Final Project[*]
Linearization Solution

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24-677 Special Topics: Linear Control Systems

This is a recommended linearization. If you linearize your system with reasonable assumptions, you will get full points.
Under the small angle assumption, we have $\cos(\delta) = 1$. Then cluster the varibles with respect to the variables within state variables, we have
For s_1:

$$\ddot{y} = \dot{y}\left(\frac{-4C_\alpha}{m\dot{x}}\right) + \dot{\psi}\left(-\dot{x} + \frac{2C_\alpha(l_r - l_f)}{m\dot{x}}\right) + \delta\left(\frac{2C_\alpha}{m}\right)$$

$$\ddot{\psi} = \dot{y}\left(\frac{2(l_r - l_f)C_\alpha}{I_z\dot{x}}\right) + \dot{\psi}\left(-\frac{2\left(l_f^2 + l_r^2\right)C_\alpha}{I_z\dot{x}}\right) + \delta\left(\frac{2l_fC_\alpha}{I_z}\right)$$

For s_2:

$$\ddot{x} = F\left(\frac{1}{m}\right) + \dot{\psi}\dot{y} - fg$$

Write them in matrix form:

$$\frac{d}{dt}s_1 = \frac{d}{dt}\begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_\alpha}{m\dot{x}} & 0 & -\dot{x} + \frac{2C_\alpha(l_r-l_f)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2(l_r-l_f)C_\alpha}{I_z\dot{x}} & 0 & -\frac{2\left(l_f^2+l_r^2\right)C_\alpha}{I_z\dot{x}} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2l_fC_\alpha}{I_z} & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$\frac{d}{dt}s_2 = \frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix}\begin{bmatrix} \delta \\ F \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\psi}\dot{y} - fg \end{bmatrix}$$

[*]Modified on Dec.2