{2}$ sin 90° = 1

By substituting above values in (i), we get

$= \left[\frac{1}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{\sqrt{3}}{2}\right]^2 + [1]^2$

$= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1 \Rightarrow \frac{1+3}{4} + \frac{1+2}{2}$

$\Rightarrow 1 + \frac{3}{2} = \frac{2+3}{2} = \frac{5}{2}$

5. $\cos^2 30° + \cos^2 45° + \cos^2 60° + \cos^2 90°$

Sol:

$\cos^2 30° + \cos^2 45° + \cos^2 60° + \cos^2 90°$ …(i)

By trigonometric ratios we have

Cos 30° = $\frac{\sqrt{3}}{2}$ cos 45° = $\frac{1}{\sqrt{2}}$

Cos 60° = $\frac{1}{2}$ cos 90° = 0

By substituting above values in (i), we get

$\left[\frac{\sqrt{3}}{2}\right]^2 + \left[\frac{1}{\sqrt{2}}\right]^2 + \left[\frac{1}{2}\right]^2 + [1]^2$

$\frac{3}{4} + \frac{1}{2} + \frac{1}{4} = 0 \;\Rightarrow 1 + \frac{1}{2} = \frac{3}{2}$

6. $\tan^2 30° + \tan^2 60° + \tan^2 45°$

Sol:

$\tan^2 30° + \tan^2 60° + \tan^2 45°$ …(i)

By trigonometric ratios we have

Tan 30° = $\frac{1}{\sqrt{3}}$ tan 60° = $\sqrt{3}$ tan 45° = 1

By substituting above values in (i), we get

$\left[\frac{1}{\sqrt{3}}\right]^2 + \left[\sqrt{3}\right]^2 + [1]^2$

$\Rightarrow \frac{1}{3} + 3 + 1 \Rightarrow \frac{1}{3} + 4$

$\Rightarrow \frac{1+12}{3} = \frac{13}{3}$

7. $2\sin^2 30° - 3\cos^2 45° + \tan^2 60°$

Sol:

$2\sin^2 30° - 3\cos^2 45° + \tan^2 60°$ …(i)

By trigonometric ratios we have

Sin 30° = $\frac{1}{2}$ cos 45° $\frac{1}{\sqrt{2}}$ tan 60° = $\sqrt{3}$

By substituting above values in (i), we get

$2\cdot\left[\frac{1}{2}\right]^2 - 3\left[\frac{1}{\sqrt{2}}\right]^2 + \left[\sqrt{3}\right]^2$

$2.\frac{1}{4} - 3.\frac{1}{2} + 3$

$\frac{1}{2} - \frac{3}{2} + 3 \Rightarrow \frac{3}{2} + 2 = 2$

8. $\sin^2 30° \cos^2 45\,° + 4\tan^2 30° + \frac{1}{2}\sin^2 90° - 2\,cos^2\,90° + \frac{1}{24}\cos^2 0\,°$

Sol:

$\sin^2 30° \cos^2 45\,° + 4\tan^2 30° + \frac{1}{2}\sin^2 90° - 2\,cos^2\,90° + \frac{1}{24}\cos^2 0\,°$ …(i)

By trigonometric ratios we have

Sin 30° = $\frac{1}{2}$ cos 45° = $\frac{1}{\sqrt{2}}$ tan 30° = $\frac{1}{\sqrt{3}}$ sin 90° = 1 cos 90° = 0 cos 0° = 1

By substituting above values in (i), we get

$\left[\frac{1}{2}\right]^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2 + 4\left[\frac{1}{\sqrt{3}}\right]^2 + \frac{1}{2}[1]^2 - 2[0]^2 + \frac{1}{24}[1]^2$

$\frac{1}{4}\cdot\frac{1}{2} + 4\cdot\frac{1}{3} + \frac{1}{2} - 0 + \frac{1}{24}$

$\frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24} = \frac{48}{24} = 2$

9. $4(\sin^4 60° + \cos^4 30°) - 3(\tan^2 60° - \tan^2 45°) + 5\cos^2 45°$

Sol:

$4(\sin^4 60° + \cos^4 30°) - 3(\tan^2 60° - \tan^2 45°) + 5\cos^2 45°$ …(i)

By trigonometric ratios we have

Sin 60° = $\frac{\sqrt{3}}{2}$ cos 30° = $\frac{\sqrt{3}}{2}$ tan 60° = $\sqrt{3}$ tan 45° = 1 cos 45 ° = $\frac{1}{\sqrt{2}}$

By substituting above values in (i), we get

$4\left(\left[\frac{\sqrt{3}}{2}\right]^4 + \left[\frac{\sqrt{3}}{2}\right]^4\right) - 3([3]^2 - [1]^2) + 5\left[\frac{1}{\sqrt{2}}\right]^2$

$\Rightarrow 4\left[\frac{9}{16} + \frac{9}{16}\right] - 3[3-1] + 5\left[\frac{1}{2}\right]$

$\Rightarrow 4\cdot\frac{18}{16} - 6 + \frac{5}{2}$

$\Rightarrow \frac{1}{4} - 6 + \frac{5}{2}$

$= \frac{9}{2} + \frac{5}{2} - 6$

$= \frac{14}{2} - 6 = 7 - 6 = 1$

10. $(cosec^2 45° \sec^2 30°)(\sin^2 30° + 4\cot^2 45° - \sec^2 60°)$

Sol:

$(cosec^2 45° \sec^2 30°)(\sin^2 30° + 4\cot^2 45° - \sec^2 60°)$ …(i)

By trigonometric ratios we have

Cosec 45° = $\sqrt{2}$ sec 30° = $\frac{2}{\sqrt{3}}$ sin 30° = $\frac{1}{2}$ cot 45° = 1 sec 60° = 2

By substituting above values in (i), we get

$\left([\sqrt{2}]^2 \cdot \left[\frac{2}{\sqrt{3}}\right]^2\right)\left(\left[\frac{1}{2}\right]^2 + 4[1]^2 \cdot [2]^2\right)$

$\Rightarrow \left[2 \cdot \frac{4}{3}\right]\left[\frac{1}{4} + 4 - 4\right] \Rightarrow 3 \cdot \frac{4}{3} \cdot \frac{1}{4} = \frac{2}{3}$

11. $cosec^3 30° \cos 60° \tan^3 45° \sin^2 90° \sec^2 45° \cot 30°$

Sol:

$cosec^3 30° \cos 60° \tan^3 45° \sin^2 90° \sec^2 45° \cot 30°$ …(i)

