26/07/18

UNIT - 2

Discrete Transform

1. Discrete Fourier Transform (DFT).

2. Fast Fourier Transform

3. Wavelet Transform

1) DISCRETE FOURIER TRANSFORM (DFT)

The DFT of any seq. is given by,

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

$k = 0, 1, \ldots N-1.$

The IDFT of any seq. is given by

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k)\, e^{j2\pi kn/N}$$

$n = 0, 1, \ldots N-1$

Calculate the DFT of

1) $x(n) = \{\overset{x(0)}{1}, \overset{x(1)}{0}, \overset{x(2)}{1}, \overset{x(3)}{0}\}$

Soln.

$N = 4.$

The DFT is given by,

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N}$$

$k = 0, 1, \ldots, N-1$

$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi nk/4}$$

$k = 0, 1, 2, 3.$

$K=0$

$$X(0) = \sum_{n=0}^{3} x(n)\, e^{0} = \sum_{n=0}^{3} x(n) = x(0)+x(1)+x(2)+x(3)$$

$$= 1+0+1+0 = 2.$$

$K=1$

$$X(1) = \sum_{n=0}^{3} x(n)\, e^{\frac{-j\pi n}{2}} = x(0)\, e^{0} + \cancel{x(1)\, e^{\frac{-j\pi}{2}}} + x(2)\, e^{-j\pi 2/2}$$

$$+ \cancel{x(3)\, e^{-j\pi 3/2}}.$$

$$= x(0) + x(2)\, e^{-j\pi} = 1+(1)(\cos\pi - j\sin\pi)$$

$$= 1+(-1-0) = 1-1 = 0$$

$K=2$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n 2/2} = \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$

$$= x(0)\, e^{0} + \cancel{x(1)\, e^{-j\pi}} + x(2)\, e^{-j2\pi} + \cancel{x(3)\, e^{-j3\pi}}$$

$$= 1 + e^{-j2\pi} = 1+(\cos 2\pi - j\sin 2\pi) = 2.$$

$K=3.$

$$X(3) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n\frac{3}{2}} = \sum_{n=0}^{3} x(n)\, e^{-j\pi n\frac{3}{2}}.$$

$$= x(0)\, e^{0} + x(1)\, e^{-j\pi 3/2} + x(2)\, e^{-j\pi 3} + x(3)\, e^{-j 9\pi/2}.$$

$$= 1 + 0 + e^{-j3\pi} + 0$$

$$= \cancel{1+e} \quad 1+(\cos 3\pi - j\sin 3\pi).$$

$$= 1+(-1-0) = 0$$

$$\boxed{X(k) = \{2, 0, 2, 0\}}$$

#2) $x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$. → DFT & Mag of $|X(k)|$ & $\angle X(k)$

$N = 8$

$\therefore \cancel{k = 0, 2, 3,}$ $k = 0, 1, 2, 3, 4, 5, 6, 7.$

$$X(k) = \sum_{n=0}^{7} x(n)\, e^{-j\pi \frac{nk}{4}}$$

$K = 0$

$$X(0) = \sum_{n=0}^{7} x(n)\, e^{-j\pi \frac{n0}{4}} = \sum_{n=0}^{7} x(n)\, e^{0} = x(0) + x(1) + x(2) + x(3) + x(4) + x(5) + x(6) + x(7).$$

$X(0) = 1+1+1+1 = 4.$ $|X(0)| = 4$ $\angle X(0) = \tan^{-1}\left(\frac{0}{4}\right) = 0$

$K = 1$

$$X(1) = \sum_{n=0}^{7} x(n)\, e^{-\frac{j\pi n}{4}} = x(0)e^{0} + x(1)e^{-\frac{j\pi}{4}} + x(2)e^{-\frac{j\pi}{2}} + x(3)e^{-\frac{j\pi 3}{4}} + x(4)e^{-j\ldots} + x(5)\ldots + x(6)\ldots + x(7)\ldots$$

$$= 1 + \left(\cos\frac{\pi}{4} - j\sin\frac{\pi}{4}\right) + \left(\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right) + \left(\cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4}\right)$$

$$= 1 + (\cancel{0.707} - j0.707) + (0 - j) + (\cancel{-0.707} - j0.707)$$

$$= 1 - j2.414$$

$|X(1)| = \sqrt{(1)^2 + (-2.414)^2}$

$|X(1)| = 2.613$

$\angle X(1) = \tan^{-1}\left(\frac{-2.414}{1}\right)$

$\angle X(1) = -67.49$

$K = 2.$

$$X(2) = \sum_{n=0}^{7} x(n)\, e^{\frac{-j\pi n}{2}}$$

$$= x(0)\, e^{0} + x(1)\, e^{-j\frac{\pi}{2}} + x(2)\, e^{-j\pi} + x(3)\, e^{\frac{-j\pi 3}{2}}$$

$$= 1 + \left(\cos\frac{\pi}{2} - j\sin\frac{\pi}{2}\right) + \left(\cos\pi - j\sin\pi\right) + \left(\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}\right)$$

$$= 1 + (0 - j) + (-1 - 0) + (0 + j) = 0.$$

$$\cancel{= 1 - j - 1 + 1 = 1+0}$$

$$|X(2)| = 0 \qquad \angle X(2) = 0^{\circ}$$

$K = 3.$

$$X(3) = \sum_{n=0}^{7} x(n)\, e^{-j\pi n \frac{3}{4}}$$

$$= x(0)\, e^{0} + x(1)\, e^{-j\pi\frac{3}{4}} + x(2)\, e^{\frac{-j\pi 3}{2}} + x(3)\, e^{-j\pi\frac{9}{4}}$$

$$= 1 + \left(\cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4}\right) + \left(\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}\right) + \left(\cos\frac{9\pi}{4} - j\sin\frac{9\pi}{4}\right).$$

$$= 1 + (-0.707 - \cancel{0.707}\ j0.707) + (0 + j) + (0.707 - j0.707)$$

$$\cancel{= 1 + j \quad X(3)} \qquad X(3) = 1 - j0.414$$

$$= 1 - j0.414 \qquad |X(3)| = 1.08$$

$$\angle X(3) = -22.5$$

K = 4

$$X(4) = \sum_{n=0}^{7} x(n)\, e^{-j\pi n}$$

$$= x(0)e^{0} + x(1)e^{-j\cancel{0}\pi} + x(2)e^{-j2\pi} + x(3)e^{-j3\pi}$$

$$= 1 + (\cos \cancel{0}\pi - j\sin 4\pi) + (\cos 2\pi - j\sin 2\pi) + (\cos 3\pi - j\sin 3\pi$$

$$= 0.$$

$X(4) = 0$

$|X(4)| = 0 \qquad \angle X(4) = 0$

K = 5

$$X(5) = \sum_{n=0}^{7} x(n)\, e^{-j\pi n \frac{5}{4}}$$

$$= x(0)e^{0} + x(1)e^{-j\pi\frac{5}{4}} + x(2)e^{-j\pi\frac{5}{2}} + x(3)\, e^{-j\pi\frac{15}{4}}$$

