Initial Aerodynamic Blade Design of a Stall regulated Horizontal Axis Wind Turbine using BEM Theory

A project report submitted in the fulfillment of the requirement for the course.

SEE-614 WIND ENERGY

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1 Introduction to Wind Turbine

Wind energy is one of the most important sources of clean, renewable energy. Wind is an inexhaustible resource which can be used to generate electricity without burning fuel and hence does not pollute the environment. Wind turbines are designed to harness power from wind mechanically by using a turbine that spins, the turbine in turn is connected to a power generator which generates electricity.

The objective of this project was to perform "Initial Design of a Stall regulated Horizontal Axis Wind Turbine using BEM Theory". For this purpose, wind profile of 'IEC Class III' [6] was chosen by the team. The target of the team was to perform initial design analysis for a wind turbine such that it was optimized for the average wind speed of the wind profile considered. The team performed analyses for various rotational speeds and chose the design that satisfied the constraints listed below. The choice of airfoils was based on recommendations given by "National Renewable Energy Laboratory" as given in reference [1]. The team chose "Thick-Airfoil Family for Medium Blades" as it was optimized for 'Stall-Regulated Rotors'. The target of the team was to design a wind turbine rated for 100kW.

The team assumed that a minimum chord of 0.25m would be sufficient to accommodate the 'overspeed-control mechanisms' needed for Stall-Regulated Wind turbines. The team assumed that minimum chord of 1m would be necessary at the root to accommodate for the structural aspects of the blade. The team performed analyses for rotational speeds in the range of 30rpm to 90rpm as typical commercially available 100kW wind turbines usually have rotational speed within this range.

Since only initial design analyses were to be performed, the team assumed that the constraints chosen above would be sufficient considering the parameters chosen. The team designed and developed a code using 'MATLAB' software to achieve its target. Key parameters and constrains considered for this project are listed below:

A. Key Parameters
   i. Turbine Type: Stall Regulated
   ii. Target Rated Power: 100 kW.
   iii. Wind Profile: IEC Class III
   iv. Annual Average Wind Speed at hub height, V_{ave}: 7.5ms^{-1}
   v. Number of blades considered, B: 2, 3 and 4
   vi. Number of elements, N: 12
   vii. Hub Radius, R_i: 0.75m
   viii. Radius of each Blade, R: 10m
   ix. Airfoils chosen: Thick-Airfoil Family for Medium Blades
      a. Root Region Airfoil (first 4 elements): NREL S821
      b. Primary Outboard Airfoil (5th to 9th element): NREL S819
      c. Tip Region Airfoil (10th to 12th element): NREL S820

B. Constrains considered:
   i. Minimum Chord Length at root, $c_{r,min}$: 1m
   ii. Minimum Chord Length at tip, $c_{t,min}$: 0.25m
   iii. Minimum Rotational Speed, ω_{min}: 30rpm
   iv. Maximum Rotational Speed, ω_{max}: 90rpm

2 Wind Profile

The team chose to design the wind turbine for 'IEC Class III' wind profile. IEC Class III assumes a wind profile distribution such that it follows Rayleigh probability distribution function at the hub height. The probability distribution function is given by Equation 1 and probability distribution curve is given in Fig. 1. Here the value of V_{ave} is 7.5ms^{-1}. The wind profile can also be defined using Weibull distribution with parameters $C \approx 8.463$ and $k = 2$ as shown below in equation 2. The probability distribution curve is plotted for wind speeds between 2.5ms^{-1} and 20 ms^{-1}.

$$P_R(V) = 1 - e^{-\pi\left(\frac{V}{V_{ave}}\right)^2}$$

(Eq. 1)

$$P_W(V) = 1 - e^{-\left(\frac{V}{C}\right)^k}$$

(Eq. 2)

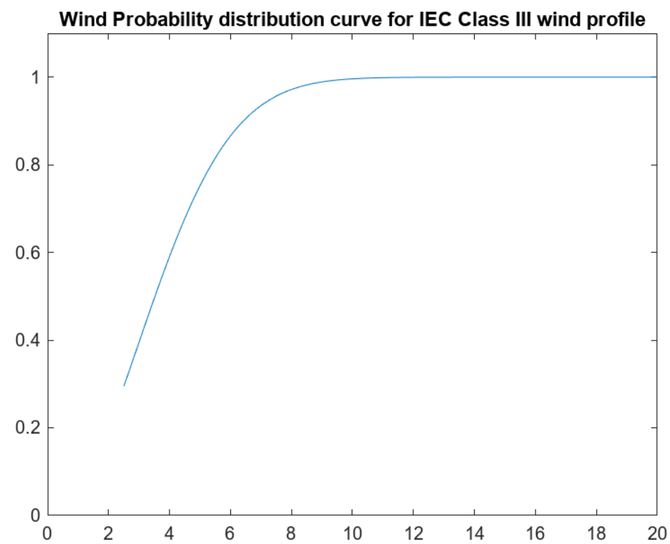

Figure 1: Wind Probability distribution curve for IEC Class III wind profile.

