Practice Sessions Astrophysical Simulations

Part 6b: Two-body problem (Solution)

GHENT UNIVERSITY

Master of Science in Physics and Astronomy
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Assignment: solve the two-body problem

Part 1

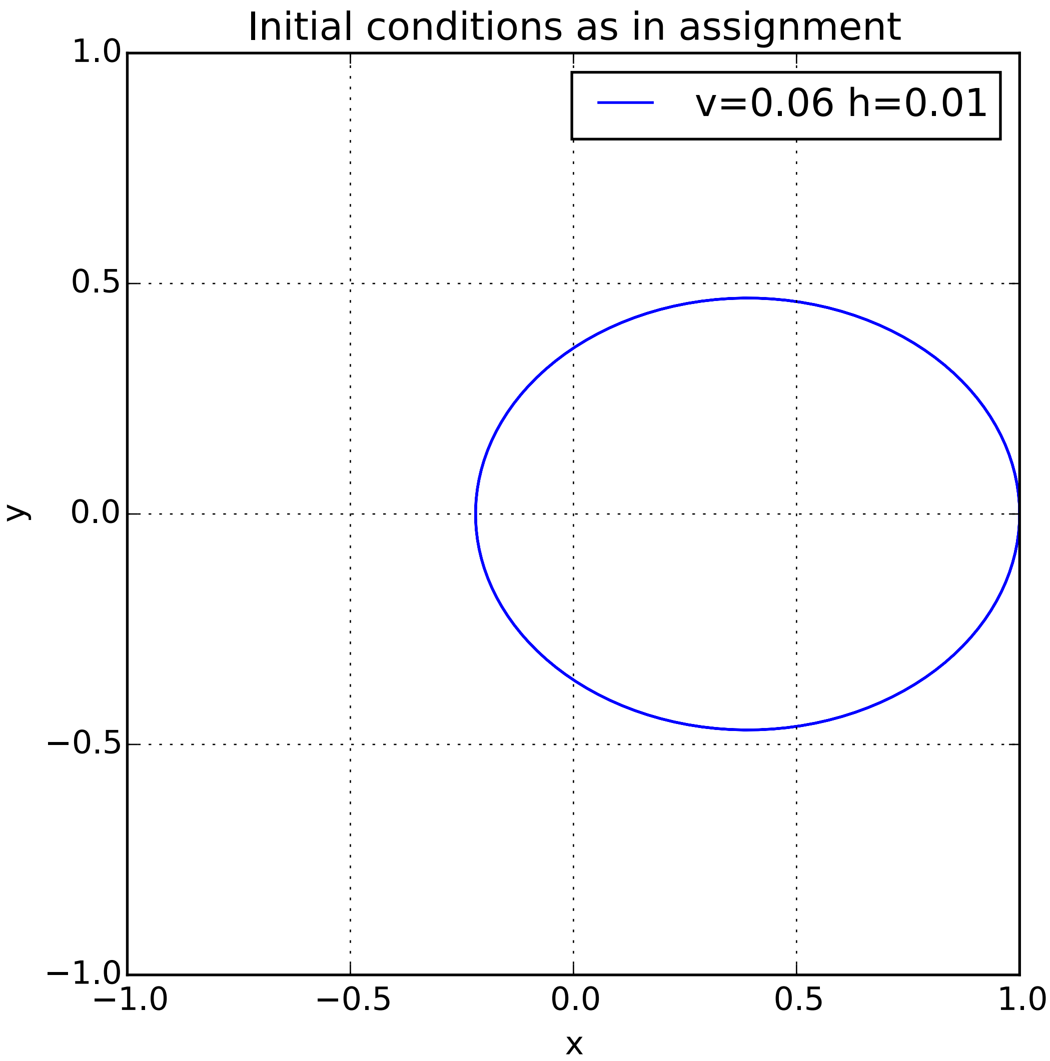
- Use the leapfrog integrator to solve the two-body problem in the xy-plane for the following initial conditions and parameters