$$= 1 + \left(\cos\frac{5\pi}{4} - j\sin\frac{\cancel{5}\pi}{4}\right) + \left(\cos\frac{5\pi}{2} - j\sin\frac{5\pi}{2}\right) + \left(\cos\frac{15\pi}{4} - j\sin\frac{15}{4}\right.$$

$$= 1 + (-\cancel{0.707}0.7 + j\,0.707) + (0 - j) + (\cancel{0.707} + j\,0.707).$$

$$= 1 + j\,0.414$$

$X(5) = 1 + j\,0.414$

$|X(5)| = \sqrt{}\ 1.08$

$\angle |X(5)| = 22.5^{\circ}$

$\underline{k=6}$

$$X(6) = \sum_{n=0}^{7} x(n) e^{-j\pi n \frac{3}{2}}$$

$$\cancel{X(3)} \quad \cancel{X(6)}$$

$$= x(0) + e^{0} + x(1) e^{-j\frac{3\pi}{2}} + x(2) e^{-j3\pi} + x(3) e^{-j\frac{9\pi}{2}}$$

$$= 1 + \left(\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}\right) + \left(\cos 3\pi - j\sin 3\pi\right) + \left(\cos\frac{9\pi}{2} - j\sin\frac{9\pi}{2}\right)$$

$$= 0$$

$X(6) = 0$

$|X(6)| = 0$

$\angle X(6) = 0°$

$\underline{k=7}$

$$X(7) = \sum_{n=0}^{7} x(n) e^{-j\pi n \frac{7}{4}}$$

$$= x(0) e^{0} + x(1) e^{-j\frac{7\pi}{4}} + x(2) e^{-j\frac{7\pi}{2}} + x(3) e^{-j\frac{21\pi}{4}}$$

$$= 1 + \left(\cos 7\frac{\pi}{4} - j\sin 7\frac{\pi}{4}\right) + \left(\cos\frac{7\pi}{2} - j\sin\frac{7\pi}{2}\right)$$

$$+ \left(\cos 21\frac{\pi}{4} - j\sin 21\frac{\pi}{4}\right)$$

$$= 1 + (0.707 + j0.707) + (0 + j) + (-0.707 + j0.707)$$

$$= 1 + 2.414j$$

$X(7) = 1 + 2.414j$

$|X(7)| = 2.61$

$\angle X(7) = 67.5°$

$$X(k) = \{4, (1-j2.414), 0, 1-0.414j, 0, 1+j0.414, 0, 1+2.414j\}.$$

Calculate 4 point IDFT of the sequence.

$X(k) = \{12, -4+j4, -4, -4-4j\}$.

Soln.

$N = 4$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi n k/N}$$

$$x(n) = \frac{1}{4} \sum_{k=0}^{3} X(k) e^{\frac{j2\pi n k}{4}} \Big\}_{n=0,1,2,3.}$$

$$x(n) = \frac{1}{4} \sum_{k=0}^{3} X(k) e^{\frac{j\pi n k}{2}}$$

n=0

$$x(0) = \frac{1}{4} \sum_{k=0}^{3} X(k) e^{0} = \frac{1}{4} X(0) + \frac{1}{4} X(1) + \frac{1}{4} (X(2)) + \frac{1}{4} X(3)$$

$$\left[\left(\frac{1}{4} \times 12\right)\right] + \left[\frac{1}{4} \times (-4+j4)\right] + \left(\frac{1}{4}(-4)\right) + \left[\frac{1}{4}(-4-4j)\right]$$

$$3 + (-1 + \cancel{j}) + (-1) + (-1 - \cancel{j}) = 0$$

$\therefore X(0) = 0.$

$n=1$

$$x(1) = \frac{1}{4} \sum_{k=0}^{3} x(k) e^{\frac{j\pi k}{2}}$$

$$= \frac{1}{4} x(0) e^{0} + \frac{1}{4} x(1) \left(\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}\right).$$

$$+ \frac{1}{4} X(2) \left(\cos\pi + j\sin\pi\right) + \frac{1}{4} x(3) \left(\cos\frac{3\pi}{2} + j\sin\frac{3\pi}{2}\right)$$

$$= \frac{1}{4}(12) + \frac{1}{4}(-4+4j)(0+j)$$

$$+ \frac{1}{4}(-4)(-1+0) + \frac{1}{4}(-4-j4)(0-j)$$

$$= 3 + (-1+j)j + 1 + (-1-j)(-j)$$

$$= 3 + (-\cancel{j} - 1) + \cancel{1} + (\cancel{j} - \cancel{1}) = 2.$$

$x(1) = 2.$

n=2.

$$X(2) = \frac{1}{4}\sum_{k=0}^{3} x(k)\, e^{j\pi k}$$

$$= \frac{1}{4}\, x(0)\, e^{0} + \frac{1}{4}\, x(1)\, e^{j\pi} + \frac{1}{4}\, x(2)\, e^{j2\pi} + \frac{1}{4}\, x(3)\, e^{j3\pi}$$

$$= \frac{1}{4}(12) + \frac{1}{4}(-4+4j)(\cos\pi + j\sin\pi) + \frac{1}{4}(-4)(\cos 2\pi + j\sin 2\pi) + \frac{1}{4}(-4-4j)(\cos 3\pi + j\sin 3\pi)$$

$$= 3 + (-1+j)(-1) + (-1)(1) + \frac{4}{4}(-1-j)(-1)$$

$$= 3 + (1-j) - 1 + (1+j)$$

$$= 4.$$

$$X(2) = 4.$$

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$n = 3.$

$$x(3) = \frac{1}{4} \sum_{k=0}^{3} X(k) e^{\frac{j\pi 3k}{2}}.$$

$$= \frac{1}{4} X(0) e^{0} + \frac{1}{4} X(1) e^{\frac{j3\pi}{2}} + \frac{1}{4} X(2) e^{j3\pi}$$

$$+ \frac{1}{4} X(3) e^{\frac{j9\pi}{2}}.$$

$$= \frac{1}{4}(12) + \frac{1}{4}(-4+4j)\left(\cos\frac{3\pi}{2} + j\sin\frac{3\pi}{2}\right)$$

$$+ \frac{1}{4}(-4)\left(\cos 3\pi + j\sin 3\pi\right)$$

$\left(\cos 3\pi\right.$

$\left.j\sin 3\pi\right)$

$$+ \frac{1}{4}(-4-4j)\left(\cos\frac{9\pi}{2} + j\sin\frac{9\pi}{2}\right)$$

$$= 3 + (-1+j)(0-j) + (-1)(-1+0)$$

$$+ (-1-j)(0+j)$$

$$= 3 + (-1+j)(-j) + (1) + (-j+1)$$

$$= 3 + (+j+1) + 1 + (1-j).$$

$$= 3 + 1 + 1 + 1 = 6.$$

$x(3) = 6.$

$$\boxed{\therefore x(n) = \{\underset{\uparrow}{0}, 2, 4, 6\}}$$

30/07/16

CIRCULAR CONVOLUTION:

i). Concentric Circle method.

ii) Matrix Method.

iii) DFT & IDFT method

Find the circular convolution of.

i) $x_1(n) = \{\overset{x(0)}{1}, \overset{x(1)}{-1}, \overset{x(2)}{-2}, \overset{x(3)}{3}, \overset{x(4)}{-1}\}$.