3 Airfoil Selection

The choice of airfoils was based on recommendations given by "National Renewable Energy Laboratory" in reference [1]. The team chose "Thick-Airfoil Family for Medium Blades" as it was optimized for 'Stall-Regulated Rotors'. This family of airfoils were designed in 1993 and have greater tip-region thickness to accommodate overspeed-control mechanisms for stall-regulated rotors at the expense of slightly higher drag. The S821 blade-root airfoil was designed to have a high $C_{l,max}$ which is largely insensitive to roughness effects. The airfoil data was calculated using Xfoil at a Reynold's number of 10^6 and for simplicity of calculations it was assumed that the airfoil data remains constant over the required range of Reynold's number for each airfoil.

Figures 2, 3 and 4 show the airfoil sections of NREL: S821[3], NREL: S819[4] & NREL: S820[5] respectively. Figures 5, 6 and 7 show the 'C_l vs α', 'C_d vs α' & 'C_l/C_d vs α' plots of the airfoils respectively. Based on the recommendation, S821 was chosen as the Root-region airfoil, S819 was chosen as the primary outboard airfoil and S820 was chosen as the tip-region airfoil. During BEM calculations, the design angle of attack, α_d, was selected corresponding to the $(C_l/C_d)_{max}$ value of each airfoil section. For S821, $\alpha_d = 6.5^o$; for S819, $\alpha_d = 7.75^o$ and for S820, $\alpha_d = 5.25^o$ were the corresponding values.

