Contract-and-Spot Hybrid Mechanism for Edge Caching Service Provisioning

Abstract—**The 5G white paper has enlisted the *edge caching* as an important measure to improve the quality of the wireless content delivery, and prior to this, Microsoft has provided the edge caching service since 2009. However, the mobile network operator(MNO) could still lack incentive to provide such service due to the extra installation and operations cost. This work provides the design of a novel contract-and-spot hybrid mechanism for the edge caching service provisioning. We first present the framework of our mechanism where a monopolistic MNO provides both contract service and spot service for multiple content providers(CPs). We then analyze the heterogeneous CPs' choices and find that the quality-sensitive CPs prefer the high-quality contract service; while the less quality-sensitive CPs prefer the dynamic spot market. We also show that the MNO can leverage this hybrid mechanism to generate considerable revenue in comparison with other traditional mechanisms(e.g., second-price auction, pure spot pricing). We finally validate our analysis through numerical simulations and show that the hybrid mechanism provides excellent revenue generation for MNO and near-optimal performance in efficiency.**

I. Introduction

The future of wireless content delivery is being shaped by more and more emerging technologies and business entities. *Edge caching* serves as an effective measure for improving the quality of wireless content delivery. Since 2009, Microsoft has provided the edge caching service, and charged the content providers on a volume-based pricing scheme [1]. The 5G white paper has also included edge caching as a key part of the future wireless content delivery network. However, as the promotion of the edge caching technology, more problems in the fields of economics and privacy appear besides the technical ones [2].

The wireless content delivery ecosystem, as is shown in Fig1, mainly includes three parties: the content providers (CPs), the mobile network operators (MNOs) and the end users (EUs). Enabling edge caching service will increase the content providers' quality of service, and improve the end users' quality of experience. However, providing edge caching service will incur considerable capital cost (e.g. storage installation at the network edge) and operations cost (e.g. the energy cost for content caching) for a large MNO albeit it could save the backhaul bandwidth. Thus a well-designed mechanism is needed for incentivizing different parties to realize the edge caching service.

A. Edge Caching Ecosystem

In contrast to the traditional content delivery network (CDN) caching, the edge caching has some special properties that have brought up new challenges for the already well-established caching techniques [3]. To achieve better performance, our mechanism needs to take the edge caching's special properties into consideration. We summarize the two main differences between the edge caching and traditional CDN caching as below.

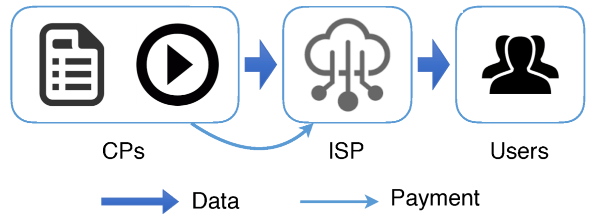

Fig. 1: The Content Delivery Ecosystem.

- *Fast Time-varying Popularity*: since the edge cache usually covers a quite limited amount of users, thus the content popularity in the edge could change steeply over time. And our mechanism for edge caching network should provide flexibility for this fast changing popularity of the contents.
- *Limited Cache Space*: the cache space at the edge is quite limited compared to the traditional CDN caching appliances. Thus our mechanism should be efficient enough to make the most of these limited resources.

Another issue our mechanism needs to consider is the incentive of MNO and CPs. First, for the MNO who provides the edge caching service, the mechanism should provide him enough revenue increase that can incentivize him to invest in edge caching and to operate the system. Second, in the perspective of CPs, the mechanism should also improve their payoffs. We then consider heterogeneous CPs who have different evaluations on edge caching due to their different needs for delivery quality. For example, a video content provider might be more sensitive to the delivery latency while a file-download service provider will be less sensitive to the latency. The quality-sensitive CPs (e.g. video providers) would like to secure some cache spaces for providing stable high-quality services, while the less quality-sensitive CPs (e.g. file downloaders) would like more dynamic access to the cache spaces. This requires our mechanism to provide different services for different CPs' preferences.

Moreover, taking the resource allocation into consideration, our mechanism needs to provide relatively good performance in efficiency and fairness. To this end, we summarize the objective of designing a mechanism for the edge caching market as follows.

- *Flexibility*: The mechanism needs to provide flexible access to the edge cache to address the stochastic content popularity and the CPs' heterogeneity.
- *Optimality*: The mechanism needs to provide high level of revenue generation for the MNO.
- *Efficiency*: The mechanism needs to allocate the resources to achieve high social welfare.

B. Motivation and Design Rationale

We illustrate the motivation of our hybrid mechanism design in the following part. We first consider the flexibility objective of mechanism design. The quite limited cache spaces make the CPs compete for cache spaces and this competition becomes more complex when popularity of contents is fast time-varying. The intense competition over the quite limited resources may make a quality-sensitive CP unable to obtain a cache space in the spot market where the MNO reallocates the cache spaces for each time slot (usually 15-30mins) [2]. Thus for some CPs with high evaluation over caching, he would like to pay a higher fee than spot price to secure the cache space for a longer time (e.g., 1 month). However, for another CP with low evaluation over caching, he only would like to cache a content when his realized short-term popularity (i.e., the request rate for the upcoming time slot) is high. Thus he would prefer to buy the cache space more dynamically. This inspires us to design a contract-and-spot hybrid mechanism which provides both contract and spot service to address the CPs' different preferences over content delivery quality.

We then consider the viability of such a hybrid mechanism in the perspective of the MNO. He could provide contract service for the quality-sensitive CPs with a higher price than spot pricing, and then secure certain amount of cache space for him throughout a finite time horizon. And he could also provide spot service for the less quality-sensitive CPs, thus these CPs can realize their popularity for the next time slot before deciding whether or not to purchase a cache space. The MNO's profit can increase since he can charge a higher contract prices to the quality-sensitive CPs. This verifies the economics viability in the MNO side, and gives the MNO more incentive to provide the service.

However, the hybrid mechanism could compromise efficiency, compared to other welfare-maximizing mechanisms like auction. In fact, the traditional second-price auction could not cater to the different CP's service preferences, and has intrinsic weakness in high complexity. Moreover, another possible problem with auction is revenue generation. We show in our analysis and experiments (Section VI) that the revenue generation can be a great concern for the auction mechanism. And hybrid mechanism shows plausible performance in revenue generation in comparison with other mechanisms, and still achieves near-optimal efficiency.

C. Contract-and-Spot Hybrid Mechanism

Based on this contract-and-spot hybrid mechanism, we study the following problems.

- **CP's strategic behavior** Which market will the CP choose to join? How much cache spaces should the CP purchase in the contract market? *We derive the membership selection of the CPs at the subgame perfect equilibrium (SPE). We found that the quality-sensitive CPs prefers contract market, as is illustrated in our design rationale.*
- **MNO's revenue generation** Which contract price and spot price should the MNO set? And what is MNO's optimal revenue? *We formulate the MNO's optimal pricing problem, and provide effective constraints on the optimal price searching. We also provide lower bound for the MNO's optimal revenue in hybrid market.*
- **Social performance** How does this mechanism influences the utilities of different parties? How does this mechanism perform in terms of efficiency and fairness? *We did comparative study through extensive numerical simulations, and found that the our mechanism has the best performance in revenue generation, and achieves near-optimal efficiency and fairness.*

Then, in Section II, we refer to related works in the field of economics and wireless network. We formulate our problem in section III, and make game-theoretic analysis over this problem in section IV. And then in section V, we discuss the generalized extension over our baseline model. We evaluate our mechanism through numerical simulations in section VI. Eventually, we conclude our work in section VII.

II. RELATED WORKS

Our work is mainly related to two groups of literatures, the pure economics study on hybrid mechanism design and the applied economics research in the field of edge caching. We first consider some classic works about the hybrid market mechanism in the field of pure economics study. Ferreira *et al*. proposed a mechanism with both futures market and spot market in [3], indicating that the hybrid market will promote the competition in the spot market. And in [4], Allaz Blaise *et al*, gave a competition analysis on the futures and spot combined market and derived the Allaz-Vila point for the market equilibrium. Further in [5], Murphy Frederic *et al*, argued that the Allaz-Vila point may not be unique and the theoretical price-of-anarchy (PoA) can be quite high in such a hybrid market.

We then consider the economics mechanism design in edge caching market. In [2], Bastug Ejder *et al*, first shed light on the economics issues in the edge caching ecosystem, and made a simple stakeholder analysis to illustrate the incentive issues. In [6], a contract-based mechanism is proposed for the market. Then in [7], the authors provided a Stackelberg game analysis for the edge-caching market. However, none of these works considered edge-caching's special charateristics as well as the MNO's revenue generation issues. And in [8], the work most relevant to ours, L. Gao *et al*, proposed a contract-and-auction hybrid mechanism for the secondary spectrum trading market with previously known secondary user's (SUs) choices over contract and auction services. In contrast, our

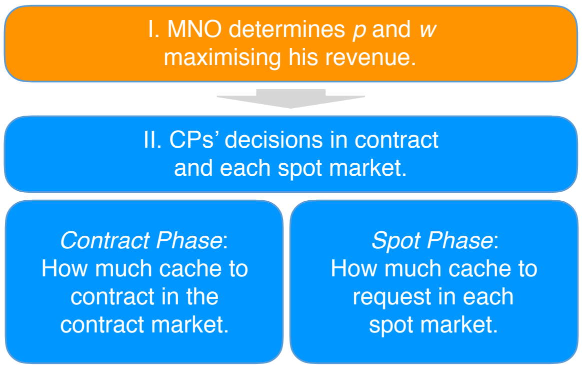

Fig. 2: The Decision Making Process.

work takes the CP's strategic service selection behaviours into account and provide a market mechanism which both caters to CPs' different preferences and generate more revenue for the MNO.

III. Edge Caching Market Model

In our model, we consider that a mobile network operator (MNO) sells the edge-caching service to a group of content providers (CPs). He provides both contract and spot services in the edge caching market to the CPs. The decision making process is shown in Fig1. The MNO first determines the prices for edge caching service in both the contract market and the spot market. Then each CP makes decision on how many cache spaces he would like to get in the *contract phase*, and how many cache spaces to request in each of the *spot phases* to maximize their utilities. Our assumption of a monopolistic MNO is due to the fact that when there are oligopolistic MNOs, they are often effective monopolists due to low churn rates [9].

