UNIT - 3

IIR Filter Design

12 m & 4 m

Realization of Digital Filters (IIR)

1) Direct Form - 1
2) Direct Form - 2.
3) Cascade Form
4) Parallel Form.

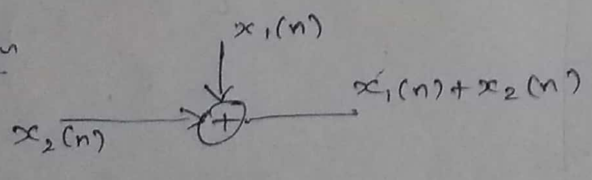
The standard form of system transfer fn is

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

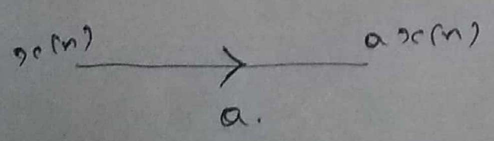
$$= \frac{b_0 z^0 + b_1 z^{-1} + b_2 z^{-2} + \dots \dots b_M z^{-m}}{1 - (a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \dots a_N z^{-1}}$$

BASIC BUILDING BLOCKS :

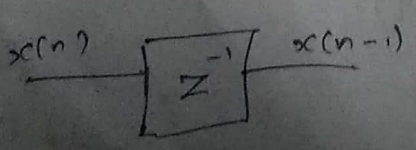
Addition



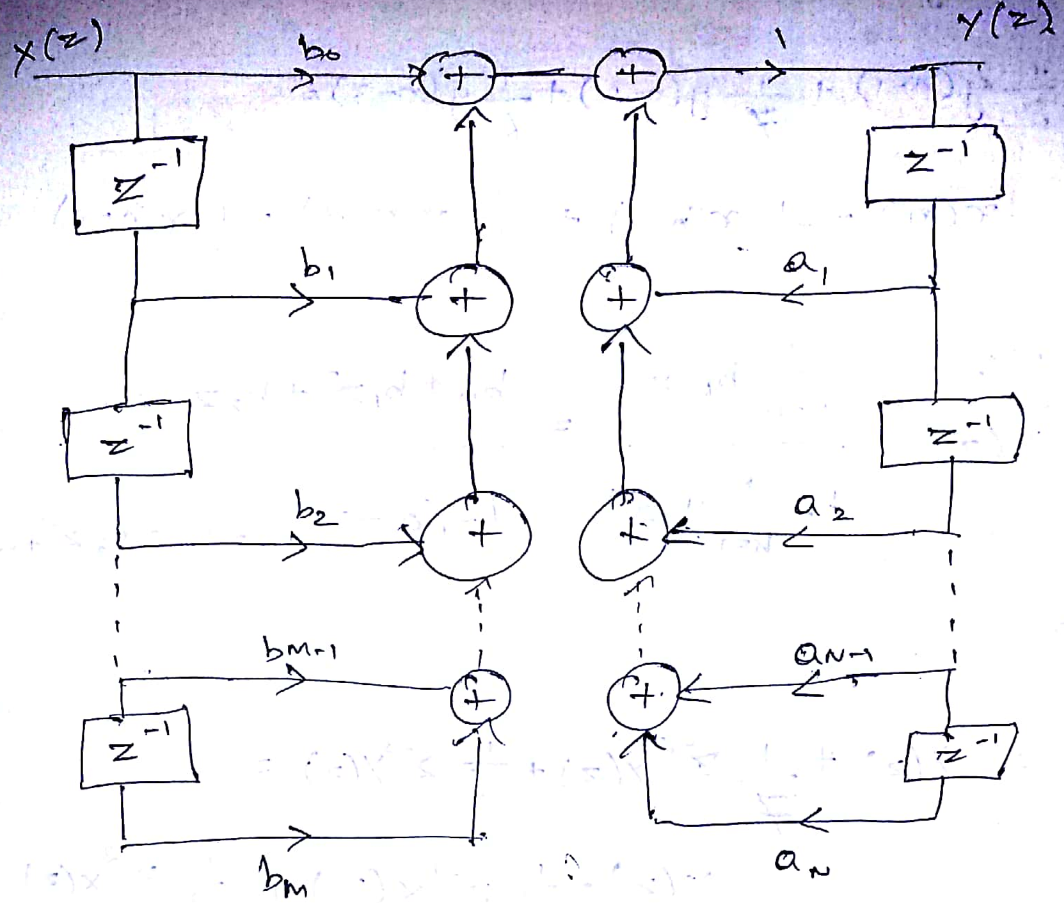
Multiplier



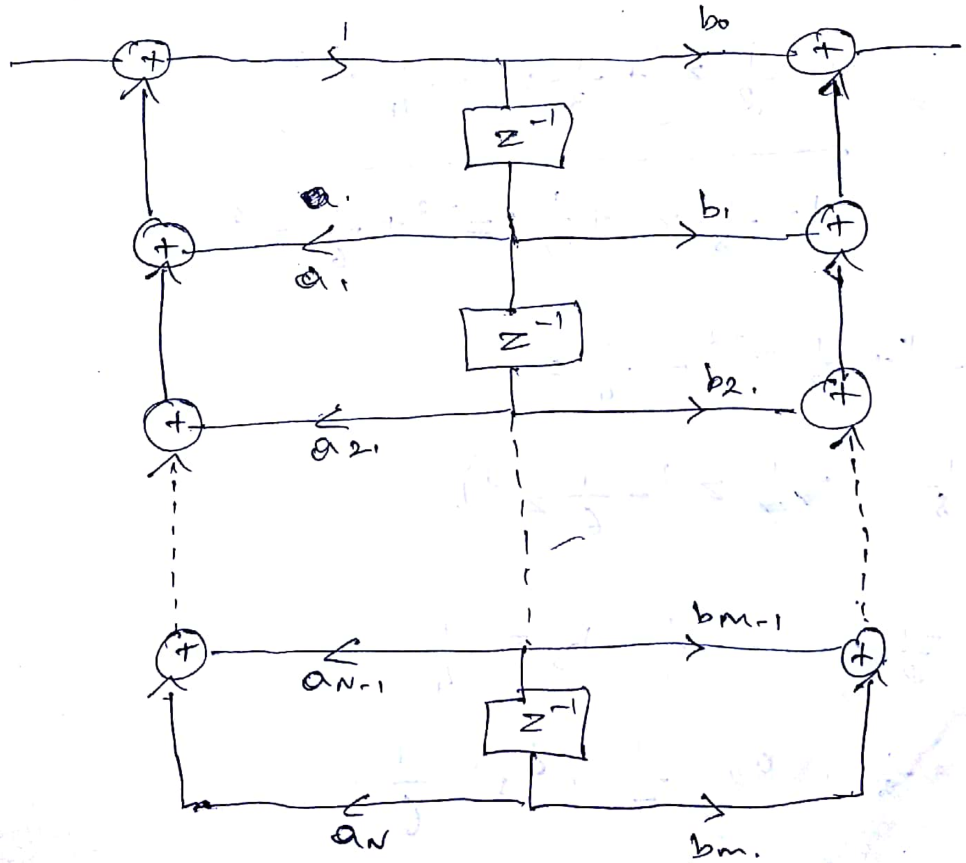
Unit delay



DIRECT FORM 1 STRUCTURE:



DIRECT FORM 2 STRUCTURE:



described by the sequence equation.

$$y(n) - \frac{1}{8}y(n-1) + \frac{1}{7}y(n-2) + \frac{1}{6}y(n-3) =$$

$$x(n) - \frac{1}{2}x(n-1) + \frac{1}{3}x(n-2) - \frac{1}{4}x(n-3)$$

Soln.

