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Probability

Ques Two dice are thrown symentaniously
(a) find the probability of getting
(i) Sum of the number on the Dice is 5.
(ii) Sum of the number on the Dice is neither 7 nor 11.
(iii) A multiple of 2 on one Dice and A multiple of 3 on the other Dice.

Ans. (i) $\frac{4}{36}$ $A = \{(1,4), (4,1), (3,2), (2,3)$

(ii) $B =$ Set of favorable Sample point (Sum is neither 7 nor 11)

$\overline{B} = \{(1,6)\ (6,1), (2,5)\ (5,2)\ (3,4)\ (4,3)\ (5,6)\ (6,5)\}$

$P(\overline{B}) = \frac{8}{36} = \frac{2}{9}$

$P(B) = 1 - P(\overline{B})$

$= 1 - \frac{2}{9} = \frac{7}{9}$

(iii) $C = \{(2,3)\ (2,6)\ (3,2)\ (3,4)\ (3,6)\ (4,3)\ (4,6)\ (6,2)\ (6,3), (6,4)\ (6,6)\}$

$P(C) = \frac{11}{36}$

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Que Urn Contain 7 red, 3 black, 5 white balls
If two balls are thrown at random
then find the probability of getting
(i) both red balls
(ii) the both balls are of the same colour
(iii) 1 is black and other is white.
(iv) 1 is black ball

Ans. Total Case $= {}^{15}C_2 = \frac{15 \times 14}{2 \times 1} = 105$

(i) $n(A) = {}^{7}C_2 = 21$

$$P = \frac{21}{105} = \frac{1}{5}$$

(ii) $n(B) = {}^{7}C_2 + {}^{3}C_2 + {}^{5}C_2$
$= 21 + 3 + 10$
$= 34$

$$P = \frac{34}{105}$$

(iii) $n(C) = {}^{3}C_1 \times {}^{5}C_1$

$$P = \frac{15}{105} = \frac{5}{35} = \frac{1}{7}$$

(iv) $n(D) = {}^{3}C_1 \times {}^{12}C_1$

$$P = \frac{3 \times 12}{105} = \frac{12}{35}$$

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Addition law of probability

→ $P(A+B) = P(A \cup B)$
= probability of happening of event A or event B or AB simultaneously.

(i) If A and B are mutually Exclusive events then
$P(A \cup B) = P(A) + P(B)$

(ii) If A and B are non-mutually Exclusive events then
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(iii) If A, B and C are non-mutually Exclusive events then

~~(iv) If A, B and C are non-mutually Exc~~
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$
$- P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

Ques One number is selected from the numbers 1 to 100. find the probability of getting the number is divisible by 2 or 5?

Ans $S = \{1, 2, 3 \ldots\ldots 100\}$

$A = \{2, 4, 6, \ldots\ldots 100\}$

$B = \{5, 10, 15, \ldots\ldots 100\}$

$A \cap B = \{10, 20, \ldots\ldots 100\}$

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$n(A) = 50, \quad n(B) = 20, \quad n(C) = 10$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{50}{100} + \frac{20}{100} - \frac{10}{100}$$
$$= \frac{3}{5}$$

Ques 2 cards are drawn from ~~A~~ a pack of 52 playing cards find the probability of getting either both are king or Black cards.

Ans Total case = ${}^{52}C_2$

$$\frac{{}^{4}C_2 + {}^{26}C_2 - {}^{2}C_2}{{}^{52}C_2}$$

Conditional Probability :-

$P(A|B) \rightarrow$ Probability of happening of event A when event B is happened

or

Probability of event A with respect to event B.

or

Probability of happening of event A when event B is taken as a Sample Space

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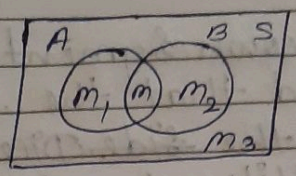

$$n(S) = m_1 + m + m_2 + m_3 = n$$

$$P(A) = \frac{m_1 + m}{n}$$

$$P(B) = \frac{m_2 + m}{n}$$

$$P(A \cap B) = \frac{m}{n}$$

$$P(A|B) = \frac{m}{m+m_2} = \frac{m/n}{(m+m_2)/n}$$

$$= \frac{P(A \cap B)}{P(B)}$$

$\rightarrow$ $$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$\rightarrow$ $$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$\rightarrow$ $$P(A|S) = \frac{P(A \cap S)}{P(S)} = P(A)$$

$*$ $$P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C)$$

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Ques Two Dice are thrown Symenteneously If is observed that the sum of the number on the ~~side~~ Dice is 6 then find the probability of getting number 2 atleast once.

Ans $S = ((1,1)(1,2) \ldots (6,6))$

$A = ((2,4)(4,2)(5,1)(1,5)(3,3))$

$B = ((2,1)(2,2)(2,3)(2,4)(2,5)(2,6)(6,2)(5,2)$
$(4,2)(3,2)(2,1))$

$A \cap B = ((2,4)(4,2))$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}$$

Multiplication law of probability

If A and B are any two events associated with a random Experiment then

(i) $P(AB) = P(A \cap B)$
$= P(A) \cdot P(B|A)$
(where $P(A) \neq 0$)

(ii) $P(A \cap B) = P(B) \cdot P(A|B)$
(where $P(B) \neq 0$)

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→ If A and B are two Independed events then

$$P(A \cap B) = P(A) \cdot P(B)$$

→ If A and B are two Independed events then

(i) A and $\bar{B}$ are Independed events

(ii) $\bar{A}$ and B are Independed events

(iii) $\bar{A}$ and $\bar{B}$ are also Independed events

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RENDOM VARIABLES

A real value function X, $X: S \to R$ is called random variable where S is sample space associated with a random experiment i.e each sample point $\omega \in S$ associated with unique real number, $X(\omega) \in R$

