Week Report (Previous)

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I. Research Background

We mainly sorts the data package service to the following categories: 1. Music Data Plans; 2. Social Data Plans; 3. Game Data Plans. Preliminaries are that the data price in China is usually CNY40-50/GB, and in Singapore is usually SGD 10/GB.

A. *Music Data Plans*

In China: The CUTC has been providing the unlimited music data plan (CNY 8 for 8G Music Data) with many music service providers including Kugou Music, Kuwo Music, NetEase, etc., since April, 2014. And we notice that the ISP also has another possibly more lucrative choice other than collaborating with the CP - providing music service as well as the unlimited music data plan by its own. In fact, the CMCC has provided the unlimited data plan on its self-operated Migu Music App, rather than cooperate with the Music CPs.

In Singapore: The Singtel has been providing the unlimited music data plan with the Spotify for 8 SGD per month.

B. *Social Data Plans*

In China: CNY 10 for 300MB 20 for 500MB, both QQ and Wechat(usually 50-70M/mth for average).

In Singapore: The Singtel has been providing the unlimited social data plan with the Wechat(SGD 6/mth), WhatsApp(SGD 6/mth), LINE App(SGD 12/mth) and Facebook(SGD 8/mth).

C. *Mobile Game Data Plans*

In China: CNY 9 for 6G Game Data(King of Glory, Dragon Valley, etc., usually roaming 300M/hour).

D. *Unlimited Data Plans in American*

However, things in America can be quite fuzzy and puzzling. The ISPs provide unlimited data plans for (Unlimited talk+text, unlimited video, unlimited hotspots, unlimited music data, 20-30GB for other data streaming) at similar prices. AT&T – USD 60, Verizon – USD 80, Sprint – USD 50, T-Mobile – USD 70.

E. *Problems Interested In*

Motivated by this CP-ISP collaboration, we have the following interesting problems would like to study about.

- Under what condition it this unlimited App(Including music, social, game, etc) data plan viable for a cooperative ISP or a self-operating ISP?
- What's the optimal subscription fee? How will it influence the different parties payoff?
- Will the ISP choose to be a self-operating music platform if it could?

* And as the ISP in China seems to be gradually developing into an App-different (or data-usage-aware) pricing system rather than simple two-part tariff data pricing for all usages of data. It's interesting how will this new direction of app-based data pricing develop in the future.

F. *Some Results up to Now*

- (Incentive for the ISP): Maybe counter-intuitively, when the users' average evaluation is lower than some threshold value, the ISP has incentive to cooperate with CP or to provide self-operating platform.
- (Self-operating ISP): When the ISP can provide high-quality service, he may consider to self-operate a music platform.
- (Revenue for ISP and CP): When the users' average evaluation is low, the revenue improvement for ISP and CP is significant(even more than 10 times), and they tend to share the revenue fairly. However, when the average evaluation increases, the CP will need to give more revenue to the ISP to incentivize him for service provisioning, and the revenue mainly flows to the ISP.
- (Incentive across content providers or service providers): For services whose users have higher elasticity(music/game users), they tend to provide provide unlimited data plans(or flat price), but for those have lower elasticity(wechat/QQ), they tend to provide usage based data plans (at least in China).

II. Model Setting

We consider a market with N users each with a single parameter θ representing their evaluation of the content consumption. And we suppose the users' distribution on θ follows the exponential distribution function $\lambda(\theta) \sim Exp(\frac{1}{\mathbb{E}(\theta)})$. The user's content consumption is measured in data volume, $z \in [0, z_m]$. The consumption is upper bounded by z_m since each user won't use infinite volume of data even if it's free. We follow [WiOpt17', infocom15', SDP14', etc] to use the following α-fair utility function to model the user's value of content consumption. Suppose the user's data is charged with unit data cost p by the ISP. We have the user's utility function as $u(z|\theta, p, \alpha) = \theta \cdot (\frac{z^{1-\alpha}}{1-\alpha}), z \in [0, z_m]$.

Furthermore, we suppose the ISP can also provide the music service as well as unlimited music data plan by his own. We characterize his music service by an extra factor, $\gamma \in (0, 1)$ for the quality discount in the music service. Thus the user using his music platform will receive the utility $u(z|\gamma, \theta, \alpha, p) = \gamma \cdot \theta \left(\frac{z^{1-\alpha}}{1-\alpha}\right), \forall z \in [0, z_m]$.

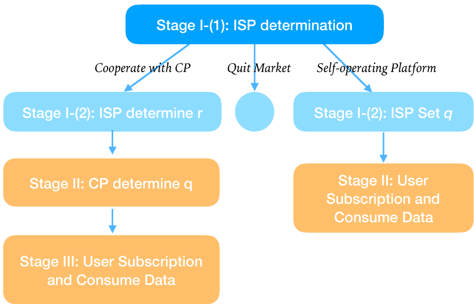

Fig. 1: The Multi-stage Stackelberg Game.

As is shown in Fig1., we capture the decision process in such a system as a three-stage Stackelberg game. In the first stage, the ISP decides whether to cooperate with the music CP, or to operate the music platform on its own, or just quit the market (in this case, the unlimited music data plan is unviable).

$$u(z|\gamma, \theta, \alpha, p, d) = \begin{cases} \theta \cdot (\frac{z^{1-\alpha}}{1-\alpha}) - pz, & \text{if } d = 0,\ x = 0; \text{ or } d = 1,\ x = 0; \text{ or } d = 2,\ x = 0, 1; \\ \theta \cdot (\frac{z^{1-\alpha}}{1-\alpha}) - q, & \text{if } d = 0, x = 1; \\ \gamma \cdot \theta \left(\frac{z^{1-\alpha}}{1-\alpha}\right) - q, & \text{if } d = 1, x = 1. \end{cases} \tag{1}$$

Here $d \in \{0, 1, 2\}$ represents the ISP's determination on whether to cooperate with CP, operate on its own or quit the market, respectively. Notice that if the ISP choose to provide self-operated music service, the users will experience a discount in his revenue by γ.

