UNIT - 4 - FINITE IMPULSE RESPONSE (FIR) FILTER

Windows.

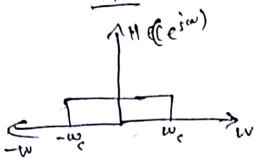
1) Rectangular window
2) Hamming "
3) Hanning "
4) Blackman. "

} LPF, HPF, Band Stop filter..
Band pass filter.

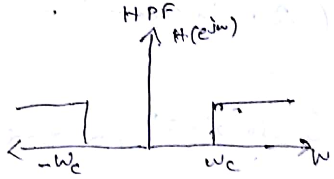
NEED FOR WINDOW: X 2m.

- To design FIR filter, one way is to truncate the infinite fourier series at $n = \pm\left(\frac{N-1}{2}\right)$
- Abrupt truncation results in oscillation in the pass band and stop band of the filter, this oscillation is called as Gibbs phenomena.
- To reduce the oscillation, fourier co-efficients of the filter $[h_d(n)]$ are modified by multiplying the infinite impulse response with a finite weight sequence $[w(n)]$ called as window.

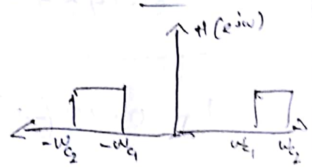
LPF



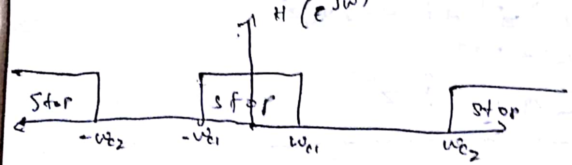
HPF



BPF



BSF



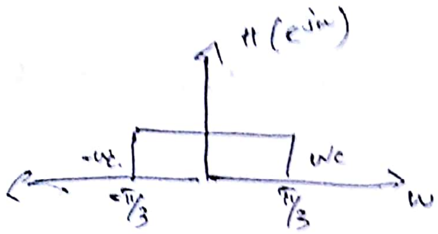
Assignment
Comparison of 4 filters.

Design an Ideal LPF, using rectangular window at N = 9 whose desired frequency response is

$$H_d(e^{j\omega}) = \begin{cases} 1 & \pi/3 \geq \omega \geq -\pi/3 \\ 0 & \text{else} \end{cases}$$

Step 1: Question STEP 2 Diagram.

LPF



Step 3:

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$n = 0, \pm 1, \pm 2 \ldots \pm\left(\frac{N-1}{2}\right)$ for odd

$n = 0, \pm 1, \pm 2 \ldots \pm\left(\frac{N}{2}\right)$ for even.

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi/3}^{\pi/3} (1) e^{j\omega n} d\omega = \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \right]_{-\pi/3}^{\pi/3}$$

$$= \frac{1}{2\pi j n} \left[e^{j\frac{\pi}{3}n} - e^{-j\pi/3 n} \right] = \frac{2 j \sin \pi/3\, n}{2\pi j n}$$

$$\boxed{h_d(n) = \frac{\sin \pi/3\, n}{\pi n}}$$

Since N is odd.

$n = 0, \pm 1, \pm 2, \pm 3, \pm 4$

For $n = 0$: $h_d(0) = \frac{\sin \pi/3 (0)}{0 (\pi)} =$.

Apply L'hospitals rule.

$$h_d(n) = \lim_{n \to 0} \frac{\cos\left[\frac{\pi}{3} n\right] \left(\frac{\pi}{3}\right)}{\pi} = \frac{\pi}{3\pi} = \frac{1}{3}$$

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For $n=1$

$$h_d(1) = \lim_{n\to 0} \frac{\sin\left(\frac{\pi}{3}(n)\right)}{\pi(1)} = \frac{0.866}{\pi} = 0.275$$

Since LPF is Symmetric h_d.

For $n=2$

$$h_d(2) = \frac{\sin\frac{2\pi}{3}}{2\pi} = 0.13 \rightarrow h_d(-2)$$

For $n=3$

$$h_d(3) = \frac{\sin\pi}{3\pi} = 0 \rightarrow h_d(-3)$$

For $n=4$

$$h_d(4) = \frac{\sin\frac{4\pi}{3}}{4\pi} = -0.06. \rightarrow h_d(-4),$$

Step 4 :

For Rectangular Window :

$$W_R(n) = \begin{cases} 1 & -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right) \\ 0 & \text{else} \end{cases}$$

$$W_R(n) = \begin{cases} 1 & -4 \le n \le 4 \\ 0 & \end{cases}$$

Step 5

Filter Coefficien.

