UNIT - 8: Implementation of Discrete-time Systems[?, ?, ?, ?]

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October 18, 2016

Implementation of discrete-time systems:: [?, ?, ?, ?, ?, ?]

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- This material is not for commercial purpose.
- This material is prepared based on Digital Signal Processing for ECE/TCE course as per Visvesvaraya Technological University (VTU) syllabus (Karnataka State, India).

Unit 8: Implementation of Discrete-time Systems:

PART - B

Realization of FIR system

- Direct form structure of FIR system
- Linear phase FIR structure
- Cascade form structure for FIR system
- Frequency sampling structure for FIR system
- Lattice structure for FIR system

Realization of IIR system

- Direct form structure for IIR system
  1. Direct form-I structure of IIR system
  2. Direct form-II structure of IIR system
- Cascade form structure for IIR system
- Parallel form structure for IIR system
- Lattice structure of IIR system

Implementation of Discrete-time Systems

System: A system is a physical device which consists of interrelated and interdependent elements which process the input signal and transform into output signal. Example: Filters, Amplifiers.

Linearity:
If input $x_1(n)$ produces response $y_1(n)$ and if $x_2(n)$ produces response $y_2(n)$ and $ax_1(n) + bx_2(n) = ay_1(n) + by_2(n)$ then the system is called **Linear system**.

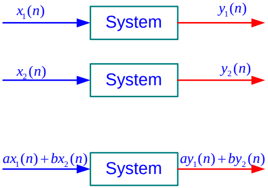


Figure 1: Linearity

Time invariance: A system is said to be time invariant if its behavior and characteristics does not change with time.
If input $x(n)$ produces response $y(n)$ then if $x(n - n_0)$ produces response $y(n - n_0)$, then the system is called as time invariant
Causality: A system is causal if the output depends only on present and past, but not future inputs. All memoryless systems are causal.

$$y[n] = x[n] + x[n-1] + x[n-2] \ldots$$

Some of the examples for such analog systems are Oscillator, regulated power supply.,,

- Digital filters are discrete Linear Time Invariant (LTI) systems and described by difference equations and are implemented in hardware or software.
- The discrete time sytems can be of finite impulse response (FIR) or infinite impulse response IIR type.
- A FIR filter is a filter whose impulse response is of finite duration, because it settles to zero in finite time, because there is no feedback in the FIR system.

The basic components of discrete-time system are, delay element, multiplier, and adder. The details of these components and their symbols with its input output relationship is as shown in Figure 2.

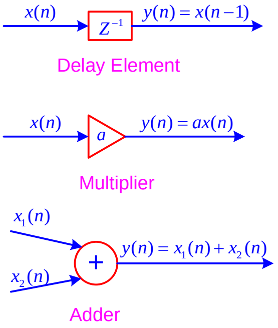


Figure 2: Basic Elements

Realization of FIR system

The difference equation is a formula for computing an output sample at time n based on past and present input samples and past output samples in the time domain. The general, causal, LTI difference equation is as follows:

$$\begin{aligned} y(n) &= b_0x(n) + b_1x(n-1) + \ldots b_kx(n-k) - a_1y(n-1) - a_2y(n-2)\ldots - a_ky(n-k) \\ &= \sum_{k=0}^{M} b_kx(n-k) - \sum_{k=1}^{N} a_ky(n-k) \end{aligned}$$

where x is the input signal, y is the output signal a_k and b_k are called the coefficients.

- The second term in this equation is usually termed as feedback for the system. This is the equation used to represent Infinite Impulse Response (IIR) system.
- If the feedback term is absent then this equation is used to represent Finite Impulse Response (FIR) system.

$$y(n) = \sum_{k=0}^{M} b_kx(n-k)$$

- LTI systems are represented by the following difference equation.

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

- By taking z-transform on both sides

$$Y(z) = -\sum_{k=1}^{N} a_k z^{-k} Y(z) + \sum_{k=0}^{M} b_k z^{-k} X(z)$$

$$Y(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

- The system function $H(z)$ is defined as

$$H(z) = \frac{Y(z)}{X(z)}$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

- An FIR system does not have feedback. Hence $y(n-k)$ term is absent in the system. FIR output is expressed as

$$y(n) = \sum_{k=0}^{M} b_k x(n-k)$$

- If there are M coefficients then

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$$

- By taking z-transform on both sides

$$Y(z) = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

- System function $H(z)$ is defined as

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{M-1} b_k z^{-k}$$

- By taking inverse z transform

$$h(n) = \begin{cases} b_n & for \quad 0 \le n \le M-1 \\ 0 & otherwise \end{cases}$$

Direct form Structure of FIR System

- Structures in which the multiplier coefficients are precisely the coefficients of the transfer function are called direct form structure.
- Since $h(n) = b_n$ then y(n) is

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

- Expanding the summation

$$y(n) = h(0)x(n) + h(1)x(n-1) + h(2)x(n-2) + \ldots h(M-1)x(n-M+1)$$

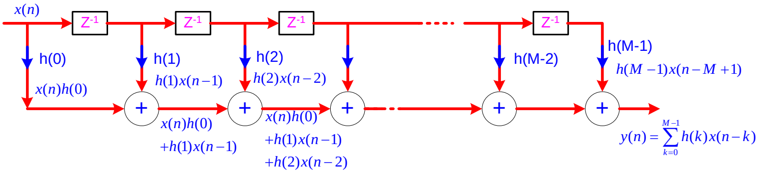


Figure 3: Direct form realization of FIR system

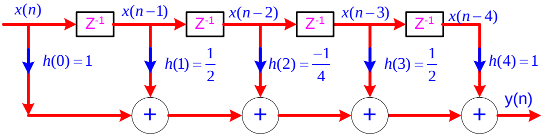
Realize a direct form FIR filter for the following impulse response.

$$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) - \frac{1}{4}\delta(n-2) + \delta(n-4) + \frac{1}{2}\delta(n-3)$$

Solution: $h(n) = \delta(n) + \frac{1}{2}\delta(n-1) - \frac{1}{4}\delta(n-2) + \frac{1}{2}\delta(n-3) + \delta(n-4)$

$$H(z) = 1 + \frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} + \frac{1}{2}z^{-3} + z^{-4}$$

$$\begin{aligned} Y(z) &= X(z)H(z) = \left[1 + \frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} + \frac{1}{2}z^{-3} + z^{-4}\right]X(z) \\ &= X(z) + \frac{1}{2}z^{-1}X(z) - \frac{1}{4}z^{-2}X(z) + \frac{1}{2}z^{-3}X(z) + z^{-4}X(z) \\ y(n) &= x(n) + \frac{1}{2}x(n-1) - \frac{1}{4}x(n-2) + \frac{1}{2}x(n-3) + x(n-4) \end{aligned}$$



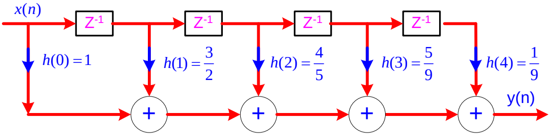
DEC-2010 EE

Realize the system function $H(z) = 1 + \frac{3}{2}z^{-1} + \frac{4}{5}z^{-2} + \frac{5}{9}z^{-3} + \frac{1}{9}z^{-4}$ using direct form II

Solution:

$$H(z) = 1 + \frac{3}{2}z^{-1} + \frac{4}{5}z^{-2} + \frac{5}{9}z^{-3} + \frac{1}{9}z^{-4}$$

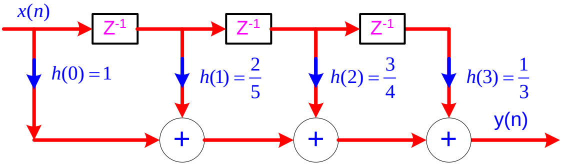
$$\begin{aligned}
Y(z) &= X(z)H(z) = \left[1 + \frac{3}{2}z^{-1} + \frac{4}{5}z^{-2} + \frac{5}{9}z^{-3} + \frac{1}{9}z^{-4}\right]X(z) \\
&= X(z) + \frac{3}{2}z^{-1}X(z) + \frac{4}{5}z^{-2}X(z) + \frac{5}{9}z^{-3}X(z) + \frac{1}{9}z^{-4}X(z) \\
y(n) &= x(n) + \frac{3}{2}x(n-1) + \frac{4}{5}x(n-2) + \frac{5}{9}x(n-3) + \frac{1}{9}x(n-4)
\end{aligned}$$



June 2012 EC

A FIR filter is given by $y(n) = x[n] + \frac{2}{5}x[n-1] + \frac{3}{4}x[n-2] + \frac{1}{3}x[n-3]$ draw the direct form.

Solution:



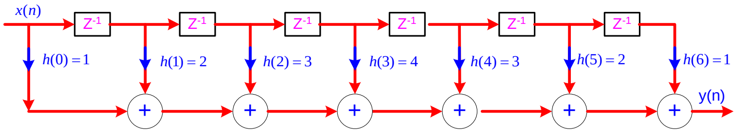
Determine a direct form realization for the following linear phase filters

$$h(n) = [1, 2, 3, 4, 3, 2, 1]$$

Solution:

$$H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 3z^{-4} + 2z^{-5} + 1z^{-6}]$$

$$\begin{aligned}
Y(z) &= X(z)H(z) = \left[1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 3z^{-4} + 2z^{-5} + 1z^{-6}\right] X(z) \\
&= X(z) + 2z^{-1}X(z) + 3z^{-2}X(z) + 4z^{-3}X(z) + 3z^{-4}X(z) + 2z^{-5}X(z) + 1z^{-6}X(z) \\
y(n) &= x(n) + 2x(n-1) + 3x(n-2) + 4x(n-3) + 3x(n-4) + 2x(n-5) + 1x(n-6)
\end{aligned}$$



Determine a direct form realization for the following linear phase filters

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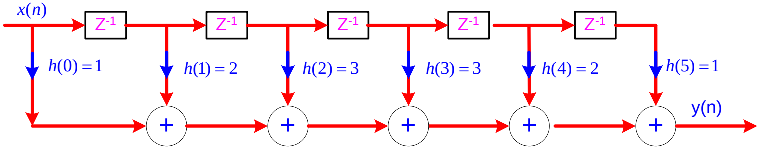
Solution:

$$H(z) = 1 + 2z^{-1} + 3z^{-2} + 3z^{-3} + 2z^{-4} + 1z^{-5}]$$

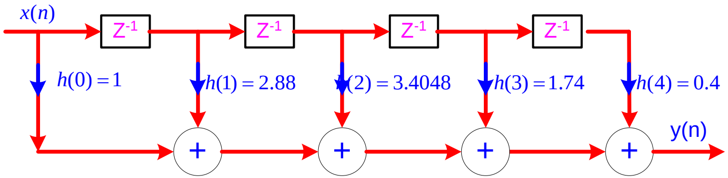
$$\begin{aligned}
Y(z) &= X(z)H(z) = \left[1 + 2z^{-1} + 3z^{-2} + 3z^{-3} + 2z^{-4} + 1z^{-5}\right] X(z) \\
&= X(z) + 2z^{-1}X(z) + 3z^{-2}X(z) + 3z^{-3}X(z) + 2z^{-4}X(z) + 1z^{-5}X(z) \\
y(n) &= x(n) + 2x(n-1) + 3x(n-2) + 3x(n-3) + 2x(n-4) + 1x(n-5) +
\end{aligned}$$



For the following FIR filter system function sketch a direct form

$$H(z) = 1 + 2.88z^{-1} + 3.4048z^{-2} + 1.74z^{-3} + 0.4z^{-4}$$

Solution:



Realize direct form FIR filter with impulse response $h(n)$ is given $h(n) = 4\delta(n) + 5\delta(n-1) + 6\delta(n-2) + 7\delta(n-3)$. With input $x(n) = [1, 2, 3]$ calculate output $y(n)$

Solution:

$$\begin{aligned} h(n) &= 4\delta(n) + 5\delta(n-1) + 6\delta(n-2) + 7\delta(n-3) \\ H(z) &= 4 + 5z^{-1} + 6z^{-2} + 7z^{-3} \end{aligned}$$

$$\begin{aligned} Y(z) &= X(z)H(z) = \left[4 + 5z^{-1} + 6z^{-2} + 7z^{-3}\right] X(z) \\ &= 4X(z) + 5z^{-1}X(z) + 6z^{-2}X(z) + 7z^{-3}X(z) \\ y(n) &= 4x(n) + 5x(n-1) + 6x(n-2) + 7x(n-3) \end{aligned}$$

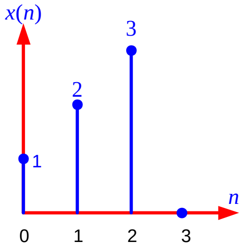
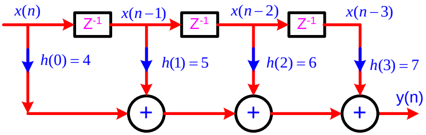


Figure 4: Input x(n) to the FIR filter



$$
\begin{aligned}
y(n) &= 4x(n) + 5x(n-1) + 6x(n-2) + 7x(n-3) \\
y(0) &= 4x(0) + 5x(0-1) + 6x(0-2) + 7x(0-3) = 4 \times 1 = 4 \\
y(1) &= 4x(1) + 5x(0) = 4 \times 2 + 5 \times 1 = 13 \\
y(2) &= 4x(2) + 5x(1) + 6x(0) = 4 \times 3 + 5 \times 2 + 6 \times 1 = 28 \\
y(3) &= 4x(3) + 5x(2) + 6x(1) + 7x(0) = 4 \times 0 + 5 \times 3 + 6 \times 2 + 7 \times 1 = 34 \\
y(4) &= 4x(4) + 5x(3) + 6x(2) + 7x(1) = 0 + 0 + 6 \times 3 + 7 \times 2 = 32 \\
y(5) &= 4x(5) + 5x(4) + 6x(3) + 7x(2) = 0 + 0 + 0 + 7 \times 3 = 21
\end{aligned}
$$

$$y(n) = [4,\ 13,\ 28,\ 34,\ 32,\ 21]$$

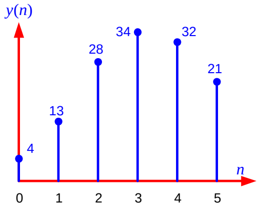
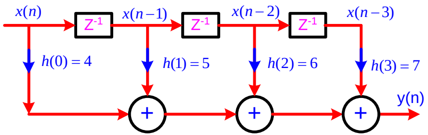


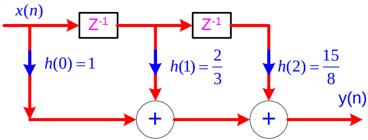
Figure 5: Output y(n) of the FIR filter



June 2015 Obtain the direct form realization of linear phase FIR system given by

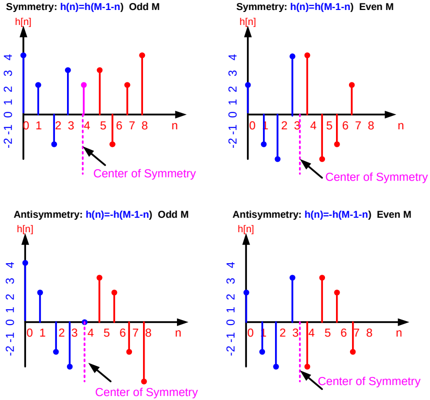
$$H(z) = 1 + \frac{2}{3}z^{-1} + \frac{15}{8}z^{-2}$$

Solution:



Linear Phase FIR structure

- Linear phase is a property of a filter, where the phase response of the filter is a linear function of frequency. The result is that all frequency components of the input signal are shifted in time (usually delayed) by the same constant amount, which is referred to as the phase delay. And consequently, there is no phase distortion due to the time delay of frequencies relative to one another.
- Linear-phase filters have a symmetric impulse response.
- The FIR filter has linear phase if its unit sample response satisfies the following condition:



$$h(n) = h(M-1-n) \qquad n = 0, 1, 2, \ldots, M-1$$

The Z transform of the unit sample response is expressed as $H(z) = \sum_{n=0}^{M-1} h(n)z^{-n}$

- For even M $\quad h(n) = h(M-1-n)$

$$H(z) = \sum_{n=0}^{M/2-1} h(n)\left[z^{-n} + z^{-(M-1-n)}\right]$$

- The system expression is

$$H(z) = \frac{Y(z)}{X(z)}$$

$$\frac{Y(z)}{X(z)} = \sum_{n=0}^{M/2-1} h(n)\left[z^{-n} + z^{-(M-1-n)}\right]$$

$$Y(z) = \sum_{n=0}^{M/2-1} h(n)\left[z^{-n} + z^{-(M-1-n)}\right] X(z)$$

- By expanding the summation

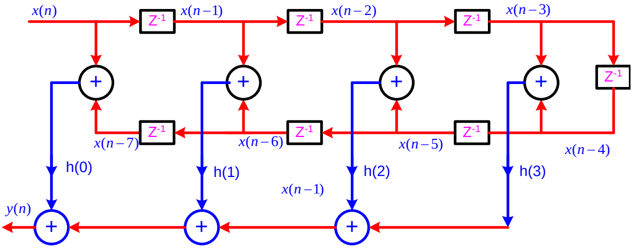
$$\begin{aligned} Y(z) &= h(0)\left[1 + z^{-(M-1)}\right] X(z) + h(1)\left[z^{-1} + z^{-(M-2)}\right] X(z) + \dots \\ &+ h(M/2-1)\left[z^{-(M/2-1)} + z^{-(M/2)}\right] X(z) \end{aligned}$$

- By taking inverse z transform

$$\begin{aligned} y(n) &= h(0)[x(n)+x(n-(M-1))]+h(1)[x(n-1)+x(n-(M-2))]+\dots \\ &+ h(M/2-1)x\left[n-(M/2-1)\right]+x\left[n-(M/2)\right] \end{aligned}$$

- By considering M=8 then $y(n)$ is

$$\begin{aligned} y(n) &= h(0)[x(n)+x(n-7)]+h(1)[x(n-1)+x(n-6)]+\dots \\ &+ h(2)\{x(n-2)+x(n-5)\}+h(3)\{x(n-3)+x(n-4)\} \end{aligned}$$



- For Odd M $h(n) = h(M-1-n)$

$$H(z) = h\left(\frac{M-1}{2}\right) z^{-\left(\frac{M-1}{2}\right)} + \sum_{n=0}^{(M-3)/2} h(n)\left[z^{-n} + z^{-(M-1-n)}\right]$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$\frac{Y(z)}{X(z)} = h\left(\frac{M-1}{2}\right) z^{-\left(\frac{M-1}{2}\right)} + \sum_{n=0}^{(M-3)/2} h(n)\left[z^{-n} + z^{-(M-1-n)}\right]$$

$$Y(z) = h\left(\frac{M-1}{2}\right) z^{\left(-\frac{M-1}{2}\right)} X(z) + \sum_{n=0}^{(M-3)/2} h(n)\left[z^{-n} + z^{-(M-1-n)}\right] X(z)$$

- By expanding the summation

$$\begin{aligned} Y(z) &= h\left(\frac{M-1}{2}\right) z^{-\left(\frac{M-1}{2}\right)} X(z) + h(0)\left[1 + z^{-(M-1)}\right] X(z) \\ &= +h(1)\left[z^{-1} + z^{-(M-2)}\right] X(z) + \dots \end{aligned}$$

- By taking inverse z transform

$$\begin{aligned} y(n) &= h\left(\frac{M-1}{2}\right)x\left[n-\frac{M-1}{2}\right]+h(0)[x(n)+x(n-(M-1))]+ \\ &+ h(1)[x(n-1)+x(n-(M-2))]+\dots \\ &+ h\left(\frac{M-3}{2}\right)\left[x\left[n-\frac{M-3}{2}\right]+x\left[n-\frac{M+1}{2}\right]\right] \end{aligned}$$

for M=9

$$\begin{aligned} y(n) &= h(4)x(n-4)+h(0)[x(n)+x(n-8)]+h(1)[x(n-1)+x(n-7)]+\dots \\ &+ h(2)\{x(n-2)+x(n-6)\}+h(3)\{x(n-3)+x(n-5)\} \end{aligned}$$