A. Edge Caching Model

We consider a finite operational horizon (e.g. one month) for the MNO, as is shown in Fig3. The whole time horizon consists of T time slots (e.g. about 2,000 time slots/month), $\mathcal{T} = \{1, 2, ..., T\}$. And in every time slot, MNO refreshes the popular content caches at the edge in the caching phase based on the recent request history of each content. And then in the request phase, the users issues requests for contents, and the contents that get hit in the cache will be delivered to the end user directly from the edge cache. And the caching phase is usually relatively short compared to the request phase.

We follow the study of [6], [8] to suppose that the MNO charges the CPs on a usage-based pricing scheme, thus the payment of each CP increases linear with the amount of cache space used. Moreover, we consider a more fine-grained operation of the MNO, thus he will allocate the cache spaces to the CPs on a time slot basis, and we study the MNO and CPs' interaction over this finite time horizon in our market design.

We then suppose the MNO has C cache spaces for the market. And we suppose there are N CPs participating in this market, $\mathcal{N} = \{1, 2, ..., N\}$. We assume that each CP has

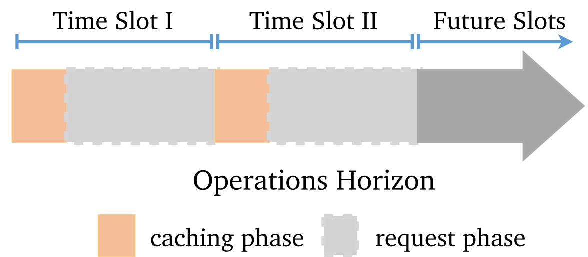

Fig. 3: Operations Horizon of the MNO.

one content to deliver in the analysis of our mechanism in Section IV, and study his strategic behaviour as a membership selection game, i.e., whether he will join the contract market, spot market or neither. And we'll generalize this condition in Section V to multiple content case where each CP has multiple contents for delivery and needs to determine an optimal content request in the contract market.

B. Contract and Spot Service Model

We illustrate the model of contract-and-spot hybrid mechanism as follows. As is shown in Fig4, the market interaction appears at the beginning of the operations horizon(*contract phase*) and before each of the caching phases(*spot phases*). We first consider the contract phase. In this phase, the MNO announces the contract price and the CPs determine whether or not to sign a contract for their contents. For CPs who have signed the contract, they can always get one cache spaces' guarantee throughout the horizon. However, these CPs also need to bear a higher contract fee for securing the cache space. Then we consider those CPs who do not sign a contract for their contents, they can still obtain a cache dynamically in the spot market. In the *spot phase* before each time slot, every CP realizes his short-term popularity(i.e., predicted request rate for his content in the upcoming time slot) and decides whether to request for a cache space in the spot market or not. When the CP's realized short term popularity is high, he will issue a cache space request in the spot phase. Since the cache spaces are limited and there could be more CPs issuing request than the cache spaces that the MNO have, thus we assume that in this case, the MNO determines the winners via a randomized allocation mechanism.

The request rate of the content could increase or decrease steeply over time due to its time-varying popularity. There have been many content popularity models trying to capture the time-varying popularity of the content and such models could be quite complex. For simplicity, we suppose that the popularity of a content over time follows a repeated Binomial distribution. And we generalize it to arbitrary popularity distribution in section V later. We denote the request rate of CP i's content in time slot t as ρ_i^t, then we have $\rho_i^t \sim \begin{pmatrix} 1-p & p \\ \underline{\rho} & \bar{\rho} \end{pmatrix}, \forall t \in \mathcal{T}$. Thus the CP's content has a probability p of having high popularity $\bar{\rho}$, and has a popularity $1-p$ of having low popularity $\underline{\rho}$. We suppose the profit that CP i can derive from a single hit is v_i. Thus delay-tolerant CPs will have low v_i, while the delay-sensitive CPs will have higher

v_i. We suppose that the MNO charges p for unit cache space in the spot market and charges w for a contracted cache space subscription.

We now consider the utility of the CPs, and we denote his membership choice $x \in \{A, C, S\}$. For CPs whose choice is $x = A$, he will join neither market and we call this kind of CP as *alien CP*. And for CPs who choose $x = C$ or $x = S$, he will join the contract market or spot market and we call them *contract CPs* or *spot CPs* respectively. Then if some CP with evaluation of v chooses not to join both market, he will get zero utility, i.e., $V(v, A) = 0$. However, if he chooses to join the contract market, we have his revenue as $v \cdot \sum_{t=1}^{T} \rho_i^t$ and his payment to the MNO for the contracted cache space as w. Thus we have this CP's utility as $V(v, C|\boldsymbol{\rho}_i) = v \cdot \sum_{t=1}^{T} \rho_i^t - w$. And since the CP has no information about exact popularity in the future time slots(i.e. hard to make long term popularity prediction) at the contract phase, thus the CP makes decisions based on the expected utility in the contract market. Thus we have his expected utility if he choose to join the contract market as

$$V(v, C) = \mathbb{E}_{\boldsymbol{\rho}_i}\{V(v, C|\boldsymbol{\rho}_i)\} = (p_0\underline{\rho} + (1-p_0)\bar{\rho}) \cdot vT - w.$$

Finally, if he chooses to join the spot market, then he will be able to perceive his short-term popularity for the upcoming time slot. And he is willing to purchase a cache space in time slot t if $\rho_i^t v_i > p$, otherwise he will have no incentive to issue a request before that time slot. The utility for in this case can be derived as follows.

$$\begin{aligned} V(v, S) =& \mathbb{E}_{\boldsymbol{\rho}}\{\sum_{t=1}^{T} \min\{\frac{C - N_C}{N_S^t}, 1\}(\rho_i^t v - p)\mathbb{1}(\rho_i^t v - p)\} \\ =& T \cdot \mathbb{E}_{\boldsymbol{\rho}}\{\min\{\frac{C - N_C}{N_S^t}, 1\}\} \cdot ((\bar{\rho} v - p) \cdot \mathbb{1}(\bar{\rho} v - p) \\ & \cdot (1 - p_0) + (\underline{\rho} v - p) \cdot \mathbb{1}(\underline{\rho} v - p) \cdot p_0) \end{aligned}$$

Here N_S^t is the number of CPs who issued a request in the spot market before time slot t. N_C is the number of contract CPs. And $\mathbb{1}(\cdot)$ is a binary indicator of the bracketed condition. Since the CP's popularity distribution is homogenous over time, thus we have $\mathbb{E}_{\boldsymbol{\rho}}\{\min\{\frac{C-N_C}{N_S^t}, 1\}\}, \forall t \in \mathcal{T}$ all are equal to some constant $\alpha \in [0, 1]$. Notice that α represents the probability of getting a cache space in the spot market, and reflects the congestion condition. When α is larger, the probability of getting a cache space in the spot market is greater, and the spot market is less congested. Then we can expand the expression of $V(v, S)$ as a piecewise function of v as follows.

$$V(v, S) = \begin{cases} 0 & \text{I} \\ \alpha T \cdot ((1 - p_0)(\bar{\rho} v - p)) & \text{II} \\ \alpha T \cdot (p_0(\underline{\rho} v - p) + (1 - p_0)(\bar{\rho} v - p)) & \text{III} \end{cases}$$

The conditions in the expression is I: $v \geq \frac{p}{\bar{\rho}}$; II: $\frac{p}{\bar{\rho}} \leq v < \frac{p}{\underline{\rho}}$; III: $v \geq \frac{p}{\underline{\rho}}$.

For the MNO, he chooses the pricing decisions p and w that maximize his overall profit. We can express the MNO's revenue as

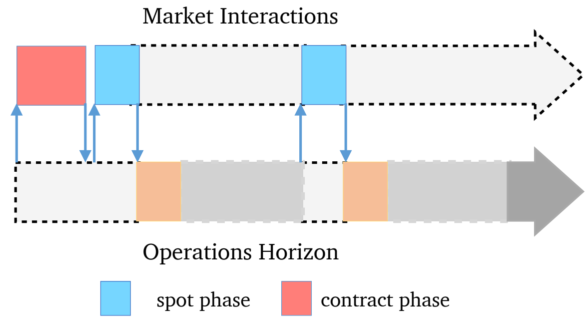

Fig. 4: Contract-and-Spot Hybrid Market.

Time Slot	1	2	3	4
CP 1	35 reqs	35 reqs	35 reqs	35 reqs
CP 2	10 reqs	20 reqs	30 reqs	40 reqs

TABLE I: Example for mechanism illustration.

$$W_{MNO} = w \cdot N_c + p \cdot \sum_{t=1}^{T} \min\{N_s^t, C - N_c\}.$$

And the cost of the MNO consists of operational cost(e.g. energy cost, maintenance cost, etc) and the capital cost(e.g. installation cost of cache space) as $C_{MNO} = C^{oper} + C^{cap}$. We'd suppose C^{oper} and C^{cap} to be constant costs for the MNO, and focus on optimal pricing for the MNO's revenue generation. we have the expected utility of the MNO as

$$V_{MNO} = w \cdot N_c + p \cdot \sum_{t=1}^{T} \mathbb{E}_{\boldsymbol{\rho}^t}\{\min\{N_s^t, C - N_c\}\} - C_{MNO}.$$

We follow the method of backward induction to solve the outcome of this game in Section IV. We first consider the CPs' best responses in Stage II. Note that CPs' decisions will also influence each other's utility and their strategic interactions in Stage II can also be delineated as a subgame. We'll use a backward-induction-like method to derive the subgame perfect equilibrium(SPE) of the CPs in Stage II. And then we'll use CPs' best responses in Stage II to solve the MNO's optimal pricing in Stage I.

Additionally, we introduce the notion of *social welfare* here. The social welfare of a system is the sum of the utilities of all parties in the system. And in our case, it is the sum of the MNO's utility and the CPs' utilities. The social welfare serves as an index for the efficiency of the resource allocation driven by the economics mechanism.