$x_2(n) = \{1, 2, 3\}$.

Soln.

$N = 5$

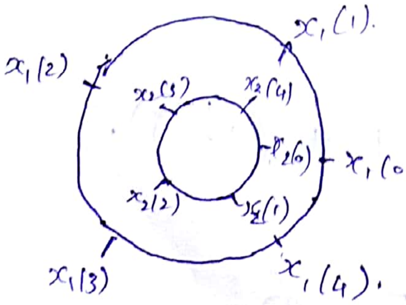
$M = 3$ $\quad N \neq M$.

Zero pad in $x_2(n) \Rightarrow 2$ Samp.

$x_2(n) = \{\overset{x_2(0)}{1}, \overset{x_2(1)}{2}, \overset{x_2(2)}{3}, \overset{x_2(3)}{0}, \overset{x_2(4)}{0}\}$

Concentric Circle

Step 1



$y(n) = x_1(n) \odot x_2(n)$

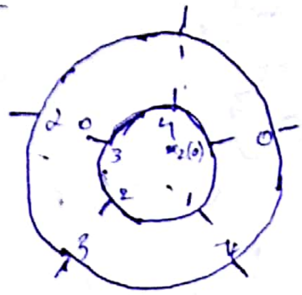
$y = x_1(n) \circledast x_2(n)$

$y(0) = x_2(0)x_1(0) + x_2(4)x_1(1) + x_2(3)\,x_1(2) + x_2(2)\,x_1(3) + x_2(1)\,x_1(4)$

$= 1\times1 + (-1\times0) + (0\times-2) + (3\times3) + (2)(-1)$

$y(0) = 8.$

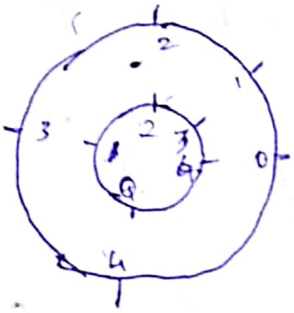
Step 2



$$y(1) = (-1)\times 1 + (-2)\times 0 + 0\times 3 + 3(-1) + 2(1)$$

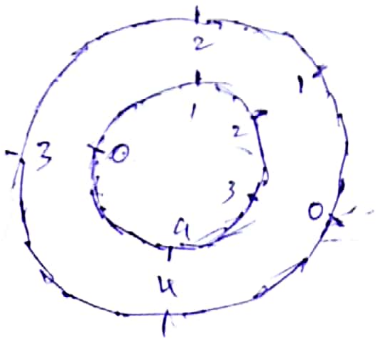
$$= -2.$$

Step 3.



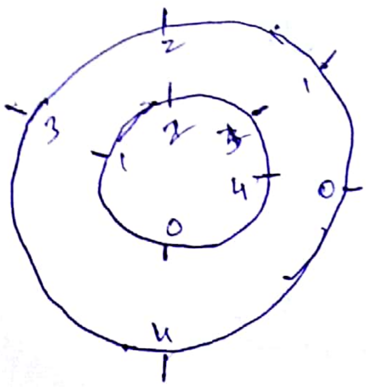
$y(2) = -1$

Step 4:



$y(3) = -1$

Step 5:



$y(4) = -1.$

ii) Matrix Method:

$x_1(n) = \{1, -1, -2, 3, -1\}.$

$x_2(n) = \{1, 2, 3, 0, 0\}.$

$$\begin{pmatrix} 1 & -1 & 3 & -2 & -1 \\ -1 & 1 & -1 & 3 & -2 \\ -2 & -1 & 1 & -1 & 3 \\ 3 & -2 & -1 & 1 & -1 \\ -1 & 3 & -2 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ -2 \\ -1 \\ -4 \\ -1 \end{pmatrix}$$

$y(n) = \{8, -2, -1, -4, -1\}.$

01/07/18

Perform the circular convolution for

$x(n) = \{1, 1, 2, 1\}$ $h(n) = \{1, 2, 3, 4\}$

using DFT, IDFT method.

$$\boxed{\begin{aligned} x_1(n) &\xrightarrow{DFT} X_1(k). \\ x_2(n) &\xrightarrow{DFT} X_2(k). \\ Y(k) &= X_1(k)\, X_2(k). \\ Y(k) &\xrightarrow{IDFT} y(n). \end{aligned}}$$

DFT
$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi nk/N} \qquad K = 0, 1, \ldots N-1$$

$N = 4$
$$X(k) = \sum_{n=0}^{3} x(n)\, e^{-j2\pi nk/4} \qquad k = 0, 1, 2, 3$$

$$\boxed{X(k) = \sum_{n=0}^{3} x(n)\, e^{-j\pi nk/2}.}$$

$K = 0$
$$X(0) = \sum_{n=0}^{3} x(n)\, e^{0} = x(0) + x(1) + x(2) + x(3)$$
$$= 1 + 1 + 2 + 1 = 5.$$

$K = 1$
$$X(1) \sum_{n=0}^{3} x(n)\, e^{-j\pi n/2} = -1$$

$\underline{K=2.}$

$$X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$$

$$= \cancel{x(0)\, e^{0} + x(1)\, e^{1}}$$

$\underline{K=3}$

$$X(3) = -1$$

$$X(k) = \{5, -1, 1, -1\}$$

$$\cancel{H}(n) = \{1, 2, 3, 4\}.$$

$$h(n) \xrightarrow[\;]{DFT} H(\cancel{k})$$

$$H(\cancel{k}) = \sum_{n=0}^{N-1} \cancel{h}(n).\, e^{-j2\pi nk/N}. \qquad K=0,1,2,3.$$

$$H(\cancel{k}) = \sum_{n=0}^{3} \cancel{h}(n)\, e^{-j\pi nk/2} \qquad \cancel{K}=0,1,2,3.$$

$\underline{K=0}$

$$H(0) = 10$$

$\underline{K=1}$

$$H(1) = -2 + j2$$

$k = 2$

$$H(2) = -2$$

$k = 3$

$$H(3) = -2 - j2$$

So: $\boxed{X(k) = \{5, -1, 1, -1\}}$

$$\boxed{H(k) = \{10, -2+j2, -2, -2-j2\}}$$

$$Y(k) = X(k)\,H(k)$$

$$= \{5 \times 10, -1(-2+j2), 1(-2), -1(-2-j2)\}$$

$$Y(k) = \{50, 2-j2, -2, 2+j2\}$$

$$Y(k) \xrightarrow{IDFT} y(n)$$

IDFT

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(k)\, e^{-j2\pi nk/N}$$

$$n = 0, 1, \ldots N-1$$

$N = 4$

$$\boxed{y(n) = \frac{1}{4} \sum_{k=0}^{3} Y(k)\, e^{j\pi nk/2} \quad n = 0, 1, 2, 3}$$

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$\underline{n=0}$

$y(0) = 13.$

$\underline{n=1}$

$y(1) = 14$

$\underline{n=2}$

$y(2) = 11$

$$\boxed{y(n) = \{13, 14, 11, 12\}.}$$

$\underline{n=3}$

$y(3) = 12.$

$x(n) = \{1, 1, 2, 1\}$ $h(n) = \{1, 2, 3, 4\}.$

Linear Convol.

