Operations Research & Optimization

A new dimension to Data Science

Andrea Taverna, PhD andrea.taverna@outlook.com



OptLab – The Optimization Laboratory @ Università degli Studi di Milano

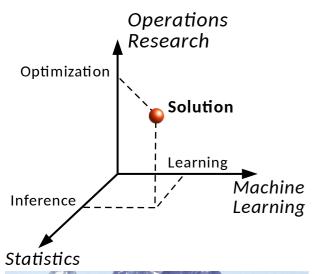


Note:

Hyperlinks seem not to work in the PDF version of the presentation.

I replaced the link symbols with numbers and added all the links in clear in the last three slides, titled "Links", with the corresponding number and description.

Key points



- 1 Operations Research (OR) is a "new dimension" of Data Science
 - New problems, new methods, new solutions



- 2 Problems that require OR are everywhere!
 - missed opportunities



- 3 You should invest in OR and Operations Researchers
 - grab those opportunities!

Outline

1 Introduction

- From Data Science to OR
- Examples of OR applications

2 Mathematical aspects of OR

- Models and algorithms
- Optimization for Data Science

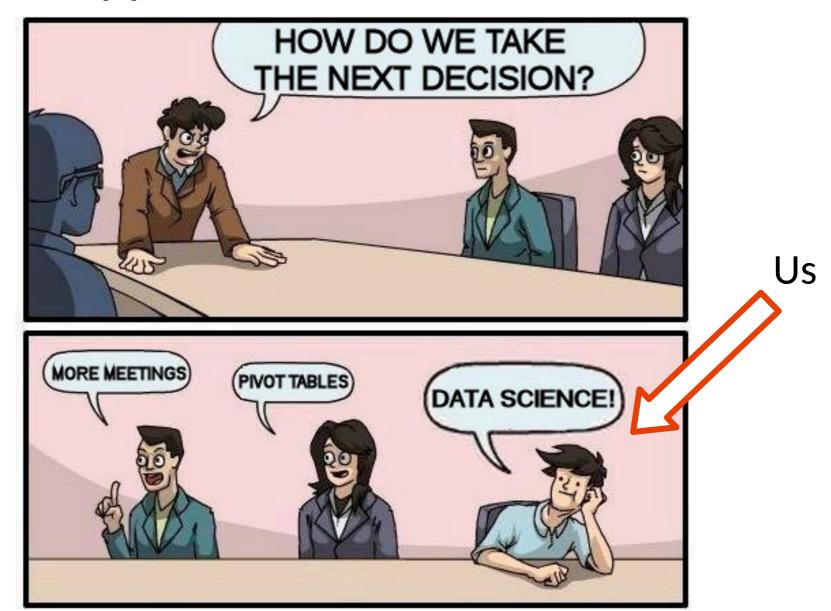
3 OR and Data Science in the industry

- ► The Analytics Stack
- OR in a Data Science team

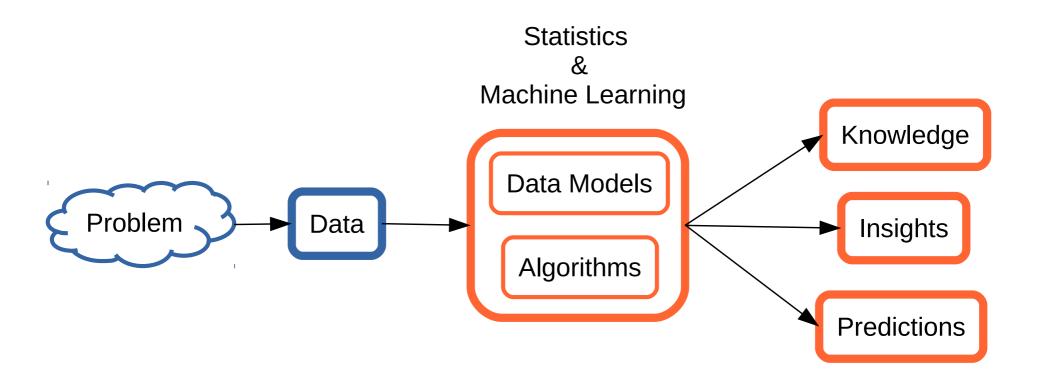
Introduction

The goal of Data Science

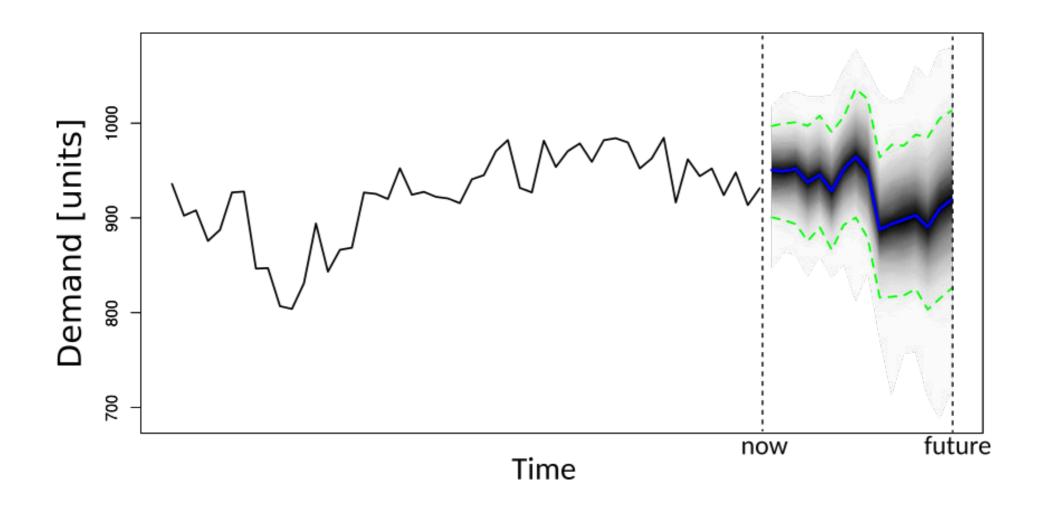
support Business Decisions!



The Data Science way

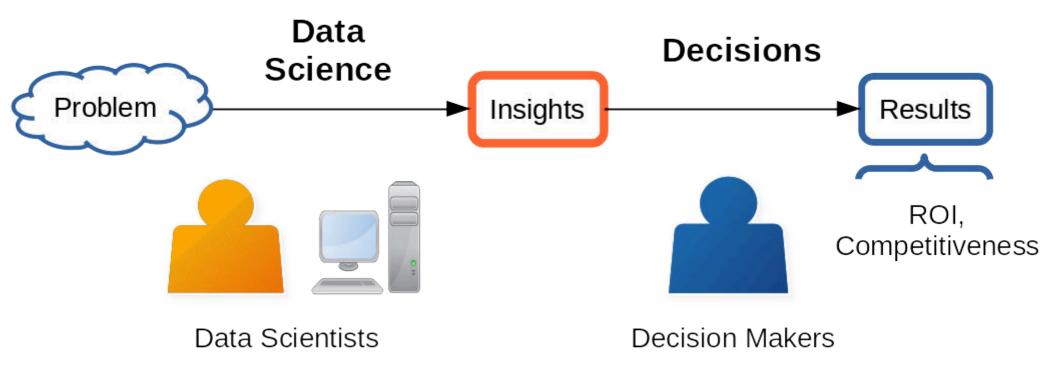


A forecast



And now what?

Take a Data-Driven Decision!



- Decisions required to obtain results
- Is data **enough** for taking **good decisions**?

Business Simulation Games

Can people take **good decisions** in a game **at least**?



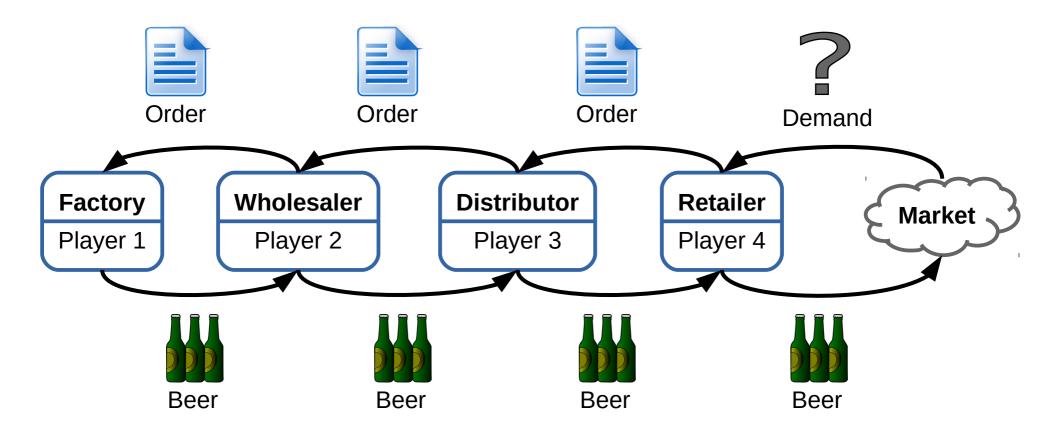
Beer Game at MIT Management School [1]



The Manufacturing Game (TM) [2]

(used in **Education** and **Consultancy**)

The Beer Distribution Game [1]