C. Illustrative Example

We now provide a simple example for our hybrid mechanism. As is illustrated in Table1, we consider an edge caching market with two content providers competing for a single cache space. Here we simply use the request rate as the CP's revenues and we consider four cycles. The request rate for CP_1 over the four cycles is constantly at 35 $requests/time\ slot$, while the request rate of CP_2 over the time slots are $\{10\text{reqs}, 20\text{reqs}, 30\text{reqs}, 40\text{reqs}\}$. Under

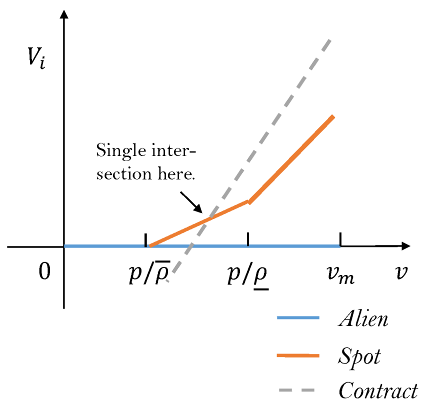

Fig. 5: The explanation for the equilibrium derivation for Stage II.

this setting, if the MNO adopts the real-time second-price auction mechanism, then the utility of the MNO would be $V_{MNO}^{auction} = 10 + 20 + 30 + 35 = 95\ reqs$, and the social welfare is $V_{SW}^{auction} = 35 + 35 + 35 + 40 = 145\ reqs$, accordingly. In contrast, if the hybrid mechanism is adopted, the MNO could charge CP_1 with an contract price of $w = 30 \times 4 = 120\ reqs$. And the CP_2 would not enter the contract market because since he would get negative utility if he chooses to join the contract market. And the revenue of the MNO will be $V_{MNO}^{hybrid} = 120\ reqs$. The social welfare becomes $V_{SW}^{hybrid} = 35 * 4 = 140\ reqs$. The social welfare loss of the hybrid mechanism is 5.0%, relatively small comparing to the 26.3% increase in the MNO's revenue. Through this example, we justify the potential of our hybrid mechanism to significantly improve the MNO's revenue with modest social welfare loss. However, later we will show that the hybrid mechanism will both in theory and practice, provide significantly better revenue generation performance with acceptable compromise in social welfare and fairness.

IV. Subgame Equilibrium In Stage II

In this section, we study the CPs' strategic behaviours in Stage II. Note that some CP's utility will be determined not only by his own membership selection but also other CPs' choices, since the other CPs' choices will influence the congestion in spot market, which is reflected by α.

And the key to solving the equilibrium of this subgame is the derivation of α. In fact, α is determined by the outcome of the subgame, namely, the number of CPs in the contract market N_S and that in the spot market N_C. However, this parameter will also influences the CPs utilities and their choices, and thus influence the outcome of the game. The correlation of α and the game outcome brings challenges for solving this subgame equilibrium. We use a backward-induction-like method to solve the CPs' membership choice equilibrium in this subgame: we first fix the α as the result of the subgame equilibrium, and consider the structural feature of the different CPs' best responses at the equilibrium; and then we use this structural feature to derive the conditions on the subgame equilibrium.

A. Structure of Subgame Equilibrium

We first fix α and plot the CPs' utilities, $V(v,x), \forall x \in \{A,S,C\}$, as in Fig5. And we denote the curve for $V(v,A), V(v,S), V(v,C)$ as *alien curve*, *spot curve* and *contract curve* respectively. Notice that the CP will choose the membership that maximizes his payoff. Thus the CP's best choice is $x = \arg \max_{x' \in \{A,S,C\}} \{V(v,x')\}$. We then have the following lemma for alien CPs.

***Lemma** 1 (Alien CPs):* The CPs with $v \in [0, v_1]$ will be alien CPs, here we have

$$v_1 = \min\left\{\frac{p}{\bar{\rho}}, \frac{w}{(p_0\underline{\rho} + (1-p_0)\bar{\rho})T}\right\}.$$

Proof: We have the conditions for CP to choose the alien market as $V(v,A) \geq V(v,S), V(v,A) \geq V(v,C)$. From the first condition we have $v < \frac{p}{\bar{\rho}}$, and from the second condition we have $v < \frac{w}{(p_0\underline{\rho}+(1-p_0)\bar{\rho})T}$. Thus the lemma establishes. ■

Since the equilibrium choices for the alien CPs is quite simple, then the key point of deriving the equilibrium exists in determining which CPs will be spot CPs and contract CPs. We then introduce the following lemma for the intersection point of spot curve and contract curve.

***Lemma** 2 (Intersection of Spot and Contract Curves):* If neither $\alpha = 1$ nor $w = pT$, the spot CP curve and the contract curve will have at most one intersection; when $\alpha = 1$ and $w = pT$, the spot and contract curve overlaps when $v \in [\frac{p}{\underline{\rho}}, v_m]$.

Proof: We prove this lemma by constructing the function $f(v) = V(v,C) - V(v,S)$. This function has the form as follows[1].

$$f(v) = \begin{cases} vT(p_0\underline{\rho} + (1-p_0)\bar{\rho}) - w & \text{I} \\ vT(p_0\underline{\rho} + (1-\alpha)(1-p_0)\bar{\rho}) - w + \alpha pT(1-p_0) & \text{II} \\ vT(1-\alpha)(p_0\underline{\rho} + (1-p_0)\bar{\rho}) + pT - w & \text{III} \end{cases}$$

We have the derivative of this piece wise function as piecewise linear function as follows.

$$f'(v) = \begin{cases} (p_0\underline{\rho} + (1-p_0)\bar{\rho})T & \text{I} \\ (p_0\underline{\rho} + (1-\alpha)(1-p_0)\bar{\rho})T & \text{II} \\ (1-\alpha)\cdot(p_0\underline{\rho} + (1-p_0)\bar{\rho})T & \text{III} \end{cases}$$

It's easy to check that $f(v) = 0, \forall v \in [\frac{p}{\underline{\rho}}, v_m]$ when $\alpha = 1, w = pT$. When neither $\alpha = 1$ and $w = pT$, we can justify that $f'(v) > 0, \forall v \in [0, v_m]$. This guarantees that $f(v)$ is a strictly increasing function, and thus $f(v)$ should have no more than one zero. Thus the two curves has at most one intersection. This ends the proof. ■

Using the Lemma1 and Lemma2, we have the following theorem for the structural character of the equilibrium.

***Theorem** 1 (Structure of the Subgame Equilibrium):* There exist threshold values $0 < v_1 \leq v_2 \leq v_m$, thus the CPs with

[1] The notation I, II, III follows the previous denotion.

value in $[0, v_1), [v_1, v_2), [v_2, v_m]$ will choose to join the alien market, the spot market and the contract market respectively.

Proof: First, from Lemma1, we have $v_1 = \min\{\frac{p}{\bar{\rho}}, \frac{w}{(p_0\underline{\rho}+(1-p_0)\bar{\rho})T}\}$. And then prove the existence of v_2 using Lemma2. When neither $\alpha = 1$ nor $w = PT$, $f(v)$ exists at most one zero. We discuss two cases as follows.

(1). *There is no intersection in* $[v_1, v_m]$.

In this case, since $f'(v) > 0, \forall v \in [v_1, v_m]$ and there is no zero for $f(v)$, then $f(v) < 0$ or $f(v) > 0$ should always establish within (v_1, v_m). And these two cases will lead to the case where all CPs in (v_1, v_m) join the spot market or the contract market directly.

(2). *There is one intersection in* $[v_1, v_m]$.

Suppose that the intersection point is v_2, then we have $V(v, C) < V(v, S)$ for CPs in (v_1, v_2), thus these CP will choose spot market; and we have $V(v, S) < V(v, C)$ for CPs in (v_2, v_m), thus these CP will choose contract market.

However, when $\alpha = 1$ and $w = pT$, we let $v_2 = \max\{v_1, \frac{p}{\underline{\rho}}\}$. Then the CPs in (v_1, v_2) will choose to join the spot market, while the CPs in (v_2, v_m) will join the contract market for securing their payoff against the other CP's choices. And this ends the proof. ■

B. Derivation of Subgame Perfect Equilibrium

After deriving the structure of the subgame equilibrium, we then try to compute the subgame equilibrium using this structural information and derive the α^* at the equilibrium. As is shown in Fig6, we consider the equilibrium under the low contract price regime(case 1), median contract price regime(case 2), high contract price regime(case 3) and ineffective contract price regime(case 4).

1) low contract price: We first consider the low contract price regime. As is shown in Fig6(case 1), the contract price is so low that no one will join the spot market. This establishes when the zero of contract curve is smaller than that of the spot curve. Thus we have $\frac{w}{(p_0\underline{\rho}+(1-p_0)\bar{\rho})\cdot T} < \frac{p}{\bar{\rho}}$, and the condition w needs to satisfy is $w < \frac{pT(p_0\underline{\rho}+(1-p_0)\bar{\rho})}{\bar{\rho}} \triangleq w_1$. And we have $v_1 = v_2 = \frac{w}{(p_0\underline{\rho}+(1-p_0)\bar{\rho})\cdot T}$ in this case. Since there is no one joining the spot market, we let $\alpha^* = 1$.

2) median contract price: Under the median contract price regime, the contract curve and the spot curve will have one intersection as v_2, we then have that the CPs in $[0, v_1)$ will become alien CPs, the CPs in $[v_1, v_2)$ will become spot CPs, the CPs in $[v_2, v_m]$ will become contract CPs. We can derive the expression of v_2 by solving $V(v_2, S|\text{II}) = V(v_2, C)$, and we have

$$\alpha^* T(1-p_0)(\bar{\rho}v_2 - p) = (p_0\underline{\rho} + (1-p_0)\bar{\rho})v_2 T - w$$

Thus we have the equation for the number of spot CPs N_C and the number of CPs with their evaluations within interval II in the spot market N_S^{II} as

$$N_C = N \cdot (1 - \frac{v_2}{v_m})$$
$$N_S^{\text{II}} = N \cdot \frac{v_2 - v_1}{v_m}$$

Notice that the spot CPs within II will request the content in the spot market when their realized request rate are high, i.e. $\rho_i^t = \bar{\rho}$. Thus these CPs will request a cache space with a probability p_0. Then we have the probability of having n cache space requests in the spot market as

$$Pr(N_S = n) = \binom{N_S^{\text{II}}}{n} \cdot p_0^n (1-p_0)^{N_S^{\text{II}} - n}, \ \forall n \in \{0, 1, ..., N_S^{\text{II}}\}$$