III. User's best response

We follow the research outline to conduct our study on the CP-ISP interactions. We denote a θ-*type* user's decision on membership as $x(\theta) \in \{0, 1\}$, with $x = 1$ representing subscribing and $x = 0$ representing unsubscribing. And we denote the total music data he will consume as $z(\theta)$. Then we have the following theorem for the CP's best responses for $d = 0, 1, 2$ respectively.

A. *Cooperating ISP, thus* $d = 0$.

Theorem *1 (User's best responses under cooperating ISP):* We have the user's best responses as

1) if $\theta < \theta_{th}$, then $x(\theta) = 0$, and $z(\theta) = \left(\frac{\theta}{p}\right)^{\frac{1}{\alpha}}$;
2) if $\theta > \theta_{th}$, then $x(\theta) = 1$, and $z(\theta) = z_m$.

Here we have θ_{th} as the unique zero the following equation in $[0, p \cdot z_m^{\alpha}]$.

$$\alpha p^{\frac{\alpha-1}{\alpha}} \cdot \theta_{th}^{\frac{1}{\alpha}} - z_m^{1-\alpha} \cdot \theta_{th} + q \cdot (1 - \alpha) = 0$$

Proof: We first consider the utility function of some user as below.

$$u(x, z|\theta, \alpha) = \begin{cases} \theta \cdot (\frac{z^{1-\alpha}}{1-\alpha}) - q & \text{if } x = 0 \\ \theta \cdot (\frac{z^{1-\alpha}}{1-\alpha}) - pz & \text{if } x = 1 \end{cases} \tag{2}$$

If he choose to subscribe, then he will determine his consumption of music data $z(\theta)$ to maximize his utility. We have $\frac{\partial u(z|x=0,\theta,\alpha)}{\partial z} = \theta z^{-\alpha} - p$, thus his best decision on $z(\theta|x = 0) = min\left\{(\frac{\theta}{p})^{\frac{1}{\alpha}}, z_m\right\}$. And his optimal revenue is

$$u^*(z|x = 0, \theta, \alpha) = \begin{cases} \frac{\alpha}{1-\alpha}\theta^{\frac{1}{\alpha}} p^{\frac{\alpha-1}{\alpha}} & \text{if } 0 \le \theta \le z_m^{\alpha} p \\ \theta \cdot (\frac{z_m^{1-\alpha}}{1-\alpha}) - z_m p & \text{if } \theta \ge z_m^{\alpha} p \end{cases} \tag{3}$$

If he choose to subscribe, notice that the function $u(z|x=1,\theta,\alpha)$ is an increasing function in $[0,z_m]$, and the maxima is reached when $z(\theta|x=1)=z_m$. And the utility is $u^*(z|x=1,\theta,\alpha)=\theta\cdot(\frac{z_m^{1-\alpha}}{1-\alpha})-q$.

Then for users with $\theta\in[z_m^{\alpha}p,+\infty)$, he will always choose to subscribe the unlimited music data plan, since $u^*(z|x=1,\theta,\alpha)-u^*(z|u=U,\theta,\alpha)=z_mp-q\geq 0$. And for a user with $\theta\in[0,z_m^{\alpha}p]$, we have

$$u^*(z|x=1,\theta,\alpha)-u^*(z|x=0,\theta,\alpha)=\theta\cdot(\frac{z_m^{1-\alpha}}{1-\alpha})-\frac{\alpha}{1-\alpha}\theta^{\frac{1}{\alpha}}p^{\frac{\alpha-1}{\alpha}}-q\triangleq\Delta u(\theta,\alpha)$$

Then we take the derivative of $u(\theta,\alpha)$ and we have

$$\frac{\partial\Delta u(\theta,\alpha)}{\partial\theta}=(\frac{z_m^{1-\alpha}}{1-\alpha})-\frac{1}{1-\alpha}\theta^{\frac{1}{\alpha}-1}p^{\frac{\alpha-1}{\alpha}}\geq 0$$

Thus we know the function $\Delta u(\theta,\alpha)$ is an increasing function in $[0,z_m^{\alpha}p]$. Moreover, we have $\Delta u(0,\alpha)=-q\leq 0$, and $\Delta u(z_m^{\alpha}p,\alpha)=z_mp-q\geq 0$. Thus due to the zero theorem, there must exist a unique $\theta_{th}\in[0,z_m^{\alpha}p]$ as the zero of the function. And when $\theta\in[0,\theta_{th}]$, $\Delta(\theta,\alpha)\leq 0$, thus the user will choose not to subscribe; and when $\theta\in[\theta_{th},z_m^{\alpha}p]$, $\Delta(\theta,\alpha)\geq 0$, thus the user will choose to subscribe to the unlimited music data plan. In summary, we have the users' best response as the theorem claims. ∎

We visualize the user's subscription problem in the following figure. Notice that users with higher evaluations tend to subscribe to the plan.

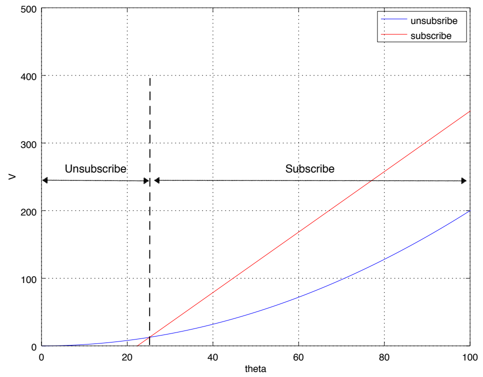

Fig. 2: The user subscription decisions under cooperating ISP.

And we have the following proposition on the dependence of θ_{th} on q.

***Proposition** 1 (Subscription fee's influence on the user behavior):* Threshold user's type θ_{th} increases with q.

Proof: We first consider the equation in Theorem1. In fact, we express q as a function of θ_{th} and get

$$q(\theta_{th})=\frac{z_m^{1-\alpha}\cdot\theta_{th}-\alpha p^{1-\frac{1}{\alpha}}\theta_{th}^{\frac{1}{\alpha}}}{1-\alpha}.$$

The derivative is

$$\frac{\partial q}{\partial\theta_{th}}=\frac{z_m^{1-\alpha}-p^{1-\frac{1}{\alpha}}\theta_{th}^{\frac{1}{\alpha}-1}}{1-\alpha}\geq 0.$$

And $\frac{\partial\theta_{th}}{\partial q}=\left(\frac{\partial q}{\partial\theta_{th}}\right)^{-1}\geq 0$, this ends the proof. ∎

The following simulation result validates our conclusion.