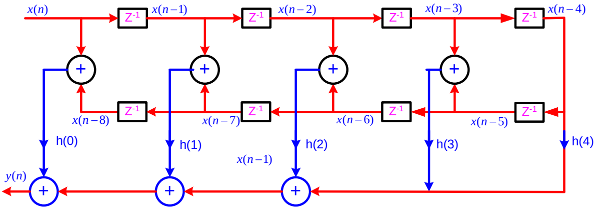


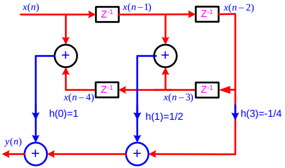
Figure 7: Linear phase FIR structure

Realize a linear phase FIR filter with the following impulse response. Give necessary equations
$h(n) = \delta(n) + \frac{1}{2}\delta(n-1) - \frac{1}{4}\delta(n-2) + \delta(n-4) + \frac{1}{2}\delta(n-3)$

Solution: $h(n) = \{1, \frac{1}{2}, -1/4, \frac{1}{2}, 1\}$. Here M=5 $h(0) = h(4)$, $h(1) = h(3)$

$$
\begin{aligned}
h(n) &= \delta(n) + \frac{1}{2}\delta(n-1) - \frac{1}{4}\delta(n-2) + \frac{1}{2}\delta(n-3) + \delta(n-4) \\
H(z) &= 1 + \frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} + \frac{1}{2}z^{-3} + z^{-4}
\end{aligned}
$$

$$
\begin{aligned}
Y(z) &= X(z)H(z) = \left[1 + \frac{1}{2}z^{-1} - \frac{1}{4}z^{-2} + \frac{1}{2}z^{-3} + z^{-4}\right] X(z) \\
Y(z) &= X(z) + \frac{1}{2}z^{-1}X(z) - \frac{1}{4}z^{-2}X(z) + \frac{1}{2}z^{-3}X(z) + z^{-4}X(z) \\
y(n) &= x(n) + \frac{1}{2}x(n-1) - \frac{1}{4}x(n-2) + \frac{1}{2}x(n-3) + x(n-4) \\
y(n) &= [x(n) + x(n-4)] + \frac{1}{2}[x(n-1) + x(n-3)] - \frac{1}{4}x(n-2)
\end{aligned}
$$

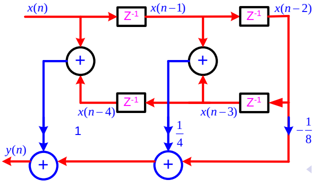


DEC 2010,2011, 2012 Realize a linear phase FIR filter having the following impulse response $h(n) = \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2) + \frac{1}{4}\delta(n-3) + \delta(n-4)$

Solution: $h(n) = \{1, \frac{1}{4}, -1/8, +\frac{1}{4}, 1\}$. Here M=5 $h(0) = h(4)$, $h(1) = h(3)$

$$\begin{aligned} h(n) &= \delta(n) + \frac{1}{4}\delta(n-1) - \frac{1}{8}\delta(n-2) + \frac{1}{4}\delta(n-3) + \delta(n-4) \\ H(z) &= 1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2} + \frac{1}{4}z^{-3} + z^{-4} \end{aligned}$$

$$\begin{aligned} Y(z) &= X(z)H(z) = \left[1 + \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2} + \frac{1}{4}z^{-3} + z^{-4}\right] X(z) \\ Y(z) &= X(z) + \frac{1}{4}z^{-1}X(z) - \frac{1}{8}z^{-2}X(z) + \frac{1}{4}z^{-3}X(z) + z^{-4}X(z) \\ y(n) &= x(n) + \frac{1}{4}x(n-1) - \frac{1}{8}x(n-2) + \frac{1}{4}x(n-3) + x(n-4) \\ y(n) &= [x(n) + x(n-4)] + \frac{1}{4}[x(n-1) + x(n-3)] - \frac{1}{8}x(n-2) \end{aligned}$$

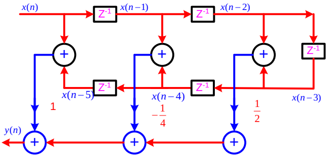


May 2010 Realize a linear phase FIR filter having the following impulse response
$h(n) = \delta(n) - \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) + \frac{1}{2}\delta(n-3) - \frac{1}{4}\delta(n-4) + \delta(n-5)$

Solution: $h(n) = \{1, -\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{4}, 1\}$. Here M=6 $h(0) = h(5), \; h(1) = h(4), \; h(2) = h(3)$

$$\begin{aligned} h(n) &= \delta(n) - \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2) + \frac{1}{2}\delta(n-3) - \frac{1}{4}\delta(n-4) + \delta(n-5) \\ H(z) &= 1 - \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{4}z^{-4} + z^{-5} \end{aligned}$$

$$\begin{aligned} Y(z) &= X(z)H(z) = \left[1 - \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2} + \frac{1}{2}z^{-3} - \frac{1}{4}z^{-4} + z^{-5}\right] X(z) \\ Y(z) &= X(z) - \frac{1}{4}z^{-1}X(z) + \frac{1}{2}z^{-2}X(z) + \frac{1}{2}z^{-3}X(z) - \frac{1}{4}z^{-4}X(z) + z^{-5}X(z) \\ y(n) &= x(n) - \frac{1}{4}x(n-1) + \frac{1}{2}x(n-2) + \frac{1}{2}x(n-3) - \frac{1}{4}x(n-4) + x(n-5) \\ y(n) &= [x(n) + x(n-5)] - \frac{1}{4}[x(n-1) + x(n-4)] + \frac{1}{2}[x(n-2) + x(n-3)] \end{aligned}$$



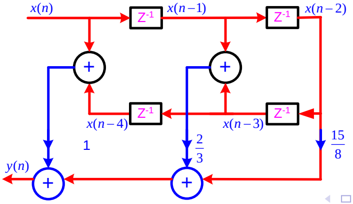
May 2010

Obtain the direct form Realization of linear phase FIR system given by
$H(z) = 1 + \frac{2}{3}z^{-1} + \frac{15}{8}z^{-2} + \frac{2}{3}z^{-3} + z^{-4}$

Solution: $h(n) = \{1, \frac{2}{3}, 15/8, \frac{2}{3}, 1\}$. Here M=5 $h(0) = h(4),\ h(1) = h(3)$

$$H(z) = 1 + \frac{2}{3}z^{-1} + \frac{15}{8}z^{-2} + \frac{2}{3}z^{-3} + z^{-4}$$

$$\begin{aligned}
Y(z) &= X(z)H(z) = \left[1 + \frac{2}{3}z^{-1} + \frac{15}{8}z^{-2} + \frac{2}{3}z^{-3} + z^{-4}\right]X(z) \\
Y(z) &= X(z) + \frac{2}{3}z^{-1}X(z) + \frac{15}{8}z^{-2}X(z) + \frac{2}{3}z^{-3}X(z) + z^{-4}X(z) \\
y(n) &= x(n) + \frac{2}{3}x(n-1) + \frac{15}{8}x(n-2) + \frac{2}{3}x(n-3) + x(n-4) \\
y(n) &= [x(n) + x(n-4)] + \frac{2}{3}[x(n-1) + x(n-3)] + \frac{15}{8}x(n-2)
\end{aligned}$$



Realize the following system function by linear phase FIR structure

$$H(z) = \frac{2}{3}z + 1 + \frac{2}{3}z^{-1}$$

Solution:

$$H(z) = 1 + \frac{2}{3}(z + z^{-1})$$

$$Y(z) = X(z)H(z) = \left[1 + \frac{2}{3}(z + z^{-1})\right] X(z)$$

$$Y(z) = X(z) + \frac{2}{3}(z + z^{-1})X(z)$$

$$y(n) = x(n) + \frac{2}{3}\left[x(n-1) + x(n+1)\right]$$

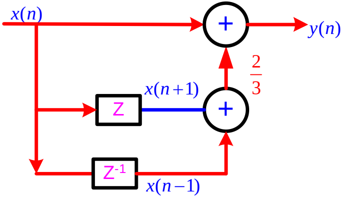


Figure 8: Linear phase FIR structure

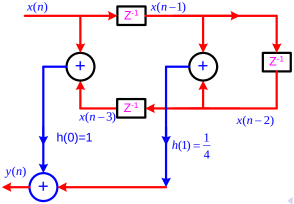
Realize the following system function by linear phase FIR structure

$$H(z) = 1 + \frac{z^{-1}}{4} + \frac{z^{-2}}{4} + z^{-3}$$

Solution:

$$H(z) = 1 + \frac{1}{4}(z^{-1} + z^{-2}) + z^{-3}$$

$$\begin{aligned} Y(z) &= X(z)H(z) = \left[1 + \frac{1}{4}(z^{-1} + z^{-2}) + z^{-3}\right] X(z) \\ &= X(z) + \frac{1}{4}(z^{-1}X(z) + z^{-2}X(z)) + z^{-3}X(z) \\ y(n) &= [x(n) + x(n-3)] + \frac{1}{4}[x(n-1) + x(n-2)] \end{aligned}$$

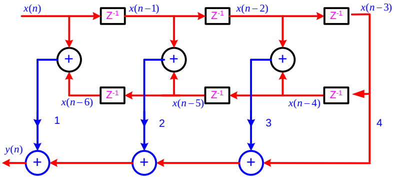


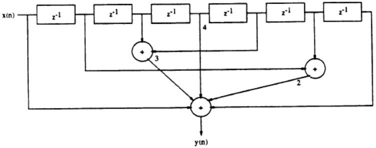
Realize the following system function by linear phase FIR structure: $h(n) = [1, 2, 3, 4, 3, 2, 1]$

Solution:

$$H(z) = 1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 3z^{-4} + 2z^{-5} + 1z^{-6}$$

$$\begin{aligned}
Y(z) &= X(z)H(z) = \left[1 + 2z^{-1} + 3z^{-2} + 4z^{-3} + 3z^{-4} + 2z^{-5} + 1z^{-6}\right] X(z) \\
&= X(z) + 2z^{-1}X(z) + 3z^{-2}X(z) + 4z^{-3}X(z) + 3z^{-4}X(z) + 2z^{-5}X(z) + 1z^{-6}X(z) \\
y(n) &= x(n) + 2x(n-1) + 3x(n-2) + 4x(n-3) + 3x(n-4) + 2x(n-5) + 1x(n-6) \\
y(n) &= 1[x(n) + x(n-6)] + 2[x(n-1) + x(n-5)] + 3[x(n-2) + x(n-4)] + 4x(n-3)
\end{aligned}$$





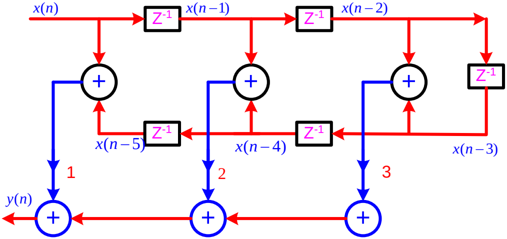
Realize a direct form for the following linear phase filters

$$h(n) = [1, 2, 3, 3, 2, 1]$$

Solution:

$$H(z) = 1 + 2z^{-1} + 3z^{-2} + 3z^{-3} + 2z^{-4} + 1z^{-5}]$$

$$\begin{aligned}
Y(z) &= X(z)H(z) = \left[1 + 2z^{-1} + 3z^{-2} + 3z^{-3} + 2z^{-4} + 1z^{-5}\right] X(z) \\
&= X(z) + 2z^{-1}X(z) + 3z^{-2}X(z) + 3z^{-3}X(z) + 2z^{-4}X(z) + 1z^{-5}X(z) \\
y(n) &= x(n) + 2x(n-1) + 3x(n-2) + 3x(n-3) + 2x(n-4) + 1x(n-5) \\
y(n) &= 1[x(n) + x(n-5)] + 2[x(n-1) + x(n-4)] + 3[x(n-2) + x(n-3)]
\end{aligned}$$



Frequency Sampling for FIR Systems

- Frequency sampling realization is used when an FIR filter is to operate on some desired frequency.
- The desired frequency may be defined and this reduces the complexity of the system.
- Consider a frequency ω

$$\omega_k = \frac{2\pi}{M}k \qquad k = 0, 1, \ldots M-1$$

- where ω_k is the frequency at discrete points.
- Let the unit sample response of FIR system be $h(n)$. The fourier transform of the $h(n)$ is defined as

$$H(\omega) = \sum_{n=0}^{M-1} h(n)e^{-j\omega n}$$

$H(\omega)$ at $\omega_{=}\omega_k = \frac{2\pi}{M}k$

$$H(\omega_k) = H\left(\frac{2\pi}{M}k\right) = \sum_{n=0}^{M-1} h(n)e^{-j2\pi kn/M}$$

- $H(\omega_k)$ is also written as H(k) and defined as

$$H(k) = \sum_{n=0}^{M-1} h(n)e^{-j2\pi kn/M} \qquad k = 0, 1, \ldots M-1$$

- This equation represents the M point DFT of h(n) and it is defined as

$$h(n) = \frac{1}{M}\sum_{n=0}^{M-1} H(k)e^{j2\pi kn/M} \qquad n = 0, 1, \ldots M-1$$

- z transform is defined as

$$H(z) = \sum_{n=0}^{M-1} h(n)z^{-n}$$

- substituting the value of h(n)

$$H(z) = \sum_{n=0}^{M-1}\left(\frac{1}{M}\sum_{n=0}^{M-1} H(k)e^{j2\pi kn/M}\right)z^{-n}$$

- Interchanging the order of summations in the above equation

$$H(z) = \sum_{n=0}^{M-1} H(k)\left(\frac{1}{M}\sum_{n=0}^{M-1} e^{j2\pi k/M}z^{-1}\right)^{n}$$

- The geometric series formula is

$$\sum_{n=0}^{N} a^n = \frac{1 - a^{N+1}}{1 - a}$$

- Using the formula where $a = e^{2\pi kn/M} z^{-1}$

$$\begin{aligned} H(z) &= \sum_{k=0}^{M-1} H(k) \frac{1}{M} \frac{\left(1 - e^{j2\pi k/M} z^{-1}\right)^M}{1 - e^{j2\pi k/M} z^{-1}} \\ &= \sum_{k=0}^{M-1} H(k) \frac{1}{M} \frac{1 - e^{j2\pi k} z^{-M}}{1 - e^{j2\pi k/M} z^{-1}} \end{aligned}$$

- $e^{j2\pi k} = cos(2\pi k) + jsin(2\pi k) = 1$

$$H(z) = \sum_{k=0}^{M-1} H(k)\frac{1}{M}\frac{1-z^{-M}}{1-e^{j2\pi k/M}z^{-1}}$$

$$H(z) = \frac{1-z^{-M}}{M}\sum_{k=0}^{M-1}\frac{H(k)}{1-e^{j2\pi k/M}z^{-1}}$$

- The equation can be considered as multiplication of two systems and defined as

$$H(z) = H_1(z).H_2(z)$$

- where

$$H_1(z) = \frac{1-z^{-M}}{M}$$

$$H_2(z) = \sum_{k=0}^{M-1}\frac{H(k)}{1-e^{j2\pi k/M}z^{-1}}$$

- $H_1(z)$ and $H_2(z)$ are realized independently. $H(z)$ is obtained by multiplication of $H_1(z)$ and $H_2(z)$.

$$H(z) = \frac{1 - z^{-M}}{M} \sum_{k=0}^{M-1} \frac{H(k)}{1 - e^{j2\pi k/M} z^{-1}}$$

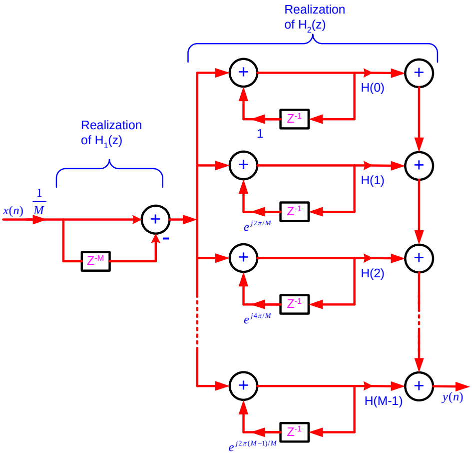


Figure 9: Frequency sampling FIR structure

Determine the transfer function H(z) of an FIR filter to implement $h(n) = \delta(n) + 2\delta(n-1) + \delta(n-2)$. Using frequency sampling technique

Solution:

$h(0) = 1,\ h(1) = 2,\ h(2) = 1$ The DFT of $h(n)$

$$H(k) = \sum_{n=0}^{M-1} h(n)e^{-j2\pi kn/M}$$

With M=3

$$H(k) = \sum_{n=0}^{2} h(n)e^{-j2\pi kn/3}$$

$$\begin{aligned} H(k) &= \sum_{n=0}^{2} h(n)e^{-j2\pi kn/3} \\ &= h(0) + h(1)e^{-j2\pi k/3} + h(2)e^{-j4\pi k/3} \end{aligned}$$

k=0,1,2

$$\begin{aligned} H(0) &= 1+2+1 = 4 \\ H(1) &= 1 + 2e^{-j2\pi/3} + e^{-j4\pi/3} = -0.5 - j0.866 = e^{-j2\pi/3} \\ H(2) &= 1 + 2e^{-j4\pi/3} + e^{-j8\pi/3} = -0.5 - j0.866 = e^{-j2\pi/3} \end{aligned}$$

The system function is

$$H(z) = \frac{1 - z^{-M}}{M} \sum_{k=0}^{M-1} \frac{H(k)}{1 - e^{2\pi k/M} z^{-1}}$$

For M=3

$$\begin{aligned} H(z) &= \frac{1 - z^{-3}}{3} \sum_{k=0}^{2} \frac{H(k)}{1 - e^{2\pi k/3} z^{-1}} \\ &= \frac{1 - z^{-3}}{3} \left[\frac{H(0)}{1 - z^{-1}} + \frac{H(1)}{1 - e^{-j2\pi/3} z^{-1}} + \frac{H(2)}{1 - e^{-j4\pi/3} z^{-1}} \right] \\ &= H_1(z) \times H_2(z) \end{aligned}$$

where

$$\begin{aligned} H_1(z) &= \frac{1 - z^{-3}}{3} \\ H_2(z) &= \left[\frac{4}{1 - z^{-1}} + \frac{e^{-j2\pi/3}}{1 - e^{-j2\pi/3} z^{-1}} + \frac{e^{-j4\pi/3}}{1 - e^{-j4\pi/3} z^{-1}} \right] \end{aligned}$$

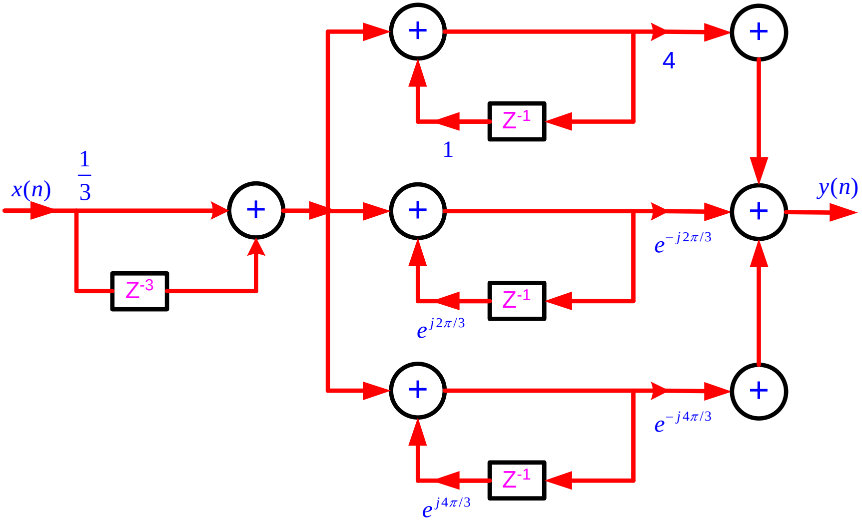


Figure 10: Frequency sampling FIR structure

Realization of IIR system

- Direct form structure for IIR system
  1. Direct form-I structure of IIR system
  2. Direct form-II structure of IIR system
- Cascade form structure for IIR system
- Parallel form structure for IIR system
- Lattice structure of IIR system

- An Infinite Impulse Response (IIR) filters are digital filters with infinite impulse response. Unlike FIR filters, they have the feedback (a recursive part of a filter) and are known as recursive digital filters therefore.
- IIR filters are computationally more efficient than FIR filters as they require fewer coefficients due to the fact that they use feedback.
- If the coefficients deviate from their true values then the feedback can make the filter unstable.

