Location Dependent Pricing in Edge Caching Market with Non-uniform Popularity

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***Abstract*—*Edge caching* is a key technology to accelerate the content delivery and to mitigate the network congestion. As a real-world example, Microsoft has been providing the edge caching service with a usage-based pricing mechanism since 2009. However, the challenge of designing a better pricing mechanism lies in balancing the finer grain size of the mechanism and the complexity of business operations it required. Our work proposes a novel location dependent pricing mechanism for the edge caching market which both improves the Internet service provider's (ISP) revenue and the social welfare. First, we introduce the location dependent pricing mechanism (LDP) that offers location-specific price, considering the popularity differences between the edge nodes(e.g. base stations, WiFi access points) at different locations. We capture the interactions between the content providers (CPs) and the ISP as a two-stage Stackelberg game and derive the CP's best responses and the ISP's optimal pricing strategy. Then we provide a partial location dependent pricing scheme (PLDP) which limits the number of price tiers to provide a balance between the optimality and economic viability. Finally, through extensive numerical simulation, we find that the LDP (or PLDP) improves the revenue of the ISP by 31%∼50% (23%∼%35) and also improves the social welfare by 46% (%32) on average, compared to the uniform pricing scheme.**

***Index Terms*—Network Economics, Game Theory, Pricing Mechanism**

I. Introduction

Edge caching is one of the effective measures taken in 5G wireless network to shorten the delivery latency and thus to improve the quality of experience (QoE) of the end users. Microsoft has been promoting its edge caching service since 2009 [1], and Amazon CloudFront also uses edge cache for higher content delivery quality [2]. However, these services are mainly based on a uniform pricing mechanism, which neglects the edge nodes' and the content providers' (CPs) heterogeneity. In this work, we take the different popularity of CPs and edge nodes into consideration, and propose a location dependent pricing mechanism that offers edge caching service in a location-based style.

A. Background and Motivation

In the economics perspective, this ecosystem is made up of three parties – the Internet service provider (ISP), the content providers (CPs) and the mobile users, as is shown in Fig1. The CPs proactively store their contents at the ISP's edge cache in order to improve the users' experience, and they also have to pay a side payment to the ISP for the cache service. And since the edge cache is quite limited, a proper mechanism is needed to allocate them in a way that optimizes system performance.

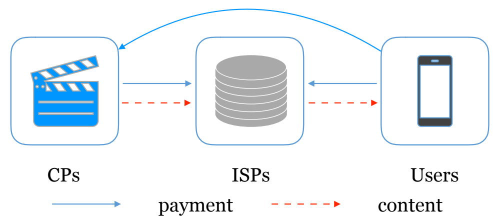

Fig. 1: Ecosystem of the edge caching market.

However, since the CPs are selfish revenue maximizers, the payment policy also needs a well design to drive the system to a social welfare maximization point.

However, the previous work mostly adopted the uniform pricing scheme, which charges the same price for edge cache at different locations, neglecting the non-uniform popularity of the edge cache and the CPs. In fact, edge cache at different locations has different values for some CPs; and different CPs also have different values for the edge cache at some location. For example, a CP who provides educational resources would prefer to cache contents at the edge cache that's closer to the educational institutions while those who provide business news would prefer the edge cache located near the business centres.

The uniform pricing mechanism fails to capture this heterogeneity of the CPs and the edge cache, which will lead to loss in social welfare and the ISP's revenue. And this motivates us to take into consideration this heterogeneity of the edge cache's popularity, and develop more appropriate and fine-grained pricing mechanism for the market. In summary, the design goals of our mechanism is as follows.

- *Robustness*: the mechanism has to guarantee stable economic equilibrium of the market.
- *Optimality*: the mechanism should optimize the ISP's revenue.
- *Efficiency*: the mechanism should maximize the social welfare.
- *Viability*: The business operations need to be simple for practical deployment.

B. Location Dependent Pricing

We propose the location dependent pricing (LDP) mechanism for this kind of service. The key idea is to charge the CPs with a location dependent price: the ISP should charge a higher price for edge cache that lies in popular area and has high request rate; while for edge cache that lies in less popular area, the ISP should charge less.

As is mentioned before, we also have to consider the economic equilibrium of this system where each player takes strategies that maximizes his own profit. We use Stackelberg game to capture the strategic interaction between the ISP and the CPs. In the first stage of the game, the ISP determines the pricing scheme maximizing his revenue; and then in the second stage, each CP determines his best response, i.e., how much cache space he wants at each node given the ISP's pricing scheme. We then analyze the equilibrium of this single-leader multi-follower game by backward induction. We first derive the CPs' best response in Stage II and then derive the ISP's optimal pricing scheme in Stage I. We prove the existence of Nash equilibrium (NE) which guarantees the robustness of our mechanism.

