CARLETON UNIVERSITY

MAAE 3004

Final Report

<u>Quick Return Mechanism</u>

Group L2 – 7

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Introduction:

We were tasked with the selection of a mechanism with a minimum of five links and one degree of freedom. The quick return mechanism fulfils each criterion, with six links and 1 degree of freedom. A kinematic analysis was performed on the mechanism as shown below.

A quick return motion mechanism converts circular motion into reciprocating motion, allowing the slider to move forwards and backwards. This device generates a reciprocating motion in which the return stroke is shorter than the forward stroke. It generally uses a system of links with three turning pairs and a sliding pair to be driven by a circular motion source [1]

This mechanism has applications in various fields, one such application is within shaping machines. The quick return mechanism minimizes the total processing time by controlling the forward action for clean and sharp cuts, and the backward action to return the slider back to its original position. The principle of the design allows for a slower forward action and therefore more refined cuts, whilst the backward action is much quicker in its return stroke, to allow for a quicker rotation.

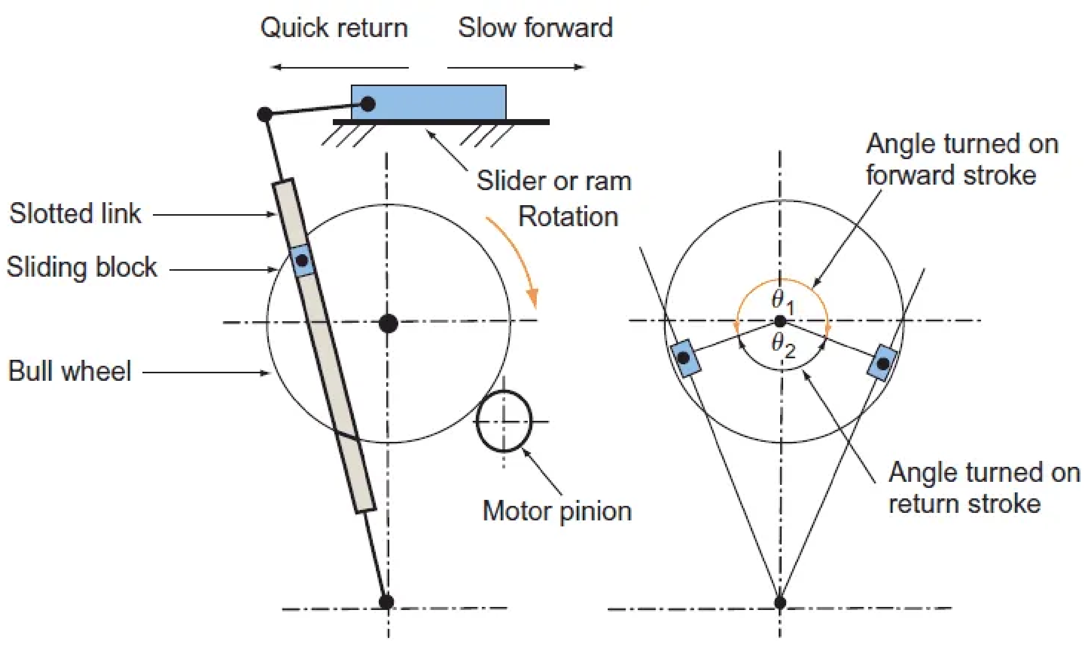


Figure 1: How a quick return mechanism operates with different stroke speeds [2]

Kutzbach Calculation:

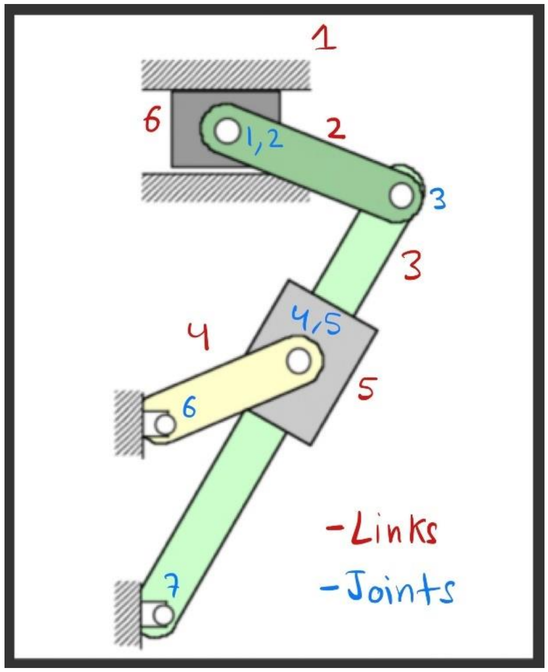


Figure 2: Mechanism with links and joints labelled

The mobility of the mechanism was calculated using the Kutzbach equation.

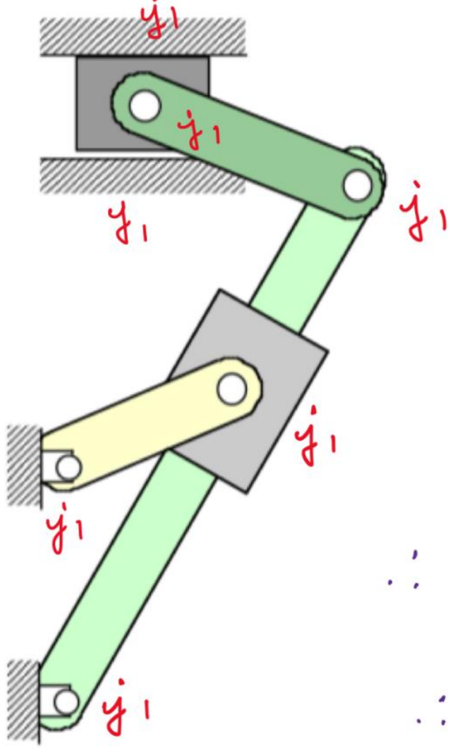


Figure 3: Mechanism with labelled joints

$$
\begin{aligned}
m &= 3(n-1) - 2j_1 - j_2 \\
n &= 6, j_1 = 7 \\
\therefore m &= 3(6-1) - 2 \times 7 - 0 \\
\therefore m &= 15 - 14 \\
\therefore m &= 1
\end{aligned}
$$

The push pull mechanism has a mobility factor of 1

Kinematic Analysis

Cross section of each link is 1m x 1m

Mass of links 2, 4, and 5:

$$\text{mass of link } 2 = 10\,\frac{\text{kg}}{\text{m}^3} \times 2.5\text{m}^3$$
$$m_2 = 25\text{kg}$$
$$\text{mass of link } 4 = 10\,\frac{\text{kg}}{\text{m}^3} \times 7.5\text{m}^3$$
$$m_4 = 75\text{kg}$$
$$\text{mass of link } 5 = 10\,\frac{\text{kg}}{\text{m}^3} \times 3\text{m}^3$$
$$m_5 = 30\text{kg}$$
$$m_{s1} = 0$$
$$m_{s2} = 0$$

Centre of gravity of links 2, 4, 5:

$$I_2 = \frac{1}{12} \times 25 \times 2.5^2$$
$$I_2 = 13.02$$
$$I_4 = \frac{1}{12} \times 75 \times 7.5^2$$
$$I_4 = 351.56$$
$$I_5 = \frac{1}{12} \times 30 \times 9$$
$$I_5 = 22.5$$

State 1:

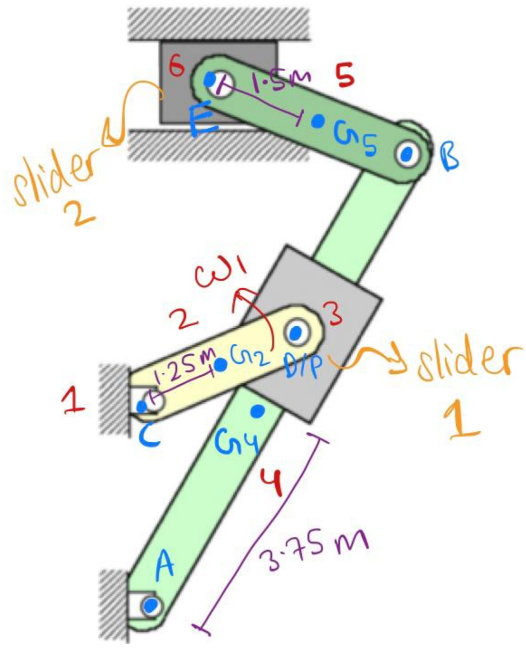


Figure 4: 1st state of mechanism

Coincident point at slider but on link 3

CD = 2.5 AB = 7.5 BE = 3 AD = 4.5 w_1 = 20

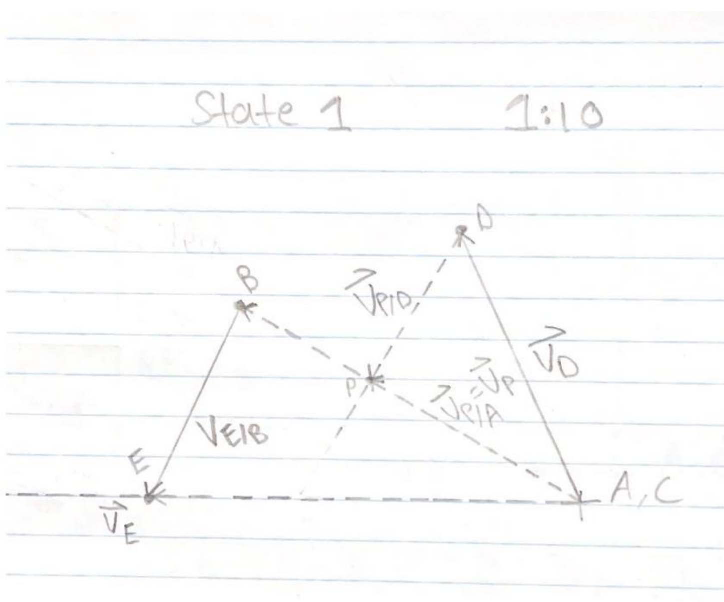


Figure 5: Velocity polygon for 1st state

$\vec{V}_D = \vec{V}_C + \vec{V}_{D/C}$
$\vec{V}_C = 0$ m/s
$\vec{V}_D = \vec{V}_{D/C} = \vec{\omega}_1 \times \vec{r}_{D/C} = 50$ m/s
$\vec{V}_P = \vec{V}_D + \vec{V}_{P/D} = \vec{V}_A + \vec{V}_{P/A}$
$\vec{V}_A = 0$, $\vec{V}_{P/D} = 29$ m/s

$\vec{v}_P = \vec{v}_D + \vec{v}_{P/D} = 41\text{m/s}$
$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = 0 + \vec{v}_{B/A}$
$\vec{v}_{B/A} = \vec{\omega}_3 \times \vec{r}_{B/A}$
$\vec{v}_P = \vec{\omega}_3 \times \vec{r}_{P/A}$
$\vec{\omega}_3 = \frac{\vec{v}_P}{\vec{r}_{P/A}} = \frac{41}{4.5} = 9.11\text{rad/s}$
$\vec{v}_{B/A} = \vec{v}_B = 68.33\text{m/s}$
$\vec{v}_E = \vec{v}_B + \vec{V}_{E/B} = 74\text{m/s}$
$\vec{\omega}_2 = \frac{\vec{v}_{E/B}}{\vec{r}_{E/B}} = \frac{36}{3} = 12\text{m/s}$
$Therefore$
$\vec{v}_{E/B} = 36\text{m/s}$

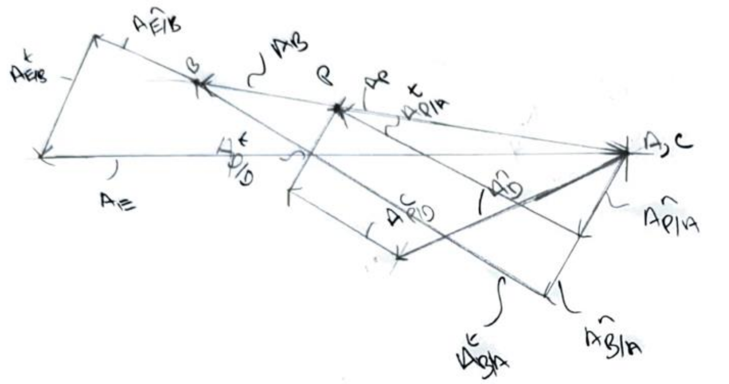


Figure 6: Acceleration polygon for 1st state with a scale of 1:200

Point P coincident to D

$$A_D = A_C + A_{\frac{D}{C}} \qquad A_C = 0$$
$$A_D = A^n{}_{D/C} + A^t{}_{D/C}$$
$$A_D = \omega^2(r_{DC}) = -(20)(2.5)$$
$$A_D = -1000\frac{m}{s}\ \ parallel\ to\ CD$$

$$A_P = A_D + A_{\frac{D}{P}} = A_D + A^n_{\frac{D}{P}} + A^t_{\frac{D}{P}} + A^C_{\frac{D}{P}}$$
$$A^n_{\frac{D}{P}} = 0$$
$$A_P = A_A + A_{\frac{P}{A}} = A^t_{\frac{P}{A}} + A^n_{\frac{P}{A}} \ , A_A = 0$$
$$\therefore A_D + A^T_{\frac{D}{P}} + A^c_{\frac{D}{P}} = A^t_{\frac{p}{A}} + A^n_{\frac{P}{A}}$$
$$A^c_{\frac{D}{p}} = 2(\omega_3)V_{\frac{D}{p}} = 2(9.11)(25)$$
$$A^c_{\frac{D}{P}} = 528.38\text{m/s}^2 \perp \text{ to } AB$$
$$A^n_{\frac{P}{A}} = -(\omega_3^2)r_{PA} = -9.11^2(4.5) = -373.46\, m/s^2 \ \ Parallel\ to\ AB$$

