# Class 6 Maths Playing With Numbers Introduction

**Introduction**

John’s uncle gave him 24 muffins to distribute among his friends. This means john has 24 ÷ 1 = 24 muffins.

He wants to equally distribute it among 6 children. How will he do that?

24 ÷ 6 = 4    i.e 24 = 6 x 4

Now what if 2 more children come to his place?

How will he distribute the same number of muffins among 8 children?

24 ÷ 8 = 3    i.e 24 = 8 x 3

Suppose 4 more children visit his place at the same time. Can he distribute 24 muffins equally among all children?

Yes, he can!!!    24 ÷ 12 = 2    i.e 24 = 12 x 2

From this calculation we can see that 24 can be written as a product of two numbers in different ways as

24 = 6 × 4; 24 = 8 × 3; 24 = 12 × 2;

This means 2, 3, 4, 6, 8 and 12 are exact divisor of 24. They are known as factors of 24.

## Class 6 Maths Playing With Numbers Factors

**Factors**

A factor of a number is defined as the number which is an exact divisor of that number.

Suppose we want number 12. Think about the numbers you can multiply together to get 12.

3 x 4 = 12 ; 2 x 6 = 12 ; 1 x 12 = 12

This shows that 1, 2, 3, 4, 6 and 12 are factors of 12.

**Facts about factors:**

- The number 1 and the number itself are always factors of that number.
- Every factor is less than or equal to the given number.
- Every factor of a number divides the given number exactly.
- The number of factors of a given number are finite.

**Common Factors**

When two (or more) numbers have the same factor, that factor is known as a common factor.

Let's find put common factors of 6 and 18.

6 = 1 x 6, 6 = 2 x 3

Therefore, factors of 6 = 1, 2, 3, 6

18 = 1 x 18, 18 = 2 x 9, 18 = 3 x 6,

Therefore, factors of 18 = 1, 2, 3, 6, 9, 18

The numbers which appear in both the list are: 1, 2, 3, 6.

Hence, common factors of 6 and 18 are 1, 2, 3 and 6

**Question:**

Write all the factors of the following numbers:

(a) 23 (b) 36

Solution:

(a) 23 = 1, 23

(b) 36 = 1 x 36,

36 = 2 x 18,

36 = 3 x 12,

36 = 4 x 9

36 = 6 x 6

Therefore, factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

**Perfect number**

A perfect number is defined as a number for which sum of all its factors is equal to twice the number.

The factors of 6 are 1, 2, 3 and 6

1 + 2 + 3 + 6 = 12 = 2 x 6

Therefore, 6 is a perfect number.

The factors of 496 are 1, 2, 4, 8, 16, 31, 62, 124, 248 and 496.
1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 + 496 = 992 = 2 x 496

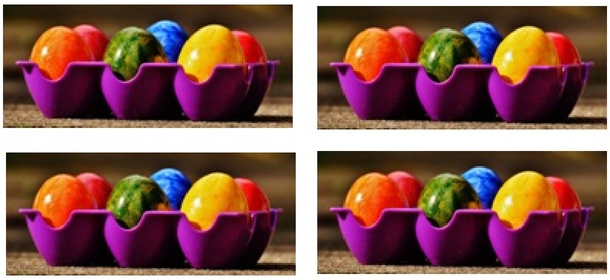
Therefore, 496 is a perfect number.

# Class 6 Maths Playing With Numbers Multiples

**Multiples**

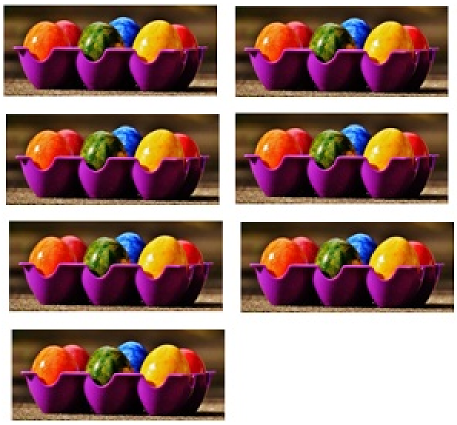
6 eggs are present in 1 container.