Eg. Three coins are tossed symultaneously

$$S \xrightarrow{X} R$$

ω_1 HHH
ω_2 HHT
ω_3 HTH
ω_4 THH
ω_5 TTT
ω_6 TTH
ω_7 THT
ω_8 HTT

~~Mean deviation about mean~~

(i) $X(\omega)$ = no. of heads in ω

$X(\omega_1) = 3$

$X(\omega_2) = X(\omega_3) = X(\omega_4) = 2$

$X(\omega_5) = X(\omega_6) = X(\omega_7) = 1$

$X(\omega_8) = 0$

$$X(\omega) = \{\underset{x_1}{0}, \underset{x_2}{1}, \underset{x_3}{2}, \underset{x_4}{3}\}$$

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(2) $X(\omega) = 2\,(\text{no. of tails}) - (\text{no. of heads})$

$X(\omega_1) = -3$

$X(\omega_2) = X(\omega_3) = X(\omega_4) = 0$

$X(\omega_5) = X(\omega_6) = X(\omega_7) = 3$

$X(\omega_8) = 6$

$X(\omega) = \{-3, 0, 3, 6\}$

(3) $X(\omega) = \dfrac{\sqrt{\text{no. of heads}}}{\text{no. of heads} + \text{no. of tails}}$

$X(\omega_1) = \frac{1}{\sqrt{3}}$

$X(\omega_2) = X(\omega_3) = X(\omega_4) = \frac{\sqrt{2}}{3}$

$X(\omega_5) = X(\omega_6) = X(\omega_7) = \frac{1}{3}$

$X(\omega_8) = 0$

Discrete Random Variable

A random variable is said to be discrete random variable if it ~~answer~~ assumes a finite no. of values of countably infinite values.

Probability distribution of a discrete random variable.

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Let x be the discrete Random Variable which assumes values $x_1, x_2, x_3 \ldots x_n$ and $P_1, P_2, P_3 \ldots P_n$ are their Respective probabilities. The probability distribution of x is given by

x	x_1	x_2	$\ldots$	x_n
P	P_1	P_2	$\ldots$	P_n

where $P_i \geq 0$

$$\sum_{i=1}^{n} P_i = 1$$

Ques The probability distribution of a random variable x is given by

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) find k
(ii) $P(3)$
(iii) $P(x \leq 4)$
(iv) $P(1 < x < 6)$
(v) $P\left(\frac{1.5 < x < 4}{x \geq 2}\right)$

Ans (i) $9k + 10k^2 = 1$

$10k^2 + 9k - 1 = 0$

$(10k-1)(10k+1) = 0$

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$$K = \frac{-1}{10} \times \qquad \boxed{K = \frac{1}{10}}$$

(ii) $P(3) = 2K = \frac{2}{10} = \frac{1}{5}$

(iii) $P(x \leq 4) = P(0) + P(1) + P(2) + P(3) + P(4)$

$$8K = 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} + \frac{3}{10}$$

$$= \frac{8}{10} = \frac{4}{5}$$

(iv) $P(1 < x < 6) = P(2) + P(3) + P(4) + P(5)$

$$= K^2 + 7K$$

$$= \frac{1}{100} + \frac{7}{10} = \frac{71}{100}$$

(v) $P(1.5 < x < 4 \mid x \geq 2)$

$$= \frac{P(2 \leq x < 4)}{P(x \geq 2)}$$

$$= \frac{4K}{1 - \{P(0) + P(1)\}} = \frac{4K}{1-K}$$

$$= \frac{4/10}{9/10} = \frac{4}{9}$$

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Continuous Random Variable

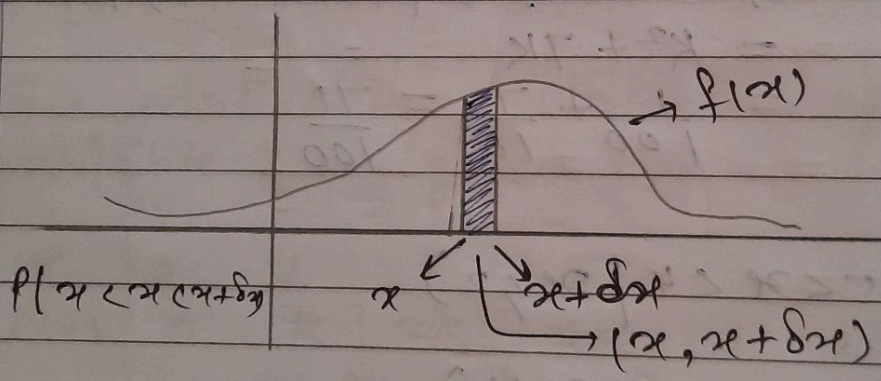
A random variable which can take infinite number of values in an interval is known as continuous random variable.

OR

A random variable which can take all posible values between certain limits

Ex. $x \in [0,1]$, $x \in (-\infty, \infty)$
$x \in [1,3)$

Probability Mass function (Pdf) :-



Consider the small interval $(x, x+\delta x)$ of length δx around the point x. Let $f(x)$ be any continuous function of x so that $f(x).\delta x$ represents the probability that x fall in the infinitesimal interval $(x, x+\delta x)$

$$P(x < x < x+\delta x) = f(x).\delta x$$

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Def. P.d.f. $f(x)$ of a random variable X is defined as

$$f(x) = \lim_{\delta x \to 0} \frac{P(x < X < x + \delta x)}{\delta x}$$

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x) = 1$

** $P(a \leq x \leq b) = \int_a^b f(x)\,dx$

* $P(x = a) = 0$

* $P(a < x < b) = P(a \leq x < b)$
$= P(a < x \leq b)$
$= P(a \leq x \leq b)$

Ques The diameter of an electric cable, say X is assumed to be a continuous random variable with p.d.f. $f(x) = 6x(1-x),\ 0 \leq x \leq 1$

(a) Check that $f(x)$ is p.d.f.

