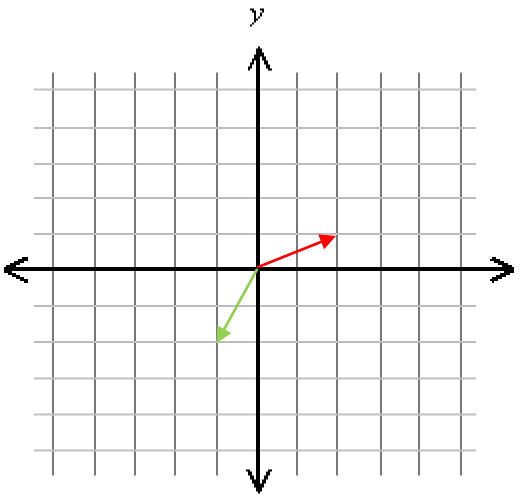
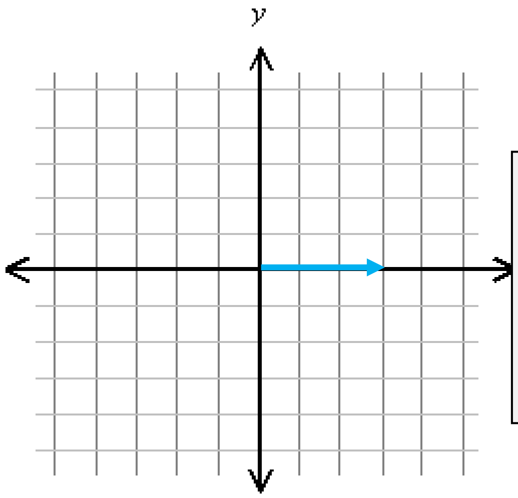
Linear Algebra Practical

1. Given the following traditional Cartesian planes, on the left one, draw the vectors **v** = (2, 1) and **w** = (-1, -2) on it. On the right Cartesian plane, draw the result of adding 2**v**+**w**.





2v+w

= 2(2, 1) +(-1, -2)

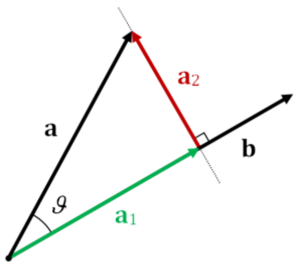
= (4, 2) + (-1, -2)

= (3, 0)

2. What is the length or 2nd norm (L_2) of vector **z** = (3, 4)?

$L_2 = \sqrt[2]{3^2 + 4^2} = \sqrt[2]{9 + 16} = \sqrt[2]{25} = 5$

3. If ϑ is 30 degrees and **a** = (3,4), what is the magnitude of the projection (a_1) of **a** upon **b**?



a = 5 (see L_2 calculation above)

θ = 30

$a_1 = \|a\| \cos\theta$ = 5cos30 (degrees not radians) = 4.3301

4. Given $\boldsymbol{u}$ = (5,2,3) and **v**=(1,-1,2), find $\boldsymbol{u}\cdot$**v** (the scalar product/inner product/dot product).

$u \cdot v = (5, 2, 3) \cdot (1, -1, 2) = (5{\cdot}1 + 2{\cdot}{-1} + 3{\cdot}2) = (5 - 2 + 6) = 9$

5. Let

$$A = \begin{pmatrix} 4 & -1 \\ 6 & 9 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 3 \\ 3 & -2 \end{pmatrix}$$

Find

(i) $A + B$,

(ii) $2A - B$

(iii) AB

(iv) BA

(i) A + B

$$= \begin{pmatrix} 4+0 & -1+3 \\ 6+3 & 9-2 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 9 & 7 \end{pmatrix}$$

(ii) 2A - B

$$= 2\begin{pmatrix} 4 & -1 \\ 6 & 9 \end{pmatrix} - \begin{pmatrix} 0 & 3 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} 8 & -2 \\ 12 & 18 \end{pmatrix} - \begin{pmatrix} 0 & 3 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} 8 & -5 \\ 9 & 20 \end{pmatrix}$$

