Abelson and Sussman *Structure and Interpretation of Computer Programs 2nd Edition.* Exercise 2.5.

Solution

We show that $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ defined by $f(a, b) = 2^a 3^b$, is 1-1.

Suppose that $f(a, b) = f(c, d)$. Then $2^a 3^b = 2^c 3^d$. Assume, for the sake of obtaining a contradiction, that $a \neq c$. Then without loss of generality we may suppose that $a > c$. This implies that $2^{a-c} 3^b = 3^d$ is even, since $a - c > 0$. This is our contradiction. Hence $a = c$. It follows immediately that $b = d$. Hence f is 1-1.