How many eggs must be there in 4 such containers.



There will be total 6 x 4 = 24 eggs.

If there are 7 containers then number of eggs in 7 containers will be



So number of eggs in 7 containers will be 7 x 4 = 28.

So, to find the total number of eggs here we have multiplied the number of eggs in 1 container by the number of containers.

This is the concept of multiple.

A multiple of a number is defined as a number which is a product of that number and any other whole number.

For example, the multiples of 3 are 3, 6, 9, 12, 15, 18 and so on.

- Every multiple of a number is greater than or equal to that number.
- The number of multiples of a given number is infinite.
- Every number is a multiple of itself.

**Common Multiples**

The Common multiples of two or more numbers are the multiples that are common to every given number.

Let’s find the common multiples of 4 and 6.

Write tables of 4.            Write table of 6.

| |
|---|
| 6 x 1 = 6 |
| 6 x 2 = 12 |
| 6 x 3 = 18 |
| 6 x 4 = 24 |
| 6 x 5 = 30 |
| 6 x 6 = 36 |
| 6 x 7 = 42 |
| 6 x 8 = 48 |
| 6 x 9 = 54 |
| 6 x 10 = 60 |

| |
|---|
| 4 x 1 = 4 |
| 4 x 2 = 8 |
| 4 x 3 = 12 |
| 4 x 4 = 16 |
| 4 x 5 = 20 |
| 4 x 6 = 24 |
| 4 x 7 = 28 |
| 4 x 8 = 32 |

| |
|---|
| 4 x 9 = 36 |
| 4 x 10 = 40 |

First 10 multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40

First 10 multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60

The common numbers which appear in both the list are 12, 24 and 36.

Therefore, the common multiples of 4 and 6 are 12, 24 and 36.

**Question:** Find all the multiples of 9 upto 100.

***Solution***: Multiples of 9 = 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99

**Prime and Composite numbers**

**Prime number**

The numbers other than 1 whose only factors are 1 and the number itself are called prime number.

E.g. 5 = 1 x 5, 23 = 1 x 23

**Composite number**

A composite number is defined as a number having more than two factors.

E.g. Factors of 9 are 1, 3 and 9. Therefore it is a composite number.

Factors of 14 are 1, 2, 7 and 14. Therefore it is a composite number.

**Note:**

1 is neither prime nor composite number.

Any whole number greater than 1 is either a prime or composite number.

**Question:**

Write seven consecutive composite numbers less than 100 so that there is no prime number between them.

Solution:

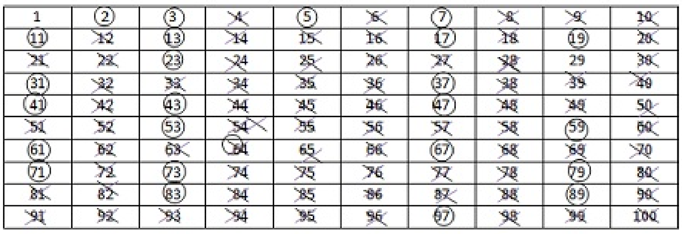
89 and 97 are the prime numbers. 90, 91, 92, 93, 94, 95, 96 lies in between 89 and 97. All these numbers have more than 2 factors. Therefore, all the numbers are composite numbers.

# Eratosthenes sieve to find prime numbers

**Eratosthenes sieve to find prime numbers**

Eratosthenes was a Greek mathematician, astronomer, and geographer in Egypt in 200 B.C.