(b) $P(0 < x < \frac{1}{2})$

(c) $P(x \geq \frac{1}{3})$

Soln (a) $f(x) = 6x(1-x),\quad 0 \leq x \leq 1$

(i) when $0 \leq x \leq 1$ $\quad x \geq 0,\ 1-x \geq 0$

i.e. $6x(1-x) \geq 0$ when ~~Ea~~ $x \in [0,1]$

$f(x) \geq 0$

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$$\int_{-\infty}^{\infty} f(x)\,dx$$

$$= \int_{0}^{1} f(x)\,dx$$

$$= 6\int_{0}^{1} x(1-x)\,dx$$

$$= 6\int_{0}^{1} (x-x^2)\,dx$$

$$= 6\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1 = 6\left(\frac{1}{2} - \frac{1}{3}\right) = 6 \times \frac{1}{6} = 1$$

(ii) $P\left(0 < x < \frac{1}{2}\right) = \int_{0}^{1/2} f(x)\,dx$

$$= \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_0^{1/2}$$

$$= \frac{1}{8} - \frac{1}{24} = \frac{1}{12}$$

(iii) $P\left(x \geq \frac{1}{3}\right) = \int_{1/3}^{1} f(x)\,dx = \left(\frac{x^2}{2} - \frac{x^3}{3}\right)_{1/3}^{1}$

$$P(x=3) = \frac{{}^3C_3 \times {}^7C_1}{{}^{10}C_4} = \frac{1}{10}$$

X :	0	1	2	3
$P(X)$:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

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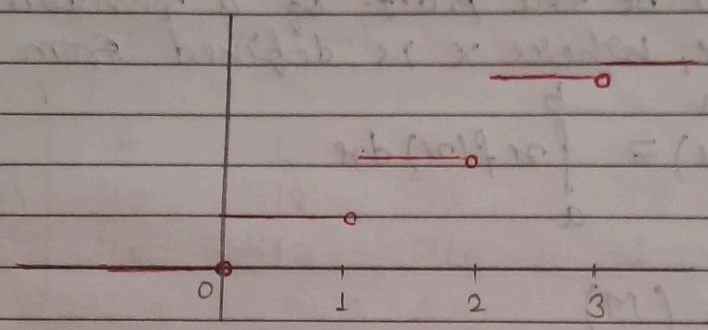
(ii) $F_X(x) = \begin{cases} 0 & -\infty < x < 0 \\ 1/6 & 0 \le x < 1 \\ 2/3 & 1 \le x < 2 \\ 20/30 & 2 \le x < 3 \\ 1 & x \geq 3 \end{cases}$



(iii) $P(x \le 1) = P(0) + P(1)$

$= \frac{1}{6} + \frac{1}{2}$

$= \frac{2}{3}$

(iv) $P(0 < x \le 2) = F(2) - F(0)$

$= \frac{29}{30} - \frac{1}{6}$

$= \frac{24}{30}$

#

X	x_1	x_2	.....	x_n
$P(X)$	P_1	P_2	.....	P_n

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$$\bar{X} = E(X) = \sum_{i=1}^{n} P_i x_i$$

$$Var(X) = \sum P_i x_i^2 - (\sum P_i x_i)^2$$

$$= \sum (X^2) - [E(X)]^2$$

* Let $f(x)$ be the p.d.f. of a random variable X, where X is defined from a to b. then

(i) Mean $(\mu) = \int_a^b x f(x)\, dx$

(ii) Median (M)

$$\int_a^M f(x)\, dx = \int_M^b f(x)\, dx = \frac{1}{2}$$

(iii) Mode (The value of x when $f(x)$ atains maximum ~~variable~~ value)

$f(x)$

$f'(x)$

$f''(x)$

for Extreme points $f'(x) = 0$

$x = a$

and $f''(a) < 0$

$\Rightarrow x = a$ is the mode of X.

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Ques Three coins are tossed simultaneously. If x is the no. tails then find the probability distribution of X.

Sol^n. $X = \{0, 1, 2, 3\}$

$A = \{HHH\}$

$B = \{HHT, HTH, THH\}$

$C = \{HTT, THT, TTH\}$

$D = \{TTT\}$

X	0	1	2	3
$P(X)$	$1/8$	$3/8$	$3/8$	$1/8$
C.F.	$1/8$	$4/8$	$7/8$	1

$p_i \geq 0$, $\sum_i p_i = 1$

$P(x \leq 2) = P(0) + P(1) + P(2)$

$= 7/8$

$P(x = 1) = 3/8$

→ Probability mass function or Probability distribution function

Commulative Distribution function or Distribution function

Let X be a random variable associated with a random ~~variable~~ experiment. Distribution function of X is denoted by $F_X(x)$ and defined as.

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$F_X(x) = P(X \leq x)$

Domain $= (-\infty, \infty)$

i.e $-\infty < x < \infty$

* $F_X(x) \in [0, 1]$ i.e $0 \leq F_X(x) \leq 1$

* $\lim_{x \to -\infty} F_X(x) = 0$

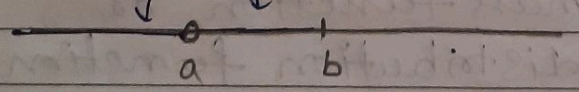
* $\lim_{x \to \infty} F_X(x) = 1$

* If $F_X(x)$ is the distribution function of the random variable x and if $a < b$ then

$$P(a < x \leq b) = F(b) - F(a)$$

Proof $P(a < x \leq b)$

$$P(a < x \leq b) + P(x \leq a) = P(x \leq b)$$



$$P(a < x \leq b) = P(x \leq b) - P(x \leq a)$$
$$= F(b) - F(a)$$

* $P(a \leq x \leq b) = P(x = a) + F(b) - F(a)$

* $P(a < x < b) = F(b) - F(a) - P(x = b)$

* If x is a discrete ~~function~~ random variable

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than graph of $F(x)$ is a step function.