Matrix mtd.

$x(n) \backslash h(n)$	1	2	3	4
1	1	2	3	4
1	1	2	3	4
2	2	4	6	8
1	1	2	3	4

Linear $y(n) = \{1, 3, 7, 12, 12, 11, 4\}$ Circular $y(n) = \{13, 14, 11, 12\}$

No. of samples in $x(n) = N = 4$

" " $h(n) = M = 4$

To avoid aliasing, we need to do zero padding in both sequences. $N+M-1 = 4+4-1 = 7$

$\therefore x(n) = \{1, 1, 2, 1, 0, 0, 0\}$

$h(n) = \{1, 2, 3, 4, 0, 0, 0\}.$

Checking Using Circular Convolution - Matrix Meth.

$y(n) = x(n) \circledast h(n).$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 2 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 2 \\ 2 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 7 \\ 12 \\ 12 \\ 11 \\ 4 \end{pmatrix}$$

$x(n) = \{1, 1, 1, 1\}$

$h(n) = \{1, 2, 1, 1\}.$

Linear Convolution Using Matrix.

	1	2	1	1
1	1	2	1	1
1	1	2	1	1
1	1	2	1	1
1	1	2	1	1

$y(n) = \{1, 3, 4, 5, 4, 2,$

Circular. Convolution.

$y(n) = x(n) \odot h(n)$.

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ 5 \\ 5 \end{pmatrix}$$

After zero padding. $\rightarrow$ $N+M-1 = 7$.

$x(n) = \{1, 1, 1, 1, 0, 0, 0, \cancel{0}\}$.

$h(n) = \{1, 2, 1, 1, 0, 0, 0, \cancel{0}\}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 4 \\ 5 \\ 4 \\ 2 \\ 1 \end{pmatrix}$$

$\therefore y(n) = \{1, 3, 4, 5, 4, 2, 1\}$

02/08/18

FAST FOURIER TRANSFORM (FFT).

* Decimation in time (DIT) FFT
* Decimation in frequency (DIF) FFT

$\Bigg\}\; x(n) \to x(k)$

Twiddle factor :

$$W_N^k = e^{\frac{-j2\pi k}{N}}$$

Decimal.	Binary.	Bit reversal	Decimal.
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7.

Decimation in time FFT.

Stage 1 : $N = N/4$

$$W_{N/4}^{0} = e^{\frac{-j2\pi k}{N}} = e^{0} = 1.$$

Stage 2 : $N = N/2$.

$$W_{N/2}^{0} = e^{-j2\pi 0/N/2} = e^{0} = 1.$$

$$W_{N/2}^{1} = e^{-j2\pi(1)/N/2} = e^{-j\pi/2} = -j$$

Stage 3: $N = N$

$$W_N^0 = e^0 = 1$$

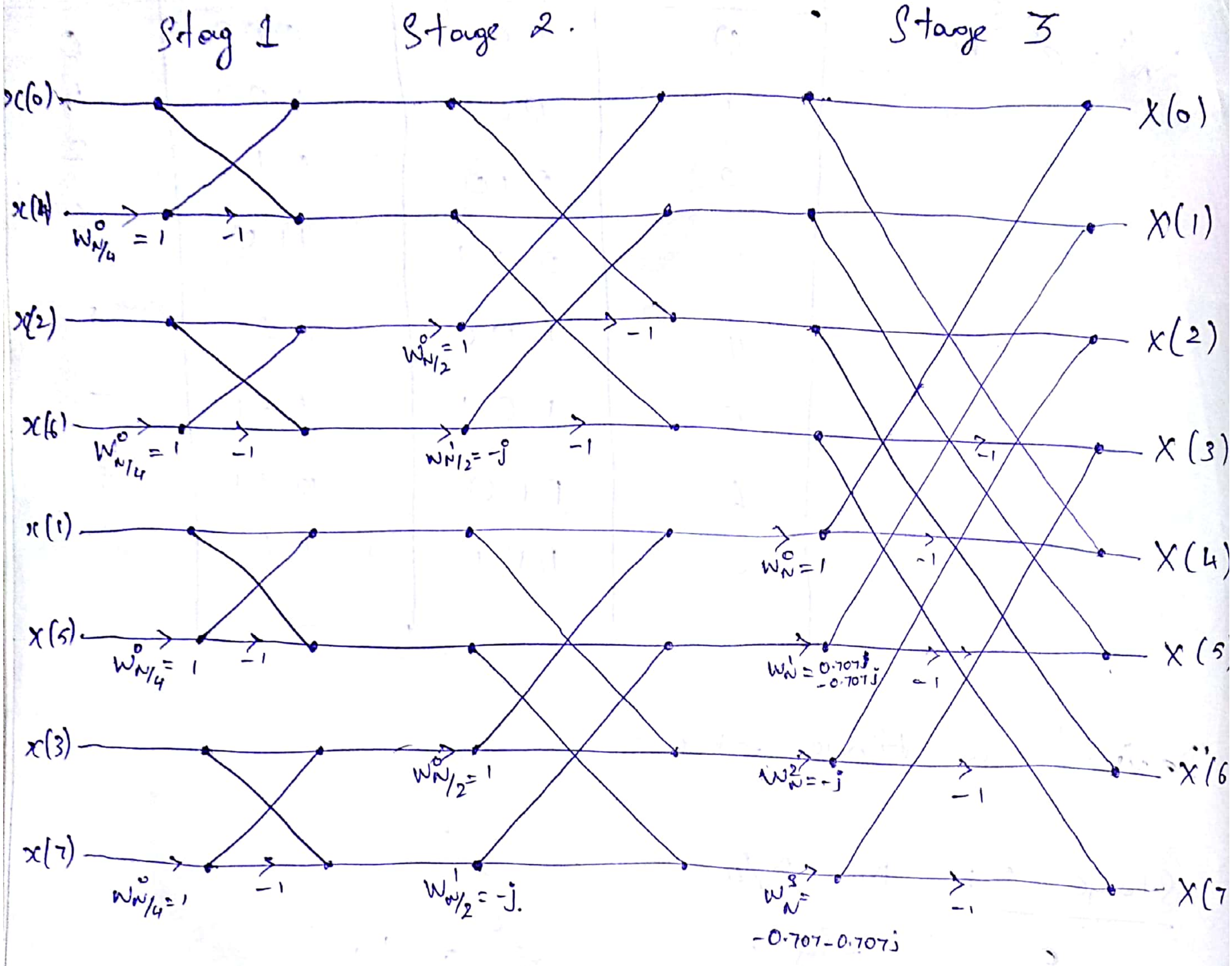
$$W_N^1 = e^{-j\frac{2\pi}{8}(1)} = e^{-j2\pi/8} = 0.707j - j0.707.$$

$$W_N^2 = e^{-j2\pi(2/8)} = -j \qquad W_N^3 = e^{-j2\pi(3)/8} = -0.707j - 0.707$$

Explanation

BUTTERFLY DIAGRAM (RADIX ALGORITHM)

$N = 8$



Compute the DFT of $x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$ by using DIT FFT algorithm.

Soln.

