Burn-and-Mint Tokenomics: Deflation and Strategic Incentives

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***Abstract*—We provide a game-theoretic analysis of deflationary burn-and-mint tokenomics, studying individual incentives, theoretically and through numerical simulations, to identify the circumstances under which economic collapse can be avoided. We identify a necessary deflation threshold to guarantee preservation of value and propose to fix rewards in fiat as a remedy to the asymptotic underincentivization of network contributors occurring in deflationary burn-and-mint.**

***Index Terms*—cryptocurrency, tokenomics, IoT**

I. Introduction

Digital money is important for enabling the Internet of Things (IoT). IoT network costs are relatively high in both capital and operating expenditures [1], so that building and operating IoT networks by one single, centralized party is often cost-prohibitive. For this reason, community-owned and community-operated networks have recently arisen as a potential solution. Currently, the most successful example of such a network has been that of Helium [2], an IoT network of individually-owned and -operated wireless hotspots connected to a blockchain infrastructure that incentivizes users to foster network growth and maintenance. The general term for a network of physical devices, joined together with the use of distributed ledger technology (DLT), is a decentralized physical infrastructure network (DePIN). DePIN projects typically use tokens to reward users and incentivize cooperation between participants who own and operate physical devices, enabling the permissionless immutability of tokens through the use of DLT. This immutability strengthens trust between participants as it empowers token holders to trust in the token as a store of value; delegating the accounting and governance to the DLT infrastructure, owners and operators of physical devices are empowered to nurture the health of the network.

To align network value with token value, several DePIN projects have followed Helium's lead and implemented burn-and-mint tokenomics, which were pioneered by Factom [3]. In burn-and-mint, the burning of tokens transfers network value to the token by decreasing token supply as demand accumulates, while minting transfers value away from the token by increasing token supply so as to incentivize contributors. The feature added by Helium is the introduction of *deflation*, in which the mint amount issued to service providers decreases with time, as opposed to remaining constant.

The purpose of this work is to provide a rigorous analysis of the deflationary burn-and-mint model. We provide results regarding the value of burn-and-mint economies and use game theory to point out fundamental risks associated with burn-and-mint models that affect both original as well as deflationary variants. Our analysis indicates that deflation needs to exceed a critical threshold of sufficient deflation, above which the burn-and-mint economy can sustain value corresponding to the net present value of present and future cash flows while, under insufficient deflation, the demand for the token collapses. This clearly highlights the benefit of deflation, but comes with its own problems as deflation leads to lower rewards over time. As a result, the rewards, when measured in the unit of account, become smaller, which implies that the supply eventually becomes underincentivized leading to network collapse. Our analysis identifies a remedy to this dilemma which is to reward participants by a constant amount, fixed in the unit of account, *i.e.*, fiat.

Our analysis is useful to improve understanding of incentive issues surrounding burn-and-mint tokenomics. This contribution is timely as new DePIN projects continue to emerge based on variants of existing proposals such as Helium's, without providing a rigorous sustainability analysis of their tokenomics. By providing such an analysis, we complement the analysis of [4] and [5], whose focus is on the original form of burn-and-mint mechanisms and their impact on the velocity of money. Other related work includes [6], which presents a control-theoretic method for achieving burn-and-mint equilibrium under certain modeling assumptions, and our own preliminary work [7], in which we present the underlying tokenomics of the onocoy network. The problem of underincentivization in a deflationary burn-and-mint economy has also been considered in the Helium Improvement Proposal which introduced deflationary burn-and-mint [8].

The rest of the paper is organized as follows. In Section II, we introduce the dynamics of deflationary burn-and-mint. In Section III, we use a game-theoretic framework to derive the value of a deflationary burn-and-mint economy. In Section IV, we introduce the concept of fiat-fixed rewards. Section V presents numerical simulations. Section VI is the conclusion.

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II. Deflationary Burn-and-Mint

The burn-and-mint tokenomic model, summarized in [9] and pioneered by Factom, is a model consisting of a work token with a floating exchange value that is "minted" *ex nihilo* to pay for services provided to the economy and "burned" *in nihilum* to pay for services provided by the economy. When minted, tokens are earned by contributors; when burned, tokens are spent by consumers.

A contributor earns tokens in exchange for providing a service; a consumer spends tokens in exchange for receiving a service. Between contributor and consumer is the market, which serves the function of determining a fair value for the services provided, priced in terms of tokens. Functionally, this is done by determining an exchange rate between the token and market participants' unit of account. The units of account of all participants in a burn-and-mint economy are assumed to be something other than the token, *e.g.*, a fiat currency, a stablecoin, or even a dominant cryptocurrency like Bitcoin.