The available cache for the spot market is $C - N_C$, and then we can get the expression of α^* as

$$\begin{aligned}\alpha^* =& \boldsymbol{E}\left\{\min\{\frac{C - N_C}{N_S}, 1\}\right\} \\ =& \sum_{n=1}^{N_S^{\text{II}}} Pr(N_S = n) \cdot \min\left\{\frac{C - N_C}{n}, 1\right\} + Pr(N_S = 0) \\ \triangleq& \delta(N_C, N_S^{\text{II}}) = \delta\left(N \cdot \frac{v_m - v_2}{v_m}, N \cdot \frac{v_2 - v_1}{v_m}\right)\end{aligned}$$

Here we introduced the notation of $\delta(\cdot)$ to simplify the expression. Finally, we can use the equation set as follows to solve the equilibrium. Note that it's an equation set with two unknown variables α^* and v_2.

$$\begin{aligned}\alpha^* =& \delta\left(N \cdot \frac{v_m - v_2}{v_m}, N \cdot \frac{v_2 - v_1}{v_m}\right) \\ \alpha^* =& \frac{(p_0\underline{\rho} + (1-p_0)\bar{\rho})v_2 T - w}{T(1-p_0)(\bar{\rho}v_2 - p)}\end{aligned}$$

And we can derive the boundary for this median contract price setting. The boundary case occurs when the contract curve intersects with the spot curve at the turning point of the spot curve. We denote the boundary contract price as w_2. Then we have $v_2 = \frac{p}{\underline{\rho}}$ for this case, and using the equation set for α^* and v_2, we can derive the expression of w_2 as

$$\begin{aligned}w_2 =& p_0 pT + (1-p_0)\frac{\bar{\rho}}{\underline{\rho}}pT + (1 - p + 0)pT(1 - \frac{\bar{\rho}}{\underline{\rho}}) \\ &\cdot \delta\left(\frac{(v_m\underline{\rho} - p)N}{v_m\underline{\rho}}, \frac{pN(\bar{\rho} - \underline{\rho})}{v_m\bar{\rho}\underline{\rho}}\right).\end{aligned}$$

3) high contract price: Under high contract price, the contract curve intersects the spot curve at the third segment (interval III). Thus we have

$$\alpha^* T \cdot ((p_0\underline{\rho} + (1-p_0)\bar{\rho})v_2 - p) = (p_0\underline{\rho} + (1-p_0)\bar{\rho})v_2 T - w$$

And we can calculate the number of contract CPs N_C, the number of spot CPs with II N_S^{II} and the number of spot CPs within III N_S^{III} as follows.

$$\begin{aligned}N_C =& N \cdot (1 - \frac{v_2}{v_m}) \\ N_S^{\text{II}} =& N \cdot \frac{\frac{p}{\bar{\rho}} - v_1}{v_m} \\ N_S^{\text{III}} =& N \cdot \frac{v_2 - \frac{p}{\bar{\rho}}}{v_m}\end{aligned}$$

Notice that the spot CPs within III will always request for a cache space in the spot market. Thus we have

$$\begin{aligned}Pr(N_S = n) =& \binom{N_S^{\text{II}}}{n - N_S^{\text{III}}} \cdot p_0^n (1-p_0)^{N_S^{\text{II}} + N_S^{\text{III}} - n} \\ &, \forall n \in \{N_S^{\text{III}}, ..., N_S^{\text{II}} + N_S^{\text{III}}\}\end{aligned}$$

And hence we have the α^* as

$$\alpha^* = \sum_{n=N_S^{\text{III}}}^{N_S^{\text{II}}+N_S^{\text{III}}} Pr(N_S = n) \cdot \min\left\{\frac{C-N_C}{n}, 1\right\} + Pr(N_S = 0) \triangleq \gamma(N_C, N_S^{\text{II}}, N_S^{\text{III}}).$$

Here we used the notation $\gamma(\cdot)$ to simplify the expression. Thus we have the following equation set in this case.

$$\alpha^* = \frac{(p_0\underline{\rho} + (1-p_0)\bar{\rho})v_2T - w}{T\cdot((p_0\underline{\rho} + (1-p_0)\bar{\rho})v_2 - p)}$$

$$\alpha^* = \gamma\left(N\cdot(1-\frac{v_2}{v_m}), N\cdot\frac{\frac{p}{\bar{\rho}} - v_1}{v_m}, N\cdot\frac{v_2 - \frac{p}{\bar{\rho}}}{v_m}\right)$$

And we can derive the boundary condition between case 3 and case 4 when the contract curve and the spot curve intersect at v_m as

$$w_3 = \bar{\rho}(p_0\underline{\rho} + (1-p_0)\cdot\gamma\left(0, \frac{pN(\bar{\rho}-\underline{\rho})}{v_m\bar{\rho}\underline{\rho}}, \frac{(v_m\underline{\rho}-p)N}{v_m\underline{\rho}}\right).$$

4) ineffective contract price: Under this condition, the contract price is so high that no one joins the contract market, then under this case $v_2 = v_m$ and

$$\alpha^* = \gamma\left(0, N\cdot\frac{\frac{p}{\bar{\rho}} - v_1}{v_m}, N\cdot\frac{v_m - \frac{p}{\bar{\rho}}}{v_m}\right).$$

In summary, we have the following theorem for the equilibrium of the subgame between the CPs.

***Theorem** 2 (Subgame Perfect Equilibrium):* The equilibrium of the subgame between the CPs is determined as follows.

1) If $0 < w < w_1$, $\alpha^* = 1$, $v_1 = v_2 = \frac{w}{(p_0\underline{\rho}+(1-p_0)\bar{\rho})\cdot T}$.
2) If $w_1 < w < w_2$, $v_1 = \frac{p}{\bar{\rho}}$, α^* and v_2 can be determined through the equation set in case 2.
3) If $w_2 < w < w_3$, $v_1 = \frac{p}{\bar{\rho}}$, α^* and v_2 can be determined through the equation set in case 3.
4) If $w > w_3$, $\alpha^* = \gamma\left(0, N\cdot\frac{\frac{p}{\bar{\rho}}-v_1}{v_m}, N\cdot\frac{v_m-\frac{p}{\bar{\rho}}}{v_m}\right)$, $v_1 = \frac{p}{\bar{\rho}}$, $v_2 = v_m$.

To further simplify the notation, in fact we have $\delta(N_C, N_S^{\text{II}}) = \gamma(N_C, N_S^{\text{II}}, 0)$, thus the notation of δ can be eliminated. Notice that the CPs with higher evaluations are willing to join the contract market, and the others with lower evaluation are willing to join the spot market, which proves the mechanism's flexibility to cater to different CPs' preferences.

V. MNO's Optimal Pricing in Stage I

After the CPs have decided their best responses in Stage II, we then consider the optimization problem for the MNO in Stage I. In fact, directly solving the optimization problem can be quite hard due to the inexplicit expression of outcome of the subgame equilibrium. However, we first derive the constraints to effectively accelerate the searching of the MNO's optimal price, and then we derive a lower bound for the MNO's optimal revenue.

A. Constraints on MNO's Optimal Price Searhing

We first use backward induction to construct the MNO's utility function in Stage I. Using the theorem for subgame equilibrium we have the MNO's utility as

$$V_{MNO} = \begin{cases} wN\cdot(1-\frac{w}{(p_0\underline{\rho}+(1-p_0)\bar{\rho})\cdot v_mT}) & (i), \\ w\cdot N_C + pT\cdot\sum_{k=0}^{N_S^{\text{II}}}\{\min\{k, C-N_C\} \\ \cdot p_0^{N_S^{\text{II}}-k}(1-p_0)^k\} & (ii), \\ w\cdot N_C + pT\cdot\sum_{k=0}^{N_S^{\text{II}}}\{\min\{N_S^{\text{III}}+k, C-N_C\} \\ \cdot p_0^{N_S^{\text{II}}-k}(1-p_0)^k\} & (iii), \\ pT\cdot\sum_{k=0}^{N_S^{\text{II}}}\{\min\{N_S^{\text{III}}+k, C\} \\ \cdot p_0^{N_S^{\text{II}}-k}(1-p_0)^k\} & (iv). \end{cases}$$

Here the cases are those discussed in Theorem2. The according $N_S^{\text{II}}, N_S^{\text{III}}, N_C$ can be calculated corresponding to the cases. Thus we have the MNO's optimal prices as $(p^*, w^*) = \arg\max_{p,w\geq 0} V_{MNO}$. And the constraint for the feasible price setting is $N_C \leq C$, which means the number of contract CPs the MNO can serve should not exceed the total number of cache spaces. And we also notice that when the spot price p is too high or contract price w is too high, there will be no one joining the spot market or the contract market. Thus we have that the *valid price setting* should satisfy the following two conditions.

1) *Feasibility*: the number of contract users should not exceed the number of cache spaces.
2) *Effectivity*: the price should not be so high that no one will join the corresponding market.

As is shown in Fig6, when the number of contract users exceed the service capacity, the price setting (p, w) becomes infeasible(Region III). And when the spot price is high(Region II), there will be no user joining the spot market, thus the spot price will not be effective. Similarly, when the contract price is high(Region I), there will be no user joining the contract market, thus the contract price will be ineffective. And we have the following theorem for the valid price setting(Region IV).

***Theorem** 3 (Valid Price Setting):* The valid price setting (w, p) should satisfy

1) $w \leq T(p_0\underline{\rho} + (1-p_0)\bar{\rho})$ (boundary between Region IV and Region I);
2) $p \leq \bar{\rho}v_m$ (boundary between Region IV and Region II);
3) $w \geq v_mT(1-\frac{C}{N})\cdot(p_0\bar{\rho} + (1-p_0)\underline{\rho})$ (second segment of boundary between Region III and IV);
4) $w \geq pT((1-p_0)\frac{\underline{\rho}}{\bar{\rho}} + p_0)$ (first segment of boundary between Region III and IV).

Proof: First for constraints on effectivity, we have $V(v_m, C) \geq 0$ and $V(v_m, S) \geq 0$. Thus we have

$$w \leq T(p_0\underline{\rho} + (1-p_0))\bar{\rho}, \; p \leq \bar{\rho}v_m.$$

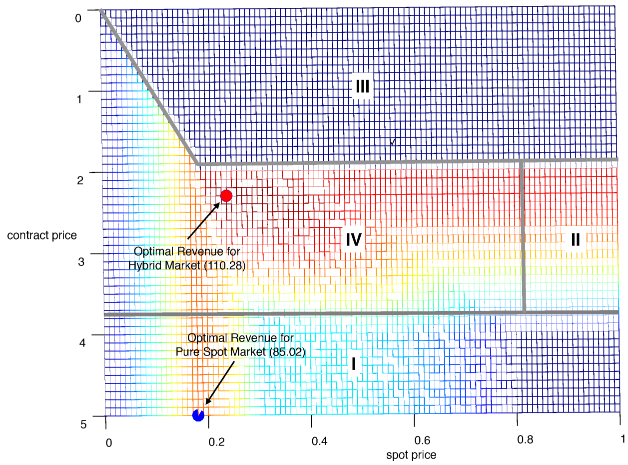

Fig. 6: The Stage I Hybrid Optimization.

