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A1) $(9\times1 + 7\times3 + 8\times1 + 0\times3 + 3\times1 + 0\times3 + 6\times1 + 4\times3 + 0\times1 + 6\times3 + 1\times1 + 5\times3) \bmod 10$

$= 93 \bmod 10$

$= 3.$

So, the check digit of 978-0-306-40615 will be $10-3 = 7$. The given check digit is also 7, so the given ISBN code is <u>valid</u>.

A2) a) If the first 9 digits are correct, then the check digit will be :

$= (0\times1 + 4\times2 + 5\times3 + 2\times4 + 3\times5 + 3\times6 + 5\times7 + 2\times8 + 7\times9) \bmod 11$

$= 203 \bmod 11 = 5$

The correct check digit will be 5.

b) If we interchange 8 and 2, the check digit of the number will be :=

$(0\times1 + 3\times2 + 1\times3 + 2\times4 + 8\times5 + 4\times6 + 3\times7 + 0\times3 + 8\times9) \bmod 11$

$= 187 \bmod 11 = X$

So, the correct ISBN will be 0-912843-08-X

A3) So, the journey took 28 hrs = 24 hrs + 4 hrs

a) The journey ~~should~~ started from 3 pm, after 24 hrs or 1 day, It will still be 3 pm and after 4 hrs more, it will be 7 pm. So, when the journey ended, was one day after the journey started.

b) All 7 days will occur once in first 7 days in a 30 day month, After 14 days, all the 7 days will occur twice. Likewise, all the 7 days will occur 4 times after the first 28 days of a 30 day month. Now, we have 2 more days left in the 30 day month. So, this two days will occur once more and hence in a 30 day month two days will occur exactly 5 times

A3) It its wednesday, after 7 days it will be wednesday again. So, after 14×7 days = 98 days, it will be a wednesday.

It will be a ~~Sl~~ Saturday after 3 days. Hence $(98+3) = 101$ days after wednesday, it will be saturday.

A4) a) Let, the statement be $P(n)$.

now, for $n = 1$

$L.H.S. = 1^5 = 1$

$R.H.S = 1/12 \; 1^2 (1+1)^2 (2\times1^2 + 2\times1 - 1)$

$= 4/12 \times 3 = 1$

so, $L.H.S = R.H.S.$

Let, for $n = k$, $P(n)$ satisfies.

so, $1^5 + 2^5 + \cdots + k^5 = \frac{1}{12} k^2 (k+1)^2 (2k^2 + 2k - 1)$

adding $(k+1)^5$ in both sides.

$1^5 + 2^5 + \cdots + k^5 + (k+1)^5 = \frac{1}{12}(k+1)^2 (2k^4 + 2k^3 - k^2) + (k+1)^5$

$= \frac{1}{12}(k+1)^2 (2k^4 + 2k^3 - k^2 + 12(k+1)^3)$

$= \frac{1}{12}(k+1)^2 (2k^4 + 2k^3 - k^2 + 12k^3 + 36k^2 + 36k + 12)$

$= \frac{1}{12}(k+1)^2 ((2k^4 + 8k^3 + 8k^2) + (6k^3 + 24k^2 + 24k) + (3k^2 + 12k + 12))$

$\Rightarrow \frac{1}{12}(k+1)^2(k+2)^2((2k^2)+(6k)+3)$

$\Rightarrow \frac{1}{12}(k+1)^2(k+2)^2(2k^2+4k+2+2k+2-1)$

$\Rightarrow \frac{1}{12}(k+1)^2(k+2)^2(2(k+1)^2+2(k+1)-1)$

$\therefore$ For every $n=k$, which satisfies $P(n)$, $n=k+1$ also satisfies $P(n)$.

$\therefore$ $P(n)$ is true for $n \in N$.

b) Let, the statement be $P(n)$,

for $n=1$,

L.H.S $= 1^4 = 1$

R.H.S. $= \frac{1}{30} \times 1 \times (2) \times (3) \times (3+3-1) = 1$

So, L.H.S. = R.H.S., $n=1$ satisfies $P(n)$.

now, for $n=k$, we assume it satisfies $P(n)$.

So, $1^4 + 2^4 + \ldots + k^4 = \frac{1}{30}k(k+1)(2k+1)(3k^2+3k-1)$

adding $(k+1)^4$ in both sides.

$1^4+2^4+\ldots+k^4+(k+1)^4 =$

$\frac{1}{30}k(k+1)(2k+1)(3k^2+3k-1)(k+1)$

$= \frac{1}{30}(k+1)(k(2k+1)(3k^2+3k-1)+30(k+1)^3)$

$= \frac{1}{30}(k+1)(6k^4+6k^3+2k^2+3k^2-k+30k^3+90k^2+90k+30)$

$\to \frac{1}{30}(k+1)(6k^4 + 39k^3 + 91k^2 + 89k + 30)$

$\to \frac{1}{30}(k+1)((6k^4 + 21k^3 + 18k^2) + (18k^3 + 63k^2 + 59k)$

$+ (10k^2 + 35k + 30))$

$\to \frac{1}{30}(k+1)(2k^2 + 7k + 6)((3k^2) + (9k) + 5)$

$\to \frac{1}{30}(k+1)(k+2)(2k+3)(3k^2 + 6k + 3 + 3k + 3 - 1)$

$\to \frac{1}{30}(k+1)(k+2)(2(k+1)+1)(3(k+1)^2 + 3(k+1) - 1)$

so, for every $n=k$, satisfying $P(n)$, $(k+1)$ also satisfies $P(n)$. and $P(1)$ satisfies $P(n)$, so $P(n)$ is true