By trigonometric ratios we have

Cosec 30° = 2, cos 60° = $\frac{1}{2}$, tan 45° = 1 sin 90° = 1 sec 45° = $\sqrt{2}$ cot 30° = $\sqrt{3}$

By substituting above values in (i), we get

$[2]^3 \cdot \frac{1}{2} \cdot (1)^3 \cdot (1)^2 (\sqrt{2})^2 \cdot \sqrt{3}$

$\Rightarrow 8 \cdot \frac{1}{2} \cdot 1 \cdot 2 \cdot \sqrt{3} \Rightarrow 8\sqrt{3}$

12. $\cot^2 30° - 2\cos^2 60° - \frac{3}{4}\sec^2 45° - 4\sec^2 30°$

Sol:

$\cot^2 30° - 2\cos^2 60° - \frac{3}{4}\sec^2 45° - 4\sec^2 30°$ …(i)

By trigonometric ratios we have

cot 30° = $\sqrt{3}$ cos 60° = $\frac{1}{2}$ sec 45° = $\sqrt{2}$ sec 30° = $\frac{2}{\sqrt{3}}$

By substituting above values in (i), we get

$(\sqrt{3})^2 - 2\left[\frac{1}{2}\right]^2 - \frac{3}{4}(\sqrt{2})^2 - 4\left[\frac{2}{\sqrt{3}}\right]^2$

$3 - 2 \cdot \frac{1}{4} - \frac{3}{4} \cdot 2 - 4 \cdot \frac{4}{3}$

$3 - \frac{1}{2} - \frac{3}{2} - \frac{8}{3} \Rightarrow -\frac{5}{3}$

13. $(\cos 0^\circ + \sin 45° + \sin 30°)(\sin 90° - \cos 45° + \cos 60°)$

Sol:

$(\cos 0^\circ + \sin 45° + \sin 30°)(\sin 90° - \cos 45° + \cos 60°)$ …(i)

By trigonometric ratios we have

Cos 0° = 1, sin 45° = $\frac{1}{\sqrt{2}}$, sin 30° = $\frac{1}{2}$, sin 90° = 1, cos 45° = $\frac{1}{\sqrt{2}}$ cos 60° = $\frac{1}{2}$

By substituting above values in (i), we get

$\left(1+\frac{1}{\sqrt{2}}+\frac{1}{2}\right)\left(1-\frac{1}{\sqrt{2}}+\frac{1}{2}\right)$

$\left[\frac{3}{2}+\frac{1}{\sqrt{2}}\right]\left[\frac{3}{2}-\frac{1}{\sqrt{2}}\right] \Rightarrow \left[\frac{3}{2}\right]^2-\left[\frac{1}{\sqrt{2}}\right]=\frac{9}{4}-\frac{1}{2}=\frac{7}{4}$

14. $\frac{\sin 30°-\sin 90°+2\cos 0°}{\tan 30° \tan 60°}$

Sol:

$\frac{\sin 30°-\sin 90°+2\cos 0°}{\tan 30° \tan 60°}$ …(i)

By trigonometric ratios we have

Sin 30° = $\frac{1}{2}$ sin 90° = 1 cos 0° = 1 tan 30° = $\frac{1}{\sqrt{3}}$ tan 60° = $\sqrt{3}$

By substituting above values in (i), we get

$\frac{\frac{1}{2}-1+2}{\sqrt{3}\cdot\frac{1}{\sqrt{3}}}=\frac{\frac{3}{2}+1}{1}=\frac{3}{2}$

15. $\frac{4}{\cot^2 30°}+\frac{1}{\sin^2 60°}-\cos^2 45°$

Sol:

$\frac{4}{\cot^2 30°}+\frac{1}{\sin^2 60°}-\cos^2 45°$ …(i)

By trigonometric ratios we have

Cot 30° = $\sqrt{3}$ sin 60° = $\frac{\sqrt{3}}{2}$ cos 45° = $\frac{1}{\sqrt{2}}$

By substituting above values in (i), we get

$\frac{4}{(\sqrt{3})^2}+\frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}-\left(\frac{1}{\sqrt{2}}\right)^2$

$\frac{4}{3}+\frac{4}{3}-\frac{1}{2}=\frac{13}{6}$

16. $4(\sin^4 30°+\cos^2 60°)-3(\cos^2 45°-\sin^2 90°)-\sin^2 60°$

Sol:

$4(\sin^4 30°+\cos^2 60°)-3(\cos^2 45°-\sin^2 90°)-\sin^2 60°$ …(i)

By trigonometric ratios we have

Sin 30° = $\frac{1}{2}$ cos 60° = $\frac{1}{2}$ cos 45° = $\frac{1}{\sqrt{2}}$ Sin 90° = 1 sin 60° = $\frac{\sqrt{3}}{2}$

By substituting above values in (i), we get

$4\left[\left(\frac{1}{2}\right)^4+\left(\frac{1}{2}\right)^2\right]-3\left[\left[\frac{1}{\sqrt{2}}\right]^2-1\right]-\left[\frac{\sqrt{3}}{2}\right]^2$

$4\left[\frac{1}{16}+\frac{1}{4}\right]-3\left[\frac{1-[\sqrt{2}]}{(\sqrt{2})^2}\right]-\frac{3}{4}$

$\frac{1}{4}+1-3\left[\frac{1-[\sqrt{2}]}{[\sqrt{2}]}\right]^2-\frac{3}{4}$

$=\frac{1}{4}+1-\frac{3}{4}+\frac{3}{2}=2$

17. $\frac{\tan^2 60°+4\cos^2 45°+3\sec^2 30°+5\cos^2 90°}{cosec\ 30°+\sec 60°-\cot^2 30°}$

Sol:

$\frac{\tan^2 60°+4\cos^2 45°+3\sec^2 30°+5\cos^2 90°}{cosec\ 30°+\sec 60°-\cot^2 30°}$ …(i)

By trigonometric ratios we have

Tan 60° = $\sqrt{3}$ cos 45° = $\frac{1}{\sqrt{2}}$ sec 30° = $\frac{2}{\sqrt{3}}$

cos 90° = 0 cosec 30° = 2 sec 60° = 2 cot 30° = $\sqrt{3}$

By substituting above values in (i), we get

$\frac{(\sqrt{3})^2+4\cdot\left(\frac{1}{\sqrt{3}}\right)^2+2+\left[\frac{2}{\sqrt{3}}\right]^2+5(0)^2}{2+2\sqrt{2}(+\sqrt{3})^2}$

