05/07/16

UNIT - 1 - DISCREET TIME SIGNALS (S/g) & SYSTEMS

IMPORTANT FORMULA:

- $\sin^2\theta = \frac{1-\cos 2\theta}{2}$
- $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\frac{e^{j\theta}+e^{-j\theta}}{2} = \cos\theta$

$\frac{e^{j\theta}-e^{-j\theta}}{2j} = \sin\theta$

- $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$

Fundamental Period $N = \frac{2\pi}{\omega}$

Even Component in s/g $x_e(n) = \frac{x(n)+x(-n)}{2}$

Odd Component in s/g $= \frac{x(n)-x(-n)}{2}$

Energy of the s/g $= E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$

Power of the s/g $= P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$; $\sum_{n=k}^{\infty} a^n = \frac{a^k}{1-a}$

$\sum_{n=-N}^{N} 1 = 2N+1$; $\sum_{n=0}^{N} 1 = N+1$

Signal

* Conveys or Carries message.

i) Continuous s/g $\rightarrow x(t)$

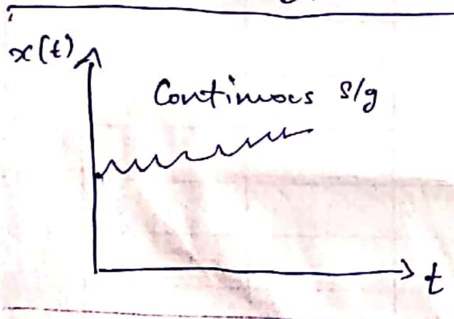
ii) Discreet s/g $\rightarrow x[n]$ (or) $x(n)$

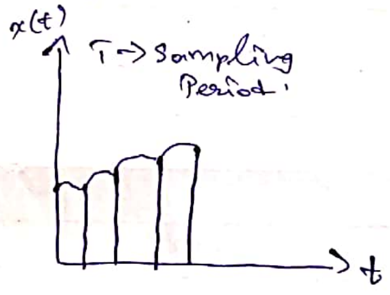
Eg. $x(n) = \{1, 2, \underset{\uparrow}{3}, 5, 6\}$.

$\uparrow \rightarrow$ zeroth location.

$x(-2)=1$; $x(-1)=2$; $x(0)=3$; $x(1)=5$; $x(2)=6$.

Generation of Discreet s/g:





Representation of Discreet s/g. (2m)

i) Functional Repren.

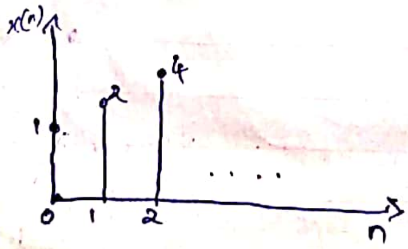
$x(n) = 2^n$

(i.e) $x(0) = 1$,

$x(1) = 2$

$x(2) = 4$.

ii) Graphical Repn.



iii) Sequencial Repn.

$x(n) = \{\underset{\uparrow}{1}, 2, 4 \ldots\}$

iv) Tabular Repn.

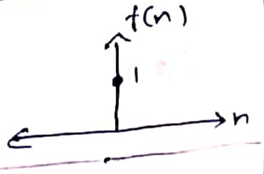
n	0	1	2	...
x(n)	1	2	4	...

Standard Test s/g

i) Digital Impulse S/g.

$f(n) = 1 \quad n = 0$

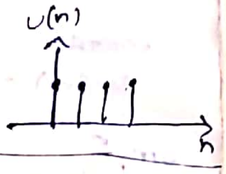
$\quad " \quad = 0 \quad n \neq 0$



ii) Unit Step.

$u(n) = 1 \;;\; n \geq 0$

$\quad " \quad = 0 \;;\; n < 0$



iii) Ramp s/g.

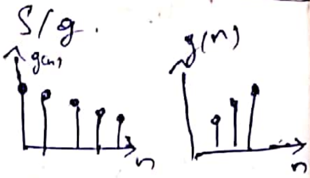
$u_r(n) = n \;;\; n \geq 0$

$\quad " \quad = 0 \;;\; n < 0$

iv) Exponential S/g.

$g(n) = a^n \;;\; n \geq 0$

$\quad " \quad = 0 \;;\; n < 0$



MATHEMATICAL OPERATIONS ON DISCRETE TIME S/g. (DT)

1) Scaling of DT s/g.

Amplitude Scaling.

$y(n) = A\,x(n).$

Let $A = 2$, $x(0) = 5$, $x(1) = 6$.