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-1} + \dots}{1 - (a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + \dots}$$

Taking ZT

$$Y(z) - \frac{1}{8}z^{-1}Y(z) + \frac{1}{7}z^{-2}Y(z) + \frac{1}{6}z^{-3}Y(z) =$$

$$X(z) - \frac{1}{2}z^{-1}X(z) + \frac{1}{3}z^{-2}X(z) - \frac{1}{4}X(z)z$$

$$Y(z)\left\{1 - \frac{1}{8}z^{-1} + \frac{1}{7}z^{-2} + \frac{1}{6}z^{-3}\right\} = X(z)\left\{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2} - \frac{1}{4}\right.$$

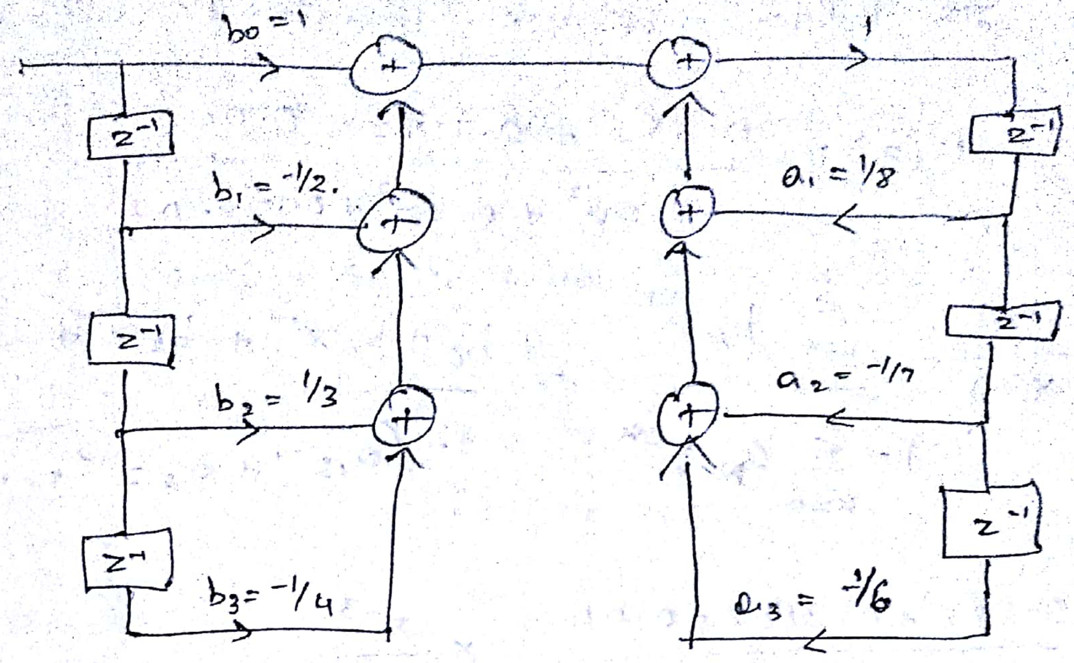
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2} - \frac{1}{4}z^{-3}}{1 - \frac{1}{8}z^{-1} + \frac{1}{7}z^{-2} + \frac{1}{6}z^{-3}}$$

$$= \frac{1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2} - \frac{1}{4}z^{-3}}{1 - (\frac{1}{8}z^{-1} - \frac{1}{7}z^{-2} - \frac{1}{6}z^{-3})}$$

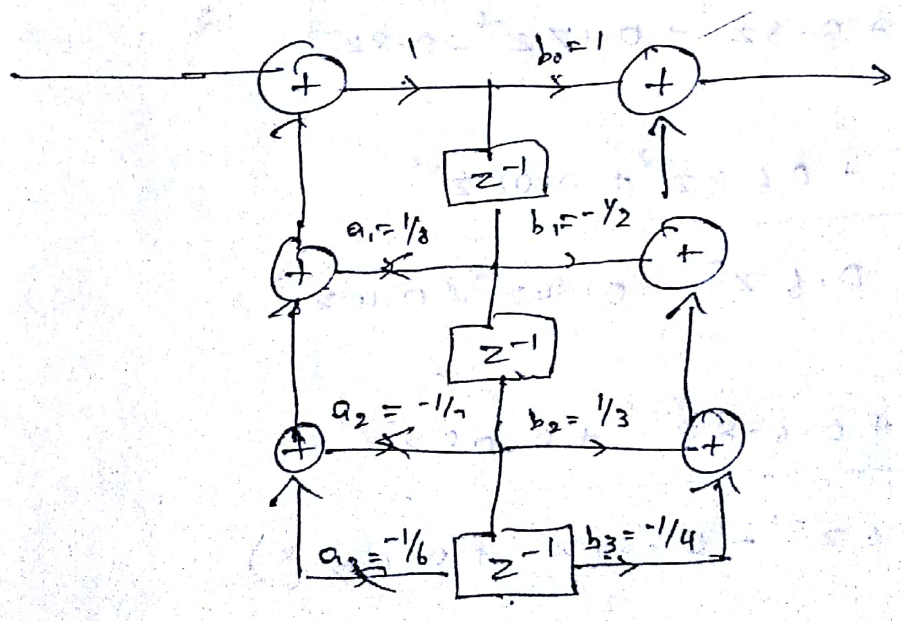
$b_0 = 1$, $b_1 = \frac{-1}{2}$, $b_2 = \frac{1}{3}$, $b_3 = \frac{-1}{4}$

$a_1 = \frac{1}{8}$, $a_2 = \frac{-1}{7}$, $a_3 = \frac{-1}{6}$.

Direct Form 1



Direct Form 2



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Determine the Direct form 1 & 2 Realization for the 3rd Order IIR filter whose Transfer fn is given by.

$$H(z) = \frac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2}$$

Soln:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=0}^{N} a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots}{1 - (a_1 z^{-1} + a_2 z^{-2} + \cdots)}$$

$$H(z) = \frac{0.28z^2 + 0.319z + 0.04}{0.5z^3 + 0.3z^2 + 0.17z - 0.2} \times \frac{z^{-3}}{z^{-3}}$$

$$= \frac{0.28z^{-1} + 0.319z^{-2} + 0.04z^{-3}}{0.5 + 0.3z^{-1} + 0.17z^{-2} - 0.2z^{-3}} \times \frac{2}{2}.$$

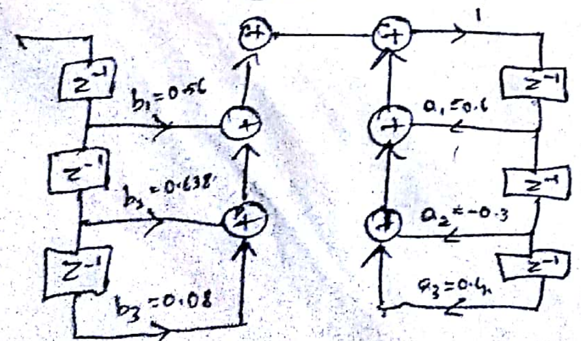
$$= \frac{0.56z^{-1} + 0.638z^{-2} + 0.08z^{-3}}{1 + 0.6z^{-1} + 0.34z^{-2} - 0.4z^{-3}}$$

$$= \frac{0.56z^{-1} + 0.638z^{-2} + 0.08z^{-3}}{1 - (-0.6z^{-1} - 0.34z^{-2} + 0.4z^{-3})}$$

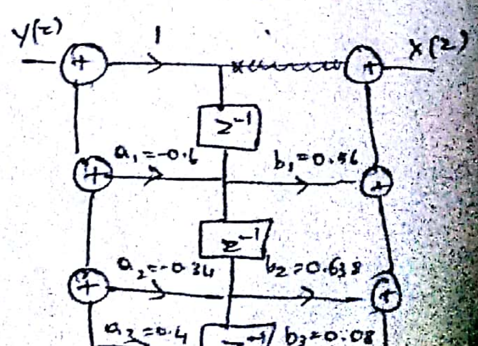
$\therefore$ $b_0 = 0.$, $b_1 = 0.56$, $b_2 = 0.638$, $b_3 = 0.08$

$a_1 = -0.6$, $a_2 = -0.34$, $a_3 = 0.4$.