Then we consider the feasibility constraint. From the Theorem2, we know that when $0 \le w \le pT((1-p_0)\frac{\rho}{\bar{\rho}}+p_0)$ (case 1), the number of contract CPs is $N_C = (1-\frac{v_2}{v_m})\cdot N$, and $v_2 = \frac{w}{T(p_0\underline{\rho}+(1-p_0)\bar{\rho})}$. We let $N_C \ge C$, and get the feasible condition as $w \ge pT((1-p_0)\frac{\rho}{\bar{\rho}}+p_0)$. Thus either $w \ge pT((1-p_0)\frac{\rho}{\bar{\rho}}+p_0)$ (the first segment of the boundary between Region III and Region IV) or $w \ge v_m T(1-\frac{C}{N})\cdot(p_0\bar{\rho}+(1-p_0)\underline{\rho})$ (the second segment of the boundary between Region III and Region IV) should establish. Moreover, we have the intersection point of these two constraints (the turning point connecting the two segments) as $(v_m\bar{\rho}(1-\frac{C}{N}), v_m T(1-\frac{C}{N})\cdot(p_0\bar{\rho}+(1-p_0)\underline{\rho}))$. This ends the proof. ■

B. Lower Bound of Optimal Revenue

Although the optimal pricing is hard to derive, we can still try to derive the lower bound for the optimal revenue of the MNO, V^*_{MNO}. Before deriving the lower bound for V^*_{MNO}, we first consider two boundary cases, where the hybrid market converges to the *pure spot market* ($w \ge w_3$) and the *pure contract market* ($w \le w_1$). Notice that in these special cases, there only exist the alien CPs and the spot CPs or the contract CPs. We can first derive the optimal revenue in the pure contract market and the lower bound of optimal revenue in the pure spot market as follows.

***Lemma** 3 (Optimal Revenue in Pure Contract Market):* The optimal revenue in the pure contract market is

$$V^*_C = \frac{v_m T}{4}(p_0\bar{\rho}+(1-p_0)\underline{\rho}).$$

Proof: In a pure contract market, there are only contract CPs and alien CPs. The revenue of the MNO is $V_C = wN\cdot(1-\frac{w}{v_m T(p_0\underline{\rho}+(1-p_0)\bar{\rho})})$, and the condition for the pure contract market is $w > v_m T(1-\frac{C}{N})(p_0\bar{\rho}+(1-p_0)\underline{\rho})$. We have the derivative of V_C as

$$\frac{\partial V_C}{\partial w} = N\cdot(1-\frac{2w}{v_m T(p_0\underline{\rho}+(1-p_0)\bar{\rho})}).$$

Hence when $0 \le w \le \frac{v_m T(p_0\bar{\rho}+(1-p_0)\underline{\rho})}{2}$, $\frac{\partial V_C}{\partial w} \ge 0$, the revenue increases with the contract price; and when $\frac{v_m T(p_0\bar{\rho}+(1-p_0)\underline{\rho})}{2} \le w \le v_m T(1-\frac{C}{N})(p_0\bar{\rho}+(1-p_0)\underline{\rho})$, the revenue decreases with the contract price. Thus the revenue is maximized when $w = \frac{v_m T(p_0\bar{\rho}+(1-p_0)\underline{\rho})}{2}$, and the optimal revenue is $V^*_C = \frac{v_m T(p_0\bar{\rho}+(1-p_0)\underline{\rho})}{2}\cdot\frac{N}{2} = N\cdot(1-\frac{2w}{v_m T(p_0\underline{\rho}+(1-p_0)\bar{\rho})})$. This completes the proof. ■

And we have the following lemma for the lower bound of optimal revenue in the pure spot market.

***Lemma** 4 (Lower Bound of Optimal Revenue in Pure Spot Market):* The optimal revenue in the pure spot market, V^*_S, is lower bounded by

$$V^b_S = \max\{\underline{\rho} v_m CT(1-\frac{C}{N}), p_0 v_m CT(1-\frac{C}{N})\}.$$

Proof: The revenue for the pure spot market is $V_S = pT\cdot\sum_{k=0}^{N_S^{\text{II}}}\{\min\{N_S^{\text{III}}+k, C\}p_0^{N_S^{\text{II}}-k}(1-p_0)^k\}$. We consider the three cases as follows.

(i). *If* $0 \le N_S^{II}+N_S^{III} \le C$.

In this case, we have

$$\begin{aligned}
V_S =& pT\cdot\sum_{k=0}^{N_S^{\text{II}}}\{\min\{N_S^{\text{III}}+k, C\}p_0^{N_S^{\text{II}}-k}(1-p_0)^k\}\\
=& pT\cdot\sum_{k=0}^{N_S^{\text{II}}}\{(N_S^{\text{III}}+k)\cdot p_0^{N_S^{\text{II}}-k}(1-p_0)^k\}\\
=& pT\cdot(N_S^{\text{III}}+p_0 N_S^{\text{II}})\\
\ge& pT\cdot p_0(N_S^{\text{III}}+N_S^{\text{II}})
\end{aligned}$$

The second step is due to $N_S^{\text{III}}+k \le N_S^{\text{III}}+N_S^{\text{II}} \le C$; the third step is due to $\sum_{k=0}^{N_S^{\text{II}}}\{p_0^{N_S^{\text{II}}-k}(1-p_0)^k\} = 1$ and $\sum_{k=0}^{N_S^{\text{II}}}\{k\cdot p_0^{N_S^{\text{II}}-k}(1-p_0)^k\} = p_0 N_S^{\text{II}}$; the fourth step is due to $p_0 \le 1$. And since $N_S^{\text{II}}+N_S^{\text{III}} = N\cdot\frac{v_m-\frac{p}{\bar{\rho}}}{v_m}$, we have $p = \bar{\rho}v_m\cdot(1-\frac{N_S^{\text{II}}+N_S^{\text{III}}}{N})$, substitute it into the inequality above and we get

$$V_S \ge p_0\bar{\rho}v_m T\cdot\left(1-\frac{N_S^{\text{II}}+N_S^{\text{III}}}{N}\right)(N_S^{\text{II}}+N_S^{\text{III}}).$$

Thus for this case, we have $V^*_S \ge \max\{p_0\bar{\rho}v_m T\cdot\left(1-\frac{N_S^{\text{II}}+N_S^{\text{III}}}{N}\right)(N_S^{\text{II}}+N_S^{\text{III}})\} = \bar{\rho}p_0 v_m CT(1-\frac{C}{N})$, and the maximum is attained when $N_S^{\text{II}}+N_S^{\text{III}} = C$.

(ii). *If* $N_S^{II}+N_S^{III} \ge C$ *and* $N_S^{III} \le C$.

In this case, we have

$$\begin{aligned}
V_S =& pT\cdot\sum_{k=0}^{N_S^{\text{II}}}\{\min\{N_S^{\text{III}}+k, C\}p_0^{N_S^{\text{II}}-k}(1-p_0)^k\}\\
\ge& pT\cdot\sum_{k=0}^{N_S^{\text{II}}}N_S^{\text{III}}\cdot p_0^{N_S^{\text{II}}-k}(1-p_0)^k\\
=& v_m\underline{\rho}T\cdot(1-\frac{N_S^{\text{III}}}{N})\cdot N_S^{\text{III}}
\end{aligned}$$

The second step is due to $\min\{N_S^{\text{III}}+k, C\} \ge N_S^{\text{III}}$. And the third step is due to $N_S^{\text{III}} = N\cdot\frac{v_m-\frac{p}{\underline{\rho}}}{v_m}$.Thus for this case, we have $V_S^* \geq \max\{v_m \underline{\rho} T \cdot (1 - \frac{N_S^{\text{III}}}{N}) \cdot N_S^{\text{III}}\} = v_m \underline{\rho} CT \cdot (1 - \frac{C}{N})$. And the maximum is obtained when $N_S^{\text{III}} = C$.

(iii). *If* $N_S^{III} \geq C$.

In this case, we have $V_S = pTC = v_m \underline{\rho} TC(1 - \frac{N_S^{\text{III}}}{N})$, and thus $V_S^* \geq \max\{v_m \underline{\rho} TC(1 - \frac{N_S^{\text{III}}}{N})\} = v_m \underline{\rho} CT \cdot (1 - \frac{C}{N})$. The maximum is obtained when $N_S^{\text{III}} = C$.

In summary, we have $V_S^* \geq \underline{\rho} v_m CT(1 - \frac{C}{N})$ and $V_S^* \geq p_0 v_m CT(1 - \frac{C}{N})$. This ends the proof. ∎

Based on the Lemmas above, we have the following theorem for the lower bound of optimal revenue of the MNO.