- LTI systems are represented by the following difference equation.

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k)$$

- By taking z-transform on both sides

$$Y(z) = -\sum_{k=1}^{N} a_k z^{-k} Y(z) + \sum_{k=0}^{M} b_k z^{-k} X(z)$$

$$Y(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

- The system function $H(z)$ is defined as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

- The general expression of an IIR filter can be expressed as follows:

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$H_1(z) = \sum_{k=0}^{M} b_k z^{-k}$$

$$H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

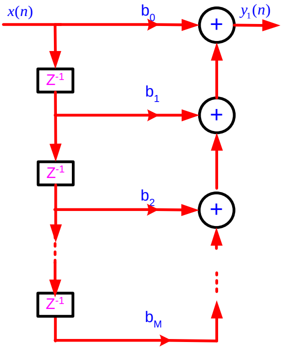
$$H(z) = H_1(z).H_2(z)$$

Direct form Structure of IIR System

Structures in which the multiplier coefficients are precisely the coefficients of the transfer function are called direct form structure.

- The direct form structures for $H_1(z)$ $H_2(z)$ are

$$\begin{aligned} H_1(z) &= \sum_{k=0}^{M} b_k z^{-k} \\ &= b_0 + b_1 z^{-1} + \ldots b_M z^{-M} \end{aligned}$$



- $H_1(z)$ is defined as

$$H_1(z) = \frac{Y_1(z)}{X_1(z)}$$

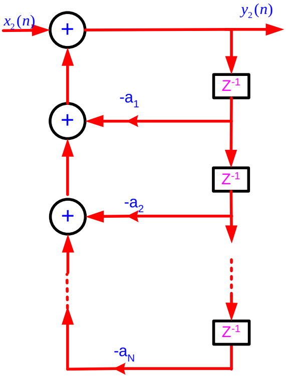
$$Y_1(z) = b_0 X_1(z) + b_1 z^{-1} X_1(z) + \ldots b_M z^{-M} X_1(z)$$

- Its inverse z transform is

$$y_1(n) = b_0 x_1(n) + b_1 x_1(n-1) + \ldots b_M x_1(n-M)$$

- $H_2(z)$ is the all pole system and is given by

$$H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$



- $H_2(z)$ is also expressed in terms of system function

$$H_2(z) = \frac{Y_2(z)}{X_2(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$Y_2(z)[1 + \sum_{k=1}^{N} a_k z^{-k}] = X_2(z)$$

$$Y_2(z) = -\sum_{k=1}^{N} a_k z^{-k} Y_2(z) + X_2(z)$$

Expanding the above function

$$Y_2(z) = -a_1 z^{-1} Y_2(z) - a_2 z^{-2} Y_2(z) - a_3 z^{-3} Y_2(z) + \ldots - a_N z^{-N} Y_2(z) + X_2(z)$$

Its inverse z transform is

$$y_2(n) = -a_1 y_2(n-1) - a_2 y_2(n-2) - a_3 y_2(n-3) - \ldots - a_N y_2(n-N) + x_2(n)$$

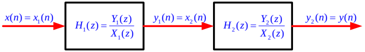


Figure 11: Direct form I realization of IIR system

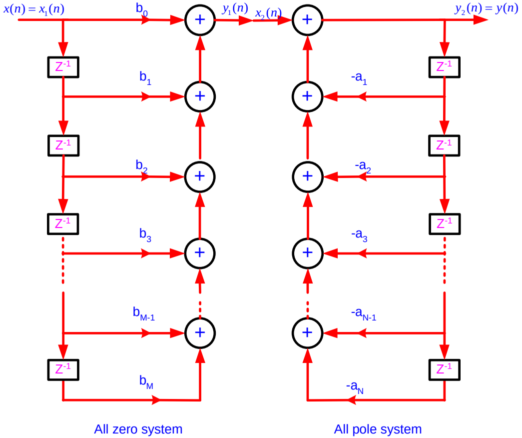


Figure 12: Direct form I realization of IIR system

Direct form-II Structure of IIR System

- Direct form II is also called as canonic form because the number delay elements is same as the order of difference equation.

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{X(z)} \frac{W(z)}{W(z)} = \frac{W(z)}{X(z)} \frac{Y(z)}{W(z)} = H_1(z).H_2(z)$$

$$H_1(z) = \frac{W(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

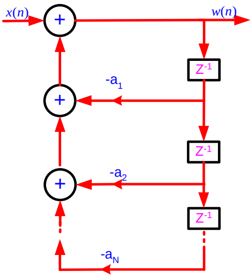
$$\begin{aligned}
W(z)[1 + \sum_{k=1}^{N} a_k z^{-k}] &= X(z) \\
W(z) &= X(z) - \sum_{k=1}^{N} a_k z^{-k} W(z) \\
&= X(z) - a_1 z^{-1} W(z) - a_2 z^{-2} W(z) - - \ldots a_N z^{-N} W(z) \\
w(n) &= x(n) - a_1 w(n-1) - a_2 w(n-2) - \ldots - a_N w(n-N)
\end{aligned}$$

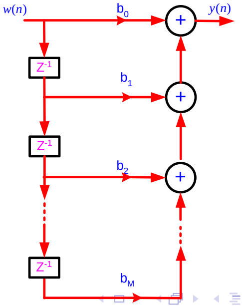
$$H_2(z) = \frac{Y(z)}{W(z)} = \sum_{k=0}^{M} b_k z^{-k}$$

$$\begin{aligned} Y(z) &= \sum_{k=0}^{M} b_k z^{-k} W(z) \\ &= b_0 W(z) + b_1 z^{-1} W(z) + b_2 z^{-2} W(z) + \ldots + b_M z^{-M} W(z) \end{aligned}$$

By inverse z transform

$$y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2) + \ldots + b_M w(n-M)$$





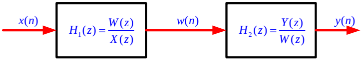


Figure 13: Cascade connection of $H_1(z)$ $H_2(z)$

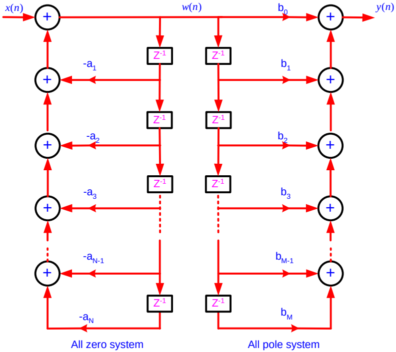


Figure 14: Direct form-II Structure

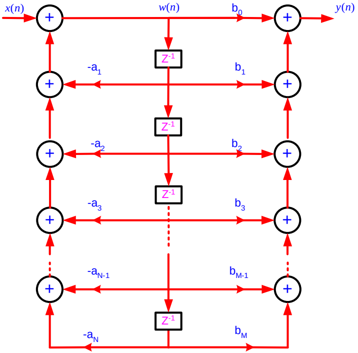


Figure 15: Direct form-II Structure

A difference equation describing a filter is given below

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n) + \frac{1}{2}x(n-1)$$

Draw the direct form I and direct form II structures

Solution:

$$y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n) + \frac{1}{2}x(n-1)$$

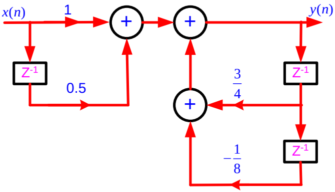


Figure 16: Direct form-I

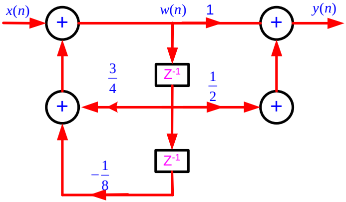


Figure 17: Direct form-II

[July 2013]:

$y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$obtain the the direct form I and direct form II structures

Solution:

$$y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$$

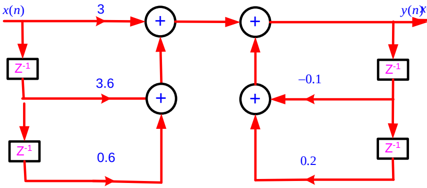


Figure 18: Direct form-I

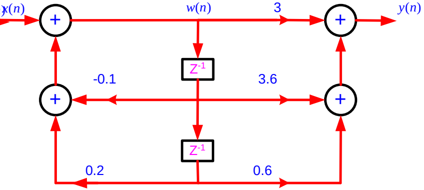


Figure 19: Direct form-II

June 2010 EC

Obtain direct form I and direct form II for the system described by

$$y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$$

Solution:

$$y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$$

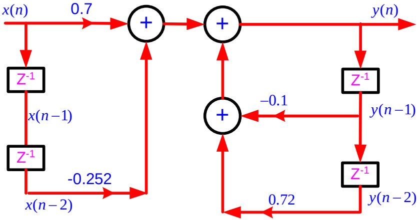


Figure 20: Direct form-I

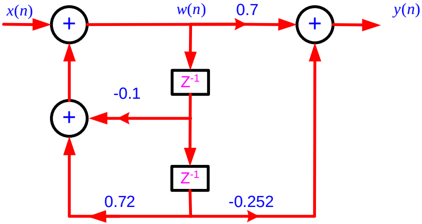


Figure 21: Direct form-II

A system is represented by transfer function $H(z)$ is given by

$$H(z) = 3 + \frac{4z}{z - \frac{1}{2}} - \frac{2}{z - \frac{1}{4}}$$

(i) Does this H(z) represent a FIR or IIR filter ? Why ?

(ii) Give a difference equation realization of this system using direct form I

(iii) Draw the block diagram for the direct form II canonic realization, and give the governing equations for implementation

Solution:

$$\begin{aligned} H(z) &= 3 + \frac{4z}{z - \frac{1}{2}} - \frac{2}{z - \frac{1}{4}} = 3 + \frac{4z}{z - 0.5} - \frac{2}{z - 0.25} \\ &= \frac{7z^2 - 5.25z + 1.375}{z^2 - 0.75z + 0.125} \end{aligned}$$

(i): By observing the system function it has numerator of polynomial of order 2 as well as denominator of polynomial of order 2. The system function has poles as well zeros, hence it represents IIR filter.

(ii): Direct form I

$$H(z) = \frac{7z^2 - 5.25z + 1.375}{z^2 - 0.75z + 0.125} = \frac{7 - 5.25z^{-1} + 1.375z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}} = \frac{Y(Z)}{X(Z)}$$

$$Y(Z)[1 - 0.75z^{-1} + 0.125z^{-2}] = 7X(Z) - 5.25z^{-1}X(Z) + 1.375z^{-2}X(Z)$$

$$y(n) - 0.75y(n-1) + 0.125y(n-2) = 7x(n) - 5.25x(n-1) + 1.375x(n-2)$$

$$y(n) = 0.75y(n-1) - 0.125y(n-2) + 7x(n) - 5.25x(n-1) + 1.375x(n-2)$$

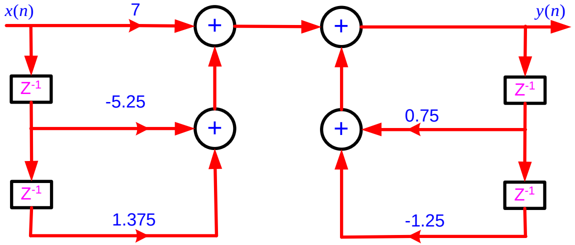


Figure 22: Direct form-I Structure

(iii): Direct form II canonic realization

$$H(z) = \frac{7 - 5.25z^{-1} + 1.375z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}} = \frac{Y(z)}{X(z)}$$

$$H(z) = \frac{Y(z)}{W(z)} \frac{W(z)}{X(z)} = \frac{7 - 5.25z^{-1} + 1.375z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$H(z) = H_1(z).H_2(z) = \frac{1}{1 - 0.75z^{-1} + 0.125z^{-2}}[7 - 5.25z^{-1} + 1.375z^{-2}]$$

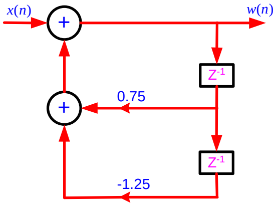
$$H_1(z) = \frac{W(z)}{X(z)} = \frac{1}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$W(z)[1 - 0.75z^{-1} + 0.125z^{-2}] = X(z)$$

$$W(z) = X(z) + 0.75z^{-1}W(z) - 0.125z^{-2}W(z)$$

Taking inverse z transform

$$w(n) = x(n) + 0.75w(n-1) - 0.125w(n-2)$$



$$H_2(z) = \frac{Y(z)}{W(z)} = 7 - 5.25z^{-1} + 1.375z^{-2}$$

$$\frac{Y(z)}{W(z)} = 7 - 5.25z^{-1} + 1.375z^{-2}$$

$$Y(z) = 7W(z) - 5.25z^{-1}W(z) + 1.375z^{-2}W(z)$$

Taking inverse z transform

$$y(n) = 7w(n) - 5.25w(n-1) + 1.375w(n-2)$$

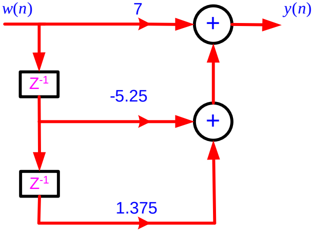


Figure 23: Direct form of $H_2(z)$

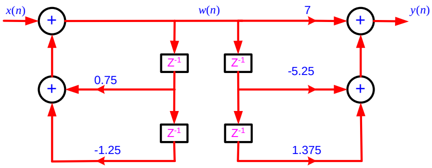


Figure 24: Realization of $H(z) = H_1(z).H_2(z)$

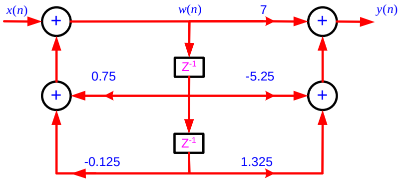


Figure 25: Direct form-II, canonic form

2012 DEC, 2012 June

Obtain the direct form II (canonic) realization for the following system.

$$H(z) = \frac{(z-1)(z^2+5z+6)(z-3)}{(z^2+6z+5)(z^2-6z+8)}$$

Solution:

$$\begin{aligned}
H(z) &= \frac{(z-1)(z^2+5z+6)(z-3)}{(z^2+6z+5)(z^2-6z+8)} \\
&= \frac{(z^2-4z+3)(z^2+5z+6)}{(z^2+6z+5)+(z^2-6z+8)} \\
&= \frac{z^4+z^3-11z^2-9z+18}{z^4-23z^2+18z+40} \\
&= \frac{1+z^{-1}-11z^{-2}-9z^{-3}+18z^{-4}}{1-23z^{-2}+18z^{-3}+40z^{-4}}
\end{aligned}$$

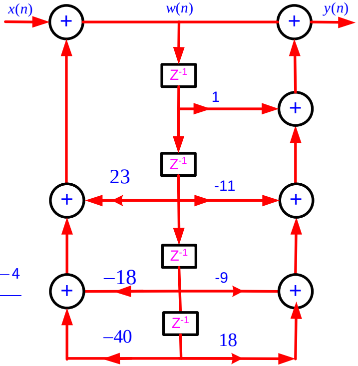


Figure 26: Direct form-II

2011 July

Obtain the direct form II realizations of the following system.

$$H(z) = \frac{(1+z^{-1})}{(1-\frac{1}{4}z^{-1})(1-z^{-1}+\frac{1}{2}z^{-2})}$$

Solution:

$$\begin{aligned} H(z) &= \frac{(1+z^{-1})}{(1-\frac{5}{4}z^{-1}+\frac{3}{4}z^{-2}-\frac{1}{8}z^{-3})} \\ &= \frac{1}{(1-\frac{5}{4}z^{-1}+\frac{3}{4}z^{-2}-\frac{1}{8}z^{-3})} \times \frac{(1+z^{-1})}{1} = H_1(z) \times H_2(z) \end{aligned}$$

$$H_1(z) = \frac{W(z)}{X(z)} = \frac{1}{(1-\frac{5}{4}z^{-1}+\frac{3}{4}z^{-2}-\frac{1}{8}z^{-3})}$$

$$W(z)[1-\frac{5}{4}z^{-1}+\frac{3}{4}z^{-2}-\frac{1}{8}z^{-3}] = X(z)$$

$$\begin{aligned} W(z) &= X(z) + \frac{5}{4}z^{-1}W(z) - \frac{3}{4}z^{-2}W(z) + \frac{1}{8}z^{-3}W(z) \\ w(n) &= x(n) + \frac{5}{4}w(n-1) - \frac{3}{4}w(n-2) + \frac{1}{8}w(n-3) \end{aligned}$$

$H_2(z) = (1 + z^{-1})$

$$\begin{aligned} H_2(z) &= \frac{Y(z)}{W(z)} = (1 + z^{-1}) \\ Y(z) &= W(z) + z^{-1}W(z) \\ y(n) &= w(n) + w(n-1) \end{aligned}$$

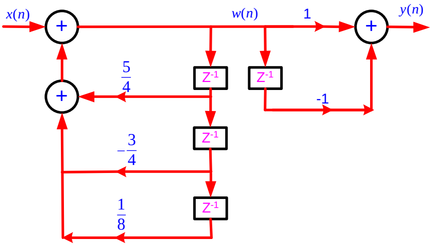


Figure 27: Direct form-II

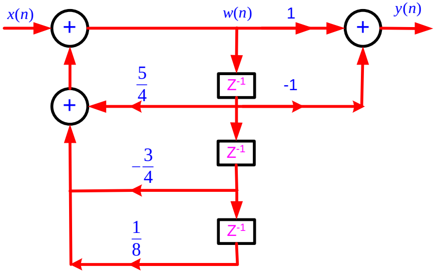


Figure 28: Direct form-II

June 2010 EE

Realize the following system function in direct form I

$$H(z) = \frac{1+\frac{1}{5}z^{-1}}{(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2})(1+\frac{1}{4}z^{-1})}$$

Solution:

$$H(z) = \frac{1+\frac{1}{5}z^{-1}}{(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2})(1+\frac{1}{4}z^{-1})}$$

$$H(z)=\frac{Y(z)}{X(z)} = \frac{1+\frac{1}{5}z^{-1}}{(1-\frac{1}{4}z^{-1}+\frac{5}{24}z^{-2}+\frac{1}{12}z^{-3})}$$

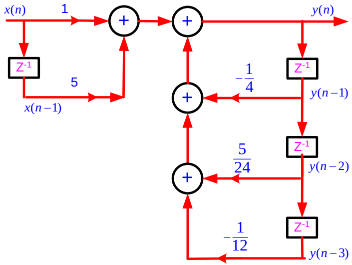


Figure 29: Direct form-I

$$Y(z)-\frac{1}{4}z^{-1}Y(z)+\frac{5}{24}z^{-2}Y(z)-\frac{1}{12}z^{-3}Y(z)=X(z)+5z^{-1}X(z)$$

By taking inverse Z transform on both sides

$$y(n)-\frac{1}{4}y(n-1)+\frac{5}{24}y(n-2)-\frac{1}{12}y(n-3)=x(n)+5x(n-1)$$

Obtain direct form II for the system described by $H(z) = \frac{8Z^3-4Z^2+11Z-2}{(Z-\frac{1}{4})(Z^2-Z+\frac{1}{2})}$ [December 2010 EC]

Solution:

$$
\begin{aligned}
H(z) &= \frac{8Z^3 - 4Z^2 + 11Z - 2}{(Z-\frac{1}{4})(Z^2 - Z + \frac{1}{2})} = \frac{Y(z)}{X(z)} \\
&= \frac{8Z^3 - 4Z^2 + 11Z - 2}{Z^3 - 1.25Z^2 + 0.75Z - 0.125} \\
&= \frac{8 - 4Z^{-1} + 11Z^{-2} - 2Z^{-3}}{1 - 1.25Z^{-1} + 0.75Z^{-2} - 0.125Z^{-1}}
\end{aligned}
$$