From polygon:

$$A^t_{D/P} = 500\text{m/s}^2, A^t_{D/\text{A}} = 1100\text{m/s}^2, A_D = 1180\text{m/s}^2$$
$$A^t_{P/A} = \alpha_3 r_{P/A}$$
$$\alpha_3 = \frac{\text{A}^t_{\text{P/A}}}{r_{P/A}} = \frac{1100}{4.5}$$
$$\alpha_3 = 244.44\text{rad/s}^2, \alpha_1 = 0$$
$$A_B = A_A + A_{\frac{B}{A}} = A_A + A^n_{\frac{B}{A}} + A^t_{\frac{P}{A}}, \qquad A_A = 0$$
$$A^n_{\frac{B}{A}} = -\omega_3^2 r_{\frac{B}{A}} = -(9.11)^2(7.5)$$
$$A^n_{\frac{B}{A}} = 622.44\frac{\text{m}}{\text{s}^2}\ parallel\ to\ AB$$

From polygon:

$$A_B = 1640\frac{\text{m}}{\text{s}^2}\ parallel \text{ to } A_p$$

$$A^t_{B/A} = 1600\frac{\text{m}}{\text{s}^2} \text{ perpendicular to } AB$$

$$\therefore A_E = A_B + A^n_{E/B} + A^t_{E/B}$$
$$A^n_{E/B} = -\omega_2^2 r_{E/B} = -(12^2)(3)$$
$$A^n_{E/B} = 432\text{m/s}^2$$

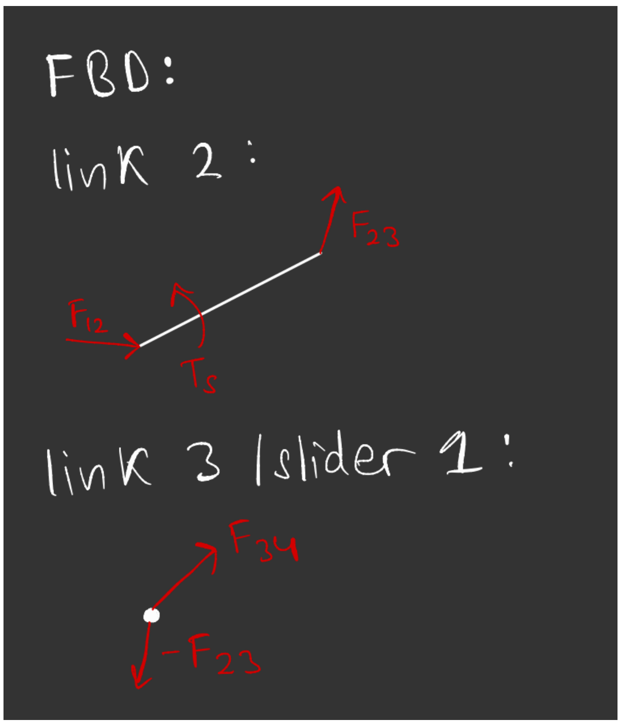
From polygon:

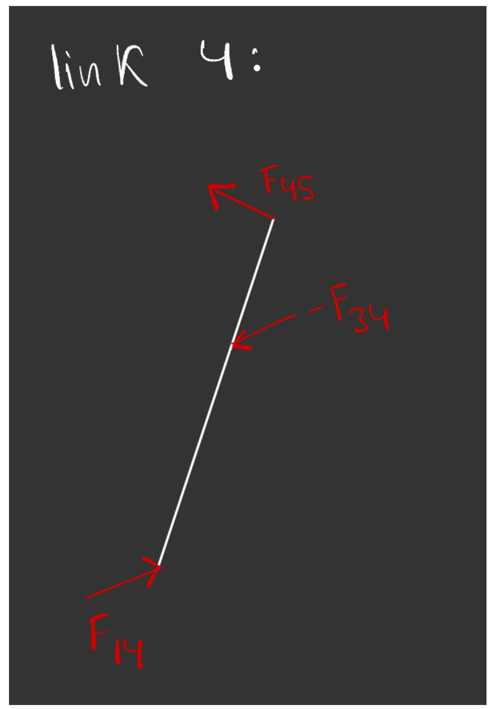
$$A_E = 2340\text{m/s}^2$$
$$A^t_{E/B} = 520\text{m/s}^3$$

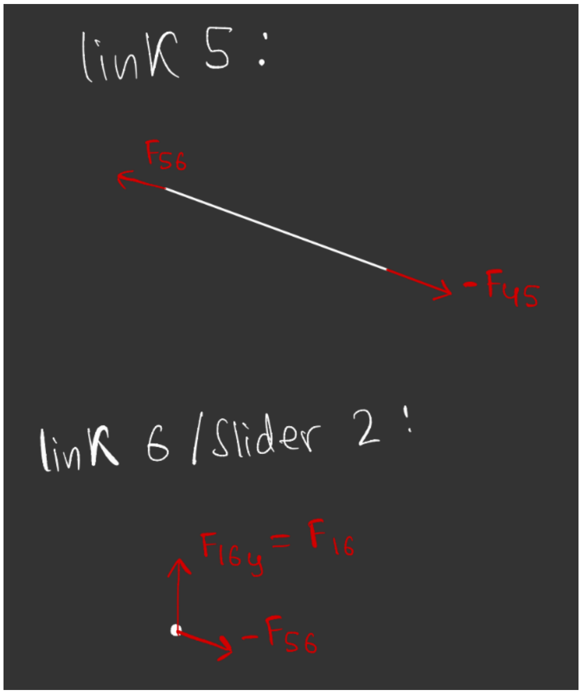
$$\therefore A_{E/B}^{t} = \alpha_2 r_{E/B}$$
$$\alpha_2 = \frac{A_{E/B}^{t}}{r_{E/B}}$$
$$\alpha_2 = \frac{520}{3}$$
$$\alpha_2 = 173.33\, rad/s^2$$

Force Analysis:

Free body diagrams for links 2, 3, 4, 5, and 6:







Acceleration of centroid at G_2, G_4, and G_5:

$$
\begin{aligned}
A_{G_2} &= \sqrt{(A_{G2}^N)^2 + (A_{G2}^T)^2} \\
&= -\omega_2^2 \times 1.25 + 0 \\
&= -20^2 \times 1.25 \\
&= -500\text{m/s}^2
\end{aligned}
$$

$$
\begin{aligned}
A_{G_4} &= \sqrt{\left(A_{G_4}^n\right)^2 + \left(A_{G_4}^t\right)^2} \\
&= \sqrt{(-\omega_3^2 \times 3.75)^2 + (\alpha_3 \times 3.75)^2} \\
&= \sqrt{(-9.11^2 \times 3.75)^2 + (244.44 \times 3.75)^2} \\
&= 968.04\text{m/s}^2
\end{aligned}
$$

$$A_{G5} = A_B + A_{G5}^N + A_{G5}^T$$

$$A_{G5} = 1640 - \omega_2^2 \times 1.5 + \alpha_2 \times 1.5$$

Through vector addition: $A_{G5} = 410$ m/s^2

Equations on link 2:

$$
\begin{aligned}
&\sum F_x = m_2 A_{G_{2x}} \\
&F_{12x} + F_{23x} = m_2 A_{G_2} \cos(25°) \\
&F_{12y} + F_{23y} = m_2 A_{G_2} \sin(25°) \\
&(F_{12x} \times 1.13) + \left(F_{12y} \times 0.53\right) + (F_{23x} \times 1.13) + \left(F_{23y} \times 0.53\right) + T_5 = I_2 \alpha_1 = 0
\end{aligned}
$$

Equations on link 3 slider 1:

$$-F_{23x} + F_{34x} = 0$$
$$-F_{23y} + F_{34y} = 0$$
$$I_4\alpha_3 = 0$$
$$F_{23} = F_{34}$$

Equations on link 4:

Using Pythagoras's theorem of A_{G4}^{N} and A_{G4}^{T} $\theta = 101.25$

Equations on link 4:

$$F_{14x} - F_{34x} + F_{45x} = m_4 A_{G4x}$$
$$A_{G_4x} = A_{G_4} \times \cos\ 101.25°$$

$$A_{G_{4y}} = A_{G_4} \times \sin\ 101 \cdot 25°$$
$$F_{14y} - F_{34y} + F_{45y} = m_4 A_{G4y}$$

$$(F_{14x} \times 3.75\sin\ 60°) + (-F_{34x} \times 0.75\sin 60°) + (F_{45x} \times 3.75\sin\ 60°) + \left(F_{14y} \times 3.75\cos 60°\right)$$

$$+ \left(-F_{34y} \times 0.75\cos\ 60°\right) + \left(F_{45y} \times 3.75\sin\ 60°\right) = I_4\alpha_3 = 85935$$

Equations on link 5:

$$-F_{45x} + F_{56x} = m_5 A_{G_5x}$$
$$-F_{45y} + F_{56y} = m_5 A_{G_5y}$$
$$A_{G_{5x}} = A_{G_5} \times \cos\ 25.3°$$
$$A_{G_{5y}} = A_{G_5} \times \sin\ 25.3°$$

$$(-F_{45} \times 1.5\sin 25) + (F_{56x} \times 1.5\sin 25) + (-F_{45} \times 1.5\cos 25) + \left(F_{56y} \times 1.5\cos 25°\right)$$
$$I_5\alpha_2 173.33 = 3899.925$$

Equations on link 6:

$$-F_{56x} + F_{16x} = 0$$
$$-F_{56y} + F_{16y} = 0$$
$$I_6 \propto_6 = 0$$

Matrix for pose 1:

$$x = \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{23x} \\ F_{23y} \\ F_{34x} \\ F_{14x} \\ F_{14y} \\ F_{45x} \\ F_{45y} \\ F_{56x} \\ F_{56y} \\ F_{16y} \\ F_5 \end{bmatrix} \quad b = \begin{bmatrix} 11328.8 \\ 5282.7 \\ 0 \\ 0 \\ 0 \\ -14164.14 \\ 71207.95 \\ 85935 \\ 11120.2 \\ 5256.5 \\ 3899.925 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.13 & 0.53 & 1.13 & 0.53 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.65 & 3.25 & 1.875 & 3.25 & 1.875 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.63 & 1.36 & 0.63 & 1.36 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

Torque = -0.039 x 10^5 N.m

F_{12x} *(10^5N)	F_{12y}*(10^5N)	F_{23x}*(10^5N)	F_{23y}*(10^5N)	F_{34x}*(10^5N)	F_{14x}*(10^5N)
0.1133	0.0528	0.1560	0	0	0.7177

F_{14y}*(10^5N)	F_{45x}*(10^5N)	F_{45y}*(10^5N)	F_{56x}*(10^5N)	F_{56y}*(10^5N)	F_{16y}*(10^5N)
0.8233	3.8762	0.8068	0.0722	0.7788	-0.0526

State 2:

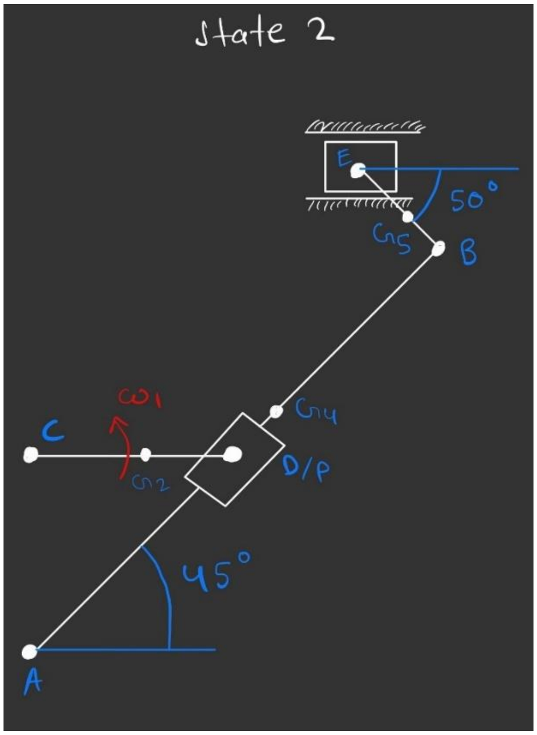


Figure 7: 2nd state of mechanism

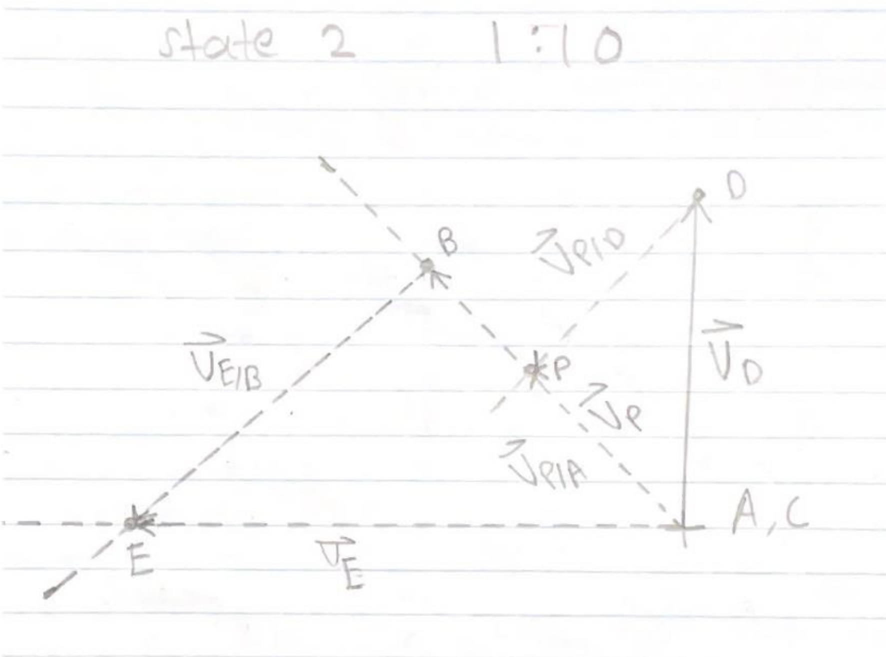


Figure 8: Velocity polygon for 2nd state

Coincident point at slider but on link 3

$\vec{V}_D = \vec{V}_C + \vec{V}_{D/c}$
$\vec{v}_C = 0\text{m/s}$
$\vec{V}_D = \vec{V}_{D/C} = \vec{\omega}_1 \times \vec{r}_{D/C} = 50\text{m/s}$
$\vec{v}_P = \vec{V}_D + \vec{V}_{P/D} = \vec{v}_A + \vec{V}_{P/A}$
$\vec{V}_A = 0 \qquad \vec{V}_{P/D} = 35\text{m/s}$

$\vec{v}_P = \vec{v}_D + \vec{v}_{P/D} = 35\text{m/s}$
$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = 0 + \vec{v}_{B/A}$
$\vec{v}_{B/A} = \vec{\omega}_3 \times \vec{r}_{B/A}$
$\vec{v}_P = \vec{\omega}_3 \times \vec{r}_{P/A}$
$\vec{\omega}_3 = \dfrac{\vec{v}_P}{r_{P/A}} = \dfrac{35}{4.5} = 7.78\text{rad/s}$
$\vec{v}_{B/A} = \vec{V}_B = 58\text{m/s}$
$\vec{v}_E = \vec{v}_B + \vec{V}_{E/B} = 87\text{m/s}$
$\vec{\omega}_2 = \dfrac{\vec{v}_{E/B}}{\vec{r}_{E/B}} = \dfrac{61}{3} = 20.3\text{m/s}$

$\vec{v}_{E/B} = 61\text{m/s}$

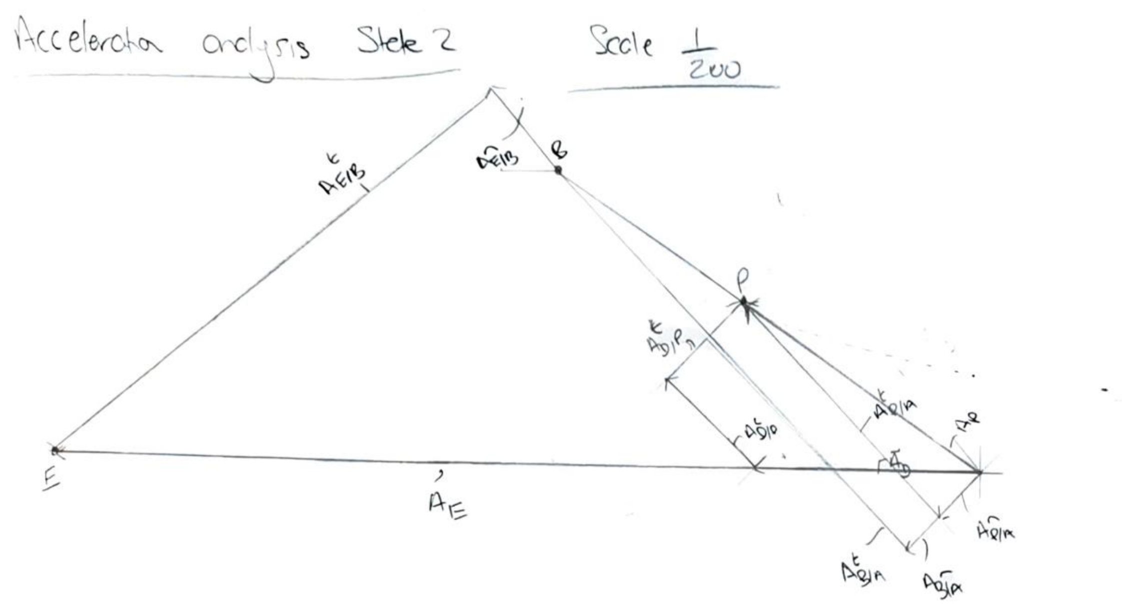


Figure 9: Acceleration polygon for 2nd state.

$$A_D = A_C + A_{\frac{D}{C}}, \quad A_C = 0$$
$$A_D = A^n_{\mathrm{D/C}} + A^t_{\mathrm{D/C}}$$
$$A_D = -\omega_1^2 r_{D/C} = -(20^2)(25)$$
$$A_D = -1000\frac{m}{s^2}\ parallel \text{ to } CD$$

Point P coincident to D

$$A_P = A_D + A_{\frac{D}{p}} = A_D + A^n_{\frac{D}{p}} + A^t_{\frac{D}{p}} + A^c_{\frac{D}{p}}$$
$$A^n_{\frac{D}{p}} = 0$$
$$A_P = A_A + A_{\frac{P}{A}} = A^t_{\frac{P}{A}} + A^n_{\frac{P}{A}}$$
$$A_A = 0$$
$$\therefore A_D + A^t_{D/P} + A^c_{D/P} = A^t_{P/A} + A^n_{P/A}$$
$$A^c_{DP} = 2\omega_3 V_{P/D} = 2(7.78)(35)$$
$$A^c_{D/P} = 544.6\mathrm{m/s^2} \text{ perpendicular to } AD$$
$$A^n_{P/A} = -\omega_3^2 r_{\mathrm{P/A}} = -(7.78^2)(4.5)$$
$$A^n_{\frac{D}{A}} = -272.3778\mathrm{m/s^2} \; parallel \text{ to } AB$$

From polygon:

$$A^t_{\frac{P}{D}} = 460\frac{\mathrm{m}}{\mathrm{s}^2}, \qquad A_P = 1260\frac{\mathrm{m}}{\mathrm{s}^2}, \qquad A^t_{P/A} = 1240\mathrm{m/s^2}$$
$$A^t_{P/A} = \alpha_3 r_{P/A}$$
$$\alpha_3 = \frac{A^t_{P/A}}{r_{P/A}} = \frac{1240}{4.5}$$
$$\alpha_3 = 275.6\ \mathrm{rad/s^2} \quad , \alpha_1 = 0$$
$$\therefore A_B = A_A + A_{\frac{B}{A}} = A_A + A^n_{\frac{B}{A}} + A^t_{\frac{B}{A}}$$
$$A_A = 0$$
$$A^n_{B/A} = -\omega_3^2 r_{BA} = -(7.78^2)(7.5)$$
$$A^n_{\frac{B}{A}} = 453.96\mathrm{m/s^2} \quad parallel \text{ to } AB$$

From polygon:

$$A_B = 2240\ \frac{\mathrm{m}}{\mathrm{s}^2} \ \ A^t_{B/A} = 2200\frac{\mathrm{m}}{\mathrm{s}^2}$$

$$\therefore A_E = A_B + A^n_{E/B} + A^t$$
$$A^n_{E/B} = -\omega_2^2 r_{E/B} = -(20.3)^2(3)$$
$$A^n_{E/B} = -\frac{1236.27\mathrm{m}}{\mathrm{s}^2} \ parallel\ to\ EB$$

From polygon:

$$A_E = 4600\mathrm{m/s^2}$$
$$A^t_{\frac{E}{B}} = 2420\ \mathrm{m/s^2}$$

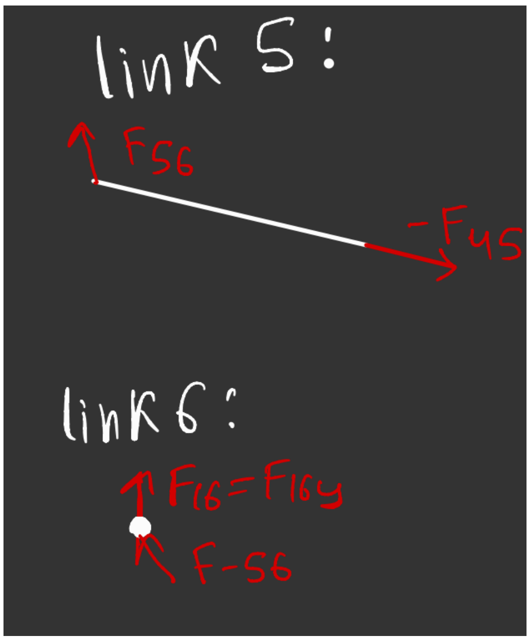
$$\therefore A_{E/B}^{t} = \alpha_2 \times r_{E/B}$$

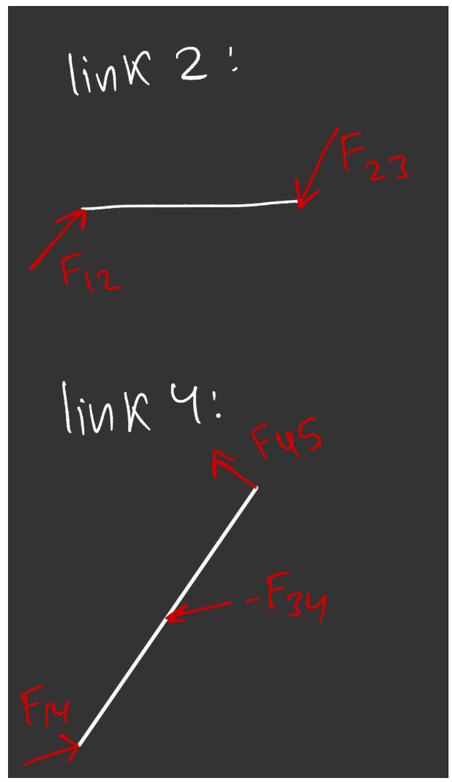
$$\alpha_2 = \frac{A_{E/B}^{t}}{r_{E/B}} = \frac{1420}{3}$$

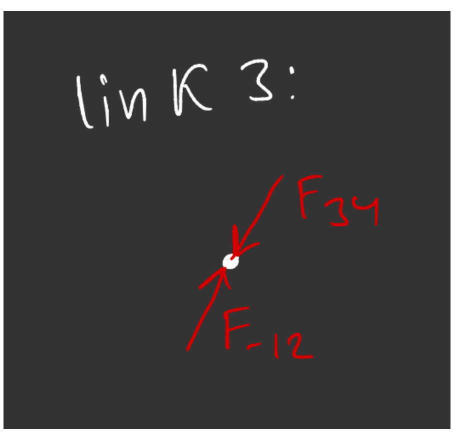
$$\alpha_2 = 473.33\text{rad/s}^2$$

Force Analysis:

Free body diagram for links 2, 3, 4, 5, and 6:







Acceleration of centroid at G_2, G_4, and G_5:

$$A_{G_2} = \sqrt{(A_{G2}^N)^2 + (A_{G2}^T)^2}$$
$$= -\omega_2^2 \times 1.25 + 0$$
$$= -20^2 \times 1.25$$
$$= -500 \text{m/s}^2$$

$$A_{G_4} = \sqrt{\left(A_{G_4}^n\right)^2 + \left(A_{G_4}^t\right)^2}$$
$$= \sqrt{(-\omega_3^2 \times 3.75)^2 + (\alpha_3 \times 3.75)^2}$$
$$= \sqrt{(7.78^2 \times 3.75)^2 + (275.6 \times 3.75)^2}$$
$$= 1058.13 \text{m/s}^2$$

$$A_{G_5} = A_B + A_{G_S}^n + A_{G_5}^t$$
$$= 2240 - \omega_2^2 \times 1.5 + \alpha_2 \times 1.5$$