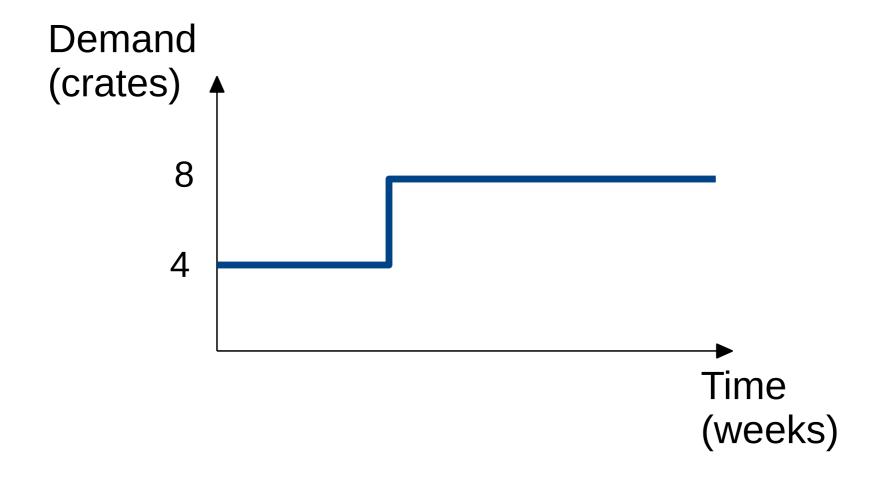
- 4 players, 4 roles
- Costs for production, inventory, backlog
- Goal: satisfy demand and minimize cost

Beer Distribution Game: results



- Average cost 10x times the optimal one
- Even experienced professionals perform poorly!

Beer Distribution Game: Demand



Forecast with persistence ...

Causes of poor performance

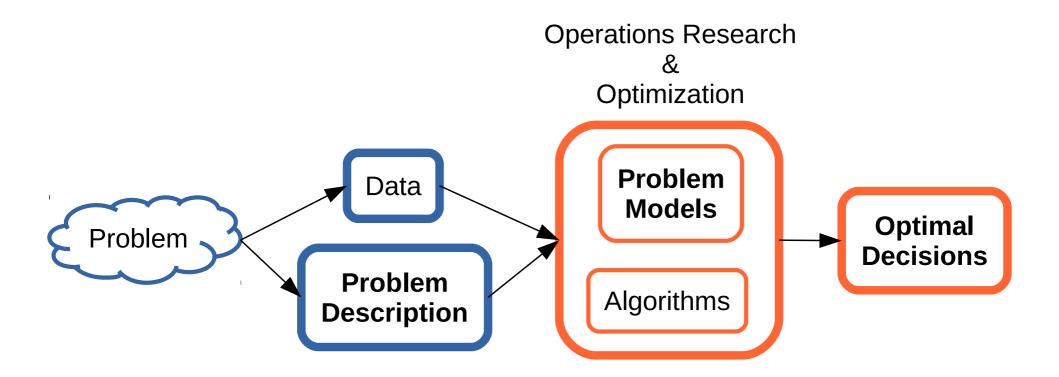
- Not a data problem. Data is trivial!
- Costs from players' suboptimal decisions

Can't we do **better**?

Yes, with **Operations Research**!



The Operations Research way



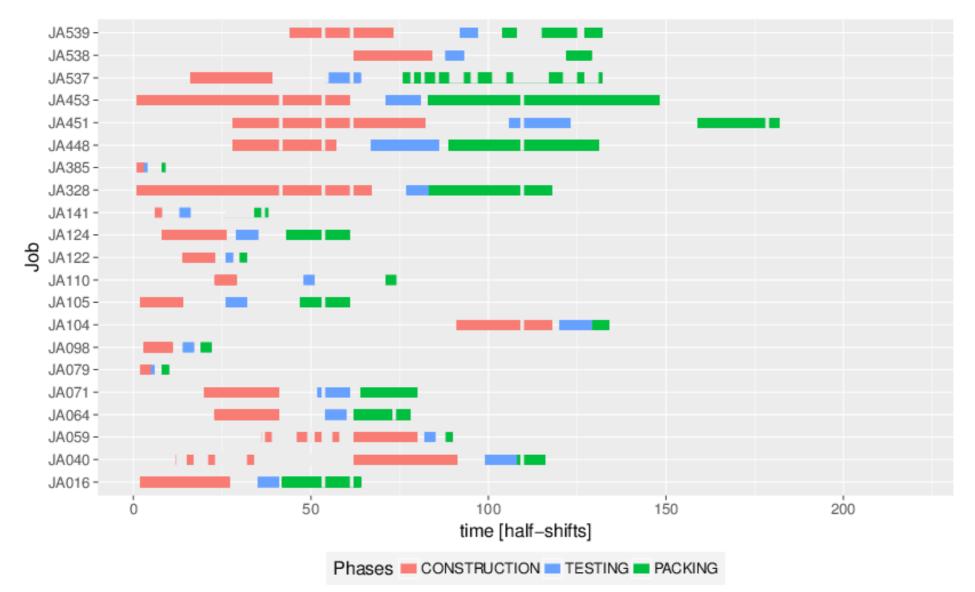
Example: Jobs planning (real-world ex.)



