1. Implement DFS, BFS for 8-puzzle problem

Step 1: What is the 8-puzzle?

The 8-puzzle is a sliding puzzle consisting of:

- A 3×3 grid
- 8 numbered tiles (1–8) and one empty space (also called the blank or 0). It looks like this:

1 2 3

4 _ 6

7 5 8

Where _ represents the blank space.

Step 2: Objective of the puzzle

Given an **initial configuration**, the goal is to move the tiles **one at a time** (sliding them into the blank space) until you reach the **goal state**:

1 2 3
4 5 6
7 8 _

Step 3: How can you move?

- You can slide a **tile adjacent to the blank** into the blank space.
- The blank can move:
  - **up**
  - **down**
  - **left**
  - **right**

For example:

Start: Move blank up:

1 2 3 1 2 3

4 _ 6 → _ 4 6

7 5 8 7 5 8

Step 4: State Space

Every **arrangement of tiles + blank** is a **state**.

- Total possible states = 9!=362,880
- But only **half of these (181,440)** are solvable because of the parity of inversions.

Step 5: Approach to solve the problem

You search for a sequence of moves from **initial state** → **goal state**.

Types of Search Algorithms:

Uninformed Search

- Breadth-First Search (BFS)
- Depth-First Search (DFS)

Informed (Heuristic) Search

- A* algorithm
- Greedy Best First Search

Step 6: Cost of a move

- Typically, each move costs 1.
- Goal: Find the **shortest path (least number of moves)** from start to goal.

Step 7: Example of solving the puzzle

Initial State:

```
1 2 3
4 5 6
_ 7 8
```

Goal State:

```
1 2 3
4 5 6
7 8 _
```

Moves:

Move 7 left:

```
1 2 3
```

4 5 6
7 _ 8

Move 8 left:

1 2 3
4 5 6
7 8 _

Solved in **2 moves**!

Why is this problem important?

- Small but challenging.
- Demonstrates search strategies & heuristics.
- Foundation for more complex AI planning & robotics.

What a **search space tree** is — especially in the context of problems like the **8-puzzle**, pathfinding, or optimization problems.

🌳 What is a Search Space Tree?

A **search space tree** is a conceptual tree that represents:

- All **possible states** of a problem
- And all the **paths (or sequences of actions)** you can take from the **initial state** to reach the **goal state**.

It helps us systematically explore all possible solutions to a problem.

Key Components of a Search Space Tree:

Term	**Meaning**
Root Node	Represents the **initial state** of the problem.
Child Nodes	States reachable by applying one action to the parent state.
Edges	Actions or moves that transform one state into another.

Term	Meaning
Leaf Nodes	States with no further moves OR the goal state.
Path	Sequence of moves (edges) from the root to any node.

Why is it called a "tree"?

Because:

- Each state can lead to one or more next states.
- These states "branch out" like a tree.
- No cycles in this abstract tree (although the real state space might have cycles — we handle that separately in algorithms).

Example: Search space tree for a simple 8-puzzle

Let's take a very simple **initial state**:

1 2 3

4 5 6

_ 7 8

- The **root node** is this initial state.
- From here, the blank (_) can move **right** or **down**, generating two children.
- Each child becomes a new node and can generate its own children.
- This continues until we find the **goal state**.

Visually:

```
        Initial State
          (root)
          /    \
    State 1       State 2
```

```
   /      \            /     \
 State 3   State 4   State 5   State 6
       ...
```