$$h(n) = h_d(n) \times W_R(n).$$

$n = 0, \pm 1, \pm 2, \pm 3, \pm 4$

$$h(0) = h_d(0) \times W_R(0) = 0.33 \times 1 = 0.33$$

$$h_d(1) = h_d(1) \times W_R(1) = 0.27 \times 1 = 0.27$$

$$h(2) = h_d(2) \times W_R(2) = 0.13 \times 1 = 0.13$$

$$h(3) = h_d(3) \times W_R(3) = 0 \times 1 = 0$$

$$h(4) = h_d(4) \times W_R(4) = -0.06 \times 1 = -0.06.$$

Step 4

$$H(z) = h(0) + \sum_{n=1}^{(\frac{N-1}{2})} h(n)\left[z^{n} + z^{-n}\right]$$

$$H(z) = h(0) + \sum_{n=1}^{4} h(n)\left[z^{n} + z^{-n}\right]$$

$$H(z) = h(0) + \Big[h(1)\left[z^{1} + z^{-1}\right] + h(2)\left[z^{2} + z^{-2}\right] + h(3)\left[z^{3} + z^{-3}\right] + h(4)\left[z^{4} + z^{-4}\right]\Big]$$

$$H(z) = 0.33 + 0.27(z^{1} + z^{-1}) + 0.13(z^{2} + z^{-2}) + 0 - 0.06(z^{4} + z^{-4})$$

$$= 0.33 + 0.27z + 0.13z^{2} - 0.06z^{4} + 0.27z^{-1} + 0.13z^{-2} - 0.06z^{-4}$$

$$\therefore H(z) = 0.33 + 0.27z + 0.13z^{2} - 0.06z^{4} + 0.27z^{-1} + 0.13z^{-2} - 0.06z^{-4}$$

$$\overline{H}Z = z^{-(\frac{N-1}{2})} H(z)$$

$$= z^{-4}H(z) = 0.33z^{-4} + 0.27z^{-3} + 0.13z^{-2} - 0.06 + 0.27z^{-5} + 0.13z^{-6} - 0.06z^{-8}$$

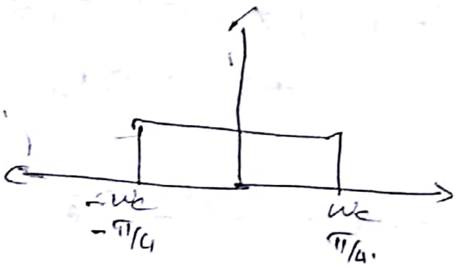
$$\boxed{\therefore \overline{H}Z = -0.06 + 0.13z^{-2} + 0.27z^{-3} + 0.33z^{-4} + 0.27z^{-5} + 0.13z^{-6} - 0.06z^{-8}}$$

28/09/18

Design an LPF using Hanning window for the given specifications.

$$H(e^{j\omega}) = \begin{cases} e^{-2j\omega} & \frac{-\pi}{4} \le \omega \le \frac{+\pi}{4} \\ 0 & \text{else.} \end{cases} \quad \text{For } N=7$$

STEP 1



STEP 2

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{-2j\omega}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{j(-2+n)}\, d\omega.$$

$$= \frac{1}{2\pi}\left[\frac{e^{j(n-2)\omega}}{j(n-2)}\right]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{2\pi\, j(n-2)}\left[e^{j(n-2)\pi/4} - e^{-j(n-2)\pi/4}\right]$$

$$= \frac{1}{2\pi\, j(n-2)}\; 2j\, \sin(n-2)\pi/4.$$

$$\boxed{h_d(n) = \frac{1}{\pi(n-2)}\sin(n-2)\pi/4}$$

STEP 3

$n = 0, \pm 1, \pm 2, \pm 3$

$$h_d(0) = \frac{1}{\pi(-2)}\sin(-\pi/2) = \frac{1}{2\pi} = 0.159$$

$$h_d(1) = \frac{1}{\pi(-1)} \sin\left(-\frac{\pi}{4}\right) = \frac{0.707}{\pi} = 0.225 = h_d(-1)$$

$$h_d(2) = \frac{1}{\pi(2-2)} \sin(0) = \frac{0}{0}$$

Applying L'hopital rule.

$$h_d(2) = \frac{\underset{n\to 2}{Lt} \left[\cos(n-2)\pi/4\right] \pi/4}{\pi} = 0.25 = h_d(-2)$$

$$h_d(3) = \frac{1}{\pi(3-2)} \sin(3-2)\pi/4 = 0.225 = h_d(-3).$$

Step 4

For Hanning window.

$$w_{hn}(n) = \begin{cases} 0.5 + 0.5 \cos\left(\frac{2\pi n}{N-1}\right) & -\left(\frac{N-1}{2}\right) \leq n \leq \left(\frac{N-1}{2}\right) \end{cases}$$

$$W_{hn}(n) = \begin{cases} 0.5 + 0.5 \cos\left(\frac{2\pi n}{6}\right) \end{cases}$$

$$W_{hn}(0) = 0.5 + 0.5 \cos 0 = 1$$

$$W_{hn}(1) = 0.5 + 0.5 \cos\left(\frac{\pi}{3}\right) = 0.75 = W_{hn}(-1)$$

$$W_{hn}(2) = 0.5 + 0.5 \cos\left(\frac{2\pi}{3}\right) = 0.25 = W_{hn}(-2)$$

$$W_{hn}(3) = 0.5 + 0.5 \cos(\pi) = 0 = W_{hn}(-3).$$

$$h(n) = h_d(n) \cdot W_{hn}(n)$$

$$h(0) = h_d(0) \cdot W_{hn}(0) = 0.159$$

$$h(1) = h_d(1) \cdot W_{hn}(1) = 0.168 = h(-1)$$

$$h(2) = h_d(2) \cdot W_{hn}(2) = 0.0625 = h(-2)$$

$$h(3) = h_d(3) \cdot W_{hn}(3) = 0 = h(-3).$$

Step 5:

$$H(z) = h(0) + \sum_{n=1}^{3} h(n)\left[z^{n} + z^{-n}\right]$$

$$= h(0) + h(1)(z^{1}+z^{-1}) + h(2)\left[z^{2}+z^{-2}\right] + h(3)\left[z^{3}+z^{-3}\right]$$