B. *Self-operating ISP, thus $d=1$.*

We can further get the CP's best response under self-operating ISP due to the following theorem.

***Theorem** 2 (CP's best response under self-operating ISP):* We have the CP's best responses as

- when $\gamma>\left(\frac{q}{z_mp}\right)^{1-\alpha}$, we have
  1) if $\underline{\theta}<\theta<\bar{\theta}$, then $x(\theta)=1$, and $z(\theta)=z_m$;
  2) if $0<\theta<\underline{\theta}$ or $\theta>\bar{\theta}$, then $x(\theta)=0$, and $z(\theta)=\left(\frac{\theta}{p}\right)^{\frac{1}{\alpha}}$.
- when $\gamma\leq\left(\frac{q}{z_mp}\right)^{1-\alpha}$, all the users will choose $x(\theta)=1$, and $z(\theta)=z_m$.

Here we have $\underline{\theta}$ and $\bar{\theta}$ as the only two zeros of $\Delta u(\theta)=0$, where $\Delta u(\theta)$ is defined in the proof below.

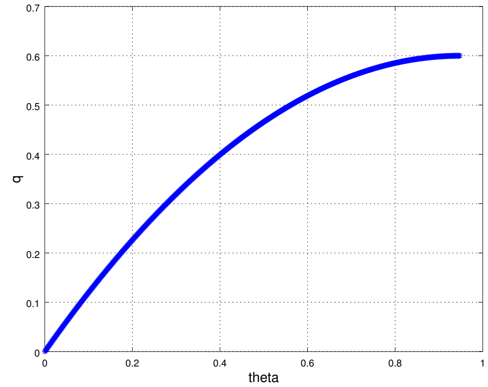

Fig. 3: θ_{th} increases with q.

Proof: We first construct a function

$$\Delta u = \begin{cases} \left(\gamma \cdot \theta \cdot (\frac{z_m^{1-\alpha}}{1-\alpha}) - q\right) - \left(\frac{\alpha}{1-\alpha}\theta^{\frac{1}{\alpha}} p^{\frac{\alpha-1}{\alpha}}\right) & \text{if } 0 \le \theta \le z_m^{\alpha} p \\ \left(\gamma \cdot \theta \cdot (\frac{z_m^{1-\alpha}}{1-\alpha}) - q\right) - \left(\theta \cdot (\frac{z_m^{1-\alpha}}{1-\alpha}) - z_m p\right) & \text{if } \theta \ge z_m^{\alpha} p \end{cases} \tag{4}$$

Notice that when $\Delta u > 0$, the user will join the unlimited data plan; and vice versa. Thus we Take derivative of this piecewise continuous function and get

$$\frac{\partial \Delta u}{\partial \theta} = \begin{cases} \left(\gamma \cdot (\frac{z_m^{1-\alpha}}{1-\alpha}) - \frac{1}{1-\alpha}\theta^{\frac{1}{\alpha}-1} p^{\frac{\alpha-1}{\alpha}}\right) & \text{if } 0 \le \theta \le z_m^{\alpha} p \\ (\gamma - 1) \cdot (\frac{z_m^{1-\alpha}}{1-\alpha}) & \text{if } \theta \ge z_m^{\alpha} p \end{cases} \tag{5}$$

Notice that when $0 < \theta < \theta_0$, $\Delta u < 0$ thus the function increases; while when $\theta > \theta_0$, $\Delta u > 0$ thus the function decreases. And here $\theta_0 = \gamma^{\frac{\alpha}{1-\alpha}} \cdot z_m^{\alpha} p$. Since we have $\Delta u(0) = -q \le 0$ and we have $\lim_{\theta \to +\infty} \Delta u(\theta) \le 0$, thus when $\Delta u(\theta_0) = \gamma^{\frac{1}{1-\alpha}} \cdot z_m p - q > 0$, there are two zeros in $(0, \theta_0)$ and $(\theta_0, +\infty)$ respectively, and we denote them as $\underline{\theta}$ and $\bar{\theta}$. Then when $\theta \in (\underline{\theta}, \bar{\theta})$, we have $\Delta u > 0$, while when $\theta \in (0, \underline{\theta}) \cup (\bar{\theta}, +\infty)$, we have $\Delta u < 0$. And when $\Delta u(\theta_0) < 0$, there is no zero and the function Δu is always negative. This together with the fact that $sign(\Delta u)$ reflects the users' willingness to join the market guarantees the establishment of the theorem. ■

We visualize the users' selection as follows. Notice that only the users with evaluations that's neither too high or too low will join the plan.

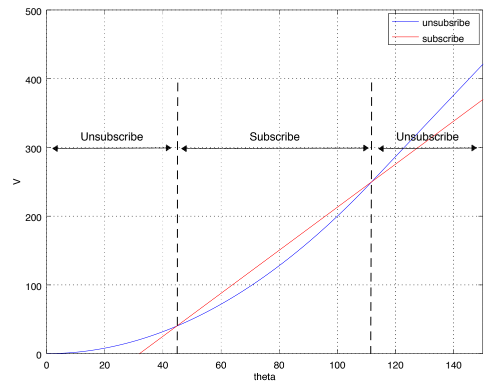

Fig. 4: The user's subscriptions under self-operating ISP.

Similarly, we can extend proposition 1 to this case, the proof method is similar so we omit it here.

IV. CP's OPTIMAL SUBSCRIPTION PRICE

In the competitive ISP's case, we consider the CP pays a side payment to the ISP as in the sponsored data plan[infocom16', SDP15']. We follow the ground setting in these previous works to assume that the side payment from the CP is linear to the subscribed users' data usage. And the ISP will determine a contract price r with the CP. We then have the demand from all the subscribed users' music data consumption as $D(q) = z_m N \cdot (1 - \Lambda(\theta_{th}))$. Then the CP's payment is $P_{CP} = r \cdot D(q)$, and we have the CP's payoff as

$$V_{CP} = W_{CP} - P_{CP} = (q - r z_m) N \cdot (1 - \Lambda(\theta_{th}))\,.$$

The CP then maximizes her payoff deciding on q. We have the following theorem on her optimal subscription fee.

