NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2020

TECHNICAL MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

1.1 1.1.1

$$2x^2 - 5x = 12$$
$$2x^2 - 5x - 12 = 0$$
$$(2x+3)(x-4) = 0$$
$$x = -\frac{3}{2} \text{ or } x = 4$$

1.1.2

$$4x + 7 = -2x^2$$
$$2x^2 + 4x + 7 = 0$$
$$x^2 + 2x + \frac{7}{2} = 0$$

OR

Option 1

Use formula

$$x = \frac{-4 \pm \sqrt{16-56}}{2}$$
$$= \frac{-4 \pm \sqrt{-40}}{2}$$
$$= \frac{-2 \pm i\sqrt{10}}{2} \text{ (use calculator)}$$
$$= -1 \pm \frac{\sqrt{10}}{2} i \text{ (mark given if left at *)}$$

Option 2

$$(x+1)^2 = 1 - \frac{7}{2} = \frac{-5}{2} = \frac{-10}{4}$$
$$x + 1 = \pm \frac{\sqrt{10}i}{2}$$
$$x = -1 \pm \frac{\sqrt{10}i}{2}$$

1.2 Let energy consumption of motor $= x$ and heater $= y$

$4x + 2y = 25$ ① and $2x + 3y = 18$ ②

② × 2: $4x + 6y = 36$

Subtr. $-4y = -11$

$$y = \frac{11}{4} \text{ kJ/hr}$$

Subst ① $4x + \frac{11}{2} = 25$

$$4x = \frac{39}{2}$$
$$x = \frac{39}{8} \text{ kJ/hr}$$

1.3

$$\Delta = (-3)^2 - 4(1)(9k)$$
$$= 9 - 36k$$

For real, diff roots $\Delta > 0$

$$9 - 36k > 0$$
$$-36k > -9$$
$$k < \frac{1}{4}$$

QUESTION 2

2.1

$$\frac{3^{2014}+9^{1007}}{27^{671}}$$

$$=\frac{3^{2014}+3^{2014}}{3^{2013}}$$

$$=\frac{2.3^{2014}}{3^{2013}} \qquad \textbf{OR} \qquad \frac{3^{2014}}{3^{2013}}+\frac{3^{2014}}{2^{2013}}$$

$$=2.3=6 \qquad\qquad =3+\ \ 3=6$$

2.2 2.2.1

$$5-\sqrt{4x+1}=x$$

$$5-x=\sqrt{4x+1} \qquad \textbf{OR}$$

$$25-10x+x^2=4x+1 \qquad 5-x\geq 0 \text{ and } 4x+1\geq 0$$

$$x^2-14x+24=0 \qquad -x\geq -5 \text{ and } x\geq -\frac{1}{4}$$

$$(x-2)(x-12)=0 \qquad x\leq 5$$

$$x=2 \text{ or } x=12 \qquad \textbf{OR} \qquad \text{Check solution}$$

N/A

2.2.2

$$2\log x=\log 4+\log(x-1)$$

$$\log x^2=\log(4x-4) \quad x>1$$

$$x^2=4x-4$$

$$x^2-4x+4=0$$

$$(x-2)^2=0$$

$$x=2$$

QUESTION 3

3.1 $\frac{(3-2i)}{(1+5i)} \cdot \frac{(1-5i)}{(1-5i)}$

$$= \frac{3-17i+10i^2}{1-25i^2}$$

$$= \frac{3-17i-10}{1+25} \qquad (i^2=-1)$$

$$= \frac{-7-17i}{26}$$

$$= \frac{-7}{26} - \frac{17}{26}i$$

3.2 3.2.1 $V = 2(\cos 120° + i\sin 120°)$ **or** $2(-\cos 60° + i\sin 60°)$

3.2.2 $V = 2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$$= -1 + \sqrt{3}i$$

3.3 11001_2

$$= 2^4 + 2^3 + 2^0$$
$$= 16 + 8 + 1$$
$$= 25$$

QUESTION 4

4.1 $= 4\ 800 - \left(4\ 800 \times \dfrac{13{,}5}{100}\right)$

$= \text{R}4\ 152$

OR $\text{R}4\ 800 \times 86{,}5\%$

$= \text{R}4\ 152$

4.2 4.2.1

$$m = 4$$
$$i = 0{,}09$$

$$1 + i = \left(1 + \frac{i^{(m)}}{m}\right)^m$$

$$1 + 0{,}09 = \left(1 + \frac{i^{(4)}}{4}\right)^4$$
$$\sqrt[4]{1{,}09} = 1 + \frac{i^{(4)}}{4}$$
$$\sqrt[4]{1{,}09} - 1 = \frac{i^{(4)}}{4}$$
$$4\left(\sqrt[4]{1{,}09} - 1\right) = i^{(4)}$$
$$\therefore i^{(4)} = 8{,}71\%$$

OR The nominal interest rate is 8,71% p.a. compounded quarterly

4.2.2 Let amount invested $= x$

$$3x = x(1 + 15i)$$
$$3 = 1 + 15i$$
$$i = \frac{2}{15} = 0{,}133$$

i.e. $13\frac{1}{3}\%$ p.a. (or 13,3%)

4.3 Original value $= x$

$$\frac{x}{2} = x(1 - 0{,}13)^n$$
$$0{,}5 = 0{,}87^n$$
$$\log_{0{,}87} 0{,}5 = n$$
$$n = 4{,}977\ldots \text{ i.e. 5 years}$$