$=\frac{3+4\cdot\frac{1}{2}+3\cdot\frac{4}{3}}{4-3}=\frac{3+2+4}{1}=9$

18. $\frac{\sin 30°}{\sin 45°}+\frac{\tan 45°}{\sec 60°}-\frac{\sin 60°}{\cot 45°}-\frac{\cos 30°}{\sin 90°}$

Sol:

$\frac{\sin 30°}{\sin 45°}+\frac{\tan 45°}{\sec 60°}-\frac{\sin 60°}{\cot 45°}-\frac{\cos 30°}{\sin 90°}$ …(i)

By trigonometric ratios we have

Sin 30° = $\frac{1}{2}$ sin 45° = $\frac{1}{\sqrt{2}}$ tan 45° = 1 sec 60° = 2 sin 60° = $\frac{\sqrt{3}}{2}$

cot 45° = 1 cos 30° = $\frac{\sqrt{3}}{2}$ sin 90° = 1

By substituting above values in (i), we get

$\frac{1}{2}\cdot\sqrt{2}+\frac{1}{2}-\frac{\sqrt{3}}{2}\cdot 1-\frac{\sqrt{3}}{2}.1$

$=\frac{2+1-\frac{2}{3}}{2}$

19. $\frac{Tan\ 45°}{cosec\ 30°}+\frac{\sec 60°}{\cot 45°}-\frac{5\sin 90°}{2\cos 0°}$

Sol:

$\frac{Tan\ 45°}{cosec\ 30°}+\frac{\sec 60°}{\cot 45°}-\frac{5\sin 90°}{2\cos 0°}$ …(i)

By trigonometric ratios we have

Tan 45° = 1 cosec 30° = 2 sec 60° = 2 cot 45° = 1 sin 90° = 1 cos 0° = 1

By substituting above values in (i), we get

$\frac{1}{2}+\frac{2}{1}-5\cdot\frac{1}{2}$

$-\frac{4}{2}+2=-2+2=0$

20. 2sin 3x = $\sqrt{3}$ s = ?

Sol:

Sin 3x = $\frac{\sqrt{3}}{2}$

Sin 3x = sin 60°

Equating angles we get,

3x = 60°

x = 20°

21. 2 sin $\frac{x}{2} = 1$ x = ?

Sol:

Sin $\frac{x}{2} = \frac{1}{2}$

Sin $\frac{x}{2} = \sin 30°$

$\frac{x}{2} = 30°$

$x = 60°$

22. $\sqrt{3} \sin x = \cos x$

Sol:

$\sqrt{3}$ tan x = 1

$\tan x = \frac{1}{\sqrt{3}}$

∴ Tan x = Tan 30°

x = 30°

23. Tan x = sin 45° cos 45 ° + sin 30°

Sol:

Tan x = $\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2}$ [∵ sin 45° = $\frac{1}{\sqrt{2}}$ cos 45° = $\frac{1}{\sqrt{2}}$ $\sin 30° = \frac{1}{2}$]

Tan x = $\frac{1}{2} + \frac{1}{2}$

Tan x = 1

Tan x = tan 45°

x = 45°

24. $\sqrt{3} \tan 2x = \cos 60° + \sin 45° \cos 45°$

Sol:

$\sqrt{3} \tan 2x = \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$ $\left[\because \cos 60° = \frac{1}{2} \sin 45° = \cos 45° = \frac{1}{\sqrt{2}}\right]$

$\sqrt{3} \tan 2x = \frac{1}{\sqrt{3}} \Rightarrow \tan 2x = \tan 30°$

2x = 30°

x = 15°

25. Cos 2x = cos 60° cos 30° + sin 60° sin 30°

Sol:

Cos 2x = $\frac{1}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{1}{2}$ $\left[\because \cos 60° = \sin 30° = \frac{1}{2} \sin 60° = \cos 30° = \frac{\sqrt{3}}{2}\right]$

Cos 2x = $2\cdot\frac{\sqrt{3}}{4}$

⇒ cos 2x = $\frac{\sqrt{3}}{2}$

Cos 2x = cos 30°

$2x = 30°$

$x = 15°$

26. If θ = 30° verify

(i) Tan 2θ = $\frac{2\tan\theta}{1-\tan^2\theta}$

Sol:

Tan 2θ = $\frac{2\tan\theta}{1-\tan^2\theta}$ …(i)

Substitute θ = 30° in (i)

LHS = Tan 60° = $\sqrt{3}$

RHS = $\frac{2\tan 30°}{1-\tan^2 30°} = \frac{2\cdot\frac{1}{\sqrt{3}}}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

= $\frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \sqrt{3}$

∴ LHS = RHS

(ii) Sin θ = $\frac{2\tan\theta}{1-\tan^2\theta}$

Substitute θ = 30°

Sin 60° = $\frac{2\tan 30°}{1+(\tan 30°)^2}$

= $\frac{\sqrt{3}}{2} = \frac{2\cdot\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$

= $\frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}}\cdot\frac{3}{4} \Rightarrow \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

∴ LHS = RHS

(iii) Cos 2θ = $\frac{1-\tan^2\theta}{1+\tan^2\theta}$

Substitute θ = 30°

LHS = cosec θ RHS = $\frac{1-\tan^2\theta}{1+\tan^2\theta}$

= cos 2(30°) = $\frac{1-\tan^2 30°}{1+\tan^2 30°}$

Cos 60° = $\frac{1}{2}$ = $\frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{1}{2}$

∴ LHS = RHS

(iv) Cos 30θ = 4 cos$^3\theta$ – 3 cos θ

LHS = Cos 30° RHS 4 cos^3 θ – 3 cos θ

Substitute $\theta = 30°$ 4cos^3 30° - 3 cos 30°

$\text{Cos } 3\ (30°) = \cos 90°$ $4\cdot\left[\frac{\sqrt{3}}{2}\right]^3 - 3\cdot\frac{\sqrt{3}}{2}$

$= 0$ $\Rightarrow \frac{3\sqrt{3}}{2} - \frac{3\sqrt{2}}{2} = 0$

27. If A = B = 60°. Verify

(i) Cos (A – B) = Cos A cos B + sin A sin B

Sol:

Cos (A – B) = Cos A cos B + sin A sin B …(i)

Substitute A & B in (i)

⇒ cos (60 - 60°)= cos 60° cos 60° + sin 60° sin 60°

Cos 0° = $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

1 = $\frac{1}{4} + \frac{3}{4} = 1 = 1$ LHS = RHS

(ii) Substitute A & B in (i)