  $r_0 = (1, 0) \qquad v_0 = (0, 0.06) \qquad \mu = 0.01 \qquad h = 0.01$
- Plot the orbits in the xy-plane, assuming that one of the bodies stays at rest in the origin
- Plot the relative error on the energy in function of time for a couple consecutive orbits, and interpret the results

Part 2

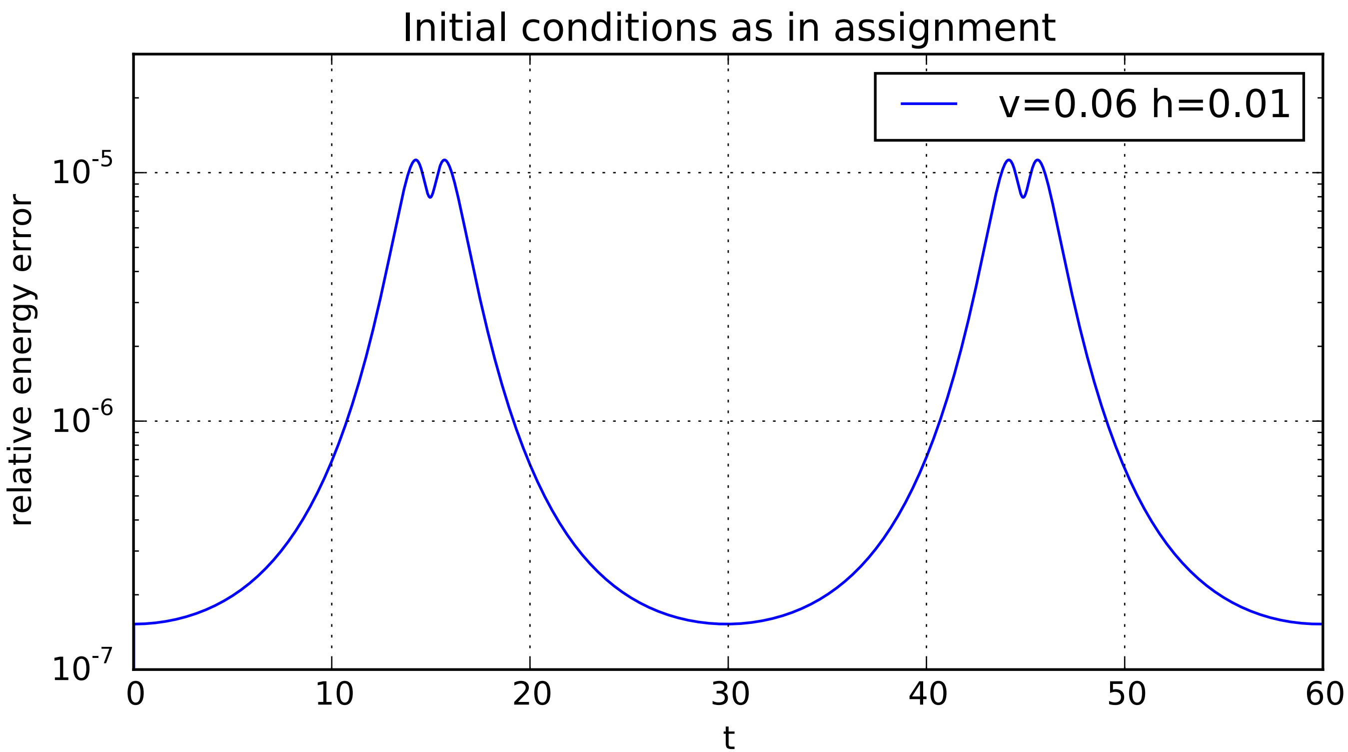
- Change the initial conditions to obtain a much more eccentric orbit, and integrate over 15 or more orbits still using h=0.01; make the same plots as above
- Change the time step to h=0.001 and note the differences