Ques From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. If the sample is drawn without replacement and the random variable X denotes the number of defective items found find

(i) the probability distribution of X

(ii) Distribution function of X

(iii) $P(x \leq 1)$

(iv) $P(0 < x \leq 2)$

Soln X = No of defective items

$X = \{0, 1, 2, 3\}$

$$P(0) = \frac{{}^7C_4}{{}^{10}C_4} = \frac{1}{6}$$

$$P(x=1) = \frac{{}^3C_1 \cdot {}^7C_3}{{}^{10}C_4} = 1/2$$

$$P(X=2) = \frac{{}^3C_2 \cdot {}^7C_2}{{}^{10}C_4} = 3/10$$

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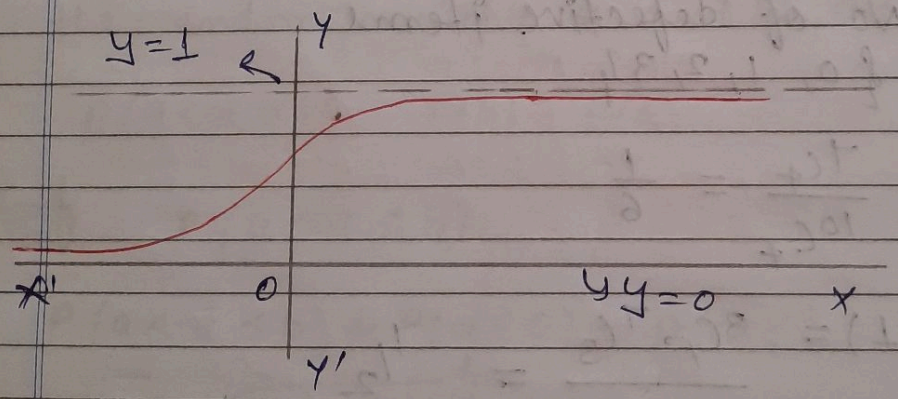
Distribution function or Commulative distribution function of a continuous random variable :-

Let X be the continuous random variable. A function $F_X(x)$ or $F(x)$ is called distribution function of X if

$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f(u)\,du$$

* $0 \le F_X(x) \le 1$

* if $x_1 > x_2$ then $F(x_1) > F(x_2)$



$$F_X(x) = \int_{0}^{x} f(u)\,du$$

$$\boxed{\frac{d}{dx} F(x) = f(x)}$$

Ques Verify that the following function is a distribution function

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$$F(x) = \begin{cases} 0 & \text{if } x < -a \\ \frac{1}{2}\left(\frac{x}{a}+1\right) & \text{if } -a \le x \le a \\ 1 & \text{if } x > a \end{cases}$$

Soln:- $$\frac{d}{dx} F(x) = \begin{cases} 0 & \text{if } x < -a \\ \frac{1}{2a} & \text{if } -a \le x < a \\ 0 & \text{if } x > a \end{cases}$$

$$f(x) = \begin{cases} \frac{1}{2a} & \text{if } -a \le x \le a \\ 0 & \text{elsewhere} \end{cases}$$

$a > 0 \quad \frac{1}{2a} > 0 \Rightarrow f(x) \geq 0 \ \forall x$

$$\int_{-a}^{a} f(x) = \int_{-a}^{a} \frac{1}{2a}\, dx = \frac{1}{2a} \cdot 2a = 1$$

Ques find the distribution function of a random variable X who's probability density function is given by →

$$F(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} \quad (\lambda > 0)$$

Ans Let $F(x)$ is the distribution function of X.

$$F(X) = P(X \le x)$$
$$= \int_{-\infty}^{x} f(u)\, du$$

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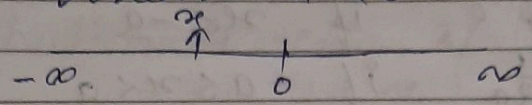
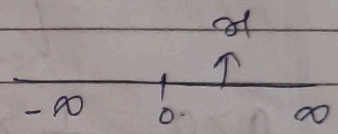
$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

Case I. When $x < 0$



$$F(x) = \int_{-\infty}^{x} f(u)\,du = 0$$

Case II When $x \geq 0$



$$F(x) = \int_{-\infty}^{x} f(u)\,du$$

$$= \int_{-\infty}^{0} f(u)\,du + \int_{0}^{x} f(u)\,du$$

$$= 0 + \int_{0}^{x} \lambda e^{-\lambda u}\,du$$

$$= \left(-e^{-\lambda u}\right)_0^x$$

$$= 1 - e^{-\lambda x}$$

$$\rightarrow F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-\lambda x} & \text{if } x \geq 0 \end{cases}$$

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To find Mean, Median, Mode, Varience etc of a continuous distribution

Let X be the continuous random variable define on interval $[a, b]$ and $F(x)$ is the probability density function of X.

(i) Mean $(\mu) = \int_a^b x f(x)\, dx$

(2) Median $(M) = \int_0^M f(x)\, dx = \int_M^b f(x)\, dx = \frac{1}{2}$

(3) Mode $= F(x) \quad x \in [a, b]$

$F'(x) = 0 \Rightarrow x = x_1 \in [a, b)$

$F''(x_1) < 0$

$\Rightarrow$ at $x = x_1$ $f(x)$ attains max value

$\Rightarrow$ $x = x_1$ is mode of the distribution

(4) Mean deviation from a part A

$$= \int_a^b |X - A|\, f(x)\, dx$$

Mean deviation from mean (μ)

$$= \int_a^b |x - \mu|\, f(x)\, dx$$

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(5) Variance $\sigma^2 = \int_a^b (x-\mu)^2 f(x)\,dx$

(6) Moments

r^{th} moment about point A

$$\Rightarrow \int_a^b (x-A)^r f(x)\,dx = \mu_r'$$

Imp → r^{th} moment about origin (μ_r')

$$= \int_a^b x^r f(x)\,dx$$

r^{th} moment about mean $= (\mu_r) =$

$$\int_a^b (x-\mu)^r f(x)\,dx$$

(7) relation between μ_r' and μ_r

$$\mu_r = \mu_r' - {}^rC_1 \mu_{r-1}' \mu_1' + {}^rC_2 \mu_{r-2}' \mu_1'^2 + \cdots$$

$$\cdots + (-1)^r \mu_1'^r$$

$\boxed{\mu_1 = 0}$ $\boxed{\mu_2 = \mu_2' - (\mu_1')^2}$

↓

varience

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$\mu_1' = \text{Mean}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