Sin (60° - 60°) = Sin 60° Cos 60° − cos 60° sin 60°

= sin 0° = 0 = 0

LHS = RHS

(iii) Tan (A – B) = $\frac{Tan\ A - \tan B}{1 + \tan A \tan B}$

A = 60° B = 60° we get

Tan (60° - 60°) = $\frac{\tan 60° - \tan 60°}{1 - \tan 60 \tan 60°}$

Tan 0° = 0

0 = 0

LHS = RHS

28. If A = 30° B = 60° verify

(i) Sin (A + B) = Sin A Cos B + cos A sin B

Sol:

A = 30°, B = 60° we get

Sin (30° + 60°) = Sin 30° cos 60° + cos 30° sin 60°

Sin 90° = $\frac{1}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}$

Sin 90° = 1 ⇒ 1 = 1

LHS = RHS

(ii) Cos (A + B) = cos A cos B – Sin A Sin B

A = 30° B = 60°

Cos (90°) = Cos 30° cos 60° $-$ sin 30° sin 60°

= cos 90° = $\frac{1}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$

0 = 0

LHS = RHS

29. Sin (A – B) = Sin A Cos B – cos A sin B

Cos (A – B) = cos A Cos B – sin A sin B

Find sin 15° cos 15°

Sol:

Sin (A – B) = Sin A Cos B – cos A sin B …(i)

Cos (A – B) = cos A Cos B – sin A sin B …(ii)

Let A = 45° B = 30° we get on substituting in (i)

⇒ Sin(45° $-$ 30°) = Sin 45° cos 30°

Sin 15° = $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

∴ Sin 15° = $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(ii) A = 45° B = 30° in equation (ii) we get

Cos (45° $-$ 30°) cos 45° cos 30° + sin 45° sin 30°

Cos 15° - $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$

Cos 15° ⇒ $\frac{\sqrt{3}+1}{2\sqrt{2}}$

30. In right angled triangle ABC. $\angle C = 90°$, $\angle B = 60°$. AB = 15units. Find remaining angles and sides.

Sol:

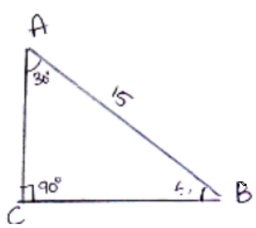
In a ∆le sum of all angles = 180°

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 90° + 60° + \angle A = 180°$

$\angle A = 180° - 150°$

$\therefore \angle A = 30°$



From above figure

Cos B $= \frac{BC}{AB}$

Cos 60° $= \frac{BC}{15}$

$\frac{1}{2} = \frac{BC}{15}$

BC = $\frac{15}{2}$

Sin B = $\frac{AC}{15}$

Sin 60° = $\frac{AC}{15}$

$\frac{\sqrt{3}}{2} = \frac{AV}{15} = AC = \frac{15\sqrt{3}}{2}$

31. In ΔABC is a right triangle such that $\angle C = 90°$ $\angle A = 45°$, $BC = 7$ units find $\angle B$, AB and AC

Sol:

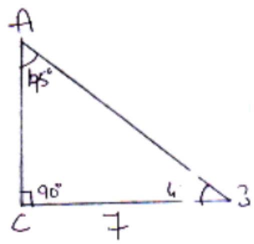
Sum of angles in Δle = 180°

$\angle A + \angle B + \angle C = 180°$

$45° + \angle B + 90° = 180°$

$\angle B = 180° - 135°$

$\angle B = 45°$



From figure $\cos B = \frac{BC}{AB}$

Cos 45° = $\frac{7}{AB}$

$\frac{1}{\sqrt{2}} \cdot \frac{7}{AB}$

AB = $7\sqrt{2}$ units

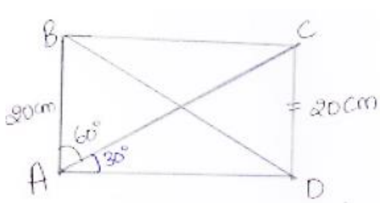
From figure sin B = $\frac{AC}{AB}$

Sin 45° = $\frac{AC}{7\sqrt{2}}$

$\frac{1}{\sqrt{2}} = \frac{AC}{7\sqrt{2}}$ $\therefore AC = 7\ units$

32. In rectangle ABCD AB = 20cm ∠BAC = 60° BC, calculate side BC and diagonals AC and BD.

Sol:



Consider Δle ABC we get

Cos A = $\frac{AB}{AC}$ Sin A = $\frac{BC}{AC}$

∴ cos 60° = $\frac{20}{AC}$ Sin 60° = $\frac{BC}{AC}$

$\frac{1}{a} = \frac{20}{AC}$ ∴ AC = 40 cm $\frac{\sqrt{3}}{2} = \frac{BC}{40}$

∴ AC = 40 cm ∴ BC = $20\sqrt{3}$ cm

Consider Δle ACD we know ∠CAD = 30°

∴ Tan 30° = $\frac{CD}{AD} = \frac{1}{\sqrt{3}} = \frac{20}{AC} = AD = 20\sqrt{3}$

In rectangle diagonals are equal in magnitude

∴ BD = AC = 40 cm

33. If Sin (A + B) = 1 and cos (A – B) = 1, 0° < A + B ≤ 90° A ≥ B. Fin A & B

Sol:

Sin(A + B) = 1

∴ Sin (A + B) = Sin 90°

A + B = 90° …(i)

Cos (A – B) = 1

Cos (A – B) = cos 0°

A – B = 0° …(ii)

Adding (i) & (ii) we get

A + B = 90°

<u>A – B = 0°</u>

A = 90° A = 45°

A – B = 0

A = B ⇒ B = 45°

34. If Tan (A – B) = $\frac{1}{\sqrt{3}}$ and Tan (A + B) = $\sqrt{3}$, $0° < A + B \le 90°$, A ≥ B, Find A & B

Sol:

Tan (A – B) = Tan 30° Tan (A + B) = Tan 60°

∴ A – B = 30° …(i) A + B = 60° …(ii)

Add (i) & (ii)

A – B = 30°

<u>A + B = 60°</u>

2A = 90° A = 40°

A – B = 30° 45° - B = 30°

B = 45° − 30° = 15°

35. If Sin (A – B) = $\frac{1}{2}$ and Cos (A + B) = $\frac{1}{2}$, 0° < A + B $\leq$ 90°, A $>$ B, Find A & B

Sol:

Sin (A – B) = sin 30° Cos (A + B) = cos 60°

A – B = 30° …(i)

A + B = 60° …(ii)

Add (i) & (ii) we get

2A = 90°, A = 45°.