However, the business operations required by the LDP could be too complex for practical employment, since it assigns a specific price for each edge node. Consider the case where there are 1000 nodes, then the LDP will assign 1000 specific prices in a one-node-one-price fashion. However, we may limit the tiers of prices it offers, and group similar nodes under one tier and assign them the same price. Motivated by this idea, we then provide a partial location dependent pricing scheme (PLDP), which is a modification from LDP but only assigns several tiers of price for different groups of edge nodes to effectively control the mechanism complexity. In summary, our contribution on the location dependent pricing scheme is mainly shown as follows.

- **Game-theoretic Analysis**: we provide game-theoretic analysis for the location dependent pricing scheme. We compute the Nash equilibrium of the game and the existence of NE point guarantees the economic stability of our mechanism.
- **Practical Complexity**: we provide a modification of LDP that improves the market complexity of LDP with mild compromise in system performance.
- **Revenue and Efficiency**: the numerical simulation results show a 23%$\sim$35% increment in ISP's revenue for the proposed mechanism, and we observe an average increase of 32% in social welfare.

The rest of this paper is organized as below. In Section II, we refer to some related works in the field of pricing mechanism design for the edge caching market. In Section III, we propose the network and economics models for the edge caching market. And then we solve the Nash equilibrium of the Stackelberg game in Section IV. In Section V, we develop a partial location dependent pricing mechanism to control the mechanism complexity. Evaluation of our mechanism is shown in Section VI. And finally we conclude our work in Section VII.

II. Related Works

The economics of caching has been intensively studied in wired content delivery network(CDN) [3]. Broadly used pricing schemes include the aggregate usage-based pricing, percentile-based pricing and so forth. In [4], a more flexible pricing scheme is proposed based on the congestion pricing theory for congestible goods. And a pricing scheme that takes quality-of-seivice (QoS) differentiation into consideration is put forward in [5], [6]. Many other pricing schemes mostly are variations of these pricing schemes.

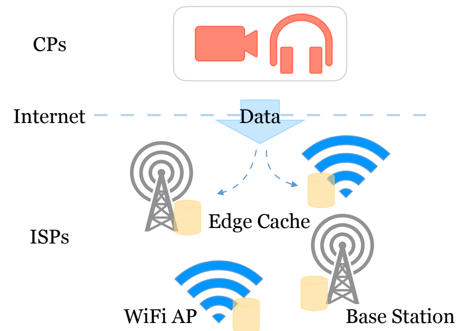

Fig. 2: architecture of edge caching network.

However, the edge-caching is different from the wired CDN caching in the sensitivity to contents' time-varying popularity and limited cache space [7]. Such difference has brought great challenge for the caching techniques used in the edge caching. In the economics' perspective, the authors in [7] also troubleshot that the incentive issues for stakeholders in a edge caching ecosystem needs well consideration. And it's important to give the ISPs enough motivation for edge caching service provisioning through well-designed economics mechanisms.

There have been several works studying the incentive issues in edge caching [8]–[10]. A CP differentiation pricing scheme is proposed in [9]. And in [10], a contract-based mechanism is proposed for the edge caching market. However, neither of these economic mechanisms take the heterogeneous edge nodes' popularity into consideration.

Moreover, smart data pricing is more and more popular these years. It can shape users' demand in different time and locations and at the same time generate more revenue for the operator. To the best of our knowledge, this is the first work that considers location dependent pricing in an edge caching network, and such pricing mechanism improves both the ISP's revenue and the social welfare.

III. Network and Economic Model

We consider an edge caching market with one monopolistic ISP and multiple CPs. We first introduce the background setting for the edge caching market, and then introduce the economics terms used in our work. We consider a monopolistic ISP because when there are duopolistic ISPs, they are usually effective monopolistic operators due to low churn rate [11].

A. Edge Caching Model

As is shown in Fig2, we suppose the ISP owns S edge nodes(e.g. small base stations or WiFi access points), $\mathcal{S} = \{1, 2, ..., S\}$. We suppose the edge cache installed at each edge

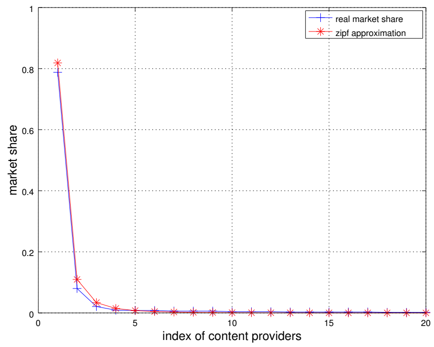

Fig. 3: Zipf approximation of the CPs' popularity.

node has storage capacity of C. And the ISP refreshes the cached contents proactively in a time-slot basis. Each time slot is divided into two parts: the caching phase and the request phase. And in the caching phase, the popular contents are cached in the nodes; later in the request phase, the users issue request for the contents, and the requests that get hit in cache will be directly served from the edge node, while others will be served via backhaul.

