

BMTH120

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Introduction

Welcome to **Business Math 120**, a concise and practical guide designed to help students excel in their exams and quizzes. This book addresses common challenges faced by students: in class, questions are often complex, critical testing points are difficult to summarize, and professors may lack time to provide one-on-one guidance. To bridge this gap, I created this book to empower students to learn independently.

Key Features of This Book:

- **Example and Formula-Based:** Every concept is explained through practical examples and formulas. There are no unnecessary words or distractions—only what you need to succeed.
- **Based on Real Teaching Experience:** This book is built on my experience teaching over a hundred Business Math 120 students. Every example aligns with test scenarios but with different numbers, ensuring direct relevance to your assessments.
- **High-School Knowledge Friendly:** No advanced math knowledge is required. The book is accessible even for students who feel less confident in math.
- **Short and Efficient:** The book contains 9 units across approximately 40 pages. Unlike traditional textbooks, it's designed for quick learning and review. You can master the course content in 5 hours or less.
- **Open Source and Editable:** This is just the first version of the book. It is open source, allowing anyone to contribute edits or improvements in the future.
- **For Future Tutors and Learners:** Math tutors, especially future Humber Math and Writing Centre co-ops, can use this book to quickly familiarize themselves with the course material.

This book is designed to be a fast, effective, and student-centered tool for success in Business Math 120.

Unit 1

Computing Future Value

The most common and fundamental type of question in BMTH 120 involves calculating the **Future Value**. Let's begin with a straightforward example to help you understand the basic concept and see how this calculation works.

Example

Sarah invested \$25,000 for 5 years at 6.8% compounded monthly.

- (a) How much was the accumulated value after 5 years?
- (b) How much interest was earned during the investment term?

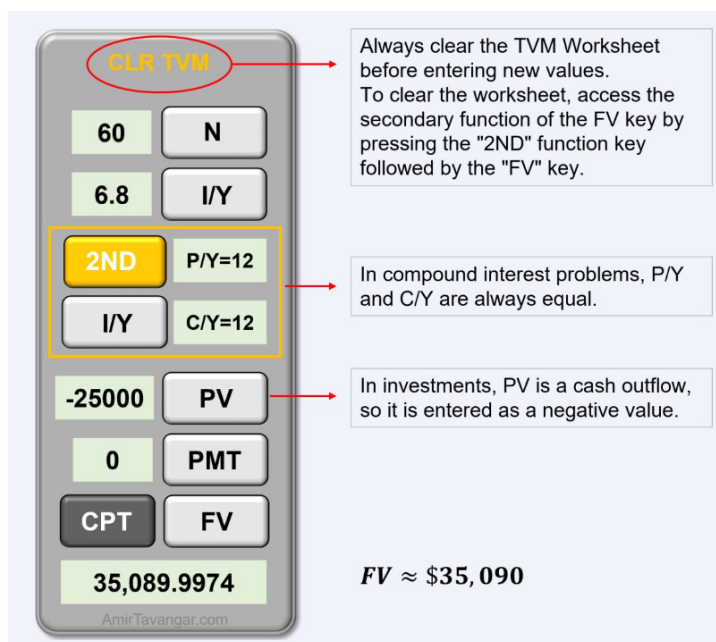
To solve this question, it is essential to first recognize that every Business Math problem involves **eight** fundamental elements:

Fundamental Elements

- **PV**: Present Value
 - **FV**: Future Value
 - **I/Y**: Nominal interest rate
 - **C/Y**: Compounding Periods per Year
 - **P/Y**: Payment Periods per Year
 - **t**: Investment Term
 - **N**: Number of compounding periods in the term
 - **PMT**: Payment
-
- **PV** = -25000 (negative since Sarah is giving out money)
 - **FV** = ? (Unfortunately, we don't know this)
 - **I/Y** = 6.8%
 - **C/Y** = 12 (Monthly means compounding 12 times in one year)
 - **P/Y** = 12 (When **PMT** is zero, **P/Y** always equals to **C/Y**)

- $t = 5$
- $N = C/Y \times t = 12 \times 5 = 60$
- $PMT = 0$ (Because there are no payments)

Now, it's time to use the BA-II calculator to solve for the Future Value!



This guide may be somewhat complex, so an explanation is provided below:

Begin by examining your calculator. Notice the small yellow labels above each button. You can access these functions by pressing the yellow '2ND' button, located at the top left of the calculator, followed by pressing the button corresponding to the desired yellow label.

1. Press 2ND and CLR TVM button.
2. Now, enter the values for N and I/Y . Note that you must first input the number and then select the corresponding function and press enter. For example, enter 60 and then press N . There is no need to clear anything—simply enter 6.8 and press I/Y and press enter.
3. For P/Y and C/Y , the process is slightly different. Instead of entering the value first, press 2ND and then I/Y to access P/Y . You can now input the desired value for P/Y . Next, press the down arrow key, and you will notice that C/Y is automatically set to match the value of P/Y . This occurs because this is a compounding problem, and there is no PMT . Always remember to press 'Enter' to confirm your input and see the equal sign. Then we will need to quite the worksheet, so press 2ND and *Quit*
4. For PV and PMT , input them like what you did to N

5. The final step is to calculate **FV** (Future Value). Press the "CPT" (Compute) button, followed by the "FV" button. The calculator will then display the value of **FV**.

Formula for Interest

$$I = FV - PV$$

Now you have grasped a general approach to solving these types of questions. Note that some of the eight fundamental elements may be presented in different formats. Several examples are provided below:

1. Different terms for **P/Y**

| Payment Interval | Length of Payment Period | Number of Payments per Year (P/Y) |
|------------------|--------------------------|-----------------------------------|
| Annually | Every 12 months (1 year) | 1 |
| Semi-annually | Every 6 months | 2 |
| Quarterly | Every 3 months | 4 |
| Monthly | Every month | 12 |
| Bi-weekly | Every two weeks | 26 |
| Weekly | Every week | 52 |

2. **t** is given in years and months or years and days.
Assume given *X* years and *Y* months, then

Formula for *t*

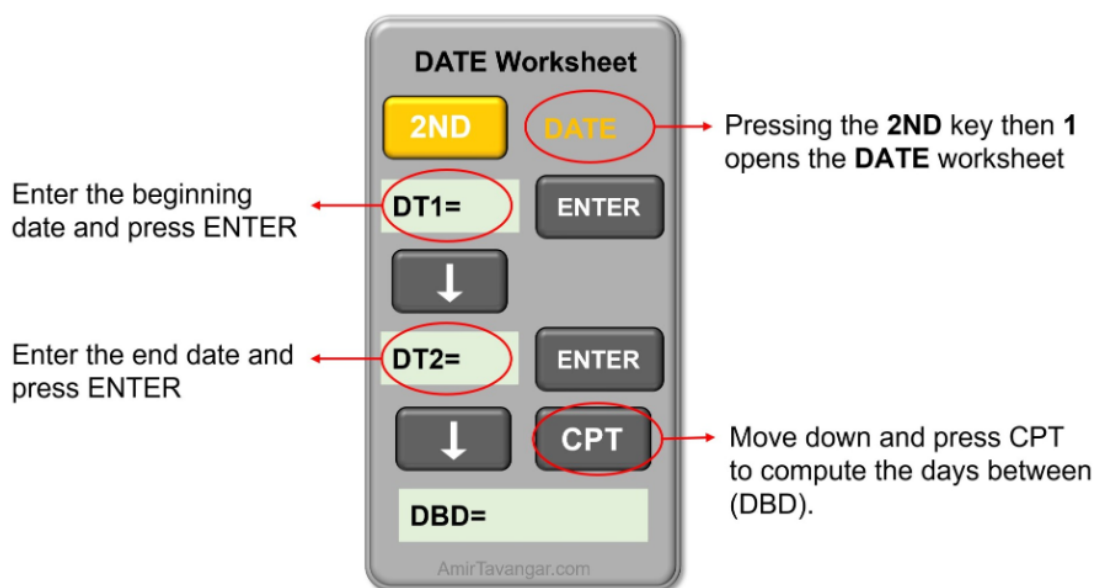
$$t = X + \frac{Y}{12}$$

Assume given *X* years and *Y* days, then

Formula for *t*

$$t = X + \frac{Y}{365}$$

3. **t** is given by two specific dates, for example, 08/2012 to 09/2021. We will use the calculator to determine the number of days between these dates, so manual counting is not necessary.