- Assign staff and machines to jobs
- Objectives:
 - 1 Maximize number of completed jobs
 - 2 Minimize schedule interruptions

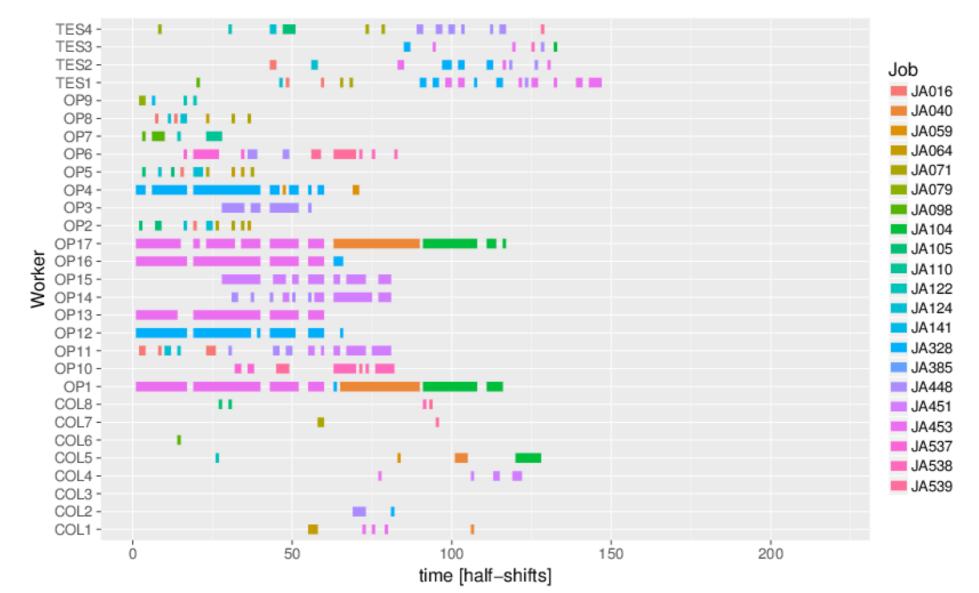
Jobs planning: Solution (jobs)

- 1 Jobs completion within 94% of true optimum
- 2 Optimal number of interruptions

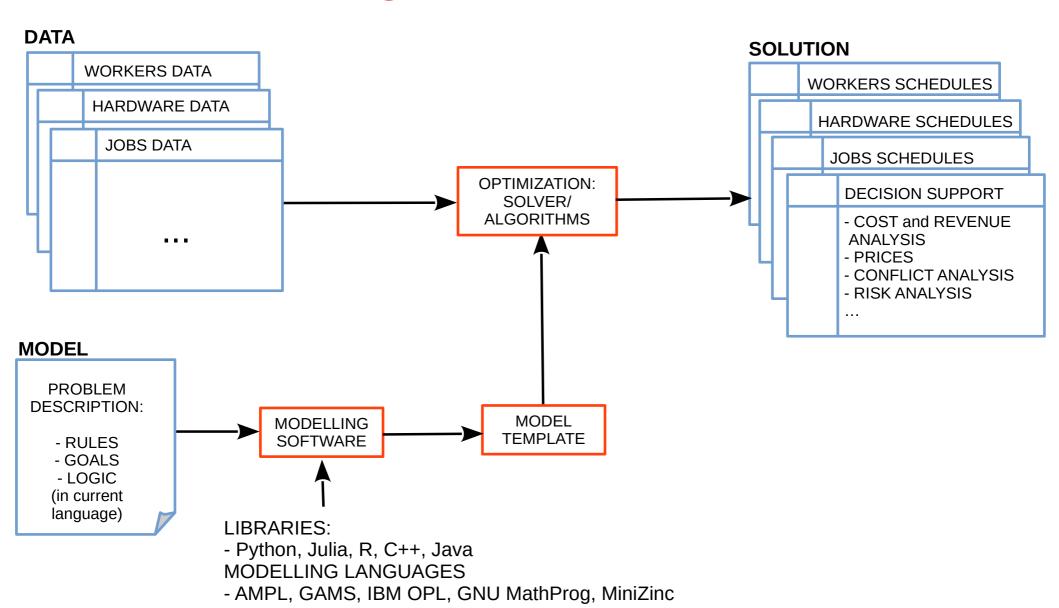


Jobs planning: Solution (workers)

- 1 Jobs completion within 94% of true optimum
- 2 Optimal number of interruptions



Jobs planning with OR: workflow



Exploiting OR and Optimization

OR **empowers** decision makers, it **does not** replace them!