He invented a method for finding prime numbers that is still used today.



| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | ~~4~~ | 5 | ~~6~~ | 7 | ~~8~~ | ~~9~~ | ~~10~~ |
| 11 | ~~12~~ | 13 | ~~14~~ | ~~15~~ | ~~16~~ | 17 | ~~18~~ | 19 | ~~20~~ |
| ~~21~~ | ~~22~~ | 23 | ~~24~~ | ~~25~~ | ~~26~~ | ~~27~~ | ~~28~~ | 29 | ~~30~~ |
| 31 | ~~32~~ | ~~33~~ | ~~34~~ | ~~35~~ | ~~36~~ | 37 | ~~38~~ | ~~39~~ | ~~40~~ |
| 41 | ~~42~~ | 43 | ~~44~~ | ~~45~~ | ~~46~~ | 47 | ~~48~~ | ~~49~~ | ~~50~~ |
| ~~51~~ | ~~52~~ | 53 | ~~54~~ | ~~55~~ | ~~56~~ | ~~57~~ | ~~58~~ | 59 | ~~60~~ |
| 61 | ~~62~~ | ~~63~~ | ~~64~~ | ~~65~~ | ~~66~~ | 67 | ~~68~~ | ~~69~~ | ~~70~~ |
| 71 | ~~72~~ | 73 | ~~74~~ | ~~75~~ | ~~76~~ | ~~77~~ | ~~78~~ | 79 | ~~80~~ |
| ~~81~~ | ~~82~~ | 83 | ~~84~~ | ~~85~~ | ~~86~~ | ~~87~~ | ~~88~~ | 89 | ~~90~~ |
| ~~91~~ | ~~92~~ | ~~93~~ | ~~94~~ | ~~95~~ | ~~96~~ | 97 | ~~98~~ | ~~99~~ | ~~100~~ |

It filters out numbers to find the prime numbers.

Step 1: Cross out 1, since it is neither prime nor a composite number.

Step 2: Encircle 2. All the numbers divisible by 2 are even numbers. So cross out all the multiples of 2.

Step 3: Encircle 3; cross out multiples of 3.

Step 4: Encircle 5; cross out multiples of 5.

Step 5: Encircle 7; cross out multiples of 7.

Step 6: Encircle 11; cross out multiples of 11. All the encircled numbers are prime numbers. Rest all the crossed out numbers except 1 are composite numbers.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

**Co-prime numbers**

Any set of numbers which do not have any other common factor other than 1 are called co-prime or relatively prime numbers.

E.g. Factors of 5 = 1, 5

Factors of 6 = 1, 2, 3, 6

This shows that 5 and 6 have no common factor other than 1. Therefore, they are co-prime numbers.

Properties of co-prime numbers:

- All prime numbers are co-prime to each other.
- Any consecutive whole numbers are always co-primed.
- Sum of any two co-prime numbers is always co-primed.
- Co-prime numbers need not to be prime numbers.

**Twin primes**

Twin primes are a pair of primes which differ by 2. First few twin primes are

E.g. (3, 5); (5, 7); (11, 13); (17, 19); (29, 31); (41, 43)

**Question :** Express 44 as the sum of two odd primes.

***Solution***: 44 = ____ + ____

Here we have to find 2 numbers which are odd as well as prime numbers and whose sum is 44.

Odd prime numbers upto 44 are 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43.

Now let’s find out a pair of numbers whose sum is 44.

Sum of 3 and 41 is 44.

So, 44 = 3 + 41

**Question:** Give three pairs of prime numbers whose difference is 2.

***Solution***: 5 and 7, 11 and 13, 41 and 43.

**Tests for divisibility of numbers**

Divisibility by 2

If the number ends with 2, 4, 6, 8 or 0, it is divisible by 2.

Example: 28, 54, 96

Here 28, 54 and 96 ends with 8, 4 and 6 respectively.

Therefore, 28, 65 and 96 are divisible by 2.

Divisibility by 3

If the sum of the digits of any number is divisible by 3 then that number is divisible by 3.

Example: 429 ; $4 + 2 + 9 = 15$ ; $15 \div 3 = 5$

Therefore, 429 is divisible by 3.