As services are typically priced in the unit of account, determining a fair rate of exchange allows the discovery of fair market value for services being provided and enables services to be priced in tokens. In ordinary economic analysis, the unit of account also serves as the internal medium of exchange in the economy with the economy keeping a credit balance in that unit. The token in the burn-and-mint model is purpose-built [10], however, designed to be used only for the functions relating to the services provided to or provided by the economy. Establishing that the token has any value and determining a fair value for the token, even if under idealized conditions, is therefore important to gaining an understanding of how such an economy functions.

We now define primitives from the perspective of individual agents. At time t, each agent i holds an amount of tokens $M^i(t)$ where the marginal change in the amount of tokens held is given by,

$$\dot{M}^i(t) = -u^i(t) + w^i(t),$$

where u^i is the amount of tokens burned and $w^i(t)$ is the amount of tokens minted. The mint is generated according to a reward mechanism which will be defined later. The burn is determined according to individual choice: upon burn, an agent i receives $y^i(t)$, a part of revenue generated by the network $y(t)$ proportional to the amount of tokens individually burned relative to the total amount burned,

$$y^i(t) = \frac{u^i(t)}{u^i(t) + u^{-i}(t)} y(t).$$

In this way, the individual decision on token burn underpins a proportional revenue-sharing mechanism between all agents, anticipating trade-offs between possible payoff strategies, expectations of future revenue generated, and expectations of other agents' payoff strategies solving an individual optimization problem in the strategy space. As this work serves as a first step in performing a strategic analysis of a burn-and-mint economy, we restrict ourselves to the assumption that agents openly share information regarding payoffs, that future cash flows are fully known, and that agents follow their optimal strategy. From an agent's perspective, the extractable value of the burn-and-mint economy is given by the amount of revenue that can be captured over the course of the entire game, discounted by a factor γ_i. The objective function for each agent i is given by,

$$J^i(t; u^0, \ldots, u^{N-1}; y) = \int_t^\infty \frac{u^i(\tau)}{u^i(\tau) + u^{-i}(\tau)} \gamma_i^{\tau - t} y(\tau) d\tau.$$

III. Strategic Analysis

Each agent maximizes the objective function by varying the token burn u. To avoid complicating the analysis, we assume that revenue is monotonically non-decreasing, *i.e.*, y is monotonically non-decreasing. Note that for a variable with superscript i, denoting a variable corresponding to agent i, we denote with superscript $-i$ the sum of variables corresponding to all agents not i.

A. *No mint $w = 0$*

We begin by assuming that the minting of tokens is zero, *i.e.*, $w^i = 0$, so that no new tokens are introduced into the game at any time. We let $\bar{M}(t)$ be the amount of tokens available at time t which, in this case, is equal to $M(t)$.

The optimization to be solved by each agent is therefore,

$$\max_{u^i} \; J^i(t; u^0, \ldots, u^{N-1}; y), \tag{1a}$$

$$\text{sub. to} \int_t^\infty u^i(\tau) d\tau \leq \bar{M}^i(t). \tag{1b}$$

In dual form, the optimization becomes,

$$\max_{u^i} \mu^{-i}(t)\bar{M}^i(t) + \int_t^\infty \left[\frac{u^i(\tau)}{u^i(\tau) + u^{-i}(\tau)} \gamma_i^{\tau-t} y(\tau) - \mu^{-i}(t) u^i(\tau) \right] d\tau, \tag{2}$$

where $\mu^i(t) \geq 0$. The optimal allocation is given by the result below.

Proposition 1. *Consider the N-agent game* (1) *with $N \geq 2$. The optimal allocation for each agent i is given by,*

$$u^i(\tau) = \frac{\mu^i(\tau)}{\mu(\tau)} \frac{y(\tau)}{\mu(\tau)}, \tag{3}$$

for all $\tau \geq t$, where,

$$\begin{aligned} \mu^i(\tau) &:= \mu(\tau) - \mu^{-i}(\tau), \\ \mu(\tau) &:= \frac{1}{N-1} \sum_j \mu^{-j}(\tau), \end{aligned} \tag{4}$$

$$\mu^{-i}(\tau) = \gamma_i^{-(\tau - t)} \mu^{-i}(t), \tag{5}$$

with initial conditions obtained by solving,

$$\bar{M}^{-i}(t) = \int_t^\infty \frac{(N-1)^2 \mu^{-i}(t)}{\left(\sum_j \mu^{-j}(t)(\gamma_i/\gamma_j)^{\tau - t}\right)^2} \gamma_i^{\tau - t} y(\tau) d\tau. \tag{6}$$