Orbit for initial conditions in assignment



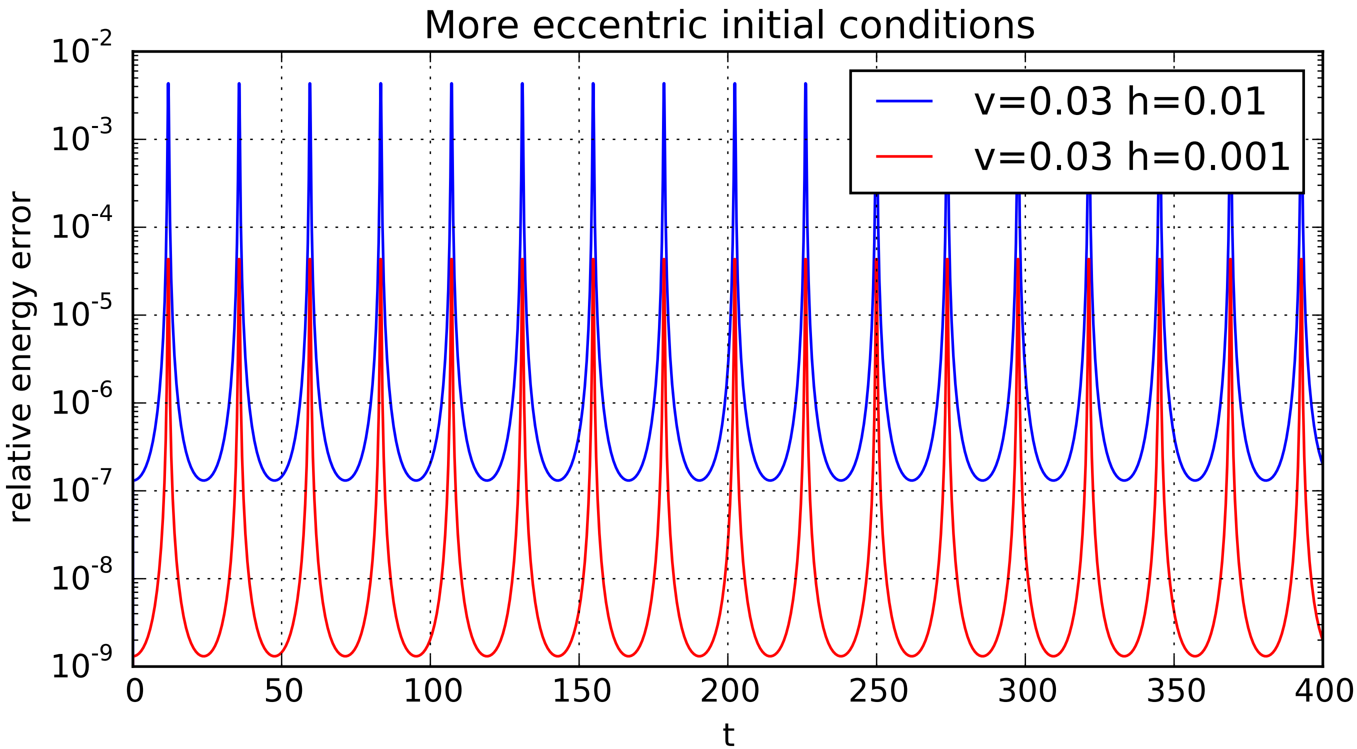
Bound system ->
elliptical orbit

Energy for initial conditions in assignment



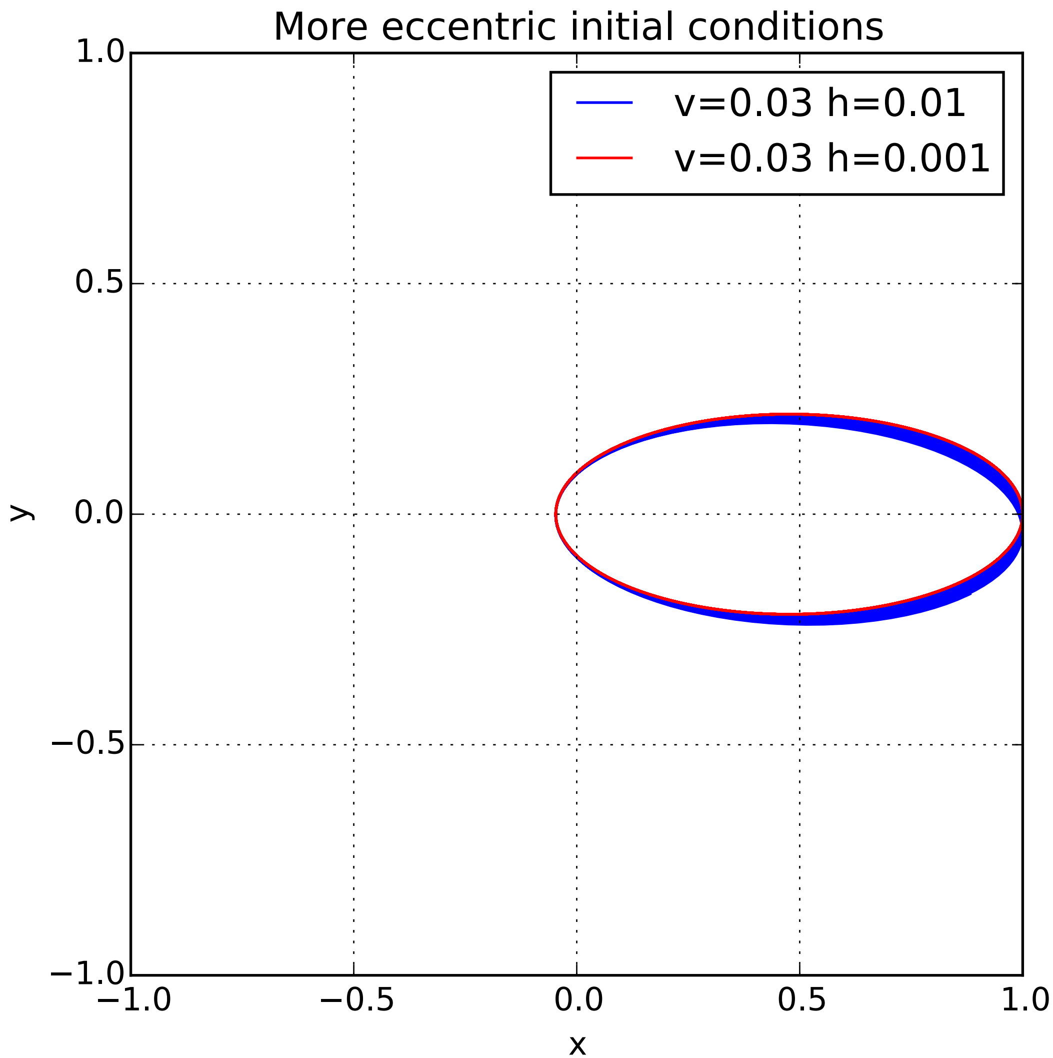
- Error on energy is high when objects are close and have large acceleration
  - » Peaks are about 2 order of magnitudes above valleys
- Error on energy returns to the same value after each orbit (because the leapfrog integrator is simplectic)

Energy for more eccentric system



- Error on energy is still periodic, however the peak error is a lot higher (more than 4 order of magnitudes, compared to 2 in previous plot)
- Decreasing the time step by 1 order of magnitude causes the energy error to drop by 2 order of magnitudes (as expected for the leapfrog integrator)

Orbit for more eccentric system



- For h=0.01 the orbit is wrong even after a few periods (although the closest approach remains correct)
- Using a smaller time step helps, but for a large number of orbits this simple integration scheme breaks down inevitably

Questions?