$\cancel{+h(3)}$

$$= 0.159 + 0.168\left[z^{1}+z^{-1}\right] + 0.0625\left[z^{2}+z^{-2}\right]$$

$$\boxed{\therefore H(z) = 0.159 + 0.168z + 0.168z^{-1} + 0.0625z^{2} + 0.0625z^{-2}}$$

To Causal.

Step 6:

$$\overline{H(z)} = z^{-\left(\frac{N-1}{2}\right)} H(z) = z^{-3} H(z).$$

$$\overline{H(z)} = 0.159z^{-3} + 0.168z^{-2} + 0.168z^{-4} + 0.0625z^{-1}$$
$$+ 0.0625z^{-5}$$

$$\boxed{\overline{H(z)} = 0.0625z^{-1} + 0.168z^{-2} + 0.159z^{-3} + 0.168z^{-4} + 0.0625z^{-5}}$$

Step 7

For Symmetric case

$$h(n) = h(N-1-n)$$

$h(0) = h(6) = 0.159$

$h(1) = h(5) = 0.168$

$h(2) = h(4) = 0.0625$

$h(3) = h(3) = 0$

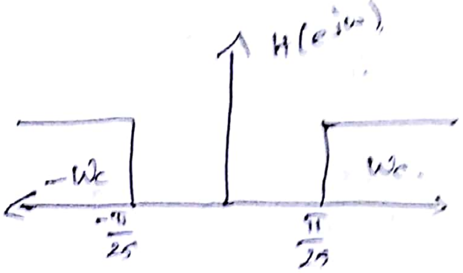
Design a HPF of length 7 with cut off frequency of 2 Hz using Hamming window. The sampling frequency is 100 Hz.

STEP 1: HPF, Hamming window, length $N=7$.

$f_c = 2\,Hz$ $\quad F_s = 100\,Hz$.

STEP 2:

$$W_c = \frac{2\pi f_c}{F_s} = \frac{2\pi \times 2}{100} = \frac{\pi}{25}.$$



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{jw})\, e^{jwn}\, dw$$

$n = 0, \pm1, \pm2, \pm3$

※ Since $H_d(e^{jw})$ is not given, Assume it as 1

$$h_d(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\frac{\pi}{25}} e^{jwn}\, dw + \int_{\pi/25}^{\pi} e^{jwn}\, dw\right]$$

$$= \frac{1}{2\pi}\left[\left[\frac{e^{jwn}}{jn}\right]_{-\pi}^{-\frac{\pi}{25}} + \left[\frac{e^{jwn}}{jn}\right]_{\pi/25}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{jn}\left(e^{-j\pi n} - e^{\frac{-j\pi n}{25}}\right) + \frac{1}{jn}\left(e^{\frac{j\pi n}{25}} - e^{j\pi n}\right)\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{jn}\left(e^{\frac{-j\pi n}{25}} - e^{-j\pi n} + e^{j\pi n} - e^{\frac{-j\pi n}{25}}\right)\right]$$

$$= \frac{1}{2\pi jn}\left[\left(\frac{e^{j\pi n} - e^{-j\pi n}}{2j}\right) - \left(e^{j\pi/25} - e^{-j\pi/25}\right)\right]$$

$$\boxed{h_d(n) = \frac{1}{\pi n}\left\{\sin \pi n - \sin \frac{\pi}{25} n\right\} = h_d(n)}$$

<u>STEP 3</u>

$n = 0, \pm 1, \pm 2, \pm 3$

$$h_d(0) = \frac{1}{\pi \times 0}\{0 - 0\} = \frac{0}{0}$$

Applyin L'Hopital rule.

$$h_d(0) = \lim_{n \to 0} \frac{\cos \pi n\,(\pi) - \cos \frac{\pi}{25} n \left(\frac{\pi}{25}\right)}{\pi}$$

$$= \frac{\pi - \frac{\pi}{25}}{\pi} = \frac{24\pi}{25\pi} = 0.96.$$

$h_d(0) = 0.96$ ~~ck~~

$$h_d(1) = \frac{1}{\pi}\left\{\sin \pi - \sin \frac{\pi}{25}\right\} = -0.039 = h_d(-1)$$

$$h_d(2) = \frac{1}{2\pi}\left[\sin 2\pi - \sin \frac{2\pi}{25}\right] = \cancel{\frac{4}{25}} \quad -0.039$$

$$h_d(3) = \frac{1}{3\pi}\left[\sin 3\pi - \sin \frac{3\pi}{25}\right] = -0.039.$$

<u>STEP 4</u>:

~~$h(n) = h_d(n)$~~ For Hamming window

$$W_{hm}(n) = \begin{cases} 0.54 + 0.46 \cos\left(\frac{2\pi n}{63}\right) & -3 \le n \le 3 \\ 0 & \text{else} \end{cases}$$