How to Calculate Days Between Dates (DBD) using a Calculator

1. Activate the DATE worksheet by pressing the "2ND" function key followed by "1".
 2. Input the initial date (DT1) using the **MM.DDYY** format. This means you should type the month (MM) followed by a decimal point, then the day (DD) as a two-digit number, and the last two digits of the year (YY). For instance, enter "11.0308" for November 3, 2008. After pressing "ENTER", the calculator will display the date in the MM-DD-YYYY format.
- Remember to use two digits for the day (DD) even if it's a single-digit day. For example, input "03" for the third day of the month.
3. Navigate to the second date (DT2) using the down arrow key, and enter the second date in the same MM.DDYY format, and then press "ENTER".
 4. To calculate the number of days between the two dates, move down to "DBD" with the down arrow key and press the "CPT" (compute) button to get the result.

► [Watch Video](#)

After we get **DBD** it is not over! We'll have to use the formula above to get **t**

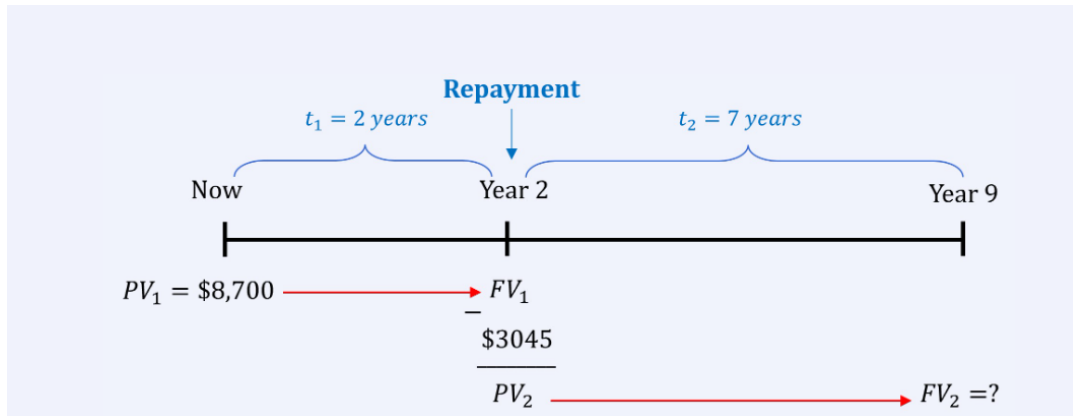
Formula for **t**

$$t = \frac{DBD}{365}$$

4. **t** has two value(two compounding periods)

Example

Latasha initially borrowed 8,700 from RBC Bank at 3.59% compounded monthly. After 2 years she repaid \$3,045. If she pays off the debt 9 years after the \$8,700 was initially borrowed, how much should her final payment be to clear the debt completely?



First period: 4 years

- Present value (cash outflow): $PV_1 = -\$84,000$
- Nominal interest rate: $I/Y_1 = 6.5\%$
- Interest is compounded semi-annually so $C/Y_1 = 2$
- For compound problems, $P/Y = C/Y$, so $P/Y_1 = 2$
- Time period: $t_1 = 4$ years
- Number of compounding periods in the term: $N_1 = C/Y_1 \cdot t_1 = 2(4) = 8$

Second period (after the rate change): 3 years

- Present value (cash outflow): $PV_2 = FV_1 = -108,492.513$
- Nominal interest rate: $I/Y_2 = 4.44\%$
- Interest is compounded semi-annually so $C/Y_2 = 4$
- For compound problems, $P/Y = C/Y$, so $P/Y_2 = 4$
- Time period: $t_2 = 3$ years
- Number of compounding periods in the term: $N_2 = C/Y_2 \cdot t_2 = 4(3) = 12$

a) Compute FV

Computing FV_1

CLR TVM

8 N

6.5 I/Y

2ND P/Y=2

I/Y C/Y=2

-84000 PV

0 PMT

CPT FV

108,492.513

AmirTavangar.com

Computing FV_2

CLR TVM

12 N

4.44 I/Y

2ND P/Y=4

I/Y C/Y=4

-108,492.513 PV

0 PMT

CPT FV

123,859.4346

AmirTavangar.com

$FV_2 \approx \$123,859.43$

Note that your second PV is your first FV .

b) The total interest earned during the 7-year term is the difference between the original principal (PV_1) and the final future value (FV_2):

$$I = FV_2 - PV_1$$

$$I = 123,859.43 - 84,000$$

$$= \$39,859.43$$

Unit 2

Calculating Elements Other Than Future Value

1. Computing PV

To calculate the PV value, enter all the other elements into the calculator in any sequence. The final step is to press **CPT** followed by PV . Remember, when determining any element, always input all known values first, and then press **CPT** for the element you want to calculate.