We suppose there are N CPs, $\mathcal{N} = \{1, 2, ..., N\}$, that purchase the cache spaces from the ISP to improve their service quality. To delineate the heterogeneity of popularity over the CPs, we suppose that the request rates of the CPs follow the Zipf's law with parameters γ. The Fig3. shows that the CPs' popularity can be well approximated by Zipf's law. The data comes from the statistics of the 20 most popular video service providers' market share in the US.

We further suppose that the request rates of the contents also follows the Zipf's law with different parameter, β. The contents' Zipf popularity has been verified and widely used in content modelling [12]. Then using the Zipf's notation, we have the i^{th} popular CP's proportion of total requests as $f(i, \gamma, N) = i^{\gamma} / \sum_{i'=1}^{N} i'^{\gamma}$. And we have the cumulative function for the first c_{ij} contents' total popularity as $F(c_{ij}, \beta, R_i) = \sum_{j'=1}^{c_{ij}} j'^{\gamma} / \sum_{j'=1}^{R_i} j'^{\gamma}$, here $f(\cdot)$ and $F(\cdot)$ are *pdf* and *cdf* of Zipf distribution. We assume that CPs are numbered according to their rank of popularity, and the i^{th} CP has R_i contents for delivery. And we denote the average request rate received by some node j as n_j. Note that n_j is an edge-node-specific parameter which reflects the edge nodes' different popularity.

B. Economics System Model

In our location dependent pricing scheme, we set a specific price for each edge node to improve the ISP revenue as well as to facilitate better cache space allocation. And we have the *pricing profile* of the ISP as follows.

***Definition** 1:* (*pricing profile*) The *pricing profile* of the ISP $\boldsymbol{p}$ is a vector $\boldsymbol{p} = (p_1, p_2, ..., p_S)$, with p_j denoting the price for a unit cache space on node j.

And we also denote the *caching profile* of each CP as follows.

***Definition** 2:* (*caching profile*) The *caching profile* of the CP i is $\boldsymbol{c}_i = (c_{i1}, c_{i2}, ..., c_{iS})$, with c_{ij} denoting the CP's demand of cache spaces on node j.

Now we derive the formulation for the ISP and the CPs' utilities. We have the request rate for the CP i at node j as $n_j \cdot f(i, \gamma, N)$. And since the CP will always caches the most popular contents at some node to maximize his benefit, then the number of hits the CP get at node j is $n_j \cdot f(i, \gamma, N) \cdot F(c_{ij}, \beta, R_i)$. And when a content is served by the cache, the CP gains benefit through the shortened latency and improved delivery quality. Assume that the average benefit the CP can get from each hit is w. Then the CP i's revenue from node j can be expressed as $W_{CP_i}^{(j)} = wn_j \cdot f(i, \gamma, N) \cdot F(c_{ij}, \beta, R_i)$. Thus the total revenue of CP i from all nodes is

$$W_{CP_i} = \sum_{j=1}^{S} w \cdot n_j \cdot f(i, \gamma, N) \cdot F(c_{ij}, \beta, R_i).$$

And we have CP i's cost as his payment to the ISP, $T_{CP_i} = \sum_{j=1}^{S} p_j \cdot c_{ij}$. The utility of the CP is $V_{CP_i} = Revenue - Cost = W_{CP_i} - T_{CP_i}$, and we have this utility as

$$V_{CP_i} = \sum_{j=1}^{S} (wn_j \cdot f(i, \gamma, N) \cdot F(c_{ij}, \beta, R_i) - p_j \cdot c_{ij}).$$

And the ISP's revenue is the total payment that he receives from all the CPs. And his overall cost, C_{ISP}, is made up of capital cost(e.g., the installation of edge storages) and operational cost(e.g energy cost), which is a constant cost for the ISP in our model. Then we have the ISP's utility as

$$V_{ISP} = \sum_{i=1}^{N} \sum_{j=1}^{S} p_j \cdot c_{ij} - C_{ISP}.$$

Finally, we introduce the notion of social welfare, which serves as an index for the system performance as follows.

***Definition** 3:* (*social welfare*) The *social welfare* of a system is the sum of the utilities of all parties in the system.