$N = 8$

Weight Analysis:

Stage 1: $N = N/4$.

$$W_{N/4}^{0} = e^{-j 2\pi(0)/(N/4)} = e^{0} = 1.$$

Stage 2: $N = N/2$.

$$W_{N/2}^{0} = 1$$

$$W_{N/2}^{1} = -j$$

Stage 3: $N = N$

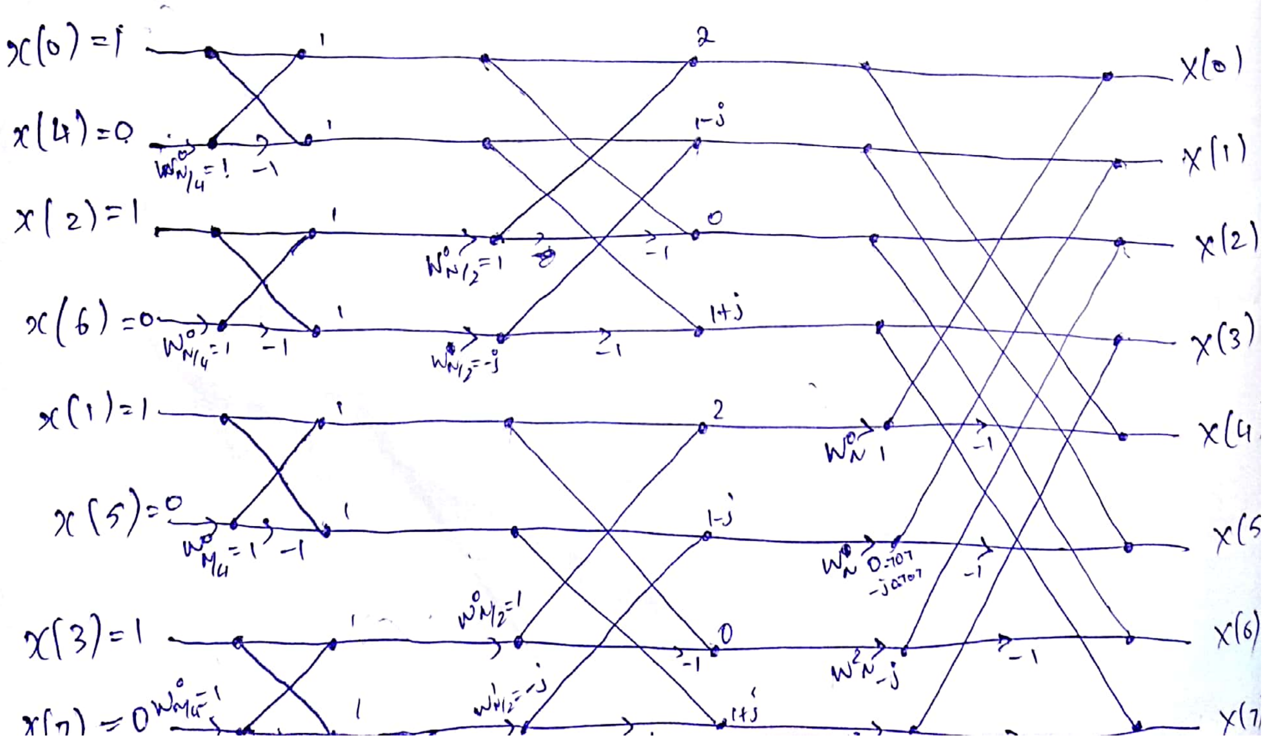
$$W_N^0 = 1$$

$$W_N^1 = 0.707 - 0.707j$$

$$W_N^2 = -j$$

$$W_N^3 = -0.707 - j0.707.$$

BUTTERFLY DIAGRAM.



Stage Calculation:

$$x(n) = \{\overset{x(0)}{1}, \overset{x(1)}{1}, \overset{x(2)}{1}, \overset{x(3)}{1}, \overset{x(4)}{0}, \overset{x(5)}{0}, \overset{x(6)}{0}, \overset{x(7)}{0}\}$$

I/P	Stage 1	Stage 2	Stage 3
1	$1+0=1$	$1+1=2$	$2+2=4$
0	$1+0=1$	$1+(-j)= 1-j$	$(1-j)+\{(1-j)(0.707-j0.707)\} = 1-j2.414$
1	$1+0=1$	$1+(-1) = 0$	$0+0 = 0$
0	$1+0=1$	$1+j$	$(1+j)+\{1+j(-0.707-j0.707)\} = \cancel{2.414} \; \cancel{j2.414}$
1	$1+0=1$	2	$2+(-2) = 0$
0	$1+0=1$	$1-j$	$(1-j)+\{(1-j)(-1)(0.707-j0.707)\} =$
1	$1+0=1$	0	0
0	$1+0=1$	$1+j$	$(1+j)+\{(1+j)(-1)(-0.707-j0.707)\} = 1+2.414j$

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09/08/18 Revision

Determine whether the given time s/g is Static or Dynamic

i) Static or dynamic.

ii) Linear or non linear.

iii) Time var or not

iv) Causal (or) non-causal.

$$y(n) = \cos(x(n))$$

$$y(0) = \cos(x(0))$$

$$y(1) = \cos(x(1)).$$

$$y(-1) = \cos(x(-1))$$

$\boxed{\therefore \text{Static}}$

$\boxed{\therefore \text{Causal}}$

$$y_1(n) = \cos(x_1(n))$$

$$y_2(n) = \cos(x_2(n))$$

$$x_3(n) = a x_1(n) + b x_2(n)$$

$$y_3(n) = \cos(x_3(n))$$

$$= \cos(a x_1(n) + b x_2(n))$$

$$y(n) = \cos(x(n-m))$$

$$y(n-m) = \cos(x(n-m)),$$

$$\therefore y(n) = y(n-m)$$

$\boxed{\therefore \text{Time Invariant}}$

Represent the seq $x(n) = \{4, 2, -1, \underset{\uparrow}{1}, 3, 2, 1\}$ as sum of shifted unit ~~pulses~~ impulses.

Soln.

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{else.} \end{cases}$$

$$x(n) = \sum_{K=-\infty}^{\infty} x(n)\, \delta(n-K)$$

Eg. $x(n) = \{1, 2, 3, 4\}$.

$$y(n) = \underset{\text{zeroth}}{1\,\delta(n)} + \underset{\text{First}}{3\,\delta(n-1)} + 4\,\delta(n-2)$$

