HW 5 of ME5701. Sun Zhanhong. A0225282J.

9.9

1. (a) PDF: $f(x) = \begin{cases} 1 & -\frac{1}{2} \le x \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

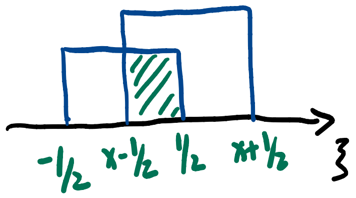
$$\mu = \int_{-\infty}^{+\infty} x f(x)\,dx = \frac{1}{2}x^2 \Big|_{x=-1/2}^{1/2} = 0$$

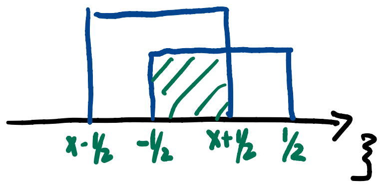
$$\sigma^2 = \int_{-\infty}^{+\infty} (x-\mu)^2 f(x)\,dx = \int_{-1/2}^{1/2} x^2\,dx = \frac{1}{3}x^3 \Big|_{x=-1/2}^{1/2} = \frac{1}{12}.$$

(b) $(f*f)(x) = \int_{-\infty}^{+\infty} f(\xi) f(x-\xi)\,d\xi$

Direct convolution $f(\xi) = \begin{cases} 1 & -\frac{1}{2} \le \xi \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$

$$f(x-\xi) = \begin{cases} 1 & -\frac{1}{2} \le x-\xi \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} = \begin{cases} 1 & x-\frac{1}{2} \le \xi \le x+\frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$





case 1: $-\frac{1}{2} \le x-\frac{1}{2} \le \frac{1}{2} \Rightarrow 0 \le x \le 1$

$$f(\xi) f(x-\xi) = \begin{cases} 1 & x-\frac{1}{2} \le \xi \le \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{so, } \int_{-\infty}^{+\infty} f(\xi) f(x-\xi)\,d\xi = \int_{x-1/2}^{1/2} 1\,d\xi = 1 - x$$

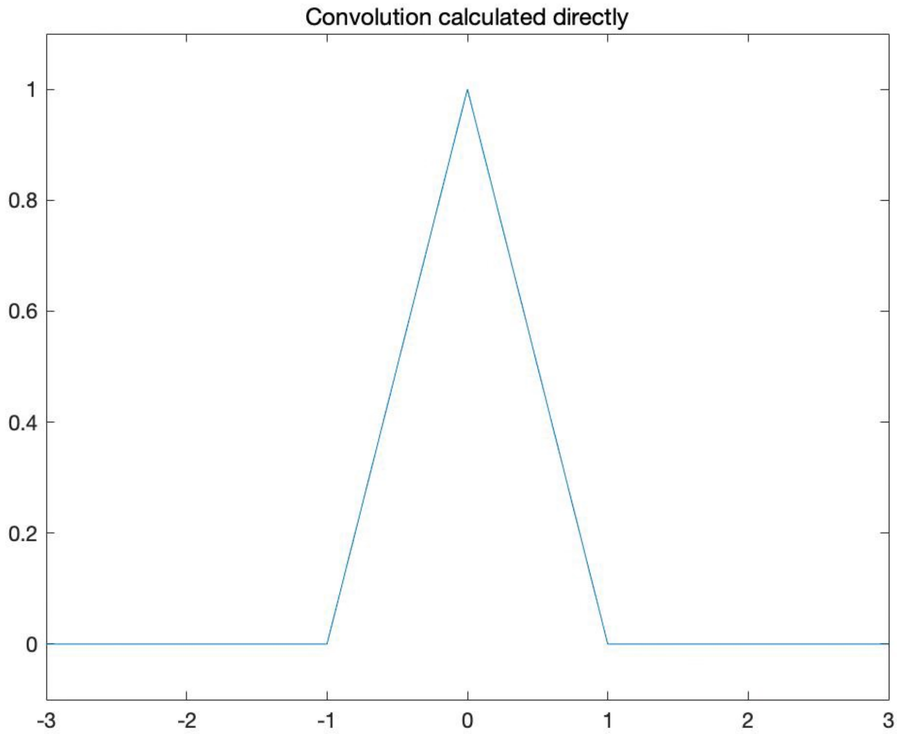
case 2: $x-\frac{1}{2} < -\frac{1}{2} \le x+\frac{1}{2} \Rightarrow -1 \le x < 0$

$$f(\xi) f(x-\xi) = \begin{cases} 1 & -\frac{1}{2} \le \xi \le x+\frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$\text{so, } \int_{-\infty}^{+\infty} f(\xi) f(x-\xi)\,d\xi = \int_{-1/2}^{x+1/2} 1\,d\xi = x + 1$$

Otherwise, the integral is zero.

Therefore, by hand. $(f*f)(x) = \begin{cases} x+1, & -1 \le x < 0 \\ 1-x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$



(c)