***Theorem** 3 (Optimal subscription fee):* The optimal subscription fee of the CP

$$q^* = \frac{z_m^{1-\alpha} \cdot \theta_{th}^* - \alpha p^{1-\frac{1}{\alpha}} \theta_{th}^{*\frac{1}{\alpha}}}{1-\alpha}.$$

, here θ_{th}^* is the single root of the following equation.

$$(\mathbb{E}(\theta) - \theta_{th}^*) z_m^{1-\alpha} - (\mathbb{E}(\theta) - \alpha\theta_{th}^*) p^{1-\frac{1}{\alpha}} \theta_{th}^{*\frac{1}{\alpha}-1} + (1-\alpha) r z_m = 0$$

Proof: Since we have the derivative of the CP's payoff to q as

$$\begin{aligned}\frac{\partial V_{CP}}{\partial q} =& N\left(1 - \Lambda(\theta_{th}) + (rz_m - q) \cdot \lambda(\theta_{th}) \cdot \frac{\partial \theta_{th}}{\partial q}\right) \\ =& N\lambda(\theta_{th}) \cdot \frac{\partial \theta_{th}}{\partial q} \cdot \left(\mathbb{E}(\theta) \cdot \frac{\partial q}{\partial \theta_{th}} + (rz_m - q)\right)\end{aligned}$$

Note that $\lambda(\theta_{th})$ and $\frac{\partial \theta_{th}}{\partial q}$ is always non-negative. And we have that $f(\theta_{th}) = H(\theta_{th})^{-1} \cdot \frac{\partial q}{\partial \theta_{th}} + (rz_m - q)$ is a decreasing function in θ_{th}. And $f(q^{-1}(rz_m)) > 0$, $f(z_m^\alpha p) < 0$. Thus there exists a zero for $f(\theta_{th})$, where the subscription fee is optimal. ■

We have the following proposition for the ISP's side data price's influence on the CP's optimal subscription price.

***Proposition** 2 (Side data price's influence on subscription fee):* The CP's subscription fee q increases with the side data price r.

Proof: We have $\frac{\partial q}{\partial r} = \frac{\partial q}{\partial \theta_{th}} \cdot \frac{\partial \theta_{th}}{\partial r}$, and since the derivative $\frac{\partial q}{\theta_{th}} \geq 0$, thus it suffices to prove that $\frac{\partial r}{\partial \theta_{th}} = \left(\frac{\partial \theta_{th}}{\partial r}\right)^{-1} \geq 0$ to get the claim establish. Since we have

$$r = \frac{1}{(1-\alpha) z_m}\left((\mathbb{E}(\theta) - \alpha\theta_{th}) p^{1-\frac{1}{\alpha}} \theta_{th}^{*\frac{1}{\alpha}-1} - (\mathbb{E}(\theta) - \theta_{th}) z_m^{1-\alpha}\right),$$

then the derivative is

$$\frac{\partial r}{\partial \theta_{th}} = \frac{1}{(1-\alpha) z_m}\left(\left(\frac{1}{\alpha} - 1\right)\mathbb{E}(\theta) p^{1-\frac{1}{\alpha}} \theta_{th}^{\frac{1}{\alpha}-1} - p^{1-\frac{1}{\alpha}} \theta_{th}^{\frac{1}{\alpha}-1} + z_m^{1-\alpha}\right).$$

We claim that the derivative is larger than zero since the first term in the bracket is non-negative, and the absolute value of the second term is no larger than that of the third term due to $\theta_{th} \leq z_m^\alpha p$. ■

The following simulation result validates our conclusion.

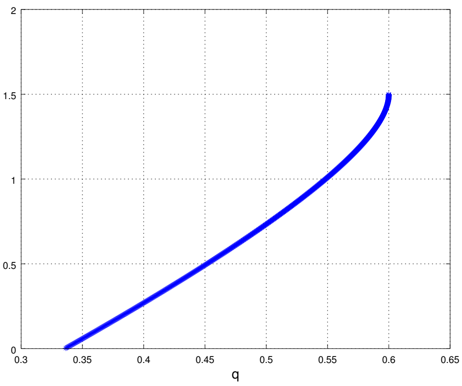

Fig. 5: q increases with r.

Notice that due to the chain rule, θ_{th} also increases with r.

V. ISP's Optimal Fee Setting

A. Cooperating ISP, thus $d = 0$

The ISP's payoff is made up the unsubscribed users' data payment and the side payment from the CP. We have the unsubscribed users' payment as $P_{data} = p \cdot \left(\int_0^{\theta_{th}} (\frac{\theta}{p})^{\frac{1}{\alpha}} \lambda(\theta) d\theta\right)$. And the ISP's payoff is

$$V_{ISP} = P_{CP} + P_{data} = (r-c) z_m N \cdot (1 - \Lambda(\theta_{th})) + (p-c) N \cdot \left(\int_0^{\theta_{th}} (\frac{\theta}{p})^{\frac{1}{\alpha}} \lambda(\theta) d\theta\right).$$

Notice that for any given r, there will be an unique q and θ_{th} determined by Theorem1 and Theorem2. Thus we have the ISP will choose the optimal side data price r maximizing his payoff $r^* = \arg\max_r (V_{ISP})$. And when $r^* < z_m p$, the ISP has incentive to provide this plan with the CP. And we have the following proposition for the optimal side data price.

***Proposition** 3 (optimal side data price):* We have the following cases for determining the ISP's optimal side data price.