Divisibility by 4

If the last 2 digits of any number are divisible by 4, then that number is divisible by 4.

Example: 628; Last 2 digits are 28 and $28 \div 4 = 7$

Therefore, 628 is divisible by 4.

Divisibility by 5

If the digit in the ones place of a number is 5 or 0, then it is divisible by 5.

Example 1: 95 ends in 5;

Therefore, 95 is divisible by 5.

Example 2: 680 ends in 0.

Therefore, 680 is divisible by 5.

Divisibility by 6

If a number is divisible by 2 and 3, then that number is divisible by 6.

Example: 246. It is divisible by 2 as it ends with 6. Now, $2 + 4 + 6 = 12$. 12 is divisible by 3, so 246 is divisible by 3 also.

This shows that 246 is divisible by 2 and 3.

Therefore, 246 is divisible by 6.

Divisibility by 8

A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

Example: 2544 Last 3 digits are 544.

544 ÷ 8 = 68

Therefore, 2544 is divisible by 8.

Divisibility by 9

A number is divisible by 9 if the sum of its digits is divisible by 9.

Example: 42,471. 4 + 2 + 4 + 7 + 1 = 18 is divisible by 9.

Therefore 42,471 is divisible by 9.

Divisibility by 10

A number is divisible by 10 if it ends with a ZERO.

Example: 1570. Here the last digit is 0.

Therefore, 1570 is divisible by 10.

Divisibility by 11

A number is divisible by 11 if the difference of the sums of the alternate digits is 0 or a multiple of 11.

Example 1: 9724

9 + 2 = 11. 7 + 4 = 11. Difference between the two sums is 0.

Therefore, 9724 is divisible by 11.

Example 2: 45958

4 + 9 + 8 = 21 ; 5 + 5 = 10 ; 21 – 10 = 11. This is a multiple of 11.

Therefore, 45958 is divisible by 11.

**Question:** Using divisibility tests, determine whether 1258 is divisible by 6 or not.

***Solution***: Last digit of the given number is 8. Therefore 1258 is divisible by 2.

When we add all the digits of 1258, the sum is 16. 16 is not divisible by 3

Since the number is not divisible by both 2 and 3, hence it is not divisible by 6.

**Some more divisibility rules**

If number is divisible by another number, then it is also divisible by each of the factors of that number.

Example: 18 and 72.

72 ÷ 18 = 4 This show 72 is divisible by 18,

Factors of 18 = 1, 2, 3, 6, 9, 18

72 ÷ 1 = 72, 72 ÷ 2 = 36, 72 ÷ 3 = 24, 72 ÷ 6 = 12, 72 ÷ 9 = 8, 72 ÷ 18 = 4

Therefore, 72 is divisible by each of the factors of 18.

If a number is divisible by two co-prime numbers, then it is also divisible by their product.

Example: Let's say 90 is divisible by 5 and 9. As we know 5 and 9 are co-prime numbers.

So 90 must be divisible by their products. i.e 5 x 9 = 45

90 ÷ 45 = 2

Therefore, 90 is divisible by product of the co-primes i,e 5 and 9.

If two given numbers are divisible by a number, then, their sum is also divisible by that number.

Example: 21 and 18 are divisible by 3.

21 ÷ 3 = 7, 18 ÷ 3 = 6

Sum of the two numbers is 21 + 18 = 39. Also 39 ÷ 3 = 13

Therefore, if 24 and 18 are divisible by 3, then their sum I,e 39 is also divisible by 3.

If two given numbers are divisible by a number, then their difference is also divisible by that number.

Example: 58 and 54 are divisible by 2.

58 ÷ 2 = 29, 54 ÷ 2 = 27.

Difference of the two numbers i,e 58 – 54 = 4 and 4 ÷ 2 = 2

Therefore, if 54 and 58 are divisible by 2, then their difference I,e 4 is also divisible by 2.