Using vector addition: A_{G5} = 450 m/s^2

Equations on link 2:

$$\sum F_x = m_2 A_{G_{2x}}$$
$$F_{12x} + F_{23x} = m_2 A_{G2}^n$$
$$F_{12y} + F_{23y} = m_2 A_{G2}^t = 0$$
$$(F_{12x} \times 0) + \left(F_{12y} \times 1.25\right) + (F_{23x} \times 0) + \left(F_{23y} \times 1.25\right) + T_5 = I_2 \alpha_1 = 0$$

Equations on link 3: Slider 1

$$-F_{23x} + F_{34x} = 0$$
$$-F_{23y} + F_{34y} = 0$$
$$I_4 \alpha_3 = 0$$
$$F_{23} = F_{34}$$

Equations on link 4:

Using Pythagoras's theorem of A_{G4}^N and A_{G4}^T $\theta = 62.4$

$$F_{14x} - F_{34x} + F_{45x} = m_4 A_{G4x}$$
$$A_{G_4 x} = A_{G_4} \times \cos{(62.4°)}$$

$$A_{G_{4y}} = A_{G_4} \times \sin{(62.4°)}$$
$$F_{14y} - F_{34y} + F_{45y} = m_4 A_{G4y}$$

$$(F_{14x} \times 3.75\sin{(40°)}) + (-F_{34x} \times 0.25\sin(40°)) + (F_{45x} \times 3.75\sin{(40°)}) + \left(F_{14y} \times 3.75\cos{(40°)}\right)$$
$$+\left(-F_{34y} \times 0.25\cos\ 40°\right) + \left(F_{45y} \times 3.75\sin\ 40°\right) = I_4\alpha_3 = 96889.936$$

Equations on link 5:

$$-F_{45x} + F_{56x} = m_5 A_{G_5x}$$
$$-F_{45y} + F_{56y} = m_5 A_{G_5y}$$
$$A_{G_{5x}} = A_{G_5} \times \cos{(1.04°)}$$
$$A_{G_{5y}} = A_{G_5} \times \sin(1.04°)$$

$$(-F_{45} \times 1.5\sin 50) + (F_{56x} \times 1.5\sin 50) + (-F_{45} \times 1.5\cos 50) + \left(F_{56y} \times 1.5\cos 50°\right)$$
$$I_5\alpha_2 = 10649.925$$

Equations on link 6:

$$-F_{56x} + F_{16x} = 0$$
$$-F_{56y} + F_{16y} = 0$$
$$I_6 \propto_6 = 0$$

Matrix for pose 2:

$$x = \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{23x} \\ F_{23y} \\ F_{34x} \\ F_{14x} \\ F_{14y} \\ F_{45x} \\ F_{45y} \\ F_{56x} \\ F_{56y} \\ F_{16y} \\ T_5 \end{bmatrix} b = \begin{bmatrix} 12500 \\ 0 \\ 0 \\ 0 \\ 0 \\ 36767.1 \\ 70328.89 \\ 96889.936 \\ 13497.78 \\ 245.03 \\ 10649.93 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.25 & 0 & 1.25 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.16 & 2.41 & 2.87 & 2.41 & 2.87 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.15 & 0.96 & 1.15 & 0.96 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

F_{12x}*(10^5N)	F_{12y}*(10^5N)	F_{23x}*(10^5N)	F_{23y}*(10^5N)	F_{34x}*(10^5N)	F_{14x}*(10^5N)
0.125	0	0	0	0	1.3366

F_{14y}*(10^5N)	F_{45x}*(10^5N)	F_{45y}*(10^5N)	F_{56x}*(10^5N)	F_{56y}*(10^5N)	F_{16y}*(10^5N)
0.8383	5.6270	-0.9664	-0.0285	1.3489	-0.0025

Table x: Force values for position 2

Torque = -0.1065 x 10^5 Nm

State 3:

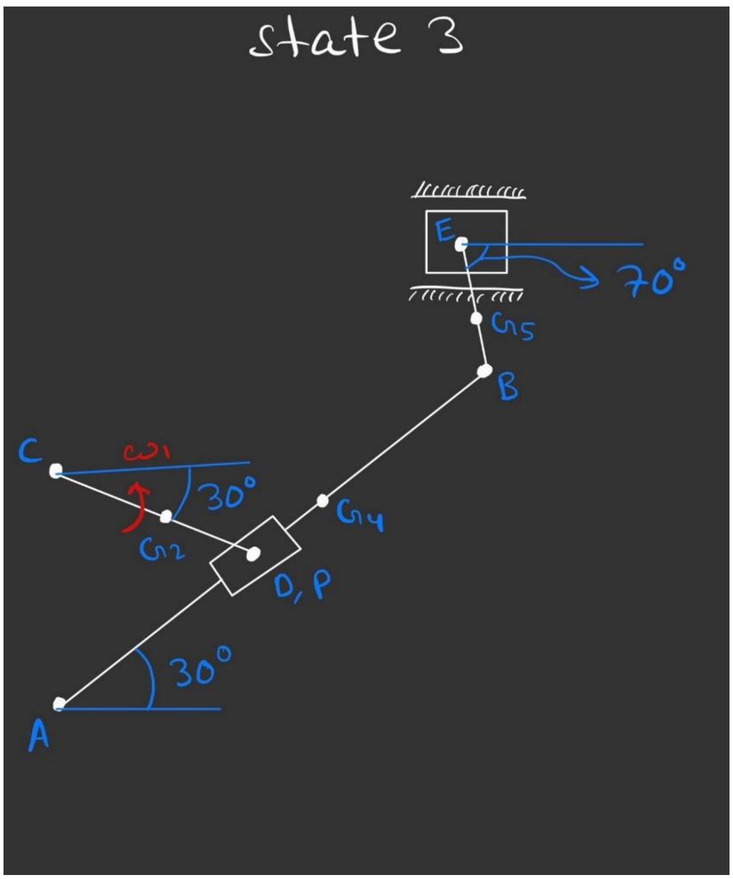


Figure 10: 3rd state of mechanism

Coincident point at slider but on link 3

$\vec{v}_D = \vec{v}_C + \vec{v}_{D/c}$
$\vec{v}_c = 0\text{m/s}$
$\vec{v}_D = \vec{v}_{D/C} = \vec{\omega}_1 \times \vec{r}_{D/C} = 50\text{m/s}$
$\vec{v}_P = \vec{v}_D + \vec{v}_{P/D} = \vec{v}_A + \vec{v}_{P/A}$
$\vec{v}_A = 0 \qquad \vec{V}_{P/D} = 43\text{m/s}$

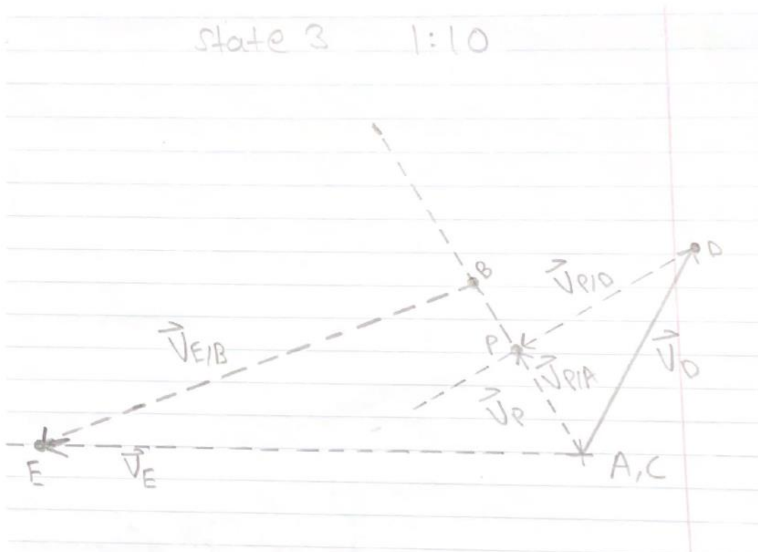


Figure 11: velocity polygon for 3rd state

$\vec{v}_P = \vec{v}_D + \vec{v}_{P/D} = 25\text{m/s}$
$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = 0 + \vec{v}_{B/A}$
$\vec{v}_{B/A} = \vec{\omega}_3 \times \vec{r}_{B/A}$
$\vec{v}_P = \vec{\omega}_3 \times \vec{r}_{P/A}$
$\vec{\omega}_3 = \dfrac{\vec{v}_P}{\vec{r}_{P/A}} = \dfrac{25}{4.5} = 5.56\text{rad/s}$
$\vec{v}_{B/A} = \vec{v}_B = 41.7\text{m/s}$
$\vec{v}_E = \vec{v}_B + \vec{v}_{E/B} = 115\text{m/s}$
$\vec{\omega}_2 = \dfrac{\vec{v}_{E/B}}{\vec{r}_{E/B}} = \dfrac{102}{3} = 34\text{m/s}$
$\vec{v}_{E/B} = 102\text{m/s}$

Figure 12: Acceleration polygon for 3rd state with a scale of 1:600.

$$A_D = A_C + A_{\frac{D}{C}} \quad , A_C = 0$$
$$A_D = A^n_{\frac{D}{C}} + A^t_{\frac{D}{C}} \quad , A^t_{\frac{D}{C}} = 0$$
$$A_D = -\omega_1^2 r_{DC} = -(20^2)(2.5)$$
$$A_D = -1000m/s^2 \quad parallel \text{ to } CD$$

Point P coincident to D

CHECK FOR P'S/D'S

$$A_P = A_D + A_{D/P} = A_D + A^n_{D/P} + A^t_{D/P} + A^c_{D/P}$$
$$A_D = A_A + A_{\frac{P}{A}} = A^t_{\frac{P}{A}} + A^n_{\frac{P}{A}}, \quad A_A = 0, A^n_{\frac{D}{P}} = 0$$
$$A_D + A^t_{D/P} + A^c_{D/P} = A^t_{P/A} + A^n_{P/A}$$
$$A^C_{D/P} = 2\omega_3 V_{P/D} = 2(5.56)(43)$$
$$A^c_{\frac{D}{P}} = \frac{478.16\text{m}}{\text{s}^2} \quad perpendicular \text{ to AD}$$
$$A^n_{P/A} = -\omega_3^2 r_{P/A} = -(5.56^2)(4.5)$$

$$A^n_{P/A} = \frac{139.1112\text{m}}{\text{s}^2} \; parallel \text{ to } AB$$

From Polygon:

$$A^t_{P/A} = 1440\text{m/s}^2 \quad A_P = \frac{1440\text{m}}{\text{s}^2} \quad A^t_{D/p} = 720\text{m/s}^2$$

$$A^t_{P/n} = \alpha_3 r_{D/A}$$
$$\alpha_3 = \frac{A^t_{\mathrm{P/A}}}{r_{P/A}} = \frac{1440}{4.5}$$
$$\alpha_3 = 320 \text{ rad/s}^2 \quad \alpha_1 = 0$$
$$A_B = A_A + A_{\frac{B}{A}} = A_A + A^n_{\frac{B}{A}} + A^t_{\frac{b}{A}}$$
$$A_A = 0$$
$$A^n_{\mathrm{B/A}} = -\omega_3^2 r_{B/A} = -(5.56)^2(7.5)$$
$$A^n_{\mathrm{B/A}} = -\frac{231.852\mathrm{m}}{\mathrm{s}^2} \; parallel \text{ to } AB$$

From polygon:

$$A_B = \frac{2400\mathrm{m}}{\mathrm{s}^2} \qquad A^t_{B/\mathrm{A}} = 2340\mathrm{m/s}^2$$

$$\therefore A_E = A_B + A^n_{E/B} + A^t_{E/B}$$
$$A^n_{EB} = -\omega_2^2 r_{E/B} = -34^2(3)$$
$$A^n_{\frac{E}{B}} = -\frac{3468\mathrm{m}}{\mathrm{s}^2} \quad parallel \text{ to } EB$$

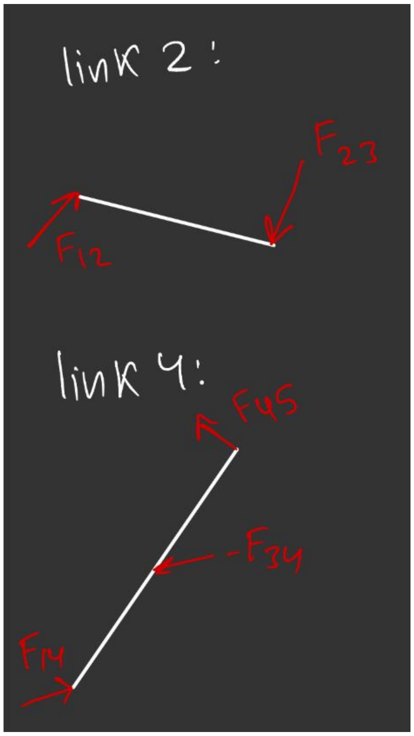
From polygon:

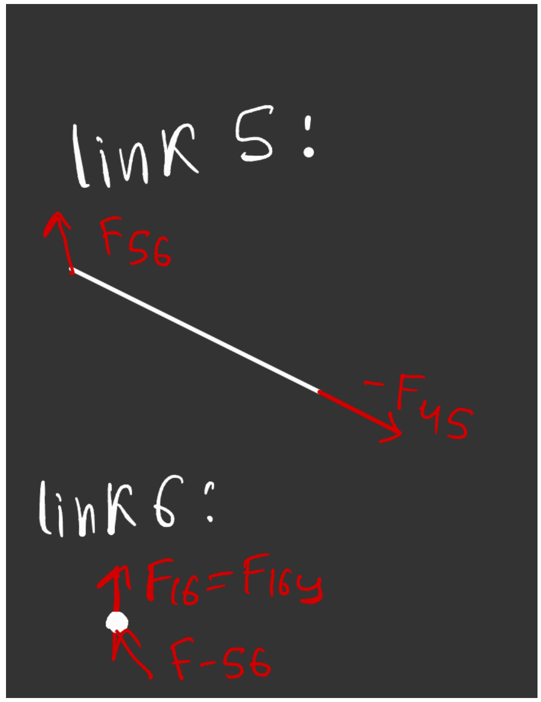
$$A^t_{E/B} = 13620\mathrm{m/s}^2 \qquad A_E = 15180\mathrm{m/s}^2$$

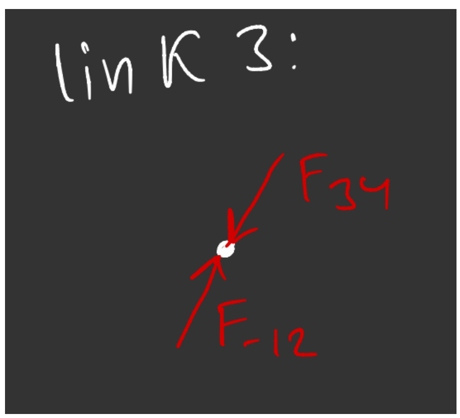
$$A^t_{\mathrm{E/B}} = \alpha_2 r_{E/B}$$
$$\alpha_2 = \frac{A^t_{E/B}}{r_{EB}} = \frac{13620}{3}$$
$$\alpha_2 = 4540 \text{ rad/s}^2$$

Force Analysis:

Free body diagrams for links 2, 3, 4, and 5:







Acceleration of centroid at G_2, G_4, and G_5:

$$A_{G_2} = \sqrt{(A_{G2}^{N})^2 + (A_{G2}^{T})^2}$$
$$= -\omega_2^2 \times 1.25 + 0$$
$$= -20^2 \times 1.25$$
$$= -500\mathrm{m/s^2}$$

$$A_{G_4} = \sqrt{\left(A_{G_4}^{n}\right)^2 + \left(A_{G_4}^{t}\right)^2}$$
$$= \sqrt{(-\omega_3^2 \times 3.75)^2 + (\alpha_3 \times 3.75)^2}$$
$$= \sqrt{(5.56^2 \times 3.75)^2 + (320 \times 3.75)^2}$$
$$= 1205.59\mathrm{m/s^2}$$

$$A_{G_5} = A_B + A_{G_S}^{n} + A_{G_5}^{t}$$
$$= 2400 - \omega_2^2 \times 1.5 + \alpha_2 \times 1.5$$

Using vector addition: A_{G5} = 470 m/s^2

Equations on link 2:

$$\sum F_x = m_2 A_{G_{2x}}$$
$$F_{12x} + F_{23x} = m_2 A_{G2} \cos(30)$$
$$F_{12y} + F_{23y} = m_2 A_{G2} \sin(30) =$$
$$(F_{12x} \times 0.75) + \left(F_{12y} \times 1.29\right) + (F_{23x} \times 0.75) + \left(F_{23y} \times 1.29\right) + T_5 = I_2 \alpha_1 = 0$$

Equations on link 3: Slider 1

$$-F_{23x} + F_{34x} = 0$$
$$-F_{23y} + F_{34y} = 0$$
$$I_4 \alpha_3 = 0$$
$$F_{23} = F_{34}$$

Equations on link 4:

Using Pythagoras's theorem of A_{G4}^{N} *and* A_{G4}^{T} $\theta = 65.51$

$$F_{14x} - F_{34x} + F_{45x} = m_4 A_{G4x}$$
$$A_{G_4x} = A_{G_4} \times \cos(65.51°)$$

$$A_{G_4y} = A_{G_4} \times \sin(65.51°)$$
$$F_{14y} - F_{34y} + F_{45y} = m_4 A_{G4y}$$

$$(F_{14x} \times 3.75\sin(30°)) + (-F_{34x} \times 0.25\sin(30°)) + (F_{45x} \times 3.75\sin(30°)) + (F_{14y} \times 3.75\cos(30°))$$
$$+(-F_{34y} \times 0.25\cos 30°) + (F_{45y} \times 3.75\sin 30°) = I_4\alpha_3 = 112499.2$$

Equations on link 5:

$$-F_{45x} + F_{56x} = m_5 A_{G_5 x}$$
$$-F_{45y} + F_{56y} = m_5 A_{G_5 y}$$
$$A_{G_{5x}} = A_{G_5} \times \cos(5.71°)$$
$$A_{G_{5y}} = A_{G_5} \times \sin(5.71°)$$

$$(-F_{45} \times 1.5\sin 70) + (F_{56x} \times 1.5\sin 70) + (-F_{45} \times 1.5\cos 70) + (F_{56y} \times 1.5\cos 70°)$$
$$I_5\alpha_2 = 102150$$

Equations on link 6:

$$-F_{56x} + F_{16x} = 0$$
$$-F_{56y} + F_{16y} = 0$$
$$I_6 \propto_6 = 0$$

Matrix for pose 3:

$$x = \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{23x} \\ F_{23y} \\ F_{34x} \\ F_{14x} \\ F_{14y} \\ F_{45x} \\ F_{45y} \\ F_{56x} \\ F_{56y} \\ F_{16y} \\ T_5 \end{bmatrix} b = \begin{bmatrix} 10825 \cdot 3 \\ 6250 \\ 0 \\ 0 \\ 0 \\ 37481 \cdot 9 \\ 82284.56 \\ 112499 \cdot 2 \\ 14030.04 \\ 1402.86 \\ 102150 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.75 & 1.29 & 0.75 & 1.29 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.13 & 1.88 & 3.25 & 1.88 & 3.25 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.41 & 0.51 & 1.41 & 0.51 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

F_{12x}*(10^5N)	F_{12y}*(10^5N)	F_{23x}*(10^5N)	F_{23y}*(10^5N)	F_{34x}*(10^5N)	F_{14x}*(10^5N)
0.1083	0.0625	0.1618	0	0	1.4998

F_{14y}*(10^5N)	F_{45x}*(10^5N)	F_{45y}*(10^5N)	F_{56x}*(10^5N)	F_{56y}*(10^5N)	F_{16y}*(10^5N)
0.9631	5.9499	-1.1110	0.8812	2.1985	-0.00140

Torque = -1.0215 x 10^5 Nm

State 4:

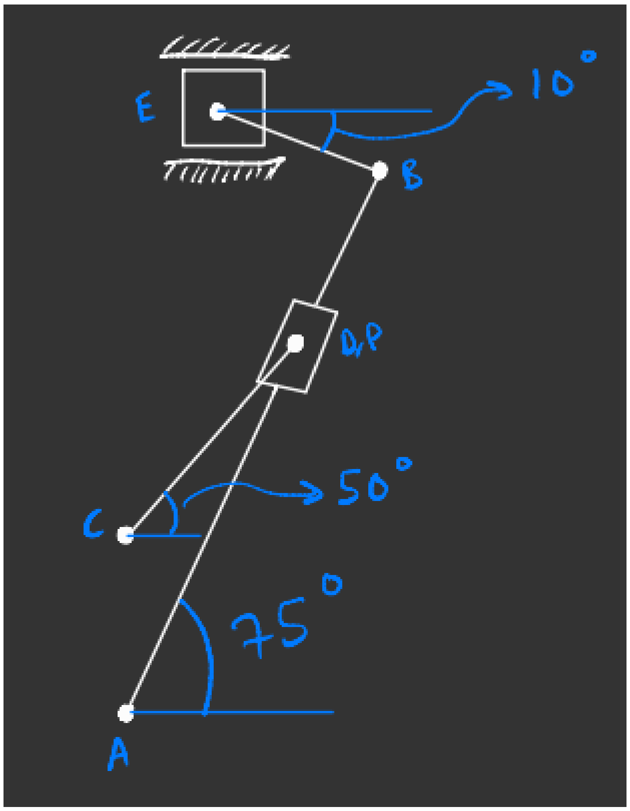


Figure 13: 4th state of mechanism

Coincident point at slider but on link 3

$\vec{V}_D = \vec{V}_C + \vec{V}_{D/C}$
$\vec{v}_C = 0\text{m/s}$
$\vec{V}_D = \vec{V}_{D/C} = \vec{\omega}_1 \times \vec{r}_{D/C} = 50\text{m/s}$
$\vec{V}_P = \vec{v}_D + \vec{V}_{P/D} = \vec{v}_A + \vec{V}_{P/A}$
$\vec{V}_A = 0 \qquad \vec{V}_{P/D} = 22\text{m/s}$

Velocity Polygon 4

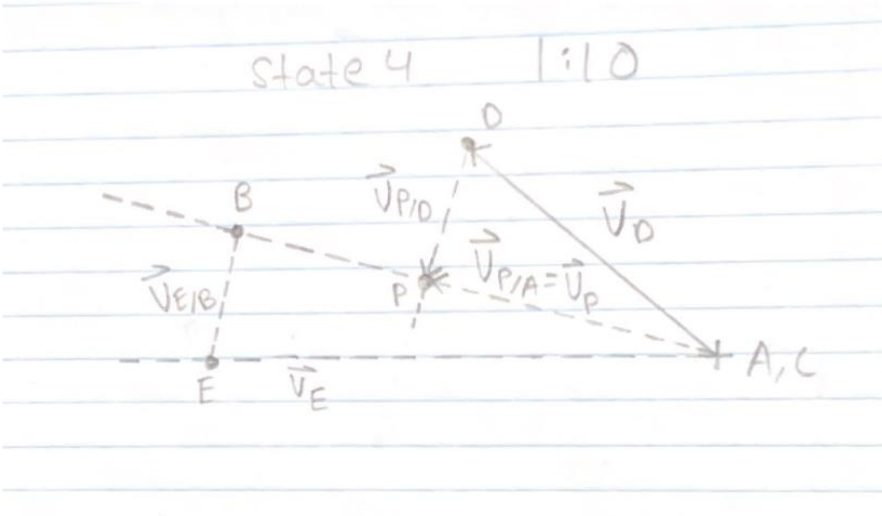


Figure 14: Rough velocity polygon for 4th state

$\vec{v}_P = \vec{v}_D + \vec{v}_{P/D} = 46\text{m/s}$
$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = 0 + \vec{v}_{B/A}$
$\vec{V}_{B/A} = \vec{\omega}_3 \times \vec{r}_{B/A}$
$\vec{v}_P = \vec{\omega}_3 \times \vec{r}_{P/A}$

$$\vec{\omega}_3 = \frac{\vec{v}_P}{\vec{r}_{P/A}} = \frac{46}{4.5} = 10.2\text{rad/s}$$

$\vec{v}_{B/A} = \vec{V}_B = 76.7\text{m/s}$
$\vec{v}_E = \vec{V}_B + \vec{V}_{E/B} = 78\text{m/s}$

$$\vec{\omega}_2 = \frac{\vec{v}_{E/B}}{\vec{r}_{E/B}} = \frac{20}{3} = 6.67\text{m/s}$$

$\vec{v}_{E/B} = 20\text{m/s}$

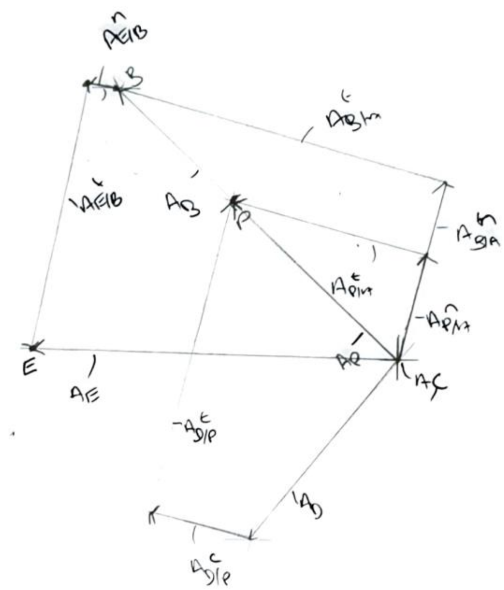


Figure 15: Acceleration polygon for 4th state with a scale of 1:200.

