Practice Sessions
Astrophysical Simulations

Part 4a: Differential equation (Assignment)

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Finite difference equations

Consider an ordinary differential equation $\dot{y}(t) = g(y,t)$ to be solved for the function $y(t)$. We choose a time step h defining discrete time points $t_n = t_0 + nh$ (with n integer), and we write $y_n \equiv y(t_n)$, $\dot{y}_n \equiv \dot{y}(t_n)$ and $g_n \equiv g(y_n, t_n)$.

We can then approximate the ODE using one of the following finite difference equations:

Scheme	Order	Equation
Forward difference	$O(h^2)$	$y_{n+1} = y_n + hg_n$
Backward difference	$O(h^2)$	$y_{n+1} = y_n + hg_{n+1}$
Centered difference	$O(h^3)$	$y_{n+1} = y_{n-1} + 2hg_n$
Centered time difference	$O(h^3)$	$y_{n+1} = y_n + h(g_n + g_{n+1})/2$

Assignment: solve differential equation

Part 1

- Implement the four finite difference methods listed previously to numerically solve the ODE $\dot{y} = -y$ with $y(0)=1$ for $y(t)$ in the interval $t \in [\,0,1\,]$ using time step $h=0.01$

- Plot the solutions and the errors relative to the analytical solution, in function of t

Hint – The backward difference and centered time difference methods are implicit: to find y_{n+1} you need g_{n+1} for which you need y_{n+1}. Fortunately in this case, after substituting $g_n = -y_n$ you can rearrange the equation to resolve the problem.

Part 2

- Adjust (a copy of) your program to use a sequence of time steps $h = 1, 0.1, 0.01, \ldots 10^{-9}$ (do not output all the computed points!)

- Plot the relative error at the end of the interval ($t_{end} = 1$) in function of h, and interpret the results

Assignment: further guidance

Work in steps, and verify your results step by step

- Start from the tabulate example
- First implement an easy finite difference method (e.g. forward difference) and develop a plotting script to visualize the results
- Make a backup of your working code; then add the other methods
- After finishing part 1, make a copy of your code to complete part 2

Keep it simple

- Hardcode all formulas and constants (i.e. no user input)
- Write data to output file as it is being computed (no memory arrays)
- Do not use C++ language facilities you don't understand

Seek help

- Use the slides
- Look up things on the web
- Ask me to review your code or to help when you get stuck

Questions?