DF 1



DF 2.



Cascade Form:

$$H(z) = H_1(z)\, H_2(z)\, H_3(z) \ldots$$

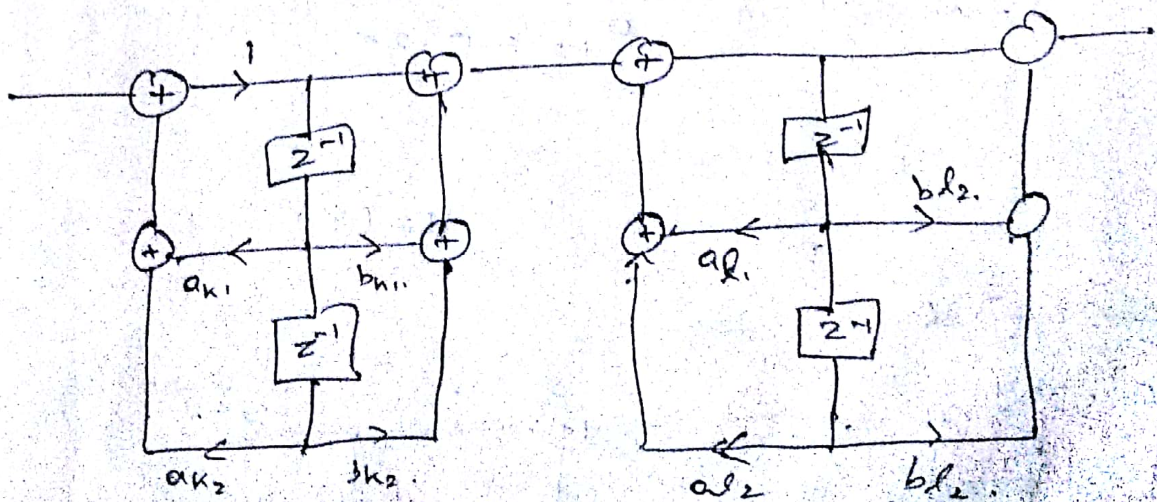
$$= \frac{(b_{k_0} + b_{k_1} z^{-1} + b_{k_2} z^{-2})(b_{l_0} + b_{l_1} z^{-1} + b_{l_2} z^{-2})(b_{m_0} + b_{m_1} z^{-1} + b_{m_0} z^{-2})}{\{1-(a_{k_1} z^{-1} + a_{k_2} z^{-2})\}\{1-(a_{l_1} z^{-1} + a_{l_2} z^{-2})\}\{1-(a_{m_1} z^{-1} + a_{m_2} z^{-2})\}}$$

where,

$$H_1(z) = \frac{b_{k_0} + b_{k_1} z^{-1} + b_{k_2} z^{-2}}{1-(a_{k_1} z^{-1} + a_{k_2} z^{-2})}$$

$$H_2(z) = \frac{b_{l_0} + b_{l_1} z^{-1} + b_{l_2} z^{-2}}{1-(a_{l_1} z^{-1} + a_{l_2} z^{-2})}$$

$$H_3(z) = \frac{b_{m_0} + b_{m_1} z^{-1} + b_{m_2} z^{-2}}{1-(a_{m_1} z^{-1} + a_{m_2} z^{-2})}$$



31/08/18

Obtain the cascade realisation for the system with diff eq.

$y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n) + \frac{1}{3} x(n-1)$

Soln.

Taking z transform.

$$Y(z) = \{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\} = X(z) \{1 + \frac{1}{3} z^{-1}\}.$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$$

$$= \frac{1 + 0.33 z^{-1}}{(1 - 0.5 z^{-1})(1 - 0.25 z^{-1})}$$

$$H(z) = H_1(z) \; H_2(z) \ldots$$

$$H_1(z) = \frac{1 + 0.33 z^{-1}}{1 - 0.5 z^{-1}}$$

$\therefore b_{01} = 1, \; b_{11} = 0.33$

$a_{11} = 0.5.$

$$H_2(z) = \frac{1}{1 - 0.25 z^{-1}}$$

$b_{02} = 1$

$a_{12} = 0.25.$

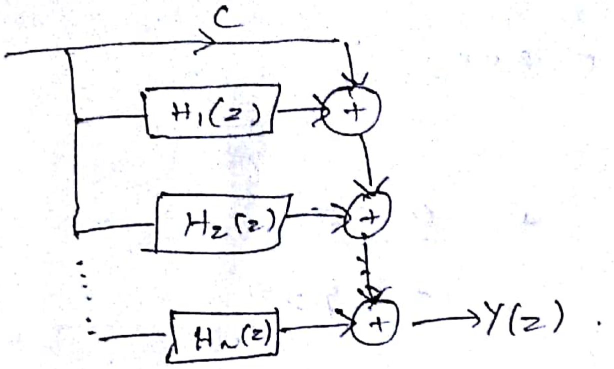
$H_1(z)$ $H_2(z)$

PARALLEL FORM:

$$H(z) = c + \sum_{k=1}^{N} \frac{c_k}{1 - P_k z^{-1}} \quad ; \quad P_k \to \text{Poles.}$$

$$H(z) = c + \frac{c_1}{1 - P_1 z^{-1}} + \frac{c_2}{1 - P_2 z^{-1}} + \cdots \frac{c_N}{1 - P_N z^{-1}}$$

$$H(z) = c + H_1(z) + H_2(z) + \cdots + H_N(z)$$



Realize the System given by the diff eqn.

$$y(n) = -0.1\,y(n-1) + 0.72\,y(n-2) + 0.7\,x(n) - 0.252\,x(n-2)$$

$$y(z) + 0.1\,y(z)z^{-1} - 0.72\,y(z)z^{-2} = 0.7\,x(z) + 0\,x(z)z^{-1} - 0.252\,x(z)z^{-2}$$

$$y(z)\left[1 + 0.1z^{-1} - 0.72z^{-2}\right] = x(z)\left[0.7 + 0z^{-1} - 0.252z^{-2}\right]$$

$$\therefore H(z) = \frac{0.7 + 0z^{-1} - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}} = \frac{-0.252z^{-2} + 0z^{-1} + 0.7}{-0.72z^{-2} + 0.1z^{-1} + 1}$$

To get constant, divide numerator by denominator.