$$y(n) = \{1, 3, 4\}$$

Calculate the inverse FFT of ~~$X(K) = \{3, 3,$~~

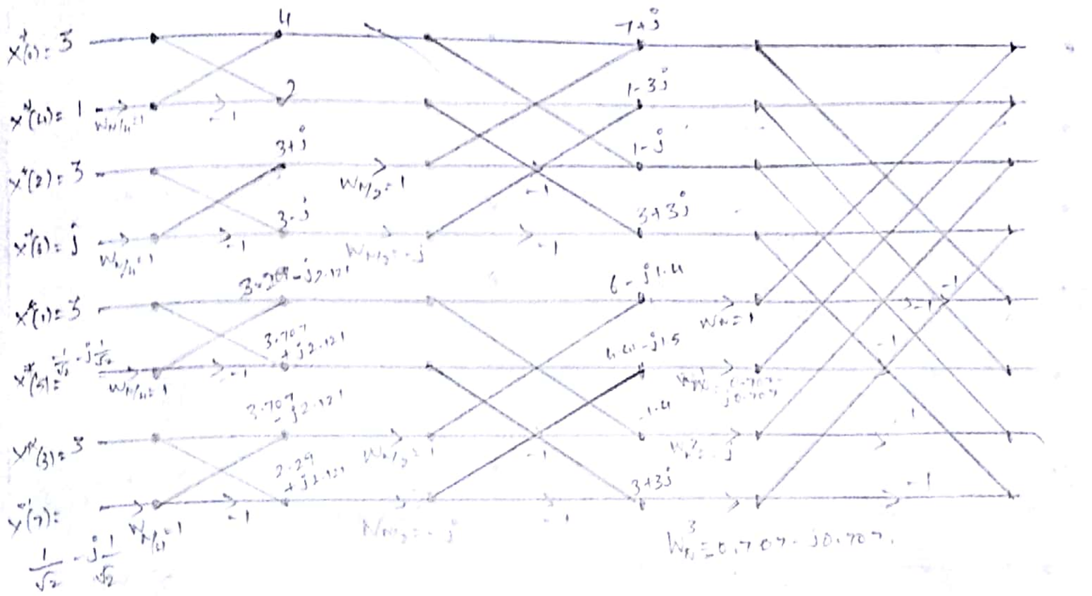
$$X(k) = \left\{3, 3, 3, 3, 1, \frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}, -j, \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right\}$$

by using DIT FFT Algorithm.

Soln:

$$X^*(k) = \left\{3, 3, 3, 3, 1, \frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}, j, \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right\}$$

BUTTERFLY DIAGRAM:



Stage 1	Stage 2	Stage 3.
$3+1 = 4$	$7+j$	$(7+j) + (6-j1.4) = 13-0.4j$
$-1+3 = 2$	$2+(-3j+j^2) = 2+(-3j-1) = 1-3j$	$(1-3j)+[(4.41-j1.5)(0.707-j0.7)]$ $(1-3j)+(2.05-j)$ $= 2.96 - j7.2$
$3+j$	$-1(3+j)+4 = -3-j+4 = 1-j$	$1-0.4j$
$3-j$	$3+3j$	$3-1.2j$
$3+\left(\frac{-1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) = 2.29 - j2.121$	$6-j1.4$	$2.6-0.6j$
$3+\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = 3.707 + j2.121$	$4.414-j1.5$	$-0.96+1.2j$
$3+\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) = 3.707 - j2.121$	$-1-4$	$1-2.4j$
$3+\left(\frac{-1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = 2.29 + j2.121$	$3+3j$	$2+7.2j$

$$x(n) = \frac{X(K)}{N} = \frac{1}{8}\{13-0.4j,\ 2.96-j7.2,\ 1-j0.4,\ 3-j1.2,\ 2.6-j0.6,\ -0.96+j1.2,\ 1-j2.4,\ 3+j7.$$

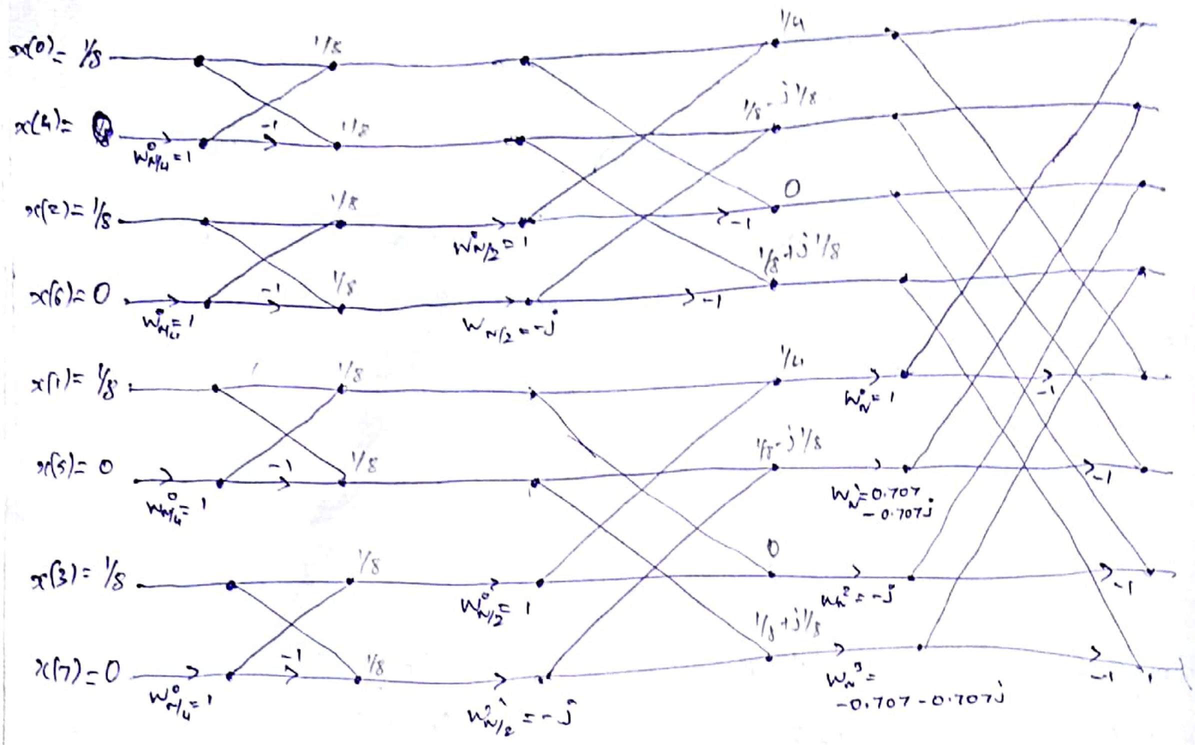
$$x(n) = \{1.625-j0.05,\ 0.37-j0.9,\ 0.125-j0.05,\ 0.375-j0.15,\ 0.125-j0.3,\ 0.375+j0.\ 0.325-j0.075,\ -0.12+j0.15,$$

20/08/18

Compute 8 point DFT by using DIT $x(n) = \frac{1}{8}$ $0 \le n \le 3$

else 0 $n > 3$.