- Case I. The equation (*) has no zero, thus the price $r^* = p$.
- Case II. The equation (*) has two zeros corresponding to $0 \le r_1 < r_2 \le p$, thus $r^* = \arg\max_{r_1,p} V_{ISP}(r)$.
- Case III.The equation (*) has four zeros corresponding to $0 \le r_1 < r_2 < r_3 < r_4 \le p$, thus $r^* = \arg\max_{r_1,r_3,p} V_{ISP}(r)$.

$$\left(\frac{\partial q}{\partial \theta_{th}} - \mathbb{E}(\theta) \cdot \frac{\partial^2 q}{\partial \theta_{th}^2}\right) \cdot \mathbb{E}(\theta) + (p-c) \cdot \left(\frac{\theta_{th}}{p}\right)^{\frac{1}{\alpha}} - r \cdot z_m + c \cdot z_m = 0 \ (*)$$

Note that the mapping from θ to r is determined by Theorem 3.

Proof: (sketch) The full proof is a little bit lengthy and we provide some understanding and proof method here. This proposition is just saying that the function $V_{ISP}(r)$ has no more than two local maxima in the interval $[0, p]$ which could be global maxima, and the right side point at $r = p$ is also a possible global maxima. Thus the global maxima only have no more than three candidates and can be efficiently computed with appropriate analysis.

The proof method is by derivate analysis. A useful trick we use here is that, since $\frac{\partial V_{ISP}}{\partial r} = \frac{\partial V_{ISP}}{\partial \theta_{th}} \cdot \frac{\partial \theta_{th}}{\partial r}$, thus the monotonicity of V_{ISP} towards r is the same to that of V_{ISP} towards θ_{th} (also note that θ_{th}, q, r are completely interdependent). We take the derivatives of V_{ISP} towards θ_{th}, and use $r = \frac{q - \mathbb{E}(\theta) \cdot \frac{\partial q}{\partial \theta_{th}}}{z_m}$ from Stage II analysis to get

$$\frac{\partial V_{ISP}}{\partial \theta_{th}} = \left(\left(\frac{\partial q}{\partial \theta_{th}} - \mathbb{E}(\theta) \cdot \frac{\partial^2 q}{\partial \theta_{th}^2}\right) \cdot \mathbb{E}(\theta) + (p-c) \cdot \left(\frac{\theta_{th}}{p}\right)^{\frac{1}{\alpha}} - r \cdot z_m + c \cdot z_m\right) \cdot \frac{\lambda(\theta_{th})}{\mathbb{E}(\theta)} \triangleq g(\theta_{th}) \cdot \frac{\lambda(\theta_{th})}{\mathbb{E}(\theta_{th})}.$$

Since $\frac{\lambda(\theta_{th})}{\mathbb{E}(\theta)}$ is always non-negative, we denote the left part in the bracket as $g(\theta_{th})$, and we have $sign(\frac{\partial V_{ISP}}{\partial \theta_{th}}) = sign(g(\theta_{th}))$

$$\frac{\partial g}{\partial \theta_{th}} = 2\mathbb{E}(\theta) \cdot \frac{\partial^2 q}{\partial \theta_{th}^2} - \mathbb{E}(\theta)^2 \cdot \frac{\partial^3 q}{\partial \theta_{th}^3} + \left(\frac{p-c}{\alpha}\right) \cdot p^{-\frac{1}{\alpha}} \theta^{\frac{1}{\alpha}-1} - \frac{\partial q}{\partial \theta_{th}};$$
$$\frac{\partial^2 g}{\partial \theta_{th}^2} = 2\mathbb{E}(\theta) \cdot \frac{\partial^3 q}{\partial \theta_{th}^3} - \mathbb{E}(\theta)^2 \cdot \frac{\partial^4 q}{\partial \theta_{th}^4} - \left((\frac{1}{\alpha} - 1) \cdot (1 - \frac{c}{p}) + 1\right) \cdot \frac{\partial^2 q}{\partial \theta_{th}^2}.$$

The detailed analysis is quite lengthy, and we do not show it here. The main idea is analysing the sign of $\frac{\partial V_{ISP}}{\partial \theta_{th}}$ by discussing the properties of $\frac{\partial g}{\partial \theta_{th}}$ and $\frac{\partial^2 g}{\partial \theta_{th}^2}$ as well as the values of these functions at the end points of the interval $[0, p]$. ■

We visualize the results in the proposition above as follows. In case I, the function $V_{ISP}(r)$ has no local maxima, and attains maximum at $r^* = p$, which also means that the unlimited data plan is not viable. However, in case II, the function $V_{ISP}(r)$ has one local maxima at $r_1 = 30.5$, one local minima at $r_2 = 49.1$, and the optimal pricing should be $r^* = \arg\max_{r_1,p} V_{ISP}(r) = 30.5$. Case III covers only a quite small range of parameters and the simulation results show that the changes are usually quite subtle in this case, which means that most ground setting would fall to Case I and Case II per se.

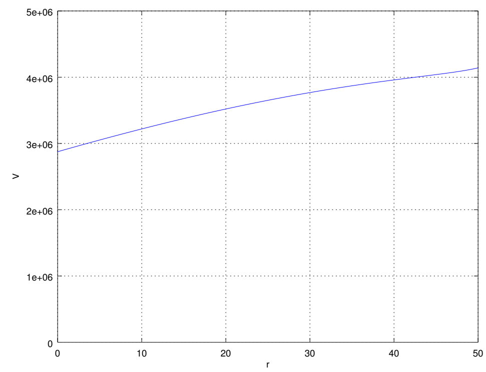

Fig. 6: Case I. V_{ISP} increases in r.

It's important to notice that all the threshold values in all three cases has nice analytical properties and is vey efficient to solve by bi-partition search. Thus computing the optimal price for the ISP is easy. Note that when $r^* = p$, the unlimited data plan is not viable. We have the following theorem for the economics viability of this system.

***Theorem** 4 (Economics viability under a cooperative ISP):* There exists some threshold value Θ. Thus when $\mathbb{E}(\theta) < \Theta$, the unlimited music data plan is viable; otherwise it's not viable.

Proof: We prove the existence of such a Θ by proving the following more stronger claim. If for some $\Theta_1 > 0$, there exists a better price than p, i.e., $\exists p^* < p, V_{ISP}(p^*) > V_{ISP}(p)$, then for any $\Theta_2 < \Theta_1$, this is also a better price than p.