A – B = 30°

45 – B = 30° B = 45 – 30°

B = 15°

36. In right angled triangle ΔABC at B, ∠A = ∠C. Find the values of

(i) Sin A cos C + Cos A Sin C

Sol:

In Δle ABC ∠A + ∠B + ∠C = 180°

$\angle A + 90° + \angle A = 180°$

$2\angle A = 90°$

$\angle A = 45°$

$\therefore \angle A = 45°$

(ii) $\text{Sin } 45° \cos 45° + \cos 45° \sin 45°$

$\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{2} \cdot \frac{1}{2} = 1$

$(ii) Sin\ A\ Sin\ B + \cos A \cos B$

∠A = 45° sin 90° + cos 45° cos 90°

= $\frac{1}{\sqrt{2}} \cdot 1 + 0$

= $\frac{1}{\sqrt{2}}$

37. Find acute angles A & B, if sin (A + 2B) = $\frac{\sqrt{3}}{2}$ Cos (A + 4B) = 0, A > B.

Sol:

Sin (A + 2B) = Sin 60°

Cos (A + 4B) = cos 90°

A + 2B = 60° …(i)

A + 4B = 90° …(ii)

Subtracting (ii) from (i)

A + 4B = 90°

$\underline{-A - 2B = -60}$

2B = 30° ∴ B = 15°

A + 4B = 90°

4B = 4(15°) = 4B = 60°

∴ A + 60° = 90° ∴ A = 30°

38. If A and B are acute angles such that Tan A = $\frac{1}{2}$ Tan B = $\frac{1}{3}$ *and* Tan (A + B) = $\frac{\tan A+Tan\ B}{1-\tan A\ Tan\ B}$ A + B = ?

Sol:

Tan A = $\frac{1}{2}$ Tan B = $\frac{1}{3}$

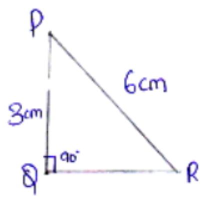
Tan (A + B) = $\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = \frac{\frac{5}{6}}{1-\frac{1}{6}} = 1$

Tan (A + B) = Tan 45°

∴ A – B = 45°

39. In ΔPQR, right angled at Q, PQ = 3cm PR = 6cm. Determine ∠P = ? ∠R = ?

Sol:



From above figure

Sin R = $\frac{PQ}{PR}$

Sin R = $\frac{3}{6} = \frac{1}{2}$

∴ Sin R = Sin 30°

R = 30°

We know in Δle ∠P + ∠Q + ∠R = 180°

∠P + 90° + 30° = 180°

∠P = 60°

Exercise 5.3

Evaluate the following:

1. $\frac{Sin\ 20°}{\cos 70°}$

Sol:

(i)

$\Rightarrow \frac{Sin\ (90°-70°)}{\cos 70°} \Rightarrow \frac{\cos 70°}{\cos 70°}$ $[\because Sin\ (90° - \theta) = \cos\theta]$

$\Rightarrow \frac{\cos 70°}{\cos 70°} = 1$

(ii)

$\frac{\cos 19°}{\sin 71°}$

$\Rightarrow \frac{\cos(90°-71°)}{\sin 71°} \Rightarrow \frac{\sin 71°}{\sin 71°}$ $[\because \cos(90° - \theta) = \sin\theta]$
= 1
(iii)
$\frac{\sin 21°}{\cos 69°} \Rightarrow \frac{\sin(\cos 69°)}{\cos 69°} = \frac{\cos 69°}{\cos 69°}$ $[\because \sin(90° - \theta) = cos\theta]$
= 1
(iv)
$\frac{Tan\ 10°}{Cot\ 80°} \Rightarrow \frac{\tan(90°-80°)}{\cot 80°} = \frac{\cot 80°}{\cot 80°}$ $[\because \tan(90 - \theta) = \cot\theta]$
= 1
(v)
$\frac{\sec 11°}{cosec\ 79°} \Rightarrow \frac{\sec(90°-79°)}{cosec\ 79°} = \frac{cosec\ 79°}{cosec\ 79°}$ $[\because \sec(90 - \theta) \cdot cosec\ \theta\ ]$
= 1

Evaluate the following:

2. (i) $\left[\frac{\sin 49°}{\cos 45}\right]^2 + \left[\frac{\cos 41°}{\sin 49°}\right]$

Sol:

We know that sin(49°) = sin(90° – 41°) = cos 41° similarly cos 41° = sin 49°

$\Rightarrow \left[\frac{\cos 41°}{\cos 41°}\right]^2 + \left[\frac{\sin 49°}{\sin 49°}\right]^2 = 1^2 + 1^2 = 2$

(ii)
Cos 48° - sin 42°

Sol:

Cos 48° = cos (90° − 42°) sin 42°
∴ sin 42° − sin42° = 0

(iii)
$\frac{\cot 40°}{\cos 35°} - \frac{1}{2}\left[\frac{\cos 35°}{\sin 55°}\right]$

Sol:

Cot 40° − cot (90° - 50°) = tan 50°
Cos 35° = cos (90° - 55°) = sin 55°
$\Rightarrow \frac{\tan 50°}{\tan 50°} - \frac{1}{2}\left[\frac{\sin 55°}{\sin 55°}\right]$
$= 1 - \frac{1}{2}[1]$
$= \frac{1}{2}$

(iv)
$\left[\frac{\sin 27°}{\cos 63°}\right] - \left[\frac{\cos 63°}{\sin 27°}\right]^2$

Sol:

Sin 27° = sin (90° - 63°) = cos 63° [∵ sin (90° - θ) = cos θ]

⇒ sin 27° = cos 63°

$\left[\frac{\sin 27°}{\sin 27°}\right]^2 - \left[\frac{\cos 63°}{\cos 63°}\right]^2 = 1 - 1 = 0$

(v)

$\frac{\tan 35°}{\cot 55°} + \frac{\cot 63°}{\cos 63°} - 1$

Sol:

Tan 35° = tan (90° - 55°) = cos 55°

Cot 78° = cot (90° - 12°) = tan 12°

⇒ $\frac{\cot 55°}{\cot 55°} + \frac{\tan 12°}{\tan 12°} - 1$

= tan 1 – 1 = 1

(vi)

$\frac{\sec 70°}{cosec\ 20°} + \frac{\sin 59°}{\cos 31°}$

Sol:

Sec 70° = sec (90° - 20°) = cosec 20° [∵ sec (90 – θ) = cosec θ]

Sin 59° = sin (90° - 31°) = cos 31° [∵ sin (90 - θ) = cos θ]