$f(x) \to \hat{f}(w)$

$\to \widehat{(f*f)}(w)$

$\to (f*f)(x)$

$$\hat{f}(w) = \int_{-\infty}^{+\infty} f(x)\, e^{-iwx}\, dx$$

$$= \int_{-1/2}^{1/2} e^{iwx}\, dx$$

$$= \frac{i}{w}\left(e^{-\frac{1}{2}wi} - e^{\frac{1}{2}wi}\right)$$

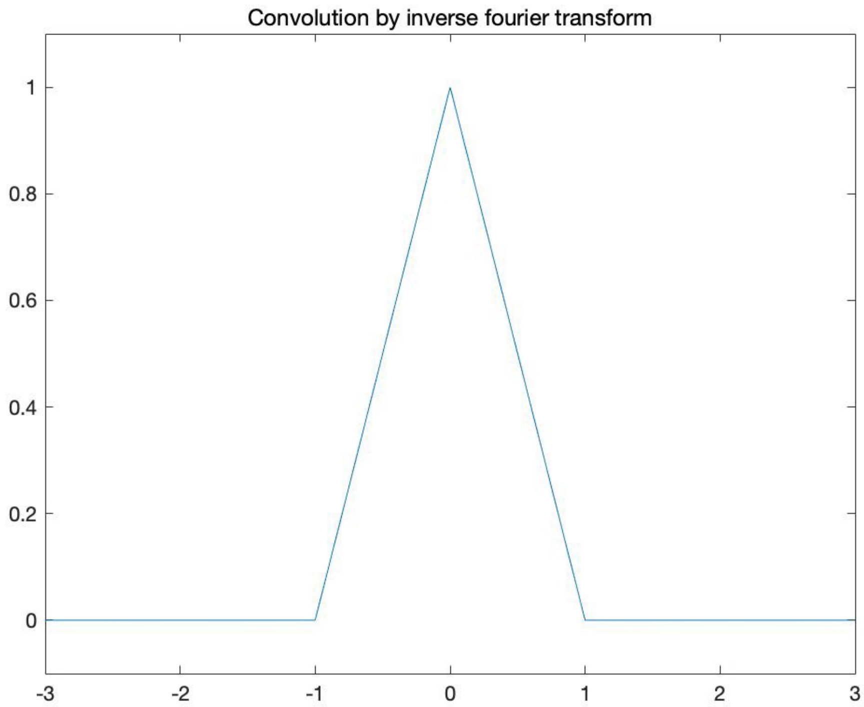
$$\widehat{(f*f)}(w) = \hat{f}(w)\,\hat{f}(w)$$

$$= -\frac{1}{w^2}\left(e^{-\frac{1}{2}wi} - e^{\frac{1}{2}wi}\right)^2$$

$$= \frac{-1}{w^2}\left(e^{-iw} + e^{iw} - 2\right)$$

$$(f*f)(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \widehat{(f*f)}(w)\, e^{iwx}\, dw$$

$$= \frac{-1}{2\pi}\int_{-\infty}^{+\infty} \frac{1}{w^2}\left(e^{i(x-1)w} + e^{i(x+1)w} - 2e^{ixw}\right) dw$$



(d) $f(x)$ in fourier expansion form:

$$f(x) = \sum_{k=-\infty}^{+\infty} \hat{f}(k)\, e^{2\pi i k x / L}.$$

So we calculate $\hat{f}(k)$ first:

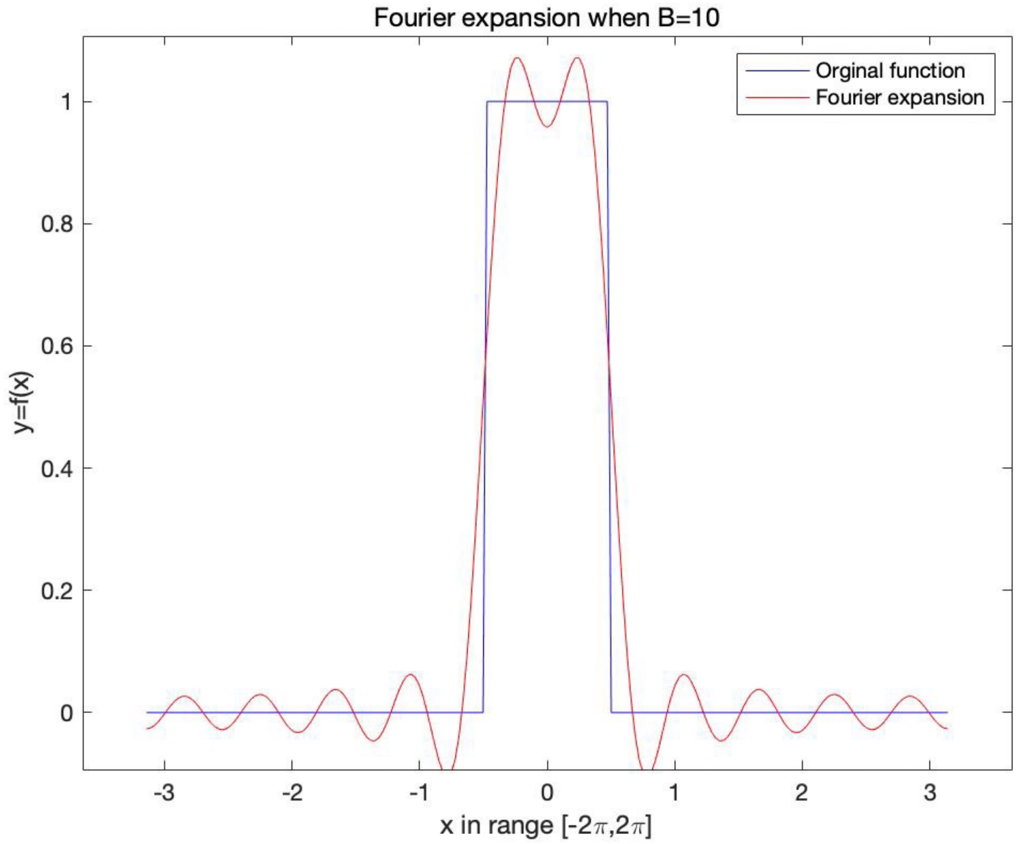
$$\hat{f}(k) = \frac{1}{L}\int_0^L f(x)\, e^{-2\pi k x i / L}\, dx$$

$$= \frac{1}{2\pi}\left(\int_0^{1/2} e^{-ikx}\, dx + \int_{2\pi - 1/2}^{2\pi} e^{-ikx}\, dx\right)$$

$$= \frac{1}{2\pi}\left(\frac{i}{k} e^{-ikx}\Big|_{x=0}^{1/2} + \frac{i}{k} e^{-ikx}\Big|_{x=2\pi-1/2}^{2\pi}\right)$$

$$= \frac{i}{2\pi k}\left(e^{-\frac{1}{2}ki} + e^{-2\pi k i} - e^{-(2\pi - \frac{1}{2})ki} - 1\right)$$