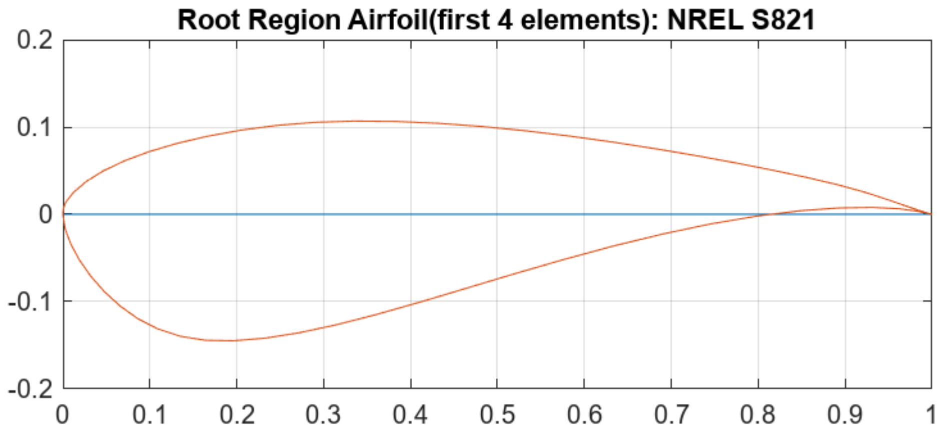

Figure 2: Root Region Airfoil (first 4 elements): NREL S821

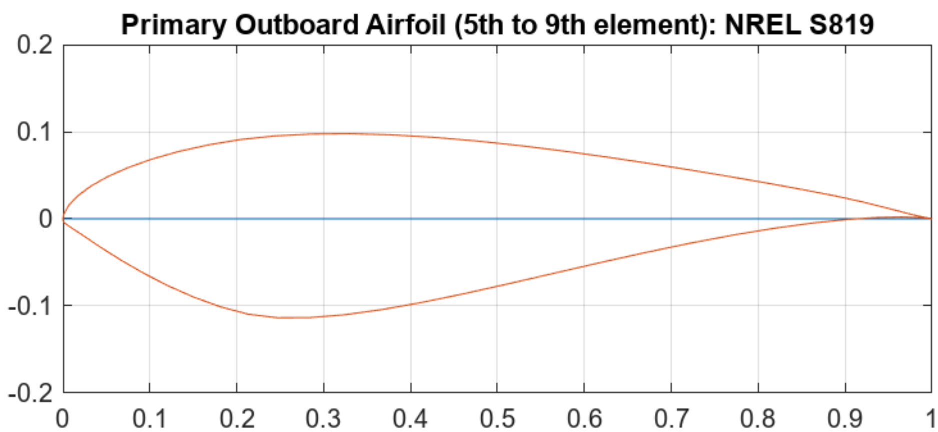

Figure 3: Primary Outboard Airfoil (5th to 9th element): NREL S819

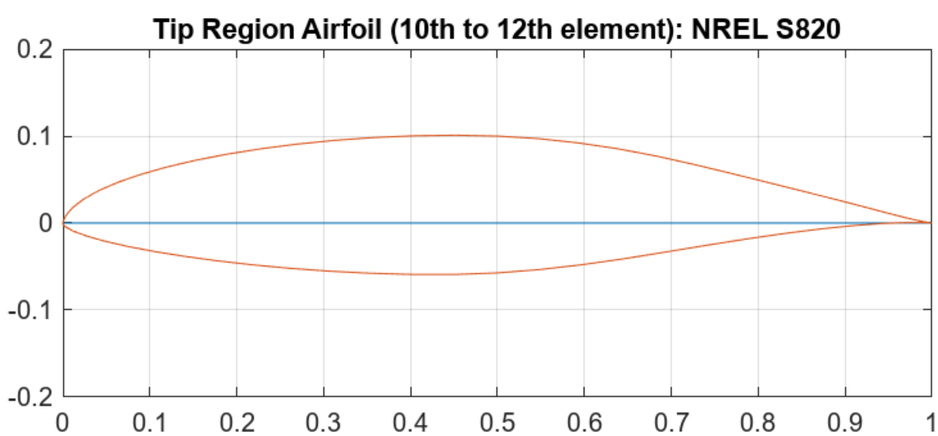

Figure 4: Tip Region Airfoil (10th to 12th element): NREL S820

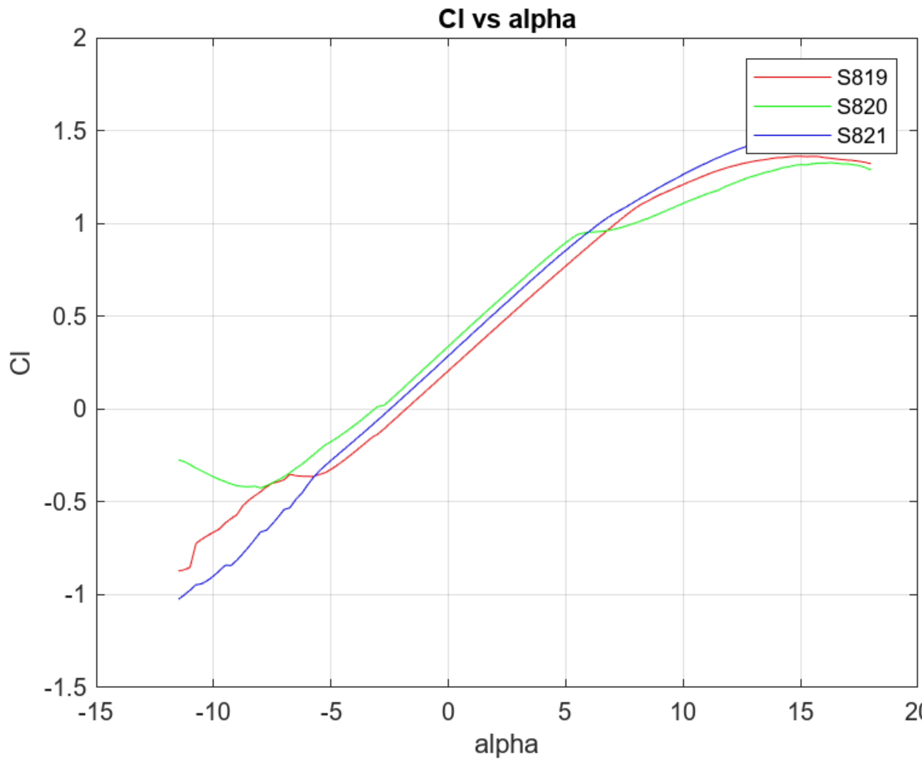

Figure 5: C_l vs alpha of the airfoils.

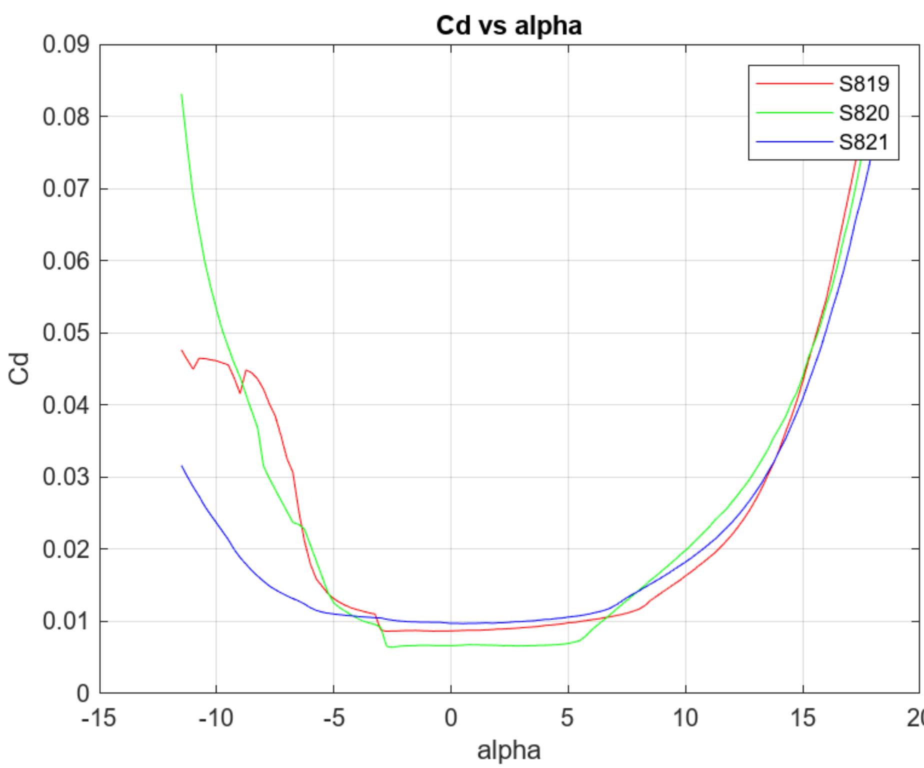

Figure 6: C_d vs alpha of the airfoils.