Proof. According to the Pontryagin Maximum Principle [11],

$$u^i(\tau) = \arg\max_{u^i} \frac{u^i}{u^i + u^{-i}(\tau)} \gamma_i^{\tau - t} y(\tau) - \mu^{-i}(t) u^i,$$

for any t, τ satisfying $t \leq \tau$, the solution to which is one of three possibilities,

$$\begin{cases} \frac{u^{-i}(\tau)}{u(\tau)} \frac{\gamma_i^{\tau-t} y(\tau)}{u(\tau)} = \mu^{-i}(t) & \text{if } y(\tau) > 0, \\ u^i(\tau) = 0 & \text{if } y(\tau) = 0,\ \mu^i(t) > 0, \\ u^i(\tau) \text{ is undefined} & \text{if } y(\tau) = 0,\ \mu^i(t) = 0. \end{cases} \tag{7}$$

To show that $\mu^{-i}(t) \neq 0$, we assume the opposite, *i.e.*, that the constraint is not active. In this case, the maximum does not exist because the objective function becomes unbounded as there is no bound on each agent's choice u^i, contradicting the assumption that the constraint is inactive. Therefore, only the first two cases of (7) are possible.

Consider the case where $y(\tau) \neq 0$ and let $\tau = t$. Summing (7) over all i: $(N-1)\frac{y(t)}{u(t)} = \sum_i \mu^{-i}(t)$. Using the definition (4) for $\mu(t)$,

$$u(t) = \frac{y(t)}{\mu(t)}. \tag{8}$$

Solving for $u^{-i}(t)$ and $u^i(t)$,

$$\begin{aligned} u^{-i}(t) &= \mu^{-i}(t) u(t) \frac{u(t)}{y(t)} = \frac{\mu^{-i}(t)}{\mu(t)} \frac{y(t)}{\mu(t)}, \\ u^i(t) &= u(t) - u^{-i}(t) = \frac{\mu^i(t)}{\mu(t)} \frac{y(t)}{\mu(t)}, \end{aligned} \tag{9}$$

which agrees with (3)-(4) for $\tau = t$. Note that, because $\mu(t) \neq 0$, this equation still holds whenever $y(t) = 0$. According to the principle of optimality, the equation also holds for any $\tau > t$.

To find the time evolution of $\mu^{-i}(\tau)$ consider that (9) is true for any $\tau \geq t$ and therefore,

$$\begin{aligned} \mu^{-i}(t) = \frac{u^{-i}(\tau)}{u(\tau)} \frac{\gamma_i^{\tau-t} y(\tau)}{u(\tau)} &= \frac{\mu^{-i}(\tau)}{\mu(\tau)} \gamma_i^{\tau-t} \mu(\tau) \\ &= \gamma_i^{\tau-t} \mu^{-i}(\tau). \end{aligned}$$

Integrating (9) over $[t, \infty)$, using (4)-(5), we obtain (6). $\square$

The proposition provides the full solution to the game in the form of a system of N equations (6) and N unknowns $\mu^{-i}(t)$.

The terms $\mu^i(t)$ may be interpreted as individual contributions of agent i to the price $\mu(t)$. The fact that $\mu(t)$ is the token price is justified by (8) which implies that $\mu(t)$ is the ratio $\frac{y(t)}{u(t)}$ whenever $u(t) \neq 0$.

Proposition 2 (Price interpretation). *Let $\bar{V}(t) = \mu(t)\bar{M}(t)$ be the value of all tokens $\bar{M}(t)$. The evolution of the value function is then given by,*

$$-\dot{\bar{V}}(t) = -\frac{\dot{\mu}(t)}{\mu(t)} \bar{V}(t) + y(t), \tag{10}$$

which has the solution,

$$\bar{V}(t) = \mu(t) \int_t^\infty \frac{y(\tau)}{\mu(\tau)} d\tau. \tag{11}$$

The price interpretation is further strengthened when we make an assumption that the discount factor γ_i is the same for all agents. Doing so implies that $\bar{V}(t)$ is the net present value of all tokens $\bar{M}(t)$.