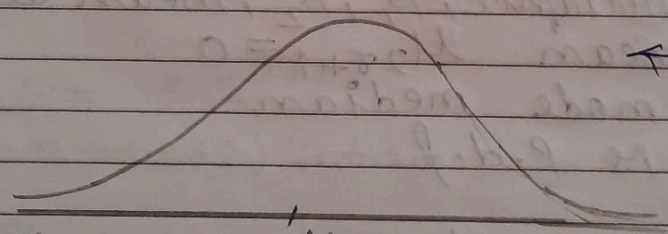
$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

(B) $\beta_1 = \dfrac{\mu_3^2}{\mu_2^3}$, $\beta_2 = \dfrac{\mu_4}{\mu_2^2}$

(Symmetricity) peak

Skewness and kurtosis

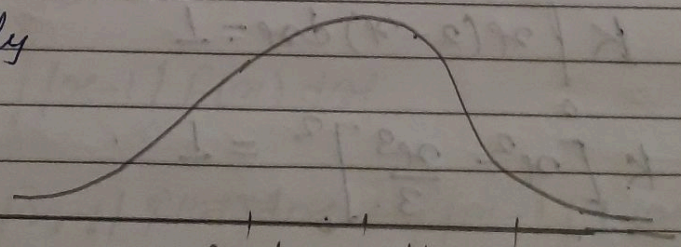
Skewness - lack of symmetry is called Skewness



$\leftarrow \boxed{\beta_1 = 0}$

mean = median = mode $\leftarrow \boxed{\beta_1 = 0}$

Positively Skewed Curve



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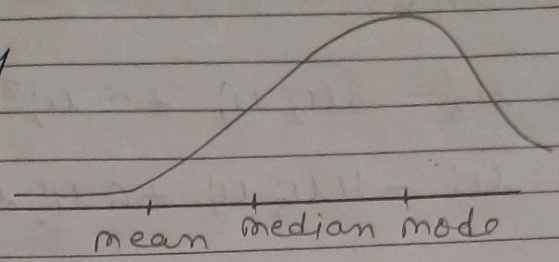
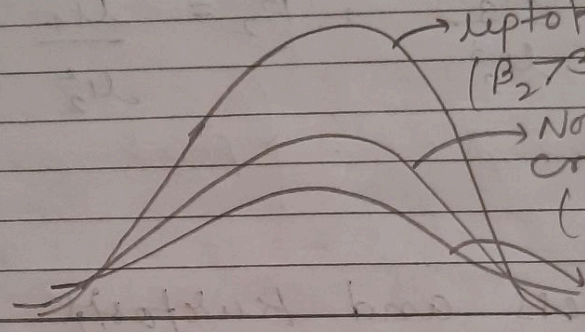
Negatively Skewed curve



$\boxed{\beta_1 = 0}$



→ Lepto kurtic ($\gamma_2 > 0$, $\beta_2 > 3$)

→ Normal curve Meso kurtic ($\beta_2 = 3$ or $\gamma_2 = 0$)

→ Platy kurtic $\boxed{\beta_2 < 3}$

Ques $f(x) = kx(-x+2) \quad 0 \le x \le 2$

find k ?

mean, variance, β_1, β_2, mean deviation about mean $\mu_{2r+1} = 0$

find mode median

Ans $f(x)$ is p.d.f.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$k\int_0^2 x(2-x)\,dx = 1$$

$$k\left[x^2 - \frac{x^3}{3}\right]_0^2 = 1$$

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$$K\left[1-\frac{8}{3}\right]=1$$

$$\boxed{K=\frac{3}{4}}$$

$$f(x)=\frac{3}{4}x(2-x) \qquad x\in[0,2]$$

$$\mu_r'=\int_{-\infty}^{\infty}x^r f(x)\,dx$$

$$=\int_0^2 x^r\,\frac{3}{4}x(2-x)\,dx$$

$$=\frac{3}{4}\int_0^2\left(2x^{r+1}-x^{r+2}\right)dx$$

$$=\frac{3}{4}\left[2\frac{x^{r+2}}{r+2}-\frac{x^{r+3}}{r+3}\right]_0^2$$

$$\mu_r'=\frac{3\cdot 2^{r+1}}{(r+2)(r+3)}$$

if $r=1$, $\mu_1'=1$ [1 Page before Mean.... is after 2 Page]

→ Mean deviation about mean

$$=\int_0^2|x-1|\,f(x)\,dx$$

$$=\int_0^1|x-1|\,f(x)\,dx+\int_1^2|x-1|\,f(x)\,dx$$

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$$= \int_0^1 (1-x) f(x)\,dx + \int_1^2 (x-1) f(x)\,dx$$

$$= \int_0^1 (1-x) \frac{3}{4} x(2-x)\,dx + \int_1^2 (x-1) \frac{3}{4} x(2-x)\,dx$$

$$= \frac{3}{4} \int_0^1 (x^3 - 3x^2 + 2x)\,dx - \frac{3}{4} \int_1^2 (x^3 - 3x^2 + 2x)\,dx$$

$$= \frac{3}{4} \left[\frac{x^4}{4} - x^3 + x^2\right]_0^1 - \frac{3}{4} \left[\frac{x^4}{4} - x^3 + x^2\right]_1^2$$

$$= \frac{3}{4} \left[\left(\frac{1}{4} - 1 + 1\right) - (0)\right] - \frac{3}{4} \left[\left(\frac{2^4}{4}\right) - 8 + 4 - \left(\frac{1}{4} - 1 + 1\right)\right]$$

$$= \frac{3}{4} \left[\frac{1}{4}\right] - \frac{3}{4} \left[(4 - 8 + 4) - \frac{1}{4}\right]$$

$$= \frac{3}{16} + \frac{3}{16} = \frac{3}{8}$$

now, mean deviation about mean

$$\mu_{2r+1} = 0$$

LHS $\int_0^2 (x-1)^{2r+1} f(x)\,dx$

$$= \int_0^2 (x-1)^{2r+1} \frac{3}{4} x(2-x)\,dx$$

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$$= \frac{3}{4}\int_0^2 x(2-x)(x-1)^{2r+1}\,dx$$

