Theorem

The expression $\frac{0}{0}$ is undefined because it is indeterminate: it does not correspond to a unique real number.

Proof

Assume, for contradiction, that there exists a real number $c \in \mathbb{R}$ such that:

$$\frac{0}{0} = c$$

By the definition of division, we know:

$$\frac{a}{b} = c \iff a = b \cdot c, \quad \text{provided that } b \neq 0$$

Applying this to our assumption:

$$\frac{0}{0} = c \Rightarrow 0 = 0 \cdot c$$

However, it is also true that:

$$\forall c \in \mathbb{R}, \quad 0 \cdot c = 0$$

Hence, the equation $0 = 0 \cdot c$ is satisfied by **every** real number c. That is:

$$\exists c_1, c_2 \in \mathbb{R}, \quad c_1 \neq c_2, \quad \text{such that } 0 = 0 \cdot c_1 = 0 \cdot c_2 \Rightarrow \frac{0}{0} = c_1 \quad \text{and} \quad \frac{0}{0} = c_2$$

This contradicts the requirement that the result of division be **unique**.

Therefore, the assumption that $\frac{0}{0}$ equals any specific real number leads to contradiction.

Conclusion

$$\boxed{\frac{0}{0} \text{ is undefined because it does not yield a unique real value; it is indeterminate.}}$$