**Question:** A number is divisible by both 5 and 12. By which other number will that number be always divisible?

***Solution***: The number is divisible by 5 and 12.

Since 5 and 12 are co-prime numbers so the number must be divisible by the product 5 × 12 = 60.

So, the given number will always be divisible by 60.

**Highest Common Factor**

(HCF) of two or more given numbers is the highest (or greatest) of their common factors. It is also known as Greatest Common Divisor (GCD).

To find the HCF of two or more numbers, we can use any of the following method.

Common factor method
Prime factorization method

**Common factor method**

Let's find the HCF of 18, 24 and 42.

First find all factors of the given numbers individually.

Factors of 18 = 1, 2, 3, 6, 9, 18

Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24

Factors of 42 = 1, 2, 3, 6, 7, 14, 21, 42

Therefore, common factors of 18, 24 and 42 are 1, 2, 3 and 6.

HCF of 18, 24 and 42 is 6.

**Prime factorization method**

Let's find the HCF of 27 and 45.

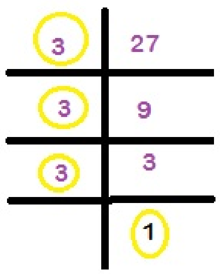
First we have to find the factors of 27.

Write a pair of factors.

27 = 3 x 9

Now further factorize the composite factor 9 as 3 and 3.

Repeat the process till you get the prime factors of all the composite factors. Since 3 is a prime number we cannot factorize it further.




27 = 1 * 3 * 3 * 3

Therefore, 27 as a product of its prime factors is written as

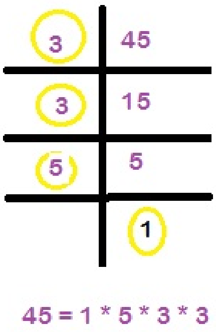
27 = 3 x 3 x 3

Let's find the factors of 45.

Write a pair of factors.

45 = 5 x 9

Further factorize the composite factor 9 as 3 and 3.




Therefore, 45 as a product of its prime factors is written as

45 = 5 x 3 x 3

Multiply all the factors which appear in both the list.

27 = 3 x 3 x 3

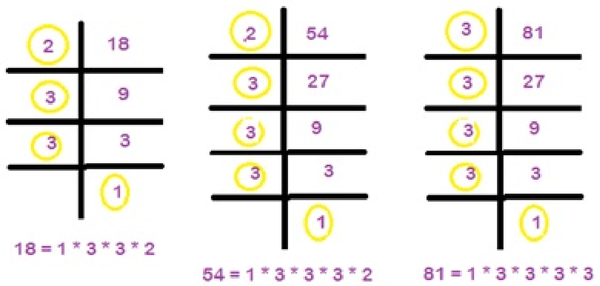
45 = 3 x 3 x 5

i.e 3 x 3 = 9

Therefore, HCF of 27 and 45 is 9.

**Question:** Find the HCF of the following numbers. 18, 54, 81

***Solution***: Let's find the HCF by prime factorization method.




18 = 2 x 3 x 3

54 = 2 x 3 x 3 x 3

81 = 3 x 3 x 3 x 3

Multiply all the factors which appear in both the list. i.e 3 x 3 = 9

The HCF of 18, 54 and 81 is 9.

**Word problem**

A fruit seller has 24 apples, 40 papaya and 56 strawberries which he must use to create fruit baskets. What is the largest number of fruit baskets she can make without having any fruit left over?

***Solution***:Here, we have to find the largest number of fruit baskets. So, we find HCF.

Apples: 24 = $2^3$ x 3

Daisies: 40 = $2^3$ × 5

Lilies: 56 = $2^3$ × 7

HCF = $2^3$ = 8

A fruit seller can make 8 fruit baskets.

Each basket will have 3 apples (Since 24 ÷ 8 = 3); 5 Papaya (Since 40 ÷ 8 = 5) and 7 strawberries: 7 (Since 56 ÷ 8 = 7)

**Lowest Common multiple (LCM)**

The Lowest Common Multiple (LCM) of two or more given numbers is the lowest (or smallest or least) of their common multiples.