$$
\begin{array}{r|l}
 & 0.35 \\
-0.72z^{-2}+0.1z^{-1}+1 & -0.252z^{-2} + 0.z^{-1} + 0.7 \\
 & -0.252z^{-2} + 0.035z^{-1} + 0.35 \\
\hline
 & 0 - 0.035z^{-1} + 0.35
\end{array}
$$

$$\left[Q + \frac{R}{D}\right]$$

$$H(z) = 0.35 + \frac{-0.035z^{-1} + 0.35}{1 + 0.1z^{-1} - 0.72z^{-1}}$$

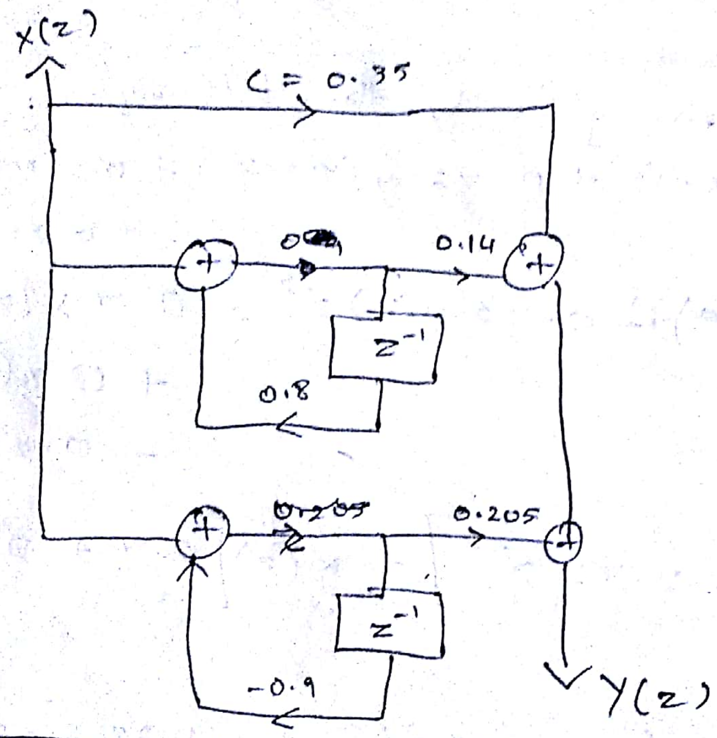
$$H(z) = 0.35 + \frac{-0.035 z^{-1} + 0.35}{(1-0.8z^{-1})(1+0.9z^{-1})}$$

By partial Fractions

$$\frac{-0.035 z^{-1} + 0.35}{(1-0.8z^{-1})(1+0.9z^{-1})} = \frac{A}{(1-0.8z^{-1})} + \frac{B}{(1+0.9z^{-1})}$$

$\therefore A = 0.14 \qquad B = 0.205$

$$H(z) = 0.35 + \frac{0.14}{1-0.8z^{-1}} + \frac{0.205}{1+0.9z^{-1}}$$



05/09/18

INFINITE IMPULSE RESPONSE (IIR) FILTER:

1. Butterworth Filter.
2. Chebyshev Filter

Bilinear Transformation

Infinite ~~response~~ invariant technique.

Design a digital low-pass butterworth filter using impulse invariant transformation. with pass band and stop band cut off frequency 200Hz & 500 Hz. The pass band & Stop band attenuations are -5dB & -12dB respectively. Sampling frequency is 5KHz.

STEP 1 :

Collect the parameters from question.

LPF, Butterworth, impulse inv.

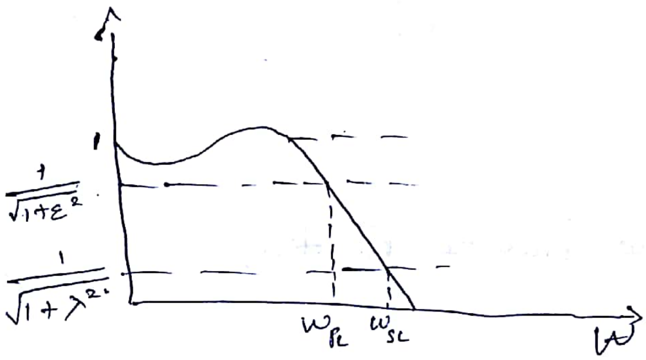
$\alpha_p = -5dB$, $\alpha_s = -12dB$.

$f_p = 200Hz$, $f_s = 500Hz$.

$F_s = 5KHz$.

STEP 2 :

Diagram.



STEP 3 :

$\varepsilon = \sqrt{10^{0.1|\alpha_p|} - 1}$ $\varepsilon = 1.47$

$\lambda = \sqrt{10^{0.1|\alpha_s|} - 1}$ $\lambda = 3.85$

STEP 4 :

$\omega_p = \frac{2\pi f_p}{F_s}$ $\omega_s = \frac{2\pi f_s}{F_s}$.

$\omega_p = 0.2512$ rad $\omega_s = 0.628$ rad.

STEP 5 :

Impulse inv technique

$\Omega_s = \frac{\omega_s}{T}$ $\Omega_p = \frac{\omega_p}{T}$

where = $T = \frac{1}{F_s}$ $\Omega_s = 3140$ rad s^{-1} $\Omega_p = 1256$ rad s^{-1}

Calculating the order of the filter

$$N \geq \frac{\log(\lambda/\varepsilon)}{\log(\Omega_s/\Omega_p)}$$

$N \geq 1.050$

If N is a decimal, write next number

$\boxed{N = 2 \text{ is the order.}}$

STEP 7: To calculate Butterworth polynomial.

For $N=2$. $s^2 + \sqrt{2}\, s + 1$.

STEP 8: To calculate Transfer function

$$H(s) = \frac{1}{\text{Polynomial}}.$$

$$H(s) = \frac{1}{s^2 + \sqrt{2}\, s + 1}$$

STEP 9: Denormalize the transfer function

LPF to LPF.

Replace S with $\frac{s}{\Omega_c}$. where $\Omega_c = \frac{\Omega_p}{\varepsilon^{1/N}}$

$$\Omega_c = \frac{1256}{(1.47)^{1/2}} = 1035.9 \simeq 1036$$

$$S \rightarrow \frac{s}{1036}$$

$$\therefore H(s) = \frac{1}{\left(\frac{s}{1036}\right)^2 + \frac{\sqrt{2}\, s}{1036} + 1} = \frac{(1036)^2}{\cancel{s^2 + \sqrt{2}\, s + 1036}}$$

$$s^2 + \sqrt{2}\, s(1036) + (1036)^2$$

$$\boxed{H(s) = \frac{1073296}{s^2 + 1465.1 s + 1073296}}$$

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STEP 10:

Transforming Analog to digital for impulse inveln.

$$H(z) = \frac{1}{S - P_K} = \frac{1}{1 - e^{P_K T} z^{-1}}$$

※ Calci in COMPLX Mode, Go in $s^2+s+Con$ format. For imaginary root click ~~Shift+Equal~~ Shift + Equal