⇒ $\frac{cosec\ 20}{cosec\ 20} + \frac{\cos 31°}{\cos 31°} = 1 + 1 = 2$

(vii)

Sec 50° Sin 40° + Cos 40° cosec 50°

Sol:

Sec 50° = sec (90° - 40°) = cosec 40°

Cos 40° = cos (90° - 50°) = sin 50°

∴ Sin θ cosec θ = 1

⇒ cosec 40° sin 40° + sin 50° cosec 50°

1 + 1 = 2

3. Express each one of the following in terms of trigonometric ratios of angles lying between 0° and 45°

(i) Sin 59° + cos 56°

Sol:

Sin 59° = sin (90° - 59°) = cos 31°

Cos 56° = cos (65° - 34°) = Sin 34°

⇒ cos 31° + sin 34°

(ii)

Tan 65° + cot 49°

Sol:
Tan 65° = tan (90° - 25°) = cot 25°
Cot 49° = cot (90° - 41°) = tan (41°)
⇒ cot 25° + tan 41°
(iii)
Sec 76° + cosec 52°
Sol:
Sec 76° = sec (90° - 14°) = cosec 14°
Cosec 52° = cosec (90° - 88°) = sec 38°
⇒ Cosec 14° + sec 38°
(iv)
Cos 78° + sec 78°
Sol:
Cos 78° = cos (90° - 12°) = sin 12°
Sec 78° = sec (90° - 12°) = cosec 12°
⇒ sin 12° + cosec 12°
(v)
Cosec 54° + sin 72°
Sol:
Cosec 54° = cosec (90° - 36°) = sec 36°
Sin 72° = sin (90° - 18°) = cos 18°
⇒ sec 36° + cos 18°
(vi)
Cot 85° + cos 75°
Sol:
Cot 85° = cot (90° - 5°) = tan 5°
Cos 75° = cos (90° - 15°) = sin 15°
= tan 5° + sin 15°
(vii)
Sin 67° + cos 75°
Sol:
Sin 67° = Sin (90° − 23°) = cos 23°
Cos 75° = cos (90° −15°) = sin 15°
= cos 23° + sin 15°

4. Express Cos 75° + cot 75° in terms of angles between 0° and 30°.
Sol:
Cot 75° = cos (90° - 15°) = sin 15°
Cot 75° = cot (90° - 15°) = tan 15°
= sin 15° + tan 15°

5. If Sin 3A = cos (A – 26°), where 3A is an acute angle, find the value of A = ?

Sol:

Cos θ = sin (90° - θ)

⇒ Cos (A – 26) = sin (90° −(A− 26°))

⇒ Sin 3A = sin (90° − (A – 26))

Equating angles on both sides

3A = 90° − A + 26°

4A = 116° A = $\frac{116}{4} = 29°$

∴ A = 29°

6. If A, B, C are interior angles of a triangle ABC, prove that (i) tan $\left(\frac{C+A}{2}\right) = \cot\frac{B}{2}$

Sol:

(i) Tan $\left[\frac{c+A}{2}\right] = \cot\frac{B}{2}$

Sol:

Given A + B + C = 180°

C + A = 180° − B

⇒ Tan $\left[\frac{180-B}{2}\right] \Rightarrow Tan\left[90° - \frac{B}{2}\right]$

⇒ cot $\frac{B}{2}$ $[\because \tan(90° - \theta) = \cot\theta]$

∴ LHS = RHS

(ii) Sin $\left[\frac{B+C}{2}\right] = \cos\frac{A}{2}$

Sol:

A + B + C = 180°

B + C = 180° - A

LHS = sin $\left[\frac{180°-A}{2}\right] \Rightarrow \sin\left[90° - \frac{A}{2}\right]$

Cos $\frac{A}{2}$ $[\because Sin\ (90° - \theta) \cdot cos\theta]$

∴ LHS =RHS

7. Prove that

(i)

Tan20° Tan 35° tan 45° tan 55° Tan 70° = 1

Sol:

Tan 20° = tan (90° - 70°) = cot 70°

Tan 35° = tan (90° - 70°) = cot 55°

Tan 45° = 1

⇒ cot 70° tan 70° x cot 55° tan 55° x tan 45° · cot θ = tan θ = 1

⇒ 1 × 1 × 1 = 1 Hence proved.

(ii)
Sin 48° sec 42° + cosec 42° = 2
Sol:
Sin 48° = sin (90° - 42°) = cos 42°
Cos (45°) = cos (90° - 42°) = sin 42°
Sec θ · cos θ = 1 · sin θ cosec θ = 1
⇒ cos 42° sec 42° + sin 42° cosec 42°
⇒ 1 + 1 = 2
∴ LHS = RHS
(iii)
$\frac{\sin 70°}{\cos 20°} + \frac{cosec\ 20°}{\sec 70°} - 2\cos 70°\ cosec\ 20° = 0$
Sol:
Sin (70°) = sin (90° - 20°) = cos 20°
Cosec 20° = cosec (90° - 70°) = sec 70°
Cos 70° = cos (90° - 20°) = sin 20°
⇒ $\frac{\cos 20°}{\cos 20°} + \frac{\sec 70°}{\sec 70°} - 2\sin 20\ cosec\ 20°$
1 + 1 – 2(1) = 0
∴ LHS = RHS Hence proved
(iv)
$\frac{\cos 80°}{\sin 10°} + \cos 59°\ cosec\ 31° = 2$
Sol:
Cos 80° = cos (90° - 10°) = sin 10°
Cos 59° = cos (90° - 31°) = sin 31°
⇒ $\frac{\sin 10°}{\sin 10°} + \sin 31°\ cosec\ 31°$
= 1 + 1 = 2 [∵ Sin θ cosec θ = 1]
Hence proved

8. Prove the following:
(i) Sin θ sin (90 - θ) – cos θ cos (90 - θ) = 0
Sol:
Sin (90 – θ) = cos θ
Cos (90 – θ) – cos θ sin θ
= 0
∴ LHS = RHS
Hence proved

(ii) $\frac{\cos(90°-\theta)\sec(90°-\theta)\tan\theta}{cosec\ (90°-\theta)\sin(90°-\theta)\cot(90°-\theta)} + \frac{\tan(90°-\theta)}{\cot\theta} = 2$
Sol:
Cos (90° - θ) = sin A cosec (90 - θ) = sec θ

Sec (90° - θ) = cosec θ sin (90 - θ) = cos θ

Cot (90 - θ) = tan θ

$\Rightarrow \frac{\sin\theta \ cosec\ \theta \ \boldsymbol{tan}\,\theta}{\sec\theta.\cos\theta.\tan\theta} = \frac{\sin\theta \ cosec\ \theta}{\sec\theta\cos\theta}$ $[\because \sin\theta \ \ cosec\ \theta = 1]$