$y(0) = 2 \times 5 = 10$

$y(1) = 2 \times 6 = 12.$

Time Scaling

Down Scaling.

$x(n) \longrightarrow x(Dn)$ where D = integer.

Let $x(0)=1$, $x(1)=2$, $x(3)=4$; $x(4)=8$.
$D = 2$.

$x(0) = x(2\times0) = x(0) = 1$

$x(1) = x(2\times1) = x(2) = 3.$

$x(2) = x(2\times2) = x(4) = 8.$

Up Scaling.

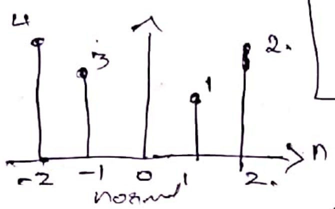
$x(n) \longrightarrow x(n/D).$

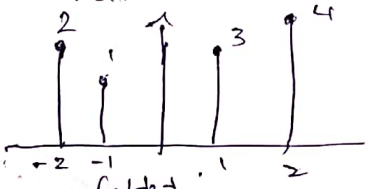
$x(4) = x(4/2) = x(2) = 3$

$x(6) = x(6/2) = x(3) = 4.$

2) Folding.

$x(n) \longrightarrow x(-n)$

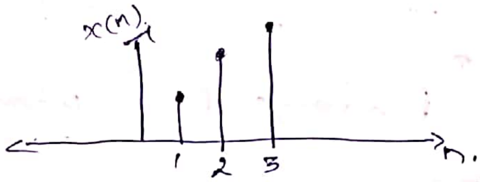




3) Time Sifting.

$x(n) \longrightarrow x(n-m).$ (or) $x(n) = x(n+m).$

Eg.

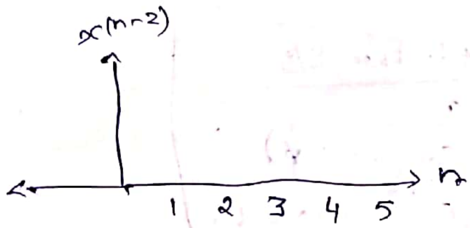


$x_1(n) = x(n-2).$

↓ delayed s/g

When $n=4$, $x_1(4) = x(4-2) = x(2).$

$n=5$, $x_2(5) = x(5-2) = x(3).$



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4) Addition of DT

$x_1(n) = \{2, 1, 1\}$; $x_2(n) = \{2, 0, 1\}$.

$y(n) = x_1(n) + x_2(n)$. $= \{2+2, 0+1, 1+1\} = \{4, 1, 2\}$.

5). Multiplication of DT

$x_1(n) = \{\underset{\uparrow}{2}, 1, 3\}$; $x_2(n) = \{2, \underset{\uparrow}{1}, 1\}$.

$y(n) = x_1(n) \times x_2(n)$.

*. Note.
If arrow not marked assume first sample as zeroth location

If any position is missing, then add zeros.

$x_1(n) = \{0, \underset{\uparrow}{2}, 1, 3\}$. $x_2(n) = \{2, \underset{\uparrow}{1}, 1, 0\}$

$y(n) = \{0, \underset{\uparrow}{2}, 1, 0\}$.

Types of Signals

1) Deterministic ~~and random~~ s/g can be represented in mathematical form

eg . step, ramp, AC etc.

Random s/g : It can't represent in mathematical form eg. noise, No. of accidents in a year etc.

2) Periodic & aperiodic s/g.

If the DT s/g $x(n)$, satisfies the condition,

$x(n+N) = x(n)$ for integer values of N,

Then its periodic. If not aperiodic s/g (or)

Fundamental period $N = \frac{2\pi}{\omega}$ = rational.
$\longrightarrow$ Periodic.

$N = \frac{2\pi}{\omega}$ = irrational
$\longrightarrow$ aperiodic.

Std. Elec S/G

$\cos \omega t$

$\cos(\omega n + \phi)$

$e^{j\omega t}$

$e^{j\omega t + \phi}$

Check whether the given signal is periodic or aperiod. Calculate the fundamental period for the given signal. a) $x(n) = \cos\left(\frac{5\pi n}{9} + 1\right)$.

Soln. $N = \frac{2\pi}{\omega}$ Comparing $x(n)$ with $\cos(\omega t + \phi)$.

$\omega = \frac{5\pi}{9}$ $N = \frac{2\pi}{\frac{5\pi}{9}} = \frac{18}{5}$ = Rational No.