$W_{hm}(0) = 0.54 + 0.46 \cos(0) = 1$

$W_{hm}(1) = 0.54 + 0.46 \cos\left(\frac{\pi}{3}\right) = 0.77$

$W_{hm}(2) = 0.54 + 0.46 \cos\left(\frac{2\pi}{3}\right) = 0.31$

$W_{hm}(3) = 0.54 + 0.46 \cos(\pi) = 0.08$

STEP 5:

$h(n) = h_d(n)\, W_{hm}(n)$

$h(0) = h_d(0) \times W_{hm}(0) = 0.96 \times 1 = 0.96$

$h(1) = h_d(1) \times W_{hm}(1) = -0.039 \times 0.77 = -0.03$

$h(2) = h_d(2) \times W_{hm}(2) = -0.039 \times 0.31 = -0.012$

$h(3) = h_d(3) \times W_{hm}(3) = -0.039 \times 0.08 = -0.00312$

$$H(z) = h(0) + \sum_{n=1}^{3} h(n)\left[z^{n} + z^{-n}\right]$$

$$= 0.96 + \left[h(1)\left[z^{1} + z^{-1}\right] + h(2)\left[z^{2} + z^{-2}\right] + h(3)\left[z^{3} + z^{-3}\right] \right]$$

$$= 0.96 + \left[-0.03 z^{1} - 0.03 z^{-1} - 0.012 z^{2} - 0.012 z^{-2} - 0.00312 z^{3} - 0.00312 z^{-3} \right]$$

$$\boxed{= 0.96 - 0.03 z^{1} - 0.012 z^{2} - 0.00312 z^{3} - 0.03 z^{-1} - 0.012 z^{-2} - 0.00312 z^{-3}}$$

$\overline{H(z)} = z^{-3} H(z)$

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$\overline{H(z)} = 0.96 z^{-3} - 0.03 z^{-2} - 0.012 z^{-1} - 0.00312 - 0.03 z^{-4}$
$- 0.012 z^{-5} - 0.00312 z^{-6}$

$\overline{H(z)} = -0.00312 - 0.012 z^{-1} - 0.03 z^{-2} + 0.96 z^{-3}$
$- 0.03 z^{-4} - 0.012 z^{-5} - 0.00312 z^{-6}$

STEP 6

For Sym filter.

$h(n) = h(N-1-n)$

$h(0) = h(6) = 0.96$

$h(1) = h(5) = -0.03$

$h(2) = h(4) = -0.03$

$h(3) = h(3) = -0.03$

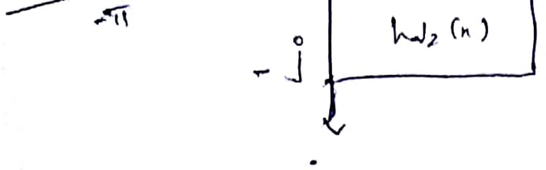
Design an Ideal Hailbert transform having the frequency response.

$$H(e^{j\omega}) = \begin{cases} j & -\pi \le \omega \le 0 \\ -j & 0 \le \omega \le \pi \end{cases}$$

Using Blackman window, take $N = 7$

STEP 1

$N = 7$, Blackman window



$$\therefore h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{jw})\, e^{jwn}\, dw.$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{0} j\, e^{jwn}\, dw + \int_{0}^{\pi} -j\, e^{jwn}\, dw \right]$$

$$= \frac{1}{2\pi} \left[j \left[\left[\frac{e^{jwn}}{jn} \right]_{-\pi}^{0} - \left[\frac{e^{jwn}}{jn} \right]_{0}^{\pi} \right] \right]$$

$$= \frac{1}{2\pi n} \left[(e^{0} - e^{-j\pi n}) - (e^{j\pi n} - e^{0}) \right]$$

$$= \frac{1}{2\pi n} \left[e^{0} - e^{-j\pi n} - e^{j\pi n} + e^{0} \right]$$

$$= \frac{1}{2\pi n} \left[2 - (e^{j\pi n} + e^{-j\pi n}) \right]$$

$$= \frac{1}{2\pi n} \left[2 - (2 \cos \pi n) \right] = \frac{2(1 - \cos \pi n)}{2\pi n}$$

$$\boxed{\therefore h_d(n) = \frac{1 - \cos \pi n}{\pi n}.}$$

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Step 3:

$$h_d(n) = \frac{1}{\pi n}\{1-\cos \pi n\} \quad n = 0, \pm 1, \pm 2, \pm 3$$

$$h_d(0) = \cancel{\frac{0}{0}} \quad \frac{0}{0} \quad \times$$

Applying L'hospital rule.

$$h_d(0) = \lim_{n \to 0} \cancel{\frac{0}{0}} \quad \frac{0 + \sin \pi n (\pi)}{\pi} = \frac{0}{\pi} = 0$$

$$h_d(0) = 0$$

$$h_d(1) = \frac{1}{\pi}[1-\cos\pi] = \frac{1}{\pi}[1+1] = \frac{2}{\pi}$$

$$h_d(1) = 0.63.$$

$$h_d(2) = \frac{1}{2\pi}[1-\cos 2\pi] = \frac{1-1}{2\pi} = 0$$

$$h_d(3) = \frac{1-\cos 3\pi}{3\pi} = \frac{1+1}{3\pi} = \frac{2}{3\pi} = 0.212.$$