$s^2 + 1465.1s + 1073296$ roots $\to -732.5 + j732.5$, $\to -732.5 - j732.5$

$$\therefore H(s) = \frac{1073296}{(s+732.5-j732.5)(s+732.5+j732.5)}$$

$$\frac{1073296}{(s+732.5-j732.5)(s+732.5+j732.5)} = \frac{A}{(s+732.5-j732.5)} + \frac{B}{(s+732.5+j732.5)}$$

$$A(s+732.5+j732.5) + B(s+732.5-j732.5) = 1073296$$

Substitute $s = -732.5 - j732.5 \Rightarrow B = 732.2j$

Substitute $s = -732.5 + j732.5 \Rightarrow A = -732.62j$

$$H(s) = \frac{-732.62j}{s+732.5-j732.5} + \frac{732.2j}{s+732.5+j732.5}$$

$\therefore P_{k1} = -732.5 + j732.5$ $\quad P_{k2} = -732.5 - j732.5$.

$$H(z) = \frac{-732.62j}{1 - e^{(-732.5+j732.5)\times 2\times 10^{-4}} z^{-1}} + \frac{732.62j}{1 - e^{(-732.5-j732.5)\times 2\times 10^{-4}} z^{-1}}$$

$$H(z) = \frac{-732.62j}{1 - e^{(0.1465+j0.1465)} z^{-1}} + \frac{732.2j}{1 - e^{(-0.1465-j0.1465)} z^{-1}}$$

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$$H(z) = \frac{732.62j}{1-e^{-0.1465}e^{j0.1465}z^{-1}} + \frac{732.2j}{1-e^{-0.1465}e^{-j0.1465}z^{-1}}$$

$$= \frac{-732.62j}{1-0.86\,e^{j0.1465}z^{-1}} + \frac{732.2j}{1-0.86\,e^{-j0.146}z^{-1}}$$

$$= \frac{-732.65j(1-0.86e^{-j0.1467}z^{-1}) + 732.2j(1-0.86e^{j0.1465}z^{-1})}{1-0.86e^{-j0.1465}z^{-1} - 0.86e^{j0.1465}z^{-1} + 0.86e^{j0.1465}z^{-1} \cdot 0.8e^{-j0.1465}z^{-1}}$$

$$= \frac{630.08jz^{-1}(e^{-0.1465j} \cdot e^{0.1465j})}{1-0.86(e^{-j0.1465}+e^{j0.1465})z^{-1}+0.64z^{-2}}$$

$$= \frac{+j630.08z^{-1}(e^{-0.1465j} \; e^{0.1465j})}{1-0.86(e^{-j0.1465}+e^{j0.1465})z^{-1}+0.64z^{-2}}$$

$$= \frac{-j630.08z^{-1}(e^{j0.1465} - e^{-j0.1465})}{1-0.86(2\cos 0.1465)z^{-1}+0.64z^{-2}}$$

$=$

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11/09/18

Design a digital butterworth L.P filter whose Tr. fn is given by.

$$0.7 \le |H(e^{j\omega})| \le 1 \qquad 0 \le \omega \le 0.2\pi$$

$$0 \le |H(e^{j\omega})| \le 0.3 \qquad 0.2\pi \le \omega \le 0.6\pi$$

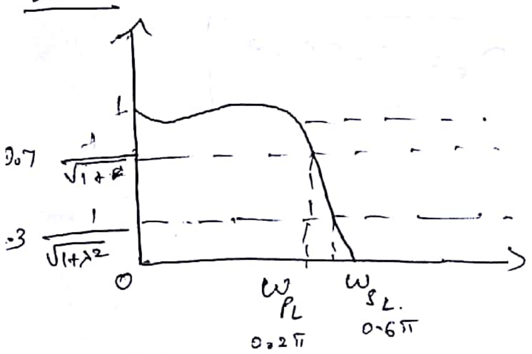
by Bilinear transformation.

Soln:

STEP 1:

Butterworth, LPF, bilinear Transformation

STEP 2:



STEP 3:

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.7$$

$$\frac{1}{1+\varepsilon^2} = \frac{49}{100}$$

$$100 = 49 + 49\varepsilon^2$$

$$49\varepsilon^2 = 51$$

$$\varepsilon^2 = 1.0408$$

$$\boxed{\varepsilon = 1.02}$$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.3$$

$$\frac{1}{1+\lambda^2} = \frac{9}{100}$$

$$100 = 9 + 9\lambda^2$$

$$9\lambda^2 = 91$$

$$\lambda^2 = 10.111$$

$$\boxed{\lambda = 3.179}$$

$$\boxed{\omega_p = 0.2\pi}$$

$$\boxed{\omega_s = 0.6\pi}$$

STEP 4

Bilinear Transform:

$\Omega_s = \frac{2}{T} \tan\left(\frac{w_s}{2}\right)$ $\Omega_p = \frac{2}{T} \tan\left(\frac{w_p}{2}\right)$

$\boxed{\text{※ Since } F_s \text{ not given, Assume } T = 1 \text{ second}}$

$\Omega_s = 2 \tan\left(\frac{0.6\pi}{2}\right)$ $\Omega_p = 2 \tan\left(\frac{0.2\pi}{2}\right)$

~~$\Omega_p = 2(0.726) = 1.453$~~

~~$\Omega_s = 2(3.077)$~~
~~$= 6.155$~~

$\boxed{\Omega_s = 2.75}$ $\boxed{\Omega_p = 0.3249.}$

STEP 5:

Calculating Order.

$$N \geq \frac{\log(\lambda/\varepsilon)}{\log(\Omega_s/\Omega_p)}$$

$$N \geq \frac{\log(3.116)}{\log(8.464)} = \frac{0.4936}{0.9276} = 0.53 \simeq 1$$

$\boxed{N = 1}$

STEP 6:

Butterworth Polynomial. for N=1.

$S+1$.

$H(s) = \frac{1}{S+1}$

$Tr. fn = \frac{1}{Polyn}$

STEP 7:

Denormalize.

$$s \longrightarrow \frac{s}{\Omega_c}$$

$$\Omega_c = \frac{\Omega_p}{\varepsilon^{1/N}}$$

$$\Omega_c = 0.6362.$$

$$s \longrightarrow \frac{s}{0.6362}.$$

$$H(s) = \frac{1}{\frac{s}{0.6362}+1} = \frac{0.6362}{s+0.6362}.$$

STEP 8

Converting to Digital.

Bilinear Transformation.

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \qquad T=1$$

$$H(s) = \frac{0.6362}{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.6362} = \frac{0.6362(1+z^{-1})}{2(1-z^{-1})+0.6362(1+z^{-1})}$$

$$= \frac{0.6362+0.6362z^{-1}}{2-2z^{-1}+0.6362+0.6362z^{-1}} = \frac{0.6362+0.6362z^{-1}}{2.6362-1.3638z^{-1}}$$

$$= \frac{0.6362+0.6362z^{-1}}{2.6362(1-0.517z^{-1})} = \frac{0.2413+0.2413z^{-1}}{1-0.517z^{-1}}$$

Draw D2 diagram.

$b_0 = 0.2413 \quad b_1 = 0.2413$

$a_1 = 0.517$

* Design a digital L.P Butterworth Filter using Bilinear Transformation with pass band & stop band cut off frequency 800 rad & 1800 rad/s. The pass band attenuation is -3 dB & stop band attenuation is -10 dB.

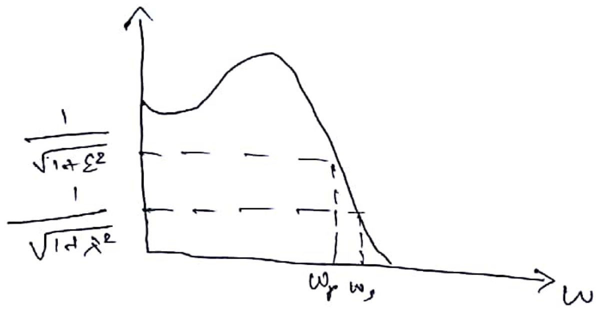
STEP 1 :

LPF, Butterworth, Bilinear.