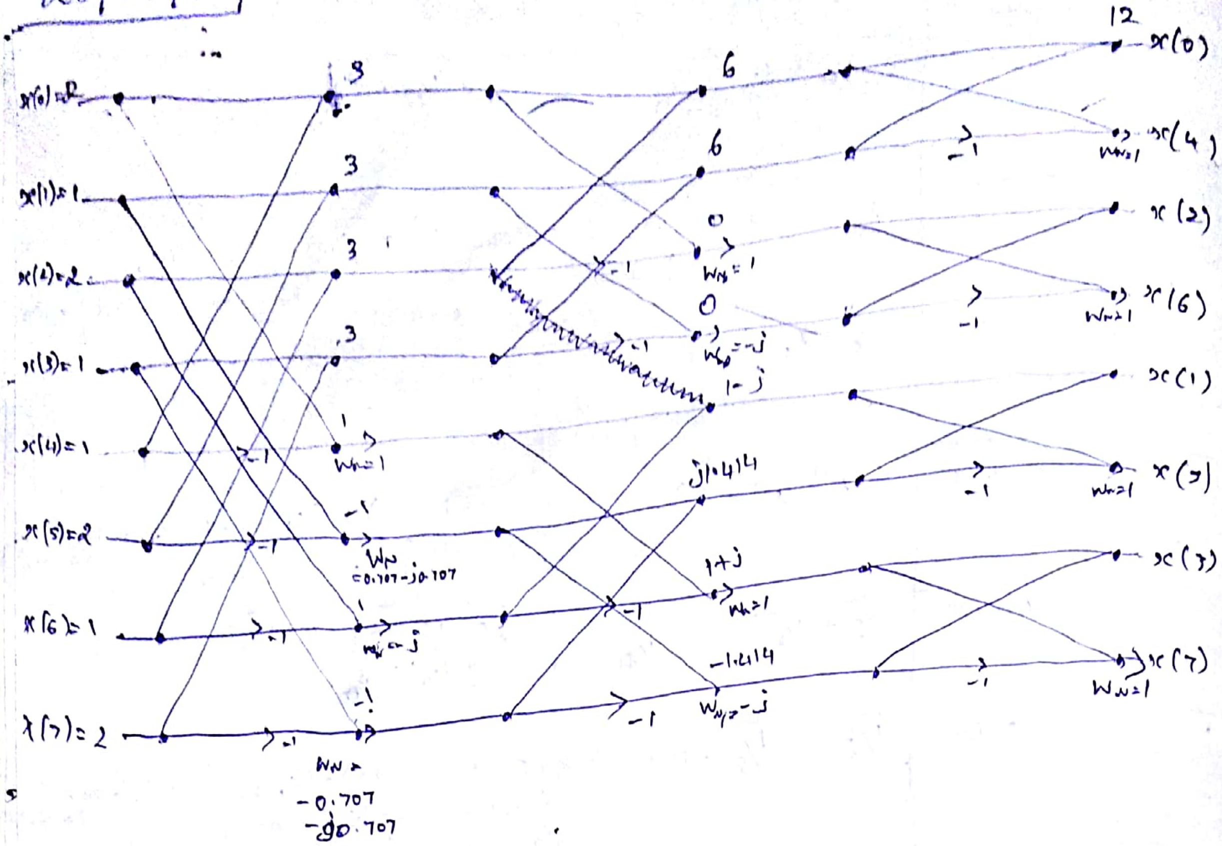
$$x(n) = \{\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, 0, 0, 0, 0\}.$$



Stage 1	Stage 2	Stage 3
$0.125+0 = 0.125$	$1/8 + 1/8 = \frac{1}{4}$	$1/4 + 1/4 = \frac{1}{2}$.
0.125	$1/8 - j\,1/8 = 0.125 - j0.125$	$\left(\frac{1}{8} - j\frac{1}{8}\right) + [(0.125 - j0.125)(0.707 - 0.707j)]$ $(0.125 - j0.125) + [0.088 - 0.088j - 0.088j - 0.088]$ $\boxed{0.125 - j0.301}$
0.125	$-1/8 + 1/8 = 0$.	$0+0 = 0$
0.125	$j\,1/8 + 1/8 = 0.125 + j0.125$	$(0.125+j0.125) + [(0.125+j0.125) \times (-0.707 - 0.707j)]$ $(0.125 - j0.125] + [-0.088 - 0.088j - 0.088j + ?$ $\boxed{0.125 - 0.051j}$
0.125	$1/8 + 1/8 = 1/4$	$1/4 - 1/4 = 0$
0.125	$1/8 - j\,1/8 = 0.125 - j0.125$	$(0.125 - j0.125) + [(0.125 - j0.125)(-0.707 + 0.707j)]$ $(0.125 - j0.125) + [-0.088 + 0.088j + 0.088j + 0.088]$ $(0.125 - j0.125) + [0.176j]$ $\boxed{0.125 + 0.051j}$
0.125	$-1/8 + 1/8 = 0$	0
0.125	$j\,1/8 + 1/8 = 0.125 + j0.125$	$0.176j + 0.125 + 0.125j$ $= 0.125 + 0.301j$

$$X(K) = \left\{ \frac{1}{2},\ 0.125 - j0.301,\ 0,\ 0.125 - j0.051,\ 0,\ 0.125 + j0.051,\ 0,\ 0.125 + j0.301 \right\}.$$

28/08/18 DIF FFT (Decimation in frequency FFT)



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Calculate 8 point DFT of $x(n) = \{2,1,2,1,1,2,1,2\}$ by using DIF FFT.

$(-j1.9 - 1.9) +$

$(-1.414 \cancel{-j1.414} \cancel{+j1.414} - 1.414)$

$(-4.728 - j1.9)$

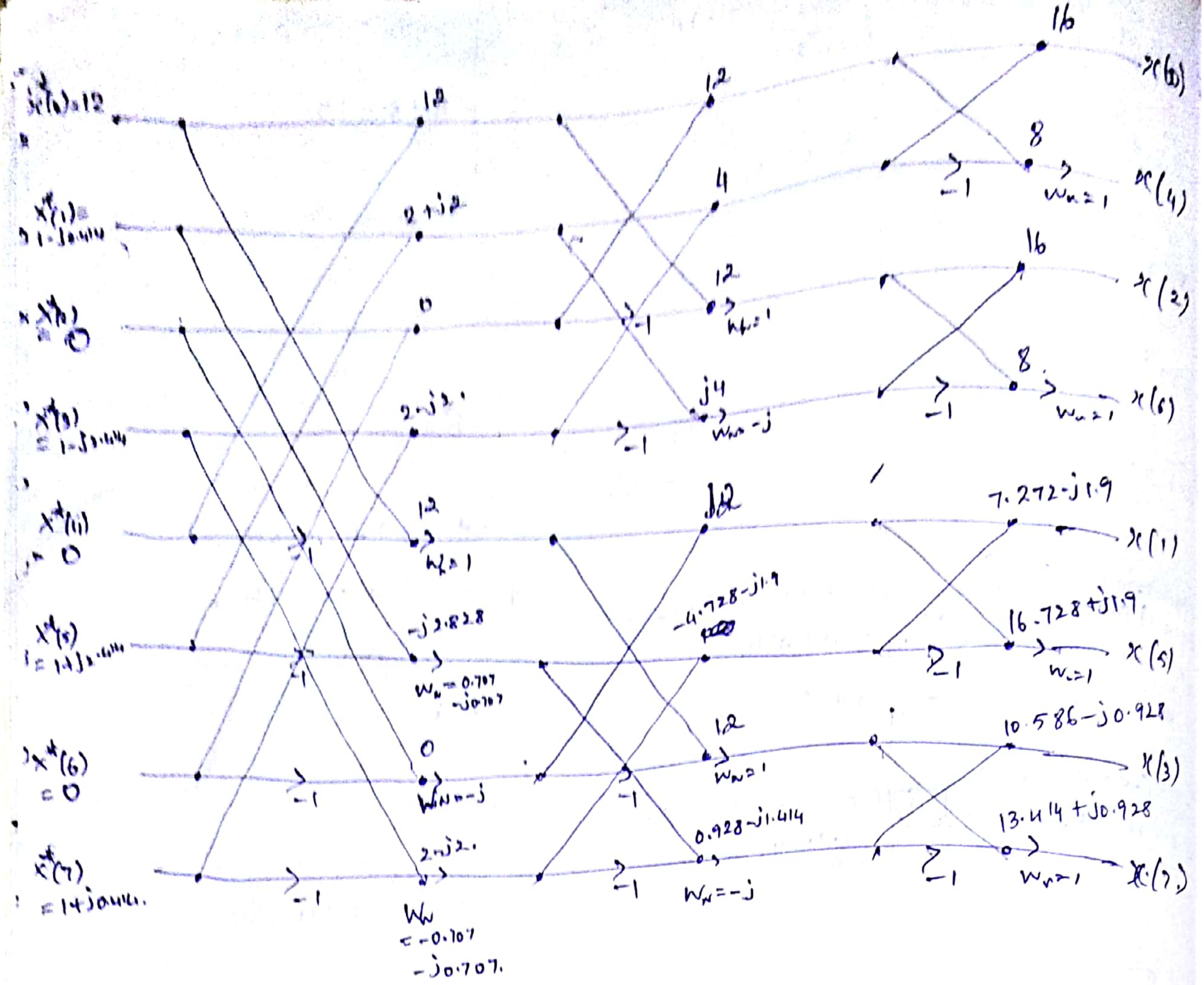
STAGE 1	STAGE 2	STAGE 3
$2+1=3$	$3+3=6$	$6+6=12$
$1+2=3$	$3+3=6$	$6-6=0$
$2+1=3$	$3-3=0$	0
$1+2=3$	$3-3=0$	$\cancel{8}$ 0
$2-1=1$	$1-j$	$(1-j)+j1.414$ $= 1+j0.414.$
$\cancel{-2(0.707-j0.707)}$ $(1-2+1)(0.707-j0.707)$ $\boxed{-1}$ $-0.707+j0.707$	$(-0.707+j0.707)+(0.707+j0.707)$ $= j1.414$	0 ~~1+j2.414~~.
1 $\boxed{1}$	$1+j$	0
$(-1)(-0.707-j0.707)$ $= 0.707+j0.707.$ $\boxed{-1}$	$(-0.707-j0.707)+(-0.707+j0.707)$ -1.414	$1-j0.414.$