Corollary 1 (Net present value). *Let $\gamma_i = \gamma$ for all agents i. Then $\bar{V}(t)$ is the net present value of the cash flow y with discount factor γ.*

Proof. According to (4)-(5), $\mu(\tau) = \gamma^{-(\tau-t)}\mu(t)$. Then, according to (11), $\bar{V}(t) = \int_t^\infty \gamma^{\tau-t} y(\tau) d\tau$. $\square$

B. Non-zero mint $w \neq 0$

The results obtained thus far give a general result on the optimal burn rate of each agent but do not take into account the mint. In reality, agents do not begin with a large amount of tokens without the expectation of earning more because tokens are disbursed periodically in the form of a mint. In this process, minted tokens $w^i(t)$ are received according to some schedule,

$$w^i(\tau) = \gamma_w^\tau w_0^i,$$

where $0 < \gamma_w < 1$ is a deflation factor, so that at any time t, an agent expects to obtain $\int_t^\infty w^i(\tau) d\tau$ tokens over the remainder of the game. We redefine $\bar{M}(t)$ as tokens-to-go, *i.e.*, the amount of tokens available for burn over all present and future time, satisfying,

$$\bar{M}^i(t) = M^i(t) + \int_t^\infty w^i(\tau) d\tau = M^i(t) + \frac{\gamma_w^t w_0^i}{-\ln \gamma_w}. \tag{12}$$

If we assume that each agent optimizes the objective in (1), the solution is then the same as before with a slight difference in meaning given the redefinition of $\bar{M}^i(t)$.

Proposition 3. *Let $V(t) = \mu(t)M(t)$ be the value of all tokens $M(t)$. The evolution of the value function is then given by,*

$$-\dot{V}(t) = -\frac{\dot{\mu}(t)}{\mu(t)} V(t) - \mu(t) w(t) + y(t). \tag{13}$$

Proof. Since $\dot{M}(t) = -u(t) + w(t)$, $-\dot{V}(t) = -\dot{\mu}(t)M(t) - \mu(t)\dot{M}(t) = -\dot{\mu}(t)\frac{V(t)}{\mu(t)} - \mu(t)w(t) + \mu(t)u(t)$. Note that, according to (8), $\mu(t)u(t) = y(t)$ and the result follows. $\square$

The introduction of a mint introduces the possibility that an agent may wish to spend tokens that are yet to be mined, *i.e.*, while the total amount of tokens expected $\bar{M}^i(t)$ is always positive, the currently held amount $M^i(t)$ may become negative. When $M^i(t)$ is negative, it is still possible that the lack of demand by one agent could be balanced out by the remainder of agents. Practically speaking, agents might choose to lend tokens to one another, making the agent with negative equity effectively short, *i.e.*, borrowing tokens in order to burn them and returning them when enough have been minted.[1]

The main concern here is that the amount of tokens present in the whole economy $M(t)$ may become negative. In this undesirable scenario, there are no creditors to balance out would-be debtors, resulting in economic collapse. Although

[1]Although debt carries risk on the part of the creditor, implying that debt tokens should be valued less than par, this consideration is out of scope of this work.

it might be possible to create tokens as needed to provide liquidity to the economy, without structural demand to support the token price, *i.e.*, without creating a reason for agents to hold tokens rather than immediately burning them, this would only lead to hyperinflation. We consider then how we may avoid monetary collapse through choice of the deflation factor γ_w.

Proposition 4. *Let $\gamma_w \leq \gamma_i$ for all agents i. Let $u(t)$ be given as the solution to* (1) *where $\bar{M}(t)$ and $M(t)$ are defined by* (12). *Then $M(t) \geq 0$ for all time $\tau \geq t$.*

Proof. We begin by presenting two useful results.

Lemma 1. *Let f and fg be integrable over $[0,\infty)$, where f is positive and g is a non-negative, monotonically non-decreasing function, with $\lim_{t\to\infty} g(t)$ lower-bounded by a positive constant. Then,*

$$\frac{\int_0^t f(\tau)g(\tau)d\tau}{\int_t^\infty f(\tau)g(\tau)d\tau} \leq \frac{\int_0^t f(\tau)d\tau}{\int_t^\infty f(\tau)d\tau}. \tag{14}$$

Proof. Since g is non-decreasing, if $g(t) > 0$, then,

$$\int_0^t f(\tau)g(\tau)d\tau \leq g(t)\int_0^t f(\tau)d\tau,$$
$$\left(\int_t^\infty f(\tau)g(\tau)d\tau\right)^{-1} \leq \frac{1}{g(t)}\left(\int_t^\infty f(\tau)d\tau\right)^{-1}.$$