$$A_D = A_C + A_{\frac{D}{C}} \ , A_C = 0$$

$$A_D = A^n_{\frac{D}{C}} + A^t_{\frac{D}{C}} \ , A^t_{\frac{D}{C}} = 0$$

$$A_D = -\omega_1^2 r_{D/C} = -(20^2)(25)$$

$$A_D = -\frac{1000m}{s^2} \ \ parallel \text{ to } CD$$

Point P coincident to point D

$$A_P = A_D + A_{\frac{D}{P}} = A_D + A^n_{\frac{D}{P}} + A^t_{\frac{D}{P}} + A^c_{\frac{D}{P}}$$

$$A^n_{\frac{D}{P}} = 0$$

$$A_P = A_A + A_{\frac{P}{A}} = A^t_{\frac{P}{A}} + A^n_{\frac{P}{A}}$$

$$A_A = 0$$

$$A_D + A^t_{\frac{D}{P}} + A^c_{\frac{D}{P}} = A^t_{\frac{P}{A}} + A^n_{\frac{P}{A}}$$

$$A^c_{\frac{D}{P}} = 2\omega_3 V_{P/D} = 2(102)(22)$$
$$A^c_{\frac{D}{P}} = \frac{448.8\text{m}}{\text{s}^2}\ \textit{perpendicular}\text{ to }AB$$
$$A^n_{P/A} = -\omega_3^2\left(r_{P/A}\right) = -(10.2)^2(4.5)$$
$$A^n_{P/A} = -\frac{468.18\text{m}}{\text{s}^2}\ \ \textit{parallel}\text{ to AB}$$

From Polygon:

$$A^t_{P/A} = 900\text{m/s}^2 \qquad A_P = 1000\text{m/s}^2 \qquad A^t_{D/\text{p}} = 1380\text{m/s}^2$$
$$A^t_{P/A} = \alpha_3 r_{P/A}$$
$$\alpha_3 = \frac{\text{A}^t_{P/A}}{r_{P/A}} = \frac{900}{4.5}$$
$$\alpha_3 = 200\frac{rad}{s^2} \qquad \alpha_1 = 0$$

$$A_B = A_A + A_{\frac{B}{A}} = A_A + A^n_{\frac{B}{A}} + A^t_{\frac{B}{A}}$$
$$A_A = 0$$
$$A^n_{\frac{B}{A}} = -\omega_3^2 r_{B/A} = -(10.2)^2(7.5)$$
$$A^n_{B/A} = -\frac{780.3\text{m}}{\text{s}^2}\ \textit{parallel}\text{ to }AB$$

From polygon:

$$A_B = 1700\text{m/s}^2 \qquad A^t_{B/A} = 1500\text{m/s}^2$$

$$\therefore A_E = A_B + A^n_{E/B} + A^t_{E/B}$$
$$A^n_{E/B} = -\omega_2^2 r_{E/B} = -(6.67)^2(3)$$
$$A^n_{\frac{E}{B}} = -133.46\frac{\text{m}}{\text{s}^2}\text{ parallel to }EB$$

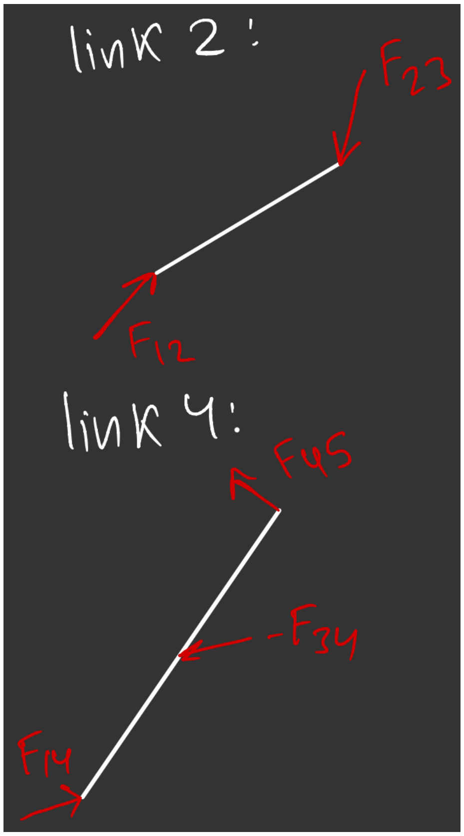
From polygon:

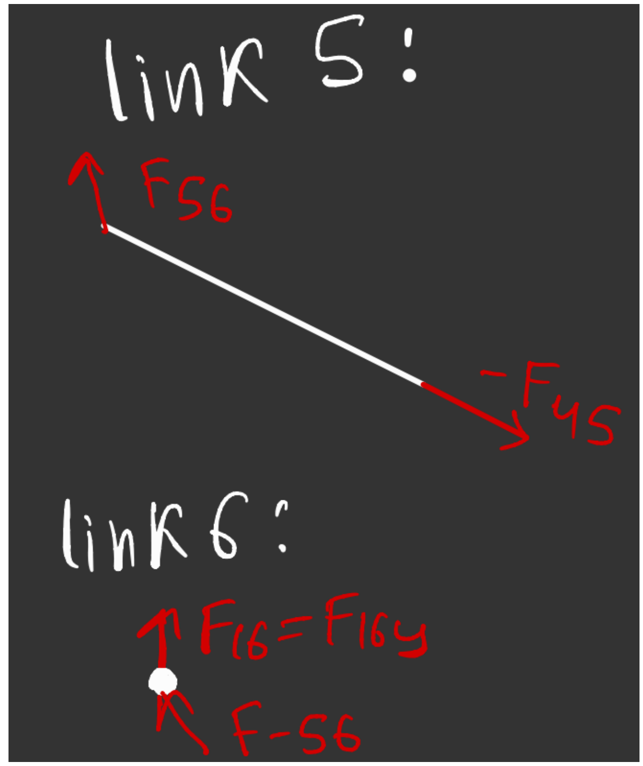
$$A_E = 1600\text{ m/s}^2 \qquad A^t_{\frac{\text{E}}{\text{B}}} = 1060\ m/s^2$$

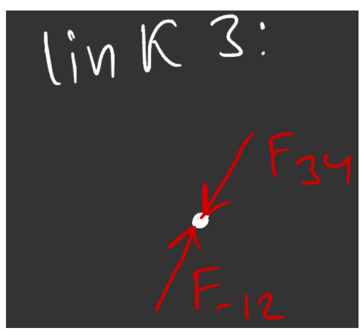
$$\therefore A^t_{E/B} = \alpha_2 r_{EB}$$
$$\alpha_2 = \frac{A^t_{E/B}}{r_{EB}} = \frac{1060}{3}$$
$$\alpha_2 = 353.33\text{ rad/s}^2$$

Force Analysis:

Free body diagrams for links 2, 3, 4, and 5:







Acceleration of centroid at G_2, G_4, and G_5:

$$
\begin{aligned}
A_{G_2} &= \sqrt{(A_{G2}^{N})^2 + (A_{G2}^{T})^2} \\
&= -\omega_2^2 \times 1.25 + 0 \\
&= -20^2 \times 1.25 \\
&= -500\text{m/s}^2
\end{aligned}
$$

$$A_{G_4} = \sqrt{\left(A_{G_4}^n\right)^2 + \left(A_{G_4}^t\right)^2}$$
$$= \sqrt{(-\omega_3^2 \times 3.75)^2 + (\alpha_3 \times 3.75)^2}$$
$$= \sqrt{(10.2^2 \times 3.75)^2 + (200 \times 3.75)^2}$$
$$= 845.41 \mathrm{m/s}^2$$

$$A_{G_5} = A_B + A_{G_S}^n + A_{G_5}^t$$
$$= 1500 - \omega_2^2 \times 1.5 + \alpha_2 \times 1.5$$

Using vector addition: A_{G5} = 385 m/s^2

Equations on link 2:

$$\sum F_x = m_2 A_{G_{2x}}$$
$$F_{12x} + F_{23x} = m_2 A_{G2} \cos(50)$$
$$F_{12y} + F_{23y} = m_2 A_{G2} \sin(50) =$$
$$(F_{12x} \times 1.15) + \left(F_{12y} \times 0.96\right) + (F_{23x} \times 1.15) + \left(F_{23y} \times 0.96\right) + T_5 = I_2 \alpha_1 = 0$$

Equations on link 3: Slider 1

$$-F_{23x} + F_{34x} = 0$$
$$-F_{23y} + F_{34y} = 0$$
$$I_4 \alpha_3 = 0$$
$$F_{23} = F_{34}$$

Equations on link 4:

Using Pythagoras's theorem of A_{G4}^N *and* A_{G4}^T $\theta = 42.48$

$$F_{14x} - F_{34x} + F_{45x} = m_4 A_{G4x}$$
$$A_{G_4 x} = A_{G_4} \times \cos(42.48°)$$

$$A_{G_4 y} = A_{G_4} \times \sin(42.48°)$$
$$F_{14y} - F_{34y} + F_{45y} = m_4 A_{G4y}$$

$$(F_{14x} \times 3.75 \sin(75°)) + (-F_{34x} \times 1.25 \sin(75°)) + (F_{45x} \times 3.75 \sin(75°)) + \left(F_{14y} \times 3.75 \cos(75°)\right)$$
$$+\left(-F_{34y} \times 1.25 \cos 75°\right) + \left(F_{45y} \times 3.75 \sin 75°\right) = I_4 \alpha_3 = 70312$$

Equations on link 5:

$$-F_{45x} + F_{56x} = m_5 A_{G_5 x}$$
$$-F_{45y} + F_{56y} = m_5 A_{G_5 y}$$
$$A_{G_{5x}} = A_{G_5} \times \cos(72.48°)$$
$$A_{G_{5y}} = A_{G_5} \times \sin(72.48°)$$

$$(-F_{45} \times 1.5 \sin 10) + (F_{56x} \times 1.5 \sin 10) + (-F_{45} \times 1.5 \cos 10) + \left(F_{56y} \times 1.5 \cos 10^{\circ}\right)$$
$$I_5\alpha_2 = 7949.925$$

Equations on link 6:

$$-F_{56x} + F_{16x} = 0$$
$$-F_{56y} + F_{16y} = 0$$
$$I_6 \propto_6 = 0$$

Matrix for pose 4:

$$x = \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{23x} \\ F_{23y} \\ F_{34x} \\ F_{14x} \\ F_{14y} \\ F_{45x} \\ F_{45y} \\ F_{56x} \\ F_{56y} \\ F_{16y} \\ F_5 \end{bmatrix} b = \begin{bmatrix} 8034.85 \\ 9575.56 \\ 0 \\ 0 \\ 0 \\ 46762.57 \\ 42819.98 \\ 70312 \\ 3411.58 \\ 11034.66 \\ 7949.925 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.15 & 0.96 & 1.15 & 0.96 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 1.21 & 3.62 & 0.97 & 3.62 & 0.97 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.26 & 1.47 & 0.26 & 1.47 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

F_{12x}*(10^5N)	F_{12y}*(10^5N)	F_{23x}*(10^5N)	F_{23y}*(10^5N)	F_{34x}*(10^5N)	F_{14x}*(10^5N)
0.0803	0.0958	0.1843	0	0	1.1707

F_{14y}*(10^5N)	F_{45x}*(10^5N)	F_{45y}*(10^5N)	F_{56x}*(10^5N)	F_{56y}*(10^5N)	F_{16y}*(10^5N)
0.4623	4.6865	-0.5928	0.0454	0.3785	-0.1103

Torque = -0.0795 x 10^5 Nm

Final Values Summation:

Table 1: Velocity Summary

Vector	State 1 (m/s)	State 2 (m/s)	State 3 (m/s)	State 4 (m/s)
V_D	50	50	50	50
V_C	0	0	0	0
V_A	0	0	0	0
$V_{P/D}$	29	35	43	22
$V_{P/A} = V_P$	41	35	25	46
$V_B = V_{B/A}$	68.33	58	41.7	10.2
V_E	74	87	115	78
$V_{E/B}$	36	61	102	20

When the rise angle of link CD is lowered the velocities of point P and B decrease and for the other points it increases.

Table 2: Angular Velocity Summary

Vector	State 1 (rad/s)	State 2 (rad/s)	State 3 (rad/s)	State 4 (rad/s)
ω_1 (Link CD)	20	20	20	20
ω_2 (Link BE)	12	20.3	34	6.67
ω_3 (Link AB)	9.11	7.78	5.56	10.2

When the rise angle link CD decreases angular velocity of link AB decreases and the angular velocity of link EB increases.

Table 3: Acceleration Summary

Vector	State 1 $(\mathrm{m/s^2})$	State 2 $(\mathrm{m/s^2})$	State 3 $(\mathrm{m/s^2})$	State 4 $(\mathrm{m/s^2})$
A_D	-1000 $\parallel$ CD	-1000 $\parallel$ CD	-1000 $\parallel$ CD	-1000 $\parallel$ CD
$A^{C}_{\frac{D}{P}}$	528.38 $\perp$ AB	544.6 $\perp$ AD	478.16 $\perp$ AD	448.8 $\perp$ AB
$A^{n}_{\frac{D}{A}}$	-373.46 $\parallel$ AB	-272.3778 $\parallel$ AB	-139.1112 $\parallel$ AB	468.18 $\parallel$ AB
$A^{t}_{\frac{D}{P}}$	500	460	1300	1380
$A^{t}_{\frac{P}{A}}$	1100	1240	1440	900
A_D	1180	1260	1440	1000
$A^{n}_{\frac{B}{A}}$	-622.44 $\parallel$ AB	453.96 $\parallel$ AB	-231.85 $\parallel$ AB	-780.3 $\parallel$ AB
$A^{t}_{\frac{B}{A}}$	1600 $\perp$ AB	2200	2340	1500
A_B	1640 $\parallel$ AP	2240	2400	1700
$A^{n}_{\frac{E}{B}}$	432	-1236.27 $\parallel$ EB	-3468 $\parallel$ EB	-133.46 $\parallel$ EB
$A^{t}_{\frac{E}{B}}$	520	2420	13620	1060
A_E	2340	4600	15180	1600

When the rise angle of link CD is lowered the accelerations increase for majority of the tangential and Coriolis components and decrease for the rest.