$W_p = 800$ rad. $\quad W_s = 1800$ rad.

$\alpha_p = -3$ dB $\quad \alpha_s = -10$ dB.

STEP 2 :



STEP 3

$$\varepsilon = \sqrt{10^{0.1|\alpha_p|} - 1} \qquad \lambda = \sqrt{10^{0.1|\alpha_s|} - 1}$$

$$\cancel{\varepsilon = \sqrt{0.5012 - 1}} \qquad \cancel{\lambda = \sqrt{10.1 - 1}}$$

$$\varepsilon = 0.997 \qquad \lambda = \sqrt{9} = 3$$

STEP 4 :

For bilinear transform,

$$\Omega_s = \frac{2}{T} \tan \frac{W_s}{2} \qquad \Omega_p = \frac{2}{T} \tan \frac{\omega_p}{2}$$

$$\boxed{\Omega_s = 30.12} \qquad \boxed{\Omega_p = 3.24}$$

STEP 5

$$N \geq \frac{\log(\lambda/\varepsilon)}{\log(\Omega_s/\Omega_p)} = \frac{\log(3.0090)}{\log(9.2963)} = \frac{0.4784}{0.9683} = 0.49$$

$N \geq 0.49 \simeq 1.$

$N = 1$

STEP 6

$\therefore$ Polynomial $= s + 1.$

STEP 7 $\quad H(s) = \frac{1}{s+1}.$

STEP 8

Denormalise.

a). Replace s with $\frac{s}{\Omega_c}$. $\quad \Omega_c = \frac{\Omega_p}{\varepsilon^{1/N}}$

$$\Omega_c = \frac{3.24}{0.997^{1/1}} = 3.249$$

$$s \longrightarrow \frac{s}{3.249}$$

$$H(s) = \frac{3.249}{s + 3.249}$$

STEP 9 $\quad s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right).$

$$H(z) = \frac{3.249}{\frac{2(1-z^{-1})}{(1+z^{-1})} + 3.249} = \frac{3.249 + 3.249z^{-1}}{2 - 2z^{-1} + 3.249 + 3.249z^{-1}}$$

$$H(z) = \frac{3.249 + 3.249z^{-1}}{5.249 + 1.249z^{-1}} = \frac{3.249 + 3.249z^{-1}}{5.249(1 + 0.2379z^{-1})}$$

Design a digital L.P Butterworth & stop Bilinear Transformation with pass band 800 rad & 1800 rad. band cut off Frequency & stop band. The pass band attenuation is −3 dB & stop band attenuation is −10 dB.

STEP 1 :

LPF, Butterworth, Bilinear.

$W_p = 800$ rad. $\quad W_s = 1800$ rad.

$\alpha_p = -3$ dB $\quad \alpha_s = -10$ dB.

STEP 2 :

STEP 3

$$\varepsilon = \sqrt{10^{0.1|\alpha_p|} - 1} \qquad \lambda = \sqrt{10^{0.1|\alpha_s|} - 1}$$

$$\cancel{\varepsilon = \sqrt{0.5012 - 1}} \qquad \cancel{\lambda = \sqrt{10.1 - 1}}$$

$$\varepsilon = 0.997 \qquad \lambda = \sqrt{9} = 3$$

STEP 4:

For bilinear transformation,

$$\Omega_s = \frac{2}{T} \tan \frac{W_s}{2}. \qquad \Omega_p = \frac{2}{T} \tan \frac{\omega_p}{2}.$$

$$\boxed{\Omega_s = 30.12.} \qquad \boxed{\Omega_p = 3.24}$$

STEP 5

$$N \geq \frac{\log(\lambda/\varepsilon)}{\log(\Omega_s/\Omega_p)} = \frac{\log(3.0090)}{\log(9.2963)} = \frac{0.4784}{0.9683} = 0.49$$

$N \geq 0.49 \simeq 1.$

$N = 1$

STEP 6

$\therefore$ Polynomial $= s + 1.$

STEP 7 $\quad H(s) = \frac{1}{s+1.}$

STEP 8

Denormalisation

a). Replace s with $\frac{s}{\Omega c}$.. $\quad \Omega c = \frac{\Omega p}{\varepsilon^{1/N}}$

$$\Omega c = \frac{3.24}{0.997^{1/1}} = 3.249$$

$$s \longrightarrow \frac{s}{3.249}$$

$$H(s) = \frac{3.249}{s + 3.249}$$

STEP 9

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right).$$

$$H(z) = \frac{3.249}{\frac{2(1-z^{-1})}{(1+z^{-1})} + 3.249} = \frac{3.249 + 3.249z^{-1}}{2 - 2z^{-1} + 3.249 + 3.249z^{-1}}$$

$$H(z) = \frac{3.249 + 3.249z^{-1}}{5.249 + 1.249z^{-1}} = \frac{3.249 + 3.249z^{-1}}{5.249(1 + 0.2379z^{-1})}$$

$$H(z) = \frac{0.619 + 0.619 z^{-1}}{(1 + \cancel{1.249}\ 0.2375 z^{-1})}$$

$b_0 = 0.619$ $b_1 = 0.6$

$\cancel{a_1 = -1.249}$

$a_1 = -0.2375.$

17/09/18

Design a Chebyshev Low Pass filter, whose Tr. fn. is

$0.8 \le |H(e^{j\omega})| \le 1$ $\quad 0 \le \omega \le 0.2\pi$

$0 \le |H(e^{j\omega})| \le 0.2.$ $\quad 0.6\pi \le \omega \le \pi$

using i) Bilinear Transformation.

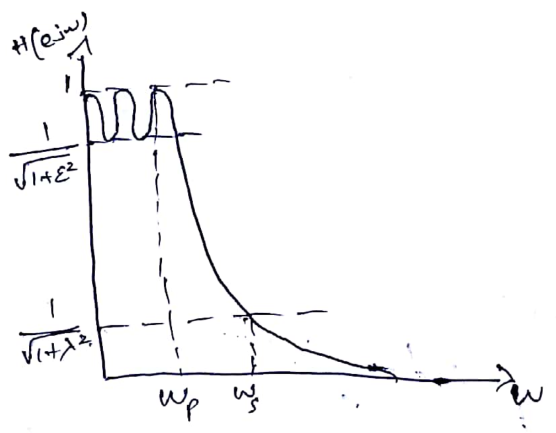
ii) Impulse invariant transformation.

STEP 1: Chebyshev LPF.

i) Bilinear Transfor. Tech.

STEP 2:

Better use Type 1.