If $g(t) = 0$, then $0 \leq \int_0^t f(\tau)g(\tau)d\tau \leq g(t)\int_0^t f(\tau)d\tau = 0$ and (14) holds in either case. ☐

Lemma 2. *Let f, g be positive and integrable over $[0,\infty)$. If,*

$$\frac{f(t)}{g(t)} \leq \frac{f(\tau)}{g(\tau)},\ t \leq \tau, \tag{15}$$

for all $t, \tau \in [0,\infty)$, then,

$$\frac{\int_0^t f(\tau)d\tau}{\int_t^\infty f(\tau)d\tau} \leq \frac{\int_0^t g(\tau)d\tau}{\int_t^\infty g(\tau)d\tau}, \tag{16}$$

for any $t \in [0,\infty)$.

Proof. Multiplying (15) by $\frac{g(t)}{f(\tau)}$, resp. $\frac{g(\tau)}{f(t)}$, and integrating over $[0,\tau]$, resp. $[t,\infty)$,

$$\frac{1}{f(\tau)}\int_0^\tau f(t)dt \leq \frac{1}{g(\tau)}\int_0^\tau g(t)dt,$$
$$f(t)\left(\int_t^\infty f(\tau)d\tau\right)^{-1} \leq g(t)\left(\int_t^\infty g(\tau)d\tau\right)^{-1}.$$

We set $\tau = t$ and the result follows. ☐

Consider that,

$$\frac{\mu(t)^{-1}}{\gamma_w^t} = \frac{N-1}{\gamma_w^t \sum_j \mu^{-j}(0)\gamma_j^{-t}} = \frac{N-1}{\sum_j \mu^{-j}(0)(\gamma_j/\gamma_w)^{-t}}$$
$$\leq \frac{N-1}{\sum_j \mu^{-j}(0)(\gamma_j/\gamma_w)^{-\tau}} = \frac{\mu(\tau)^{-1}}{\gamma_w^\tau},\ t \leq \tau, \tag{17}$$

and,

$$\frac{\int_0^t \frac{y(\tau)}{\mu(\tau)}d\tau}{\int_t^\infty \frac{y(\tau)}{\mu(\tau)}d\tau} \leq \frac{\int_0^t \frac{1}{\mu(\tau)}d\tau}{\int_t^\infty \frac{1}{\mu(\tau)}d\tau} \leq \frac{\int_0^t \gamma_w^\tau d\tau}{\int_t^\infty \gamma_w^\tau d\tau}, \tag{18}$$

where the first inequality is implied by Lemma 1 and the second is implied by Lemma 2 and (17). Since (12), $\dot{M}(t) = -u(t) + w(t)$ and,

$$\frac{M(0) + \frac{w_0}{-\ln\gamma_w}}{\int_t^\infty u(\tau)d\tau} = \frac{\int_0^\infty \frac{y(\tau)}{\mu(\tau)}d\tau}{\int_t^\infty \frac{y(\tau)}{\mu(\tau)}d\tau} = \frac{\int_0^t \frac{y(\tau)}{\mu(\tau)}d\tau}{\int_t^\infty \frac{y(\tau)}{\mu(\tau)}d\tau} + 1$$
$$\leq \frac{\int_0^t \gamma_w^\tau d\tau}{\int_t^\infty \gamma_w^\tau d\tau} + 1 = \frac{\int_0^\infty \gamma_w^\tau d\tau}{\int_t^\infty \gamma_w^\tau d\tau} = \gamma_w^{-t},$$

where the inequality is implied by (18). Therefore $M(t) = \int_t^\infty u(\tau)d\tau - \frac{\gamma_w^t w_0}{-\ln\gamma_w} \geq \gamma_w^t M(0) \geq 0$. ☐

Proposition 5. *Let $\gamma_w > \gamma_i$ for some agent i. Let $u(t)$ be given as the solution to* (1) *where $\bar{M}(t)$ and $M(t)$ are defined by* (12). *Let $\lim_{t\to\infty} y(t)$ be bounded. Then there exists a time t' where $M(t) < 0$ for all $t > t'$.*