$\therefore$ S/g is periodic.

b) $x(n) = \sin\left(\frac{n}{9} - \pi\right)$.

$N = \frac{2\pi}{\omega}$ $\omega = 1/9$ $\therefore N = 18\pi$ = irrat.

$\therefore$ S/g is aperiodic.

a) $x(n) = e^{j7\pi n/4}$

$N = \frac{2\pi}{\omega}$ $\omega = \frac{7\pi}{4}$ $\therefore N = \frac{8}{7}$ $\therefore$ Periodic S/g.

e) $x(n) = 2\cos\frac{5n}{3} + 3e^{j3\pi n/4}$.

$= 2\left(\cos\frac{3\pi n}{4} + j\sin\frac{3\pi n}{4}\right)$

$N_1 = \frac{2\pi}{\omega_1}$ $N_2 = \frac{2\pi}{\omega_2}$. $\omega_1 = \frac{5}{3}$ $\omega_2 = \frac{3\pi}{4}$

$\therefore N_1 = \frac{6\pi}{5}$ $N_2 = \frac{8}{3}$

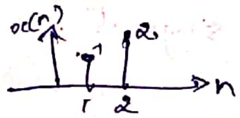
$N = \frac{N_1}{N_2} = \frac{\frac{6\pi}{5}}{\frac{8}{3}} = \frac{18\pi}{40}$ irrational.

$\therefore$ S/g is aperiodic.

09/07/18

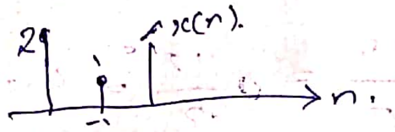
Causal, Non-Causal & Anti Causal s/g.

Causal: s/g defined for $n \geq 0$ eg.



Anti Causal: s/g defined for $n \leq 0$.

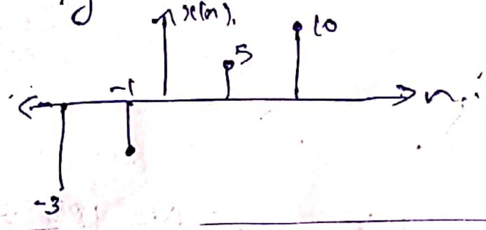
eg



$$\frac{1}{\infty} = 0$$

$$\frac{1}{0} = \infty$$

Non-Causal: s/g defined for $n \leq 0$. and $n > 0$.



Even & Odd Signal.

Even: $x(n) = x(-n)$ Odd: $x(n) = -x(-n)$.

Even part $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

Odd part $x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

Determine the even & odd parts of the signal.

\# 1) $x(n) = 3^n$

Soln. $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$x(-n) = 3^{-n}$

$$\boxed{x_e(n) = \frac{1}{2}[3^n + 3^{-n}]}$$

$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

$$\boxed{x_o(n) = \frac{1}{2}[3^n - 3^{-n}]}.$$

2) $x(n) = e^{\frac{j\pi}{5}n}$.

Soln. $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

$x(-n) = e^{\frac{j\pi}{5}(-n)} = e^{-\frac{j\pi}{5}n}$.

$$\boxed{x_e(n) = \frac{1}{2}\left[e^{\frac{j\pi}{5}n} + e^{-\frac{j\pi}{5}n}\right]}$$

$= \frac{1}{2}\left(2\cos\frac{\pi}{5}n\right) = \cos\frac{\pi}{5}n$.

$$\boxed{\therefore x_o(n) = j\sin\left(\frac{\pi}{5}n\right)}$$

Energy & Power s/g

$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$ J

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$ W

For energy s/g E = finite value

$P = 0$ $0 < E < \infty$

For Power s/g $E = \infty$

P = finite value

$0 < P < \infty$

$\therefore$ If given, $e^{jn\pi}$, make it like $\cos\theta + j\sin\theta$.

Determine whether the following s/g are Energy / Power.

a) $x(n) = \left(\frac{1}{4}\right)^n u(n)$.

Soln. $E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$

$= \lim_{N\to\infty} \sum_{n=-N}^{N} \left(\frac{1}{4}\right)^{2n} u(n)$.

$= \lim_{N\to\infty} \sum_{n=0}^{N} \left(\frac{1}{4^2}\right)^n = \sum_{n=0}^{\infty} \left(\frac{1}{16}\right)^n$

Using $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$ $\quad a = \frac{1}{16}$

$= \frac{1}{1-1/16} = \frac{16}{15}$ = finite Value.

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$= \lim_{N\to\infty} \frac{1}{2N+1} \left\{ \frac{16}{15} \right\}$.