$$x-1=t$$
$$dx=dt$$

$$= \frac{3}{4}\int_{-1}^{1}(t+1)(1-t)(t)^{2r+1}\,dt$$

$$= \frac{3}{4}\int_{-1}^{1}(1-t^2)t^{2r+1}\,dt$$

$$= \frac{3}{4}\int_{-1}^{1}\left(t^{2r+1}-t^{2r+3}\right)dt$$

$$= \frac{3}{4}\left[\frac{t^{2r+2}}{2r+2}-\frac{t^{2r+4}}{2r+4}\right]_{-1}^{1}$$

$$= \frac{3}{4}\left[\left(\frac{1}{2r+2}-\frac{1}{2r+4}\right)-\left(\frac{1}{2r+2}-\frac{1}{2r+4}\right)\right]$$

$$= 0 = RHS$$

So $[\mu_{2r+1}=0]$

[this answer should write before 2 page.]

$\mu_2' = \frac{6}{5}$

$\mu_3' = \frac{8}{5}$ $\qquad$ $\mu_4' = \frac{16}{7}$

mean $= \mu_1' = 1$

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variance $\mu_2 = \mu_2' - \mu_1'^2$

$= 6/5 - 1 = 1/5$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$

$= 0$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= \frac{16}{7} - 4 \cdot \frac{8}{5} \cdot 1 + 6 \times \frac{6}{5} \times 1 - 3 \times 1$

$= \frac{16}{7} - \frac{11}{5} = \frac{3}{35}$

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0$

$\Rightarrow$ distribution is symmetric
so mean, median, mode are same

[mean = median = mode = 1]

$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{3/35}{1/25} = \frac{15}{7}$

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Mathamatical Expectation

let X be random variable the mean of X is called Expectation of X and it is generally denoted by $E(X)$.

$$E(X) = \begin{cases} \sum_{x} x_i P(x = x_i) \\ \int_{-\infty}^{\infty} x f(x) dx \end{cases}$$

* $E(C) = C$
* $E(aX + b) = aE(X) + b$
* $E(a_1 X_1 + a_2 X_2 + \dots + a_n X_n)$

$= a_1 E(X_1) + a_2 E(X_2) + \dots + a_n E(X_n)$

* If X and Y are two Indep ended random variables then Expectation of $E(XY) = E(X) \cdot E(Y)$

* In ~~Ge~~ General
$E(XY) \leq E(x) E(Y)$

Example. let x be a random variable whose probability mass function is given by

$$P(x = x_i) = \frac{e^{-1}}{x_i!}$$

$$x = 0, 1, 2, \dots \infty$$

find $E(x!)$

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Ans $P(X = x_y) = \frac{e^{-1}}{x!}$

$$\sum_{x=1}^{\infty} \frac{e^{-1}}{x!} = e^{-1}\left(\frac{1}{\lfloor 0} + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \cdots \infty\right)$$

$$= e^{-1} \cdot e = 1$$

$$E(x!) = \sum_{x=0}^{\infty} (x!)\, P(X = x)$$

$$= \sum_{x=0}^{\infty} (x!)\left(\frac{e^{-1}}{x!}\right)$$

$$= e^{-1} \sum_{x=0}^{\infty} 1 \longrightarrow \infty$$

$\Rightarrow E(x!)$ does not exist.

Let X be a random variable and $g(x)$ be any continuous function

$$E(g(x)) = \begin{cases} \sum_{x} g(x) \cdot F(x) & \text{for discrete random variable} \\ \int_{-\infty}^{\infty} g(x) F(x)\, dx & \text{for continuous random variable} \end{cases}$$

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Moment generating function

Let x be a random variable. moment generating function of x is denoted by $M_x(t)$ and defined as:-

$$M_x(t) = E(e^{tx})$$

$$= \begin{cases} \sum e^{tx} f(x) \\ \int_{-\infty}^{\infty} e^{tx} f(x)\,dx \end{cases}$$

$$M_x(t) = E(e^{tx})$$

$$= E\left\{1 + tx + \frac{t^2x^2}{\lfloor 2} + \frac{t^3x^3}{\lfloor 3} + \frac{t^4x^4}{\lfloor 4} + \cdots\right\}$$

$$= E(1) + t\,E(x) + \frac{t^2}{\lfloor 2}E(x^2) + \frac{t^3}{\lfloor 3}E(x^3) + \cdots$$

$$= 1 + t\mu_1' + \frac{t^2}{\lfloor 2}\mu_2' + \frac{t^3}{\lfloor 3}\mu_3' + \cdots$$

$$= \text{Coefs. of } \frac{t^r}{\lfloor r} = \mu_r'$$

$$* \quad \mu_r' = \left[\frac{d^r M_x(t)}{dt^r}\right]_{t=0}$$

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Some Special descrete probability distribution

(i) Barnaulli distribution

x	0	1
$P(x)$	q	p

where $p+q=1$

$$\mu_r' = E(x^r) = \sum_i x_i^r P(x=x_i)$$
$$= 0 + 1(p)$$

$$\boxed{\mu_r' = p}$$

$\mu_1' = p$, $\mu_1 = 0$
$\mu_2' = p$, $\mu_2 = \mu_2' - \mu_1'^2 = pq$
$\mu_3' = p$
$\mu_4' = p$