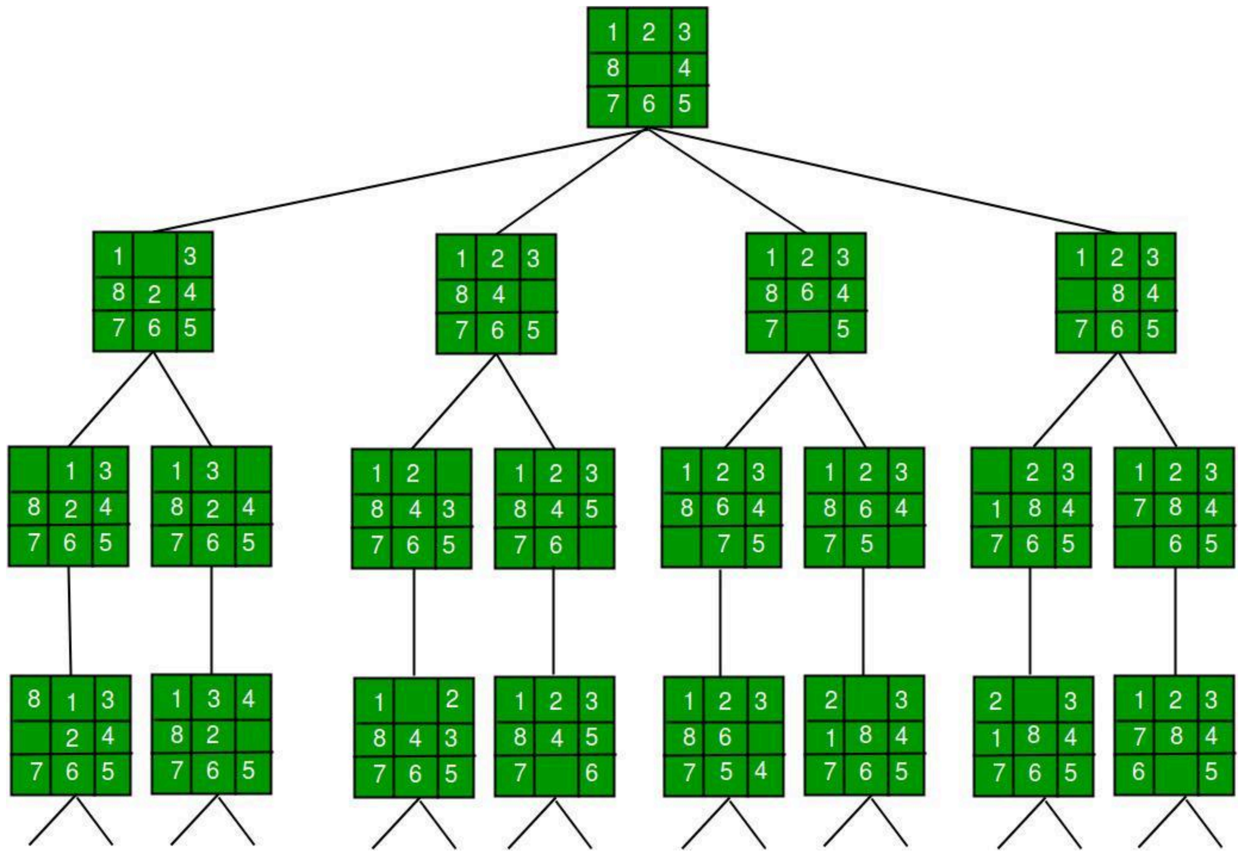
Each level of the tree represents one more **move** away from the root.

Search in this tree

- **Breadth-First Search (BFS)** explores the tree level by level.
- **Depth-First Search (DFS)** explores as deep as possible along a branch before backtracking.
- **A*** and other informed algorithms explore the most promising nodes first using heuristics.

Benefits:

Helps us **visualize** the problem-solving process.
Ensures we don't miss any possible solution.
Makes it clear how states are connected.
Useful for designing and understanding search algorithms.