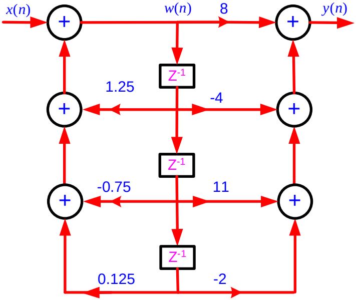


Figure 30: Direct form-II

By taking inverse Z transform on both sides

$$y(n) - 1.25y(n-1) + 0.75y(n-2) - 0.125y(n-3) = 8x(n) - 4x(n-1) + 11x(n-2) - 2x(n-3)$$

$$y(n) = 8x(n) - 4x(n-1) + 11x(n-2) - 2x(n-3) + 1.25y(n-1) - 0.75y(n-2) + 0.125y(n-3)$$

Obtain direct form II for the system described by

$$H(z) = \frac{(z-1)(z-2)(z+1)z}{[z-(\frac{1}{2}+j\frac{1}{2})][z-(\frac{1}{2}-j\frac{1}{2})](z-\frac{j}{4})(z+\frac{j}{4})}$$

[December 2010 EC]

Solution:

$$\begin{aligned} H(z) = \frac{Y(z)}{X(z)} &= \frac{(Z^2-1)(Z^2-2Z)}{(Z^2-Z+\frac{1}{2})(Z^2+\frac{1}{16})} \\ &= \frac{Z^4-2Z^3-Z^2+2Z}{Z^4-Z^3+\frac{9}{16}Z^2-\frac{1}{16}Z+\frac{1}{32}} \\ &= \frac{1-2Z^{-1}-Z^{-2}+2Z^{-3}}{1-Z^{-1}+\frac{9}{16}^{-2}-\frac{1}{16}Z^{-3}+\frac{1}{32}Z^{-4}} \end{aligned}$$

By taking inverse Z transform on both sides

$$y(n)-y(n-1)+\frac{9}{16}y(n-2)-\frac{1}{16}y(n-3)+\frac{1}{32}y(n-4) = x(n)-2x(n-1)-x(n-2)+2x(n-3)$$

$$y(n) = x(n)-2x(n-1)-x(n-2)+2x(n-3)+y(n-1)-\frac{9}{16}y(n-2)+\frac{1}{16}y(n-3)-\frac{1}{32}y(n-4)$$

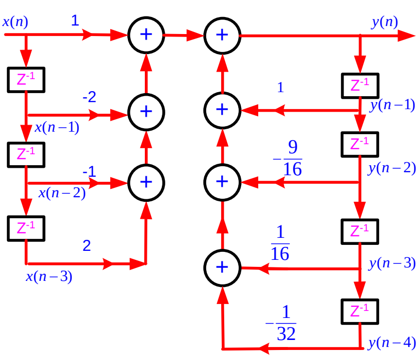


Figure 31: Direct form-I

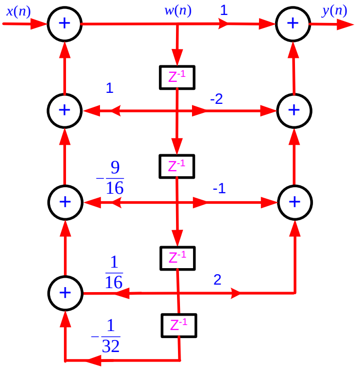


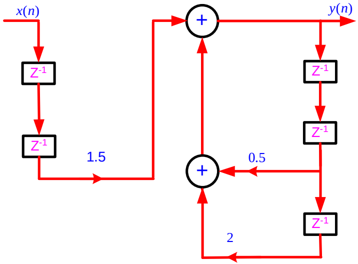
Figure 32: Direct form-II

Realize direct form-I and form -II for linear time invariant system which is described by the following input output relation

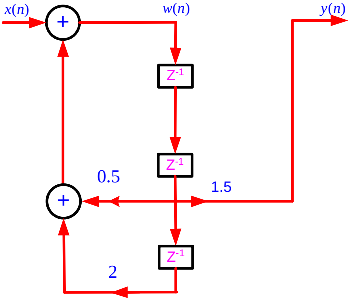
$$2y(n) - y(n-2) - 4y(n-3) = 3x(n-2)$$

Solution:

$$y(n) = 0.5y(n-2) + 2y(n-3) + 1.5x(n-2)$$



(a) Direct form-I



(b) Direct form-II

Figure 33: Realization of IIR systems

Realize direct form-I and form -II for system given by

$$H(z) = \frac{z^{-1} - 3z^{-2}}{(10 - z^{-1})(1 + 0.5z^{-1} + 0.5z^{-2})}$$

Solution:

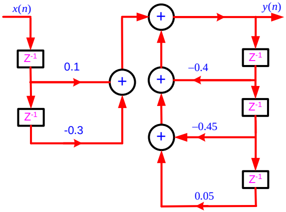
$$H(z) = \frac{z^{-1} - 3z^{-2}}{(10 - z^{-1})(1 + 0.5z^{-1} + 0.5z^{-2})} = \frac{z^{-1} - 3z^{-1}}{(10 + 4z^{-1} + 4.5z^{-2} - 0.5z^{-3})}$$

$$\frac{Y(z)}{X(z)} = \frac{0.1z^{-1} - 0.3z^{-1}}{(1 + 0.4z^{-1} + 0.45z^{-2} - 0.05z^{-3})}$$

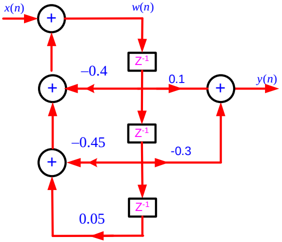
By taking inverse z transform on both sides

$$y(n) + 0.4y(n-1) + 0.45y(n-2) - 0.05y(n-3) = 0.1x(n-1) - 0.3x(n-2)$$

$$y(n) = -0.4y(n-1) - 0.45y(n-2) + 0.05y(n-3) + 0.1x(n-1) - 0.3x(n-2)$$



(a) Direct form-I



(b) Direct form-II

Cascade Form Structure

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_N z^{-N}}$$

The system can be factored into a cascade of second order subsystems such that H(z) can be expressed as

$$H(z) = H_1(z) \times H_2(z) \times H_3(z) \ldots \times H_k(z) = \prod_{k=1}^{K} H_k(z)$$

where K is the integer part of (N+1)/2 and $H_k(z)$ has the general form

$$H_k(z) = \frac{b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2}}{1 + a_{k1} z^{-1} + a_{k2} z^{-2}}$$

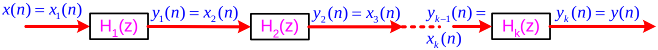


Figure 35: Cascade realization

In cascade each $H_k(z)$ are represented as

$$\begin{aligned} H_k(z) &= \frac{Y_k(z)}{X_k(z)} \\ &= \frac{W_k(z)}{X_k(z)}\frac{Y_k(z)}{W_k(z)} \\ &= H_{k1}(z).H_{k2}(z) \end{aligned}$$

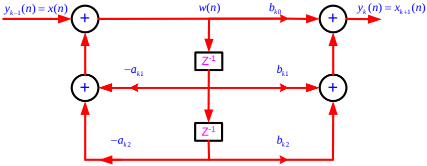


Figure 36: Cascade realization

where $H_{k1}(z)$ and $H_{k2}(z)$ defined as

$$H_{k1}(z) = \frac{W_k(z)}{X_k(z)} = \frac{1}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}$$

$$H_{k2}(z) = b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}$$

$$\begin{aligned} w_k(n) &= -a_{k1}w_k(n-1) - a_{k2}w_k(n-2) + -y_{k-1}(n) \\ y_k(n) &= b_{k0}w_k(n) + b_{k1}w_k(n-1) + b_{k2}w_k(n-2) \\ y_{k-1}(n &= x_k(n) \\ y_k(n) &= x_{k+1}(n) \\ y_0(n) &= x(n) \\ y(n) &= x_k(n) \end{aligned}$$

June 2010 EE

Realize the following system function in cascade form

$$H(z) = \frac{1+\frac{1}{5}z^{-1}}{(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2})(1+\frac{1}{4}z^{-1})}$$

Solution:

$$H(z) = H_1(z) \times H_2(z) = \frac{1}{(1+\frac{1}{4}z^{-1})} \cdot \frac{1+\frac{1}{5}z^{-1}}{(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2})}$$

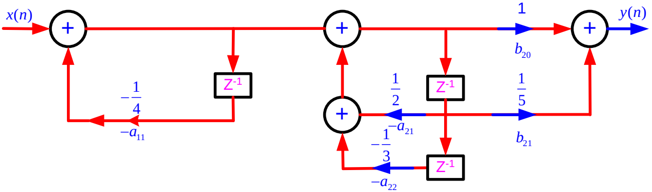


Figure 37: Cascade realization

Realize the following system function in cascade form

$$H(z) = \frac{1 - \frac{1}{2}z^{-1}}{(1 - \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2})(1 - \frac{1}{5}z^{-1} + \frac{1}{6}z^{-2})}$$

Solution:

$$H(z) = H_1(z) \times H_2(z) = \frac{1}{(1 - \frac{1}{5}z^{-1} + \frac{1}{6}z^{-2})} \cdot \frac{1 - \frac{1}{2}z^{-1}}{(1 - \frac{1}{4}z^{-1} + \frac{1}{2}z^{-2})}$$

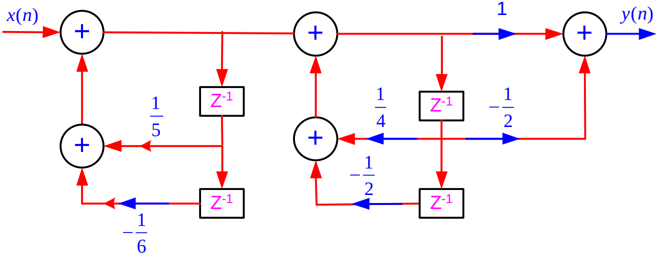


Figure 38: Cascade realization

Realize the following system function in cascade form

$$H(z) = \frac{1 + \frac{1}{3}z^{-1}}{(1 - \frac{1}{5}z^{-1})(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2})}$$

Solution:

$$H(z) = H_1(z) \times H_2(z) = \frac{1}{(1 - \frac{1}{5}z^{-1})} \times \frac{1 + \frac{1}{3}z^{-1}}{(1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2})}$$

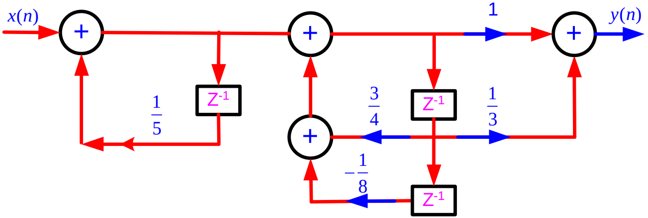


Figure 39: Cascade realization

The transfer function of a discrete system is given as follows

$$H(z) = \frac{1 - z^{-1}}{(1 - 0.2z^{-1} - 0.15z^{-2})}$$

Draw the cascade realization

Solution:

$$H(z) = H_1(z) \times H_2(z) = (1 - z^{-1}) \frac{1}{(1 - 0.2z^{-1} - 0.15z^{-2})}$$

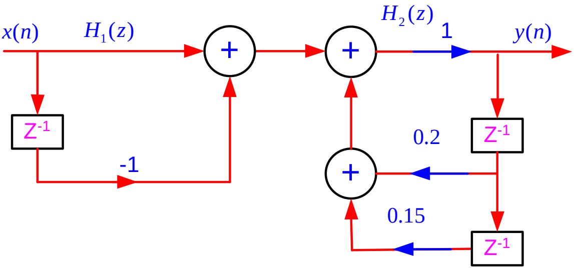


Figure 40: Cascade realization

December 2010 EE

Draw the cascade realization for the following system

$$y(n) = 0.75y(n-1) - 0.125y(n-2) + 6x(n) + 7x(n-1) + x(n-2)$$

Solution:

$$Y(z) = 0.75z^{-1}Y(z) - 0.125z^{-2}Y(z) + 6X(z) + 7z^{-1}X(z) + z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{6 + 7z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$\begin{aligned} H(z) &= \frac{Y(z)}{X(z)} = \frac{6 + 7z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}} \\ &= \frac{6z^2 + 7z + 1}{z^2 - 0.75z + 0.125} \\ &= \frac{(6z+1)(z+1)}{(z-0.5)(z-0.25)} \\ &= \frac{(6+z^{-1})(1+z^{-1})}{(1-0.5z^{-1})(1-0.25z^{-1})} = H_1(z) \times H_2(z) \end{aligned}$$

$a = 1$, $b = -0.75$, $c = 0.125$
Roots of the quadratic equation for $ax^2 + bx + c = 0$ are
$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\begin{aligned} &= \frac{0.75 \pm \sqrt{0.75^2 - 4(0.125)}}{2} \\ &= \frac{0.75 \pm \sqrt{0.0625}}{2} = \frac{0.75 \pm 0..25}{2} \\ &= \Rightarrow 0.5,\ 0.25 \end{aligned}$$

$$H_1(z) = \frac{6 + z^{-1}}{1 - 0.5z^{-1}}$$

$$H_2(z) = \frac{(1 + z^{-1})}{(1 - 0.25z^{-1})}$$

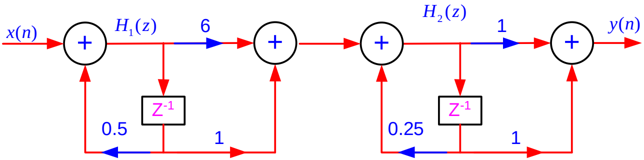


Figure 41: Cascade realization

A system function is specified by its transfer function H(z) given by

$$H(z) = \frac{(z-1)(z-2)(z+2)z}{[z-(\frac{1}{2}+j\frac{1}{2})][z-(\frac{1}{2}-j\frac{1}{2})](z-\frac{j}{4})(z+\frac{j}{4})}$$

Realize the cascade of two biquadratic sections.

Solution:

$$\begin{aligned} H(z) &= \frac{(z-1)(z-2)(z+2)z}{[z-(\frac{1}{2}+j\frac{1}{2})][z-(\frac{1}{2}-j\frac{1}{2})](z-\frac{j}{4})(z+\frac{j}{4})} \\ &= \frac{z^2-3z+2}{z^2-z+0.5} \times \frac{z^2+2z}{z^2+0.0625} \\ &= \frac{1-3z^{-1}+2z^{-2}}{1-z^{-1}+0.5z^{-2}} \times \frac{1+2z^{-1}}{1+0.0625z^{-2}} \\ &= H_1(z) \times H_2(z) \end{aligned}$$

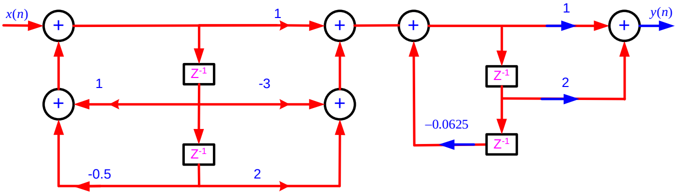


Figure 42: Cascade realization

Obtain the cascade realization of the following system. The system should have two biquadratic sections. [EC December 2012]

$$H(z) = \frac{(z-1)(z^2+5z+6)(z-3)}{(z^2+6z+5)(z^2-6z+8)}$$

Solution:

$$\begin{aligned} H(z) &= \frac{(z-1)(z^2+5z+6)(z-3)}{(z^2+6z+5)(z^2-6z+8)} \\ &= \frac{z^2-4z+3}{z^2+6z+5} \times \frac{z^2+5z+6}{z^2-6z+8} \\ &= \frac{1-4z^{-1}+3z^{-2}}{1+6z^{-1}+5z^{-2}} \times \frac{1+5z^{-1}+6z^{-2}}{1-6z^{-1}+8z^{-2}} \end{aligned}$$

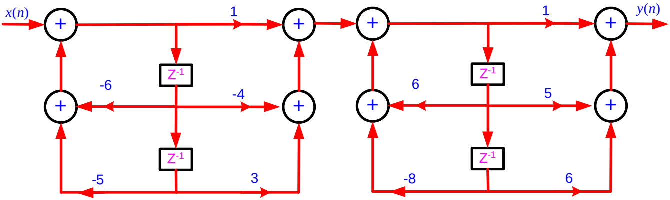


Figure 43: Cascade realization

Obtain the cascade realization using second order sections.

$$H(z) = \frac{10(1-\frac{1}{2}z^{-1})(1-\frac{2}{3}z^{-1})(1+2z^{-1})}{(1-\frac{3}{4}z^{-1})(1-\frac{1}{8}z^{-1})[1-(\frac{1}{2}+j\frac{1}{2})z^{-1}][1-(\frac{1}{2}-j\frac{1}{2})z^{-1}]}$$

Solution:

$$\begin{aligned}
H(z) &= \frac{10z(z-0.5)(z-0.6667)(z+2)}{(z-0.75)(z-0.125)[z-(0.5+j0.5)][z-(0.5-j0.5)]} \\
&= \frac{10z(z-0.5)}{(z-0.75)(z-0.125)} \times \frac{(z-0.6667)(z+2)}{[z-(0.5+j0.5)][z-(0.5-j0.5)]} \\
&= \frac{10z^2-5z}{z^2-0.875z+0.0938} \times \frac{z^2+1.333z-1.333}{z^2-z+0.5} \\
&= \frac{10-5z^{-1}}{1-0.875z^{-1}+0.0938z^{-2}} \times \frac{1+1.333z^{-1}-1.333z^{-2}}{1-z^{-1}+0.5z^{-2}}
\end{aligned}$$

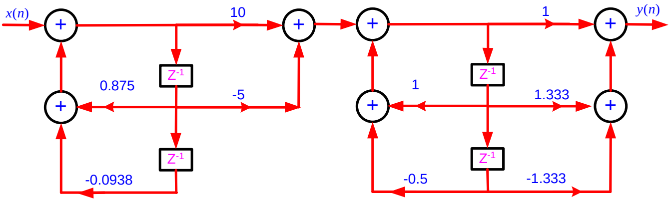


Figure 44: Cascade realization

EC December 2011

Obtain the cascade realization for the following system.

$$H(z) = \frac{1+\frac{1}{4}z^{-1}}{(1+\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2})}$$

Solution:

$$\begin{aligned} H(z) &= \frac{1+\frac{1}{4}z^{-1}}{(1+\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2})} \\ &= \frac{1+\frac{1}{4}z^{-1}}{1+\frac{1}{2}z^{-1}} \times \frac{1}{1+\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2}} \end{aligned}$$

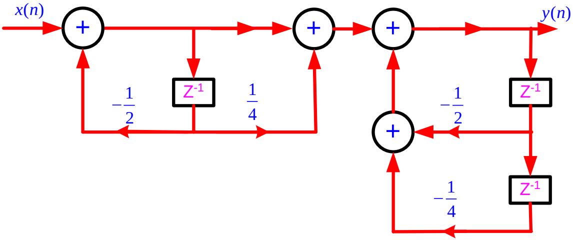


Figure 45: Cascade realization

Obtain the cascade realization for the following system.

$$H(z) = \frac{(1+z^{-1})^3}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2})}$$

Solution:

$$\begin{aligned} H(z) &= \frac{(1+z^{-1})^3}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1}+\frac{1}{2}z^{-2})} \\ &= \frac{1+z^{-1}}{(1-\frac{1}{4}z^{-1})} \times \frac{1+2z^{-1}+z^{-2}}{1-z^{-1}+\frac{1}{2}z^{-2}} \\ &= H_1(z) \times H_2(z) \end{aligned}$$

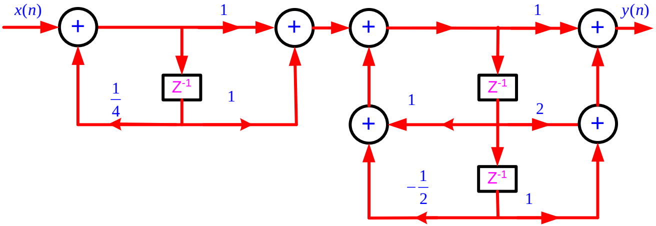


Figure 46: Cascade realization

EC 2011 July

Obtain the direct form II realizations of the following system.