Thus the social welfare in the edge caching market is $V_{SW} = V_{ISP} + \sum_{i=1}^{N} V_{CP_i}$. Notice that the payment terms diminishes from the expression of social welfare. Higher social welfare means higher efficiency of the mechanism.

IV. Game Formulation

In the following discussion, we formulate the strategic interactions between the ISP and the CPs as a two-stage Stackelberg game. As is shown in Fig4, in the first stage, the ISP determines an optimal pricing profile $\boldsymbol{p}$ maximizing his revenue; and then in the second stage, each CP i determines his caching profiles $\boldsymbol{c}_i$ to maximize his utility. We solve the Nash equilibrium using the backward induction. We first derive the CPs' best responses in Stage II, and then use the CPs' best responses to determine the ISP's optimal pricing scheme in Stage I.

A. Stage II. CPs' Best Caching Decisions

In Stage II, the CPs will determine their caching profiles maximizing their utilities, given the ISP's pricing profile in Stage I. The maximization problem for CP i can be formulated as follows.

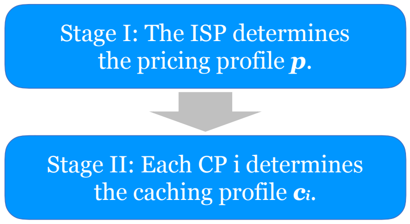

Fig. 4: Decision Making in the Stackelberg Game.

$$\begin{aligned}\max_{\boldsymbol{c}_i} \quad & \sum_{j=1}^{N} wn_j \cdot f(i,\gamma,N) \cdot F(c_{ij},\beta,R_i) - p_j \cdot c_{ij} \\ \text{s.t.} \quad & 0 \le c_{ij} \le R_i, \qquad \forall j \in S\end{aligned}$$

In this stage, CP i solves the problem above to make a best caching decision. Here we have the best response of the CPs determined by the following theorem.

***Theorem** 1 (CP's best response):* The CP i's best choice of the cache space at node j, c_{ij}^* is

$$c_{ij}^* = \left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor,$$

here θ_i is

$$\theta_i = \frac{w}{\sum\limits_{t=1}^{N} \frac{1}{t^\gamma} \sum\limits_{k=1}^{R_i} \frac{1}{k^\beta}}.$$

Proof: We consider the monotonicity of the series, $\{V_{CP_i}^{(j)}(\tau)\}, \tau = 0,1,...,C$. We can derive its monotonicity as follows.

Since we have

$$V_{CP_i}^{(j)}(\tau+1) - V_{CP_i}^{(j)}(\tau) = \frac{\theta_i n_j}{(\tau+1)^\beta \cdot i^\gamma} - p_j \triangleq \Delta_i^{(j)}(\tau)$$

Thus when $\tau \le \left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor - 1$, we have

$$\begin{aligned}\Delta_i^{(j)}(\tau) =& \frac{\theta_i n_j}{(\tau+1)^\beta \cdot i^\gamma} - p_j \ge \frac{\theta_i n_j}{\left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor^\beta \cdot i^\gamma} - p_j \\ \ge& \frac{\theta_i n_j}{\left(\left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}}\right)^\beta \cdot i^\gamma} - p_j = 0\end{aligned}$$

Notice that we've used the assumption in the second step, and we used the inequality $\lfloor x \rfloor \le x$ in the third step. Thus we know that $\{V_{CP_i}^{(j)}(\tau)\}$ is an increasing series in $[0,1,..., \left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor]$.

Similarly, when $\tau \ge \left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor$, we have

$$\begin{aligned}\Delta_i^{(j)}(\tau) \le& \frac{\theta_i n_j}{\left(\left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor + 1\right)^\beta \cdot i^\gamma} - p_j \\ \le& \frac{\theta_i n_j}{\left(\left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}}\right)^\beta \cdot i^\gamma} - p_j = 0\end{aligned}$$

Here we've used the assumption in the second step, and we used the inequality $\lfloor x \rfloor + 1 \ge x$ in the third step. Thus we know that $\{V_{CP_i}^{(j)}(\tau)\}$ is a decreasing series in $[\left\lfloor \left(\frac{\theta_i n}{p \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor, C]$.