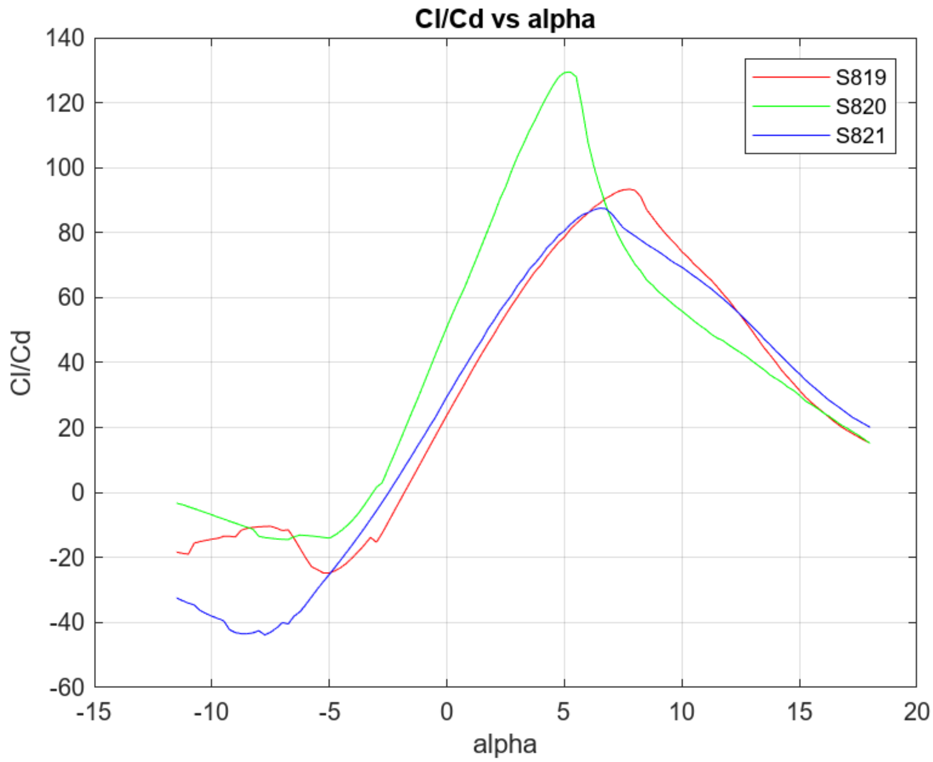

Figure 7: C_l/C_d vs alpha of the airfoils.

4 Aerodynamic Blade Design using BEM Theory [2]

The design of turbine blades depends strongly upon whether the wind turbine would be stall regulated, pitch regulated or variable speed pitch regulated. Irrespective of the type, we initially calculate the power co-efficient for the average wind speed V_{ave} for every element and then we take the average of these values as the required power co-efficient. For each element, the value of Power co-efficient, C_p, is maximized by choosing appropriate values of the chord, c, and local pitch angle, θ.

These calculations were made over a range of different rotational speeds and for different number of blades as shown below. After calculations, the results were compared and the results giving the highest value of power co-efficient were selected for each combination of blades. Finally, the power coefficients of each optimized blade combination are compared, and the most optimized combinations were selected.

Before starting the calculations following parameter selection had to be made by the team:

i. Turbine Type: Stall Regulated
ii. Annual Average Wind Speed at hub height, V_{ave}: 7.5ms^{-1}
iii. Number of blades considered, B: 2, 3 and 4
iv. Number of elements, N: 12
v. Hub Radius, R_i: 0.75m
vi. Radius of each Blade, R: 10m
vii. Airfoils chosen: Thick-Airfoil Family for Medium Blades
   a. NREL S821: $\alpha_{d,1} = 6.50^o$, $(C_l/C_d)_{max,1}$ = 87.54, $C_{l,1}$ = 1.0059 and $C_{d,1}$ = 0.0115
   b. NREL S819: $\alpha_{d,2} = 7.75^o$, $(C_l/C_d)_{max,2}$ = 93.38, $C_{l,2}$ = 1.0627 and $C_{d,2}$ = 0.0114
   c. NREL S820: $\alpha_{d,3} = 5.25^o$, $(C_l/C_d)_{max,3}$ = 129.48, $C_{l,3}$ = 0.9206 and $C_{d,3}$ = 0.0071
viii. Minimum Chord Length at root, $c_{r,min}$: 1m

ix. Minimum Chord Length at tip, $c_{t,min}$: 0.25m
x. Minimum Rotational Speed, ω_{min}: 30rpm
xi. Maximum Rotational Speed, ω_{max}: 90rpm