$$H(z) = \frac{(1+z^{-1})}{(1-\frac{1}{4}z^{-1})(1-z^{-1}+\frac{1}{2}z^{-2})}$$

$$\begin{aligned} H(z) &= \frac{(1+z^{-1})}{(1-\frac{1}{4}z^{-1})(1-z^{-1}+\frac{1}{2}z^{-2})} \\ &= \frac{1+z^{-1}}{(1-\frac{1}{4}z^{-1})} \times \frac{1}{1-z^{-1}+z^{-2}} \\ &= H_1(z) \times H_2(z) \end{aligned}$$

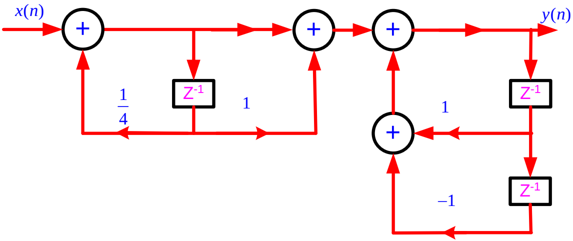


Figure 47: Cascade realization

June 2010 EC

Obtain cascade form for the system described by

$$y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$$

Solution:

$$Y(z) = -0.1z^{-1}Y(z) + 0.72z^{-2}Y(z) + 0.7X(z) - 0.252z^{-2}X(z) =$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}} = (0.7 - 0.252z^{-2}) \times \frac{1}{1 + 0.1z^{-1} - 0.72z^{-2}}$$

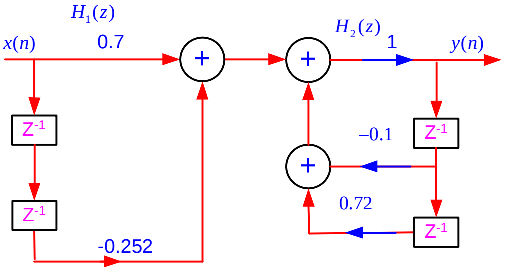
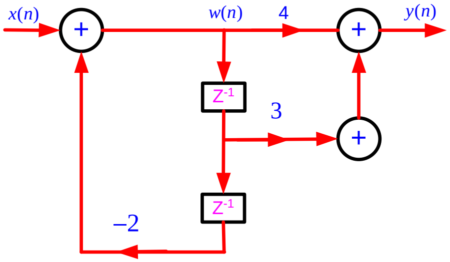


Figure 48: Cascade realization

June 2015 EC

Find the transfer function and difference equation realization shown in Figure



Solution:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{4 + 3z^{-1}}{1 + 2z^{-2}}$$

$$Y(z) = -2z^{-2}Y(z) + 4X(z) + 3z^{-1}X(z)$$

The difference equation is

$$y(n) = -2y(n-2) + 4x(n) + 3x(n-1)$$

Parallel Form Structure for IIR System

- The system function for IIR sytem is

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$= \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_N z^{-N}}$$

The function can be expanded in partial fractions as follows:

$$H(z) = C + H_1(z) + H_2(z) \ldots + H_k(z)$$

C is constant and each $H_1(z) + H_2(z) \ldots + H_k(z)$ are the second order system which is as shown in Figure 50

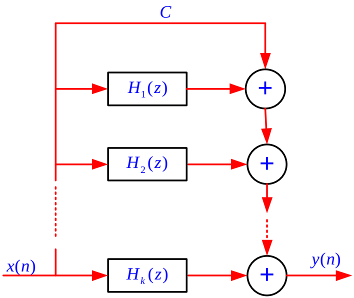


Figure 49: Parallel form realization for IIR systems

$$H_k(z) = \frac{b_{k0} + b_{k1}z^{-1}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}$$

$$\begin{aligned} w_k(n) &= a_{k1}w_k(n-1) - a_{k2}w_k(n-2) + x(n) \\ y_k(n) &= b_{k0}w_k(n) - b_{k1}w_k(n-1) \\ y(n) &= C\ x(n) + \sum_{k=1}^{K} y_k(n) \end{aligned}$$

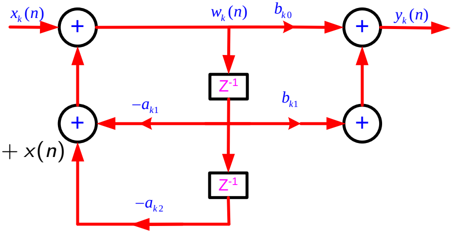


Figure 50: Direct form II of second order subsystem

Realize the following System in parallel form:

$$H(z) = 3 + \frac{4z}{z - \frac{1}{2}} + \frac{2}{z - \frac{1}{4}}$$

Solution:

$$H(z) = 3 + \frac{4}{1 - 0.5z^{-1}} + \frac{2z^{-1}}{1 - \frac{1}{4}z^{-1}}$$

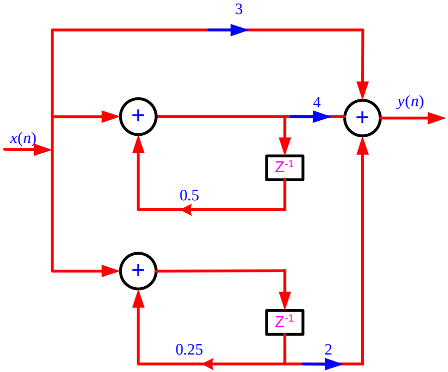


Figure 51: Parallel realization

Realize the following System in parallel form:

$$H(z) = \frac{(1+z^{-1})(1+2z^{-1})}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1+\frac{1}{8}z^{-1})}$$

Solution:

$$\begin{aligned}
H(z) &= \frac{z^{-2}(z+1)(z+2)}{z^{-3}(z+0.5)(z-0.25)(z+0.125)} \\
H(z) &= \frac{z(z+1)(z+2)}{(z+0.5)(z-0.25)(z+0.125)} \\
\frac{H(z)}{z} &= \frac{(z+1)(z+2)}{(z+0.5)(z-0.25)(z+0.125)} \\
\frac{H(z)}{z} &= \frac{A}{(z+0.5)} + \frac{B}{(z-0.25)} + \frac{C}{(z+0.125)} \\
\frac{(z+1)(z+2)}{(z+0.5)(z-0.25)(z+0.125)} &= \frac{A}{(z+0.5)} + \frac{B}{(z-0.25)} + \frac{C}{(z+0.125)}
\end{aligned}$$

Coefficients A B and C are determined

$$
\begin{aligned}
A &= \frac{H(z)}{z}[z+0.5]|_{z=-0.5} = \frac{(z+1)(z+2)}{(z-0.25)(z+0.125)} \\
&= \frac{(-0.5+1)(-0.5+2)}{(-0.5-0.25)(-0.5+0.125)} = 2.66
\end{aligned}
$$

$$
\begin{aligned}
B &= \frac{H(z)}{z}[z-0.25]|_{z=0.25} = \frac{(z+1)(z+2)}{(z+0.5)(z+0.125)} \\
&= \frac{(0.25+1)(0.25+2)}{(0.25+0.5)(0.25+0.125)} = 10
\end{aligned}
$$

$$
\begin{aligned}
C &= \frac{H(z)}{z}[z+0.125]|_{z=-0.125} = \frac{(z+1)(z+2)}{(z+0.5)(z-0.25)} \\
&= \frac{(-0.125+1)(-0.125+2)}{(-0.125+0.5)(-0.125-0.25)} = -11.66
\end{aligned}
$$

$$\frac{H(z)}{z} = \frac{2.66}{(z+0.5)} + \frac{10}{(z-0.25)} + \frac{-11.66}{(z+0.125)}$$

$$H(z) = \frac{2.66}{(1+0.5z^{-1})} + \frac{10}{(1-0.25z^{-1})} + \frac{-11.66}{(1+0.125z^{-1})}$$

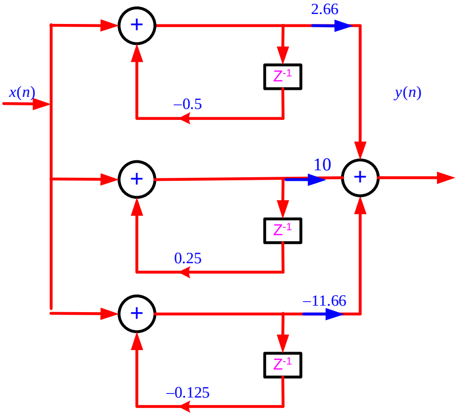


Figure 52: Parallel realization

Realize the following System in parallel form:

$$H(z) = \frac{1 - z^{-1}}{(1 - 0.2z^{-1} - 0.15z^{-2})}$$

Solution

$$H(z) = \frac{1 - z^{-1}}{(1 - 0.2z^{-1} - 0.15z^{-2})}$$

$$H(z) = \frac{z^2 - z}{(z^2 - 0.2z - 0.15)}$$

$a = 1, b = -0.2, c = -0.15$

Roots of the equation $ax^2 + bx + c = 0$ are $= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{0.2 \pm \sqrt{0.2^2 - 4(-0.15)}}{2} = \frac{0.2 \pm \sqrt{0.64}}{2} = \frac{0.2 \pm 0.8}{2} \Rightarrow 0.5, \quad -0.3$$

$$\frac{H(z)}{z} = \frac{z - 1}{(z - 0.5)(z + 0.3)}$$

$$\frac{H(z)}{z} = \frac{A}{(z - 0.5)} + \frac{B}{(z + 0.3)}$$

The values of A and B are determined by the following procedure.

$$A = \frac{H(z)}{z}(z-0.5)|_{z=0.5} = \frac{z-1}{(z+0.3)} = \frac{0.5-1}{(0.5+0.3)} = -0.625$$

$$B = \frac{H(z)}{z}(z+0.3)|_{z=-0.3} = \frac{z-1}{(z-0.5)} = \frac{-0.3-1}{(-0.3-0.5)} = 1.625$$

$$\frac{H(z)}{z} = \frac{-0.625}{z-0.5} + \frac{1.625}{z+0.3}$$

$$H(z) = \frac{-0.625}{1-0.5z^{-1}} + \frac{1.625}{1+0.3z^{-1}} = H_1(z) + H_2(z)$$

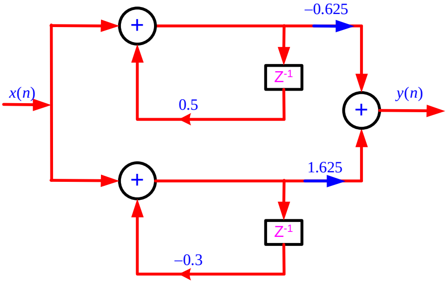


Figure 53: Parallel realization

Realize the following System in parallel form:

$$H(z) = \frac{1 + 0.33z^{-1}}{(1 - 0.75z^{-1} + 0.125z^{-2})}$$

Solution

$$\begin{aligned} H(z) &= \frac{1 + 0.33z^{-1}}{(1 - 0.75z^{-1} + 0.125z^{-2})} = \frac{z^{-1}(z + 0.33)}{z^{-2}(z^2 - 0.75z^{-1} + 0.125z^{-2})} \\ \frac{H(z)}{z} &= \frac{z + .33}{(z^2 - 0.75z + 0.125)} \end{aligned}$$

$a = 1, b = -0.75, c = 0.125$
Roots of the equation are

$$= \frac{0.75 \pm \sqrt{0.75^2 - 4(0.125)}}{2} = \frac{0.75 \pm \sqrt{0.0625}}{2} = \frac{0.75 \pm 0.25}{2} \Rightarrow 0.5,\ 0.25$$

$$\begin{aligned} \frac{H(z)}{z} &= \frac{z + 0.33}{(z - 0.5)(z - 0.25)} \\ &= \frac{A}{(z - 0.5)} + \frac{B}{(z - 0.25)} \end{aligned}$$

The values of A and B are determined by the following procedure.

$$A = \frac{H(z)}{z}(z-0.5)|_{z=0.5} = \frac{z+0.33}{(z-0.25)} = \frac{0.25+0.33}{(0.5-0.25)} = 3.33$$

$$B = \frac{H(z)}{z}(z-0.25)|_{z=0.25} = \frac{z+0.33}{(z-0.5)} = \frac{0.25+0.33}{(0.25-0.5)} = -2.33$$

$$\frac{H(z)}{z} = \frac{3.33}{(z-0.5)} + \frac{-2.33}{(z-0.25)}$$

$$H(z) = \frac{3.33}{1-0.5z^{-1}} + \frac{-2.33}{1-0.25z^{-1}} = H_1(z) + H_2(z)$$

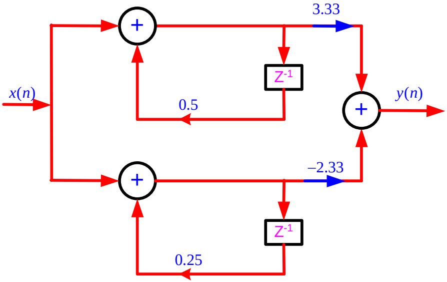


Figure 54: Parallel realization

Realize the following System in parallel form:

$$y(n) = 0.75y(n-1) - 0.125y(n-2) + 6x(n) + 7x(n-1) + x(n-2)$$

Solution

$$Y(z) = 0.75z^{-1}Y(z) - 0.125z^{-2}Y(z) + 6X(z) + 7z^{-1}X(z) + z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{6 + 7z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

$$H(z) = \frac{z^{-2}[6z^2 + 7z + 1]}{z^{-2}[z^2 - 0.75z + 0.125]}$$

$$\begin{aligned} H(z) &= \frac{z^{-2}[6z^2 + 7z + 1]}{z^{-2}[z^2 - 0.75z + 0.125]} \\ &= \frac{[6z^2 + 7z + 1]}{[z^2 - 0.75z + 0.125]} \end{aligned}$$

$a = 1, b = -0.75, c = 0.125$
Roots of the equation are

$$= \frac{0.75 \pm \sqrt{(-0.75)^2 - 4(0.125)}}{2} = \frac{0.75 \pm \sqrt{0.0625}}{2} = \frac{0.75 \pm 0.25}{2}$$

$0.5, 0.25$

$$H(z) = \frac{6z^2 + 7z + 1}{(z-0.5)(z-0.25)}$$

$$\frac{H(z)}{z} = \frac{6z^2 + 7z + 1}{z(z-0.5)(z-0.25)}$$

$$\frac{H(z)}{z} = \frac{A}{z} + \frac{B}{(z-0.5)} + \frac{C}{(z-0.25)}$$

$$A = H(z)z|_{z=0} = \frac{6z^2 + 7z + 1}{z(z-0.5)(z-0.25)}z = \frac{1}{(-0.5)(-0.25)} = 8$$

$$B = H(z)(z-0.5)|_{z=0.5} = \frac{6z^2 + 7z + 1}{z(z-0.5)(z-0.25)}(z-0.5z) = \frac{6(0.5)^2 + 7(0.5) + 1}{0.5[0.5-0.25]} = 48$$

$$C = H(z)(z-0.25z)|_{z=0.25} = \frac{6z^2 + 7z + 1}{z(z-0.5)(z-0.25)}(z-0.25)$$

$$= \frac{6(0.25)^2 + 7(0.25) + 1}{0.25[0.25-0.5]} = -50$$

$$H(z) = 8 + \frac{48}{(1-0.5z^{1})} + \frac{-50}{(1-0.25z^{1})} = H_1(z) + H_2(z) + H_3(z)$$

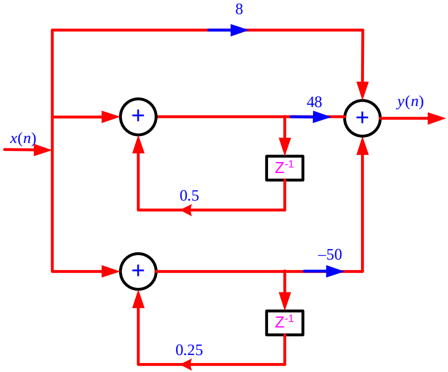


Figure 55: Parallel realization

Realize the following System in parallel form:

$$H(z) = \frac{8z^3 - 4z^2 + 11z - 2}{[z - \frac{1}{4}][z^2 - z + \frac{1}{2}]}$$

$$\frac{H(z)}{z} = \frac{8z^3 - 4z^2 + 11z - 2}{z[z - \frac{1}{4}][z^2 - z + \frac{1}{2}]}$$

$$\begin{aligned}
\frac{H(z)}{z} &= \frac{8z^3 - 4z^2 + 11z - 2}{z[z - \frac{1}{4}][z^2 - z + \frac{1}{2}]} \\
\frac{H(z)}{z} &= \frac{A}{z} + \frac{B}{z - 0.25} + \frac{Cz + D}{z^2 - z + .5}
\end{aligned}$$

$$\begin{aligned}
A &= \frac{H(z)}{z} z|_z = 0 = \frac{8z^3 - 4z^2 + 11z - 2}{[z - 0.25][z^2 - z + 0.5]} \\
&= \frac{-2}{[-0.25][0.5]} = 16
\end{aligned}$$

$$
\begin{aligned}
B &= \frac{H(z)}{z}[z-0.5]|_{z=0.5} = \frac{8z^3-4z^2+11z-2}{z[z^2-z+0.5]} \\
&= \frac{8(0.5)^3-4(0.5)^2+11(0.5)-2}{(0.5)[(0.5)^2-(0.5)+0.5]} = 8
\end{aligned}
$$

$$
\begin{aligned}
\frac{H(z)}{z} &= \frac{8z^3-4z^2+11z-2}{z[z-0.25][z^2-z+0.5]} \\
\frac{8z^3-4z^2+11z-2}{z[z-0.25][z^2-z+0.5]} &= \frac{A}{z}+\frac{B}{z-0.25}+\frac{Cz+D}{z^2-z+.25} \\
8z^3-4z^2+11z-2 &= A[z-0.25][z^2-z+0.5]+ \\
&\quad +Bz[z^2-z+.25]+(Cz+D)z[z-0.25] \\
8z^3-4z^2+11z-2z &= z^3[A+B+C]+z^2[-1.25A-B-0.25C+D]+ \\
&\quad +z[0.5A+.25B-.25D]-0.0625A
\end{aligned}
$$

$$
\begin{aligned}
8 &= A+B+C = 16+8+C \\
C &= -16
\end{aligned}
$$

$$
\begin{aligned}
-4 &= [-1.25A-B-0.25C+D] = [-20-8+4+D] \\
D &= 20
\end{aligned}
$$

$$\frac{H(z)}{z} = \frac{A}{z} + \frac{B}{z-0.25} + \frac{Cz+D}{z^2-z+.5}$$

$$\frac{H(z)}{z} = \frac{16}{z} + \frac{8}{z-0.25} + \frac{-16z+20}{z^2-z+.5}$$

$$H(z) = 16 + \frac{8}{1-0.25z^{-1}} + \frac{-16+20z^{-1}}{1-z^{-1}+.5z^{-2}}$$

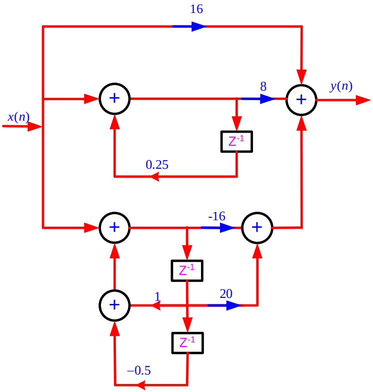


Figure 56: Parallel realization

EE 2010 May

Realize the following System in parallel form:

$$H(z) = \frac{1 - \frac{1}{5}z^{-1}}{[1 - \frac{1}{2}z^{-1} + \frac{1}{3}z^{-2}][1 + \frac{1}{4}z^{-1}]}$$