To conclude, the series increases in $[0,1,..., \left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor]$, and decreases in $[\left\lfloor \left(\frac{\theta_i n}{p \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor, C]$, so we have the best response of the CP at the turning point, as the theorem contends. ∎

B. Stage I. Optimal Pricing Scheme

Then back in Stage I, the ISP maximizes his payoff by determining the optimal pricing profile $\boldsymbol{p}^*$. We can formulate the optimal pricing problem of the ISP as below.

$$\begin{aligned}\max_{\boldsymbol{p}} \quad & \sum_{i=1}^{N} \sum_{j=1}^{S} p_j \cdot c_{ij} \\ \text{s.t.} \quad & \sum_{i=1}^{N} c_{ij} \le C, \qquad \forall j \in S \\ & p_j \ge 0, \qquad \forall j \in S\end{aligned}$$

The first group of constraints are due to the limited capacity of edge nodes. Using the result of stage II's analysis, we can transform the ISP's objective function to the following form.

$$V_{ISP} = \sum_{i=1}^{N} \sum_{j=1}^{S} p_j \cdot \left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor$$

We note that the constraints and objective of ISP's optimization problem is completely decoupled at each node. Furthermore, the pricing on different nodes is not correlated. Thus, we divide the ISP's optimization problem into S independent subproblems. We have the subproblem of finding an optimal pricing on node j with the following objective function.

$$V_{ISP}^{(j)} = \sum_{i=1}^{N} p_j \cdot \left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor$$

And we have the constraints in this subproblem as $\sum\limits_{i=1}^{N} c_{ij} \le C$ and $p_j \ge 0$. We then consider the property of the objective function. We notice that the rounding will cause segmentations in this objective function. And we can derive the set of points where this function becomes discontinuous by setting the rounded part of this equation to some positive integer.

$$\left(\frac{\theta_i n_j}{p_j \cdot i^\gamma}\right)^{\frac{1}{\beta}} = \delta, \ \forall \delta \in \{1,2,...\}$$

Then we have the set of the break points $\mathcal{E}_j$ as

$$\mathcal{E}_j = \left\{ \frac{\theta_i n_j}{i^\gamma \delta^\beta}, \forall i \in \{1,2,...,N\}, \forall \delta \in \{1,2,...\} \right\}.$$

So between each two neighbouring break points, the payoff function of the CP should be continuous. Based on the discussion above, we have the following lemma for determining the best pricing strategy.

***Lemma** 1:* The optimal pricing p_j^* for the ISP must satisfy $p_j^* \in \mathcal{E}_j$.

Proof: Consider the payment of the i^{th} CP $T_{CP_i}^{(j)}$, we try to express it as a piecewise function. First, when $\frac{\theta_i n_j}{i^\gamma(\delta+1)^\beta} < p_j \le \frac{\theta_i n_j}{i^\gamma \delta^\beta}, \forall \delta \ge 1$, the payment of the i^{th} CP is

$$T_{CP_i}^{(j)} = p_j \cdot \left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma} \right)^{\frac{1}{\beta}} \right\rfloor = p_j \cdot \delta.$$

The second equality can be derived via simple transformation of the condition.

And when $p_j > \frac{\theta_i n_j}{i^\gamma}$, we have

$$T_{CP_i}^{(j)} = p_j \cdot \left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma} \right)^{\frac{1}{\beta}} \right\rfloor = 0$$

In conclusion, the function can be expressed in the following piecewise linear form.

$$T_{CP_i}^{(j)} = \begin{cases} p_j \cdot \delta & , p_j \in \left(\frac{\theta_i n_j}{i^\gamma(\delta+1)^\beta}, \frac{\theta_i n_j}{i^\gamma \delta^\beta} \right], \forall \delta \ge 1 \\ 0 & , p_j > \frac{\theta_i n_j}{i^\gamma} \end{cases}$$

The ISP's total payoff in this subproblem is the sum of all the CPs' payments on the edge node j. Since the linear combination of finite number of piecewise linear functions should still be a piecewise linear function, the ISP's total payoff on node j, $V_{ISP}^{(j)}$, which is sum of $T_{CP_i}^{(j)}, j \in \{1, 2, ..., S\}$, should also be piecewise linear. Due to the monotonicity of linear functions, the maximun of the piecewise linear function can only be attained at the break points, thus we have $p_j^* \in \mathcal{E}_j$. ■

However, since the number of break points is infinite, it seems impossible to find the maximum value tractably. In the following lemma, we derive a upper bound for the optimal price, thus subtracting the possible point set to a finite size.

***Lemma** 2:* The parameters in the optimal price p_j^* must satisfy

$$\delta^* \le \left\lfloor \frac{C}{i^*} \right\rfloor + 1.$$

Proof: Notice we have

$$c_{ij} = \left\lfloor \left(\frac{\theta_i n_j}{p_j \cdot i^\gamma} \right)^{\frac{1}{\beta}} \right\rfloor.$$

And the following inequality establishes for any $i > k$,

$$\left(\frac{\theta_i n_j}{p_j \cdot i^\gamma} \right)^{\frac{1}{\beta}} < \left(\frac{\theta_k n_j}{p_j \cdot k^\gamma} \right)^{\frac{1}{\beta}}.$$