2. Computing I/Y , N

Same as above

3. Computing t

Formula for t

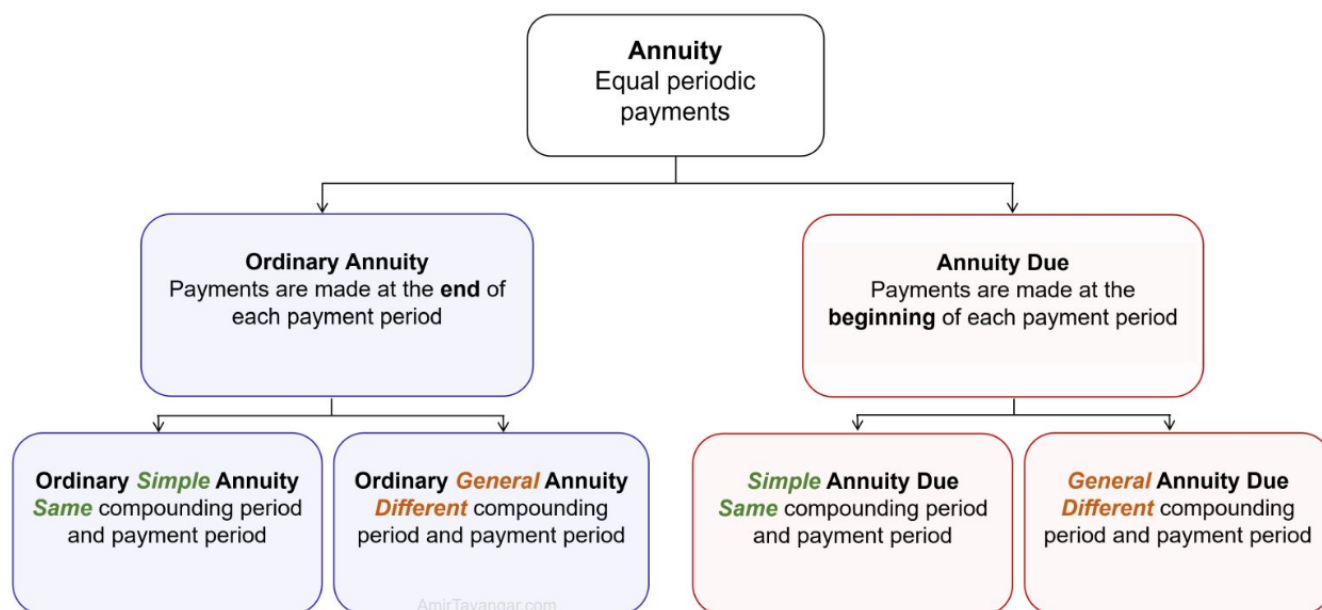
$$t = \frac{N}{C/Y}$$

According to the formula, we first need to **CPT** N and then use the formula to determine the value of t .

Unit 3

Annuities: Concepts, Future Value calculations

There are 4 kinds of Annuities



Same compounding period and payment period means $P/Y = C/Y$
 Different compounding period and payment period means $P/Y \neq C/Y$

When $P/Y \neq C/Y$, we have a new formula for N

Formula for N

$$N = P/Y \times t$$

For annuity problems, you will usually have a **PMT** value, which represents the payment amount for each period. The term P/Y indicates the number of payments made in one year. Therefore, P/Y and C/Y might differ in this case. To determine P/Y , follow the same process as for C/Y . Look for keywords such as "paying X money monthly, quarterly,..." to identify the frequency of payments.

Note that **PMT** is usually a negative value because it represents money being paid.

Additionally, before entering all the elements into the calculator, you must determine whether the problem involves an Ordinary Annuity or an Annuity Due. Then, set your calculator to the appropriate mode: **END** for Ordinary Annuity or **BGN** for Annuity Due. Here is an example:

Example

Dario recently retired and has \$500,000 in his retirement fund, which earns an annual interest rate of 8% compounded annually. He intends to withdraw \$4,000 at the beginning of every month.

- (a) Calculate the total number of withdrawals Dario will make.
- (b) How long will it take for the fund to be depleted?

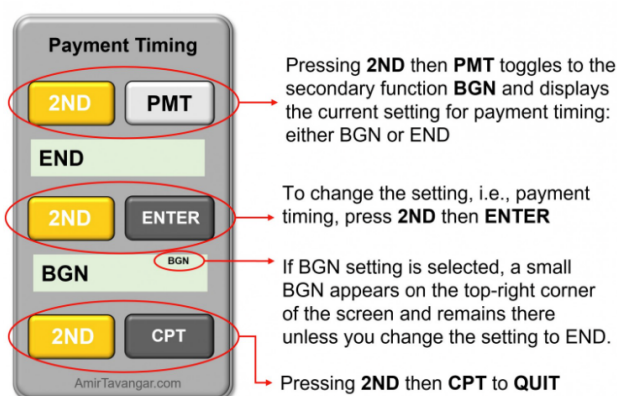
Given Information

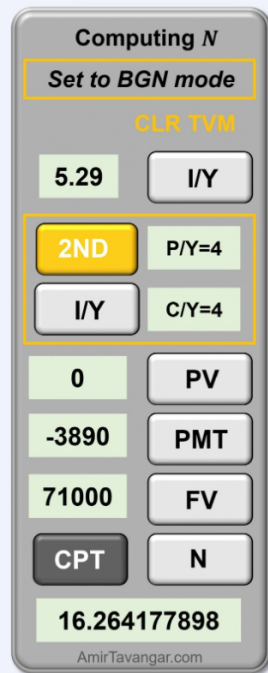
- Annuity Due: Set your calculator to BGN mode
- Interest is compounded annually so $C/Y = 1$
- Payments are withdrawn at the beginning of every month so $P/Y = 12$

$$C/Y \neq P/Y \Rightarrow \text{General Annuity Due}$$

- Nominal interest rate: $I/Y = 8\%$
- Payments are received, so they are cash inflow: $PMT = \$4,000$
- Present value is invested in a bank, so it is a cash outflow: $PV = -\$500,000$
- The fund balance at the end of the term will be zero, so $FV = 0$

How to set your calculator to **BGN** mode? Here is the guide:



a) $N = ?$ *Rounded up*

$$N = 17$$

The number of payments in the term, N, is rounded up to 17 deposits.

b) $t = ?$

To find the time period of the term, we substitute N and P/Y into Formula [3.1b](#):

$$t = \frac{N}{P/Y}$$

$$t = \frac{17}{4}$$

$$= 4.25 \text{ years}$$

$$= 4 \text{ years} + 0.25(12) \text{ months}$$

$$= 4 \text{ years and } 3 \text{ months}$$

Introducing two formulas for Annuity Problems:

Formula for sum of all payments

$$\text{Sum of all payments} = N \times PMT$$

Formula for Interest(I)

$$I = FV - (N \times PMT) - PV$$

Here is an example:

Example

Jayceon purchased a car by making a down payment of \$6,000 and monthly payments of \$425 at the end of every month for 5 years.

- (a) If interest was 2.95% compounded monthly, what was the purchase price of the car?
- (b) What was the cost of financing?

Round all answers to two decimal places if necessary. Enter only positive values for the "Purchase Price of the Car", and "Cost of Financing".

Hint for solution

1. Calculate **PV**
2. Purchase price = **PV** + Down payment
3. Cost of Financing = **I** = $FV - (N \times PMT) - PV$

You also might see other terms instead of **Sum of all payments**; other terms with the same meaning include: **total amount paid**.

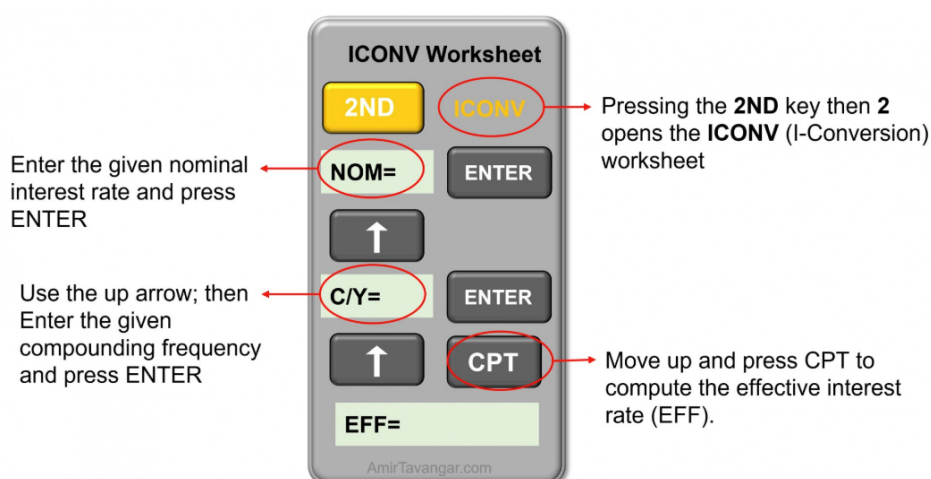
Unit 4

Computing the Periodic and Nominal Interest Rates

Formula for periodic interest rate

$$i = \frac{I/Y}{C/Y}$$

In this unit, you will learn how to determine the **EFF** (Effective Interest Rate) and **NOM** (Nominal Interest Rate). Here is a general guide:



You will use the ICONV worksheet to solve questions.