- ► easy decisions → automatic
 - Prescriptive analytics
- ► hard decisions → clear
 - coordinate and communicate

Solvers, Modelling Software and Associations













































GLPK Gecode **GNU MathProg**



SCIP



Minizinc



Success stories at INFORMS' Impact [3]

- ► NBC: optimize advertising slots sales
 - + 50 mln/y, reduce replanning by 80%
- ► UPS: delivery optimization
 - + 300 000 mln\$/y and -100 000 CO₂ ton/y
- ► Chile Football League: optimize game schedules
 - + 6 bln \$/y by increasing audience
- Disney World

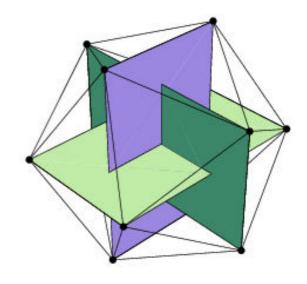
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Optimisation in the Real World [4] Friendly introduction to practical OR problems



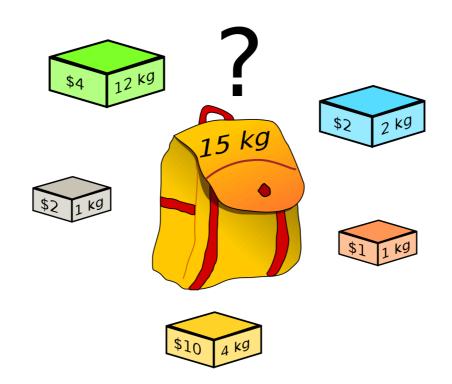
Mathematical Aspects of OR

Models



The Binary Knapsack Problem (BKP)

- Items with different values and weights
- A knaspack with limited capacity



Problem:

► Put items in the knapsack to maximize its value

Ingredients of an optimization problem

- ▶ Data → Parameters
 - ightharpoonup Weight W_i and value e_i of item $i \in I$
 - Knapsack capacity W
- **Decisions** → Variables
 - X_i binary: $X_i = 1$ if *i* is in the knapsack
- **► Rules** → Constraints
 - ► Knapsack Capacity: $\sum_{i \in I} w_i x_i \leq W$
- ► Goals → Objective functions
 - ► Knapsack value: $\max f(\mathbf{x}) = \sum_{i \in I} e_i \mathbf{x}_i$

The Binary Knapsack Model

$$\max f(\mathbf{x}) = \sum_{i \in I} e_i \mathbf{x}_i \tag{1}$$

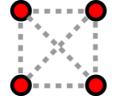
$$\sum_{i \in I} w_i x_i \le W \tag{2}$$

$$\mathbf{x}_{i} \in \{0, 1\} \qquad \forall i \in I \tag{3}$$

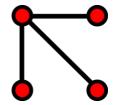
Solved by either a **general-purpose solver** or an **ad-hoc algorithm**

Mathematical Aspects of OR

Algorithms







Types of optimization problems

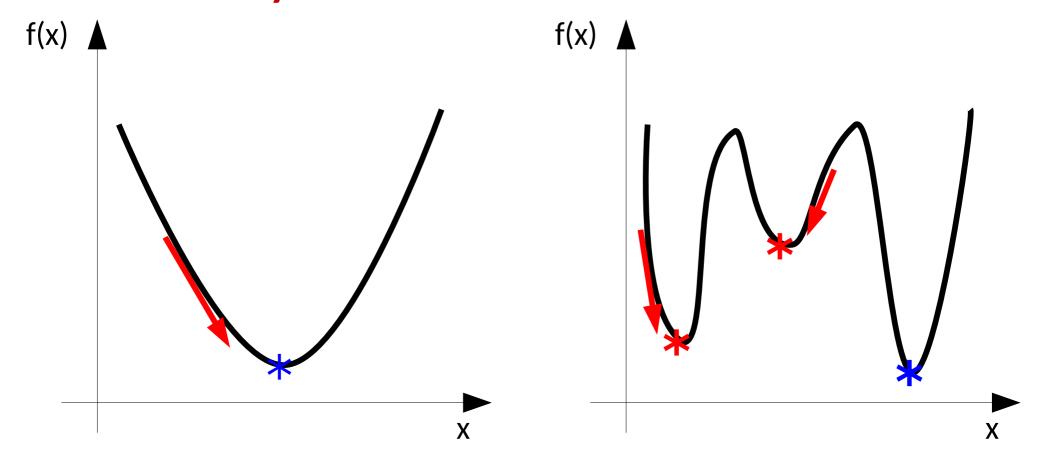
Problem
$$P : \min f(\mathbf{x})$$

s.t. $\mathbf{x} \in \mathcal{X} \subset \mathbb{R}^{n \times m}$

Classification:

- ► Convexity → shape of functions and sets
 - Convex
 - Non-convex
- ► Integrality → type of variables
 - **Continuous:** all continuous
 - Mixed-integer: some integer, logical or categorical