$\therefore$ $X(K) = \{12, 1+j0.414, 0, 1+j2.414, 0, 1-j2.414, 0, 1-j0.414\}.$

ii) For the given sequence of $X(K) = \{12, 1+j0.414, 0, 1+j2.414, 0, 1-j2.414, 0, 1-j0.414\}.$

calculate inverse DFT

$X^*(K) = \{12, 1-j0.414, 0, 1-j2.414, 0, 1+j2.414, 0, 1+j0.414\}$



Stage 1	Stage 2	Stage 3
12 + 0 = 12.	12 + 0 = 12.	12 + 4 = 16
(1 - j0.414) + (1 + j2.414) 2 + j2.	(2 + j2) + (2 - j2) = 4	-4 + 12 = 8
0 + 0 = 0	12 + 0 = 12	$12 + (-j^2 4) = 12 + 4 = 16$
(1 - j2.414) + (1 + j0.414) 2 - j2.	(2 + j2)(-2 + j2) j4	$12 + (+j^2 4) = 12 - 4 = 8$
0 + 12 = 12.	12 + 0 = 12	12 - 4.728 - j1.9 = 7.272 - j1.9
(-1 - j2.414) + (1 - j0.414) -j2.828	(-j2.828)(0.707 - j0.707) + (2 - j2)(-0.707 - j0.707) = [-4.728 - j1.9]	12 + 4.728 + j1.9 = 16.728 + j1.9
0 + 0 = 0	12 + 0 = 12.	12 + (-0.928j - 1.414) = 10.586 - j0.928
(1 + j0.414) + (1 - j2.414) 2 - j2.	(-j2.828)(+0.707 - j0.707) + (-2 + j2)(-0.707 - j0.707) = -j1.414 - 1.4 + (1.414 - j1.414 - j1.414 + 1.414) $\boxed{0.928 - j1.414}$	12 + (0.928j + 1.414) = 13.414 + j0.928

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$X(k) = \{16, 7.272 - j1.9, 16, 10.586 - j0.928, 8, 16.728 + j1.9, 8, 13.414 + j0.928\}$

$$x(n) = \frac{X(k)}{8} = \frac{1}{8}\{16, 7.272 - j1.9, 16, 10.586 - j0.928, 8, 16.728 + j1.9, 8, 13.414 + j0.928\}$$

$x(n) = \{2, 0.9 - j0.2, 2, 1.3 - j0.1, 1, 2.1 + j0.2, 1, 1.7 + j0.1\}$

29/08/16

Find the DFT of the sequence $x(n) = \{1, 0, 0, 1\}$, using DIT FFT.

Soln.

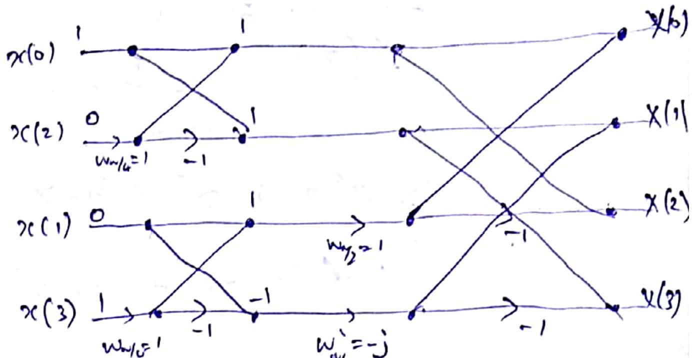
STAGE 1

$W_{N/4}^{0} = 1$

Stage 2

$W_{N/2}^{0} = 1$

$W_{N/2}^{1} = -j$

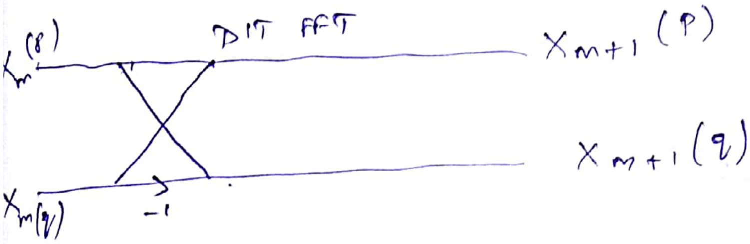


STAGE 1	STAGE 2
1+0 = 1	1+1 = 2
1+0 = 1	1+j
0+1 = 1	1−1 = 0
0−1 = −1	1−j

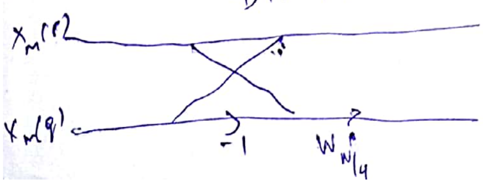
$\therefore X(n) = \{2, 1+j, 0, 1-j\}$.

BASIC BUTTERFLY STRUCTURE '×' 2m.

DIT FFT



DIF FFT.



Assignment.
Property of DFT with proof.