Example

Convert the interest rate of 7.42% compounded quarterly to an equivalent interest rate compounded monthly. Express your answer as a percent rounded to two decimal places.

solution:

Given information:

- Original compounded frequency is quarterly so $C/Y_1 = 4$
- Original nominal interest rate: $NOM_1 = 7.42\%$
- Desired compounded frequency is monthly so $C/Y_2 = 12$

The diagram illustrates a two-step process on a financial calculator to find the nominal rate compounded monthly from a given effective rate.

Screen 1: Finding EFF

- Buttons: **2ND** (yellow), **ICONV** (yellow)
- Display: **NOM=7.42**
- Buttons: **ENTER**, **↑**
- Display: **C/Y=4**
- Buttons: **ENTER**, **↑**, **CPT**
- Display: **EFF=7.62902658**

Screen 2: Finding NOM_2

- Buttons: **↓**
- Display: **EFF=7.62902658**
- Buttons: **C/Y=12**, **ENTER**
- Buttons: **↓**, **CPT**
- Display: **NOM=7.3745867**

Final Result: $NOM_2 \approx 7.37\%$ Compounded monthly

For this question, the key step is to use **EFF** twice. First, calculate the **EFF**, then adjust **C/Y** and press **CPT** to determine the new **NOM**, which will be the final answer.

Unit 5

Amortization of Loans and Mortgages

In this unit, you will use tables to perform amortization calculations. During quizzes, you will be required to complete missing entries in these tables. The structure of the table is as follows:

| Payment Number | Payment Amount PMT (\$) | Interest Portion INT (\$) | Principal Portion PRN (\$) | Loan Balance BAL (\$) |
|----------------|----------------------------|------------------------------|-------------------------------|--------------------------|
| 0 | n/a | n/a | n/a | Loan Amount (PV) |
| 1 | PMT | INT_1 | PRN_1 | BAL_1 |
| 2 | PMT | INT_2 | PRN_2 | BAL_2 |
| \vdots | \vdots | \vdots | \vdots | \vdots |
| N-1 | PMT | INT_{N-1} | PRN_{N-1} | BAL_{N-1} |
| N | PMT_N | INT_N | PRN_N | 0 |
| Totals | Total Amount Paid | Total Interest Portion | Total Principal Portion | n/a |

Each payment number corresponds to the following values: **PMT** (payment amount), **INT** (interest portion), **PRN** (principal portion), and **BAL** (remaining balance). The **payment number** refers to the sequence of payments, such as the first, second, third, and so on, up to the n -th payment.

For PMT, INT, PRN and BAL, we have several formulas:

Formula for PMT

For the final payment: $PMT_N = PRN_N + INT_N$
 $PRN_N = BAL_{N-1}$

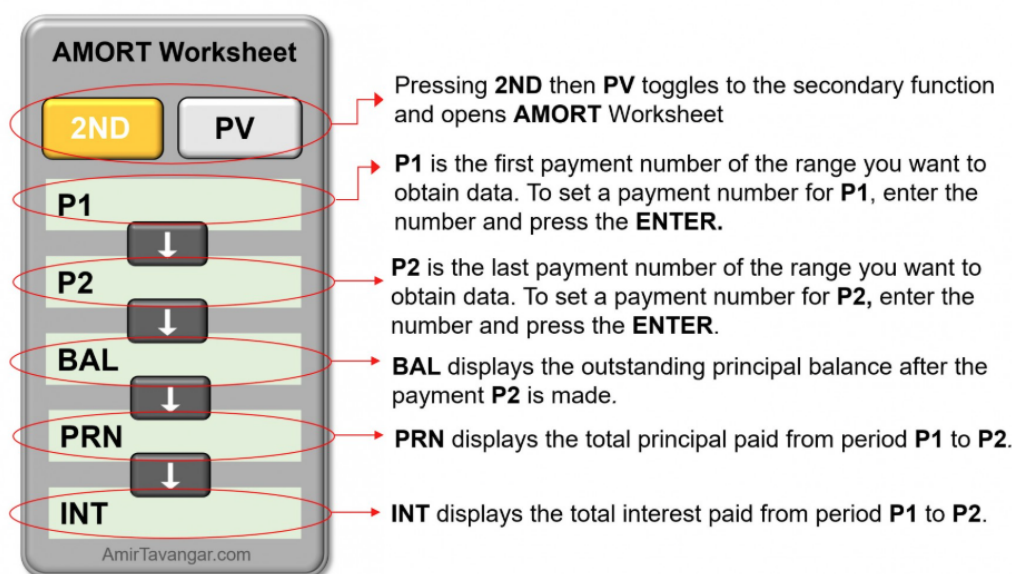
Formula for Total Principle Portion

Total Principle Portion = Original Loan Amount

Formula for Total Amount Paid

Total Amount Paid = Total Interest Portion + Total Principle Portion

For PMT, INT, PRN and BAL besides the final payment, you will have to use calculator to solve these values. Specifically, we will use **AMORT worksheet**



Once you input the correct values for P_1 and P_2 , press the down arrow key. The calculator will automatically compute the **BAL**, **PRN**, and **INT** for you.

Finding P_1 and P_2 can be challenging. Typically, there are two types of P_1 and P_2 :

1. **For a Specific Payment:** To find the interest and principal portion of a specific payment, in the AMORT worksheet, set both P_1 and P_2 to that **specific payment number**. Then, review the details provided to see the breakdown of interest and principal for that specific payment.
2. **For a Specific Period:** To calculate the total principal or interest paid during a specific period of a loan, such as the X th year (where X is a positive integer, e.g., the 4th year or the 9999th year), two formulas are required to determine P_1 and P_2 :


Formulas for P_1 and P_2


$$P_2 = \text{loan time} \times P/Y$$

$$P_1 = P_2 - P/Y + 1$$

Here are some examples:

Chad has a mortgage of \$416,000 through his bank for property purchased. The loan is repaid by end of month payments of \$2,378.57 with an interest rate of 3.34% compounded monthly over 20 years. What is the interest paid in the 4th year of the mortgage? **Enter a POSITIVE VALUE for the answer, rounded to two decimal places.**

$P_1 =$  $P_2 =$ 

Interest paid in 4th year = \$  (enter a **positive** value)

solution

For this question, we will use the formulas for P_1 and P_2 to calculate their values since we are given a period rather than a specific payment. The loan period is the 4th year, so we substitute 4 into the formulas:

$$P_2 = 4 \times 12 = 48$$

$$P_1 = 48 - 12 + 1 = 37$$

Input the values for FV , PV , PMT , I/Y , P/Y , and C/Y into your calculator. Open the **AMORT** worksheet, enter P_1 and P_2 , and press the down key. The calculator will display the interest for the specified period.

Another example is here:

Example

Hakeem has a mortgage of \$514,000 through his bank for a property purchase. The loan is repaid through end-of-month payments of \$4,689.92 with an interest rate of 6.58% compounded monthly over 14 years. What is the interest included in the 31st payment of the mortgage? Enter a **positive value** rounded to two decimal places.