The Design Algorithm using BEM Theory to design HAWT turbine blade can be summarized as follows for a single blade element:

a. Guess the values of axial induction factor, *a,* and tangential induction factor, *a'*. Usually, they're guessed as 0 and 0 respectively at the start of each iterative calculation.
b. Find the flow angle, ϕ, using equations 3 & 4 as given below:

$$\phi = \tan^{-1}\left(\frac{(1-a)R}{(1+a')\lambda r}\right)$$

(Eq. 3)

$$\lambda = \frac{\omega R}{V_{\text{ave}}}$$

(Eq. 4)

c. Find local pitch angle, θ, using equation 5. Here, the design angle of attack, α_d, is selected corresponding to the $(C_l/C_d)_{max}$ value of the respective airfoil section:

$$\theta = \phi - \alpha_d$$

(Eq. 5)

d. Find Normal force coefficient, C_n, using equation 6 and tangential force co-efficient, C_t using equation 7. Here, C_l and C_d values are selected corresponding to the $(C_l/C_d)_{max}$ value of the respective airfoil section:

$$C_n = C_l.Cos\phi + C_d.Sin\phi$$

(Eq. 6)

$$C_t = C_l.Sin\phi - C_d.Cos\phi$$

(Eq. 7)

e. Find Prandtl's tip loss factor, F, using equation 8. Prandtl's tip loss factor is used to correct the assumption of infinite number of rotors made in 1-D momentum theory. This correction is derived from the 'Prandtl's Lifting Line Theory'; hence, BEM Theory predicts a solution that is very similar to that found using the 'Prandtl's Lifting Line Theory' for a finite number of blades, B.

$$F = \frac{2}{\pi}Cos^{-1}\left(e^{\left(-\frac{B}{2}\right)\left(\frac{R-r}{rSin\phi}\right)}\right)$$

(Eq. 8)

f. Predict new axial induction factor, *a,* and tangential induction factor, *a'*, which are corrected to account for the finite number of blades, B, using equations 9, 10, 11 and 12 or 13. Here, we first find Thrust co-efficient, C_T, using equation 9, where the solidity, σ, is found using equation 10. Then, we find tangential induction factor, *a'*, using equation 11. Finally, we find axial induction factor, *a,* using equation 12 if $a < 0.3$; if $a > 0.3$ then we find *a* using equation 13.

$$C_T = \frac{\sigma(1-a)^2 C_n}{Sin^2\,\phi}$$

(Eq. 9)

$$\sigma = \frac{Bc}{2\pi r}$$

(Eq. 10)

$$a' = \frac{1}{\left(\frac{4F.Sin\phi.Cos\phi}{\sigma.C_t} - 1\right)}$$

(Eq. 11)

$$a(1-a) = \frac{C_T}{4F}$$

(Eq. 12)

$$C_T = 4aF\left(1 - \frac{(5-3a)a}{4}\right)$$

(Eq. 13)

g. Check for convergence of axial induction factor, *a*, and tangential induction factor, *a'*. If not converged, then take the new values of *a* and *a'* found in step 'f' as the new guessed value of *a* and *a'* and recalculate *a* and *a'* until convergence is achieved within a certain tolerance.

h. Find Local Power co-efficient, dC_P, using equation 14.

$$dC_P = \frac{B.\lambda^2(1-a)(1+a').\frac{c}{R}.\frac{r}{R}.C_t}{2\pi.Sin\phi.Cos\phi}$$

(Eq. 14)

After calculating power coefficients for every element, an average of the calculated power coefficients is found, which is the predicted Power co-efficient without 3-D corrections for a Wind turbine with 'B' number of blades.