Using $x < y \Rightarrow \lfloor x \rfloor \le \lfloor y \rfloor$, we have $c_{ij} \le c_{kj}, \forall i > k$. Then we have

$$C \ge \sum_{i=1}^{N} c_{ij} = \sum_{i=1}^{i'} c_{ij} + \sum_{i=i'+1}^{N} c_{ij}$$

$$\ge \sum_{i=1}^{i'} c_{ij} \ge i' c_{i'j}$$

The first inequality is due to the capacity constraint and the last inequality is due to $c_{ij} \le c_{kj}, \forall i > k$. Here i' can be any integer in $\{1, 2, .., N\}$. Substitute it with i^* we have

$$C \ge i^* c_{i^*j} = i^* \cdot \left\lfloor \left(\frac{\theta_{i^*} n_j}{p_j \cdot i^{*\gamma}} \right)^{\frac{1}{\beta}} \right\rfloor.$$

Relax the rounding and we get

$$\frac{C}{i^*} + 1 > \left(\frac{\theta_{i^*} n_j}{p_j \cdot i^{*\gamma}} \right)^{\frac{1}{\beta}} = \delta^*.$$

The second equality is due to the definition of δ. Rounding both sides and we finally get

$$\delta^* \le \left\lfloor \frac{C}{i^*} \right\rfloor + 1.$$

Thus the lemma is proved. ■

With Lemma1 and Lemma2, the possible optimal price should belong to a finite subset of $\mathcal{E}_j$. And we can determine the optimal pricing strategy for the ISP using the following theorem.

***Theorem** 2 (ISP's optimal pricing):* The optimal pricing strategy for the ISP at node j is

$$p_j^* = \max_{(i,\delta) \in \mathcal{E}'} \frac{\theta_i n_j}{i^\gamma \delta^\beta},$$

here $\mathcal{E}'$ is

$$\mathcal{E}' = \left\{ (i, \delta) \middle| i \in N, \delta \in \left\{ 1, 2, ..., \left\lfloor \frac{C}{i} \right\rfloor + 1 \right\} \right\}.$$

C. Nash Equilibrium Computation

In this subsection, we study the property of Nash Equilibrium in this Stackelberg game. The Nash Equilibrium in our model is defined as follows.

***Definition** 4 (Nash Equilibrium):* The *(weakly dominant) Nash Equilibrium* of this game is the pricing and caching profiles $\boldsymbol{p}^*$, $\boldsymbol{c}^*$ that satisfies the following conditions for the ISP and the CPs.

$$V_{ISP}(\boldsymbol{p}^*, \boldsymbol{c}^*) \ge V_{ISP}(\boldsymbol{p}', \boldsymbol{c}^*), \forall \boldsymbol{p}' \ne \boldsymbol{p}^*$$

$$V_{CP_i}(\boldsymbol{c}_i^*, \boldsymbol{c}_{-i}^*, \boldsymbol{p}^*) \ge V_{CP_i}(\boldsymbol{c}_i', \boldsymbol{c}_{-i}^*, \boldsymbol{p}^*), \forall \boldsymbol{c}_i' \ne \boldsymbol{c}_i^*, \forall i \in N$$

At the status of Nash equilibrium, the ISP and the CPs will have no willingness to deviate from the equilibrium point, since any unilateral deviation will only reduce his utility. We have the following theorem for the existence and uniqueness of the Nash Equilibrium for LDP mechanism.

***Theorem** 3 (existence and uniqueness of NE):* There always exists unique NE point in this game.

Proof: First, for Stage II, analytical solutions always exist and is unique. Second, for Stage I, the possible break point set, $\mathcal{E}' = \left\{ (i, \delta) \middle| i \in N, \delta \in \left\{ 1, 2, ..., \left\lfloor \frac{C}{i} \right\rfloor + 1 \right\} \right\}$, is finite and nonempty, so there is always a maxima. Thus we have the unique NE as $(\boldsymbol{p}^*, \boldsymbol{c}^*)$, with $\boldsymbol{p}^*$ uniquely determined by Theorem2, and $\boldsymbol{c}^*$ uniquely determined by Theorem1. This ends the proof. ■

Now based on Theorem1 and the Theorem2 that we have derived, we can use them to compute the NE point. We have the following computational complexity result for computing the NE.

***Theorem** 4:* The complexity of computing the NE is $\mathcal{O}(S \cdot N^3)$.

Proof: Refer to the Appendix. ■

V. Partial Location-dependent Pricing

Through location dependent pricing scheme, the ISP is able to extract relatively high profit from the CPs. However, the LDP mechanism usually incurs high market complexity and thus not applicable to the real markets. For example, for an ISP with millions of edge nodes, it would not be viable to use the location dependent pricing scheme and assign a price for each edge node, since that would dramatically complex the interactions between the ISP and the CPs.

