Problem

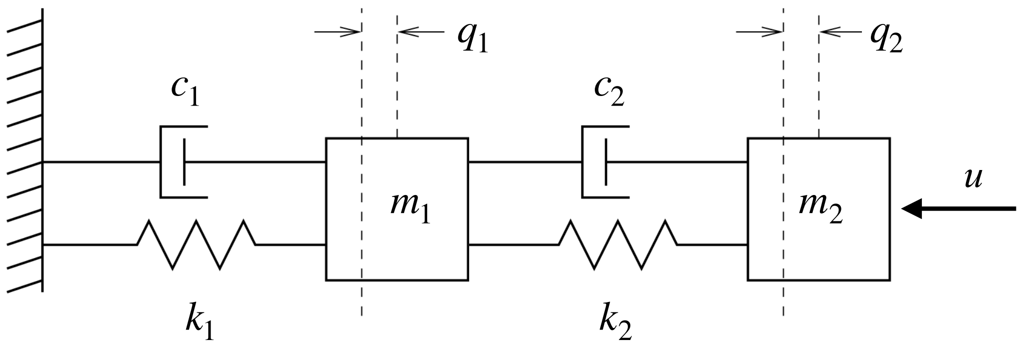

Figure 1: Two-degree-of-freedom mass-damper-spring system

Given the model depicted in figure 1 of e.g. a suspension system:

Q1. Write the linear equations of motions

Q2. Write the corresponding linear dynamical system

Q3. Solve the linear dynamical system via ODE45 in Matlab

Q4. Discuss the stability of the system for different damping parameters c_1 and c_2

Q5. Discuss the observability of the system assuming that you can measure q_1 and q_2, q_1 only and q_2 only, respectively

Q6. Discuss the controllability of the system for different damping parameters c_1 and c_2

Q7. What happens to the controllability of the system if we apply the input u on the first mass only?

Q8. Write the transfer function of the system and its impulse response in Matlab

Solutions

Q1. The linear equations of motions are as follows

$$\begin{cases} m_1\ddot{q}_1 + (c_1+c_2)\dot{q}_1 - c_2\dot{q}_2 + (k_1+k_2)q_1 - k_2 q_2 = 0 \\ m_2\ddot{q}_2 - c_2\dot{q}_1 + c_2\dot{q}_2 - k_2 q_1 + k_2 q_2 = -u. \end{cases} \tag{1}$$

Q2. By defining the following state vector

$$\mathrm{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \dot{q}_1 \\ q_1 \\ \dot{q}_2 \\ q_2 \end{bmatrix},$$

the corresponding dynamical system is

$$\frac{\mathrm{d}}{\mathrm{dt}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -(c_1+c_2)/m_1 & -(k_1+k_2)/m_1 & c_2/m_1 & k_2/m_1 \\ 1 & 0 & 0 & 0 \\ c_2/m_2 & k_2/m_2 & -c_2/m_2 & -k_2/m_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -1/m_2 \\ 0 \end{bmatrix} u$$

Q3. See attached Matlab script.

Q4. The system is asymptotically stable for c_1 and $c_2 > 0$, stable for c_1 and $c_2 = 0$ and unstable for c_1 and $c_2 < 0$. In addition, (i) for $|c_1| \gg |c_2|$, with $c_1 > 0$ and $c_2 < 0$, the system is asymptotically stable, and (ii) for $|c_2| \gg |c_1|$, with $c_2 > 0$ and $c_1 < 0$, the system is also asymptotically stable. Finally, for both (i) and (ii) there exists a combination of values of c_1 and c_2 for which the system is stable.

Q5. The system is always observable for any of the measurement combinations suggested. Matrix A guarantees enough coupling. See also attached Matlab script.

Q6. The system is always controllable regardless of c_1 and c_2. See also attached Matlab script.

Q7. The system remains controllable. See also attached Matlab script.

Q8. See the attached Matlab Script.