$h_d(0) = 0$

$h_d(1) = 0.63$

$h_d(2) = 0$

$h_d(3) = 0.212$

$h_d(-1) = -h_d(1) = -0.63$

$h_d(-2) = -h_d(2) = 0$

$h_d(-3) = -h_d(3) = -0.212$

Step 4 :

For Blackman Window :

$$W_B(n) = \begin{cases} 0.42 + 0.5 \cos\left(\frac{2\pi n}{3}\right) + 0.08 \cos\left(\frac{2\pi n}{3}\right) \\ 0 \end{cases}$$

$$W_B(0) = 0.42 + 0.5 \cos 0 + 0.08 \cos 0$$

$$= 0.42 + 0.5 + 0.08 = 1$$

$$W_B(1) = 0.42 + 0.5 \cos\frac{\pi}{3} + 0.08 \cos\left(\frac{2\pi}{3}\right)$$

$$= 0.42 + 0.25 - 0.04 = 0.63.$$

$$W_B(2) = 0.42 + 0.5 \cos\left(\frac{2\pi}{3}\right) + 0.08 \cos\left(\frac{4\pi}{3}\right)$$

$$= 0.42 - 0.25 - 0.04 = 0.13.$$

$$W_B(3) = 0.42 + 0.5 \cos\pi + 0.08 \cos 2\pi$$

$$= 0.42 - 0.5 + 0.08 = 0.$$

$W_B(-1) = -0.63$ $\quad$ $W_B(-2) = -0.13$ $\quad$ $W_B(-3) = 0$

Step 5 :

$$h(n) = h_d(n) \times W_B(n)$$

$h(0) = h_d(0) \times W_B(0) = 0$

$h(1) = h_d(1) \times W_B(1) = 0.3969$

$h(2) = h_d(2) \times W_B(2) = 0$

$h(3) = h_d(3) \times W_B(3) = 0$

$h(-1) = h_d(-1) \times W_B(-1) = 0.3969$

$h(-2) = h_d(-2) \times W_B(-2) = 0$

$h(-3) = h_d(-3) \times W_B(-3) = 0$

STEP 6 :

$$H(z) = h(0) + \sum_{n=1}^{3} h(n)\,[z^{n} + z^{-n}]$$

$$H(z) = 0 + \left[h(1)[z^{1}+z^{-1}] + \cancel{h(2)}[z^{2}+z^{-2}] + \cancel{h(3)}[z^{3}+z^{-3}] \right.$$

$$\boxed{= 0.3969\, z^{1} + 0.3969\, z^{-1}}$$

$$\overline{H(z)} = z^{-3}\, H(z)$$

$$\boxed{\overline{H(z)} = 0.3969\, z^{-2} + 0.3969\, z^{-4}}$$

For Symm Filter

$h(n) = h(N-1-n)$

$h(0) = h(6) = 0$

$h(1) = h(5) = 0.3969$

$h(2) = h(4) = 0$

$h(3) = h(3) = 0$

$h(-1) = h(7) = 0.3969$

$h(-2) = h(8) = 0$

$h(-3) = h(9) = 0$

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$[z^3 + z^{-3}]$

LINEAR PHASE FIR FILTERS

Phase & angle

Freq response $H(e^{j\omega}) = |H(e^{j\omega})| \cdot e^{j\angle H(e^{j\omega})}$

Magnitude

$= |H(e^{j\omega})| \, e^{j\theta(\omega)}$

Phase delay $\tau_p = -\dfrac{\theta(\omega)}{\omega}$ Group delay $\tau_g = -\dfrac{d\,\theta(\omega)}{d\omega}$

Determine the frequency response of FIR filter, defined by $y(n) = 0.25\,x(n) + x(n-1) + 0.25\,x(n-2)$.

Soln.

$z = e^{j\omega}$

Taking Z. Transf.

$Y(z) = 0.25\,X(z) + z^{-1}X(z) + 0.25\,z^{-2}X(z)$

$H(z) = \dfrac{Y(z)}{X(z)} = 0.25 + z^{-1} + 0.25\,z^{-2}$.

The freq response is given by.

$H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = 0.25 + e^{-j\omega} + 0.25\,e^{-2j\omega}$

Bringing $H(e^{jw})$ to the form of $|H(e^{jw})| e^{j\theta(w)}$

$$H(e^{jw}) = e^{-jw}\left[0.25 e^{jw} + 0.25 e^{-jw} + 1\right]$$

$$= e^{-jw}\left[0.25 (e^{jw} + e^{-jw}) + 1\right]$$

$$= e^{-jw}\left[0.25 (2\cos w) + 1\right]$$

$$= \underbrace{e^{-jw}}_{\text{phase ang}}\left[\underbrace{0.5 \cos w + 1}_{\text{magnitd}}\right]$$