= 1 $[\sec\theta\cos\theta = 1]$

$\frac{\tan(90°-\theta)}{\cot\theta} = \frac{\cot\theta}{\cot\theta} = 1$

$\Rightarrow$ 1 + 1 = 2

$\therefore$ LHS = RHS

Hence proved

(iii) $\frac{\tan(90-A)\cot A}{cosec^2 A} - \cos^2 A = 0$

Sol:

Tan (90 – A) = cot A

$\Rightarrow \frac{\cot A\,.\cot A}{cosec^2 A} - \cos^2 A$

$\Rightarrow \frac{\cot^2 A}{cosec^2 A} - \cos^2 A$

= $\frac{\cos^2 A}{\sin^2 A} - \cos^2 A \Rightarrow \cos^2 A \cos^2 A = 0$

Hence proved

(iv) $\frac{\cos(90°-A)\sin(90°-A)}{\tan(90°-A)} - \sin^2 A = 0$

Sol:

Cos (90° - A) = sin A Tan (90° - A) = cot A

Sin (95° A) = cos A

$\frac{\sin A\cos A}{\cot A} - \sin^2 A = 0$

$\frac{\sin A.\cos A.}{\cos A}\sin A - \sin^2 A$

$\sin^2 A - \sin^2 A = 0$

LHS = RHS

Hence Proved

(v) Sin (50° + θ) – cos (40° – θ) + tan 1° tan 10° tan 20° tan 70° tan 80° tan 89° = 1

Sol:

Sin (50 + θ) = cos (90 – (50 + θ)) = cos (40 – θ)

Tan 1 = tan (90° − 89°) · cot 89°

Tan 10° = tan (90° - 80°) = cot 80°

Tan 20° = tan (90° - 70°) = cot 70°

$\Rightarrow$ cos (40° - θ) – cos (40 - θ) = cot 89° tan 89° . cot 80° . cot 70° tan 70°

Cot . tan θ = 1

= 1 · 1 · 1 = 1

LHS = RHS
Hence proved

9. Evaluate:

(i) $\frac{2}{3}\ (\cos^4 30° - \sin^4 45\ °) - 3(\sin^2 60° - \sec^2 45°) + \frac{1}{4}\cot^2 30°$

Sol:

Cos 30° $= \frac{\sqrt{3}}{2}$ sin 60° $= \frac{\sqrt{3}}{2}$ cot 30° $= \sqrt{3}$ sin 45° = $\frac{1}{\sqrt{2}}$ sec 45° = $\frac{1}{\sqrt{2}}$

Substituting above values in (i)

$\frac{2}{3}\left[\left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^4\right] - 3\left[\left(\frac{\sqrt{3}}{2}\right)^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2\right] + \frac{1}{4}\left(\sqrt{3}\right)^2$

$\frac{2}{3}\left[\frac{9}{16} - \frac{1}{4}\right] - 3\left[\frac{3}{4} - \frac{1}{2}\right]\frac{1-3}{4}$

$\frac{2}{3}\left[\frac{9-4}{16}\right] - 3\left[\frac{3-2}{4}\right] - \frac{3}{4}$

$\Rightarrow \frac{2}{3} \cdot \frac{5}{16} - \frac{3}{4} + \frac{3}{4} \Rightarrow \frac{5}{24}$

(ii) 4 ($\sin^2$ 30 + $\cos^4$ 60°) $-\frac{2}{3}\ 3\left[\left(\sqrt{\frac{3}{2}}\right)^2 \cdot \left[\frac{1}{\sqrt{2}}\right]^2\right] + \frac{1}{4}\left(\sqrt{3}\right)^2$

Sol:

$Sin\ 30° = \frac{1}{2}\cos 60 = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$ $\cos 45° = \frac{1}{\sqrt{2}}$ $\tan 60° = \sqrt{3}$

$\Rightarrow 4\left[\left[\frac{1}{2}\right]^4 + \left[\frac{1}{2}\right]^4\right] - \frac{2}{3}\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right] + \frac{1}{2}\left(\sqrt{3}\right)^2$

$4\left[2.\frac{1}{16}\right] - \frac{2}{3}\left[\frac{3}{4} - \frac{1}{2}\right] + \frac{3}{2}$

$= \frac{1}{2} - \frac{2}{3} \cdot \frac{1}{4} + \frac{3}{2} = \frac{11}{6}$

(iii) $\frac{\sin 50°}{\cos 40°} + \frac{co\sec 40°}{\sec 50°} - 4\cos 50°\ cosec\ 40°$

Sol:

Sin 50° = sin (90° - 40°) = cos 40°

Cosec 40° = cosec (90° - 50°) = sec 50°

Cos 50° = cos (90° - 40°) = sin 40°

$\Rightarrow \frac{\cos 40°}{\cos 40°} + \frac{\sec 50°}{sec\ 50°} - 4\sin 40°\ cosec\ 40°$

1 + 1 – 4 = − 2 [∵ Sin 40° cosec 40° = 1]

(iv) Tan 35° tan 40° tan 50° tan 55°

Sol:

Tan 35° = tan (90° - 55°) = cot 55°

Tan 40° = tan (90° - 50°) = cot 55°

Tan 65° = 1

Cot 55 tan 55· cot 50 tan 50 · tan 45
1 · 1 · 1 = 1

(v) Cosec (65 + θ) – sec (25 – θ) – tan (55 – θ) + cot (35 + θ)
Sol:
Cosec (65 + θ) = sec (90 – (65 + θ)) = sec (25 – θ)
Tan (55 – θ) = cot (90 – (55 – θ) = cot (35 + θ)
⇒ sec (25 – θ) – sec (25 – θ) tan (55 – θ) + tan (55 – θ) = 0

(vi) Tan 7° tan 23° tan 60° tan 67° tan 83°
Sol:
Tan 7° tan 23° tan 60° tan (90° - 23) tan (90° - 7°)
⇒ tan 7° tan 23° tan 60° cot 23° tan 60°
$1 \cdot 1 \cdot \sqrt{3} = \sqrt{3}$