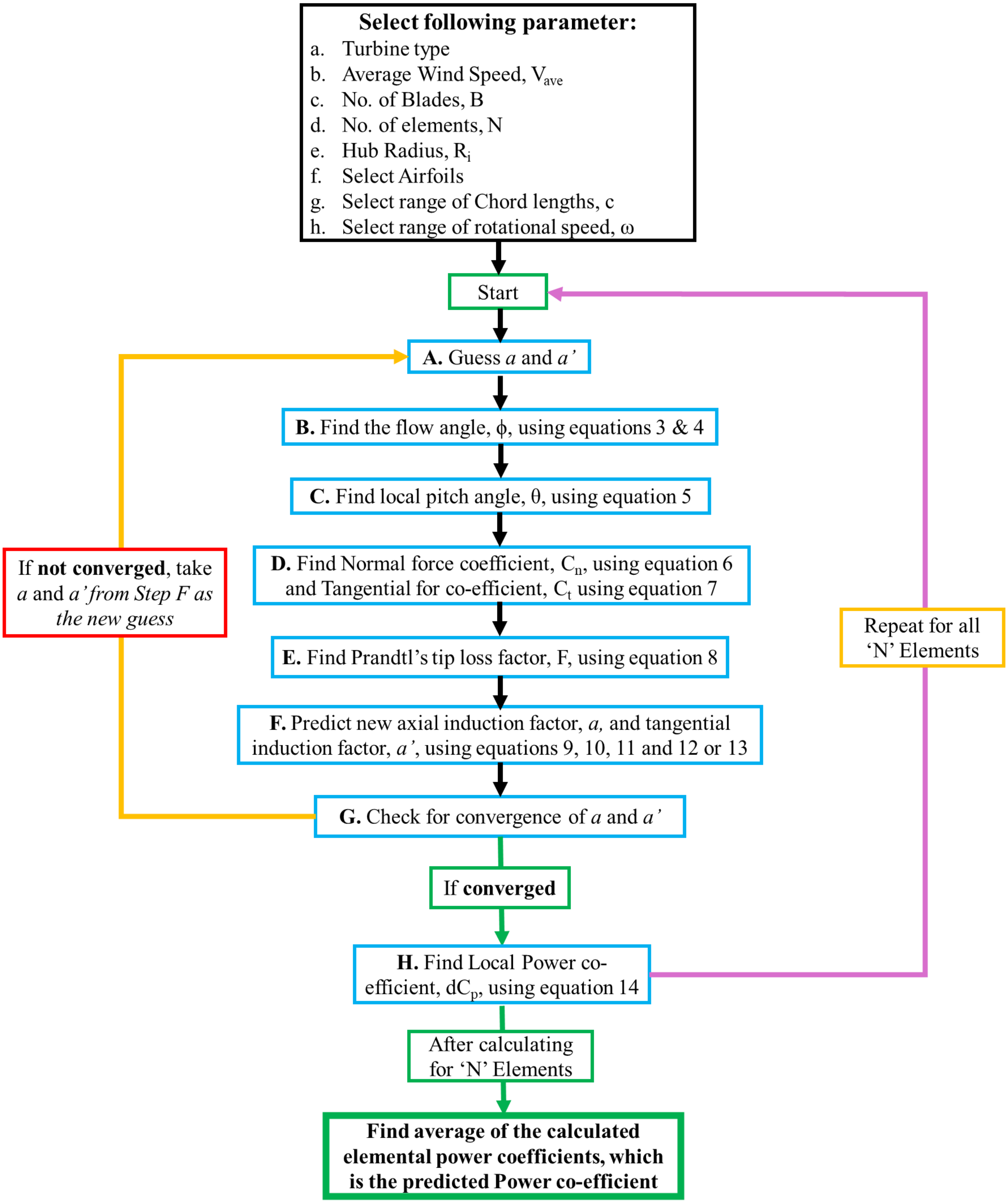

Figure 8: Flow Chart of the Design Algorithm using BEM Theory to design HAWT turbine blade.

5 Results

The calculations were done using a MATLAB code. The calculated average power co-efficients for various rotational speeds and various number of blades have been tabulated in table 1 below. The same has been plotted in Figure 9.

Rotational Speed (in rpm)	Power co-efficient		
	B = 2	B = 3	B = 4
30	0.3352	0.4212	0.4601
35	0.3910	0.4563	0.4823
40	0.4291	0.4787	0.4946
45	0.4553	0.4929	0.5026
50	0.4743	0.5020	0.5087
55	0.4878	0.5081	0.5137
60	0.4977	0.5126	**0.5170**
65	0.5048	0.5162	0.5157
70	0.5101	**0.5183**	0.4998
75	0.5139	0.5171	0.4642
80	0.5166	0.5054	0.4144
85	0.5180	0.4786	0.3588
90	**0.5180**	0.4399	0.3072

Table 1: Calculated average Power co-efficients for various rotational speeds and various number of blades.