Solution:

$$H(z) = \frac{z^{-1}(z - 0.2)}{z^{-2}[z^2 - 0.5z + 0.33]z^{-1}[z + 0.25]}$$

$$H(z) = \frac{z^2(z - 0.2)}{[z^2 - 0.5z + 0.33z][z + 0.25]}$$

$$\frac{H(z)}{z} = \frac{z(z - 0.2)}{[z^2 - 0.5z + 0.33][z + 0.25]}$$

$$\frac{H(z)}{z} = \frac{Az + B}{[z^2 - 0.5z + 0.333]} + \frac{C}{[z + 0.25]}$$

$$\frac{z(z - 0.2)}{[z^2 - 0.5z + 0.33][z + 0.25]} = \frac{Az + B}{[z^2 - 0.5z + 0.333]} + \frac{C}{[z + 0.25]}$$

Coefficients A B and C are determined

$$
\begin{aligned}
C &= \frac{H(z)}{z}[z+0.25]|_{z=-0.25} = \frac{(z-0.2)}{[z^2-0.5z+0.333]} \\
&= \frac{-0.25(-0.25-0.2)}{[(-0.25)^2-(0.5)(-0.25)+0.333]} = 0.217
\end{aligned}
$$

$$
\begin{aligned}
\frac{z(z-0.2)}{[z^2-0.5z+0.33][z+0.25]} &= \frac{Az+B}{[z^2-0.5z+0.333]} + \frac{C}{[z+0.25]} \\
z(z-0.2) &= (Az+B)[z+0.25] + C[z^2-0.5z+0.33] \\
z^2-0.2z &= Az^2+0.25Az+Bz+0.25B+Cz^2-0.5Cz+0.33C] \\
z^2-0.2z &= z^2(A+C)+z(0.25A+B-C)+0.25B+0.33C
\end{aligned}
$$

Equating coefficients

$$
\begin{aligned}
A+C &= 1 \\
A &= 1-C = 1-0.217 = 0.783
\end{aligned}
$$

$$
\begin{aligned}
0.25B+0.33C &= 0 \\
B &= \frac{-0.33C}{0.25} = -0.2864
\end{aligned}
$$

$$\frac{H(z)}{z} = \frac{0.78z + -0.2864}{[z^2 - 0.5z + 0.333]} + \frac{0.216}{[z + 0.25]}$$

$$\frac{H(z)}{z} = \frac{z(0.78 - 0.2864z^{-1})}{z^2(1 - 0.5z^{-1} + 0.333z^{-2})} + \frac{0.216}{z[1 + 0.25z^{-1}]}$$

$$H(z) = \frac{(0.78 - 0.2864z^{-1})}{(1 - 0.5z^{-1} + 0.333z^{-2})} + \frac{0.216}{(1 + 0.25z^{-1})}$$

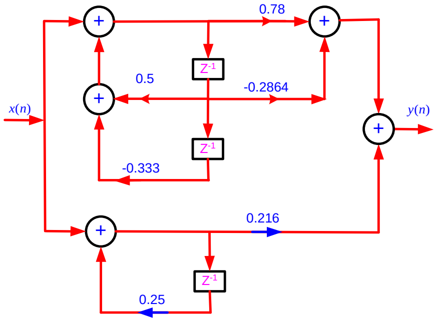


Figure 57: Parallel realization

EC 2012 December

Realize the following System in parallel form:

$$H(z) = \frac{(z-1)(z-2)(z+2)z}{[z-(\frac{1}{2}+j\frac{1}{2})][z-(\frac{1}{2}-j\frac{1}{2})](z-\frac{j}{4})(z+\frac{j}{4})}$$

Solution:

$$\begin{aligned} H(z) &= \frac{(z-1)(z-2)(z+2)z}{[z-(\frac{1}{2}+j\frac{1}{2})][z-(\frac{1}{2}-j\frac{1}{2})](z-\frac{j}{4})(z+\frac{j}{4})} \\ \frac{H(z)}{z} &= \frac{(z-1)(z-2)(z+2)z}{[z-(\frac{1}{2}+j\frac{1}{2})][z-(\frac{1}{2}-j\frac{1}{2})](z-\frac{j}{4})(z+\frac{j}{4})} \\ \frac{H(z)}{z} &= \frac{A}{z-(0.5+j0.5)} + \frac{B}{z-(0.5-j0.5)} + \frac{C}{z-j0.25} + \frac{D}{z+j0.25} \end{aligned}$$

$$\frac{H(z)}{z} = \frac{A}{z-(0.5+j0.5)} + \frac{B}{z-(0.5-j0.5)} + \frac{C}{z-j0.25} + \frac{D}{z+j0.25)}$$

$$A = \frac{H(z)}{z}[z-(0.5+j0.5)]|_{z=0.5+j0.5} = \frac{(z^2-1)(z-2)}{[z-(0.5-j0.25)][z-j0.25][z+j0.25]}$$

$$= \frac{[(0.5+j0.5)^2-1][(0.5+j0.5-2]}{[0.5+j0.5-(0.5-j0.5)][z^2+0.0625]}$$

$$= \frac{[0.25+j0.5-0.25-1][(0.5+j0.5-2]}{[j1][z^2+0.0625]}$$

$$= \frac{[-1+j0.5][(-1.5+j0.5]}{[j1][j0.5+0.0625]}$$

$$= \frac{[1.5-j0.5-j0.75+0.25]}{-0.5+j0.0625} = \frac{1.25-j1.25}{-0.5+j0.0625} = \frac{1.767\angle-45}{0.503\angle172}$$

$$= \frac{1.767\angle-45}{0.503\angle172} = 3.513\angle-217 = -2.8+j2.1$$

Similarly B, C and D are estimated.

$$\begin{aligned}\frac{H(z)}{z} &= \frac{A}{z-(0.5+j0.5)} + \frac{B}{z-(0.5-j0.5)} + \frac{C}{z-j0.25} + \frac{D}{z+j0.25)} \\ &= \frac{-2.8+j2.1}{z-(0.5+j0.5)} + \frac{-2.7683-j2.1517}{z-(0.5-j0.5)} + \frac{3.268-j7.837}{z-j0.25} + \frac{3.268+j7.837}{z+j0.25)}\end{aligned}$$

We have to design second order system, hence combine first two terms and last two terms.

$$\begin{aligned}\frac{H(z)}{z} &= \frac{-5.536z+0.612}{z^2-z+0.5} + \frac{6.5366z-3.918}{z^2+0.0625)} \\ \frac{H(z)}{z} &= \frac{z(-5.536+0.612z^{-1})}{z^2(1-z^{-1}+0.5z^{-2})} + \frac{z(6.5366-3.918z^{-1})}{z^2(1+0.0625z^{-2})} \\ H(z) &= \frac{(-5.536+0.612z^{-1})}{(1-z^{-1}+0.5z^{-2})} + \frac{(6.5366-3.918z^{-1})}{(1+0.0625z^{-2})} = H_1(z)+H_2(z)\end{aligned}$$

$$H(z) = \frac{(-5.536 + 0.612z^{-1})}{(1 - z^{-1} + 0.5z^{-2})} + \frac{(6.5366 - 3.918z^{-1})}{(1 + 0.0625z^{-2})} = H_1(z) + H_2(z)$$

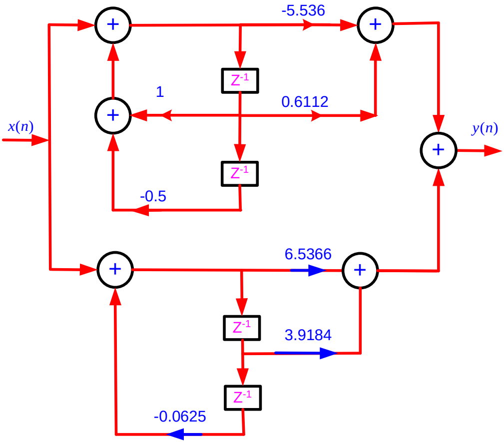


Figure 58: Parallel realization

EE 2010 May

Realize the following System in parallel form:

$$H(z)=\frac{1+\frac{1}{4}z^{-1}}{[1+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2}][1+\frac{1}{2}z^{-1}]}$$

Solution:

$$\begin{aligned}
H(z) &= \frac{z^{-1}(z-0.25)}{z^{-2}[z^2-0.5z+0.25]z^{-1}[z+0.5]}\\
H(z) &= \frac{z^2(z+0.25)}{[z^2+0.5z+0.25][z+0.5]}\\
\frac{H(z)}{z} &= \frac{z(z-0.25)}{[z^2+0.5z+0.25][z+0.25]}\\
\frac{H(z)}{z} &= \frac{Az+B}{[z^2+0.5z+0.25]}+\frac{C}{[z+0.5]}\\
\frac{z(z-0.25)}{[z^2+0.5z+0.25][z+0.5]} &= \frac{Az+B}{[z^2+0.5z+0.25]}+\frac{C}{[z+0.5]}
\end{aligned}$$

Coefficients A B and C are determined

$$\begin{aligned} C &= \frac{H(z)}{z}[z+0.5]|_{z=-0.5} = \frac{z(z-0.2)}{[z^2-0.5z+0.333]} \\ &= \frac{-0.5(-0.5-0.25)}{[(-0.5)^2-(0.5)(-0.5)+0.25]} = 0.5 \end{aligned}$$

$$\begin{aligned} \frac{z(z+0.25)}{[z^2+0.5z+0.25][z+0.5]} &= \frac{Az+B}{[z^2+0.5z+0.25]} + \frac{C}{[z+0.5]} \\ z(z+0.25) &= (Az+B)[z+0.5] + C[z^2+0.5z+0.25] \\ z^2+0.25z &= Az^2+0.5Az+Bz+0.5B+Cz^2+0.5Cz+0.25C] \\ z^2+0.25z &= z^2(A+C)+z(0.5A+B+0.5C)+0.5B+0.25C \end{aligned}$$

Equating coefficients

$$\begin{aligned} A+C &= 1 \\ A &= 1-C = 1-0.5 = 0.5 \end{aligned}$$

$$\begin{aligned} 0.25 &= 0.5A+B+0.5C = 0.5\times0.5+B+0.5\times0.5 \\ B &= -0.25 \end{aligned}$$

$$\frac{H(z)}{z} = \frac{0.5z - 0.25}{[z^2 + 0.5z + 0.25]} + \frac{0.5}{[z + 0.5]}$$

$$\frac{H(z)}{z} = \frac{z(0.5 - 0.25z^{-1})}{z^2(1 + 0.5z^{-1} + 0.25z^{-2})} + \frac{0.5}{z[1 + 0.5z^{-1}]}$$

$$H(z) = \frac{(0.5 - 0.25z^{-1})}{(1 + 0.5z^{-1} + 0.25z^{-2})} + \frac{0.5}{(1 + 0.5z^{-1})}$$

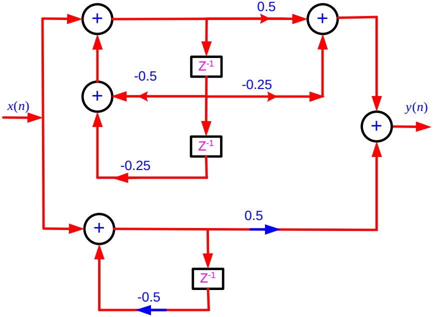


Figure 59: Parallel realization

Realize the following System in parallel form:

$$H(z) = \frac{1 - \frac{1}{2}z^{-1}}{(1 - \frac{1}{3}z^{-1})(1 - \frac{1}{4}z^{-1})}$$

Solution:

$$\begin{aligned}
H(z) &= \frac{z^{-1}(z-0.5)}{z^{-1}(z-0.33)z^{-1}(z-0.25)} \\
H(z) &= \frac{z(z-0.5)}{(z-0.33)(z-0.25)} \\
\frac{H(z)}{z} &= \frac{(z-0.33)}{(z-0.33)(z-0.25)} \\
\frac{H(z)}{z} &= \frac{A}{z-0.33} + \frac{B}{z-0.25} \\
\frac{(z-0.5)}{(z-0.33)z^{-1}(z-0.25)} &= \frac{A}{z-0.33} + \frac{B}{z-0.25}
\end{aligned}$$

Coefficients A and B are determined

$$A = \frac{H(z)}{z}[z-0.33]|_{z=0.33} = \frac{z-0.5}{z-0.25}$$
$$= \frac{0.33-0.5}{0.33-0.25} = -2$$

$$B = \frac{H(z)}{z}[z-0.25]|_{z=0.25} = \frac{z-0.5}{z-0.33}$$
$$= \frac{0.25-0.5}{0.25-0.33} = 3$$

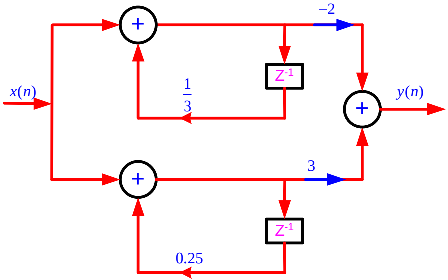


Figure 60: Parallel realization

Realize the following System in parallel form:

$$H(z) = \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

Solution:

$$\begin{aligned}
H(z) &= \frac{1+\frac{1}{3}z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} \\
H(z) &= \frac{1+0.33z^{-1}}{(1-0.5z^{-1})(1-0.25z^{-1})} \\
H(z) &= \frac{z^{-1}(z+0.33)}{z^{-1}(z-0.5)z^{-1}(z-0.25)} \\
H(z) &= \frac{z(z+0.33)}{(z-0.5)(z-0.25)} \\
\frac{H(z)}{z} &= \frac{(z+0.33)}{(z-0.5)(z-0.25)} \\
\frac{H(z)}{z} &= \frac{A}{z-0.5}+\frac{B}{z-0.25} \\
\frac{(z+0.33)}{(z-0.5)(z-0.25)} &= \frac{A}{z-0.33}+\frac{B}{z-0.25}
\end{aligned}$$

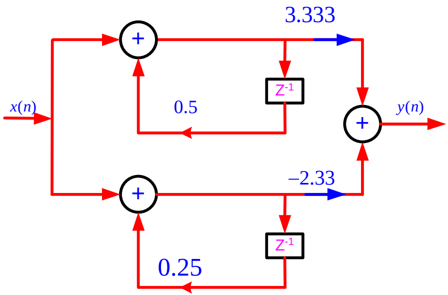
Coefficients A and B are determined

$$A = \frac{H(z)}{z}[z-0.5]|_{z=0.5} = \frac{z+0.33}{z-0.25} = \frac{0.5+0.33}{0.5-0.25} = 3.33$$

$$B = \frac{H(z)}{z}[z-0.25]|_{z=0.25} = \frac{z+0.33}{z-0.5} = \frac{0.25+0.33}{0.25-0.5} = -2.33$$

$$\frac{H(z)}{z} = \frac{3.33}{z-0.5} + \frac{-2.33}{z-0.25}$$

$$H(z) = \frac{3.33}{z-0.5} + \frac{-2.33}{z-0.25}$$



EE 2010 May

Realize the following System in parallel form:

$$H(z) = \frac{10(1-\frac{1}{2}z^{-1})(1+2z^{-1})(1-\frac{2}{3}z^{-1})}{(1-\frac{3}{4}z^{-1})(1-\frac{1}{8}z^{-1})[1-(\frac{1}{2}+j\frac{1}{2})z^{-1}][1-(\frac{1}{2}-j\frac{1}{2})z^{-1}]}$$

Solution:

$$H(z) = \frac{10(1-0.5z^{-1})(1+2z^{-1})(1-0.66z^{-1})}{(1-0.75z^{-1})(1-0.125z^{-1})[1-(0.5+j0.5)z^{-1}][1-(0.5-j0.5)z^{-1}]}$$

$$H(z) = \frac{10(1-0.5z^{-1})(1+2z^{-1})(1-0.66z^{-1})}{(1-0.75z^{-1})(1-0.125z^{-1})[1-z^{-1}+0.5z^{-2}]}$$

$$H(z) = \frac{10z^{-3}(z-0.5)(z+2)(z-0.66)}{z^{-4}(z-0.75)(z-0.125)[z^2-z+0.5]}$$

$$\frac{H(z)}{z} = \frac{10(z-0.5)(z+2)(z-0.66)}{(z-0.75)(z-0.125)[z^2-z+0.5]}$$

$$\frac{H(z)}{z} = \frac{A}{[z-0.75]} + \frac{B}{[z-0.125]} + \frac{Cz+D}{[z^2-z+0.5]}$$

$$\frac{10(z-0.5)(z+2)(z-0.66)}{(z-0.75)(z-0.125)[z^2-z+0.5]} = \frac{A}{[z-0.75]} + \frac{B}{[z-0.125]} + \frac{Cz+D}{[z^2-z+0.5]}$$

Coefficients A B and C are determined

$$A = \frac{H(z)}{z}[z-0.75]|_{z=0.75} = \frac{10(z-0.5)(z+2)(z-0.66)}{(z-0.125)[z^2-z+0.5]}$$
$$= \frac{10(0.75-0.5)(0.75+2)(0.75-0.66)}{(0.75-0.125)[0.75^2-0.75+0.5]} = 3.16$$

$$B = \frac{H(z)}{z}[z-0.125]|_{z=0.125} = \frac{10(z-0.5)(z+2)(z-0.66)}{(z-0.75)[z^2-z+0.5]}$$
$$= \frac{10(0.125-0.5)(0.125+2)(0.125-0.66)}{(0.125-0.75)[0.125^2-0.125+0.5]} = -17.46$$

$$\frac{z(z+0.25)}{[z^2+0.5z+0.25][z+0.5]} = \frac{Az+B}{[z^2+0.5z+0.25]} + \frac{C}{[z+0.5]}$$
$$z(z+0.25) = (Az+B)[z+0.5] + C[z^2+0.5z+0.25]$$
$$z^2+0.25z = Az^2+0.5Az+Bz+0.5B+Cz^2+0.5Cz+0.25C]$$
$$z^2+0.25z = z^2(A+C)+z(0.5A+B+0.5C)+0.5B+0.25C$$

Equating coefficients

$$A+C = 1$$
$$A = 1-C = 1-0.5 = 0.5$$

$$\frac{H(z)}{z} = \frac{0.5z - 0.25}{[z^2 + 0.5z + 0.25]} + \frac{0.5}{[z + 0.5]}$$

$$\frac{H(z)}{z} = \frac{z(0.5 - 0.25z^{-1})}{z^2(1 + 0.5z^{-1} + 0.25z^{-2})} + \frac{0.5}{z[1 + 0.5z^{-1}]}$$

$$H(z) = \frac{(0.5 - 0.25z^{-1})}{(1 + 0.5z^{-1} + 0.25z^{-2})} + \frac{0.5}{(1 + 0.5z^{-1})}$$

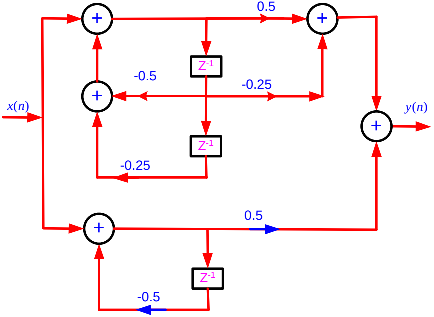


Figure 62: Parallel realization

Lattice structure for FIR Systems

- The system function of mth order FIR filter is represented by the polynomial $A_m(z)$

$$H_m(z) = A_m(z)$$

- where $A_m(z)$ is the polynomial and is defined as

$$A_m(z) = 1 + \sum_{i=1}^{m} a_m(i) z^{-i}$$

- $A_0(z) = 1$ The system function becomes

$$H_m(z) = 1 + \sum_{i=1}^{m} a_m(i) z^{-i}$$

- $H_0(z) = 1$ the z transform the input and output are related as

$$H_m(z) = \frac{Y(z)}{X(z)}$$

$$\frac{Y(z)}{X(z)} = 1 + \sum_{i=1}^{m} a_m(i) z^{-i}$$

$$Y(z) = X(z) + \sum_{i=1}^{m} a_m(i) z^{-i} X(z)$$

- Taking inverse z transform

$$y(n) = x(n) + \sum_{i=1}^{m} a_m(i) x(n-i)$$

- Consider the order of the filter m=1

$$y(n) = x(n) + a_1(1) x(n-1)$$

- From the figure y(n) is

$$y(n) = f_0(n) + k_1 g_0(n-1)$$

$f_0(n) = x(n)$ and $g_0(n-1) = x(n-1)$

$$y(n) = x(n) + k_1 x(n-1)$$

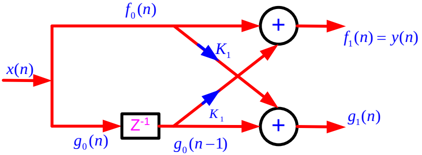


Figure 63: Single stage Lattice filter

Consider the filter of order of two.