In this section, we consider the partial location-dependent pricing scheme which reduces the market complexity of this pricing mechanism and makes it more economically viable. The limited-tier pricing technique has been previously used in some market pricing mechanism designs [13]. We show the design of such a partial pricing scheme as follows. First, the ISP categorizes the edge nodes into m levels, $\{\mathcal{L}_1, \mathcal{L}_2, ..., \mathcal{L}_m\}$, based on their popularity differences. Then a specific price p_l is assigned to the cache spaces in $\mathcal{L}_l$. The techniques used for edge node clustering can be popularity-based[1] or edge-node-based[2]. And we have the partial optimal pricing problem for the ISP in this case as below.

$$\begin{aligned} \max \quad & \sum_{i=1}^{N}\sum_{l=1}^{m} p_l \cdot \sum_{j\in\mathcal{L}_l} \left\lfloor \left(\frac{\theta_i n_j}{p_l \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor \\ \text{s.t.} \quad & 0 \le c_{ij} \le R_i, \qquad\qquad \forall j \in S \\ & p_l \ge 0, \qquad\qquad \forall l \in \{1,2,...,m\} \end{aligned} \tag{1}$$

Using similar skills in the previous game problem. We have the following theorem for determining the optimal pricing strategy of the ISP.

***Theorem** 5:* The optimal pricing in the partial location-dependent pricing scenario is

$$p_m^* = \max_{(i,j,\delta)\in\mathcal{E}_m'} \sum_{i=1}^{N} p_l \cdot \sum_{j\in\mathcal{L}_m} \left\lfloor \left(\frac{\theta_i n_j}{p_l \cdot i^\gamma}\right)^{\frac{1}{\beta}} \right\rfloor,$$

here $\mathcal{E}_m'$ is

$$\mathcal{E}_m' = \left\{(i,j,\delta)\middle| i \in N, j \in \mathcal{L}_m, \delta \in \left\{1,2,...,\left\lfloor \frac{C}{i} \right\rfloor + 1\right\}\right\}.$$

Proof: Refer to the Appendix. ■

VI. Simulation

A. Ground Setting

We consider an edge caching market that consists of $S = 1000$ edge nodes. And we set the number of CPs $N = 10$, and each CP has $R = 2,000$ files for deliver. The storage capacity at every edge node is $C = 100\ chunks$, and we set the popularity parameters $\gamma = \beta = 0.7$. We further suppose that the request rates of the edge nodes follow the Gaussian distribution, $n \sim N(n_0, \sigma)$, here we set $n_0 = 1000, \sigma = 200$. And the CP's revenue gained by cache hit is $w = \$0.1/hit$.

[1]The edge nodes' average request rates are segmented into several continuous intervals, and the edge nodes whose request rates are within the same interval belong to the same group.

[2]The edge nodes are ranked down according to their average request rates, and the first K edge nodes belong to the same group, the second K edge nodes belong to the same group, etc.

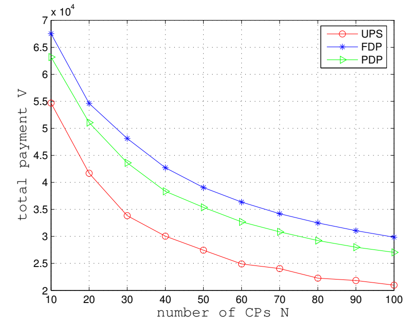

Fig. 5: ISP revenue under different numbers of CPs N

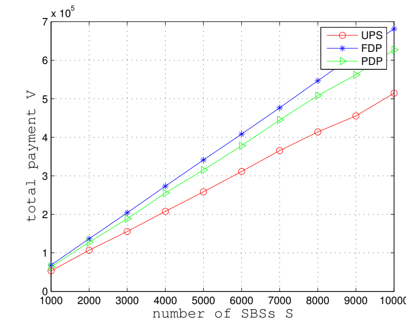

Fig. 6: ISP utility under different numbers of edge nodes S

B. Experiment Results

We consider the following following performance indexes of the pricing schemes.

- *ISP Revenue*: the mechanism that generates more revenue for the ISP will provide higher incentive for him to invest in edge cache and hence promote the service provisioning of edge cache service.
- *Social Welfare*: the mechanism that achieves high social welfare provides better system performance, meaning that the resource allocation driven by the economic mechanism is more efficient.

And we mainly consider three pricing mechanisms for comparative study.