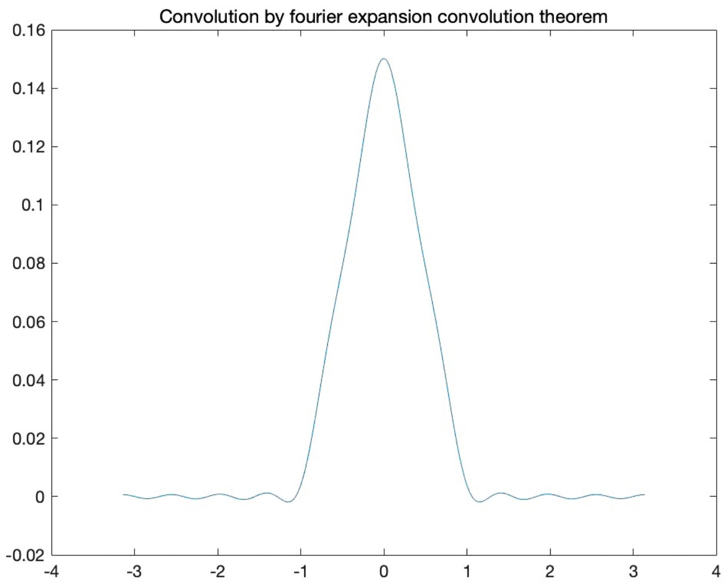
For $f(x)$, we do sum with Matlab:



(e) With fourier expansion convolution theorem

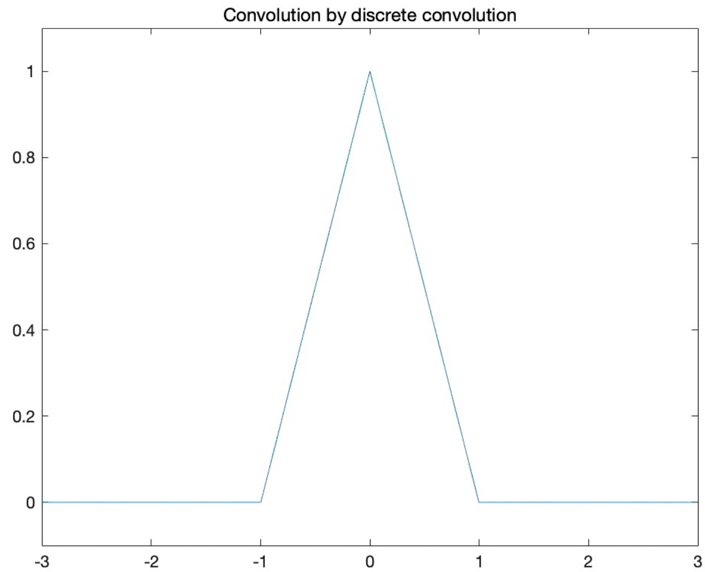
$$(f*f)(x) = \sum_{-\infty}^{+\infty} \hat{f}(k)\hat{f}(k)e^{2\pi ikx/L}$$

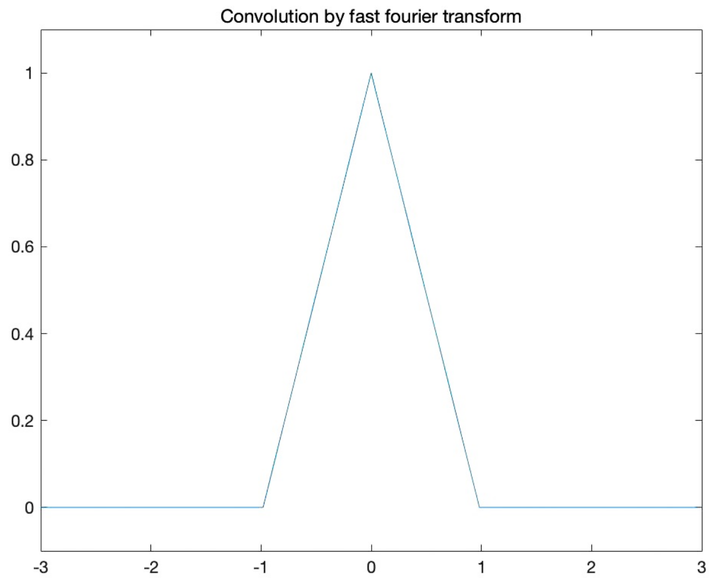
Once we have $\hat{f}(k)$, we can do sum in Matlab.



2.

```
%% Question 2
%% a
% generate sample points
jj = linspace(-pi,pi,64)
kk = 1; % counting num
for i = jj
    fcc(kk) = 1 * (i<=1/2 && i>=-1/2) %original function in int form
    kk = kk+1;
end

w = conv(fcc,fcc);
pp1 = linspace(-2*pi,2*pi,length(w));
w = w/sum(fcc==1,'all');
% the convolutional sum is influenced by the number
%of sample points in '1' area, so divide it

figure();
plot(pp1,w)
axis([-3 3 -0.1 1.1])
title('Convolution by discrete convolution')

%% b

Y = fft(fcc);
ppl = Y.*Y;
% shift to central position
ress = fftshift(ifft(ppl))./sum(fcc==1,'all')
% still has a shift because missing one point when doing fft
ress = [0 ress];
figure();
pp2 = linspace(-pi,pi,length(ress));

plot(pp2,ress)
axis([-3 3 -0.1 1.1])
title('Convolution by fast fourier transform')
```





A.2. Show $\|A\|_2^2 = \frac{1}{2}\left(\|A\|^2 + \sqrt{\|A\|^4 - 4|\det(A)|^2}\right)$

Here we use the result of A.4.

$$\|A\|_2 = \sqrt{\lambda_{max}(A^*A)}$$

, and $\|A\| = \sqrt{tr(AA^*)} = \sqrt{tr(A^*A)}$

We concern A^*A. Assume $A^*A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

where a, b, c and d are complex values.