(iii) A·B

$$= \begin{pmatrix} 4 & -1 \\ 6 & 9 \end{pmatrix} \cdot \begin{pmatrix} 0 & 3 \\ 3 & -2 \end{pmatrix}$$

$$\begin{pmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{pmatrix}$$

$$= \begin{pmatrix} (4*0) + (-1*3) & (4*3) + (-1*-2) \\ (6*0) + (9*3) & (6*3) + (9*-2) \end{pmatrix} = \begin{pmatrix} (0-3) & (12+2) \\ (0+27) & (18-18) \end{pmatrix} = \begin{pmatrix} -3 & 14 \\ 27 & 0 \end{pmatrix}$$

(iv) B·A

$$= \begin{pmatrix} 0 & 3 \\ 3 & -2 \end{pmatrix} \cdot \begin{pmatrix} 4 & -1 \\ 6 & 9 \end{pmatrix}$$

$$\begin{pmatrix} AE + BG & AF + BH \\ CE + DG & CF + DH \end{pmatrix}$$

$$= \begin{pmatrix} (0*4) + (3*6) & (0*-1) + (3*9) \\ (3*4) + (-2*6) & (3*-1) + (-2*9) \end{pmatrix} = \begin{pmatrix} (0+18) & (0+27) \\ (12-12) & (-3-18) \end{pmatrix} = \begin{pmatrix} 18 & 27 \\ 0 & -21 \end{pmatrix}$$

(v) A^T (the transpose of A)

(vi) Let $A = \begin{pmatrix} 4 & -1 \\ 6 & 9 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 3 \\ 3 & -2 \end{pmatrix}$

(i) Is AB defined? If it is defined, find it.
(ii) Is BA defined? How come?
(iii) What is the element A_{22} of A
(iv) What is the result of AI?
(v) What is the result of BB^{-1}?
(vi) Calculate the inverse of B, i.e. B^{-1}
(vii) Manually calculate the result of BB^{-1}

v) A= $\begin{pmatrix} 4 & -1 \\ 6 & 9 \end{pmatrix}$ A^T = $\begin{pmatrix} 4 & 6 \\ -1 & 9 \end{pmatrix}$

vi) ________________

i) AB is defined as: $\begin{pmatrix} (4*0)+(1*-3) & (4*3)+(-1*-2) \\ (6*0)+(9*3) & (6*3)+(9*-2) \\ (2*0)+(3*3) & (2*3)+(3*-2) \end{pmatrix} = \begin{pmatrix} (0-3) & (12+2) \\ (0+27) & (18-18) \\ (0+9) & (6-6) \end{pmatrix} = \begin{pmatrix} -3 & 14 \\ 27 & 0 \\ 9 & 0 \end{pmatrix}$

ii) No. The number of columns and rows are not equal.

E.g [1st row of B, 1st column of A] = (0*4)+(3*6)+(**?***2) ? is an undefined value

iii) A_{22} = A (Row2, Col2) = 9

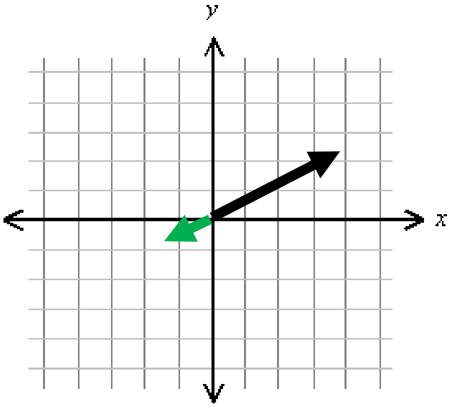
iv) AI = A

v) $BB^{-1} = B^{-1}B = I$

vi) $B^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{(0*-2)-(3*3)}\begin{pmatrix} -2 & -3 \\ -3 & 0 \end{pmatrix} = \frac{1}{-9}\begin{pmatrix} -2 & -3 \\ -3 & 0 \end{pmatrix}$

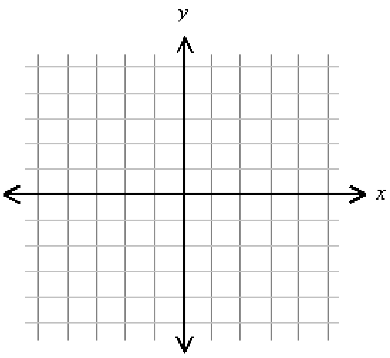
vii)

6. Draw the span of the vectors below



7. Are the following 2 vectors linearly dependent or independent? (Hint: plot them)

$$\boldsymbol{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \boldsymbol{w} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$



8. The vectors i and j below are the basis vectors in some space. Can you draw the vector (1,1) in that basis?