Solution

For this question, we have a specific payment number. Therefore, we set P_1 and P_2 to the same payment number, which is 31. The next step is to input all the other values into your calculator, open the **AMORT** worksheet, and enter P_1 and P_2 to find the interest.

After calculation, the final answer is 2,484.80






Sometimes you need to calculate N or PMT before you calculate PRN , BAL and INT , here is an example:

A **\$23,960** loan is to be settled by making payments of **\$6,698** at the end of every three months. The interest is **7.36%** compounded semi-annually.

a) Find the number of payments in the term. $N =$ 

b) Fill in the missing values of the amortization schedule below.

Round off your answers to two decimal places. Enter a positive value for all answers.

| Payment Number | Payment Amount(\$) PMT | Interest Portion (\$) INT | Principal Portion (\$) PRN | Loan Balance (\$) BAL |
|----------------|---|---------------------------|----------------------------|---|
| 0 | | | | \$23,960 |
| 1 | \$ <input type="text"/>  | \$ <input type="text"/> | \$ <input type="text"/> | \$ <input type="text"/> |
| 2 | \$ <input type="text"/>  | \$ <input type="text"/> | \$ <input type="text"/> | \$ <input type="text"/> |
| 3 | \$ <input type="text"/>  | \$ <input type="text"/> | \$ <input type="text"/> | \$ <input type="text"/> |
| 4 | \$ <input type="text"/>  | \$ <input type="text"/> | \$ <input type="text"/> | \$ <input type="text"/>  |

Solution

This question requires you to calculate N first. To do this, use the standard method to input all the other values into your calculator and compute N . Note that when you enter your N value into the blank, always round **up to the next whole number**. Some questions may incorrectly instruct you to round to the nearest whole number, or they may not provide any rounding guidance at all.

After obtaining the rounded N value, it is essential to input the newly rounded N value back into your calculator before opening the AMORT worksheet. This step is critical because failing to do so will result in incorrect answers for all subsequent subquestions.

The same logic applies to any other value. For instance, if the question asks you to compute PMT and round it to a whole number, you must re-enter the rounded PMT into your calculator.

In general, whenever you modify the value of any element, ensure that the updated value is entered into your calculator to maintain accuracy.

Full answer is here:

A **\$23,960** loan is to be settled by making payments of **\$6,698** at the end of every three months. The interest is **7.36%** compounded semi-annually.

a) Find the number of payments in the term. $N =$ ✓

b) Fill in the missing values of the amortization schedule below.

Round off your answers to two decimal places. Enter a positive value for all answers.

| Payment Number | Payment Amount(\$) PMT | Interest Portion (\$) INT | Principal Portion (\$) PRN | Loan Balance (\$) BAL |
|----------------|--|--|--|---|
| 0 | | | | \$23,960 |
| 1 | \$ <input type="text" value="6,698.00"/> ✓ | \$ <input type="text" value="436.88"/> ✓ | \$ <input type="text" value="6,261.12"/> ✓ | \$ <input type="text" value="17,698.88"/> ✓ |
| 2 | \$ <input type="text" value="6,698.00"/> ✓ | \$ <input type="text" value="322.72"/> ✓ | \$ <input type="text" value="6,375.28"/> ✓ | \$ <input type="text" value="11,323.60"/> ✓ |
| 3 | \$ <input type="text" value="6,698.00"/> ✓ | \$ <input type="text" value="206.47"/> ✓ | \$ <input type="text" value="6,491.53"/> ✓ | \$ <input type="text" value="4,832.07"/> ✓ |
| 4 | \$ <input type="text" value="4,920.18"/> ✓ | \$ <input type="text" value="88.11"/> ✓ | \$ <input type="text" value="4,832.07"/> ✓ | \$ <input type="text" value="0"/> ✓ |

Another tip for this unit is to always use a negative PMT . While your calculations might result in a positive PMT , ensure that you add a negative sign when entering PMT into your calculator.

Unit 6

DCF and NPV

For this unit, we will begin with Net Present Value (NPV). The formula for NPV is as follows:

Formula

$$NPV = PV_{\text{Cash inflows}} - PV_{\text{Cash outflows}}$$

The most important part of this unit is to identify what constitutes PV_{inflows} and PV_{outflows} .

Let us begin with some examples, after which we will summarize common inflows and outflows.

Nylah is the financial advisor for her company and is considering the purchase of excavation equipment which will cost \$55,000.

The purchase of this equipment is expected to save her company \$7,105 at the end of every year for 8 years.

At the end of the 8 years, she expects the excavation equipment to have a residual (inflow) value of \$13,400. The company requires a 5% rate of return.

Round **PV** to the nearest cent. Round **NPV** to the nearest whole number.

1) What is the Net Present Value (NPV) of this equipment **investment**?

Cash Inflows

| Cash Inflows | Payments (Savings) | Residual (Inflow) |
|--------------|-------------------------|-------------------------|
| P/Y = | 1 ✓ | 1 ✓ |
| C/Y = | 1 ✓ | 1 ✓ |
| N = | 8 ✓ | 8 ✓ |
| I/Y = | 5 ✓ % | 5 ✓ % |
| PV = | \$ <input type="text"/> | \$ <input type="text"/> |
| PMT = | \$ 7105 ✓ | \$ 0 ✓ |
| FV = | \$ 0 ✓ | \$ 13,400.00 ✓ |

(If the NPV is negative, enter it as a negative number. If the NPV is zero, enter 0.)

NPV = \$ (round to the nearest whole number)

Solution

All the values except PV are given because PV is the only unknown value. However, special attention must be paid to the values of PMT and PV , as they can be easily confused.

Typically, if you have a payment (PMT), your FV is usually 0 because you are paying off a loan, and the loan balance will reduce to zero at the end.

On the other hand, for residual inflows, your PMT is usually 0, and FV is nonzero.

For NPV , we need to calculate the PV_{inflow} and PV_{outflow} . The payment (savings) represents the first PV_{inflow} , as saving money is a form of inflow. Residual (inflow) is evidently the second PV_{inflow} . Therefore, the total inflow in this case is given by:

$$PV_{\text{inflow}} = PV_{\text{payments}} + PV_{\text{residual}}$$

For the outflow, if you read the question carefully, you will find it mentioned in the first sentence: "will cost 55000."

Thus, the NPV is calculated as:

$$NPV = PV_{\text{payments}} + PV_{\text{residual}} - 55000$$

| Cash Inflows | Payments (Savings) | Residual (Inflow) |
|--------------|--------------------|-------------------|
| P/Y = | 1 ✓ | 1 ✓ |
| C/Y = | 1 ✓ | 1 ✓ |
| N = | 8 ✓ | 8 ✓ |
| I/Y = | 5 ✓ % | 5 ✓ % |
| PV = | \$ 45,921.13 ✓ | \$ 9,069.65 ✓ |
| PMT = | \$ 7105 ✓ | \$ 0 ✓ |
| FV = | \$ 0 ✓ | \$ 13,400.00 ✓ |

(If the NPV is negative, enter it as a negative number. If the NPV is zero, enter 0.)