In order to calculate the eigenvalues of A^*A, first we write its characteristic equation

$$\begin{vmatrix} \lambda-a & b \\ c & \lambda-d \end{vmatrix} = 0 \Rightarrow \lambda^2 - (a+d)\lambda + (ad-bc) = 0.$$

And the eigenvalues are,

$$\lambda = \frac{(a+d) \pm \sqrt{(a+d)^2 - 4(ad-bc)}}{2}.$$

By observation, $a+d = tr(A^*A)$
$ad-bc = \det(A^*A)$

Therefore, the larger eigenvalue,

$$\lambda_{max}(A^*A) = \frac{1}{2}\left(tr(A^*A) + \sqrt{tr(A^*A)^2 - 4\det(A^*A)}\right)$$

Rewrite the above eqn with norms.

$$\|A\|_2^2 = \frac{1}{2}\left(\|A\|^2 + \sqrt{\|A\|^4 - 4\det(A^*A)}\right).$$

Now we concern $\det(A^*A)$

we know that

$$\det(A^*A) = \det(A^*)\det(A)$$

and

$$\det(A^*) = \overline{\det(A)}.$$

assume $\det(A) = a+bi$

then $\det(A^*) = a-bi$

$$\det(A^*A) = \det(A^*)\det(A)$$
$$= (a+bi)(a-bi)$$
$$= a^2+b^2$$
$$= |\det(A)|^2$$

Therefore,

$$\|A\|_2^2 = \|A\|^2 + \sqrt{\|A\|^4 - 4|\det(A)|^2}$$

is proved.

A.3 If $\|A\|_2^2 = \frac{1}{2}\left(\|A\|^2 + \sqrt{\|A\|^4 - 4|\det(A)|^2}\right)$,

then, $\left(2\|A\|_2^2 - \|A\|^2\right)^2 = \|A\|^4 - 4|\det(A)|^2$

$$\cancel{4}\|A\|_2^4 + \cancel{\|A\|^4} - \cancel{4}\|A\|_2^2\|A\|^2 = \cancel{\|A\|^4} - \cancel{4}|\det(A)|^2$$

$$\Rightarrow \|A\|^2 = \|A\|_2^2 + \frac{|\det(A)|^2}{\|A\|_2^2}$$

A.4 Show $\|A\|_2 = \sqrt{\lambda_{max}(A^*A)}$

We consider the original definition of $\|A\|_2$:

$$\|A\|_2 \doteq \max_{x\neq 0}\left(\frac{x^*A^*Ax}{x^*x}\right)^{\frac{1}{2}}$$

We know A^*A is hermittian matrix,

So it can be diagonalized as.

$$A^*A = U^*\begin{pmatrix}\lambda_1 & & 0\\ & \ddots & \\ 0 & & \lambda_n\end{pmatrix}U$$

, where U is unitary and λ_i are eigenvalues of A^*A

Therefore, $x^*A^*Ax = x^*U^*\Sigma Ux$

assume $Ux = y = [y_1, y_2 \dots]^T$

So, $x^*A^*Ax = y^*\Sigma y. = \lambda_1 y_1^2 + \lambda_2 y_2^2 + \dots$

And, $y^*y = x^*U^*Ux$

Since U is unitary, $U^*U = I$

So, $x^*x = y^*y = y_1^2 + y_2^2 + \dots$

Therefore,

$$\|A\|_2 = \max_{x\neq 0} \left(\frac{x^* A^* A x}{x^* x} \right)^{1/2}$$

$$= \max_{x\neq 0} \left(\frac{\lambda_1 y_1^2 + \lambda_2 y_2^2 + \cdots + \lambda_n y_n^2}{y_1^2 + y_2^2 + \cdots + y_n^2} \right)^{1/2}$$

concern the inner expression.

Actually, $\frac{\lambda_1 y_1^2 + \lambda_2 y_2^2 + \cdots + \lambda_n y_n^2}{y_1^2 + y_2^2 + \cdots + y_n^2} \leq \lambda_{max}$

with equality when $\lambda_1 = \lambda_2 = \cdots = \lambda_n = \lambda_{max}$.

Therefore, $\|A\|_2 = \max_{x\neq 0} \left(\frac{x^* A^* A x}{x^* x} \right)^{1/2}$

$$= \sqrt{\lambda_{max}(A^* A)}$$

get proved.

A.27 $A = \begin{pmatrix} -2 & 1 & 2 \\ 0 & -1 & 6 \\ 0 & 0 & -3 \end{pmatrix}$

$|A - \lambda I| = 0$

$\Rightarrow (\lambda+2)(\lambda+1)(\lambda+3) = 0$

$\lambda = -1, -2, -3.$

All eigenvalues of A are negative,

so the linear part is stable.

$g(t) = \begin{bmatrix} e^{-2t} \\ \cos 2t \\ 0 \end{bmatrix}$. $\|g(t)\| = (e^{-4t} + \cos^2 2t)^{1/2} \leq \sqrt{2}$ for $t \geq 0$.

Therefore the whole system is stable

A.28.

(a) $x(t) = \exp(At)x_0 + \int_0^t \exp(A(t-\tau))g(\tau)d\tau$.

So. $\frac{dx(t)}{dt} = Ae^{At}x_0 + \frac{d}{dt}\left(e^{At}\int_0^t e^{-A\tau}g(\tau)d\tau\right)$

(By fundamental thm of calculus)

$= Ae^{At}x_0 + Ae^{At}\int_0^t e^{-A\tau}g(\tau)d\tau + \cancel{e^{At}e^{-At}}g(t)$

$= A\left[e^{At}x_0 + e^{At}\int_0^t e^{-A\tau}g(\tau)d\tau\right] + g(t)$

$= A\left[\underbrace{\exp(At)x_0 + \int_0^t \exp(A(t-\tau))g(\tau)d\tau}_{=x(t)}\right] + g(t)$

$= Ax(t) + g(t)$

So, $x(t) = \exp(At)x_0 + \int_0^t \exp(A(t-\tau))g(\tau)d$

is a solution for $\frac{dx(t)}{dt} = Ax + g(t)$ with $x(0) = x_0$

(b) $x(t) = \exp(At)x_0 + \int_0^t \exp(A(t-\tau))B(\tau)x(\tau)d\tau$.

So. $\frac{dx(t)}{dt} = Ae^{At}x_0 + \frac{d}{dt}\left(e^{At}\int_0^t e^{-A\tau}B(\tau)x(\tau)d\tau\right)$

$= Ae^{At}x_0 + Ae^{At}\int_0^t e^{-A\tau}B(\tau)x(\tau)d\tau$
$\quad + e^{At}e^{-At}B(t)x(t)$

$= Ae^{At}x_0 + Ae^{At}\int_0^t e^{-A\tau}B(\tau)x(\tau)d\tau + B(t)x(t)$

$= A\left(\exp(At)x_0 + \int_0^t \exp(A(t-\tau))B(\tau)x(\tau)d\tau + B(t)x(t)\right.$

$= (A + B(t))x(t)$

Thus $x(t) = \exp(At)x_0 + \int_0^t \exp(A(t-\tau))B(\tau)x(\tau)d\tau$

is a solution for $\frac{d}{dt}x(t) = (A + B(t))x(t)$.