***Theorem** 4 (Lower Bound of Optimal Revenue):* The optimal revenue V_{MNO} is bounded by

$$V_{MNO}^b = \max\{V_C^*, V_S^b\}.$$

Proof: Since have

$$V_{MNO}^* = \max_{p,w \geq 0} V_{MNO} \geq \max_{p \geq 0, w \leq w_1} V_{MNO} = V_C^*,$$

$$V_{MNO}^* = \max_{p,w \geq 0} V_{MNO} \geq \max_{p \geq 0, w \geq w_3} V_{MNO} = V_S^* \geq V_S^b,$$

thus the theorem establishes. ∎

VI. Model Extension

In this section, we extend our simplified model to the generalized case. We consider an edge-caching market with N CPs, each CP i's evaluation $v_i \in [0, v_m]$. Notice that we have relaxed the uniform distribution of CPs' evaluations to arbitrary distribution. Then we suppose that CP i has n_i contents for delivery. The probability density function of the jth content's request rate of CP i is $g_{ij}(\rho_{ij}), \rho_{ij} \in [0, \bar{\rho}]$, and $G_{ij}(\rho_{ij})$ is the corresponding cumulative distribution function of ρ_{ij}. In this case, in the contract phase, the CP's decision variable becomes the number of cache spaces he would buy in the contract market, we denote it as c_i. And then in the tth spot phase, the CP i will realize the exact popularity of his contents in the upcoming time slot. Then he caches the c_i most popular contents in the contracted cache space, and decides how many cache spaces he will request in the tth spot phase r_{it}. We denoted X_{tm}^i as a random variable representing the request rate of the m^{th} popular content of CP i in tth time slot. Then we have the utility for CP i in the contract market, V_C^i as

$$\begin{aligned} V_C^i(c_i) =& v_i \boldsymbol{E}(\sum_{t=1}^{T} \sum_{m=1}^{c_i} X_{tm}^i) - wc_i \\ =& v_i \sum_{t=1}^{T} \sum_{m=1}^{c_i} \boldsymbol{E}(X_{tm}^i) - wc_i \end{aligned}$$

The second step is due to $E(X+Y) = E(X) + E(Y)$ for any random variable X, Y. For $\boldsymbol{E}(X_{tm}^i)$, we have

$$\boldsymbol{E}(X_{tm}^i) = \sum_{k_1 \neq k_2 \neq \ldots k_m} \int_0^{\bar{\rho}} \rho \prod_{r=1}^{m} (1 - G_{ik_r}(\rho)) \cdot \prod_{t \neq k_1, \ldots, k_m} G_{it}(\rho) d\rho.$$

In each spot phase, each CP i will issue requests for contents with positive gain in the spot market, except for the c_i most popular contents. Thus the payoff of CP i in the spot market should be

$$\begin{aligned} V_S^i(c_i) =& \alpha \boldsymbol{E}(\sum_{t=1}^{T} \sum_{m=c_i+1}^{n_i} \max\{v_i X_{tm}^i - p, 0\}) \\ =& \alpha \sum_{t=1}^{T} \sum_{m=c_i+1}^{n_i} \boldsymbol{E}(\max\{v_i X_{tm}^i - p, 0\}) \end{aligned}$$

Notice that only when $X_{tm}^i \geq \frac{p}{v_i}$, the CP will request a cache space for this content in the spot phase. Thus for $\boldsymbol{E}(\max\{v_i X_{tm}^i - p, 0\})$, we have

$$\boldsymbol{E}(\max\{v_i X_{tm}^i - p, 0\}) = \sum_{k_1 \neq k_2 \neq \ldots k_m} \int_{\frac{p}{v_i}}^{\bar{\rho}} (v_i \rho - p) \prod_{r=1}^{m} (1 - G_{ik_r}(\rho)) \cdot \prod_{s \neq k_1, \ldots, k_m} G_{is}(\rho) d\rho.$$

Thus we have the payoff of CP i as

$$\begin{aligned} V^i(c_i) =& V_S^i(c_i) + V_C^i(c_i) \\ =& \alpha \sum_{t=1}^{T} \sum_{m=c_i+1}^{n_i} \boldsymbol{E}(\max\{v_i X_{tm}^i - p, 0\}) \\ &+ v_i \sum_{t=1}^{T} \sum_{m=1}^{c_i} \boldsymbol{E}(X_{tm}^i) - wc_i \end{aligned}$$

Notice here we have

$$\alpha = \boldsymbol{E}\left(\min\left\{1, \frac{C - \sum_{i=1}^{N} c_i}{\sum_{i=1}^{N} r_i}\right\}\right).$$

And for the MNO's revenue, we have[2]

$$V_{MNO} = \sum_{i=1}^{N} wc_i + pT\boldsymbol{E}(\min\{\sum_{i=1}^{N} r_i, C - \sum_{i=1}^{N} c_i\}).$$

A. Subgame Equilibrium in the Generalized Scenario

Similar to the derivation of subgame equilibrium in Section IV, we first fix α, that is the outcome of the subgame, and derive the CPs best response given α; then we consider the CP's influence on α and derive the equilibrium conditions. We have the following theorem for the best response of CP i given α.

***Lemma** 5 (Best Response of CP i for given α):* The best response c_i^* of CP i, given α is

$$c_i^*(\alpha) = \arg\min_c T\boldsymbol{E}(\max\{(1-\alpha)v_i X_{(c+1)}^i + \alpha p, v_i X_{(c+1)}^i\}) \leq w.$$

Proof: We consider the difference sequence of $\{V^i(c), \forall c \in \{0, 1, ..., n_i\}\}$, which is $\Delta^i(c) = V^i(c+1) - V^i(c), \forall c \in \{0, 1, ..., n_i - 1\}$. Specially, we denote $\Delta^i(n_i) = -\infty$. Then we have

[2] Here we omitted the subscript t in r_{it} since r_{it} is homogeneous for $t \in \mathcal{T}$, the same omissions is also used in some other expressions later.

$$
\begin{aligned}
\Delta^i(c) =& v_i T\boldsymbol{E}(X^i_{(c+1)}) - \alpha T\boldsymbol{E}(\max\{v_i X^i_{(c+1)} - p, 0\}) - w \\
=& T\boldsymbol{E}(v_i X^i_{(c+1)} - \alpha \max\{v_i X^i_{(c+1)} - p, 0\}) - w \\
=& T\boldsymbol{E}(\max\{(1-\alpha) v_i X^i_{(c+1)} + \alpha p, v_i X^i_{(c+1)}\}) - w
\end{aligned}
$$

Since $\{X^i_c, \forall c \in \{1, 2, ..., n_i\}\}$ is a decreasing sequence, thus $\Delta^i(c)$ is also a decreasing function. Notice that the positive linear combination of decreasing sequences and constant sequences is still decreasing sequence, and $\max\{\cdot, \cdot\}$ doesn't change this decreasing property of the sequence. Thus $\Delta_i(c)$ is a decreasing function in c. We denote $c^* = \min\{c|\Delta^i(c) \leq 0\}$. Then $V^i(c)$ should be an increasing sequence in $[0, c^*]$; and a decreasing sequence in $[c^*, n_i]$. And the maxima is attained when $c = c^*$. This ends the proof. ■

In fact, we have the following corollary for the influence of α on c_i^*.

***Corollary** 1 (α's influence on c_i):* When α increases, the CP's best response $c_i^*(\alpha)$ will decrease.

Proof: Since we have

$$\Delta^i(c) = v_i T\boldsymbol{E}(X^i_{(c+1)}) - \alpha T\boldsymbol{E}(\max\{v_i X^i_{(c+1)} - p, 0\}) - w,$$

thus $\Delta^i(c)$ will decrease as α increases. Then $c_i^* = \min\{c|\Delta^i(c) \leq 0\}$ will also decrease as α increases, this ends the proof. ■

Then we consider how this α is influenced by the CPs' decisions $\boldsymbol{c}$. We have the following lemma for how the CP i's decision c_i for the contract market could influence the congestion level α in the spot market.

***Lemma** 6 (Influence of CPs' decisions on α):* The congestion level in the spot market, α, will decrease with CP i's request c_i in the contract market.

Proof: We denote $\sigma(c_i, \boldsymbol{c}_{-i}) = \sum_{t=1}^N c_t$ as the total number of contracted cache spaces, and denote $\epsilon(c_i, \boldsymbol{c}_{-i}, \boldsymbol{\rho}) = \sum_{t=1}^N \sum_{j=c_t}^{n_t} \mathbb{1}(v_t X^t_j(\boldsymbol{\rho}) \geq p)$ as the total request in spot market given $\boldsymbol{\rho}$ and CPs' contract decisions. Notice X^i_j is determined given $\boldsymbol{\rho}$, thus we express it as a function of $\boldsymbol{\rho}$. Then we have

$$
\begin{aligned}
\alpha(c_i, \boldsymbol{c}_{-i}) =& \boldsymbol{E}_{\boldsymbol{\rho}}\left(\min\left\{1, \frac{C - \sigma(c_i, \boldsymbol{c}_{-i})}{\epsilon(c_i, \boldsymbol{c}_{-i}, \boldsymbol{\rho})}\right\}\right) \\
=& \sum_{\boldsymbol{\rho} \in \Omega} \left(\min\left\{1, \frac{C - \sigma(c_i, \boldsymbol{c}_{-i})}{\epsilon(c_i, \boldsymbol{c}_{-i}, \boldsymbol{\rho}')}\right\}\right) Pr(\boldsymbol{\rho} = \boldsymbol{\rho}')
\end{aligned}
$$

And we consider a unit increase in c_i, then we have

$$
\begin{aligned}
&\alpha(c_i + 1, \boldsymbol{c}_{-i}) \\
=& \sum_{\boldsymbol{\rho} \in \Omega} \left(\min\left\{1, \frac{C - \sigma(c_i + 1, \boldsymbol{c}_{-i})}{\epsilon(c_i + 1, \boldsymbol{c}_{-i}, \boldsymbol{\rho}')}\right\}\right) Pr(\boldsymbol{\rho} = \boldsymbol{\rho}') \\
=& \sum_{\boldsymbol{\rho} \in \Omega} \left(\min\left\{1, \frac{C - \sigma(c_i, \boldsymbol{c}_{-i}) - 1}{\epsilon(c_i + 1, \boldsymbol{c}_{-i}, \boldsymbol{\rho}')}\right\}\right) Pr(\boldsymbol{\rho} = \boldsymbol{\rho}') \\
\leq& \sum_{\boldsymbol{\rho} \in \Omega} \left(\min\left\{1, \frac{C - \sigma(c_i, \boldsymbol{c}_{-i}) - 1}{\epsilon(c_i, \boldsymbol{c}_{-i}, \boldsymbol{\rho}') - 1}\right\}\right) Pr(\boldsymbol{\rho} = \boldsymbol{\rho}') \\
\leq& \sum_{\boldsymbol{\rho} \in \Omega} \left(\min\left\{1, \frac{C - \sigma(c_i, \boldsymbol{c}_{-i})}{\epsilon(c_i, \boldsymbol{c}_{-i}, \boldsymbol{\rho}')}\right\}\right) Pr(\boldsymbol{\rho} = \boldsymbol{\rho}') \\
=& \alpha(c_i, \boldsymbol{c}_{-i})
\end{aligned}
$$

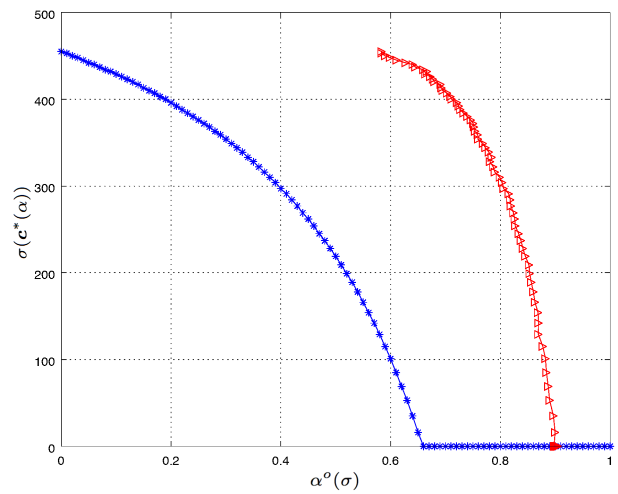

Fig. 7: An example of $\alpha^o(\sigma)$ and $\sigma(\boldsymbol{c}^*(\alpha))$.

