24-677 Final Project XingYu Wang

Task 1: model Linearization

system equation:

$$S_1 \begin{cases} \frac{d}{dt}(y) = \dot{y} \\ \frac{d}{dt}(\dot{y}) = -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left(\cos\delta\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right) \\ \frac{d}{dt}(\psi) = \dot{\psi} \\ \frac{d}{dt}(\dot{\psi}) = \frac{2l_fC_\alpha}{I_z}\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{2l_rC_\alpha}{I_z}\left(-\frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right) \end{cases}$$

$$S_2 \begin{cases} \frac{d}{dt}(x) = \dot{x} \\ \frac{d}{dt}(\dot{x}) = \dot{\psi}\dot{y} + \frac{1}{m}(F - fmg) \end{cases}$$

$$\frac{d}{dt}(X) = \dot{x}\cos\psi - \dot{y}\sin\psi$$

$$\frac{d}{dt}(Y) = \dot{x}\sin\psi + \dot{y}\cos\psi$$

Assumption:

1. δ is small angle: $\cos\delta \approx 1$
2. δ is input of S_1, $\dot{\delta} = \frac{\delta(k+1)-\delta(k)}{\Delta T}$, $\delta(k+1) = \delta(k) + \dot{\delta}\Delta T$
3. We can break the state into longitudinal and lateral state, then $\dot{x}$ becomes a constant in lateral state, and $\dot{\psi}\dot{y}$ also becomes constant in longitudinal state.

Linearization:

$$S_2: \frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \left[\frac{\partial f}{\partial x}\right]\begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \left[\frac{\partial f}{\partial u}\right]\begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$\left[\frac{\partial f}{\partial x}\right] = \begin{bmatrix} \partial f_1/\partial x & \partial f_1/\partial \dot{x} \\ \partial f_2/\partial x & \partial f_2/\partial \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\left[\frac{\partial f}{\partial u}\right] = \begin{bmatrix} \partial f_1/\partial \delta & \partial f_1/\partial F \\ \partial f_2/\partial \delta & \partial f_2/\partial F \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix}$$

For longitudinal system S_2: $\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix}}_{B}\begin{bmatrix} \delta \\ F \end{bmatrix}$

For lateral system S_1:

$$\frac{d}{dt}\begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} = \left[\frac{\partial f}{\partial x}\right]\begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} + \left[\frac{\partial f}{\partial u}\right]\begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{4C_\alpha}{m\dot{x}} & 0 & \frac{2C_\alpha}{m\dot{x}}(l_r - l_f) - \dot{x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_\alpha}{I_z\dot{x}}(l_r - l_f) & 0 & -\frac{2C_\alpha}{I_z\dot{x}}(l_f^2 + l_r^2) \end{bmatrix} = A$$

$$\frac{\partial f}{\partial u} = \left[\frac{\partial f}{\partial \delta} \;\; \frac{\partial f}{\partial F}\right] = \begin{bmatrix} 0 & 0 \\ 2C_\alpha/m & 0 \\ 0 & 0 \\ 2l_f C_\alpha/I_z & 0 \end{bmatrix} = B$$

For lateral system S_1:

$$\frac{d}{dt}\begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} = A \cdot \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} + B \cdot \begin{bmatrix} \delta \\ F \end{bmatrix}$$

δ can then be used to derive $\dot{\delta}$ using the following equation:

$$\delta(k+1) = \delta(k) \pm \dot{\delta}\Delta T$$

$$\dot{\delta} = \frac{\delta(k+1) - \delta(k)}{\Delta T}$$