$= \frac{1}{2(\infty)+1} \times \frac{16}{15} = 0$

$\therefore P = 0$.

$\therefore$ Energy s/g.

b) $x(n) = e^{j\frac{\pi}{3}n}$

Soln. $E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$

Using $e^{j\theta} = \cos\theta + j\sin\theta$.

$x(n) = \cos\frac{\pi}{3}n + j\sin\frac{\pi}{3}n$.

$|x(n)| = \sqrt{(\cos^2\frac{\pi}{3}n + \sin^2\frac{\pi}{3}n}$

$|x(n)| = 1$

$E = \lim_{N\to\infty} \sum_{n=-N}^{N} 1$

$= \lim_{N\to\infty} 2N+1$

$E = \infty$

Power $= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$

$= \lim_{N\to\infty} \sum_{n=-N}^{N} = \lim_{N\to\infty} \frac{1}{2N+1} (2N+1)$

$\therefore P = 1\,W$ = finite.

$\therefore$ The signal is Power s/g.

c) $x(n) = \sin\frac{\pi}{4}n$.

Soln. $E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$.

$= \frac{1}{2}\lim_{N\to\infty} \sum_{n=-N}^{N} \left[1 - \cos\frac{2\pi}{4}n\right]$

$= \frac{1}{2}\left\{ \lim_{N\to\infty} \sum_{n=-N}^{N} 1 - \lim_{N\to\infty} \sum_{n=-N}^{N} \cos\frac{\pi}{2}n \right\}$

$= \frac{1}{2}\lim_{N\to\infty} \sum_{n=-N}^{N} 1 = \frac{1}{2}\lim_{N\to\infty} 2N+1$.

$E = \infty$

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum |x(n)|^2$.

$\therefore P = 1/2$. $\therefore$ Power s/g.

d) $x(n) = (1/3)^n u(n-3)$.

Soln. $E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$.

$= \lim_{N\to\infty} \sum_{n=-N}^{N} (1/3)^{2n} u(n-3)$

$= \lim_{n\to\infty} \sum_{n=3}^{N} (1/3^2)^n$.

$\boxed{u(n) \quad n=0 \quad u(n-3) \quad n=3}$

$= \lim_{N\to\infty} \sum_{n=3}^{N} (1/9)^n$

$= \sum_{n=3}^{\infty} (1/9)^n$ $\quad \sum_{n=k}^{\infty} a^n = \frac{a^k}{1-a}$. $\therefore k=3 \quad a = 1/9$.

$= \frac{(1/9)^3}{1-(1/9)} = \frac{1}{648}$ = finite.

E = finite.

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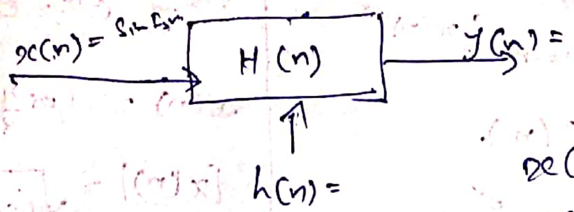
$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$ $\quad P = 0 \;,\; E = \text{finite}$

∴ Energy Signal.

SYSTEMS :



$x(n) \rightarrow$ i/p s/g

$y(n) =$ o/p s/g.

$h(n) \rightarrow$ impulsive Response,

Types of System

1) Static (memory less) [Present] & Dynamic (memory) Sys [Present / Past / Future] [B.

2) Time variant & Time invariant Sys.

3) Linear & Non-linear Sys.

4) Causal [Present / past] & Non-Causal [Present / Past / Future] Sys.

5) Stable & Unstable Sys.

STATIC & DYNAMIC :

A discreat time system is called as static or memory less system, if its output at any instant depends on the input sample at the same time.

$y(n) = a\,x(n)$.

Soln. $n = 0,\; y(0) = a\,x(0)$

$n = -2,\; y(-2) = a\,x(-2)$

$n = 4,\; y(4) = a\,x(4)$

It is a Static System as the Present Output depends only on Present Input.

$y(n) = a\,x(n+5)$

$n = 0,\; y(0) = a\,x(0+5) = a\,x(5)$

$n = -4,\; y(-4) = a\,x(-4+5) = a\,x(1)$

$n = 5,\; y(5) = a\,x(10)$

It is a Dynamic System as the Present Output depends on the future input / past / Present

Causal & Non Causal.

* Present o/p depends on present i/p or/and past i/p, it is called Causal System.

* Present o/p depends on future i/p then it's called Non-Causal System.