Convexity: Convex vs Non-Convex

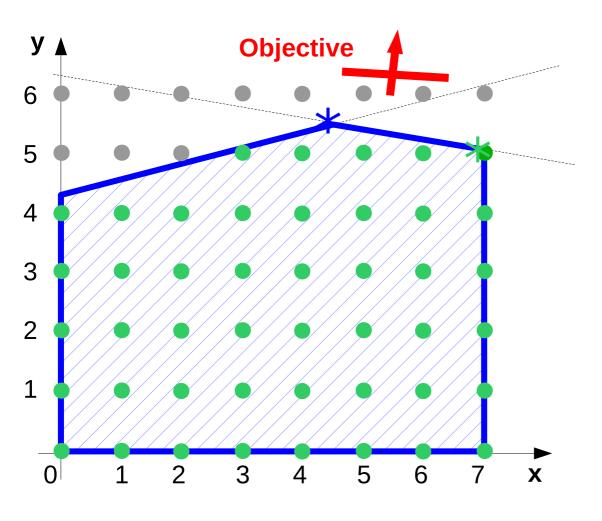


→: gradient

* : local optimum

* : global optimum

Integrality: Continuous vs Mixed Integer



- : invalid integer point
- : valid integer point
- : continuous valid space
- *: continuous optimum
- *: integer optimum

continuous optimum =/= integer optimum

→ rounding not enough!

Solving optimization problems

	Algorithm	Complexity (as spiciness)	
		Convex	Non- Convex
Continuous	gradient- based		
Mixed- Integer	smart enumeration of integer points		

A few algorithms for optimization

- Simplex
- Barrier
- Lagrangean Decomposition
- Branch&Bound
- Column Generation
- Benders Decomposition
- Ad-hoc Relaxations
- Matheuristics
- Metaheuristics: Tabu-Search, Large-Scale Neighbourhood search,...

Mathematical Aspects of OR

Optimization for Data Science



Optimization for Data Science

General **learning problem** on (X,y):

$$\min R(\theta) = \int_{\mathbf{X}} \mathcal{L}(\mathbf{y}, \mathbf{f}(\mathbf{x}; \theta)) d\mathbf{x} \quad \text{s.t. } \theta \in \Theta$$

→ An optimization problem!

Reframe learning as optimization:

- Improve existing DS methods
- Better solutions, richer models

Optimization for Data Science[5]

"[...] the best machine learning work is an attempt to re-phrase prediction as an optimization problem [...] bad machine learning papers (most of them in fact) use bad out of date ad-hoc optimization techniques.

One thing we did in the past was to use CPLEX [...] to compute support vector machines. [...] it blew away all approaches coming from machine learning as CPLEX was several orders of magnitude faster."



Addendum

Following the talk, I decided to clarify and expand the points in the two previous slides.

- OR algorithms can be successfully exploited in some optimization problems in ML
 - See [6]

In other cases, OR algorithms are likely to be ill-suited

- Furthermore, ML can be successfully exploited in OR algorithms as well
 - See [7], and [8]
- OR and ML are complementary approaches that yield the best when working in synergy!

Regression as an optimization problem (proof of concept)

(X,y) dataset \rightarrow features $X \in \mathbb{R}^{n \times m}$, labels $y \in \mathbb{R}^n$.

$$I=\{1...n\}, J=\{1...m\}.$$

$$\min \sum_{i \in I} \epsilon_i^+ + \epsilon_i^-$$

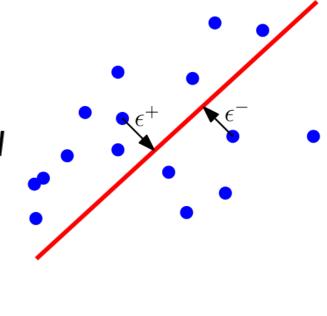
s.t.
$$\sum_{j \in J} \beta_j^\top x_{ij} + \alpha = y_i + \epsilon_i^+ - \epsilon_i^- \quad \forall i \in I$$

$$\epsilon_{i}^{+} \geq 0, \epsilon_{i}^{-} \geq 0 \quad \forall i \in I$$

 $\beta_{i} \in \mathbb{R} \ \forall j \in J, \ \alpha \in \mathbb{R}$

Where:

- \triangleright β,α : coefficients and fixed term
- ► €+, €-: residuals



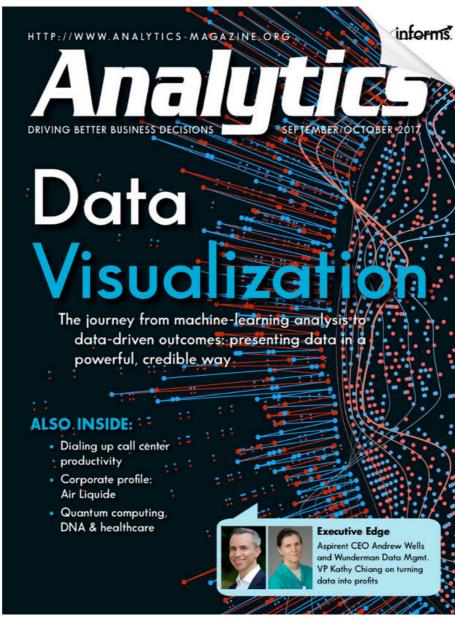
OR and Data Science in the Industry



INFORMS' Analytics [9] (since 2008)