Table 4: Angular Accelerations Summary

Vector	State 1 (rad/s^2)	State 2 (rad/s^2)	State 3 (rad/s^2)	State 4 (rad/s^2)
α_1 (Link CD)	0	0	0	0
α_2 (Link EB)	173.33	473.33	4540	353.33
α_3 (Link AB)	244.44	275.6	320	200

When the rise angle link CD decreases angular acceleration 2 and 3 increase.

Table 5: Force vectors and Torque summary

Force vector x10^5N	State 1	State 2	State 3	State 4
F_{12x}	0.1133	0.125	0.1083	0.0803
F_{12y}	0.0528	0	0.0625	0.0958
F_{23x}	0.156	0	0.1618	0.1843
F_{23y}	0	0	0	0
F_{34x}	0	0	0	0
F_{14x}	0.7177	1.3366	1.4998	1.1707
F_{14y}	0.8233	0.8383	0.9631	0.4623
F_{45x}	3.8762	5.627	5.9499	4.6865
F_{45y}	0.8068	-0.9664	-1.111	-0.5928
F_{56x}	0.0722	-0.0285	0.8812	0.0454
F_{56y}	0.7788	1.3489	2.1985	0.3785
F_{16y}	-0.0526	-0.0025	-0.0014	-0.1103
Torque x10^5N.m	-0.039	-0.1065	-1.0215	-0.0795

Kinetic Analysis

Whilst performing the force analysis some assumptions had to be made due to the limitation of choosing lengths and angular velocities. The first assumption in the force analysis was the massless slider consideration, due to multiple forces present and moving at a remarkably high rate of 20 rad/seconds the slider was kept massless thus causing two reaction forces to equal the same value. The second assumption was made that the centroids of each link are exactly in the center throughout all states and that the distances remain the same and that there were no centroids on the sliders as they are re-imagined as pin slots.

Simscape model development

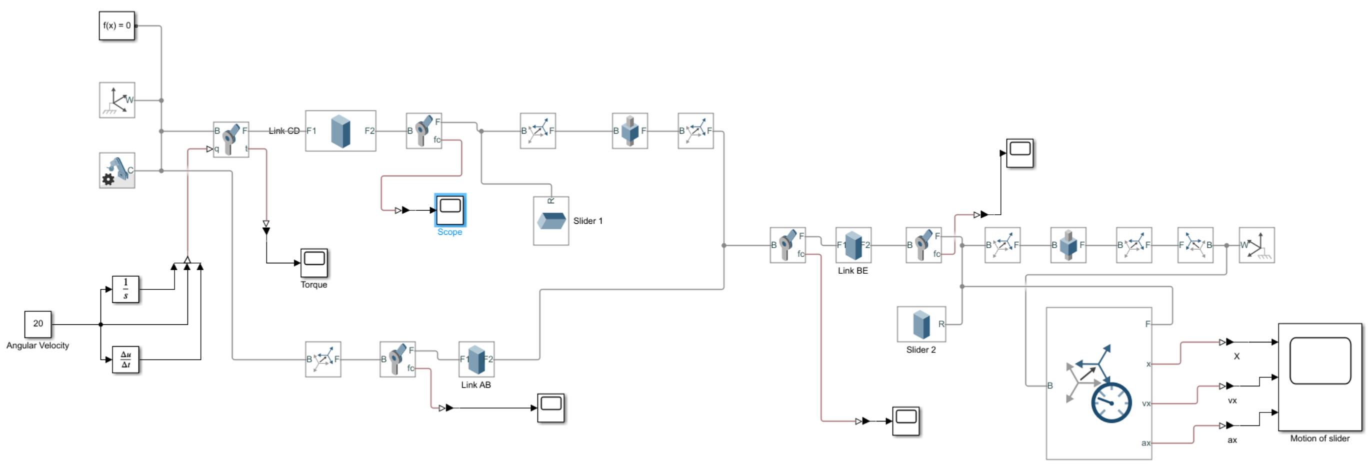


Figure 16: Simscape Multibody Schematic

The Simscape Multibody was used to get a 3D simulation of the mechanism. The final schematic can be seen below. To set up the working grounds for the system, a solver configuration, world frame, and mechanism configuration (with a negative z acceleration of 9.81 m/s^2) blocks were created. Next, a massless pin along with a revolute joint placed on the world frame were assembled, allowing the connection of the link CD. Link CD was connected to another revolute joint for the first slider. The slider was made with the help of a prismatic joint, in rotation configuration in +Y axis followed by an angle +90^o and -90^o with the help of rigid transform. The links and slider were specified in terms of density, dimensions, and position in the subsystem shown in Figure 16. Slider is assumed to be massless, 2g in the case of simulation. Link AB was off-set by 6m in negative y-direction via rigid-transform. Link AB's follower was slider to have a constrained motion. Link BE is followed with AB by a revolute joint. The other side of link BE is connected to the slider with an offset of 3m in y-direction, moved by a prismatic joint, in rotation configuration in +X axis followed by an angle +90^o and -90^o with the help of rigid transform. Reaction forces of each revolute joint is analyzed by composite force sensing and torque for input link by connecting to scope. The graph of torque and all the reaction can be seen in Appendix for comparison of results with an analytical model of the mechanism.

Results and Comparisons

When comparing the Simscape graphs with the derived values, the values have been consistent with each other with slight variation. When looking at the graphs in the appendix, the x and y direction forces are represented by the blue and yellow lines respectively. The graphs start off peaking at around 0.1 seconds as it requires more force to get the linkage into motion than to keep it in motion.

On average the force analysis had an average % error of about 56% for position 1, 48% for position 2, 67% for position 3 and 51% for position 4. Using the time array from the Simscape model and got corresponding torque and reaction forces in MATLAB workspace. This made comparison easier with analytical results. The expected force analysis was always larger than the extrapolated data from Simulink. The reason for this is because when doing the force analysis, the sliders were assumed to be massless, neglecting any inertial forces from the analysis while on Simscape the sliders were removed entirely and replaced with a solid mass of about 2 grams, this was done in order to avoid a dividing by 0 error in MATLAB when solving the force matrix. Furthermore, when taking the moments of inertia of every link, they were assumed to be cylindrical, while they were cubic in the Simscape model.

Actuator Selection

Based off Simulink data (see figure 17), the mechanism's actuator needs to be able to sustain a torque of 542036 Nm

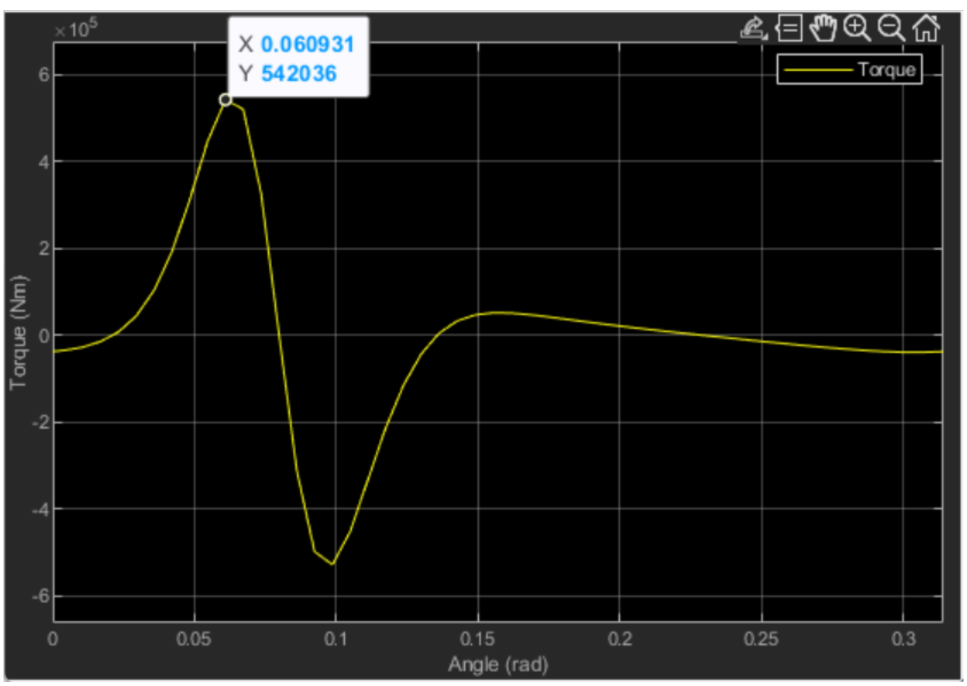


Figure 17: Simscape torque vs time curve for link 2

However, commercially there are no rotary actuators which are capable of handling such high torques. Therefore, there are two proposed solutions. First is to reduce the expected input angular velocity from 20 rad/s to around 5 rad/s, this will decrease the torque significantly. Secondly, to change the geometry of the linkage, by scaling down the linkage less force and torque will be required to put the mechanism in motion.

If the proposed solutions were to be accepted, the selected actuator would be the double piston rotary actuator type 200000 by Hunger Hydraulics [3]. This actuator uses two 180 mm pistons to create a couple moment around a rack and pinion which can produce up to 207800 Nm of torque. See technical data below.

Figure 18: Double piston rotary actuator [3]

Type	Torque (calculated) Нм	Piston Diameter мм	Absorption Capacity l/180°
3000	3180	50	0,76
6000	6060	63	1,27
13000	13030	80	2,60
28000	28500	100	5,65
36000	36600	100	7,23
55000	55660	125	11,03
85000	88180	150	17,48
100000	100770	150	19,91
145000	144280	180	28,58
185000	184680	180	36,49
200000	207800	180	40,96

Figure 19: Technical data for double piston rotary actuator [3]

Conclusion

For the 4-bar mechanism, a dynamic analysis was performed in both Simscape and by hand. Kinematic analysis was done graphically by hand where both the velocity and acceleration analysis were obtained. The results obtained from the kinematics and force analysis were compared with the generated Simscape model. The results obtained were compared and the discrepancies were discussed in the result comparison section. The actuator selected was a double piston rotary actuator with a maximum torque of 207800 Nm. This actuator would only be appropriate if the proper adjustments are made to the original design such as lowering the expected angular velocity and downscaling the entire linkage. The quick return mechanism minimizes the total processing time by controlling the forward action for clean and sharp cuts, and the backward action to return the slider back to its original position.

References

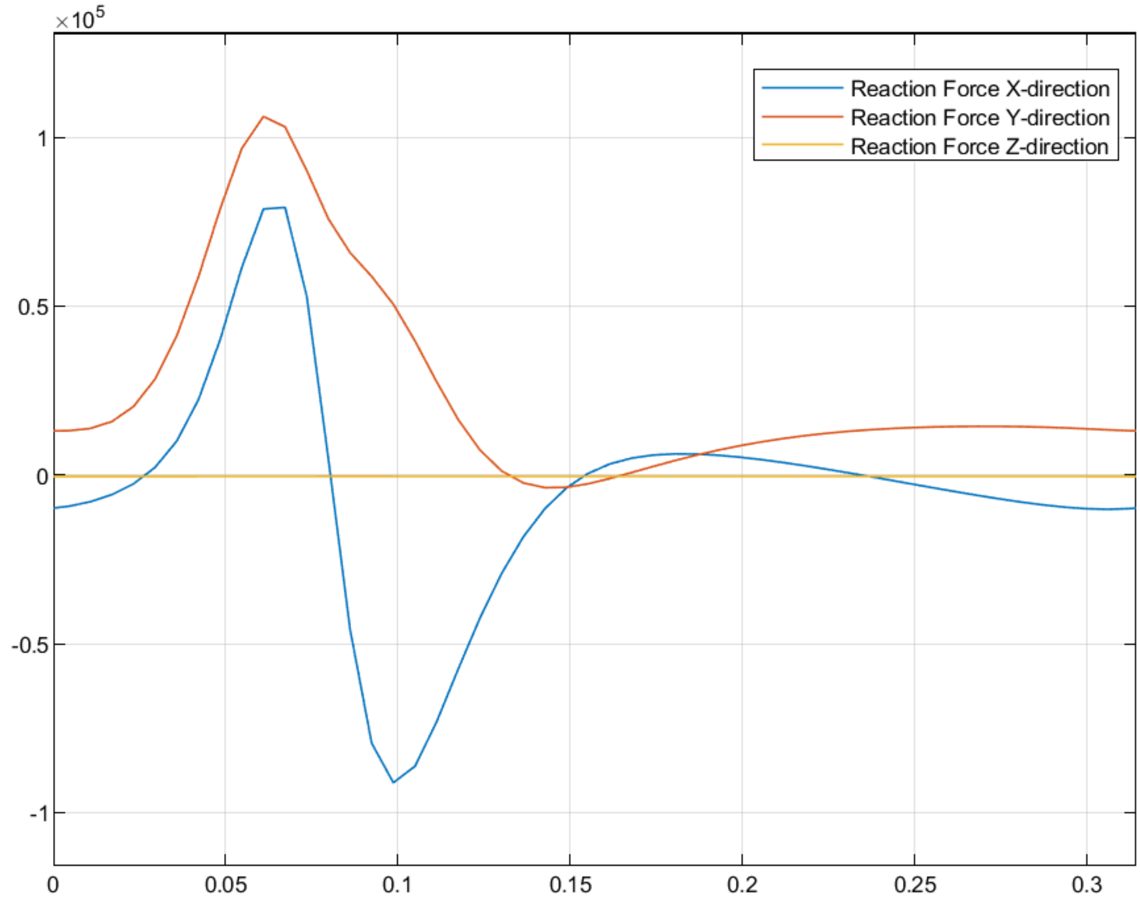
[1] “Quick return mechanism: Application, parts, types, working,” *Student Lesson*, 24-Feb-2022. [Online]. Available: https://studentlesson.com/definition-applications-components-types-working-advantages-and-disadvantages-of-quick-return-mechanism/. [Accessed: 21-Oct-2022].