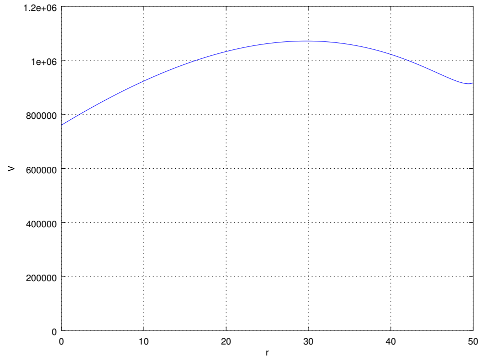

Fig. 7: Case I. V_{ISP} have one local maxima.

That is, $V_{ISP}(p^*|\mathbb{E}(\theta)=\Theta_1) > V_{ISP}(p|\mathbb{E}(\theta)=\Theta_1) \Rightarrow V_{ISP}(p^*|\mathbb{E}(\theta)=\Theta_2) > V_{ISP}(p|\mathbb{E}(\theta)=\Theta_2)$. In fact, we have that if $V_{ISP}(p^*) > V_{ISP}(p)$, then

$$V_{ISP}(p) - V_{ISP}(p^*) < 0 \Rightarrow \int_{p^*}^{p} \left(\frac{\partial V_{ISP}}{\partial q}\right) dq < 0 \Rightarrow \int_{\theta_{th}^*}^{z_m^\alpha p} \left(\frac{\partial V_{ISP}}{\partial \theta_{th}}\right) d\theta_{th} < 0$$

$$\Rightarrow \int_{\theta_{th}^*}^{z_m^\alpha p} \left(\left(\left(\frac{\partial q}{\partial \theta_{th}} - \mathbb{E}(\theta)\cdot\frac{\partial^2 q}{\partial \theta_{th}^2}\right)\cdot\mathbb{E}(\theta) + (p-c)\cdot\left(\frac{\theta_{th}}{p}\right)^{\frac{1}{\alpha}} - r\cdot z_m + c\cdot z_m\right)\cdot\frac{\lambda(\theta_{th})}{\mathbb{E}(\theta)}\right) d\theta_{th} < 0$$

$$\Rightarrow \int_{\theta_{th}^*}^{z_m^\alpha p} \left(\frac{\partial q}{\partial \theta_{th}}\mathbb{E}(\theta) - \frac{\partial^2 q}{\partial \theta_{th}^2}\cdot\mathbb{E}(\theta)^2\right)\cdot\lambda(\theta_{th})d\theta_{th} < \int_{\theta_{th}^*}^{z_m^\alpha p}\left((p-c)\cdot\left(\frac{\theta_{th}}{p}\right)^{\frac{1}{\alpha}} - r\cdot z_m + c\cdot z_m\right)\cdot\lambda(\theta_{th})d\theta_{th}$$

Notice that the *lfs* increases in $\mathbb{E}(\theta)$ since $\frac{\partial q}{\partial \theta_{th}} \geq 0$ and $-\frac{\partial^2 q}{\partial \theta_{th}^2} \geq 0$ always establishes. Thus there if this equation establishes for some $\mathbb{E}(\theta) = \Theta_1$ and θ_{th}^*, then it establishes for all $\mathbb{E}(\theta) = \Theta_2 < \Theta_1$ and the same θ_{th}^*. This implies the existence of a threshold Θ. ■

As is shown in the simulation result, when $\mathbb{E}(\theta) < 40$, the ISP will set $r^* < p$, thus the unlimited data plan is viable. However, when $\mathbb{E}(\theta) < 40$, the ISP will always set $r^* = p$, and lacks incentive to provide this plan with the CPs.

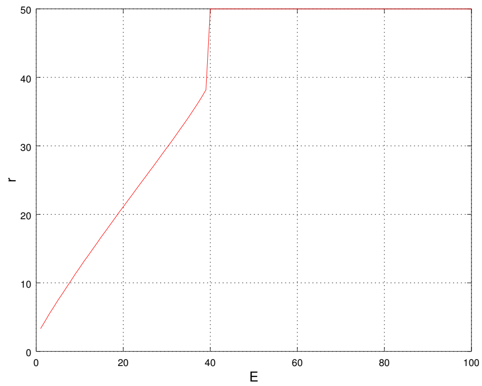

Fig. 8: The existence of a threshold value Θ.

However, when $\Theta = +\infty$, this plan is always not viable; and when $\Theta = 0$, it is always viable. And through observation, we find that Θ increases with z_m, thus when the users' intrinsic needs are higher, the plan is more viable. And we found that r^* increases in $\mathbb{E}(\theta)$, thus when the users' evaluations are higher, the optimal side data price and subscription fee is higher.

B. *Self-operating ISP, thus $d = 1$*

Under this case, the ISP should determine the subscription fee to maximize his payoff. We have his payoff as

$$V_{ISP} = N\int_0^{\underline{\theta}} (p-c)\cdot\left(\frac{\theta}{p}\right)^{\frac{1}{\alpha}}\lambda(\theta)d\theta + N(q - cz_m)\cdot\int_{\underline{\theta}}^{\bar{\theta}}\lambda(\theta)d\theta + N\int_{\bar{\theta}}^{+\infty}(p-c)\cdot\left(\frac{\theta}{p}\right)^{\frac{1}{\alpha}}\lambda(\theta)d\theta$$

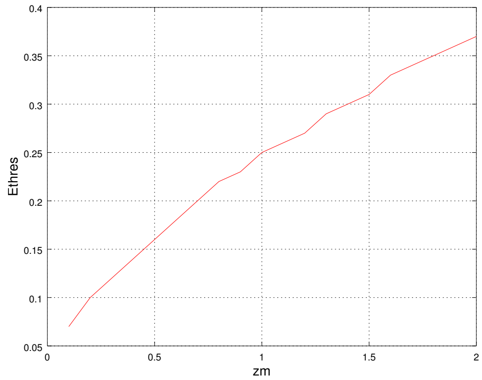

Fig. 9: Θ increases in z_m.