Free professional webzine, general scope



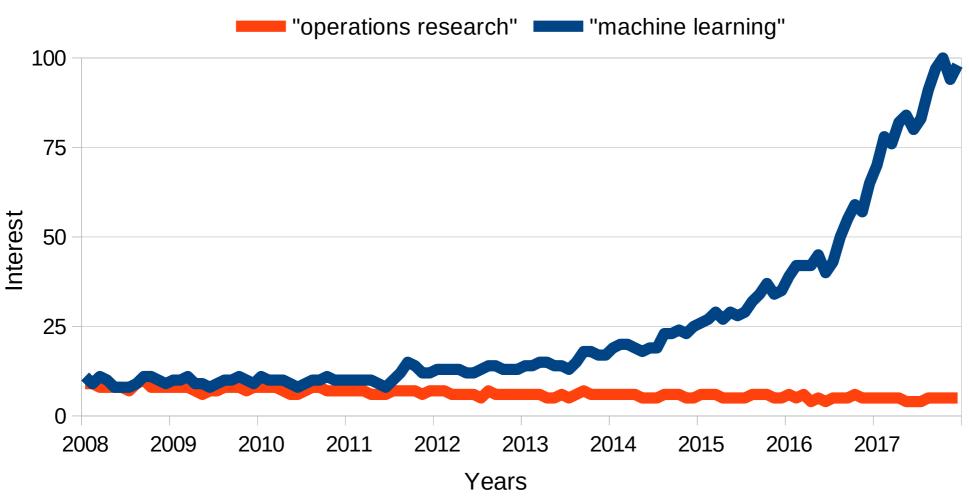


OR&Optimization: the paradox

- Highest maturity, intelligence and unique disruptive power
 - High ROI!
- Example 2 Less known, compared to other methods
 - Too much disruptive?

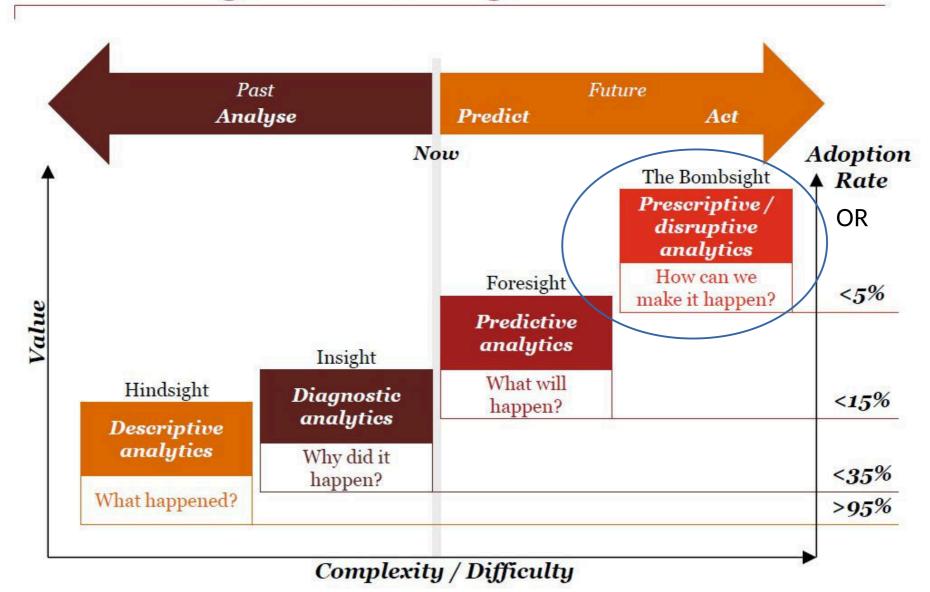
Lack of "hype"?



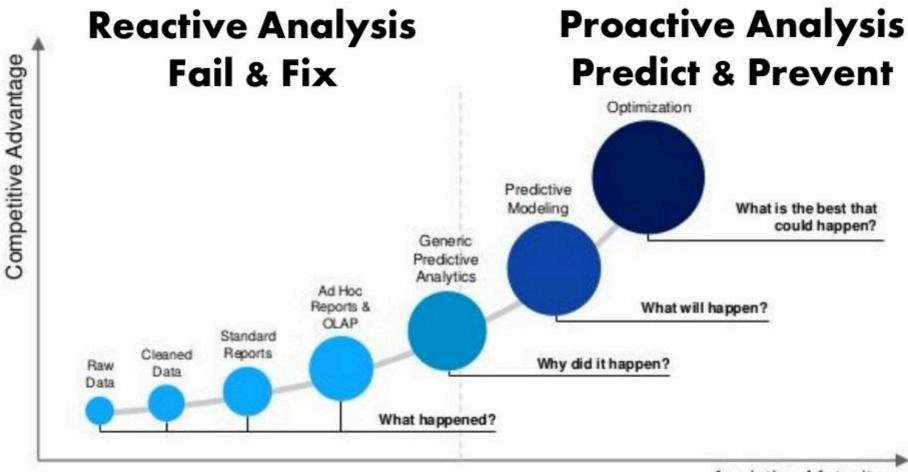


The Analytics stack (PWC [10]-2013)