[2] A. K. Dey, “Shaper machine,” *Learn Mechanical*, 28-Feb-2019. [Online]. Available: https://learnmechanical.com/shaper-machine/. [Accessed: 07-Nov-2022]

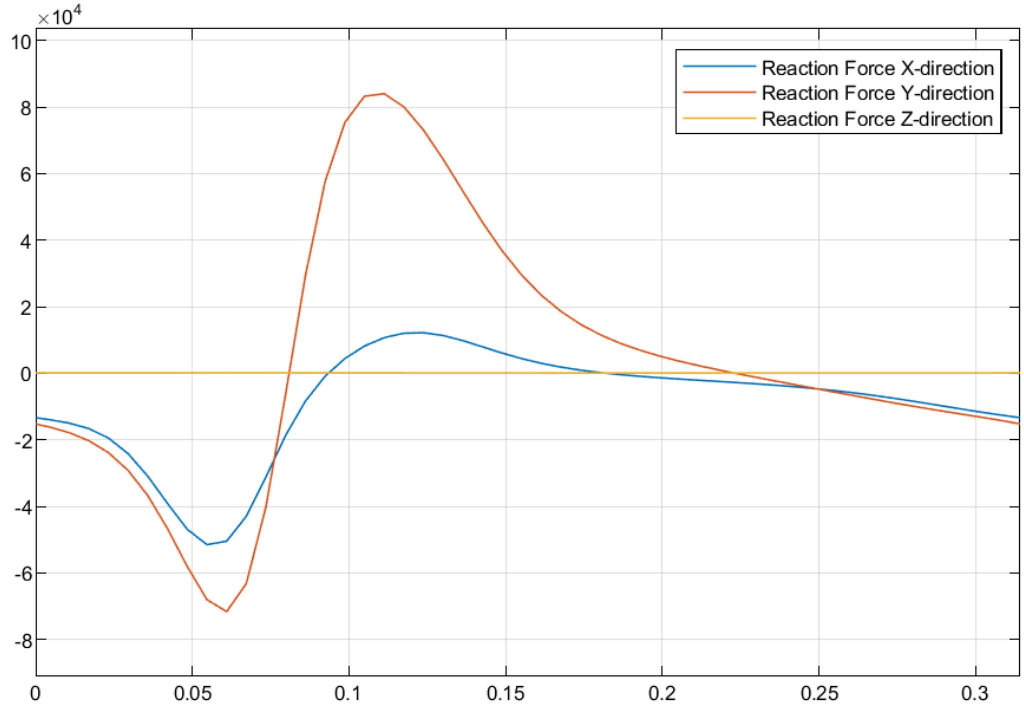
[3] “Hunger hydraulic solutions – successfull operating worldwide,” *HUNGER Hydraulics USA: Rotary actuators*. [Online]. Available: https://www.hunger-hydraulics.com/service-products/hydraulic-components/rotary-actuators. [Accessed: 06-Dec-2022].

Appendix:

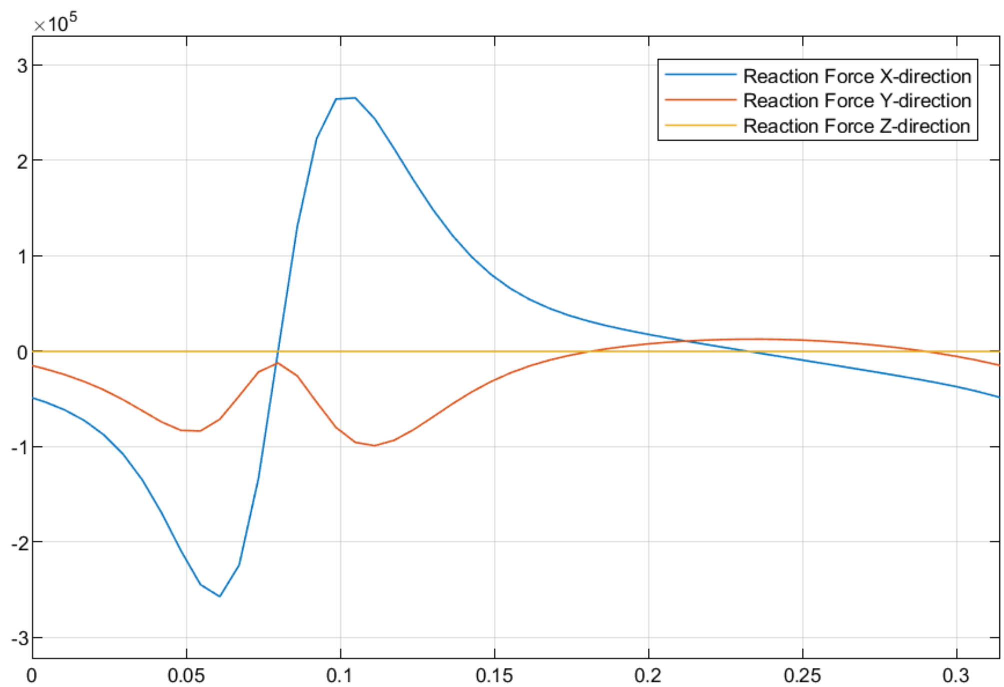
Reaction forces joint A:



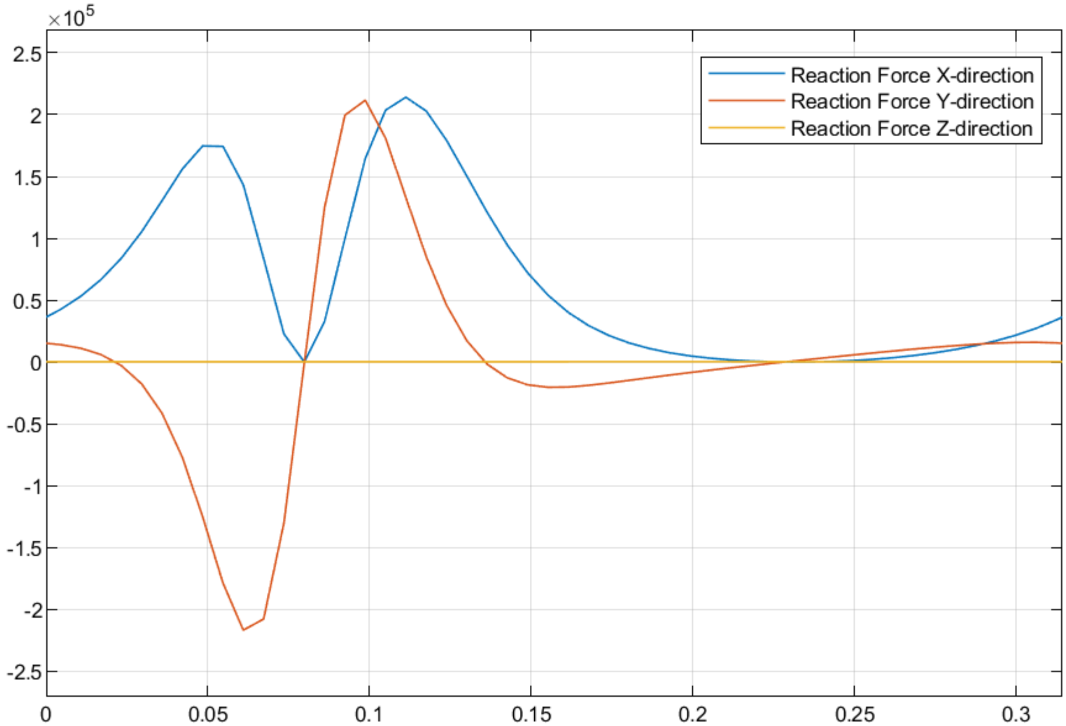
Reaction forces joint B:



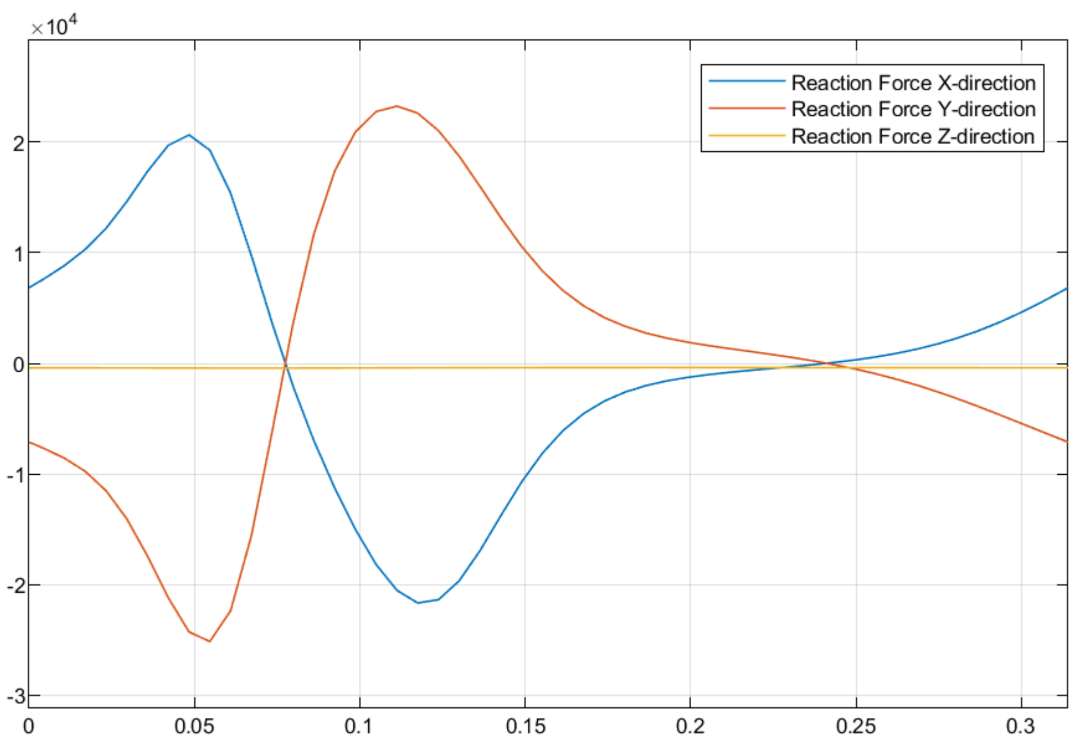
Reaction forces joint C:



Reaction forces joint D:



Reaction forces joint E:



MATLAB code for State 4 calculations:

```
A = [1 0 1 0 0 0 0 0 0 0 0 0 1; 0 1 0 1 0 0 0 0 0 0 0 0 0; 1.15 0.96 1.15 0.96 0 0 0 0 0 0 0 0
0; 0 0 -1 0 1 0 0 0 0 0 0 0 0; 0 0 0 -1 0 0 0 0 0 0 0 0 0; 0 0 0 0 -1 1 0 1 0 0 0 0 0; 0 0 0 0 0 0
1 0 1 0 0 0 0; 0 0 0 0 -1.21 3.62 0.97 3.62 0.97 0 0 0 0; 0 0 0 0 0 0 0 -1 0 1 0 0 0; 0 0 0 0 0
0 0 0 -1 0 1 0 0; 0 0 0 0 0 0 0 0.26 1.47 0.26 1.47 0 0; 0 0 0 0 0 0 0 0 0 -1 0 0 0; 0 0 0 0 0 0
0 0 0 0 -1 1 0];

b = [8034.85; 9575.56; 0; 0; 0; 46762.57; 42819.98; 70312; 3411.58; 11034.66;
7949.925; 0; 0];

x = inv(A)\b;

 disp(x)
```