NPV = \$ -9 ✓ (round to the nearest whole number)

Once the Net Present Value (NPV) is calculated, we can decide whether to accept or reject the project. The decision criterion is as follows:

Formula

NPV Decision Criteria:



- **If $NPV > 0$: Accept the project.**
The project adds value and is expected to generate returns greater than the required rate of return.
- **If $NPV = 0$: Indifference.**
The project breaks even, covering the cost of capital, with no gain or loss in value.
- **If $NPV < 0$: Reject the project.**
The project does not add value, as returns are less than the required rate of return.

Therefore, since NPV is -9, for the previous example:

2) Should this equipment purchase be made according to the NPV criterion?

☐ Yes

☒ No

Let's try another example:

Example

Hydro One has two options for upgrading a geothermal power station to meet new government standards:

Option 1: Hydro One will make the upgrades themselves. This is expected to cost \$10,300 at the end of every six months for 12 years. At the end of the operation (in 12 years), Hydro One expects to sell all equipment needed for the upgrade for \$125,000.

Option 2: Pay experienced contractors. This will cost \$29,000 up front and \$9,300 semi-annually (at the end of every six months) for 15 years.

Assume all interest is 2.24% compounded semi-annually.

Round the answers to NPV (Option 1) and NPV (Option 2) to the nearest dollar. Round all other answers to two decimal places where applicable.

1) Find the net present value of option 1:

| | Payments (Cost) | Sale of equipment (Residual) |
|--------------|-----------------|------------------------------|
| P/Y = | 2 ✓ | 2 ✓ |
| C/Y = | 2 ✓ | 2 ✓ |
| N = | 24 ✓ | 24 ✓ |
| I/Y = | 2.24 ✓ % | 2.24 ✓ % |
| PV = | \$ 215,712.43 ✓ | \$ 95,679.86 ✓ |
| PMT = | \$ 10300 ✓ | \$ 0 ✓ |
| FV = | \$ 0 ✓ | \$ 125000 ✓ |

(If the NPV is negative, enter it as a negative number. If the NPV is zero, enter 0.)

NPV (Option 1) = \$ (rounded to the nearest whole number)

Solution

To calculate NPV, we need to identify the present value (PV) of inflows and outflows:

- **PV Outflow:** Costs are always considered outflows since they represent payments or expenses.
- **PV Inflow:** The sale of equipment (residual value) is considered an inflow as it represents revenue or gains.

Thus, the Net Present Value (NPV) is calculated as:

$$NPV = PV \text{ Inflows} - PV \text{ Outflows}$$

$$NPV = 95679.86 - 2156712.43 = -120,033$$

2) Find the net present value of option 2:

| | Payments (Cost) | |
|-------|-----------------|--|
| P/Y = | 2 ✓ | |
| C/Y = | 2 ✓ | |
| N = | 30 ✓ | |
| I/Y = | 2.24 ✓ % | |
| PV = | \$ 235,855.45 ✓ | |
| PMT = | \$ 9300 ✓ | |
| FV = | \$ 0 ✓ | |

(If the NPV is negative, enter it as a negative number. If the NPV is zero, enter 0.)

NPV (Option 2) = \$ (round to the nearest whole number)

Solution

For Option 2, the present value (PV) primarily consists of costs, which are treated as outflows. The first outflow is the series of semi-annual payments. Additionally, the question states, "This will cost \$29,000," indicating an upfront cost, which represents the second outflow. These two components are added together to determine the total PV outflow.

Since there are no inflows specified for Option 2, the PV inflow is zero.

Applying the NPV formula:

$$NPV = PV \text{ Inflows} - PV \text{ Outflows}$$

$$NPV = 0 - (235,885.45 + 29,000) = -264,885$$

Thus, the Net Present Value for Option 2 is calculated as $-264,885$.

3) Which option should Hydro One choose?

- ☒ Option 1
- ☐ Option 2
- ☐ Either option could be chosen



Solution

By comparing the NPV of the two options, we select Option 1 because $NPV_1 > NPV_2$. The option with the larger NPV is preferred, as it provides the greatest financial return. Hence, Option 1 is the optimal choice.

As for Discounted Cash Flow (DCF), the following formulas are used to calculate the present value of future cash flows:

Formual

$$PV_{\text{All Cash Flows}} = PV_{\text{Cash Flow 1}} + PV_{\text{Cash Flow 2}} + \dots$$

Check an easy example here:

A company has two investment opportunities. Alternative 1 (Alt. 1) pays \$10,000 (inflow) two years from now, and \$26,000 (inflow) four years from now. Alternative 2 (Alt. 2) pays \$7,500 (inflow) at the end of every year for five years. Interest is 7.49% compounded annually. Which is the preferable alternative?

Round the values for **PV** to the **nearest cent**.

| | TWO YEARS | FOUR YEARS | FIVE YEARS |
|-------|---------------|----------------|----------------|
| P/Y = | 1 ✓ | 1 ✓ | 1 ✓ |
| C/Y = | 1 ✓ | 1 ✓ | 1 ✓ |
| N = | 2 ✓ | 4 ✓ | 5 ✓ |
| I/Y = | 7.49 ✓ % | 7.49 ✓ % | 7.49 ✓ % |
| PV = | \$ 8,654.94 ✓ | \$ 19,476.06 ✓ | \$ 30,352.20 ✓ |
| PMT = | \$ 0 ✓ | \$ 0 ✓ | \$ 7500 ✓ |
| FV = | \$ 10,000 ✓ | \$ 26000 ✓ | \$ 0 ✓ |

Write the Discounted Cash Flow (DCF) for Alt. 1 and Alt. 2.

Enter positive values for Alt. 1, and Alt. 2, rounded to the nearest dollar.

Alt. 1 = \$ Alt. 2 = \$ Choice

Solution

Given the table, all blanks have been filled, and we proceed to calculate the DCF for Alternative 1 (Alt1) and Alternative 2 (Alt2). Pay close attention to the **PMT** and **FV** values in the table and why they are 0 or non-zero, as indicated in the question.

According to the formula, the DCF is the sum of the PV of all cash flows:

- For **Alt1**, cash flows occur in years 2 and 4. We use the PV of these two time ranges and add them to compute the DCF:

$$\text{Alt1 DCF} = 8654.94 + 19476.06 = 28131$$

- For **Alt2**, the cash flow only occurs in year 5. We simply use the PV for this time range:

$$\text{Alt2 DCF} = 30352.20$$

Unit 7

Break-Even Analysis

Abbreviations

Below are abbreviations for some terms used in this unit.

Abbreviation

- **NI:** Net Income
- **TR:** Total Revenue
- **TC:** Total Cost
- **TVC:** Total Variable Cost
- **VC:** Variable Cost
- **SP:** Selling Price
- **CM:** Contribution Margin
- **FC:** Fixed Cost
- **BP:** Breakeven Point

Here are the formulas:

Formulas

$$NI = TR - TC$$

$$TR = SP \cdot X$$

$$TC = TVC + FC$$

$$TVC = VC \cdot X$$

$$TC = VC \cdot X + FC$$

$$CM = SP - VC$$

$$NI = (SP - VC) \cdot X - FC$$

$$X_{BE} = \frac{FC}{CM}$$

$$MoS = \frac{CM}{SP}$$

$$TCM = TR - TVC$$

$$SP = \frac{\text{Cost}}{1 - MoS}$$

$$\text{Percent of Capacity} = \left(\frac{\text{X value of the break-even point}}{\text{Commodity}} \right) \times 100$$

Here is a very basic example of the usage of these formulas:

Example

Margaret runs a business that makes custom-printed shirts. It will cost her \$7 each to purchase and print on shirts, and she will have to pay a rent of \$1,596 per month for her workshop. Based on market research, Margaret estimates that she can sell custom shirts for \$26 each.