c) given, $n=5$,

from (Pascal's formula):-

$6 \times (1^5 + 2^5 + 3^5 + \dots + n^5) = (n+1)^6 - (1 + n + 15\sum_{k=1}^{n} k^4$

$+ 20\sum_{k=1}^{n} k^3 + 6\sum_{k=1}^{n} k)$

$= (n+1)^6 - (1 + n + 15 \times \frac{1}{30} n(n+1)(2n+1)(3n^2 + 3n - 1)$

$+ 20\left(\frac{n(n+1)}{2}\right)^2 + 15 \times \frac{n(n+1)(2n+1)}{6}$

$+ 6\frac{n(n+1)}{2})$

$$\Rightarrow (n+1)^6 - \left(1+n+3n(n+1) + \frac{5n(n+1)(2n+1)}{2} + 5n^2(n+1)^2 + \frac{n(n+1)(2n+1)(3n^2+3n-1)}{2}\right)$$

$$\Rightarrow (n+1)^6 - 1/2\,(2+2n+6n^2+6n+5\times 2n^3+5\times 3n^2 + 5n + 10n^4 + 20n^3 + 10n^2 + (2n^2+3n+1)n\,(3n^2+3n-1))$$

$$\Rightarrow (n+1)^6 - 1/2\,(2+12n+31n^2+40n^3+25n^4+6n^5)$$

$$\Rightarrow (n+1)^6 - 1/2\,(6(n^5+2n^4+n^3) + 13n^2(n^2+2n+1) + 8n(n^2+2n+1) + 2(n^2+2n+1))$$

$$\Rightarrow (n+1)^6 - 1/2\,(n+1)^2\,(6n^3+13n^2+8n+2)$$

$$\Rightarrow 1/2\,(n+1)^2\,(2(n+1)^4 - 6n^3 - 13n^2 - 8n - 2)$$

$$\Rightarrow 1/2\,(n+1)^2\,(2(n+1)(n^3+3n^2+3n+1) - 6n^3 - 13n^2 - 8n - 2)$$

$$\Rightarrow 1/2\,(n+1)^2\,(2n^4+6n^3+6n^2+2n+2n^3+6n^2+6n+2-6n^3-13n^2-8n-2)$$

$$\Rightarrow 1/2\,(n+1)^2\,(2n^4+2n^3-n^2)$$

$$\Rightarrow 1/2\,n^2(n+1)^2\,(2n^2+2n-1)$$

So, $6 \times (1^5 + 2^5 + 3^5 + \cdots + n^5) = \frac{1}{2} n^2 (n+1)^2 (2n^2 + 2n - 1)$

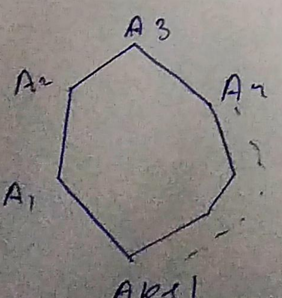
or $\boxed{(1^5 + 2^5 + 3^5 + \cdots + n^5) = \frac{1}{12} n^2 (n+1)^2 (2n^2 + 2n - 1)}$

A5) Let, $P(n)$ is the statement "The sum of the interious angles of a convex polygon with $n$ sides is $(n-2)\pi$".

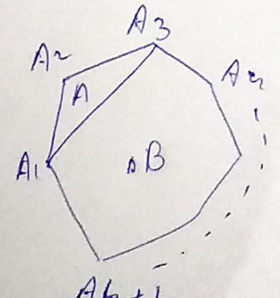
Let's take $n=3$, as base case, as we all know the sum of the sum of interious angles of a triangle is $180 = (3-2) \times \pi$.

Now, let $k \in \mathbb{N}\ [k \geq 3]$, such that $k$ satisfies $P(n)$. So, sum of all the interious angles of a $k$-gon is $(k-2)\pi$.

Now, if we take a $(k+1)$ sided convex polygon




and segment it like



It gives a polygon $B$ with $k+1-1 = k$ sides and $A_1A_2A_3$ triangle. So, the sum of the interiours of the $k+1$ sided convex polygon

= the sum of the interior angles of $B$ (a $k$ sided convex polygon) +

sum of the angles of a triangle ($A_1A_2A_3$)

$\Rightarrow (k-2)\pi + \pi = (k-1)\pi = ((k+1)-2)\pi$.

So, we have proven the base case, and for all $k \geq 3$ satisfying $P(n)$, there is $k+1$ which also satisfies $P(n)$.

So, $P(n)$ is proved.