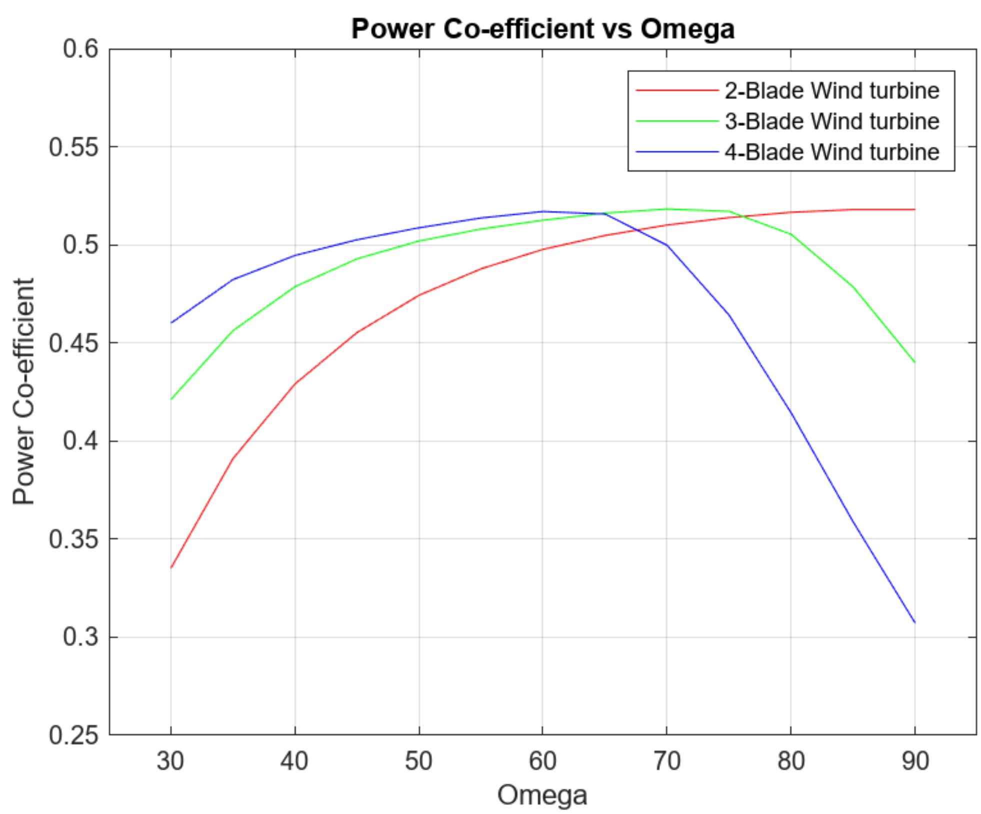

Figure 9: Calculated average Power co-efficients for various rotational speeds and various number of blades.

From Table 1, we can see that the average power co-efficients reach a maximum value of around 0.518 for B = 2, 3 & 4 albeit at different rotational speeds. The maximum value thus reached for each B has been shown in Bold letters. The calculated parameters, i.e. chord length, local pitch angle and the corresponding rotational speeds are shown in tables 2, 3 and 4 below for the optimized '2-Blade', '3-Blade' and '4-Blade' wind turbines respectively.

Rotational Speed (in rpm)	Radial Location (in m)	Elemental Power co-efficient	Local Pitch Angle (in Degrees)	Local Chord (in m)
90	1.1354	0.5339	15.7495	1.2300
	1.9063	0.5676	7.7497	0.8800
	2.6771	0.5783	3.9317	0.6600
	3.4479	0.5835	1.6152	0.5300
	4.2188	0.5861	-1.0039	0.4100
	4.9896	0.5878	-2.0386	0.3500
	5.7604	0.5890	-2.8766	0.3100
	6.5312	0.5895	-3.3790	0.2700
	7.3021	0.5879	-3.9746	0.2500
	8.0729	0.5844	-1.7107	0.2500
	8.8438	0.5515	-2.4621	0.2500
	9.6146	0.3805	-3.6190	0.2500

Table 2: Calculated Parameters for 2-Blade Wind turbine

Rotational Speed (in rpm)	Radial Location (in m)	Elemental Power co-efficient	Local Pitch Angle (in Degrees)	Local Chord (in m)
70	1.1354	0.5067	20.2757	1.1700
	1.9063	0.5543	11.1944	0.9100
	2.6771	0.5714	6.7141	0.7000
	3.4479	0.5786	3.8278	0.5700
	4.2188	0.5835	0.7711	0.4500
	4.9896	0.5853	-0.4243	0.3800
	5.7604	0.5869	-1.5370	0.3400
	6.5312	0.5882	-2.2304	0.3000
	7.3021	0.5878	-2.8295	0.2700
	8.0729	0.5876	-0.7700	0.2800
	8.8438	0.5678	-1.2017	0.2500
	9.6146	0.4255	-2.6770	0.2500

Table 3: Calculated Parameters for 3-Blade Wind turbine

Rotational Speed (in rpm)	Radial Location (in m)	Elemental Power co-efficient	Local Pitch Angle (in Degrees)	Local Chord (in m)
60	1.1354	0.4861	23.2689	1.0600
	1.9063	0.5433	13.7738	0.8700
	2.6771	0.5649	8.7192	0.6900
	3.4479	0.5744	5.4312	0.5700
	4.2188	0.5805	2.1855	0.4500
	4.9896	0.5832	0.6241	0.3900
	5.7604	0.5854	-0.4356	0.3400
	6.5312	0.5869	-1.2371	0.3000
	7.3021	0.5878	-1.9202	0.2700
	8.0729	0.5883	0.0665	0.2800
	8.8438	0.5752	-0.5706	0.2600
	9.6146	0.4506	-2.0139	0.2500

Table 4: Calculated Parameters for 4-Blade Wind turbine

Figure 10 shows the variation of dimensionless chord, c/R vs dimensionless radius, r/R for optimized wind turbine design. Figure 11 shows the variation of Pitch angle, θ(in degrees) vs dimensionless radius, r/R for optimized wind turbine design.