Comparing with

$\cancel{H(e^{jw}) = e^{j\theta(w)}}$ $\quad H(e^{jw}) = e^{j\theta(w)} |H(e^{jw})|$

$$\theta(w) = -w.$$

$$\tau_p = \frac{-\theta(w)}{w} = \frac{-(-w)}{w} = 1$$

$$\tau_g = \frac{-d\theta(w)}{dw} = 1.$$

$\boxed{\text{※ If window function not given assume it to be } 1}$

08/10/18

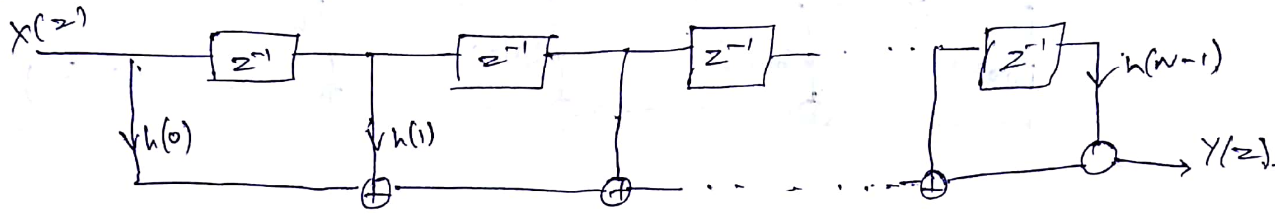
REALIZATION OF FIR Filter.

In general, $H(z) = h(0) + h(1)z^{-1} + h(2)z^{-2} + h(N-1)z^{-(N-1)}$

$$Y(z) = \{h(0) + h(1)z^{-1} + h(2)z^{-2} + \dots + h(N-1)z^{-(N-1)}\} X(z).$$

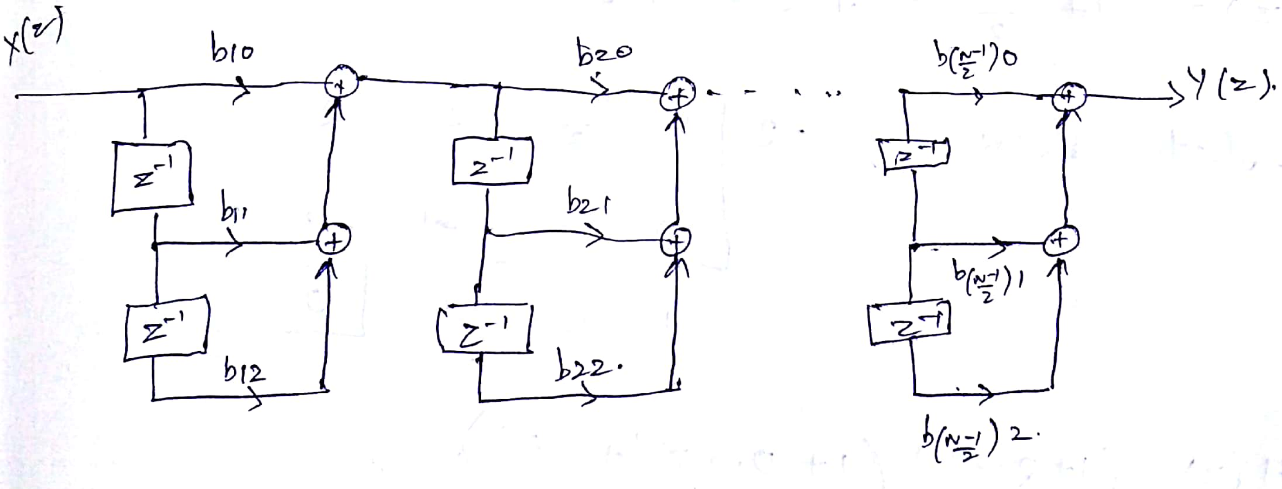
i) Transversal Structure

i) Direct Form.



ii) Cascade Form:

$$H(z) = \prod_{k=1}^{\frac{N-1}{2}} (b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}) = (b_{10} + b_{11}z^{-1} + b_{12}z^{-2})(b_{20} + b_{21}z^{-1} + b_{22}z^{-2}) \dots (b_{(\frac{N-1}{2})0} + b_{(\frac{N-1}{2})1}z^{-1} + b_{(\frac{N-1}{2})2}z^{-2})$$



Scanned by CamScanner

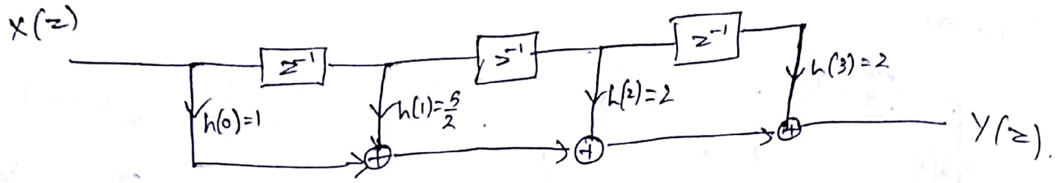
7) Obtain the Direct form, Cascade form realization for the system function using transversal structure

$$H(z) = 1 + \frac{5}{2} z^{-1} + 2 z^{-2} + 2 z^{-3}$$

Soln:

$h(0) = 1 \quad h(1) = \frac{5}{2} \quad h(2) = 2 \quad h(3) = 2.$

DIRECT FORM:



CASCADE FORM:

General form

$$H(z) = \prod_{k=1}^{\frac{N-1}{2}} (b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2}).$$

$$H(z) = 1 + \frac{5}{2} z^{-1} + 2 z^{-2} + 2 z^{-3}$$

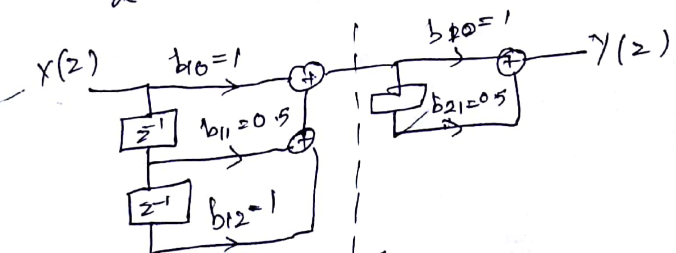
Synthetic division

-2	1	5/2	2	2
	0	-2	-1	-2
	1	0.5	1	0

$$H(z) = (1 + 2z^{-1})(1 + 0.5 z^{-1} + z^{-2}).$$

$b_{\phi 0} = 1 \quad b_{\phi 1} = 2$

$b_{20} = 1 \quad b_{21} = 0.5 \quad b_{22} = 1$

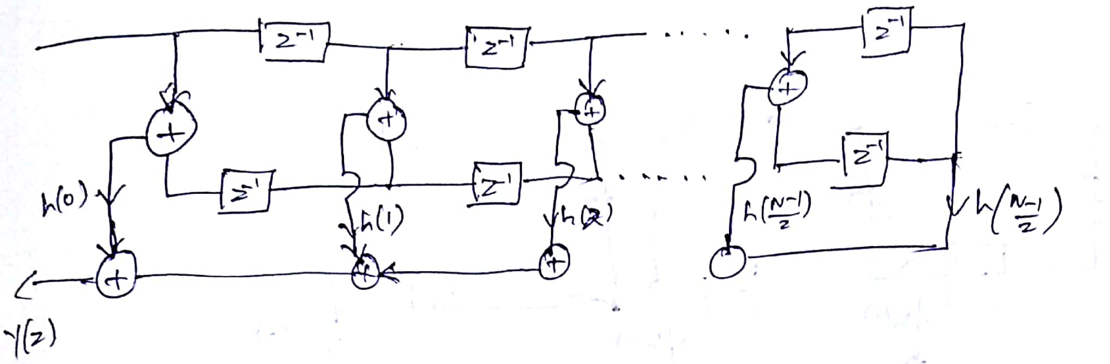


LINEAR PHASE REALIZATION

General form,

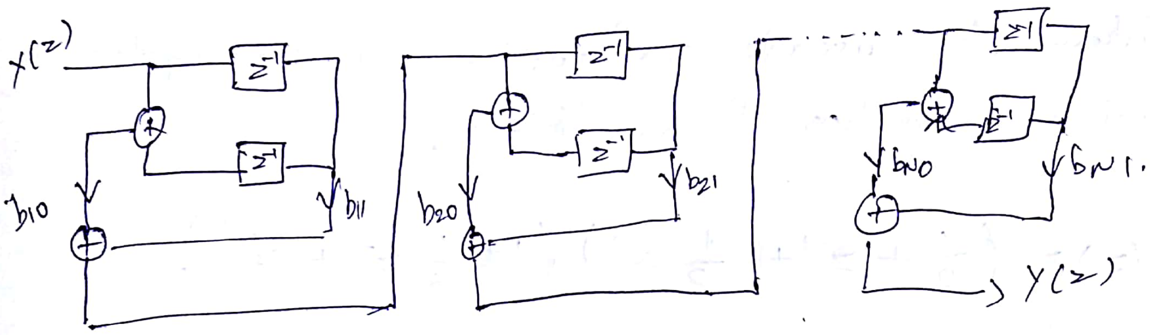
I) Direct form.

$$H(z) = h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n)\left[z^{-n} + z^{-(N-1-n)}\right]$$



II) Cascade Form :

$$H(z) = \prod_{k=1}^{(N-1)/2} \left(b_{k0} + b_{k1} z^{-1} + b_{k0} z^{-2}\right).$$



Realize the system by linear phase realization, using direct form, ~~Cascade form~~

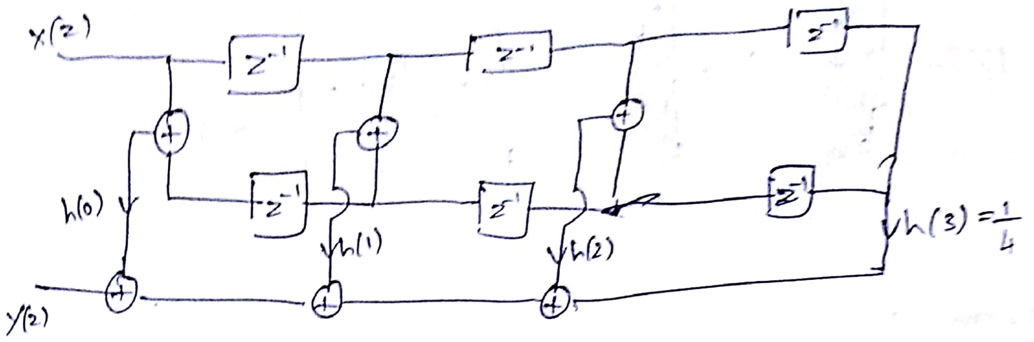
$$H(z) = \frac{1}{2} + \frac{1}{3} z^{-1} + z^{-2} + \frac{1}{4} z^{-3} + z^{-4} + \frac{1}{3} z^{-5} + \frac{1}{2} z^{-6}.$$