1) $y(n) = x(n) + x(n-3)$

$y(0) = x(0) + x(0-3) = x(0) + x(-3)$

$y(-1) = x(-1) + x(-4)$

$y(2) = x(2) + x(-1)$

∴ Its Causal System, Since the present o/p depends on past i/p.

2) $y(n) = 5x(n^2)$.

$n=0$ $y(0) = y(0) = 5x(0)$.

$n=1$ $y(1) = 5x(1)$.

$n = -1$, $y(-1) = 5x(1)$.

∴ Its Non Causal System. Since present output depends on future i/p.

A

Check whether the given signal is Static (or) Dynamic, Causal or Non-Causal.

1) $y(n) = x(n^2) + x(n+4)$.

$n=0$, $y(0) = x(0) + x(4)$.

$n=1$, $y(1) = x(1) + x(5)$

$n=-1$, $y(-1) = x(1) + x(3)$

∴ The System is Dynamic as the present output depends on future Input.

The System is also Non-Causal.

2) $y(n) = 6x(n+1) + x(n)$.

$n=0$, $y(0) = 6x(1) + x(0)$

$n=1$, $y(1) = 6x(2) + x(1)$

$n=-1$ $y(-1) = 6x(0) + x(-1)$

∴ The System is Dynamic and Non-Causal as the present Output depends on future Input.

TIME VARIANT & INVARIANT SYSTEM:

A System is said to be time invariant if its input output characteristics do not change with time.

a) $y(n) = x(n) + x(n-1)$.

Soln:

Delay the sy. i/p s/g $n \to$ "n-m".

$y(n) = x(n-m) + x(n-m-1) \longrightarrow ①$

Delay the o/p s/g $n \to$ "n-m"

$y(n-m) = x(n-m) + x(n-m-1) \to ②$

Comp ① & ②

$y(n) = y(n-m)$

∴ Time Invariant Sys

b) $y(n) = nx(n)$.

Soln.

Delay the i/p.

$y(n) = n\,x(n-m) — ①$.

Delay the o/p.

$y(n-m) = (n-m)\;x(n-m) — ②$

$y(n) \neq y(n-m)$.

∴ Time Variant System.

c) $y(n) = x(-n)$.

Delay the i/p s/g

$y(n) = x(m-n)$

Delay the o/p s/g

$y(n-m) = x(m-n)$.

∴ $y(n) = y(n-m)$.

∴ The System is Time Invariant

d) $y(n) = x(n) - b\,x(n-1)$.

Delay the i/p.

$y(n) = x(n-m) - b\,x(m-m-1)$

Delay the o/p.

$y(n-m) = x(n-m) - b\,x(n-m-1)$

∴ $y(n) = y(n-m)$.

∴ The System is Time Invariant.

LINEAR & NON-LINEAR SYSTEM:

A System is Said to be Linear, if it satisfies the Super position Theorem.

$$H\{a x_1(n) + b x_2(n)\} = H\{a x_1(n)\} + H\{b x_2(n)\}.$$

Check whether the System is Linear or Not.

a) $y(n) = n\,x(n)$

Soln.

$y_1(n) = n x_1(n)$

$y_2(n) = n\,x_2(n)$

Super Position Theorem

$x_3(n) = a x_1(n) + b x_2(n)$,

$y_3(n) = n\,x_3(n)$.

$y_3(n) = n\{a x_1(n) + b x_2(n)\}$

$y_3(n) = n\,a x_1(n) + n\,b x_2(n)$

$y_3(n) = a y_1(n) + b y_2(n)$.

∴ The System is Linear

b) $y(n) = x(n^2)$.

$y_1(n) = x_1(n^2)$.

$y_2(n) = x_2(n^2)$.

Super Position Theorem.

$x_3(n^2) = a x_1(n^2) + b x_2(n^2)$.

$y_3(n) = x_3(n^2)$.

$y_3(n) = \{a x_1(n^2) + b x_2(n^2)\}$

$y_3(n) = a\,y_1(n) + b\,y_2(n)$.

∴ The System is Linear.

c). $y(n) = x^2(n)$

$y_1(n) = x_1^2(n)$.

$y_2(n) = x_2^2(n)$

Super Position Theorem.

$x_3(n) = a\,x_1(n) + b x_2(n)$.

$y_3(n) = x_3^2(n)$.

$y_3(n) = \{a x_1(n) + b x_2(n)\}^2$

$y_3(n) = a^2 x_1^2(n) + b^2 x_2(n) + 2ab\,x_1(n) x_2(n)$

$y_3(n) = a^2 y_1(n) + b^2 y_2(n) + 2ab\,x_1(n) x_2(n)$.