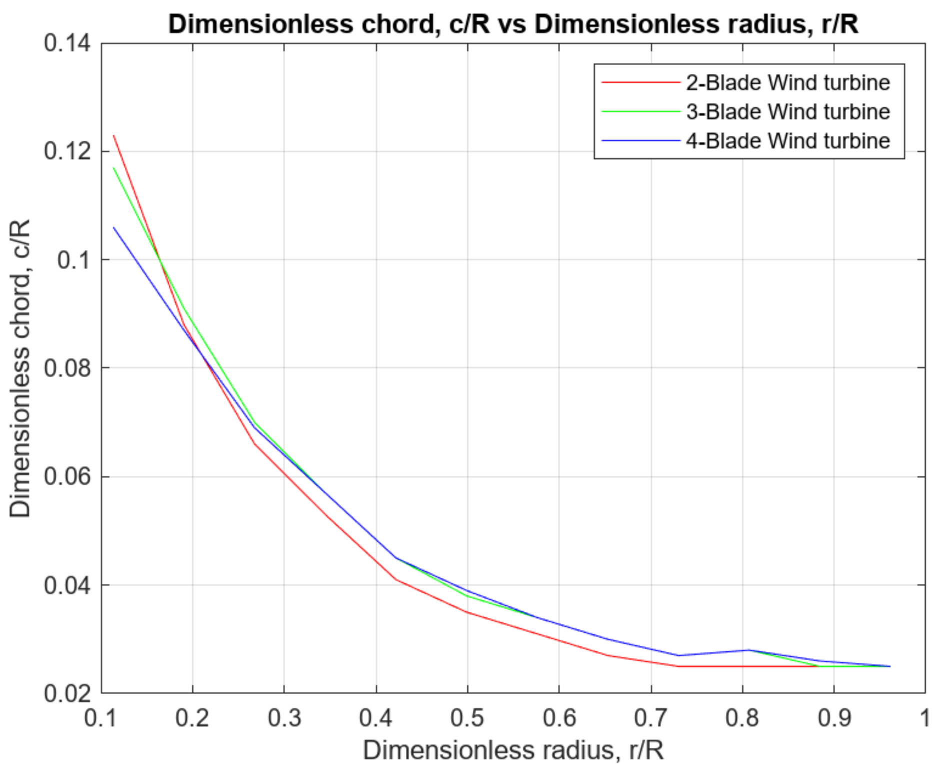

Figure 10: Variation of Dimensionless Chord, c/R vs Dimensionless radius, r/R.

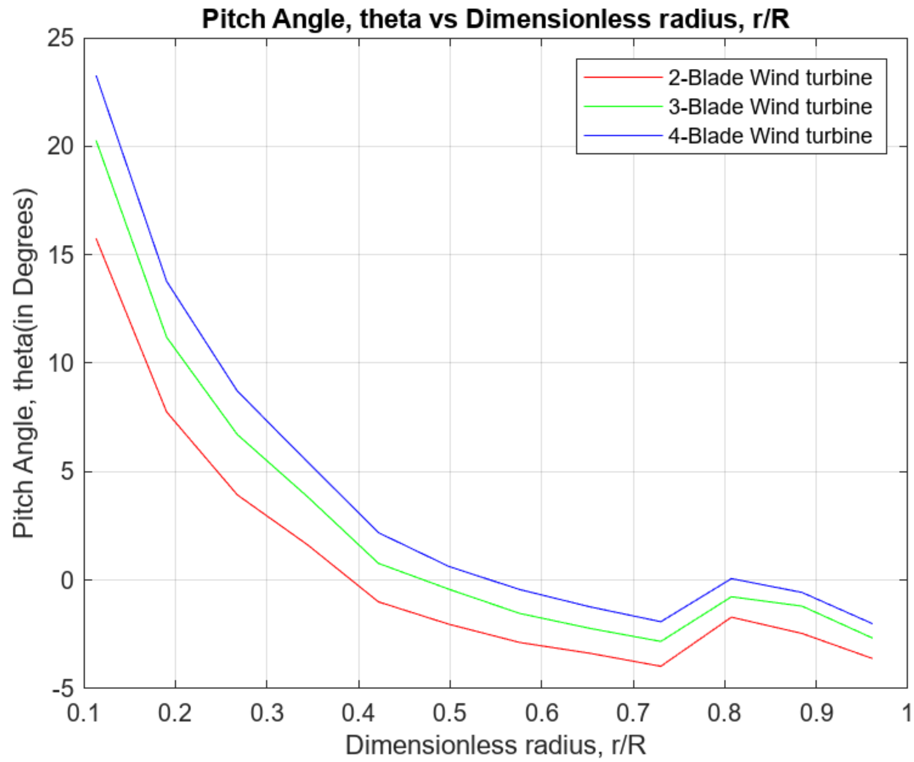

Figure 11: Variation of Pitch angle, θ(in degrees) vs Dimensionless radius, r/R.

6 Conclusion

The initial design calculations made using BEM Theory predict that the designed wind turbines have a power co-efficient that is nearly 87.5% of the Betz Limit(16/27). This would allow us to harness optimal amounts of power from wind. The calculated power co-efficient indicates that the designed blades would be a feasible choice to harness wind energy in conditions where the wind profile is expected to be like 'IEC Class III' wind profile. For a stall-regulated wind turbine, the design will depend strongly on the actual wind distribution, hence knowledge of wind distribution at the installation site is important.

The calculations shown here do not account for the 3-D effects of the turbine, hence these values may not represent actual power co-efficient values of the wind turbine. To predict the power co-efficient with better accuracy, 3D corrections must be incorporated in the design process. However, initial design is mainly used as a starting point of the design process and hence 3-D corrections are unnecessary at this stage of the design process.

7 References

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