Proof. Let $i^* = \arg\min_i \gamma_i$. According to Proposition 3, $V(t) = \mu(t)\int_t^\infty \left(\frac{y(\tau)}{\mu(\tau)} - w(\tau)\right)d\tau$. Since the limit of $y(t)$ is bounded so is the limit of $V(t)$ and therefore $\lim_{t\to\infty}\gamma_{i^*}^t V(t) = 0$. Furthermore,

$$\lim_{t\to\infty}\frac{d}{dt}\left(\gamma_{i^*}^t V(t)\right) = \lim_{t\to\infty}\gamma_w^t \mu^{-i^*}(0)w_0 - \gamma_{i^*}^t y(t),$$

where we used the facts that $\lim_{t\to\infty}\frac{\dot{\mu}(t)}{\mu(t)} = -\ln\gamma_{i^*}^t$ and $\lim_{t\to\infty}\gamma_{i*}^t\mu(t) = \mu^{-i^*}(0)$ to reduce (13) in the limit.

Since $\gamma_w > \gamma_{i^*}$, according to the above, there exists a time t'' such that $\frac{d}{dt}\left(\gamma_{i^*}^t V(t)\right)$ is positive for all $t > t''$. Since $\gamma_{i^*}^t V(t) = 0$ in the limit, this implies that the function approaches 0 from below. Implying that there exists a time $t' > t''$ such that $\gamma_{i^*}^t V(t)$ is negative for all $t > t'$. Since $\gamma_{i^*}^t > 0$, $V(t)$, $t > t'$, must be negative, implying Since $\mu(t) > 0$, $M(t) = \frac{V(t)}{\mu(t)} < 0$ for all $t > t'$. ☐

1) Drawbacks of deflationary burn-and-mint: We have thus shown that $\gamma_w \leq \gamma_i$ for all i is a necessary condition to avoid monetary collapse. This implies that the original application of burn-and-mint with no deflation, *i.e.*, $\gamma_w = 1$, theoretically leads to immediate monetary collapse as soon as the cash flow $y(t)$ becomes positive. However, sufficient deflation is not sufficient in and of itself to ensure a stable economy because, even sufficient deflation, the reward amount $w(t)$ may converge to zero faster than price can appreciate. Consider for an agent i that the expected reward from mint is given by $\mu(t)w(t)$ and evolves according to,

$$\frac{d}{dt}(\mu(t)w(t)) = \dot{\mu}(t)w(t) + \mu(t)\dot{w}(t)$$
$$\leq \left(-\ln\gamma_{i^*} + \ln\gamma_w\right)(\mu(t)w(t)),$$

where $\gamma_{i^*} = \min_i \gamma_i$. If $\gamma_w < \gamma_{i^*}$, then,

$$\frac{d}{dt}(\mu(t)w(t)) < -\varepsilon(\mu(t)w(t)),$$

for some $\varepsilon > 0$, and therefore $\mu(t)w(t)$ tends to zero over time. The implication is that, whereas the choice $\gamma_w > \gamma_{i^*}$ eventually leads to monetary collapse, the choice $\gamma_w < \gamma_{i^*}$ eventually leads to economic collapse. Therefore γ_w should equal γ_{i^*} exactly.

Nevertheless, the above theoretical analysis considers only long-run behavior and a case may be made that deflationary burn-and-mint can be sustainable over the short term. However, the choice of the ideal deflationary factor depends on many factors, most of which are unknowable to designers. Chief among them is the time preferences, or discount factors, of each agent. Compounding the problem is the fact that DePIN projects are typically spread across geographically large regions, crossing national boundaries; given that interest rates often differ from one jurisdiction to another, there is no simple way to reconcile the use of a single deflation factor γ_w in the design of a burn-and-mint economy. To remedy this, we propose a framework based on a target payment fixed in the same currency of cash flows y.

IV. Fiat-Fixed Rewards

We propose a scheme in which each agent i is paid a constant fiat amount z^i,

$$w^i(t) = \frac{z^i}{\mu(t)}. \tag{19}$$

To achieve this, we remove $z = \sum_j z^j$ from the cash flow $y(t)$ so that $y - z$ is available for burn. When the cash flow $y(t)$ is less than z, we reverse the process: instead of agents competing for a share of revenues, they instead pay $z - y(t)$ in order to obtain a share of tokens. The amount requested by agents is the negative of the burn $-u^i(t)$ so that the payout to each agent is $-u^i(t)(z - y(t))$. Thus the objective for each agent is the same regardless of whether they choose to burn or mint,

$$\max_{u^i} \int_t^\infty \gamma_i^{\tau - t} \left(\frac{u^i}{u^i + u^{-i}(\tau)} (y(\tau) - z) + z^i \right) d\tau, \tag{20a}$$

subject to the constraint,

$$\int_t^\infty u^i(\tau) d\tau \leq M^i(t). \tag{20b}$$

Comparing (20) to (1), we see that the solution to the latter is the same as the former with an affine change of variables $y(t) \mapsto y(t) - z$.