STEP 3:

$$\cancel{\epsilon = \sqrt{10^{0.1 \alpha_p}-1}}$$

$$\frac{1}{\sqrt{1+\epsilon^2}} = 0.8$$

$$\frac{1}{1+\epsilon^2} = 0.64$$

$0.64 + 0.64\epsilon^2 = 1$

$0.64\epsilon^2 = 0.36$

$\epsilon^2 = 0.5625$

$$\varepsilon = \sqrt{0.5625} = 0.75.$$ $\boxed{\varepsilon = 0.75}$

$$\frac{1}{\sqrt{1+\lambda^2}} = 0.2 \Rightarrow \frac{1}{1+\lambda^2} = 0.04$$

$$0.04 + 0.04\lambda^2 = 1 \qquad 0.04\lambda^2 = 0.96 \Rightarrow \lambda^2 = 24$$

$$\boxed{\lambda = 4.899}$$

STEP 4:

$$\omega_p = 0.2\pi$$
$$\omega_s = 0.6\pi$$

STEP 5:

For bilinear Transformation..

$$\Omega_s = \frac{2}{T} \tan\frac{\omega_s}{2} = 2.75.$$

Assume $T = 1$ sec..

$$\Omega_p = \frac{2}{T} \tan\frac{\omega_p}{2} = 0.649$$

STEP 6:

Order $N \geq \dfrac{\cosh^{-1}(\lambda/\varepsilon)}{\cosh^{-1}(\Omega_s/\Omega_p)} = \dfrac{\cosh^{-1}(4.89/0.75)}{\cosh^{-1}(2.75/0.649)} \geq 1.21$

$$\boxed{N = 2}$$

STEP 7:

The chebyshev polynomial is

$$S_k = \Omega_p\left[-\sin\phi_k \sinh\theta + j\cos\phi_k \cosh\theta\right].$$

where $\theta = \dfrac{1}{N}\sinh^{-1}\left(\dfrac{1}{\varepsilon}\right)$ $\qquad \phi_k = \dfrac{(2k-1)\pi}{2N}$, $k = 1, 2, 3 \ldots N$

Here $k = 1, 2.$

$K=1,$

$$\phi_1 = \left\{ \frac{2(1)-1}{2(2)} \right\} \pi = \frac{\pi}{4}$$

$K=2,$

$$\phi_2 = \left\{ \frac{2(2)-1}{2(2)} \right\} \pi = \frac{3\pi}{4}$$

$$\theta = \frac{1}{2} \sinh^{-1}\left(\frac{1}{0.75}\right) = 0.549.$$

<u>K=1</u>

$$S_1 = \Omega_p \left[-\sin\phi_1 \sinh\theta + j\cos\phi_1 \cosh\theta \right]$$

$$S_1 = 0.649 \left[-\sin\left(\frac{\pi}{4}\right)\sinh(0.549) + j\cos\left(\frac{\pi}{4}\right)\cosh(0.549) \right]$$

$$S_1 = 0.649 \left[-(0.707)(0.577) + j\,0.707\,(1.155) \right]$$

$$S_1 = 0.649 \left[-0.4079 + j\,0.8166 \right]$$

$$\boxed{S_1 = -0.265 + j\,0.531}$$

$$S_2 = 0.649 \left[-\sin\left(\frac{3\pi}{4}\right)\sinh(0.549) + j\cos\left(\frac{3\pi}{4}\right)\cosh(0.549) \right]$$

$$\boxed{S_2 = -0.2656 - j\,0.531}$$

<u>STEP 8</u> :

Numerator

For even N

$$\text{Numerator} = \frac{S_1 \times S_2 \times \dots S_N}{\sqrt{1+\varepsilon^2}}$$

$$= \frac{S_1 \times S_2}{\sqrt{1+(0.75)^2}} = (-0.265 + j\,0.531)\,(-0.2656 - j$$

$$= \frac{0.07 + j0.14 - j0.14 + 0.28}{\sqrt{}} = \frac{0.3519}{1.25} = 0.$$

STEP 9 :

$$H(s) = \frac{Num}{\text{Chebishev Poly.}}$$

$$\boxed{H(s) = \frac{0.2809}{(s+0.265-j0.531)(s+0.265+j0.531)}}$$

STEP 10 :

For Bilinear Transformation.

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right).$$

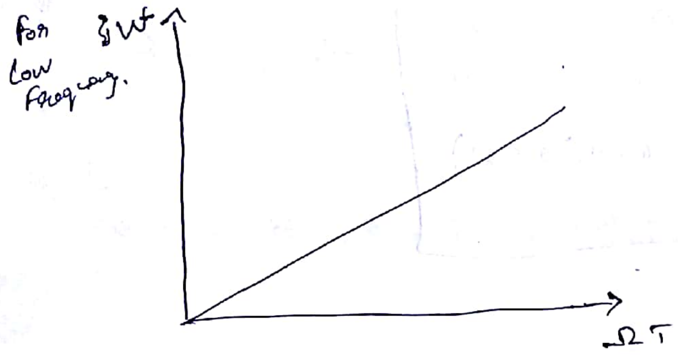
$$T = 1$$

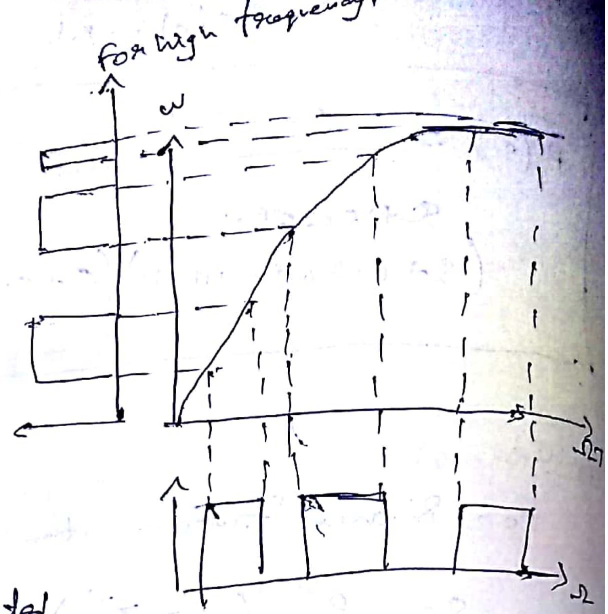
$$H(z) = \frac{0.2809}{\left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.265-j0.531\right\} \times \left\{2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+0.265+j0.531\right\}}$$

Simplify.

19/09/18 Butterworth Filter Theory:

WARPING EFFECT :





This mag-distortion is called as warping. It can be eliminated by prewarping analog filter by.

$$\Omega = \frac{2}{T} \tan \frac{\omega}{2}.$$

Questions:

Unit 2:

1. DFT Using $X(k) = \sum_{n=0}^{N-1} x(n) \, e^{-j2\pi nk/N}$ $k=0,1,2,$
2. IDFT Using $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \, e^{j2\pi nk/N}$. $n=0,1,2..N-1$
3. Butterfly diagram → DIT FFT, DIF FFT, DIT IFFT, DIF IFFT
4. Convolution → Circular & Matrix. [4m only]
5. What is Wavelet Transform & What is multiresolution [2m]

Unit 3:

1. Design of Butterworth → Bilinear / Invariant [12m]
2. Chebyshev → Bilinear.
3. $H(s) = \frac{1}{s^2+2s+1}$ → Convert to digital using Bilinear/Invariant.
4. Realisation of Filter → Direct 1 / 2 / Parallel / Cascade.

25/09/18

CHEBYSHEV FILTER:

$0.8 \leq |H(e^{j\omega})| \leq 1 \qquad 0 \leq \omega \leq 0.2\pi$

transform

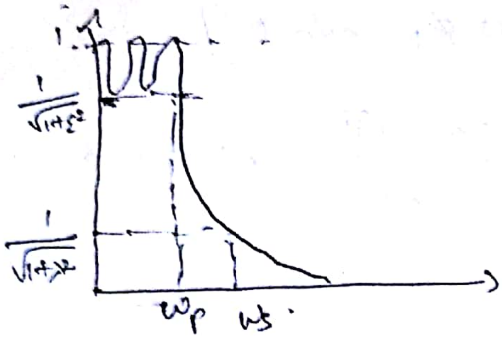
$|H(e^{j\omega})| \leq 0.2 \qquad 0.6\pi \leq \omega \leq \pi$ by impulse invariant.

