

# CS641A: Modern Cryptology

## Assignment 6

objectStrongly

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### Problem statement -

$c(x) =$

588511908193557145472758995584417156637461398472460756192707453386  
5700705569837874063774277536176889970088885808705066261431830544  
3064448898026503556757610342938490741361643696285051867260278567  
89699192735196455737497761964476363322989666851175243222252815921  
401317331985564535161939387143345550581741643299

$n =$

843644437357250348644025545338262791747038934397633433438632603  
427566786092168950937792630288092465059556475721766826694452700  
08816481771701417554768871285020442403001649254405058303439906229  
2019095993486695656975343316520195164095148002658873885392833810  
53937433496994442146419682027649079704982600857517093

$e = 5$

$c(x) = x^e \pmod n$

find  $x$  ?

### Solution Proposal

- We first noticed that the value of  $e$  is small.
- We were not completely aware of the secret message i.e. we had a certain clue with a good probability of what might have been the secret padding.
- Thus the RSA mode we had was a bit relaxed in terms that we knew with certain probability its first few starting bits.
- So we tried breaking the cipher using Coppersmith attack.

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## Coppersmith's Theorem<sup>1</sup>

Given an integer  $N$  and a monic polynomial  $F$  of degree  $d$  over integers, set  $X = N^{1/d + \epsilon}$  for  $1/d + \epsilon > 0$  then we can find all the  $x < X$  such that  $f(x) \equiv 0 \pmod{N}$ .

- By knowing the padding, our RSA model can be resolved into “ $f(x) = (m+x)^5 - c$ ” In which the known part of the message is ‘m’ and the unknown part is ‘x’.
- If  $x < N^{1/5}$ , we will find the required password as the root of the polynomial  $(m+x)^5 \equiv c \pmod{N}$ .
- To solve this above equation we made use of the code available on this Github Repository<sup>2</sup>

## Working with the code<sup>3</sup>

- Coppersmith's LLL attack states that the length of unknown part of the message has to be maximum of  $N^{1/e}$ . As the length of  $N$  is 308 decimal length (equivalent to 1024 bits in binary) and  $e=5$ , length of ‘x’ should be less than one-fifth of 1024 i.e; around 200 bits. So we tried the attack for each possible length of ‘x’.
- At first we guessed a padding text which we converted into binary using the `ascii_to_bin` utility and now we solve for the value of ‘x’ using `as6_RSA.py` provided in the assignment package.
- We tried with several paddings but none of them returned a value for x. After a few more tries, we found a padding that returned some value of x and that padding was :

“This door has RSA encryption with exponent 5 and the password is ”

- This padding returned the length of unknown message ‘x’ to be 72; and gave the value of ‘x’ to be: 2147562143725930046825

Binary form of x is: 1110100 01101011 01101001 01100111 01110010 01100100  
01110010 01100101 01101001

ASCII form of x is: “tkigrdrei”

Password to the next round : **tkigrdrei**

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<sup>1</sup> Source [https://en.wikipedia.org/wiki/Coppersmith%27s\\_attack](https://en.wikipedia.org/wiki/Coppersmith%27s_attack)

<sup>2</sup> Source (<https://github.com/mimoo/RSA-and-LLL-attacks/blob/master/coppersmith.sage>)

<sup>3</sup> To run the code install sageMath and run the `as6_RSA.py` on it.