A.29

(a) $\ddot{x} + \dot{x} + (1+e^{-t})x = 0$

assume $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$

$$\dot{X} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_2 - (1+e^{-t})x_1 \end{bmatrix}$$

$$= \Big(\underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}}_{A} + \underbrace{\begin{bmatrix} 0 & 0 \\ -e^{-t} & 0 \end{bmatrix}}_{B(t)}\Big) X$$

characteristic equation of A: $\lambda(\lambda+1)+1=0 \Rightarrow \lambda^2+\lambda+1=0$

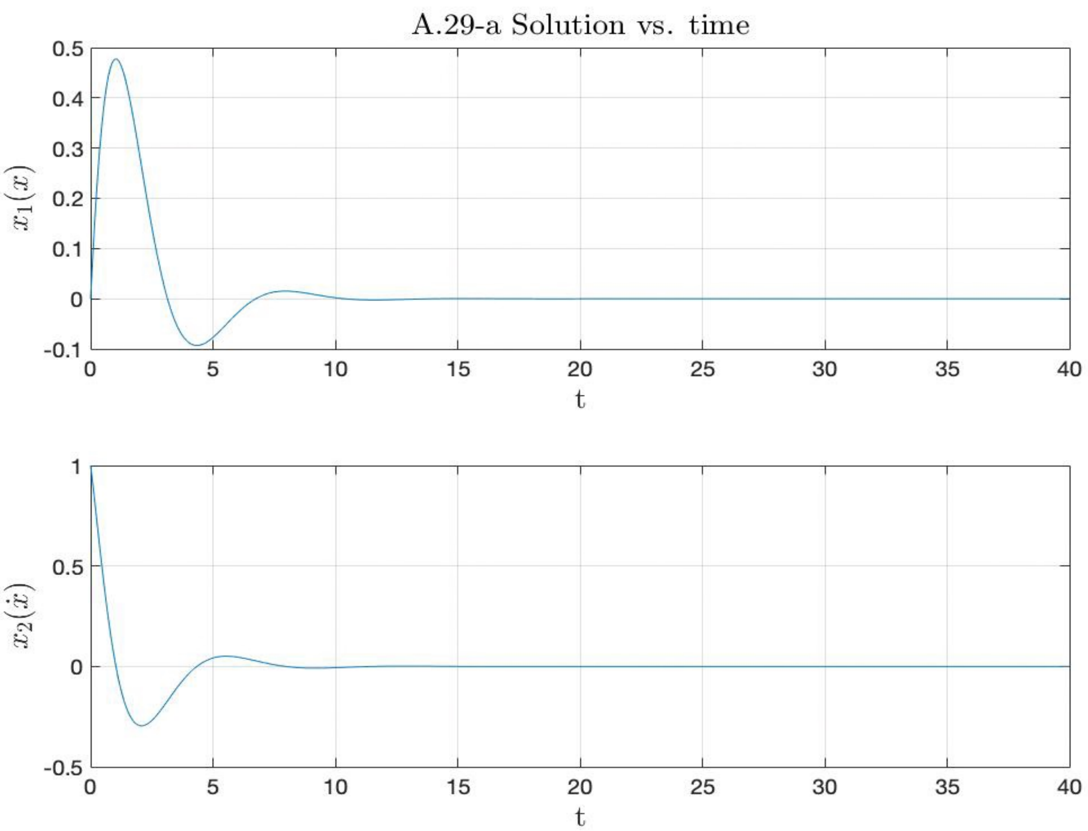
$\Rightarrow \lambda = \frac{1}{2}(-1 \pm \sqrt{\Delta})$. the real part of eigenvalue A are all negative.

So the linear part of the system is stable.

$$\|B(t)\| = (\lambda_{max}(B^* B))^{1/2} = \left(\lambda_{max}\left(\begin{bmatrix} e^{-2t} & 0 \\ 0 & 0 \end{bmatrix}\right)\right)^{1/2}$$

$$= e^{-2t} \le 1 \quad \text{for } t \ge 0.$$

Therefore $x(t)$ converges as $t \to \infty$



(b) $\ddot{x} + \dot{x} + (1 + 0.2\cos t)x = 0$

assume $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$

then $\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \Bigg(\underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}}_{A} + \underbrace{\begin{bmatrix} 0 & 0 \\ -0.2\cos t & 0 \end{bmatrix}}_{B(t)} \Bigg) X$