STEP 1:

$\omega_p = 0.2\pi \qquad \omega_s = 0.6\pi$

$\dfrac{1}{\sqrt{1+\varepsilon^2}} = 0.8 \qquad \dfrac{1}{\sqrt{1+\lambda^2}} = 0.2$

STEP 2:



STEP 3:

$\dfrac{1}{1+\varepsilon^2} = 0.64 \qquad \dfrac{1}{1+\lambda^2} = 0.04$

$0.64 + 0.64\,\varepsilon^2 = 1 \qquad 0.04 + 0.04\,\lambda^2 = 1$

$\varepsilon^2 = \dfrac{0.36}{0.64} = 0.5625 \qquad \lambda^2 = \dfrac{0.96}{0.04} = 24$

$\boxed{\varepsilon = 0.75} \qquad \boxed{\lambda = 4.89}$

STEP 4:

$\omega_p = 0.2\pi \qquad \omega_s = 0.6\pi$

STEP 5 :

$$\Omega_s = \frac{\omega_s}{T} \qquad \Omega_p = \frac{\omega_p}{T} \qquad \Omega_s = 0.6\pi$$

$$\Omega_p = 0.2\pi$$

STEP 6 :

$$N \geq \frac{\cosh^{-1}(\lambda/\varepsilon)}{\cosh^{-1}(\Omega_s/\Omega_p)} = \frac{\cosh^{-1}(6.52)}{\cosh^{-1}(3)} = \frac{2.56}{1.76} = 1.642$$

$$\boxed{\therefore N = 2}$$

STEP 7 :

$$S_K = \Omega_p \left[-\sin\phi_K \sinh\theta + j\cos\phi_K \cosh\theta \right]$$

$$\theta = \frac{1}{N}\sinh^{-1}\left(\frac{1}{\varepsilon}\right) \qquad \phi_K = \frac{(2K-1)\pi}{2N} \qquad K = 1, 2$$

$$\theta = \frac{1}{2}\sinh^{-1}(1.333) = 0.549$$

K = 1

$$\phi_1 = \frac{\pi}{4}$$

K = 2

$$\phi_2 = \frac{3\pi}{4}$$

$$S_1 = 0.2\pi \left[-\sin\frac{\pi}{4} \sinh(0.549) + j\cos\left(\frac{\pi}{4}\right)\cosh(0.549) \right]$$

$$S_2 = 0.2\pi \left[-\sin\frac{3\pi}{4} \sinh(0.549) + j\cos\left(\frac{3\pi}{4}\right)\cosh(0.549) \right]$$

$$S_1 = 0.2\pi \left[-0.707(0.576) + j(+0.707)(1.154) \right]$$

$$S_1 = 0.2\pi \left[-0.407 + j0.815 \right]$$

$$S_1 = 0.628\left[-0.407 + j0.815 \right] = \boxed{-0.255 + j0.512}$$

$S_2 = 0.2\pi\left[-0.707\,(0.576) + j\,(-0.707)\,(0.549)\right]$

$S_2 = 0.628\left[-0.407 - j\,0.815\right]$

$$\boxed{S_2 = -0.255 - j\,0.5129}$$

Step 8 :

$$\text{Numerator} = \frac{S_1 \times S_2}{\sqrt{1+\epsilon^2}}$$

$$= \frac{(-0.255 + j0.512)\,(-0.255 - j\,0.5129)}{\sqrt{1+0.5625}}$$

$$= \frac{(-0.255 + j0.512)\,(-0.255 - j0.5129)}{1.25}$$

$$= 0.261$$

$\cancel{H(s)}$ =

Step 9 :

$$H(s) = \frac{0.261}{(s+0.255-j0.512)\,(s+0.255+j0.512)}$$

Step 10 :

$$\frac{1}{s-P_k} \longrightarrow \frac{1}{1-e^{P_kT}z^{-1}}$$

$$\frac{0.261}{(s+0.255-j0.512)\,(s+0.255+j0.512)} = \frac{A}{s+0.255-j0.512} + \frac{B}{s+0.255+j0.512}$$

$$A(s+0.255+j0.512)+B(s+0.255-j0.512)=0.$$

Let

$$s=-0.255-j0.512$$

$$B(\cancel{0}.-j1.024)=0.261$$

$$B=\frac{-0.254}{j}\times\frac{j}{j}=+0.254j$$

Let $s=-0.255+j0.512$.

$$\therefore A(j1.024)=0.261$$

$$A=\frac{0.254}{j}\times\frac{j}{j}=-0.254j$$

$$H(s)=\frac{-0.254j}{s+0.255-j0.512}+\frac{0.254j}{(s+0.255+j0.512)}$$

$$\therefore P_1=-0.255+j0.512 \qquad P_2=-0.255-j0.512.$$

$$H(z)=\frac{-0.254j}{1-e^{(-0.254+j0.512)}z^{-1}}+\frac{0.254j}{1-e^{(-0.255-j0.5129)}z^{-1}}$$

$$H(z)=\frac{-j0.254}{1-e^{-0.254}e^{j0.5129}z}+\frac{j0.254}{1-e^{-0.254}e^{-j0.512}z^{-1}}$$

$$H(z) = \frac{-j0.254}{1-0.77e^{j0.5129}z^{-1}} + \frac{j0.254}{1-0.77\,e^{-j0.512}z^{-1}}$$

$$= \frac{-j0.254\left[1-0.77e^{-j0.512}z^{-1}\right] + j0.254\left[1-0.77e^{j0.5129}z^{-1}\right]}{\left(1-0.77e^{j0.5129}z^{-1}\right)\left(1-0.77\,e^{-j0.5129}z^{-1}\right)}$$

$$= \frac{-j0.254 + 0.1955\,j\,e^{-j0.512}z^{-1} + j0.254 - 0.1955j\,e^{j0.512}z^{-1}}{1-0.77e^{-j0.512}z^{-1} - 0.77e^{j0.512}z^{-1} + 0.5929\,e^{j0.529}e^{-j0.529}z^{-2}}$$

$$= \frac{0.1955z^{-1}j\left(e^{-j0.512} - e^{j0.512}\right)}{1-0.77e^{-j0.512}z^{-1} - 0.77e^{j0.512}z^{-1} + 0.5929z^{-2}}$$

$$= \frac{0.1955z^{-1}j\left(-2j\sin 0.512\right)}{1-0.77\left(e^{-j0.512}+e^{j0.512}\right)z^{-1} + 0.5929z^{-2}}$$

$$= \frac{0.1955z^{-1}\left(-j0.979\right)}{\cancel{1+0.77(-j0.979)z^{-1}+0.5929z^{-2}}\;\; 1-0.77(2)\cos 0.512z^{-1} + 0.5929z^{-2}}$$

$$= \cancel{0.1955z} \;\; \frac{-0.191j^2 z^{-1}}{\cancel{1+0.753jz^{-1}+0.5929z^{-2}}}$$

$$= \frac{+0.191z^{-1}}{1-\left(1.34z^{-1}-0.592z^{-2}\right)} \quad //$$