∴ The System is Non-Linear.

d) $y(n) = 3x(n) + \frac{1}{x(n-2)}$.

$y_1(n) = 3x_1(n) + \frac{1}{x_1(n-2)}$.

$y_2(n) = 3x_2(n) + \frac{1}{x_2(n-2)}$.

Super Position Theorem.

$x_3(n) = a x_1(n) + b x_2(n)$

$x_3(n-2) = a x_1(n-2) + b x_2(n-2)$.

$y_3(n) = 3x_3(n) + \frac{1}{x_3(n-2)}$.

$= 3a x_1(n) + 3b x_2(n) + \frac{1}{a x_1(n-2) + b x_2(n-2)}$

$y_3(n) \neq a\,y_1(n) + b\,y_2(n)$

Non-Linear System

Check whether the following are Static, Causal, Linear, Time Varied

1) $y(n) = x^2(n) + x(n+3)$.

$n=0$, $y(0) = x^2(0) + x(3)$

$n=1$, $y(1) = x^2(1) + x(4)$.

$n=-1$, $y(-1) = x^2(-1) + x(2)$.

$\therefore$ The System is Dynamic, and Non-Causal as the present output depends on future input.

Delaying the i/p sig.

$y(n) = x^2(n-m) + x(n-m+3)$.

Delaying the o/p sig

$y(n-m) = x^2(n-m) + x(n-m+3)$.

$y(n) = y(n-m)$. $\therefore$ The System is Time Invariant

$y_1(n) = x_1^2(n) + x_1(n+3)$

$(x_1(n))^2$

$y_2(n) = x_2^2(n) + x_2(n+3)$

Super Position Theor

$x_3(n) = a\,x_1(n) + b\,x_2(n)$.

$x_3(n+3) = a\,x_1(n+3) + b\,x_2(n+3)$.

$y_3(n) = x_3^2(n) + x_3(n+3)$

$= a^2 x_1^2(n) + b^2 x_2^2(n) + 2ab\,x_1(n)\,x_2(n)$
$+ a\,x_1(n+3) + b\,x_2(n+3)$.

$\therefore$ The System is Non-Linear.

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STABLE & UNSTABLE :

A System is Said to be (BIBO) STABLE, if and only if every bounded input produces a bounded output.

The Condition for Stability

$$\boxed{\sum_{n=-\infty}^{\infty} |h(n)| < \infty}$$

Test the Stability of LTI System, whose Impulse response are given by,

1). $h(n) = 0.2^n u(n)$.

Soln. Condition for Stability.

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty \qquad \sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} (0.2)^n u(n).$$

$$\sum_{n=0}^{\infty} (0.2)^n = \frac{1}{1-0.2} = \frac{1}{0.8} < \infty \qquad \sum_{n=0}^{\infty} a^n \; \frac{1}{1-a}.$$

$\therefore$ The System is Stable.

---

2). $h(n) = 4^n u(-n+3)$

Soln. Condition for Stability $\sum_{n=-\infty}^{\infty} h(n) < \infty$

$$\sum_{n=-\infty}^{\infty} 4^n u(-n+3)$$

$$\sum_{n=-\infty}^{3} 4^n. \quad \text{Reversing the fen.}$$

$$\sum_{n=3}^{\infty} 4^{-n} = \frac{\left(\frac{1}{4}\right)^3}{1-\frac{1}{4}} = \frac{\frac{1}{4^3}}{\frac{3}{4}} = \frac{1}{4^2} \times \frac{1}{3} = \frac{1}{48}$$

$$\frac{1}{48} < \infty$$

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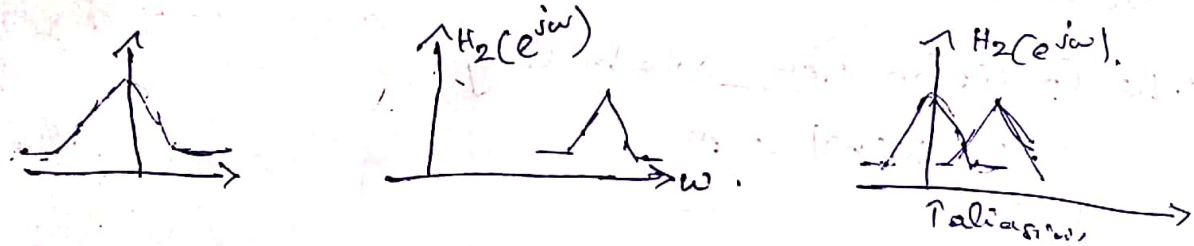
17/07/18

Aliasing

* Spectral overlap.