Moment Genrating function :-

$$M_x(t) = E(e^{tx})$$
$$= \sum_i e^{tx_i} P(x=x_i)$$
$$= e^{t \cdot 0} q + e^t \cdot p$$

$$\boxed{M_x(t) = q + pe^t}$$

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$$\mu_1' = \left[\frac{d}{dt} M_X(t)\right]_{t=0}$$

$$= \left[pe^t\right]_{t=0} = p$$

$$\mu_2' = \left[\frac{d^2}{dt^2} M_X(t)\right]$$

$$= \left[pe^t\right]_{t=0} = p$$

Similarly

$$\mu_3' = \mu_4' = p$$

(2) Binomial Distribution

A random variable X is said to follow Binomial distribution if its Probability mass function is given by:-

$$P(X=x) = {}^nC_x q^{n-x} p^x$$

where

$$x = 0, 1, 2, \ldots n$$

Q. find the mean and variance of Binomial distribution?

Ans $\mu_1' = E(X)$

$$= \sum_{x=x_i} x_i P(x = x_i)$$

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$$= \sum_{x_i=0}^{n} x_i \, {}^nC_{x_i} q^{n-x_i} p^{x_i}$$

$$= \sum_{x=0}^{n} x \, {}^nC_x q^{n-x} \cdot p^x$$

$$= \sum_{x=0}^{n} x \cdot \frac{n}{x} \cdot {}^{n-1}C_{x-1} q^{n-x} p^x$$

$$= np \sum_{x=0}^{n} {}^{n-1}C_{x-1} q^{n-x} p^{x-1}$$

$$= np(q+p)^{n-1}$$

$$= np \qquad \left\{ {}^nC_x = \frac{n}{x} \cdot {}^{n-1}C_{x-1} \right\}$$

$$\Rightarrow E(X^2) = \mu_2' = \sum_{x=0}^{n} x^2 \, {}^nC_x q^{n-x} p^x$$

$$= \sum_{x=0}^{n} \{x(x-1)+x\} \, {}^nC_x q^{n-x} p^x$$

$$= \sum_{x=0}^{n} x(x-1) \, {}^nC_x q^{n-x} \cdot p^x + np$$

$$= \sum_{x=0}^{n} n(n-1) \cdot {}^{n-2}C_{x-2} q^{n-x} p^x + np$$

$$= n(n-1)p^2 \sum_{x=0}^{n} {}^{n-2}C_{x-2} q^{n-x} p^{x-2} + np$$

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$$= n(n-1)p^2 (q+p)^{n-2} + np$$

$$= n(n-1)p^2 + np$$

(Variance) $\mu_2 = \mu_2' - \mu_1^2$

$$= n(n-1)p^2 + np - n^2p^2$$
$$= n^2p^2 - np^2 + np - n^2p^2$$
$$= np - np^2$$
$$= np(1-p)$$
$$= npq$$

$$S.D. = \sqrt{npq}$$

(Standard Deviation)

Ques find the moment Generating function of Binomial distribution also find moments about origin?

Ans $M_X(t) = E(e^{tx})$

$$= \sum_{x=0}^{n} e^{tx} P(X=x)$$

$$= \sum_{x=0}^{n} e^{tx} {}^nC_x q^{n-x} p^x$$

$$= \sum_{x=0}^{n} {}^nC_x q^{n-x} (pe^t)^x$$

$$= (q + pe^t)^n$$

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$$\Rightarrow M_x(t) = (q+pe^t)^n$$

$$\frac{d}{dt} M_x(t) = n(q+pe^t)^{n-1} pe^t$$

$$\frac{d}{dt} M_x(t) = npe^t (q+pe^t)^{n-1}$$

$$\mu_1' = \left[\frac{d}{dt} M_x(t)\right]_{t=0} = np = \text{mean}$$

$$\frac{d^2}{dt^2} M_x(t) = np\left[e^t(q+pe^t)^{n-1} + e^t(n-1)(q+pe^t)^{n-2} pe^t\right]$$

$$\mu_2' = \left[\frac{d^2}{dt^2} M_x(t)\right]_{t=0} = np\left[1+(n-1)p\right]$$

$$= np\left[np+q\right]$$

$$\{\because p+q=1\}$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= np\left[np+q\right] - n^2p^2$$

$$= n^2p^2 + npq - n^2p^2$$

$$= npq$$

$$\boxed{\text{variance} = npq}$$

Standard deviation $= \sqrt{npq}$

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Ques A coin is tossed 10 times. getting a head in a trial is consider a success find the probability of getting

(1) 7 success
(2) no success
(3) At least 1 success
(4) At most four success

Soln $n = 10$, $p = 1/2$, $q = 1/2$

$$P(x) = {}^{n}C_{x}\, q^{n-x}\, p^{x} \qquad \text{where } x = 1, 2 .. 10$$

$$= {}^{10}C_{x}\, (1/2)^{10}$$

(i) $P(x = 7)$

$$= {}^{10}C_{7} (1/2)^{10} = {}^{10}C_{3} (1/2)^{10} =$$

(ii) $P(x=0) = {}^{10}C_{0} (1/2)^{10} = (1/2)^{10} = \frac{1}{1024}$

(iii) $P(x \geq 1) = 1 - P(0)$

$$= 1 - \frac{1}{1024} = \frac{1023}{1024}$$

(iv) $P(x \leq 4) = P(0) + P(1) + P(2) + P(3) + P(4)$

$$= \left(\frac{1}{2}\right)^{10} \left[1 + {}^{10}C_{1} + {}^{10}C_{2} + {}^{10}C_{3} + {}^{10}C_{4} \right]$$

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Ques The mean and variance of a Binomial distribution are 4 and $4/3$ then find $P(x \geq 1)$?

Ans. $B(n, p)$

mean $= np = 4$ — (i)

variance $= npq = 4/3$ — (ii)

$$\frac{4}{8}(1-p) = 4/3$$

$$1-p = 1/3$$

$$1 - 1/3 = p$$

$$p = 2/3$$

$$n = \frac{4 \times 3}{2} = 6$$

$$n = 6$$

$$P(x \geq 1) = 1 - P(0)$$

$$= 1 - q^n$$

$$= 1 - (1/3)^6$$

Ques fit the Binomial distribution from the following data

x	0	1	2	3	4	5
$f(x)$	2	8	22	18	7	3

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Ans. $\Sigma f_i x_i$

$= 0 + 8 + 44 + 54 + 28 + 15$

$= 149$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{149}{60}$$

$(N = \Sigma f_i = 60)$

$\rightarrow np = \frac{149}{60}$

$n = 5$

$5 \times p = \frac{149}{60}$

$p = \frac{149}{60 \times 5} = \frac{149}{300} = 0.49$

$P(X = x) = {}^5C_x (0.503)^{5-x} (0.497)^x$

X	0	1	2	3	4	5
f(X)	$N \times P(0)$	$N \times P(1)$	$N \times P(2)$	...	...	