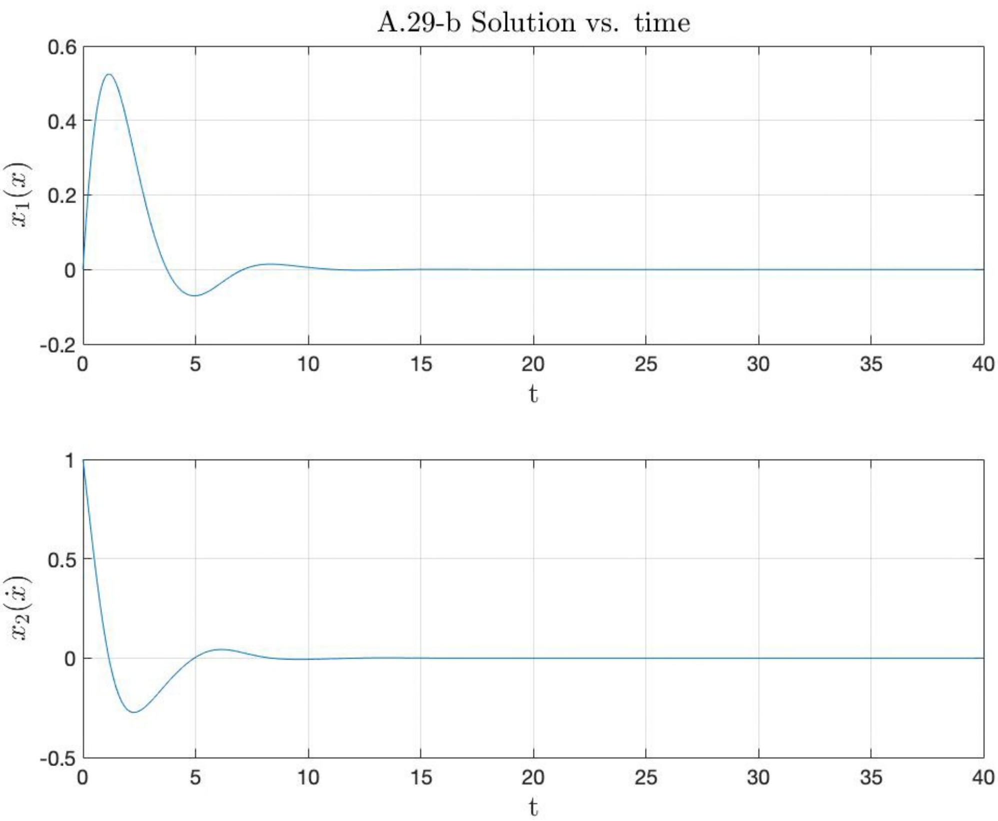
For A: $\lambda(\lambda+1)+1=0$

$\lambda_{1,2} = -\frac{1}{2} \pm \frac{1}{2}\sqrt{\Delta}$. real parts of all eigenvalues are negative, so the linear part of the system is stable

For $B(t)$:

$\lambda_{max}(B^{*}B) = 0.04\cos^2 t$, so $\|B(t)\| = 0.2|\cos t| \leq 1$ for $t \geq 0$

Therefore $x(t)$ converges as $t \to \infty$



(c) $\ddot{x} + \dot{x} + x = \cos t$

assume $X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_1 - x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \cos t \end{bmatrix}$$

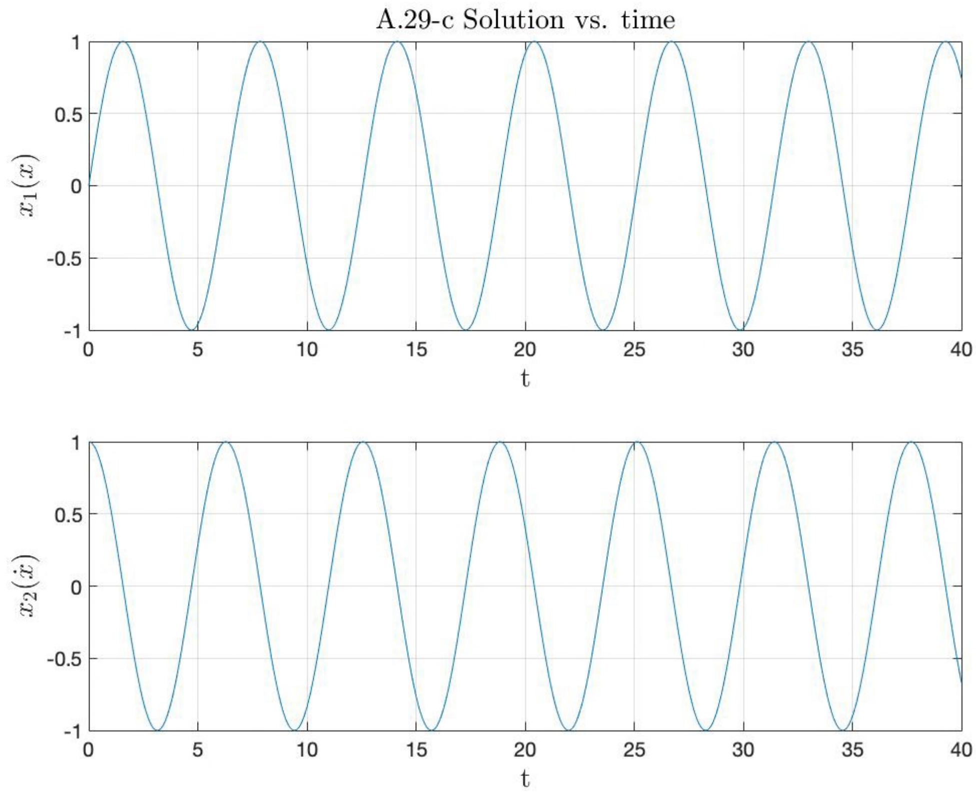
$$= \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \cos t \end{bmatrix}}_{g(t)}$$

All eigenvalues of A have negative real part.

Therefore the linear part of system is stable.

$\|g(t)\| = |\cos t| \le 1$ for $t \geq 0$

Thus $x(t)$ converges as $t \to \infty$



(d) $\ddot{x} + \dot{x} + x = e^{-t}$

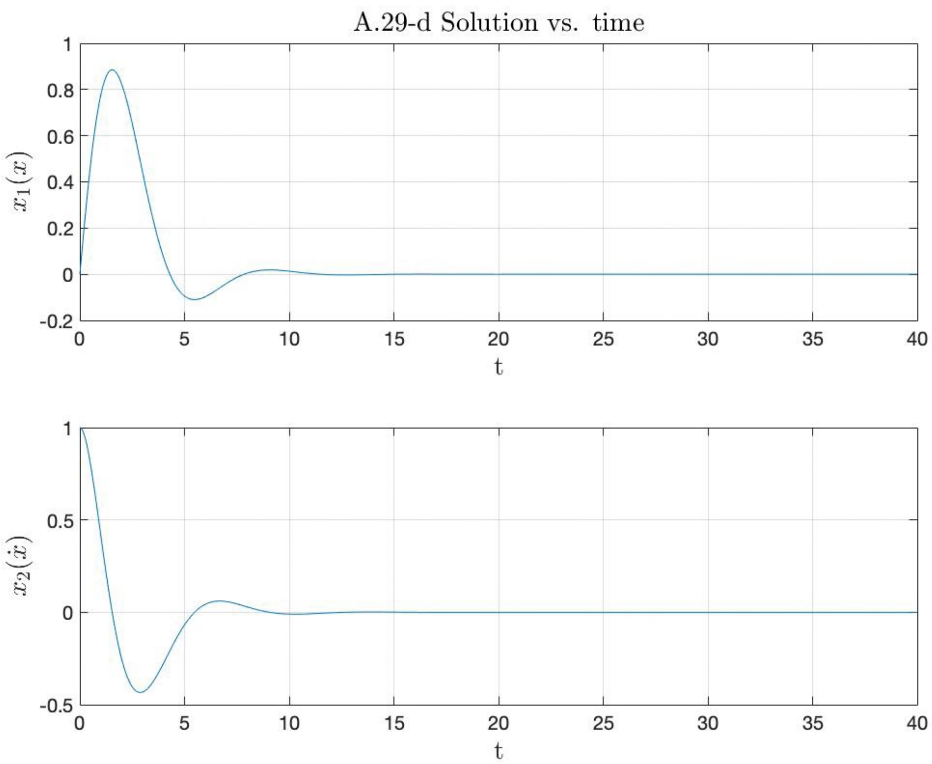
Assume $X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$.