Linear Convolution ($*$).

$\times\ \&\ + \Rightarrow *$

$$y(n) = x(n) * h(n) = \sum_{m=-\infty}^{\infty} x(m)\, h(n-m).$$

i) Graphical mtd.

ii) Matrix mtd.

iii) Tabular mtd.

---

1) Determine the o/p response of the LTI system whose i/p $x(n)$ & impulse response $h(n)$ are

$$x(n) = \{\underset{\uparrow}{1}, 2, 0.5, 1\} \quad h(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$$

$$y(n) = x(n) * h(n) = \sum_{m=-\infty}^{\infty} x(m)\, h(n-m).$$

$$y(0) = \sum_{m=-\infty}^{\infty} x(m)\, h(-m).$$

$$y(0) = x(-3) \times h(-3) + \dots$$

$$= (1\times 0) + (1\times 0) + (1\times 0) + \dots = 1$$

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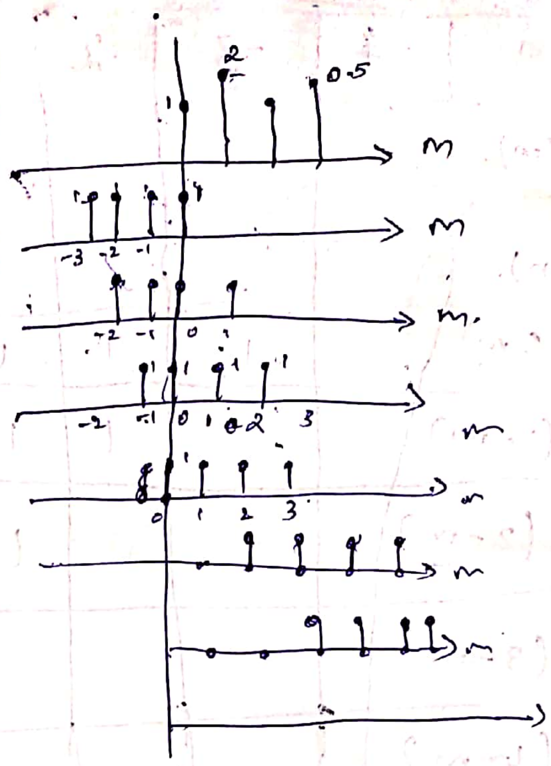


$$y(1) = \sum_{m=-\infty}^{\infty} x(m)\,h(1-m) = 2+1 = 3$$

$$y(2) = \sum_{m=-\infty}^{\infty} x(m)\,h(2-m).$$

$$= 1+2+0.5 = 3.5$$

$$y(3) = \sum_{m=-\infty}^{\infty} x(m)\,h(3-m)$$

$$= 1+2+0.5+1 = 4.5$$

$y(4) = 3.5$

$y(5) = 1.5$

$y(6) = 1$

$y(7) = 0$

MATRIX METHOD.

$x(n)$ \ $h(n)$	1	1	1	1
1	1	1	1	1
2	2	2	2	2
0.5	0.5	0.5	0.5	0.5
1	1	1	1	1

$y(n) = \{1, 3, 3.5, 4.5, 3.5, 1.5, 1\}$

18/07/18

TABULAR METHOD !

$x(n) = \{1, 2, 0.5, 1\}.$

$h(n) = \{1, 1, 1, 1\}.$

$$y(n) = \sum_{m=-\infty}^{\infty} x(m)\,h(n-m)$$

m	-4	-3	-2	-1	0	1	2	3	4	5	6
x(m)				1	2	0.5	1				
h(n).				1	1	1	1				
h(-m)		1	1	1	1						
h(1-m)			1	1	1	1					
h(2-m)				1	1	1	1				
h(3-m)					1	1	1	1			
h(4-m)						1	1	1	1		
h(5-m)							1	1	1	1	
h(6-m).								1	1	1	1