(3) Poission distribution

Poission distribution is the limiting case of Binomial distribution.

$p \rightarrow 0$

$n \rightarrow \infty$

mean = constant

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Ans A random X is said to follow poision distribution if its probability mass function is given by:-

$$P(X=x) = \frac{e^{-m} \cdot m^{x}}{\lfloor x}$$

where $x = 1, 2, 3 \ldots$
$m =$ mean

Que Prove that the poission distribution is the limiting case of Binomial distribution.

Ans $p \to 0$, $n \to \infty$
$np = m$ (constant)

Binomial

$$P(X=x) = {}^{n}C_{x} (1-p)^{n-x} p^{x}$$

$$\boxed{\text{mean} = np = m}$$

$$= \frac{\lfloor n}{\lfloor x \lfloor n-x} (1-p)^{n-x} p^{x}$$

$$= \frac{n(n-1)(n-2) \ldots \{n-(x-1)\}}{\lfloor x} (1-p)^{n-x} p^{x}$$

$$= \frac{(np)(np-p)(np-2p) \ldots (np-(x-1)p) \times (1-p)^{n-x}}{x!}$$

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$$= \frac{m(m-p)(m-2p)\ldots\ldots(m-(x-1)p)}{x!} \cdot \frac{\left(1-\frac{m}{n}\right)^n}{(1-p)^x}$$

$$\rightarrow \lim_{x\to a} \{1+f(x)\}^{g(x)} \quad (1^\infty)$$

$$= e^{\lim_{x\to a} f(x)\,g(x)}$$

$$\lim_{n\to\infty}\left(1-\frac{m}{n}\right)^n = e^{\lim_{n\to\infty}\left(\frac{-m}{n}\right)^n} = e^{-m}$$

on applying limit $p\to 0$ and $n\to\infty$

$$= \frac{m(m-0)(m-0)(\ldots\ldots(m-0)}{\lfloor x} \frac{e^{-m}}{1^x}$$

$$= \frac{m^x e^{-m}}{\lfloor x}, \quad x = 0, 1, 2, 3 \ldots\ldots$$

Que Suppose on an average one house in 1000 in a certain district has a fire during a year. If there are two thousand houses in that district what is the probability there Exactly 5 houses will have a fire during the year? $(e^{-2} = 0.1353)$

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Ans $p = \frac{1}{1000}$ $n = 2000$

$np = 2 = m$

$$P(X=x) = \frac{e^{-m} m^x}{\lfloor x}$$

$$P(X=5) = \frac{e^{-2}(2)^5}{\lfloor 5}$$

$$= \frac{0.1353 \times 32}{120}$$

$$= 0.036$$

Ques A car hire firm has two cars which it hires out day by day the number of demands for a car on each day is distributed as a poission variafent with mean 1.5. Calculate the preposen of days on which

(i) neither car is used

(ii) Some demand is refused.

$(e^{-1.5} = 0.231)$

Ans $m = 1.5$

$$P(X=x) = \frac{e^{-1.5}(1.5)^x}{\lfloor x}$$

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(i) no car is used

$$P(X=0) = \frac{e^{-1.5}(1.5)^0}{\lfloor 0}$$

$$= e^{-1.5} \Rightarrow 0.231$$

(2) $P(x>2) = 1 - \{P(0) + P(1) + P(2)\}$

$$= 1 - \left\{ e^{-m} + \frac{e^{-m} m}{\lfloor 1} + \frac{e^{-m}(m)^2}{\lfloor 2} \right)$$

$$= 1 - e^{-1.5}\left(1 + \frac{1.5}{1} + \frac{(1.5)^2}{\lfloor 2}\right)$$

$$= 1 - 0.231\left(1 + 1.5 + \frac{1.25}{2}\right)$$

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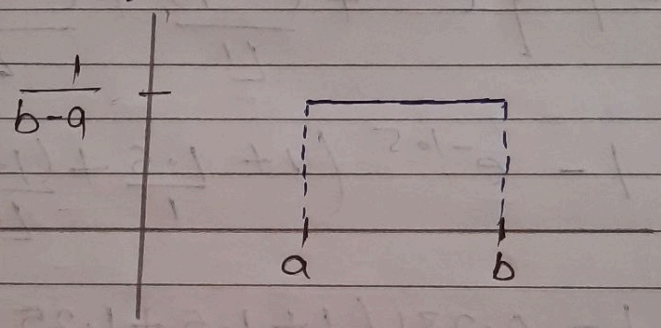
Some Continuous probability distribution

(i) Rectangular or uniform distribution

A random variable X is said to follow Rectangular distribution if its probability density function is given by :-

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{when } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$



Que find r^{th} moment about origin mean and variance of the Continuous uniform distribution?

Ans

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{when } a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_r' = E(x^r) = \int_{-\infty}^{\infty} x^r f(x)\,dx$$

$$= \int_a^b x^r \left(\frac{1}{b-a}\right) dx$$

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$$= \frac{1}{b-a}\left[\frac{x^{r+1}}{r+1}\right]_a^b$$

$$= \frac{1}{b-a}\left[\frac{b^{r+1}-a^{r+1}}{r+1}\right]$$

$$\boxed{\text{Mean} = \mu_1' = \frac{b^2-a^2}{(b-a)2} = \frac{a+b}{2}}$$

$$\mu_2' = \frac{b^3-a^3}{(b-a)3} = \frac{a^2+ab+b^2}{3}$$

$$\text{Variance} = \mu_2 = \mu_2' - (\mu_1')^2$$

$$= \frac{a^2+ab+b^2}{3} - \frac{(a+b)^2}{4}$$

$$\boxed{\text{Variance} = \frac{(b-a)^2}{12}}$$