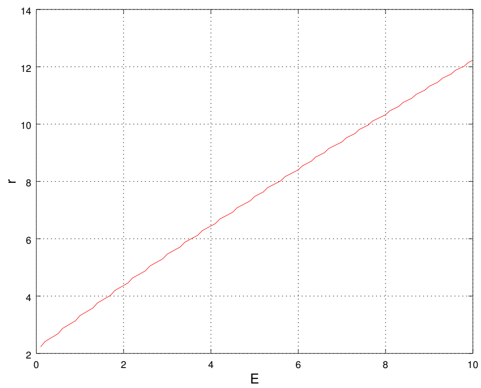

Fig. 10: r increases in $\mathbb{E}(\theta)$.

To this end, however, any analytical result seems hard to obtain, and we resort to simulations for interesting observations. As is shown in the result, the function V_{ISP} seems to be unimodal in q. Thus we can use one-dimensional search effectively to find an optimal subscription fee for the self-operating ISP based on this observation.

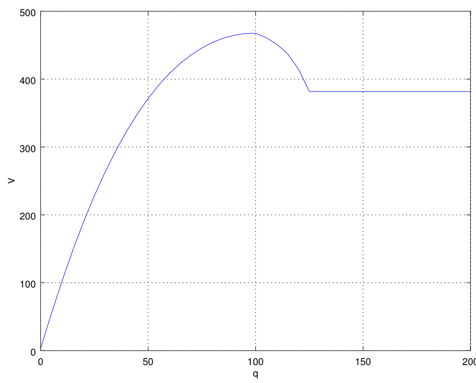

Fig. 11: V_{ISP} is unimodal in q.

C. ISP's decision on cooperation, self-operation or no provision.

The ISP finally needs to decide whether to join the cooperation with CP, or to self-operate a music platform and provide unlimited data plans, or provide no unlimited music data plan. He maximizes his revenue by making an optimal decision, and we visualize the simulation results as below.

As is shown in the figure, we have the following observation.

***Observation** 1 (Service Provisioning for ISP):* There exists some threshold values Γ_1, Γ_2, thus

- Case 1(*low discount*, thus $0 \leq \gamma \leq \Gamma_1$) The ISP will only choose to cooperate with CP when $\mathbb{E}(\theta) \leq \Theta$, and provide no unlimited data plan when $\mathbb{E}(\theta) > \Theta$;
- Case 2(*median discount*, thus $\Gamma_1 \leq \gamma \leq \Gamma_2$) The ISP will choose to cooperate with CP when $\mathbb{E}(\theta) \leq \alpha$ or $\beta \leq \mathbb{E}(\theta) \leq \Theta$, and choose to self-operate when $\alpha \leq \mathbb{E}(\theta) \leq \beta$, and no provision otherwise;

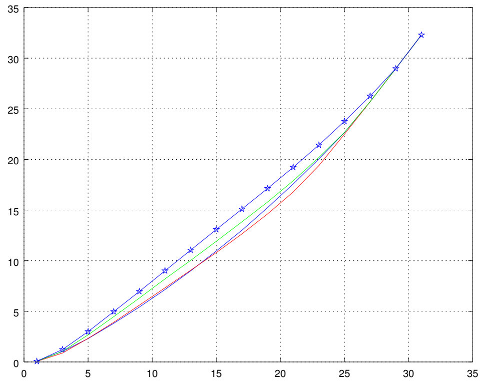

Fig. 12: Optimal ISP revenues.

- Case 3(*high discount*, thus $\Gamma_2 \leq \gamma \leq 1$) The ISP will only choose to cooperate with CP when $\mathbb{E}(\theta) \leq \alpha$, and choose to self-operate when $\alpha \leq \mathbb{E}(\theta) \leq \beta$, and no provision otherwise.

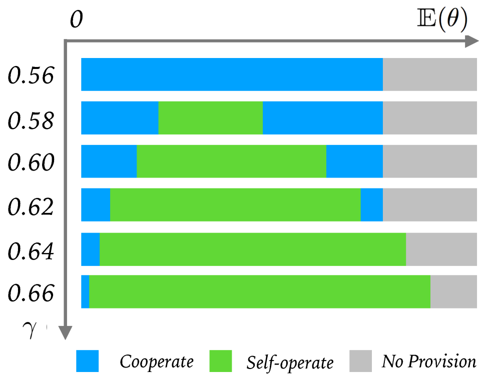

Fig. 13: Optimal ISP decisions.

We analyse each case here. In case 1, since the discount factor is too low, self-operation will lead to low user subscription and is always inferior to cooperation with CP, which attracts a lot more user subscription for high quality of music service. And more importantly, since the unlimited data service mainly attracts the low-evaluation users' consumption, thus when the user average evaluation is low, more users will be attracted and thus the ISP will have more incentive to cooperate with CP. While when the user average evaluation is high, less low-evaluation users will be attracted and the cooperation will be less lucrative for the ISP (since high-evaluation users are already paying a high data fee, they will not be the main source of profit for the unlimited data plan). In all, we give a explanation for case 1.

For case 3, we have the same explanation for why the self-operating option is optimal for $\mathbb{E}(\theta)$ lower than some threshold value β – the self-operating ISP mainly attracts low-evaluation users since high-evaluation users will not subscribe for music service discount. And for the differences between the self-operating ISP and cooperating ISP, the former fails to attract some low evaluation users and some high evaluation users. However, when the average evaluation is sufficiently low, the effect of the high-evaluation users are infinitesimal and the revenue loss from the low-evaluation users makes the self-operating ISP inferior to cooperating ISP.

For case 2, we perceive that the distribution of that under some high average user evaluations, the ISP still chooses to cooperate with the ISP(the blue bar between the green and grey bars), this is due to the revenue loss from the users with high evaluations for the self-operating ISP.

As is shown in the figure, under some cases($\mathbb{E}(\theta) < 5$), the ISP can more than double his revenue by cooperating with the CPs; and when $5 \geq \mathbb{E}(\theta) < 13.5$, he will improve his revenue by $35\% \sim 143\%$ via self-operation. However, when $13.5 < \mathbb{E}(\theta) < 26$, the mild improvement from cooperation with CP still exists. However, when $26 < \mathbb{E}(\theta)$, the ISP has no incentive in providing the unlimited data plan.

Moreover, the consumer surplus as well as social welfare increases.