(vii) $\frac{2 \sin 68}{\cos 22} - \frac{2 \cot 15°}{5 \tan 75°} - \frac{8 \tan 45° \tan 20° \tan 40° \tan 50° \tan 70°}{5}$
Sol:
Sin 68° = sin (90 - 22) = cos 22
Cot 15° = tan (90 - 75) = tan 75
$2 \cdot \frac{\cos 22}{\cos 22} - \frac{2 \tan 75°}{5 \tan 75°} - \frac{3 \tan 45° \tan 20° \tan 40° \cot 40° \cot 20°}{5}$
$= 2 - \frac{2}{5} - \frac{3}{5} = 2 - 1 = 1$

(viii) $\frac{3 \cos 55°}{7 \sin 35°} - \frac{4(\cos 70\ cosec\ 20°)}{7\ (\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°)}$
Sol:
Cos 55° = cos (90° - 35°) = sin 35°
Cos 70° = cos (90 – 20) = sin 20°
Tan 5 = cot 85° tan 25° = cot 65°
$\Rightarrow \frac{3 \sin 35°}{7 \sin 35°} - \frac{4\ (\sin 20°\ cosec\ 20°)}{7(\cot 85° \tan 85° \cot 65° \tan 65° \tan 45°)}$
$= \frac{3}{7} - \frac{4}{7} = -\frac{1}{7}$

(ix) $\frac{\sin 18°}{\cos 72°} + \sqrt{3}\ [\tan 10° \tan 30° \tan 40° \tan 50° \tan 80°]$
Sol:
Sin 18° = sin (90° - 72) = cos 72°
Tan 10° = cot 80° tan 50° = cot 40°
$\Rightarrow \frac{sin\ 18°}{\sin 18°} + \sqrt{3}\ \left[\tan 80 \cos 30 . \tan 40 \cot 40 . \frac{1}{\sqrt{3}}\right]$
$= 1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}} = 2$

(x) $\frac{\cos 58°}{\sin 32°}+\frac{\sin 22°}{\cos 68°}-\frac{\cos 38° \operatorname{cosec} 52°}{\tan 18° \tan 35° \tan 60° \tan 72° \tan 65°}$

Sol:

Cos 58° = cos (90° - 32°) = sin 32°

Sin 22° = sin (90° - 68°) = cos 68°

Cos 38° = cos (90 – 52) = sin 52°

Tan 18° = cot 72 tan 35° = cot 55°

$\Rightarrow \frac{\sin 32°}{\sin 32°}+\frac{\cos 68°}{\cos 68°}-\frac{\sin 52\ cosec\ 52}{\tan 72 .\cot 72 \tan 55 \cot 55 .\tan 60}$

= 1 + 1 $-\frac{1}{\sqrt{3}}=\frac{2\sqrt{3}-1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{6-\sqrt{3}}{3}$

10. If Sin θ = cos (θ – 45°), where θ – 45° are acute angles, find the degree measure of θ.

Sol:

Sin θ = cos (θ – 45°)

Cos θ = cos (90 – θ)

Cos (θ – 45°) = sin (90° - (θ – 45°)) = sin (90 – θ + 45°)

Sin θ = sin (135 – θ)

$\theta = 135 - \theta$

$2\theta = 135$

$\therefore \theta = 135°/2$

11. If A, B, C are the interior angles of a ΔABC, show that:

(i) Sin $\left(\frac{B+C}{2}\right)=\cos\frac{A}{2}$ (ii) cos $\left[\frac{B+C}{2}\right]=Sin\frac{A}{2}$

Sol:

A + B + C = 180

B – C = 180 – $\frac{A}{2}$

(i) Sin $\left[90-\frac{A}{2}\right]=\cos\frac{A}{2}$

∴ LHS = RHS

(ii) Cos $\left[90-\frac{A}{2}\right]=\sin\frac{A}{2}$

∴ LHS = RHS

12. If 2θ + 45° and 30° $-$ θ are acute angles, find the degree measure of θ satisfying Sin (20 + 45°) = cos (30 - θ°)

Sol:

Here 20 + 45° and 30 – θ° are acute angles:

We know that (90 – θ) = cos θ

Sin (2θ + 45°) = sin (90 – (30 – θ))

Sin (2θ + 45°) = sin (90 – 30 + θ)

Sin (20 + 45°) = sin (60 + θ)
On equating sin of angle of we get
$2\theta + 45 = 60 + \theta$
$2\theta - \theta = 60 - 45$
$\theta = 15°$

13. If θ is a positive acute angle such that sec θ = cosec 60°, find 2 $\cos^2\theta - 1$
Sol:
We know that sec (90 – θ) = $\text{cosec}^2\,\theta$
Sec θ = sec (90 – 60°)
On equating we get
Sec θ = sec 30°
$\theta = 30°$

Find $2\cos^2\theta - 1$
$\Rightarrow 2 \times \cos^2 30° - 1 \qquad \left[\cos 30 = \frac{\sqrt{3}}{2}\right]$
$\Rightarrow 2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1$
$\Rightarrow 2 \times \frac{3}{4} - 1$
$\Rightarrow \frac{3}{2} - 1$
$= \frac{1}{2}$

14. If cos 2θ = sin 4θ where 2θ, 4θ are acute angles, find the value of θ.
Sol:
We know that sin (90 – θ) = cos θ
Sin 20 = cos 2θ
Sin 4θ = sin (90 - 2θ)
$4\theta = 90 - 20$
$6\theta = 90$
$\theta = \frac{90}{6}$
$\theta = 15°$

15. If Sin 3θ = cos (θ – 6°) where 3 θ and $\theta - 6°$ are acute angles, find the value of θ.
Sol:
30, θ – 6 are acute angle
We know that sin (90 – θ) = cos θ
Sin 3θ = sin (90 – (θ - 6°))
Sin $3\theta = \sin(90 - \theta + 6°)$

Sin $3\theta = \sin(96° - \theta)$
$3\theta = 96° - \theta$
$4\theta = 96°$
$\theta = \frac{96°}{4}$
$\theta = 24°$

16. If Sec 4A = cosec (A – 20°) where 4A is acute angle, find the value of A.
Sol:
Sec 4A = sec $[90 - A - 20]$ $[\because \sec(90 - \theta) = cosec\ \theta]$
Sec 4A = sec (90 – A + 20)
Sec 4A = sec (110 – A)
4A = 110 – A
5A = 110
A = $\frac{110}{5} \Rightarrow A = 22$

17. If Sec 2A = cosec (A – 42°) where 2A is acute angle. Find the value of A.
Sol:
We know that (sec (90 – θ)) = cosec θ
Sec 2A = sec (90 – (A – 42))
Sec 2A = sec (90 – A + 42)
Sec 2A = sec (132 – A)
Now equating both the angles we get
2A = 132 – A
3A = $\frac{132}{3}$
A = 44