Proposition 6. *Consider the N-agent game (20) with $N \geq 2$. The optimal allocation for each agent i is given by,*

$$u^i(\tau) = \frac{\mu^i(\tau)}{\mu(\tau)} \frac{y(\tau) - z}{\mu(\tau)},$$

for all $\tau \geq t$, where $\mu^i(\tau)$ and $\mu^{-i}(\tau)$ satisfy (4)-(5) with initial conditions obtained by solving,

$$M^{-i}(t) = \int_t^\infty \frac{(N-1)^2 \mu^{-i}(t)}{\left(\sum_j \mu^{-j}(t)(\gamma_i/\gamma_j)^{\tau - t}\right)^2} \gamma_i^{\tau - t} (y(\tau) - z) d\tau.$$

TABLE I
SIMULATION PARAMETERS

agent i	type	parameters			
		γ_i	M_0^i (m)	w_0^i ($\ln \gamma_w^{-1}$m)	z^i (\$m)
0	LT, large	0.95	350	0	0
1	LT, miner	0.95	0	150	25
2	LT, small	0.9	150	0	0
3	ST, miner	0.8	0	150	25
4	ST, small	0.8	0	0	0

In the following result we show that, in the case of fiat-fixed rewards, it is not possible for the economy to run out of tokens, even in the case where $y(t)$ is not monotonic.

Proposition 7. *Let $u^i(t)$ be the solution to (20) where $M^i(t)$ is defined by (12) and $w^i(t)$ is defined by (19). Let $i^* = \arg\min_i \gamma_i$ and assume $\int_t^\infty \gamma_{i^*}^{\tau - t}(y(\tau) - z) d\tau > 0$ for all $t \geq 0$. Then $M(t) > 0$ for all $t \geq 0$.*

Proof. According to (11) and (5),

$$\begin{aligned} V(t) &= \mu(t) \int_t^\infty \frac{y(\tau) - z}{\mu(\tau)} d\tau \\ &= \int_t^\infty \frac{1}{N-1} \sum_j \mu^{-j}(\tau) \gamma_j^{\tau - t} \frac{y(\tau) - z}{\mu(\tau)} d\tau \\ &\geq \int_t^\infty \frac{1}{N-1} \sum_j \mu^{-j}(\tau) \gamma_{i^*}^{\tau - t} \frac{y(\tau) - z}{\mu(\tau)} d\tau \\ &= \int_t^\infty \gamma_{i^*}^{\tau - t} (y(\tau) - z) d\tau > 0. \end{aligned}$$

Then $M(t) = \frac{V(t)}{\mu(t)} > 0$. ☐

V. Numerical Simulations

We consider a numerical simulation of an economy of $N = 5$ agents, with parameters given in Table I. Agents exhibit various time preferences, both long term (LT) and short term (ST). Some are speculators and do not mine; others are miners. In particular, the largest speculator, representing the behavior of true believers, *e.g.*, core team and investors, has low time preference. The cash flow is \$0 over the first two years, linearly ramping to \$200 million (m) over the next eight years and remaining constant thereafter.

We perform three simulations of a deflationary burn-and-mint economy with $\gamma_w = 0.95, 0.85, 0.7$ and one with fiat-fixed rewards. The results are presented in Figs. 1-4. Fig. 1 shows the total value of the circulating money supply $V(t)$ and Fig. 2 shows the total money supply $M(t)$. Note how the economy collapses by year 17 in the case $\gamma_w = 0.95$ (and there is no longer a token price to speak of). In the case $\gamma_w = 0.85$, the collapse does not happen by year 40 but we are able to see that the economic value is not being sustained on the way to eventual collapse. For the other two cases, the economic value is sustained but, in the deflationary case $\gamma_w = 0.7$, as shown in Fig. 3, the rewards have been reduced to a point where contributors are no longer incentivized to participate, making the result purely theoretical. Only in the