The second equation is because $\sigma(c_i + 1, \boldsymbol{c}_{-i}) = \sigma(c_i, \boldsymbol{c}_{-i}) + 1$. The third step is due to

$$
\begin{aligned}
&\epsilon(c_i + 1, \boldsymbol{c}_{-i}, \boldsymbol{\rho}) \\
=& \sum_{t \neq i} \sum_{j=c_t}^{n_t} \mathbb{1}(v_t X^t_j(\boldsymbol{\rho}) \geq p) + \sum_{j=c_i+1}^{n_i} \mathbb{1}(v_i X^i_j(\boldsymbol{\rho}) \geq p) \\
\geq& \sum_{t \neq i} \sum_{j=c_t}^{n_t} \mathbb{1}(v_t X^t_j(\boldsymbol{\rho}) \geq p) + \sum_{j=c_i+1}^{n_i} \mathbb{1}(v_i X^i_j(\boldsymbol{\rho}) \geq p) \\
&+ (\mathbb{1}(v_i X^i_{(c_i)}(\boldsymbol{\rho}) \geq p) - 1) \\
=& \epsilon(c_i, \boldsymbol{c}_{-i}, \boldsymbol{\rho})
\end{aligned}
$$

And the final step is due to $\min\{1, \frac{x}{y}\} \geq \min\{1, \frac{x-1}{y-1}\}$. Thus we know that $\alpha(c_{-i}, c_i)$ is a decreasing function in c_i. This ends the proof. ■

Using the theorems we discussed above, we have the following theorem for the existence of the subgame perfect equilibrium(SPE).

***Theorem** 5 (Existence of Subgame Perfect Equilibrium):* The subgame perfect equilibrium exists if $\sigma(\boldsymbol{c}^*(0)) \leq C$.

Proof: We consider the best response under some α, $\boldsymbol{c}^*(\alpha)$. We have $\sigma(\boldsymbol{c}^*(\alpha))$ decreases with α, because each $c_i^*(\alpha)$ decreases with α.Then $\sigma(\boldsymbol{c}^*(\alpha))$ is a mapping from $[0, 1]$ to $[\sigma(\boldsymbol{c}^*(1)), \sigma(\boldsymbol{c}^*(0))]$. And the inverse mapping of $\sigma(\cdot)$ is viable for $\sigma \in (\sigma(\boldsymbol{c}^*(1)), \sigma(\boldsymbol{c}^*(0))]$.

We denote this inverse mapping as $\sigma^{-1} : \sigma \rightarrow (\boldsymbol{c}^*, \alpha)$, $\forall \sigma \in (\sigma(\boldsymbol{c}^*(1)), \sigma(\boldsymbol{c}^*(0))]$ with a little abuse of notations. Note that the condition in the theorem, $\sigma(\boldsymbol{c}^*(0))$ guarantees the viability of this mapping. And we denote the inverse function as $\boldsymbol{t}$ (for $\boldsymbol{c}^*$) and g (for α). Based on this inverse mapping, we further denote $\alpha^o(\sigma) = \boldsymbol{E}\left(\min\left\{1, \frac{C-\sigma}{\epsilon(\boldsymbol{t}(\sigma))}\right\}\right)$, and $\alpha^o(\sigma)$ is a mapping from $(\sigma(\boldsymbol{c}^*(1)), \sigma(\boldsymbol{c}^*(0))]$ to $[\alpha^o(\sigma(\boldsymbol{c}^*(0))), \alpha^o(\sigma(\boldsymbol{c}^*(1))))$.

Finally, we denote $f(\sigma) = g(\sigma) - \alpha^o(\sigma)$. Then we have $f(\sigma(\boldsymbol{c}^*(0))) = -\alpha(\sigma(\boldsymbol{c}^*(0))) \leq 0$, and $\lim_{\sigma' \rightarrow \sigma(\boldsymbol{c}^*(1))} f(\sigma') = 1 - \alpha(\sigma(\boldsymbol{c}^*(1))) \geq 0$ with a little extension on g to let $g(\sigma(\boldsymbol{c}^*(t))) = t, \forall t \in [0,1]$.[3] Thus according to the zero theorem, there must exist at least zero for $f(\sigma)$. We denote the zero as σ^*, then the corresponding equilibrium is $\alpha^* = g(\sigma^*), \boldsymbol{c}^* = \boldsymbol{t}(\sigma^*)$. This ends the proof. ■

In Fig 7, we provide a graph showing $\alpha^o(\sigma)$ and $\sigma(\boldsymbol{c}^*(\alpha))$ where the condition stated in Theorem5 is satisfied. It's clear that the subgame perfect equilibrium exists at the intersection point. And this equilibrium is unique in this figure. As we can observe from the result, when α increases, the CPs will move their requests from the contract market to the spot market, thus $\sigma(\boldsymbol{c}^*(\alpha))$ will decrease(Corollary1); and when σ increases, the spot market will become more congested(Lemma6), thus $\alpha^o(\sigma)$ will decrease.

B. MNO's Optimization in the Generalized Scenario

And we can naturally extend the Theorem3 and Theorem4 for the MNO's optimal price setting and optimal revenue as follows.

***Theorem** 6 (Valid Price Setting):* The valid price setting for the MNO in the extended scenario should satisfy $p \le v_m\bar{\rho}$ and $w \le v_m\bar{\rho}T$.

Proof: Since when $w > v_m\bar{\rho}T$, we have $V^i(c_i) = v_iT\sum_{m=1}^{c_i}\boldsymbol{E}(X_m^i) - wc_i \le v_mT\sum_{m=1}^{c_i}\boldsymbol{E}(X_m^i) - wc_i \le v_mT\cdot c_i\bar{\rho} - v_m\bar{\rho}T\cdot c_i = 0$, thus no one will choose to join the contract market, which makes the price setting in the contract market ineffective. Similarly, when $p > v_m\bar{\rho}$, $W_{spot_i}(c_i) = \alpha T\boldsymbol{E}(\sum_{m=c_i+1}^{n_i}\max\{v_iX_m^i - p, 0\}) \le \alpha T\boldsymbol{E}(\sum_{m=c_i+1}^{n_i}\max\{v_m\bar{\rho} - p, 0\}) = 0$, thus no one will join the spot market. ■

And we have the following lower bound for the MNO's optimal revenue.

***Theorem** 7 (Lower Bound of MNO's Revenue):* The optimal revenue of MNO is lower bounded by

$$V_{MNO}^b = CT\cdot[E(v_iX_c^i)]^{(C)},$$

here $[E(v_iX_c^i)]^{(C)}$ is the Cth largest value among $E(v_iX_c^i)$, $\forall i \in \mathcal{N}$, $\forall c \in \{1,2,...,n_i\}$.

Proof: Similarly, we consider the boundary cases where $p = v_m\bar{\rho}$. Under this scenario, there is no demand in the spot market. And the demand in the contract market is $D_C = \sum_{i=1}^N c_i^*(0)$. Thus we have

$$\begin{aligned} V_{MNO} =& w\cdot\sum_{i=1}^N \min\{c|T\boldsymbol{E}(v_iX_{c+1}^i) \le w\} \\ =& w\cdot\sum_{i=1}^N\sum_{c=1}^{n_i}\mathbb{1}(\boldsymbol{E}(v_iX_{c+1}^i) \ge \frac{w}{T}) \\ =& w\cdot\sum_{k=1}^{N\cdot(\sum_i n_i)}\mathbb{1}([E(v_iX_c^i)]^{(k)} \ge \frac{w}{T}) \end{aligned}$$

[3]In fact, this is due to the incapability of expressing the inverse mapping of inverse mapping of many-to-one mapping and is natural.

The first step is due to the best response in Lemma5. The notation of $[\cdot]^{(k)}$ is in alignment with the denotion in the theorem. Thus we let $T\cdot[E(v_iX_c^i)]^{(C)}$, then $V_{MNO} = T\cdot[E(v_iX_c^i)]^{(C)}\cdot\sum_{k=1}^{N\cdot(\sum_i n_i)}\mathbb{1}([E(v_iX_c^i)]^{(k)} \ge \frac{w}{T}) = CT\cdot[E(v_iX_c^i)]^{(C)}$. This ends the proof. ■

VII. SIMULATION

In this section, we consider the performance of our mechanism and the substitute mechanisms systematically. We first show the robustness in revenue generation of our mechanism. Then we conduct a series of comparative studies. We compare the performance of the hybrid mechanism and other possible substitutes in terms of revenue generation, social welfare and fairness. Through simulation, we show that hybrid mechanism has significantly better performance in revenue generation than other mechanisms, and also has almost optimal performance in efficiency and fairness. In the experiment conclusion part, we summarize the pros and cons of our mechanism and other mechanisms.

A. Simulation Setup

In the numerical simulation, we consider an edge caching market that consists of $N = 100$ content providers, with evaluation v uniformly distributed in $[0,1]$. We let $C = \gamma\cdot N$, and we set $\gamma = 0.5$. For the content popularity distribution, we set $p_0 = 0.5, \underline{\rho} = 0.3, \bar{\rho} = 0.8$. We consider a time horizon of $T = 10$ time slots.

B. Robustness of the Hybrid Mechanism

To show the robustness of our mechanism as follows, we consider the performance's changes under two indexes, the different *content popularity* and *resource scarcity*. The different content popularity is reflected by p_0. With higher p_0, the content popularity distribution is more biased to the high popularity; and the other way round. As we observe from the results shown in Fig8 and Fig9, the performance of our mechanism is relatively robust for different content popularity distribution, improving $30\% - 50\%$ of the MNO's revenue in comparison with pure spot market mechanism.