$h\left(\frac{N-1}{2}\right) = h(3) = \frac{1}{4}$

$h(2) = 1 = h(4).$

$h(1) = \frac{1}{3} = h(5)$

$h(0) = \frac{1}{2} = h(6)$



Obtain the cascade realization for the system with minimum number of multiplies

$H(z) = \left(\frac{1}{2} + z^{-1} + \frac{1}{2} z^{-2}\right)\left(1 + \frac{1}{3} z^{-1} + z^{-2}\right).$

~~$b_{10} = \frac{1}{2}$ $b_{11} = 1$ $b_{10} = (b_{12})$ ...~~

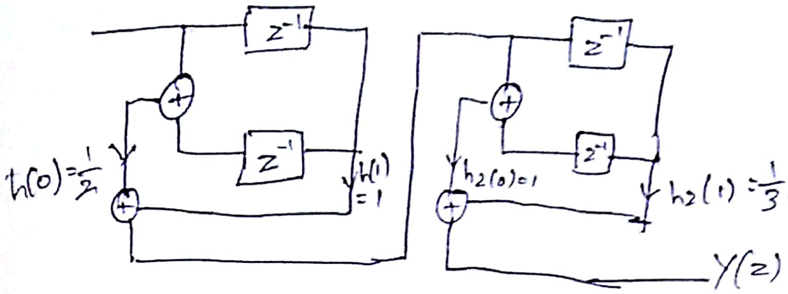
$H(z) = h_1(n)\, h_2(n)$

$h_1 \frac{(N-1)}{2} = h(1) = 1.$

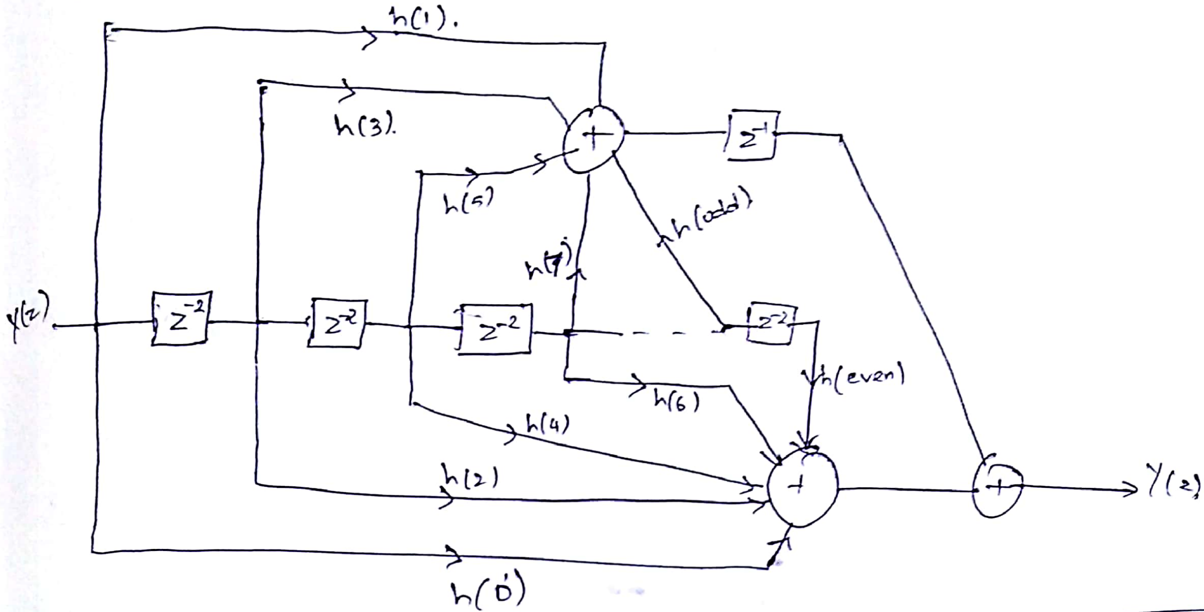
$h(0) = \frac{1}{2} = h(2).$

$h_2\left(\frac{N-1}{2}\right) = h_2(1) = \frac{1}{3}.$

$h_2(0) = 1 = h_2(2).$



Polyphase realization of FIR filter.



Realize the following filter in polyphase realization.

$$H(z) = 1 + 4z^{-1} - 3z^{-2} + 6z^{-3} - 9z^{-4} + 5z^{-5} + 7z^{-6}$$

$h(0)=1$; $h(1)=4$; $h(2)=-3$; $h(3)=6$; $h(4)=-9$; $h(5)=5$; $h(6)=7$