Another observation is that when the user demand is more sensitive to the price, i.e., the price elasticity $alpha^{-1}$ is sufficiently high, the revenue generation for the ISP can be dramatic. As is shown in the results, when $\alpha = 0.1$, the revenue for no-provision ISP is ignorable compared to the cooperative ISP. The intuitive explanation is that when the user is highly sensitive to the price, their usage is greatly suppressed by the ISP's usage-based pricing. However, under flat-pricing, the high-evaluation users' data consumption is released, and they are willing to pay a relatively economic flat price for using the data.

And another observation is that, as the data cost is higher, the incentive of the ISP to provide unlimited data plan is lower.

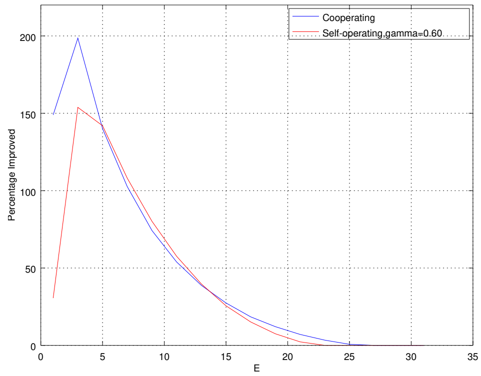

Fig. 14: Revenue Improvement of ISP.

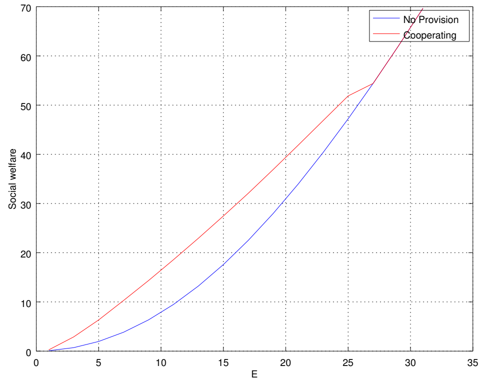

Fig. 15: Revenue Improvement of cooperative ISP.

As is shown in the figure, the revenue is relatively evenly shared between the ISP and CP when $\mathbb{E}(\theta)$ is low, this is because the revenue improvement is significant and the ISP has the incentive to join the market under even share of revenue. However, when $\mathbb{E}(\theta)$ increases, the superiority of cooperation decreases for the ISP due to the reduction in revenue improvement. The CP needs to give larger portion of the revenue to the ISP for incentivizing him for unlimited data plan provisioning.

As is shown in the figure, as $\mathbb{E}(\theta)$ increase, the number of subscribing users at the the equilibrium decreases. This is because the users with

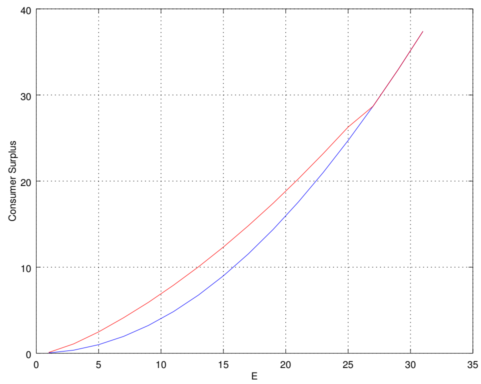

Fig. 16: Consumer Surplus under cooperative ISP.

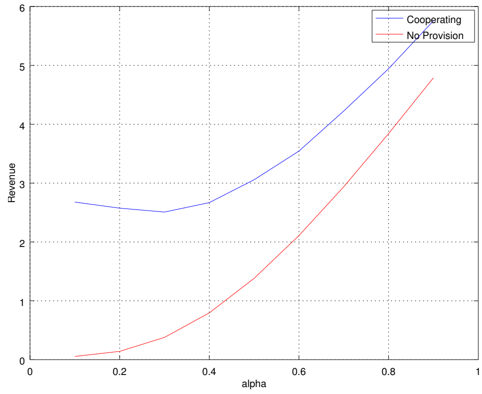

Fig. 17: Revenue Improvement under different elasticity.

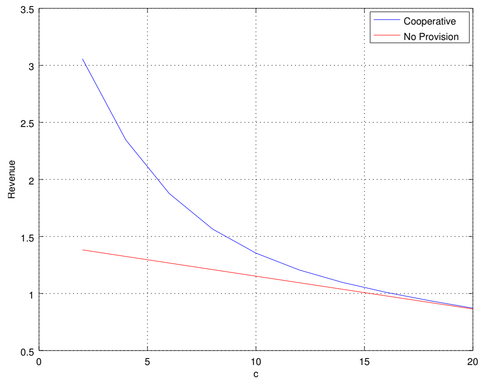

Fig. 18: Revenue Improvement under data costs.

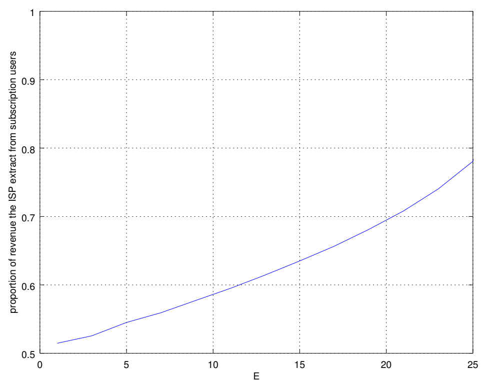

Fig. 19: The Proportion of Revenue ISP get from the Subscribing Users.

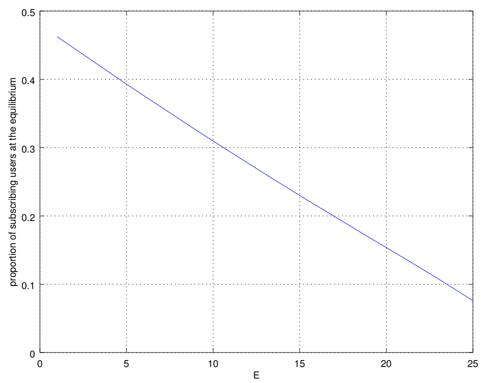

Fig. 20: The Proportion of the Subscribing Users.