$$
\begin{aligned}
y(n) &= x(n) + \sum_{i=1}^{2} a_m(i)x(n-i) \\
&= x(n) + a_2(1)x(n-1) + a_2(2)x(n-2)
\end{aligned}
$$

The first stage output is

$$
\begin{aligned}
f_1(n) &= x(n) + K_1 x(n-1) \\
g_1(n) &= K_1 x(n) + x(n-1)
\end{aligned}
$$

The second stage output is

$$
\begin{aligned}
f_2(n) &= f_1(n) + K_2 g_1(n-1) \\
g_2(n) &= K_2 f_1(n) + g_1(n-1)
\end{aligned}
$$

The output $y(n)$ is

$$
y(n) = f_1(n) + K_2 g_1(n-1)
$$

$$
\begin{aligned}
y(n) &= x(n) + K_1 x(n-1) + K_2[K_1 x(n-1) + x(n-2)] \\
&= x(n) + K_1(1 + K_2)x(n-1) + K_2 x(n-2)
\end{aligned}
$$

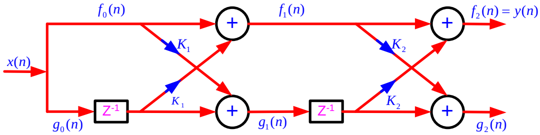


Figure 64: Two stage Lattice filter

$$y(n) = x(n) + K_1(1+K_2)x(n-1) + K_2x(n-2)$$

This equation represents the second order FIR system if:

$$a_2(1) = K_1(1+K_2) \quad and \quad a_2(2) = K_2$$

$$K_1 = \frac{a_2(1)}{(1+K_2)} \quad and \quad K_2 = a_2(2)$$

The lattice structure for mth order is obtained from direct form coefficients by the following recursive equations.

$$K_m = a_m(m)$$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)a_m(m-i)}{1-K_m^2} \qquad i = 1, 2, \ldots m-1$$

Draw the lattice structure for the following FIR filter function

$$H(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2}$$

Solution:

$$H(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2}$$

$a_2(1) = 2,\ a_2(2) = \frac{1}{3}$
$K_m = a_m(m)$
for m=2
$K_2 = a_2(2) = \frac{1}{3}$

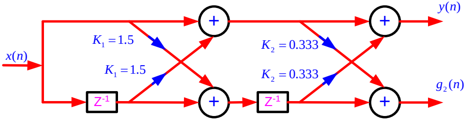


Figure 65: Lattice filter

$$\begin{aligned}
a_{m-1}(i) &= \frac{a_m(i) - a_m(m)a_m(m-i)}{1 - K_m^2} \\
a_1(i) &= \frac{a_2(i) - a_2(2)a_2(2-i)}{1 - K_2^2} \\
a_1(1) &= \frac{a_2(1) - a_2(2)a_2(1)}{1 - K_2^2} \\
&= \frac{2 - (0.333)(2)}{1 - (0.333)^2} \\
&= 1.5
\end{aligned}$$

A FIR filter is given by

$$y(n) = x(n) + \frac{2}{5}x(n-1) + \frac{3}{4}x(n-2) + \frac{1}{3}x(n-3)$$

Draw the lattice structure

Solution:

$$y(n) = x(n) + \frac{2}{5}x(n-1) + \frac{3}{4}x(n-2) + \frac{1}{3}x(n-3)$$

By taking z transform

$$\begin{aligned} Y(z) &= X(z) + \frac{2}{5}z^{-1}X(z) + \frac{3}{4}z^{-2}X(z) + \frac{1}{3}z^{-3}X(z) \\ H(z) &= \frac{Y(z)}{X(z)} = 1 + \frac{2}{5}z^{-1} + \frac{3}{4}z^{-2} + \frac{1}{3}z^{-3} \\ &= 1 + 0.4z^{-1} + 0.75z^{-2} + 0.333z^{-3} \end{aligned}$$

$a_3(1) = 0.4, \; a_3(2) = 0.75, \; a_3(3) = 0.333$

for m=3, $K_3 = a_3(3) = 0.333$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)a_m(m-i)}{1-K_m^2}$$

$$a_2(i) = \frac{a_3(i) - a_3(3)a_3(3-i)}{1-K_3^2}$$

$$a_2(1) = \frac{a_3(1) - a_3(3)a_3(2)}{1-K_3^2}$$

$$= \frac{0.4 - (0.333)(0.75)}{1-(0.333)^2} = 0.169$$

$$a_2(2) = \frac{a_3(2) - a_3(3)a_3(1)}{1-K_3^2}$$

$$= \frac{0.75 - (0.333)(0.4)}{1-(0.333)^2} = 0.693$$

for m=2, $K_2 = a_2(2) = 0.693$

$$a_1(i) = \frac{a_2(i) - a_2(2)a_2(2-i)}{1-K_2^2}$$

$$a_1(1) = \frac{a_2(1) - a_2(2)a_2(1)}{1-K_2^2}$$

$$= \frac{0.169 - (0.693)(0.169)}{1-(0.693)^2}$$

$$= 0.0998$$

for m=1, $K_1 = a_1(1) = 0.0998$

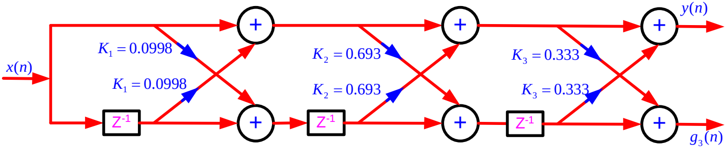


Figure 66: Lattice filter

A FIR filter is given by

$$y(n) = x(n) + 3.1x(n-1) + 5.5x(n-2) + 4.2x(n-3) + 2.3x(n-4)$$

Draw the lattice structure

Solution:

$$y(n) = x(n) + 3.1x(n-1) + 5.5x(n-2) + 4.2x(n-3) + 2.3x(n-4)$$

By taking z transform

$$\begin{aligned} Y(z) &= X(z) + 3.1z^{-1}X(z) + 5.5z^{-2}X(z) + 4.2z^{-3}X(z) + 2.3z^{-4}X(z) \\ H(z) &= \frac{Y(z)}{X(z)} = 1 + 3.1z^{-1} + 5.5z^{-2} + 4.2z^{-3} + 2.3z^{-4} \end{aligned}$$

$a_4(1) = 3.1,\; a_4(2) = 5.5,\; a_4(3) = 4.2\; a_4(4) = 2.3$

For m=4 $K_4 = a_4(4) = 2.3$

$$
\begin{aligned}
a_{m-1}(i) &= \frac{a_m(i) - a_m(m)a_m(m-i)}{1-K_m^2} \\
a_3(i) &= \frac{a_4(i) - a_4(4)a_4(4-i)}{1-K_4^2} \\
a_3(1) &= \frac{a_4(1) - a_4(4)a_4(3)}{1-K_4^2} \\
&= \frac{3.1 - (2.3)(4.2)}{1-(2.3)^2} \\
&= 1.529 \\
a_3(2) &= \frac{a_4(2) - a_4(4)a_4(2)}{1-K_4^2} \\
&= \frac{5.5 - (2.3)(5.5)}{1-(2.3)^2} \\
&= 1.6667 \\
a_3(3) &= \frac{a_4(3) - a_4(4)a_4(1)}{1-K_4^2} \\
&= \frac{4.5 - (2.3)(3.1)}{1-(2.3)^2} \\
&= 0.683
\end{aligned}
$$

For m=3 $K_3 = a_3(3) = 0.683$

$$
\begin{aligned}
a_{m-1}(i) &= \frac{a_m(i) - a_m(m)a_m(m-i)}{1-K_m^2} \\
a_2(i) &= \frac{a_3(i) - a_3(3)a_3(3-i)}{1-K_3^2} \\
a_2(1) &= \frac{a_3(1) - a_3(3)a_3(3)}{1-K_3^2} \\
&= \frac{1.529 - (0.683)(1.667)}{1-(0.683)^2} \\
&= 0.732 \\
a_2(2) &= \frac{a_3(2) - a_3(3)a_3(1)}{1-K_3^2} \\
&= \frac{1.667 - (0.683)(1.529)}{1-(0.683)^2} \\
&= 1.167
\end{aligned}
$$

For m=2, $K_2 = a_2(2) = 1.167$

For m=2, $K_2 = a_2(2) = 1.167$
For m=1

$$k_1 = a_1(1) = \frac{a_m(i) - a_m(m)a_m(m-i)}{1 - K_m^2}$$

$$\begin{aligned} k_1 = a_1(1) &= \frac{a_2(1) - a_2(2)a_2(1)}{1 - K_2^2} \\ &= \frac{0.732 - (1.167)(0.732)}{1 - (1.167)^2} = 0.338 \end{aligned}$$

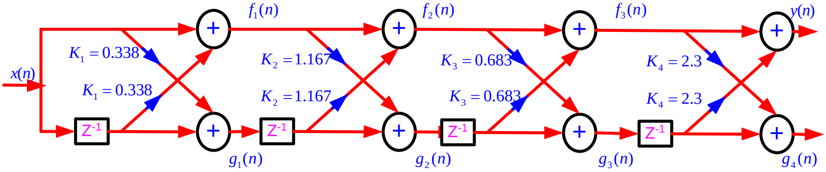


Figure 67: Lattice filter

Determine all the FIR filters which are specified by the lattice parameters $K_1 = 0.1$, $K_2 = 0.2$, and $K_3 = 0.3$ and draw the structure

Solution:

$$\begin{aligned} a_m(0) &= 1 \\ a_m(m) &= K_m \\ a_m(i) &= a_{m-1}(i) + a_m(m)a_{m-1}(m-i) \end{aligned}$$

For m=1

$$\begin{aligned} a_1(0) &= 1 \\ a_1(1) &= K_1 = 0.1 \end{aligned}$$

For m=2

$$\begin{aligned} a_2(0) &= 1 \\ a_2(2) &= K_2 = 0.2 \\ a_2(i) &= a_1(i) + a_2(2)a_1(2-i) \\ a_2(1) &= 0.1 + (0.2)0.1 = 0.12 \end{aligned}$$

For m=3

$$\begin{aligned} a_3(0) &= 1 \\ a_3(3) &= K_3 = 0.3 \\ a_3(i) &= a_2(i) + a_3(3)a_2(3-i) \\ a_3(1) &= a_2(1) + a_3(3)a_2(2) \\ &= 0.12 + (0.3)(0.2) = 0.18 \\ a_3(2) &= a_2(2) + a_3(3)a_2(1) \\ &= 0.2 + (0.3)(0.12) = 0.236 \end{aligned}$$

$$\begin{aligned} H(z) &= 1 + \sum_{i=1}^{m} a_m(i) z^{-i} \\ &= 1 + \sum_{i=1}^{3} a_3(i) z^{-i} \\ &= 1 + a_3(1) z^{-1} + a_3(2) z^{-2} + a_3(3) z^{-3} \\ &= 1 + 0.18 z^{-1} + 0.236 z^{-2} + 0.3 z^{-3} \end{aligned}$$

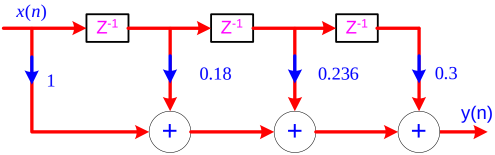


Figure 68: Lattice filter

Determine all the FIR filters which are specified by the lattice parameters $K_1 = 0.5$, $K_2 = 0.333$, and $K_3 = 0.25$ and draw the structure

Solution:

$$\begin{aligned} a_m(0) &= 1 \\ a_m(m) &= K_m \\ a_m(i) &= a_{m-1}(i) + a_m(m)a_{m-1}(m-i) \end{aligned}$$

For m=1

$$\begin{aligned} a_1(0) &= 1 \\ a_1(1) &= K_1 = 0.5 \end{aligned}$$

For m=2

$$\begin{aligned} a_2(0) &= 1 \\ a_2(2) &= K_2 = 0.333 \\ a_2(i) &= a_1(i) + a_2(2)a_1(2-i) \\ a_2(1) &= 0.5 + (0.333)0.5 = 0.665 \end{aligned}$$

For m=3

$$\begin{aligned} a_3(0) &= 1 \\ a_3(3) &= K_3 = 0.25 \\ a_3(i) &= a_2(i) + a_3(3)a_2(3-i) \\ a_3(1) &= a_2(1) + a_3(3)a_2(2) \\ &= 0.665 + (0.25)(0.333) = 0.75 \\ a_3(2) &= a_2(2) + a_3(3)a_2(1) \\ &= 0.333 + (0.667)(0.12) = 0.5 \end{aligned}$$

$$a_3(3) = k_3 = 0.25$$

$$a_3(0) = 1,\ a_3(1) = 0.75,\ a_3(2) = 0.5,\ and\ a_3(3) = 0.25$$

$a_3(0) = 1,\ a_3(1) = 0.75,\ a_3(2) = 0.5,\ and\ a_3(3) = 0.25$

$$\begin{aligned} H(z) &= 1 + \sum_{i=1}^{3} a_3(i) z^{-i} \\ &= K_3 = 1 + a_3(1)z^{-1} + a_3(2)z^{-2} + a_3(3)z^{-3} \\ &= 1 + 0.75z^{-1} + 0.5z^{-2} + 0.25z^{-3} \end{aligned}$$

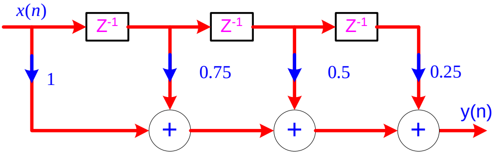


Figure 69: Lattice filter

Determine the impulse response of a FIR filter with reflection coefficients $K_1 = 0.6$, $K_2 = 0.3$, $K_3 = 0.5$ and $K_4 = 0.9$ Also draw the direct form structure

Solution:

$$\begin{aligned} a_m(0) &= 1 \\ a_m(m) &= K_m \\ a_m(i) &= a_{m-1}(i) + a_m(m)a_{m-1}(m-i) \end{aligned}$$

For m=1

$$\begin{aligned} a_1(0) &= 1 \\ a_1(1) &= K_1 = 0.5 \end{aligned}$$

For m=2

$$\begin{aligned} a_2(0) &= 1 \\ a_2(2) &= K_2 = 0.3 \\ a_2(i) &= a_1(i) + a_2(2)a_1(2-i) \\ a_2(1) &= 0.6 + (0.3)0.6 = 0.78 \end{aligned}$$

For m=3

$$\begin{aligned} a_3(0) &= 1 \\ a_3(3) &= K_3 = 0.5 \\ a_3(i) &= a_2(i) + a_3(3)a_2(3-i) \\ a_3(1) &= a_2(1) + a_3(3)a_2(2) \\ &= 0.78 + (0.5)(0.3) = 0.93 \\ a_3(2) &= a_2(2) + a_3(3)a_2(1) \\ &= 0.3 + (0.5)(0.78) = 0.69 \end{aligned}$$

$a_3(3) = k_3 = 0.25$

For m=4

$$
\begin{aligned}
a_4(0) &= 1 \\
a_4(i) &= a_3(i) + a_4(4)a_3(4-i) \\
a_4(1) &= a_3(1) + a_4(4)a_3(3) = 0.93 + (0.9)(0.5) = 1.38 \\
a_4(2) &= a_3(2) + a_4(3)a_3(2) = 0.69 + (0.9)(0.69) = 1.311 \\
a_4(3) &= a_3(3) + a_4(3)a_3(1) = 0.5 + (0.9)(0.93) = 1.337
\end{aligned}
$$

$a_4(4) = K_4 = 0.9$
$a_4(0) = 1,\ a_4(1) = 1.38,\ a_4(2) = 1.311,\ and\ a_4(3) = 1.337\ and\ a_4(4) = 0.9$

$$
\begin{aligned}
H(z) &= 1 + \sum_{i=1}^{3} a_3(i)z^{-i} = 1 + a_4(1)z^{-1} + a_4(2)z^{-2} + a_4(3)z^{-3} + a_4(4)z^{-3} \\
&= 1 + 1.38z^{-1} + 1.311z^{-2} + 1.337z^{-3} + 0.9z^{-4}
\end{aligned}
$$

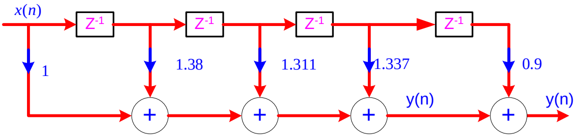


Figure 70: Lattice filter

Lattice structure for IIR Systems

- The system function of mth order IIR filter is represented by

$$H(z)=\frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

- Let us consider an all pole system with system function

$$H_1(z)=\frac{1}{1+\sum_{k=1}^{N} a_N(k) z^{-k}}=\frac{1}{A_N(z)}$$

- But the system function $H_1(z)=\frac{Y(z)}{X(z)}$

$$\frac{Y(z)}{X(z)}=\frac{1}{1+\sum_{k=1}^{N} a_N(k) z^{-k}}$$

$$Y(z)+\sum_{k=1}^{N} a_N(k) z^{-k} Y(z)=X(z)$$

$$Y(z) + \sum_{k=1}^{N} a_N(k) z^{-k} Y(z) = X(z)$$

- Taking inverse z transform

$$y(n) = -\sum_{k=1}^{N} a_N(k) y(n-k) + x(n)$$

- Interchange the input and output

$$x(n) = -\sum_{k=1}^{N} a_N(k) x(n-k) + y(n)$$

- Rearranging the above equation

$$y(n) = x(n) + \sum_{k=1}^{N} a_N(k) x(n-k)$$

- The equation is similar to the FIR

- Consider the order of the filter m=1

$$y(n) = a_1(1) y(n-1) + x(n)$$

- From the above equation

$$K_1 = a_1(1)$$

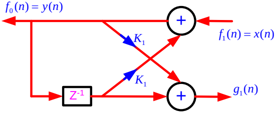


Figure 71: Single stage Lattice filter

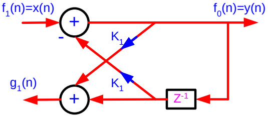


Figure 72: Single stage Lattice

Consider the filter of order of two.

$$
\begin{aligned}
f_2(n) &= x(n)\\
f_1(n) &= f_2(n) - K_2 g_1(n-1)\\
g_2(n) &= K_2 f_1(n) + g_1(n-1)\\
f_0(n) &= f_1(n) - K_1 g_0(n-1)\\
g_1(n) &= K_1 f_0(n) + g_0(n-1)\\
y(n) &= f_0(n) = g_0(n)
\end{aligned}
$$

$$
\begin{aligned}
y(n) &= -K_1(1+K_2)y(n-1) - K_2 y(n-2) + x(n)\\
g_2(n) &= K_2 y(n) + K_1(1+K_2)y(n-1) - y(n-2)
\end{aligned}
$$

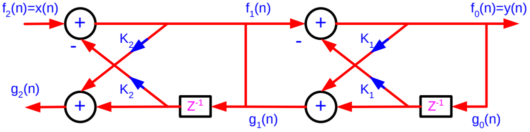


Figure 73: Single stage Lattice filter

$$y(n) = -\sum_{k=1}^{N} a_N(k)y(n-k) + x(n)$$

$$y(n) = -\sum_{k=1}^{2} a_2(k)y(n-k) + x(n)$$

$$\begin{aligned} y(n) &= -\sum_{k=1}^{2} a_2(k)y(n-k) + x(n) \\ &= -a_2(1)y(n-1) - a_2(2)y(n-2) + x(n) \end{aligned}$$

$$y(n) = x(n) + K_1(1+K_2)x(n-1) + K_2x(n-2)$$

This equation represents the second order FIR system if:

$$a_2(1) = K_1(1+K_2) \quad and \quad a_2(2) = K_2$$

$$K_1 = \frac{a_2(1)}{(1+a_2(2))} \quad and \quad K_2 = a_2(2)$$

The lattice structure for mth order is obtained by the following recursive equations.