$$y(0) = \sum_{m=-\infty}^{\infty} x(m)\, h(-m) = 1.$$

$$y(1) = \sum_{m=-\infty} x(m)\, h(1-m) = 3$$

$$y(2) = \sum_{m=-\infty} x(m)\, h(2-m) = 3.5.$$

$y(3) = 4.5$

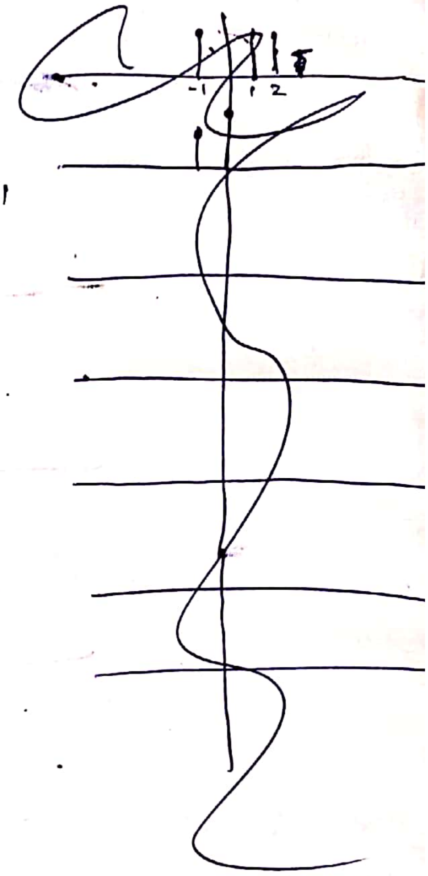
$y(4) = 3.5$

$y(5) = 1.5$

$y(6) = 1.$

\# $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$ $h(n) = \{1, \underset{\uparrow}{2}, 2, 1\}$

$$y(n) = \sum_{m=-\infty}^{\infty} x(m)\, h(-m)$$



$n = -1$

$$y(-1) = \sum_{m=-\infty}^{\infty} x(m)\, h(-1-m) = 1 \times 1 = 1$$

$n = 0$ $\sum_{m=-\infty}^{\infty} = 2 \times 1 + 2 \times 1 = 4.$

$n = 1$

$y(1) = 7.$

$n = 2$

$y(2) = 8.$

$y(3) = 6$

$y(4) = 3$

$y(5) = 1.$

23/07/18

Find the convolution of two finite sequences,

$h(n) = a^n u(n)$

$x(n) = b^n u(n)$.

Soln.

$y(n) = x(n) * h(n)$.

$$= \sum_{m=-\infty}^{\infty} x(m)\, h(n-m)$$

$$= \sum_{m=-\infty}^{\infty} b^m u(m)\, a^{n-m} u(m).$$

$$= \sum_{m=0}^{\infty} b^m a^{n-m}$$

but since finite sequence,

$$= \sum_{m=0}^{n} b^m a^{n-m}. = \sum_{m=0}^{n} b^m a^n a^{-m}$$

$$= a^n \sum_{m=0}^{n} \left(\frac{b}{a}\right)^m \qquad \left[\because \sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}\right]$$

$$= a^n \left\{\frac{1-\left(\frac{b}{a}\right)^{n+1}}{1-\frac{b}{a}}\right\}$$

---

ii) $h(n) = \left(\frac{1}{2}\right)^n u(n)$

$x(n) = 2^n u(n)$.

Soln. $y(n) = x(n) * h(n)$.

$$= \sum_{m=-\infty}^{\infty} x(m)\, h(n-m).$$

$$= \sum_{m=-\infty}^{\infty} 2^m u(m) \left(\frac{1}{2}\right)^{n-m} u(m).$$

$$= \sum_{m=0}^{n} 2^m \left(\frac{1}{2}\right)^{n-m}$$

$$= \sum_{m=0}^{n} 2^m \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^{-m}$$

Using. $\left[\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}\right]$,

$$y(n) = \left(\frac{1}{2}\right)^n \left[\frac{4^{n+1}-1}{3}\right]$$

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25/07/18

SAMPLING THEOREM:

$f_s \geq 2 f_m$ ← Max freq.

Sampling freq.

Nyquist rate $f_s = 2 f_m$

Compute the nyquist sampling frequency for the following signals

i) $x(t) = 3 \cos 4t$

Soln. $f_s = 2 f_m$ → Nyquist rate.

$\omega_s = 2 \omega_m$

$\omega_m = 4$

$\boxed{\omega_s = 8}$ rad / sec

$\omega_s \geq 8$ rad/sec → sampling freq.

ii) $x(t) = 4 \sin c \frac{3t}{\pi}$

$$\omega_s = \frac{3}{\pi} \times 2 = \frac{6}{\pi}$$

$$\omega_s \geq \frac{6}{\pi}$$

Unit - 1 :-

- Types of S/g.
- Types of System
- Convolution (Graph & Table).
- Impulse Response (Z-transform & 24/07/18).
- Aliasing, Quantisation, Coding → theory