If two numbers are co-prime then the LCM is the product of the two numbers.

Find the LCM of 7 and 13.

7 = 7 x 1 and 13 = 1 x 13

So, LCM is 7 x 13 = 91.

**Common multiple method**

Find the lowest common multiple of the numbers 8, 12 and 18.

Solution:

List the multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88

List the multiples of 12: 12, 24, 36, 48, 60, 72, 84,

List the multiples of 18: 18, 36, 54, 72, 90, 1

The common multiples of 8, 12 and 18 are 72 ...

L.C.M. of 8, 12 and 18 is 72.

This method works only when there are very small numbers.

**Prime factorization method**

Find the LCM of 90, 100 and 150.

Solution:

Prime factorisation of the given numbers

$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$

$100 = 2 \times 2 \times 5 \times 5 = 2^2 \times 5^2$

$150 = 2 \times 3 \times 5 \times 5 = 2 \times 3 \times 5^2$

Let’s find the product of all the factors with highest powers.

$2^2 \times 3^2 \times 5^2 = 4 \times 9 \times 25 = 900$

**Common division method of prime factorisation**

A very convenient method to find the LCM is the common division method. In this method of prime

factorisation we proceed as follows:

Arrange all the given numbers in a row and separated by commas.
Start with the lowest prime number which divides at least one of the given numbers exactly.

Write down the quotients and any undivided numbers in the next line.
Repeat the process as shown below until 1 is the only common factor.
Find the product of all the divisors. This is the required LCM.

Find the L.C.M. of the numbers 36, 48 and 72.

Solution:

| | | | |
|---|---|---|---|
| 2 | 36, | 48, | 72, |
| 2 | 18, | 24, | 36 |
| 2 | 9, | 12, | 18 |
| 2 | 9, | 6, | 9 |
| 3 | 9, | 3, | 9 |
| 3 | 3, | 1, | 3 |
| | 1, | 1, | 1 |

LCM = 2 x 2 x 2 x 2 x 3 x 3 = 144

**Word problem**

Find the least number which when divided by 18, 28, 32 and 42 leaves a remainder 5 in each case.

Solution:

First we should find the LCM of 18, 28, 32 and 42.

| | | | | |
|---|---|---|---|---|
| 2 | 18, | 28, | 32, | 42 |
| 2 | 9 | 14, | 16 | 21 |
| 2 | 9, | 7, | 8 | 21 |
| 2 | 9, | 7, | 4 | 21 |
| 2 | 9, | 7, | 2 | 21 |
| 3 | 9, | 7, | 1 | 21 |
| 3 | 3 | 7 | 1 | 7 |
| 7 | 1 | 7 | 1 | 7 |
| | 1 | 1 | 1 | 1 |

LCM = 2 x 2 x 2 x 2 x 2 x 3 x 3 x 7 = 2016

2016 is the least number which when divided by the given numbers will leave remainder 0 in each case. But we need the least number that leaves remainder 5 in each case.

Therefore, the required number is 5 more than 2016. The required least number = 2016 + 5 = 2021

**Question:** The traffic lights at three different road crossings change after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7 a.m., at what time will they change simultaneously again?

Solution:

The time period after which these lights will change = LCM of 48, 72 and 108

| | | | |
|---|---|---|---|
| 2 | 48, | 72, | 108, |
| 2 | 24 | 36, | 54 |
| 2 | 12, | 18, | 27 |
| 2 | 6, | 9, | 27 |
| 3 | 3, | 9, | 27 |
| 3 | 1, | 3, | 9 |
| 3 | 1 | 1 | 3 |
| | 1 | 1 | 1 |

LCM = 2 x 2 x 2 x 2 x 3 x 3 x 3 = 432

Therefore, the light will change together after every 432 seconds. i.e 7 min 12 seconds.

Hence, they will change simultaneously at 7: 07: 12 am.