$$K_m = a_m(m)$$

$$a_{m-1}(i) = \frac{a_m(i) - a_m(m)a_m(m-i)}{1-K_m^2} = \frac{a_m(i) - K_m a_m(m-i)}{1-K_m^2} \qquad i = 1, 2, \ldots m-1$$

The IIR system function is

$$H_{system}(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} = \frac{B_M(z)}{A_N(z)}$$

$$\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m a_m(m-i) \qquad i = M, M-1, M-2, \ldots 1, 0$$

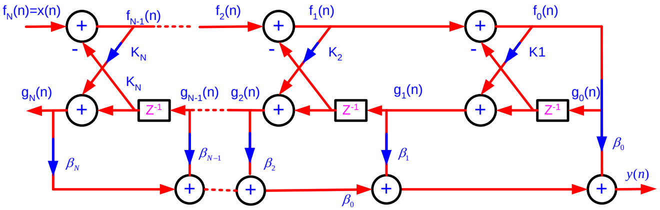


Figure 74: Lattice ladder structure

Realize the lattice structure for all pole filter given by $H(z) = \frac{1}{1+\frac{2}{5}z^{-1}+\frac{3}{4}z^{-2}+\frac{1}{3}z^{-3}}$

Solution:

m=3 $a_3(0) = 1,\ a_3(1) = \frac{2}{5},\ a_3(2) = \frac{3}{4}, a_3(3) = \frac{1}{3}$ $\quad a_{m-1}(i) = \frac{a_m(i)-a_m(m)a_m(m-i)}{1-K_m^2}$

for m=3

$$a_2(i) = \frac{a_3(i) - a_3(3)a_3(3-i)}{1-K_3^2}$$

$$a_2(1) = \frac{a_3(1) - a_3(3)a_3(2)}{1-K_3^2}$$

$$a_2(1) = \frac{0.4 - (0.333)(0.75)}{1-(0.33)^2} = 0.16875$$

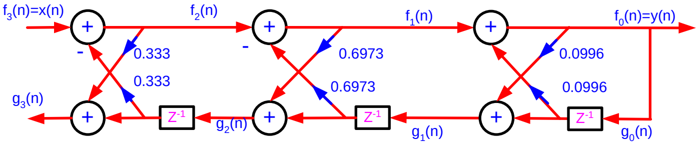
$$a_2(2) = \frac{a_3(2) - a_3(3)a_3(1)}{1-K_3^2}$$

$$a_2(2) = \frac{0.75 - (0.333)(0.4)}{1-(0.33)^2} = 0.6937$$

for m=2

$$\begin{aligned} a_1(1) &= \frac{a_2(1) - a_2(2)a_2(1)}{1-K_2^2} \\ &= \frac{0.1687 - (0.6973)(0.1687)}{1-(0.697)^2} \\ &= 0.0996 \end{aligned}$$

$K_3 = a_3(3) = \frac{1}{3}$,
$K_2 = a_2(2) = 0.6937$
$K_1 = a_1(1) = 0.0996$



Realize the lattice structure for all pole filter given by $H(z) = \frac{1}{1-0.9z^{-1}+0.64z^{-2}-0.576z^{-3}}$

Solution:

m=3 $a_3(0) = 1, \;\; a_3(1) = -0.9 \;\; a_3(2) = 0.64 \;\; a_3(3) = -0.576$

for m=3, i=1,2

$$\begin{aligned}
a_2(i) &= \frac{a_3(i) - a_3(3)a_3(3-i)}{1-K_3^2} \\
a_2(1) &= \frac{a_3(1) - a_3(3)a_3(2)}{1-K_3^2} \\
a_2(1) &= \frac{-0.9-(-0.576)(0.64)}{1-(-0.5763)^2} = -0.795 \\
a_2(2) &= \frac{a_3(2) - a_3(3)a_3(1)}{1-K_3^2} \\
a_2(2) &= \frac{0.64-(-0.576)(-0.9)}{1-(-0.5763)^2} = 0.1819
\end{aligned}$$

for m=2

$$\begin{aligned}
a_1(1) &= \frac{a_2(1) - a_2(2)a_2(1)}{1-K_2^2} \\
&= \frac{-0.795-(0.1819)(-0.795)}{1-(0.1819)^2} \\
&= -0.6726
\end{aligned}$$

$K_3 = a_3(3) = -0.576,$
$K_2 = a_2(2) = 0.1819$
$K_1 = a_1(1) = -0.6726$

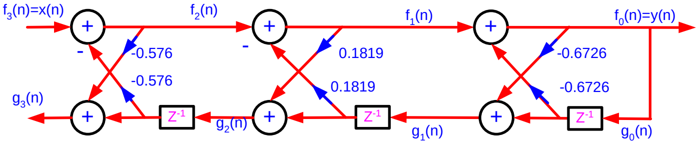


Figure 76: Lattice filter

Develop the lattice ladder structure for the filter with difference equation

$$y(n)+\frac{3}{4}y(n-1)+\frac{1}{4}y(n-2)=x(n)+2x(n-1)$$

Solution:

$$y(n)+\frac{3}{4}y(n-1)+\frac{1}{4}y(n-2)=x(n)+2x(n-1)$$

By taking z transform

$$\begin{aligned} Y(z)+\frac{3}{4}z^{-1}Y(z)+\frac{1}{4}z^{-2}Y(z) &= X(z)+2z^{-1}X(z) \\ H(z) &= \frac{Y(z)}{X(z)}=\frac{1+2z^{-1}}{1+\frac{3}{4}z^{-1}+\frac{1}{4}z^{-2}} \end{aligned}$$

$B(z)=1+2z^{-1}$
$A(z)=1+\frac{3}{4}z^{-1}+\frac{1}{4}z^{-2}$
$a_2(1)=0.75,\ a_2(2)=0.25$

$$\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m a_m(m-i)$$

for m=2
$K_2 = a_2(2) = 0.25$

$$\begin{aligned} a_{m-1}(i) &= \frac{a_m(i) - a_m(m)a_m(m-i)}{1-K_m^2} \\ a_1(i) &= \frac{a_2(i) - a_2(2)a_2(2-i)}{1-K_2^2} \\ a_1(1) &= \frac{a_2(1) - a_2(2)a_2(1)}{1-K_2^2} \\ &= \frac{0.75 - (0.25)(0.75)}{1-(0.25)^2} = 0.6 \end{aligned}$$

for m=1
$K_1 = a_1(1) = 0.6$
Ladder coefficients are
$B(z) = 1 + 2z^{-1}$ M=1
$b_0 = 1,\ \ b_1 = \beta_1 = 2$

$$\begin{aligned} \beta_0 &= b_0 - \sum_{m=1}^{1} \beta_m a_m(m) \\ &= b_0 - \beta_1 a_1(1) = 1 - 2(0.6) = -0.2 \end{aligned}$$

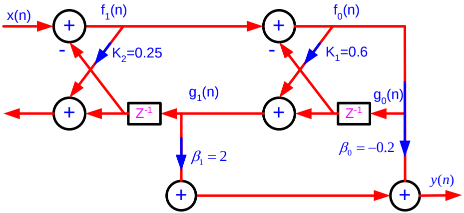


Figure 77: Lattice filter

A linear time invariant system is defined by

$$H(z) = \frac{0.129 + 0.38687z^{-1} + 0.3869z^{-2} + 0.129z^{-3}}{1 - 0.2971z^{-1} + 0.3564z^{-2} - 0.0276z^{-3}}$$

Realize the IIR transfer function using lattice ladder structure

Solution:

$$H(z) = \frac{1}{A(z)}B(z)$$

where

$$\begin{aligned} A(z) &= 1 - 0.2971z^{-1} + 0.3564z^{-2} - 0.0276z^{-3} \\ B(z) &= 0.129 + 0.38687z^{-1} + 0.3869z^{-2} + 0.129z^{-3} \end{aligned}$$

$a_3(3) = -0.0276,\ a_3(2) = 0.3564,\ a_3(1) = -0.2971$

for m=3
$K_3 = a_3(3) = -0.0276$

$$
\begin{aligned}
a_{m-1}(i) &= \frac{a_m(i) - a_m(m)a_m(m-i)}{1-K_m^2} \\
a_2(i) &= \frac{a_3(i) - a_3(3)a_3(3-i)}{1-K_3^2} \\
a_2(1) &= \frac{a_3(1) - a_3(3)a_3(2)}{1-K_3^2} \\
&= \frac{-0.2971 - (-0.0276)(0.3564)}{1-(-0.2971)^2} \\
&= -0.2875 \\
a_2(2) &= \frac{a_3(2) - a_3(3)a_3(1)}{1-K_3^2} \\
&= \frac{0.3564 - (-0.0276)(-0.2971)}{1-(-0.0.0276)^2} \\
&= 0.3485
\end{aligned}
$$

for m=2
$K_2 = a_2(2) = 0.3485$

$$
\begin{aligned}
a_1(i) &= \frac{a_2(i) - a_2(2)a_2(2-i)}{1-K_2^2} \\
a_1(1) &= \frac{a_2(1) - a_2(2)a_2(1)}{1-K_2^2} \\
&= \frac{0.2875 - (0.3485)(-0.2875)}{1-(0.3485)^2} \\
&= -0.2132
\end{aligned}
$$

for m=1
$K_1 = a_1(1) = -0.2132$

Ladder coefficients are: $B(z) = 0.129 + 0.38687z^{-1} + 0.3869z^{-2} + 0.129z^{-3}$

$$\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m a_m(m-i)$$

$b_0 = 0.129,\ b_1 = 0.38687,\ b_2 = 0.3869,\ b_3 = 0.129 = \beta_3$
$i = 2$

$$\beta_2 = b_2 - \sum_{m=3}^{3} \beta_m a_m(m-2) = b_2 - \beta_3 a_3(1) = 0.3869 - (0.129)(-0.2971) = 0.4252$$

$i = 1$

$$\begin{aligned}\beta_1 &= b_1 - \sum_{m=2}^{3} \beta_m a_m(m-1)\\ &= b_1 - \beta_2 a_2(1) - \beta_3 a_3(2) = 0.3867 - (0.4252)(-0.2875) - (0.129)(0.3564) = 0.4630\end{aligned}$$

$i = 0$

$$\begin{aligned}\beta_0 &= b_0 - \sum_{m=1}^{3} \beta_m a_m(m)\\ &= b_0 - \beta_1 a_1(1) - \beta_2 a_2(2) - \beta_3 a_3(3)\\ &= 0.129 - (0.4630)(-0.2132) - (0.4252)(0.3485) - (0.129)(-0.0276) = 0.0831\end{aligned}$$

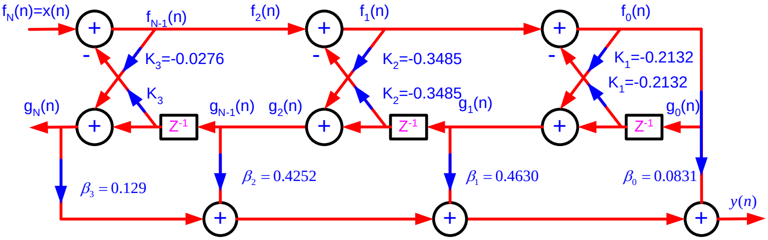


Figure 78: Lattice filter

A linear time invariant system is defined by

$$H(z) = \frac{1 + z^{-1} + z^{-2}}{(1 + 0.5z^{-1})(1 + 0.3z^{-1})(1 + 0.4z^{-1})}$$

Realize the IIR transfer function using lattice ladder structure

Solution:

$$\begin{aligned} H(z) &= \frac{1 + z^{-1} + z^{-2}}{(1 + 0.5z^{-1})(1 + 0.3z^{-1})(1 + 0.4z^{-1})} \\ &= \frac{1 + z^{-1} + z^{-2}}{1 + 1.2z^{-1} + 0.47z^{-2} + 0.06z^{-3}} \end{aligned}$$

$$H(z) = \frac{1}{A(z)}B(z)$$

where

$$\begin{aligned} A(z) &= 1 + 1.2z^{-1} + 0.47z^{-2} + 0.06z^{-3} \\ B(z) &= 1 + z^{-1} + z^{-2} \end{aligned}$$

$a_3(3) = 0.06, \; a_3(2) = 0.47, \; a_3(1) = 1.2$

for m=3
$K_3 = a_3(3) = 0.06$

$$\begin{aligned}
a_{m-1}(i) &= \frac{a_m(i) - a_m(m)a_m(m-i)}{1-K_m^2} \\
a_2(i) &= \frac{a_3(i) - a_3(3)a_3(3-i)}{1-K_3^2} \\
a_2(1) &= \frac{a_3(1) - a_3(3)a_3(2)}{1-K_3^2} \\
&= \frac{1.2 - (-0.06)(0.47)}{1-(-0.06)^2} \\
&= 1.176 \\
a_2(2) &= \frac{a_3(2) - a_3(3)a_3(1)}{1-K_3^2} \\
&= \frac{0.47 - (-0.06)(1.2)}{1-(0.06)^2} \\
&= 0.4
\end{aligned}$$

for m=2
$K_2 = a_2(2) = 0.4$

$$\begin{aligned}
a_1(i) &= \frac{a_2(i) - a_2(2)a_2(2-i)}{1-K_2^2} \\
a_1(1) &= \frac{a_2(1) - a_2(2)a_2(1)}{1-K_2^2} \\
&= \frac{1.176 - (0.4)(1.176)}{1-(0.4)^2} \\
&= 0.84
\end{aligned}$$

for m=1
$K_1 = a_1(1) = 0.84$

Ladder coefficients are
$B(z) = 1 + z^{-1} + z^{-2}$ M=1
$b_0 = 1,\ b_1 = 1,\ b_2 = 1 = \beta_2$

$$\beta_i = b_i - \sum_{m=i+1}^{M} \beta_m a_m(m-i)$$

$$\beta_2 = b_2 = 1$$

$i = 1$

$$\begin{aligned}\beta_1 &= b_1 - \sum_{m=2}^{3} \beta_m a_m(m-1) \\ &= b_1 - \beta_2 a_2(1) \\ &= 1 - 1.176 = -0.176\end{aligned}$$

$i = 0$

$$\begin{aligned}\beta_0 &= b_0 - \sum_{m=1}^{1} \beta_m a_m(m) \\ &= b_0 - \beta_1 a_1(1) - \beta_2 a_2(2) \\ &= 1 + .84(0.176) - 0.4 = 0.748\end{aligned}$$

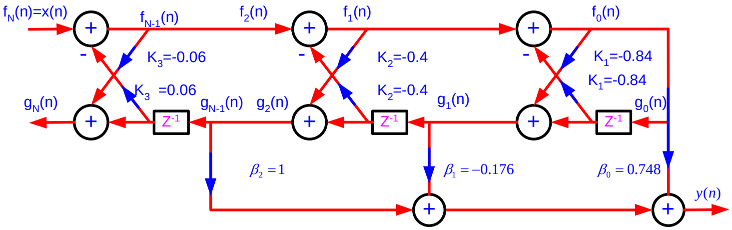


Figure 79: Lattice filter

Signal Flow Graph

- Signal flow graph (SFG) is **graphical representation** of block diagram structure.
- The basic elements of SFG are **branches and nodes**. The signal out of branch is equal to the branch gain.
- **Adders** and **pick-off points** are replaced by **nodes**. **Multipliers** and **delay elements** are indicated by the the branch transmittance.
- The **input** to the system originates at **source node** and **output signal** is extracted from a **sink node**.
- The importance of SFG in FIR or IIR structure is **transposition or flow graph reversal theorem.**
- Theorem states that "*if we reverse the directions of all branch transmittances and interchange the input and output in the flow graph, the system function remains unchanged*". The resulting structure is called a **transposed structure or a transposed form**.

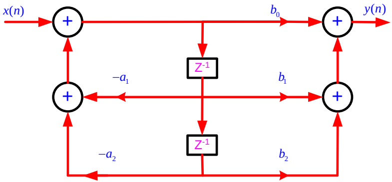


Figure 80: Direct Form II

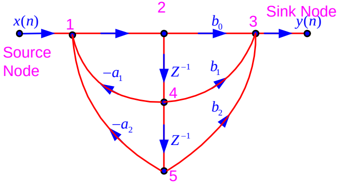


Figure 81: Signal Flow graph

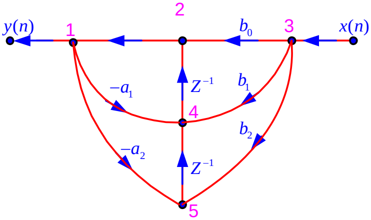


Figure 82: Transposed Structure SFG

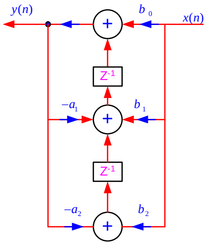


Figure 83: Direct form-II realization

- Figure 80 shows the direct form II structure while Figure 81 shows its signal flow graph
- Figure 82 shows the Transposed Structure while Figure 83 shows its realization.

A linear time invariant system is defined by

$$H(z) = \frac{1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}}{1 + \frac{1}{8}z^{-1} - \frac{1}{5}z^{-2} + \frac{1}{6}z^{-3}}$$

Draw the signal flow graph, transposed signal flow graph and transposed direct form-II structure

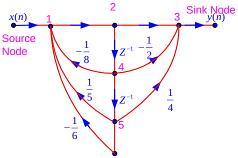
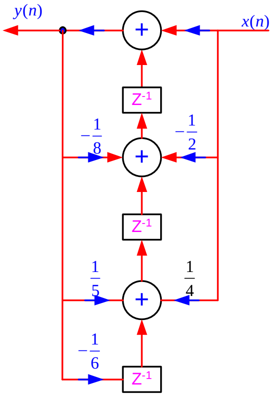


Figure 84: Signal flow graph



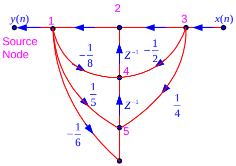


Figure 85: Transposed Structure signal flow graph

Figure 86: Direct form-II Structure

For the flow graph write difference equations and system function

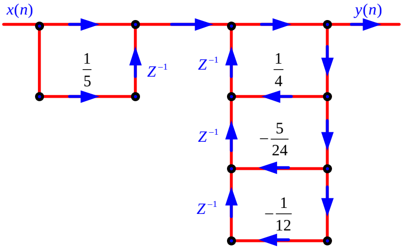


Figure 87: Direct Form I

Solution:

$$y(n) = \frac{1}{4}y(n-1) - \frac{5}{24}y(n-2) - \frac{1}{12}y(n-3) + x(n) + \frac{1}{5}x(n-1)$$

$$H(z) = \frac{1 + \frac{1}{5}z^{-1}}{1 - \frac{1}{4}z^{-1} + \frac{5}{24}z^{-2} + \frac{1}{12}z^{-3}}$$

For the flow graph write difference equations and system function

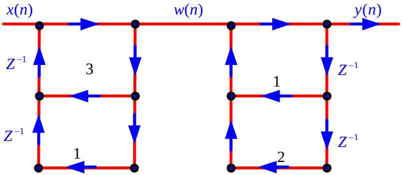


Figure 88: Direct Form II Cascade

$$H(z) = \frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)}\frac{W(z)}{X(z)}$$

$$= \frac{1}{(1-z^{-1}-2z^{-2})(1-3z^{-1}-z^{-2})}$$

$$= \frac{1}{1-4z^{-1}+7z^{-3}+2z^{-4}}$$

Solution:

$$w(n) = x(n) + 3w(n-1) + w(n-2)$$

$$\frac{W(z)}{X(z)} = \frac{1}{1-3z^{-1}-z^{-2}}$$

$$y(n) = w(n) + y(n-1) + 2y(n-2)$$

$$\frac{Y(z)}{W(z)} = \frac{1}{1-z^{-1}-2z^{-2}}$$

$$Y(z)[1-4z^{-1}+7z^{-3}+2z^{-4}] = X(z)$$

$$Y(z) = X(z) + 4z^{-1}Y(z) - 7z^{-3}Y(z) - 2z^{-4}Y(z)$$

$$y(n) = x(n) + 4y(n-1) - 7y(n-3) - 2y(n-4)$$