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ e^{-t} \end{bmatrix}}_{g(t)}$$

All eigenvalues of A have negative real part.
So the linear part of the system is stable.

$\|g(t)\| = e^{-t} \leq 1$ for $t \geq 0$.

Therefore $x(t)$ converges as $t \to \infty$



(e) $\ddot{x} + \dot{x} + x = e^{2t}$

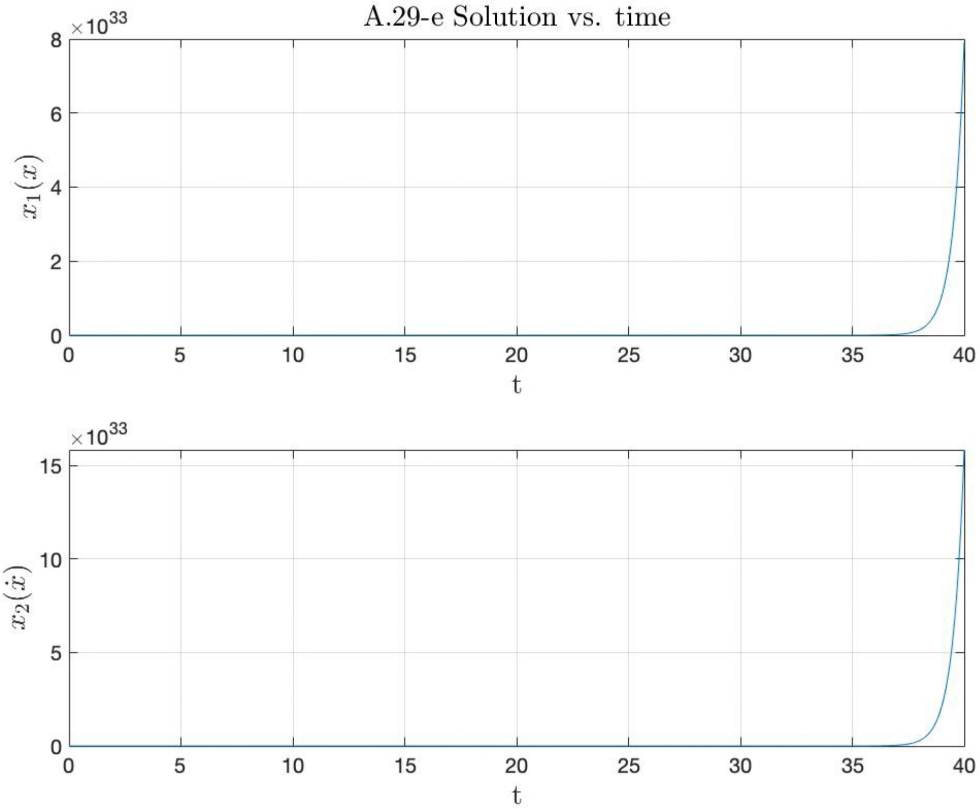
Assume $X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$

$$\dot{X} = \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}}_{A} X + \underbrace{\begin{bmatrix} 0 \\ e^{2t} \end{bmatrix}}_{g(t)}$$

$\|g(t)\| = e^{2t}$.

We cannot find a γ s.t. $\|g(t)\| \leq \gamma$.