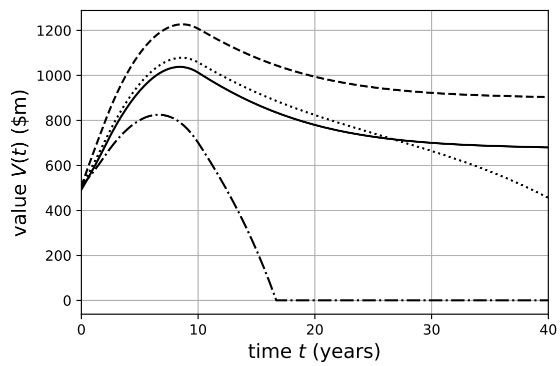

Fig. 1. Value of circulating supply $V(t)$ in the case of fiat-fixed (solid) and deflationary burn-and-mint with $\gamma_w = 0.95$ (dot-dashed), $\gamma_w = 0.85$ (dotted), and $\gamma_w = 0.7$ (dashed)

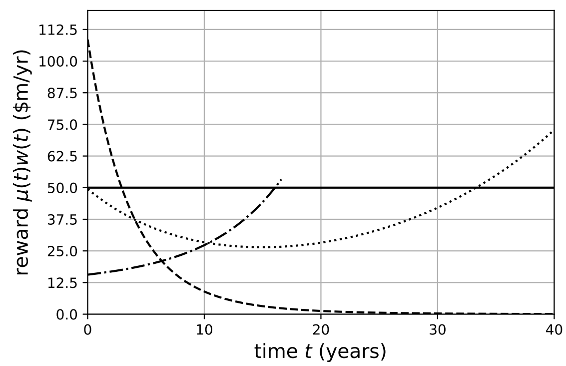

Fig. 3. Mint reward $\mu(t)w(t)$ in the case of fiat-fixed (solid) and deflationary burn-and-mint with $\gamma_w = 0.95$ (dot-dashed), $\gamma_w = 0.85$ (dotted), and $\gamma_w = 0.7$ (dashed)

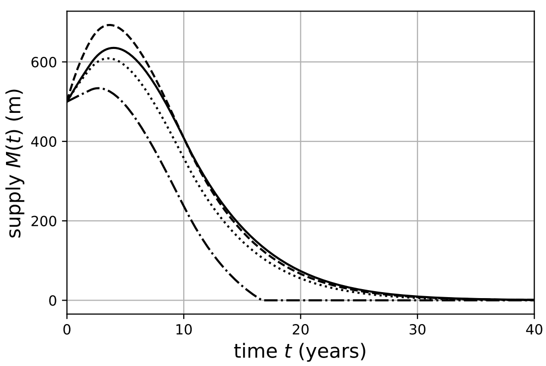

Fig. 2. Token supply $M(t)$ in the case of fiat-fixed (solid) and deflationary burn-and-mint with $\gamma_w = 0.95$ (dot-dashed), $\gamma_w = 0.85$ (dotted), and $\gamma_w = 0.7$ (dashed)

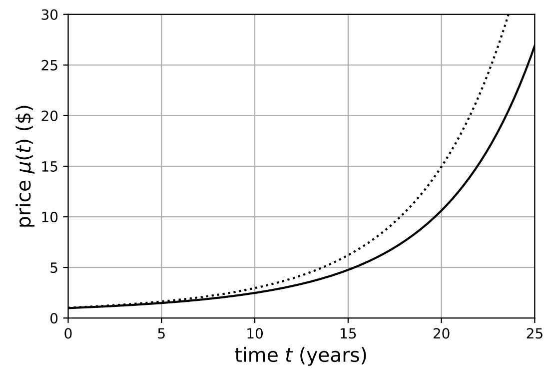

Fig. 4. Price $\mu(t)$ in the case of fiat-fixed (solid) and deflationary (dotted) burn-and-mint

case of fiat-fixed rewards are both network value and token rewards sustained. Fig. 4 presents the token price for both the deflationary (same for all choices of γ_w) and fiat-fixed cases. By design, the starting prices are similar. Because more value is kept in the hands of token holders in the deflationary case, the price theoretically rises faster. However, this observation is moot because the economy is not sustainable in that case.

We note that the simulation timescales are long and the value of the economy in the first few years is similar in all four cases. Given the recency of the DePIN concept and the realistic nature of our simulation parameters, this suggests that an insufficient amount of time has passed to provide empirical evidence for or against the claims provided in this work.

VI. Conclusion

In this work, we presented several incentive issues arising from the implementation of burn-and-mint schemes. We have shown that a sufficient amount of deflation of contributor reward is necessary to sustain token value. However, as we have also shown, deflation leads to underincentivization over time. We therefore make the recommendation that implementations of burn-and-mint schemes fix contributor rewards in the unit of account, *i.e.*, fiat.

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