- Find the **revenue function**.
- Find the **cost function**.
- Calculate the **number of shirts** she needs to sell per month to break even.
- Calculate the **break-even point in dollars** (round off to the nearest cent).

Solution

The selling price is \$26, so

$$SP = \$26$$

The variable cost per t-shirt is \$7, so

$$VC = \$7$$

The only fixed cost is the workshop rent, so

$$FC = \$1596$$

a) Revenue Function

:

$$TR = SP \cdot X$$

Substituting the values:

$$TR = 26X$$

b) Cost Function

:

$$TC = VC \cdot X + FC$$

Substituting the values:

$$TC = 7X + 1596$$

c) Break-Even Quantity

To find the quantity at break-even, we use the condition $TR = TC$:

$$26X = 7X + 1596$$

Simplify:

$$(26 - 7)X = 1596$$

$$19X = 1596$$

Solve for X :

$$X = \frac{1596}{19}$$

$$X = 84$$

Therefore, Margaret needs to sell 84 T-shirts to break even.

d) Break-Even in Dollars

The break-even revenue is the total revenue at $X = 84$:

$$TR = 26 \cdot X$$

Substituting $X = 84$:

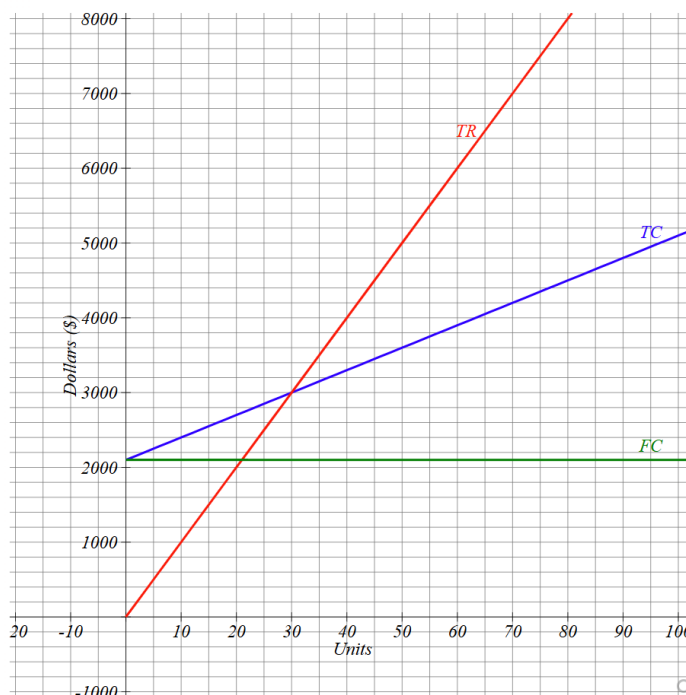
$$TR = 26 \cdot 84$$

$$TR = \$2184$$

Sometimes, we use different terms when referring to the break-even point. For example, achieving a **positive profit** is also associated with finding the break-even point. This is because a business must first reach the break-even point before it can start generating profits.

Now, let's observe the following graph:

The figure below shows graphs of the fixed cost function, total cost function and the total revenue function for a certain commodity.



If the selling price per unit is \$100, and the variable cost per unit is \$30:

Example

- (a) What is the break-even point?
Enter the answer in the form (x,y).
e.g., (295, 7650)
- (b) What are the fixed costs?
\$ _____
- (c) If the maximum production capacity of the commodity is 120, express the break-even units as a percent of capacity:

Percent of capacity = _____% (round to two decimal places if necessary).

Example

The breakeven point is the intersection point of TR and TC. So you will have to find the co-ordinate of this point, in this graph it is point (30, 3000), notice that for the Y-axis one grade in 200.

As for FC, since it is a horizontal line, you just need to find the Y value of it. The answer is 2500.
As for percent of capacity

$$\text{Percent of Capacity} = \left(\frac{\text{Break-even Units}}{\text{Maximum Capacity}} \right) \times 100$$

Substituting values:

$$\text{Percent of Capacity} = \left(\frac{30}{120} \right) \times 100 = 25\%$$

Example

A company sells one of its products for \$10.70 per unit. Its fixed costs are \$910.00 per month, and the variable cost per unit is \$4.20.

(a) The contribution margin per unit is:

\$ _____ (rounded to the nearest cent).

(b) The break-even volume, i.e., the level of output at break-even, is:

_____ units per month.

(c) The profit at a monthly output level of 166 units is:

\$ _____ (rounded to the nearest cent).

Example

(a) Contribution Margin per Unit:

The contribution margin (CM) is calculated as:

$$CM = SP - VC$$

Substituting the values:

$$CM = 10.70 - 4.20 = 6.50$$

Therefore, the contribution margin per unit is:

$$\boxed{\$6.50}$$

(b) Break-Even Volume:

The break-even volume (X_{BE}) is calculated as:

$$X_{BE} = \frac{FC}{CM}$$

Substituting the values:

$$X_{BE} = \frac{910}{6.50} \approx 140$$

Therefore, the break-even volume is:

$$\boxed{140 \text{ units per month}}$$

(c) Profit at 166 Units:

The profit (NI) is calculated as:

$$NI = (SP - VC) \cdot X - FC$$

Substituting the values:

$$NI = (10.70 - 4.20) \cdot 166 - 910$$

$$NI = 6.50 \cdot 166 - 910$$

$$NI = 1079 - 910 = 169$$

Therefore, the profit at 166 units is:

$$\boxed{\$169.00}$$

Unit 8

Trade Discount and Cash Discount

For this unit, the focus is entirely on formulas. You will need to carefully read the questions and determine both the price before and after the discount—this is often the challenging part. However, if you take the time to understand the meaning behind these formulas, you will not need to memorize them. They make sense in real-life situations, such as when we go shopping and calculate discounts or final prices.

Formulas

$$D = d \times L$$

$$N = L - D$$

$$N = L(1 - d)$$

$$N = L(1 - d_1)(1 - d_2)(1 - d_3) \dots (1 - d_n)$$

$$d_e = 1 - [(1 - d_1)(1 - d_2)(1 - d_3) \dots (1 - d_n)]$$

Explanation:

D is the amount of trade discount, d is the discount rate, L is the list price (price before discount), N is the net price (price after discount), and d_e is the single equivalent discount rate.

For this unit, there are mainly two types of questions:

1. Calculate the price after discount.
2. Calculate the price before discount.

Some examples will be provided below.

Examples

A manufacturer sells an item with a list price of \$195 to a distributor. The distributor receives a trade discount of 35%. How much will the distributor pay for the item after the discount?

Solution

In this case, we are using the formula:

$$N = L(1 - d)$$

Reasoning: The trade discount is 35%, which means the price after this discount is $(1 - 35\%)$ of the original price. Therefore, instead of calculating the discount amount D using $D = d \times L$ and subtracting it from the list price, we can directly calculate the net price N using $(1 - d)$, where d is the discount rate.

Step 1: Substitution into the formula.

$$N = 195(1 - 0.35)$$

Step 2: Simplify.

$$N = 195(0.65)$$

Step 3: Calculate.

$$N = 126.75$$

Let's check another example

Examples

A 25% discount on a smartphone amounts to \$475. We need to find the phone's list price L .