Therefore $x(t)$ diverges as $t \to \infty$



Appendix: Matlab Code

```
clc;close all;clear all;

%% Question 1
%% PDF convolution directly f*f(x) and plot b

% Define analytical variables
syms f_x x f_x_t t f_ome omega kki

% Has defined the f function as piecewise function
f_t = f(t);
f_x_t = f(x-t);

% Do integration to do convolution
result = int(f_t*f_x_t,t,-inf,inf);

% For plot
xx = -3:0.01:3;
pp1 = subs(result,x,xx);
figure(1)
plot(xx,pp1)
axis([-3 3 -0.1 1.1])
title('Convolution calculated directly')


%% Convolution by inverse Fourier transform c

x = -3:0.01:3;
f = zeros(1,length(x));
for k = 1:length(x)
    fun = @(w,x) (1i./w.*(exp(-0.5.*w.*1i)-exp(0.5.*w.*1i))).^2.*exp(1i.*w.*x);

    f(k) = 1/2/pi*integral(@(w)fun(w,x(k)),-1000,1000);
end
figure()
plot(x,f);
title('Convolution by inverse fourier transform')
axis([-3 3 -0.1 1.1])

%% Fourier series expansion d

B = 10;
i = 1i;
xq = -pi:pi/100:pi;

sums = 1/2/pi;
for k = -B:1:-1
    sums = sums + (i/(2*pi.*k)).*(exp(-0.5.*k*i)+exp(-2*pi.*k*i)-exp(-(2*pi-0.5)*k*i)-1).*exp(i.*xq.*k);
end
for k = 1:1:B
    sums = sums + (i/(2*pi.*k)).*(exp(-0.5.*k*i)+exp(-2*pi.*k*i)-exp(-(2*pi-0.5)*k*i)-1).*exp(i.*xq.*k);
end

pd = makedist('Uniform','lower',-1/2,'upper',1/2);
x = -pi:pi/100:pi;
pdff = pdf(pd,x);
figure()
plot(x,pdff,'b');
hold on

plot(xq,sums,'r');
```

```
axis([-pi-0.5 pi+0.5 -0.1 1.1])
legend('Orginal function','Fourier expansion')
xlabel('x in range [-2\pi,2\pi]')
ylabel('y=f(x)')
title('Fourier expansion when B=10')
hold off

%% Fourier expansion convolution theorem e
xp = -pi:pi/100:pi;
sums1 = 1/(4*pi^2); %when k=0
for k = -B:1:-1
    sums1 = sums1 + ((i/(2*pi.*k)).*(exp(-0.5*k*i)+exp(-2*pi.*k*i)-exp(-
(2*pi-0.5).*k*i)-1)).^2*exp(i.*xp.*k);
end
for k = 1:1:B
    sums1 = sums1 + ((i/(2*pi.*k)).*(exp(-0.5*k*i)+exp(-2*pi.*k*i)-exp(-
(2*pi-0.5).*k*i)-1)).^2*exp(i.*xp.*k);

end
sums1 = sums1;

figure()
plot(xp,sums1);
title('Convolution by fourier expansion convolution theorem')


%% Question 2
%% a
% generate sample points
jj = linspace(-pi,pi,64)
kk = 1; % counting num
for i = jj
    fcc(kk) = 1 * (i<=1/2 && i>=-1/2) %original function in int form
    kk = kk+1;
end

w = conv(fcc,fcc);
pp1 = linspace(-2*pi,2*pi,length(w));
w = w/sum(fcc==1,'all');
% the convolutional sum is influenced by the number
%of sample points in '1' area, so divide it

figure();
plot(pp1,w)
axis([-3 3 -0.1 1.1])
title('Convolution by discrete convolution')

%% b
Y = fft(fcc);
ppl = Y.*Y;
% shift to central position
ress = fftshift(ifft(ppl))./sum(fcc==1,'all')
% still has a shift because missing one point when doing fft
ress = [0 ress];
figure();
pp2 = linspace(-pi,pi,length(ress));

plot(pp2,ress)
axis([-3 3 -0.1 1.1])
title('Convolution by fast fourier transform')
```

```
clc;
close all
clear all

T0    = 0;         % initial time
TN    = 40;         % final time
dt    = 0.01;       % time-step
tspan = T0:dt:TN;  % time set
x1    = 0;         % initial condition for x1
x2    = 0;         % initial condition for x2
x3    = 0;

%% A.27
eqn1 = @(t,x)[
                  -2*x(1) + x(2) + 2*x(3) + exp(-2*t); ...
    -x(2) + 6*x(3) + cos(12*t); ...
    -3*x(3)
];

[t,sol] = ode45(eqn1,tspan,[x1 x2 x3]);
x1_s = sol(:,1);
x2_s = sol(:,2);
x3_s = sol(:,3);

figure()
subplot(3,1,1); plot(t,x1_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_1 (x)
$','Interpreter','latex','FontSize',14);
title('A.27   Solution vs. time','Interpreter','latex','FontSize',14)
subplot(3,1,2); plot(t,x2_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_{2} (x)
$','Interpreter','latex','FontSize',14)

subplot(3,1,3); plot(t,x3_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_{3} (x)
$','Interpreter','latex','FontSize',14)

%% A.29-a
eqn1 = @(t,x)[
                  x(2); ...
    -(1+exp(-t))*x(1)-x(2); ...
];

[t,sol] = ode45(eqn1,tspan,[x1 x2]);
x1_s = sol(:,1);
x2_s = sol(:,2);

figure()
subplot(2,1,1); plot(t,x1_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_1 (x)
$','Interpreter','latex','FontSize',14);
title('A.29-a   Solution vs. time','Interpreter','latex','FontSize',14)
subplot(2,1,2); plot(t,x2_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_{2}
(\dot{x}) $','Interpreter','latex','FontSize',14)

%% A.29-b
eqn1 = @(t,x)[
                  x(2); ...
    -(1+0.2*cos(t))*x(1)-x(2); ...
];

[t,sol] = ode45(eqn1,tspan,[x1 x2]);
x1_s = sol(:,1);
x2_s = sol(:,2);
```

```
figure()
subplot(2,1,1); plot(t,x1_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_1 (x) $','Interpreter','latex','FontSize',14);
title('A.29-b   Solution vs. time','Interpreter','latex','FontSize',14)
subplot(2,1,2); plot(t,x2_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_{2} (\dot{x}) $','Interpreter','latex','FontSize',14)

%% A.29-c
eqn1 = @(t,x)[
              x(2); ...
    -x(1)-x(2)+cos(t); ...
];

[t,sol] = ode45(eqn1,tspan,[x1 x2]);
x1_s = sol(:,1);
x2_s = sol(:,2);

figure()
subplot(2,1,1); plot(t,x1_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_1 (x) $','Interpreter','latex','FontSize',14);
title('A.29-c   Solution vs. time','Interpreter','latex','FontSize',14)
subplot(2,1,2); plot(t,x2_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_{2} (\dot{x}) $','Interpreter','latex','FontSize',14)

%% A.29-d
eqn1 = @(t,x)[
              x(2); ...
    -x(1)-x(2)+exp(-t); ...
];

[t,sol] = ode45(eqn1,tspan,[x1 x2]);
x1_s = sol(:,1);
x2_s = sol(:,2);

figure()
subplot(2,1,1); plot(t,x1_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_1 (x) $','Interpreter','latex','FontSize',14);
title('A.29-d   Solution vs. time','Interpreter','latex','FontSize',14)
subplot(2,1,2); plot(t,x2_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_{2} (\dot{x}) $','Interpreter','latex','FontSize',14)

%% A.29-e
eqn1 = @(t,x)[
              x(2); ...
    -x(1)-x(2)+exp(2*t); ...
];

[t,sol] = ode45(eqn1,tspan,[x1 x2]);
x1_s = sol(:,1);
x2_s = sol(:,2);

figure()
subplot(2,1,1); plot(t,x1_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_1 (x) $','Interpreter','latex','FontSize',14);
title('A.29-e   Solution vs. time','Interpreter','latex','FontSize',14)
subplot(2,1,2); plot(t,x2_s) ;grid on; xlabel('t','Interpreter','latex','FontSize',14); ylabel('$x_{2} (\dot{x}) $','Interpreter','latex','FontSize',14)
```