Data Analytics Maturity Model



Analytics Maturity Model (SAS [11]-2013)



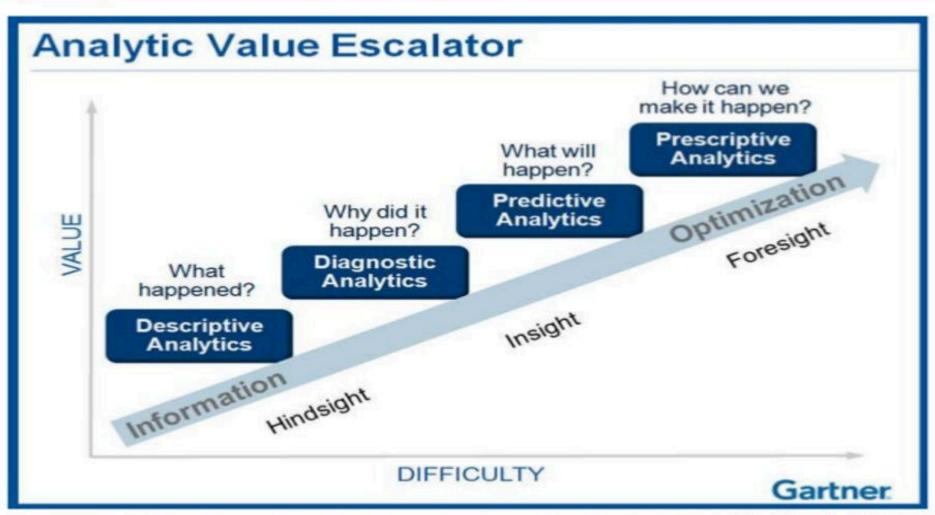
Analytics Maturity



Analytics Value Escalator (Gartner[12]-2016)

Quadrants of Analytic Value





OR&Optimization: the paradox

- Highest maturity, intelligence and unique disruptive power
 - High ROI!
- Example 2 Less known, compared to other methods
 - Too much disruptive?

Beat the averages!Do Operations Research!



The Ideal Data Science team ... has Operations Research in it!

"Based on my experience in large industrial companies, I believe that every Data Science team should have a 20% of Operations Research expertise, to effectively deliver sustainable value to the business."

Benoit Rottembourg [13], head of Pricing Analytics @
 Maersk (Copenhagen) - former Partner at EURODECISION (France), Nov 2017



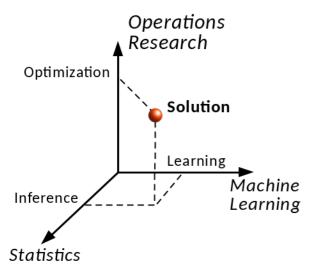
An Operations Researcher near you

Where are **your**Operations Researchers now?



(It's cold outside!)

Key points



1 Give Data Science a new dimension!



2 Bring Operations Research to the world!



3 Grab the opportunities, Get OR!

Links -I

- [1] Beer distribution game at MIT https://www.systemdynamics.org/index.php? option=com_content&view=article&id=141:beergame&catid=20:site-content&Itemid=120
- [2] The Manufacturing Game http://manufacturinggame.com/the-manufacturing-game/
- [3] Informs' Impact magazine https://www.informs.org/Impact
- [4] Optimization in the real world www.optimisationintherealworld.co.uk/
- [5] JF Puget's blog post https://www.ibm.com/developerworks/community/blogs/jfp/entry/machine_learning_and_optimization1?lang=en

Links-II

- [6] OR algorithms for ML and Data Science http://www.lnmb.nl/conferences/2018/programln mbconference/Bertsimas-1.pdf
- [7] Data Science algorithms for OR http://cerc-datascience.polymtl.ca/wp-content/uploads/2017/04/CERC_DS4DM_2017_004-1.pdf
- [8] On synergy between OR and ML https://cpaior2017.dei.unipd.it/slides/Lodi.pdf
- [9] Analytics Magazine http://analytics-magazine.org/

Links-III

- [10] Analytics Stack (PWC) http://slideplayer.com/slide/4880231/
- [11] Analytics Maturity Model (SAS)
 https://www.slideshare.net/louisfernandes/
 130812-lse-lecturev20public
- [12] Analytics Value Escalator https://www.slideshare.net/CAPHC_ACCSP/o ct-25-caphc-breakfast-symposiumsponsored-by-hitachi-cgi-evident-and-intelpaul-lewis-70281057
- [13] Benoit Rottembourg's profile https://www.linkedin.com/in/rottembourg/