Solution

Step 1: Identify the appropriate formula. The discount amount is given as $D = 475$, and the discount rate is $d = 0.25$. We use the formula:

$$D = d \cdot L$$

This formula is chosen because the keywords "*discount on*" indicate that the percentage provided refers specifically to the discount amount itself. There is no need to subtract the discount percentage from 1.

Step 2: Solve for the list price L . Rearranging the formula to isolate L , we get:

$$L = \frac{D}{d}$$

Step 3: Substitute the known values and calculate.

$$L = \frac{475}{0.25}$$

$$L = 1900$$

Summary: Choosing the Correct Formula

To determine whether you need to use the formula $N = L(1 - d)$ or $D = d \cdot L$, pay attention to the following keywords in the question:

- **Use $N = L(1 - d)$:**
Look for keywords like: “discount of $X\%$ ”, “% off the regular selling price” or “less”.
These indicate that you are finding the price **after the discount** has been applied.
- **Use $D = d \cdot L$:**
Look for keywords like: “discount is” or “ $X\%$ of the regular price ”.
These indicate that you are calculating the discount itself, which is a portion of the list price.

Cash discount fractions are often used to describe the terms of payment discounts offered by sellers to encourage early payment. These fractions typically include:

Examples

For example, the terms $6/7, 3/19, n/34$ mean:

- $6/7$: A 6% discount is available if the payment is made within 7 days.
- $3/19$: A 3% discount is available if the payment is made between 8 and 19 days.
- $n/34$: There is no discount after 19 days.

Check an example here:

Examples

Wyatt receives an invoice from Home and Office Supply dated April 4th, 2016, for \$1,350 with terms $6/7, 3/19, n/34$. This invoice was paid on April 7th, 2016.

a) **Number of Days:**

Number of days: _____ days.

b) **Amount of Discount:**

Amount of discount: _____ (round to the nearest cent).

c) **Amount Paid:**

Amount paid: _____ (round to the nearest cent).

Solution

a) **Number of Days:** Number of days between April 4th and April 7th:

$$7 - 4 = 3 \text{ days.}$$

Answer: 3 days.

b) **Amount of Discount:** The payment was made within 7 days, so a 6% discount applies:

$$\text{Discount} = d \cdot L = 0.06 \cdot 1350 = 81.00$$

Answer: \$81.00.

c) **Amount Paid:** The amount paid is the invoice amount minus the discount:

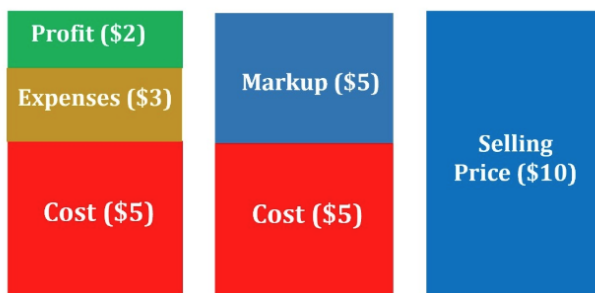
$$\text{Amount Paid} = L - \text{Discount} = 1350 - 81 = 1269.00$$

Answer: \$1,269.00.

Unit 9

Markup and Markdown

Markup is the difference between the cost price of a product and its selling price. It is usually expressed as a percentage of the cost price and is used to determine the profit margin on a product.



Formulas

$$S = C + M$$

$$M = E + P$$

$$S = C + E + P$$

$$S_{BE} = C + E$$

$$S_{\text{Red}} = S(1 - \text{MD Rate})$$

$$\text{MoS} = \frac{M}{S} \times 100$$

$$\text{MoC} = \frac{M}{C} \times 100$$

Where:

- S : Selling Price
- C : Cost Price
- M : Markup
- E : Expenses
- P : Profit
- S_{BE} : Break-Even Selling Price
- S_{Red} : Reduced Selling Price
- MD Rate: Markdown Rate
- MoS: Markup based on Selling Price (percentage)
- MoC: Markup based on Cost Price (percentage)

Examples

Canadian Tire paid \$77 for a humidifier. Expenses are 25% of cost, and the profit is 11% of cost.

- 1) What is the regular selling price?

Answer: _____

- 2) If the humidifier is sold at cost, what is the resulting profit or loss?

(Use a negative sign (-) if there is a loss.)

Answer: _____

Solution

Canadian Tire paid \$77 for a humidifier. Expenses are 25% of cost, and the profit is 11% of cost.

- 1) **Regular Selling Price (S):** Using $S = C + E + P$, where $E = 0.25 \cdot C$ and $P = 0.11 \cdot C$:

$$E = 0.25 \cdot 77 = 19.25, \quad P = 0.11 \cdot 77 = 8.47$$

$$S = 77 + 19.25 + 8.47 = 104.72$$

Answer: \$104.72

- 2) **Profit or Loss if Sold at Cost (C):** Since the selling price is the same as the cost price, there is no profit. We only need to subtract the expenses because the money gained from selling the item is just enough to cover the cost:

$$\text{Profit or Loss} = -E = -19.25$$

Answer: -\$19.25 (Loss)

Now let's try another example:

Examples

Home Hardware paid \$108 for a particular type of drill. Expenses are 18% of the selling price, and the required profit is 22% of the selling price. Round all answers to the nearest cent if applicable.

- 1) What is the regular selling price?

Answer: _____

- 2) What is the break-even selling price?

Answer: _____

- 3) During an inventory sale, the drill was marked down 21% on the regular selling price. What is the sale price?

Answer: _____

- 4) What is the operating profit or loss during the inventory sale?
(Use a negative sign (-) for a loss.)

Answer: _____

Solution

The selling price (S) is determined using the equation:

$$S = C + E + P$$

where $E = 0.18 \cdot S$ (expenses) and $P = 0.22 \cdot S$ (profit).

Step 1: Substitute the values for E and P in terms of S :

$$S = C + (0.18 \cdot S) + (0.22 \cdot S)$$

Step 2: Combine like terms:

$$S = C + (0.18 + 0.22) \cdot S$$

$$S = C + 0.4 \cdot S$$

Step 3: Rearrange to isolate S :

$$S - 0.4 \cdot S = C$$

$$(1 - 0.4) \cdot S = C$$

$$0.6 \cdot S = C$$

Step 4: Solve for S :

$$S = \frac{C}{0.6}$$

Final Answer: The regular selling price is:

$$\boxed{\$180.00}$$

Solution

2) Break-Even Selling Price (S_{BE}):

At break-even, there is no profit ($P = 0$), so:

$$S_{BE} = C + E = 140.40$$

Answer: 140.4

3) Sale Price:

The sale price is calculated as:

$$\text{Sale Price} = S(1 - \text{Markdown Rate}) = 180(1 - 0.21) = 180 \cdot 0.79 = 142.20$$

Answer: \$142.20

4) Operating Profit or Loss:

During the sale, the operating profit or loss is:

$$\text{Profit or Loss} = \text{Sale Price} - C - E$$

$$\text{Substituting } E = 0.18 \cdot \text{Sale Price} = 0.18 \cdot 142.20 = 25.60:$$

$$\text{Profit or Loss} = 142.20 - 108 - 25.60 = 1.80$$

Answer: \$1.80 (Profit)

References

Tavangar, A. (n.d.). *Mathematics of Finance* (First Edition). Creative Commons Attribution NonCommercial ShareAlike License.

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