- *Uniform Pricing Scheme ([9], [10])*: Under the UPS mechanism, the ISP offers a single usage-based price to the CPs.
- *Location Dependent Pricing(Section IV)*: The LDP offers a location-aware pricing profile to the CPs.
- *Partial Location Dependent Pricing(Section V)*: The PLDP offers a location-aware and limited-tier[3] pricing profile to the CPs.

We have the following key observation for the three pricing schemes' performance in social welfare and revenue generation for the ISP.

***Observation** 1 (Performance Comparison):*

1) The mechanisms' performance in revenue generation is $LDP > PLDP > UPS$;

[3]We consider $m = 4$ tiers of pricing in our simulation.

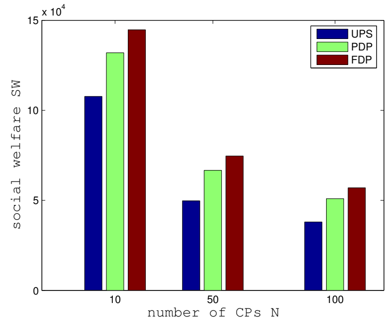

Fig. 7: Social welfare under numbers of CPs.

2) The mechanisms' performance in social welfare is $LDP > PLDP > UPS$;
3) The market complexity of the mechanisms is $UPS < PLDP < LDP$.[4]

We show in Fig5. and Fig6 that the LDP always achieves the highest revenue for the ISP. We find that the LDP and the PLDP improve the ISP's revenue by 31%-50% and 23%-35% respectively, compared to the UPS. Moreover, the social welfare could also be promoted through the LDP and the PLDP. From Fig7. we can observe that the LDP and the PLDP improves the social welfare by 46% and 32% on average.

To conclude, although the LDP has the best performance both in revenue generation and social welfare, it still has potential problem in high market complexity. And the PLDP serves as a tradeoff between the optimality and viability, with suboptimal performance and acceptable market complexity.

VII. Conclusion

In this paper, we designed a location dependent pricing (LDP) mechanism for the edge caching market. We formulated the CPs and the ISP's interaction as a two-stage Stackelberg game and derived the Nash equilibrium through game-theoretic analysis. The existence and uniqueness of the NE point guarantee the economic robustness of the mechanism. Then we proposed a partial location dependent pricing (PLDP) mechanism which provides limited tiers of prices and hence reduces the market complexity. Through simulation, we showed that the LDP improves the ISP's payoff by 23%-35%, and increases the social welfare by 32% on average, compared to the uniform pricing scheme. To conclude, the pricing schemes LDP and the PLDP shown in this work are both economically deployable and efficient.

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Appendix

A. Proof for Theorem4.

Proof: Based on the ISP's optimal price search on the finite set in Theorem2, we have the computational complexity as

$$\begin{aligned} T(N) =& S \cdot \left(\sum_{i=1}^{N} N \cdot (\left\lfloor \frac{C}{i} + 1 \right\rfloor) \cdot N + o(N^3) \right) \\ =& S \cdot \left(N^2 \cdot \sum_{i=1}^{N} \left\lfloor \frac{C}{i} \right\rfloor + \mathcal{O}(N^3) \right) \end{aligned}$$

In fact, for $N^{'} = C$ and $k = \frac{1}{C} \sum_{i=1}^{C} \lfloor \frac{C}{i} \rfloor$, when $N \geq N^{'}$ we have

$$\begin{aligned} \sum_{i=1}^{N} \left\lfloor \frac{C}{i} \right\rfloor =& \sum_{i=1}^{C} \left\lfloor \frac{C}{i} \right\rfloor + \sum_{i=C+1}^{N} \left\lfloor \frac{C}{i} \right\rfloor \\ =& \sum_{i=1}^{C} \left\lfloor \frac{C}{i} \right\rfloor \leq \left(\frac{1}{C} \cdot \sum_{i=1}^{N} \left\lfloor \frac{C}{i} \right\rfloor \right) \cdot C \leq kN \end{aligned}$$

Thus $T(N) = S \cdot \left(N^2 \mathcal{O}(N) + \mathcal{O}(N^3) \right) = \mathcal{O}(S \cdot N^3)$. ■

B. Proof for Theorem5.

Proof: Similarly as in the LDP scheme, the function of $V_{ISP}(p^*)$ is piecewise linear, thus we have the maximum value at the end points. The set of end points in this case is

$$\mathcal{E}_m = \bigcup_{j \in \mathcal{L}_m} \mathcal{E}_j = \left\{ \frac{\theta_i n_j}{i^\gamma \delta^\beta}, \forall i \in N, \forall j \in \mathcal{L}_m, \forall \delta \in \{1, 2, ...\} \right\}.$$

And since we also have similar constraint on the parameter δ. Thus the theorem is proved. ■