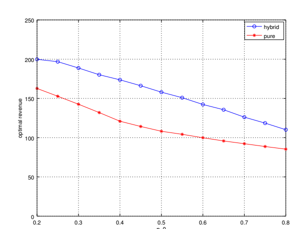

Fig. 8: performance under different content popularity distribution

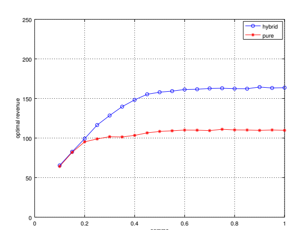

Fig. 9: performance under different resource scarcity

The resource scarcity γ is defined as the ratio shown below.

$$\gamma = \frac{\#cache\ space}{\#content\ providers} \in [0,1].$$

As we can notice from the result, when the resource scarcity is low enough, the optimal performance of the pure spot market and the hybrid market is close to each other. However, in most cases, the hybrid market offers considerable improvements in revenue generation.

C. Comparative Study

In this section, we consider several substitute mechanisms and compare their performances with our mechanism's performance. The substitute mechanisms we consider in our simulation are shown as below.

Real-time Auction: In this case, the MNO runs a multi-item auction before every time slot to determine the allocation of cache spaces. We use multi-item Vickery-Clark-Groves(VCG) auction here, which guarantees both the truthfulness and the optimal efficiency.

Pure Contract Market: In this case, only the contract service is provided. The CPs pay a contract fee to the MNO to secure some cache spaces for private use in the contract phase. The dynamic access to cache spaces in the spot market is not provided.

Pure Spot Market: In this case, only the spot market service is provided, and all the CPs have to compete for cache spaces in each time slot.

And the following are the three performance indexes we care about the most in our comparison study.

Revenue: the revenue generation is the most straightforward target for the profit-seeking MNO, and it's also the most important incentive for his investment.

Fairness: we use Jain's fairness index for indicating whether the cache space is fairly allocated. The Jain's index can be calculated as below. And with larger J, the allocation is more fair.

$$\mathcal{J} = \frac{(\sum_{i=1}^{N} V_i)^2}{N \cdot \sum_{i=1}^{N} V_i^2} \in [0, 1]$$

Efficiency: The efficiency is the proportion of social welfare achieved in comparison with the highest possible social welfare. Higher efficiency means more social-caring resource allocation.

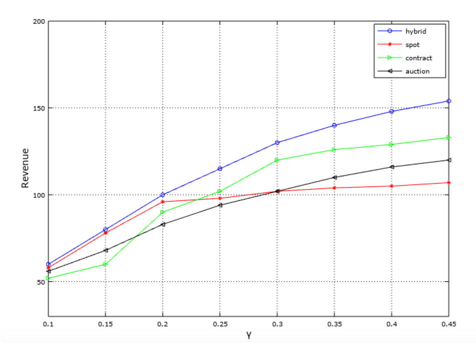

Fig. 10: Revenue Comparison Results.

1) Revenue generation: As is shown in Fig10, the optimal revenue of the hybrid market mechanism is always higher than all the other mechanisms over a wide range of parameters(γ, p_0, etc). The improvement of the MNO's revenue of the hybrid mechanism over the auction mechanism goes from mild improvement(4.6%) to significant improvement(38.1%), while the revenue gap between hybrid mechanism and pure contract mechanism stays relatively stable over different levels of resource scarcity. In terms of revenue generation, the hybrid mechanism prevails all the other possible substitutes we considered.

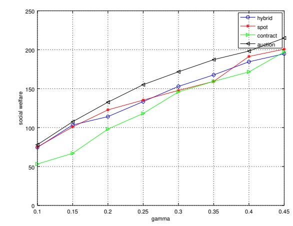

Fig. 11: Social Welfare.

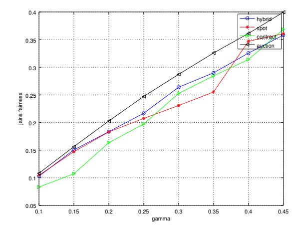

Fig. 12: Fairness.

2) Fairness and efficiency: As is shown in Fig.11 and Fig.12, the auction mechanism always guarantees the highest efficiency and the highest fairness. However, another key observation is that the efficiency loss and the fairness loss of the hybrid mechanism will not exceed 13.8% and 12.0% even for the worst case. But the revenue loss for the auction mechanism can be extremely high, with the revenue loss ratio being 38.1% compared to hybrid mechanism in the worst case.

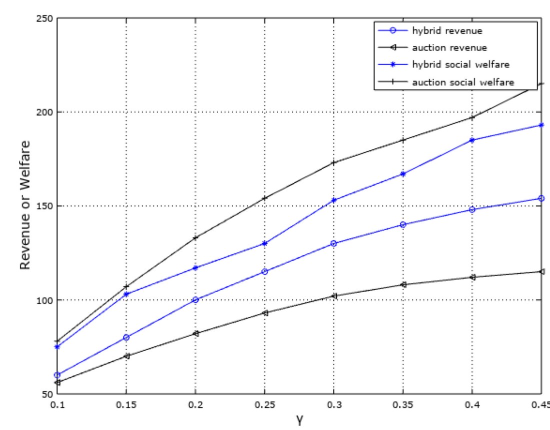

Fig. 13: Revenue Realization Comparison.

We focus on the difference between the auction mechanism and the hybrid pricing mechanism in revenue generation. The social welfare, in our case, is also the highest achievable revenue for the MNO[4], thus the ratio between the MNO's revenue and the social welfare can be an important index for the mechanism's performance in revenue generation. This ratio of the hybrid mechanism can be extremely high; while that of the auction mechanism could be quite low, as is shown in Fig13. For the auction mechanism, the achieved revenue could be almost one third of the social welfare in some worst

[4]The full price discrimination mechanism could extract all the surplus. The highest achievable revenue we considered here is similar to the idea of economic surplus.

cases, which means quite low level of revenue generation for the MNO.

Moreover, notice that with more resources, the MNO's profit doesn't show much improvement for the auction mechanism, this is due to the second-price payment policy of the traditional VCG auction. However, this can deter the MNO's investment in provisioning the edge caching service. However, the MNO's revenue under the hybrid mechanism shows significant improvement(almost linear) with the increase of the number of cache spaces, which will encourage the MNO to invest more in service provisioning. And this is quite important for the viability as well as sustainability of this ecosystem.

3) Conclusion on comparison: We summarize the comparison results as below, and we conclude that the hybrid mechanism serves as a good choice for the revenue generation of the MNO, with almost optimal performances in efficiency and fairness.

- *Hybrid vs Auction*. As is previously shown, the hybrid mechanism offers sub-optimal efficient and fairness performances, and it also generates significantly higher revenue than the auction mechanism in most cases. The hybrid mechanism also motivates the MNO to invest more for service provisioning while the auction mechanism could deter further investments.
- *Hybrid vs Pure Spot and Contract*. As is analyzed and shown by the experiments, the hybrid mechanism not only improves the service provisioning by providing different services for the CPs with different evaluations, but also promotes the MNO's revenue, compared to the pure spot and contract mechanism.

D. The Extended Case

In this subsection, we consider the extended case, where each of the CPs has 10 contents for delivery, and each of the content's popularity distribution is a randomly generated piecewise function defined on $[0, 1]$.

1) Uniqueness and Stability of Equilibrium in Stage II: Notice that we have proved the existence of Nash Equilibrium for the general case with Theorem5. However, this does not guarantee the uniqueness and the stability of the SPE. However, we have the following observations from our experiment study.

Observation1.*(Uniqueness of SPE)* There exists unique Nash Equilibrium (in pure pricing market or hybrid market) or two Nash Equilibrium(one in pure pricing market and the other in hybrid market).

As is shown in the two graphs below, in the first case, there exist one pure-pricing SPE. And in the second case, there exist two SPE. One corresponds to a hybrid market case, while the other corresponds to a pure spot market case.

However, we also have the following observation for the stability of the Subgame Perfect Equilibrium.

Observation2.*(Stability of Nash Equilibrium)* There exists one unique stable SPE.

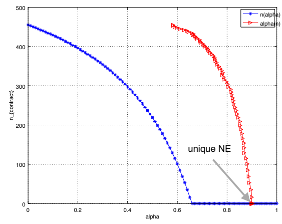

Fig. 14: Unique NE

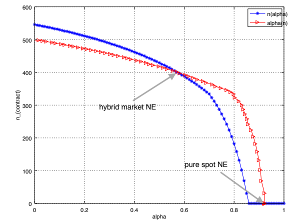

Fig. 15: Two NEs

This can be graphically proved by the evolutionary game-theoretic analysis, based on the simulation results. Notice we have the following evolutionary result in Fig. 16 for the two cases shown above. In the first case, the unique SPE is also stable, since any initial setting of α_0 will guarantee robust convergence to this SPE. In the second case, we denote the value of α in the two equilibrium by α_1, α_2, then when initial $\alpha_0 \in [0, \alpha_1) \cup (\alpha_1, 1]$ the equilibrium will converge to α_2, with α_1 being a solitary point. Thus only α_2 guarantees stable convergence. We use best-response-update algorithm to compute the SPE in the subgame to guarantee that we end in a stable equilibrium.

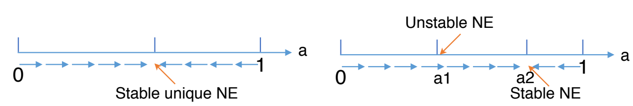

Fig. 16: Stability of Subgame Perfect Equilibrium.

2) StageI. MNO's Revenue Improvement: To be added.

VIII. Conclusion

In our paper, we designed a hybrid mechanism for the edge caching market. Using game-theoretic analysis, we conclude that the hybrid mechanism maintains stable market equilibrium. Moreover, by analysing the structure of the equilibrium, we conclude that the hybrid mechanism provides service discrimination for different CPs, with provisioning contract service for high-evaluation CPs who prefers stable and high-quality service, and provisioning spot market service for low-evaluation CPs who prefers more dynamic access to the cache spaces. In the experiment part, we showed that the hybrid market provides robust and outstanding performance in revenue generation with quite satisfying performance in efficiency and fairness.

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