https://github.com/RajPShinde/8-Puzzle-BFS_Algorithm

https://www.youtube.com/watch?v=POM4mmLctyo&authuser=0

https://www.javatpoint.com/8-puzzle-problem-in-python?authuser=0

https://github.com/RajPShinde/8-Puzzle-BFS_Algorithm?authuser=0

https://github.com/AlexP11223/EightPuzzleSolver?authuser=0

https://www.kaggle.com/code/muhammadrozy77/8-puzzle-problem-bfs-group-1?authuser=0

```
#include <iostream>

#include <vector>
```

```
#include <queue>
#include <set>
using namespace std;

// Define the dimensions of the puzzle
#define N 3

// Structure to represent the state of the puzzle
struct PuzzleState {
    vector<vector<int>> board;
    int x, y;
    int depth;

    // Constructor
    PuzzleState(vector<vector<int>> b, int i, int j, int d) : board(b), x(i), y(j),
depth(d) {}
};

// Possible moves: Left, Right, Up, Down
int row[] = {0, 0, -1, 1};
int col[] = {-1, 1, 0, 0};

// Function to check if the current state is the goal state
bool isGoalState(vector<vector<int>>& board) {
    vector<vector<int>> goal = {{1, 2, 3}, {4, 5, 6}, {7, 8, 0}};
    return board == goal;
}
```

```
// Function to check if a move is valid
bool isValid(int x, int y) {
    return (x >= 0 && x < N && y >= 0 && y < N);
}

// Function to print the puzzle board
void printBoard(vector<vector<int>>& board) {
    for (auto& row : board) {
        for (auto& num : row)
            cout << num << " ";
        cout << endl;
    }
    cout << "--------" << endl;
}

// BFS function to solve the 8-puzzle problem
void solvePuzzleBFS(vector<vector<int>>& start, int x, int y) {
    queue<PuzzleState> q;
    set<vector<vector<int>>> visited;

    // Enqueue initial state
    q.push(PuzzleState(start, x, y, 0));
    visited.insert(start);

    while (!q.empty()) {
```

```
PuzzleState curr = q.front();
q.pop();

// Print the current board state
cout << "Depth: " << curr.depth << endl;
printBoard(curr.board);

// Check if goal state is reached
if (isGoalState(curr.board)) {
   cout << "Goal state reached at depth " << curr.depth << endl;
   return;
}

// Explore all possible moves
for (int i = 0; i < 4; i++) {
   int newX = curr.x + row[i];
   int newY = curr.y + col[i];

   if (isValid(newX, newY)) {
      vector<vector<int>> newBoard = curr.board;
      swap(newBoard[curr.x][curr.y], newBoard[newX][newY]);

      // If this state has not been visited before, push to queue
      if (visited.find(newBoard) == visited.end()) {
         visited.insert(newBoard);
         q.push(PuzzleState(newBoard, newX, newY, curr.depth + 1));
```

```
                }
            }
        }
    }

    cout << "No solution found (BFS Brute Force reached depth limit)" << endl;
}

// Driver Code
int main() {
    vector<vector<int>> start = {{1, 2, 3}, {4, 0, 5}, {6, 7, 8}}; // Initial state
    int x = 1, y = 1;

    cout << "Initial State: " << endl;
    printBoard(start);

    solvePuzzleBFS(start, x, y);

    return 0;
}
```

Output

```
Initial State:
1 2 3
4 0 5
6 7 8
--------
```

Depth: 0

1 2 3

4 0 5

6 7 8

Depth: 1

1 2 3

0 4 5

6 7 8

Depth: 1

1 2 3

4 5 0

6 7 8

Depth: 1

1 0 3

4 2 5

6 7 8

-------...

imitations of DFS and BFS in the 8-Puzzle Problem

- ***DFS****: Can get stuck in deep, unproductive paths, leading to excessive memory usage and slow performance.*
- ***BFS****: Explores all nodes at the current depth level, making it inefficient as it does not prioritize promising paths.*

Optimizing with Branch and Bound (B&B)

*Branch and Bound enhances search efficiency by using a **cost function** to guide exploration.*

1. ***Intelligent Node Selection**: Prioritizes nodes **closer to the goal**, unlike DFS (blind) or BFS (equal priority).*
2. ***Pruning**: Eliminates unpromising paths to save time and memory.*

Approach:

- Use a **priority queue** to store live nodes.
- Initialize the **cost function** for the root node.
- Expand the **least-cost** node first.
- Stop when a goal state is reached or when the queue is empty.

Types of Nodes in B&B:

- **Live Node**: Generated but not yet explored.
- **E-node (Expanding Node)**: Currently being expanded.
- **Dead Node**: No longer considered (either solution found or cost too high).