QUESTION 5

5.1 Subst $(-2;5)$ in g: $5=-(-2)+k$

$3=k$

5.2 T is $(0;3)$ i.e. $c=3$

$$f(x)=-\frac{1}{2}x^2+bx+3$$

Subst $(-2;5)$: $5=-\frac{1}{2}(-2)^2+b(-2)+3$

$$2b=-4$$
$$b=-2$$

OR

$$y=-\frac{1}{2}(x+2)^2+5$$
$$=-\frac{1}{2}x^2+2x-2+5$$
$$=-\frac{1}{2}x^2-2x-3$$
$$c=3;\ b=-2$$

5.3 $\mathrm{PQ}=y_{\mathrm{P}}-y_{\mathrm{Q}}$

$$=(-x+3)-\left(-\frac{1}{2}x^2-2x+3\right)$$
$$=\frac{1}{2}x^2+x$$

5.4

$$\frac{1}{2}x^2+x=12$$
$$x^2+2x-24=0$$
$$(x+6)(x-4)=0$$
$$x=-6 \text{ or } x=4$$

P N/A

Subst in g: $y_{\mathrm{P}}=-(-6)+3=9$

P is $(-6;9)$

5.5 $g(x)\le f(x)$

$-2\le x\le 0$ **OR** $[-2;0]$

QUESTION 6

6.1 $y = 1$

6.2 At B, $x = 0$: $y = 2^{\circ} + 1$

$= 1 + 1 = 2$

i.e. $r = 2$ so $g(x) = \sqrt{4 - x^2}$

6.3 Domain: $x \in [-2\,;2]$ **OR** $-2 \le x \le 2$

Range: $y \in [0\,;2]$ $0 \le y \le 2$

6.4

$$M_{AB} = \frac{y_{\text{A}} - y_{\text{B}}}{x_{\text{A}} - x_{\text{B}}}$$

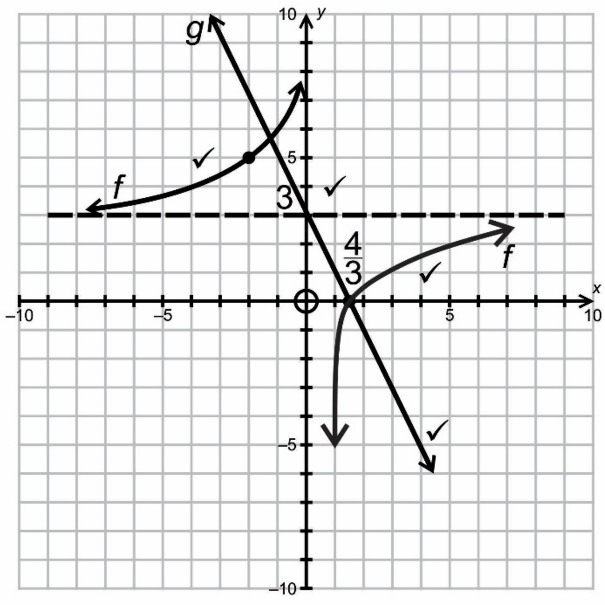
$$0,44 = \frac{k - 2}{-1,466 - 0}$$

$$0,44(-1,466) = k - 2$$

$$k \approx 1,35$$

QUESTION 7

7.1



A asymptote $y = 3$

Put $y = 0$ $\quad 0 = -\frac{4}{x} + 3 \quad \Rightarrow x = \frac{4}{3}$

Marks allocated on graph.

7.2 In $g = \frac{3-0}{0-4} = -\frac{9}{4}$

Eqn of $g : y = -\frac{9}{4}x + 3$

$$\therefore h(x) = -\frac{9}{4}x + 3 + 1$$

$$= -\frac{9}{4}x + 4$$

QUESTION 8

8.1

$$f^{/}(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{5-2x-2h-5+2x}{h}$$
$$= \lim_{h\to 0}\frac{-2h}{h}$$
$$= \lim_{h\to 0} -2$$
$$= -2$$

8.2

$$y = \frac{x^2}{x} - \frac{4x}{x} + \frac{3}{x}$$
$$= x - 4 + 3x^{-1}$$
$$\therefore \frac{dy}{dx} = 1 - 3x^{-2}$$

OR

$$= 1 - \frac{3}{x^2}$$

8.3

$$f(x) = 2x^{\frac{1}{2}} + x^{-3} - \sqrt{2}x$$
$$f^{/}(x) = x^{-\frac{1}{2}} - 3x^{-4} - \sqrt{2}$$

OR

$$= \frac{1}{\sqrt{x}} - \frac{3}{x^4} - \sqrt{2}$$

QUESTION 9

9.1 9.1.1 (a)

$$\text{Vol} = \pi r^2 h$$
$$375 = \pi r^2 h$$
$$\frac{375}{\pi r^2} = h$$

(b)

$$\text{SA} = 2\pi r^2 + 2\pi r h$$
$$= 2\pi r^2 + \frac{2\pi r.375}{\pi r^2}$$
$$= 2\pi r^2 + \frac{750}{r}$$

9.1.2

$$\text{S} = 750r^{-1} + 2\pi r^2$$
$$\frac{ds}{dr} = -750r^{-2} + 4\pi r$$

At min $\frac{ds}{dr} = 0$

$$\frac{-750}{r^2} + 4\pi r = 0$$
$$-750 + 4\pi r^3 = 0$$
$$r^3 = \frac{750}{4\pi} = \frac{375}{2\pi}$$
$$r = \sqrt[3]{\frac{375}{2\pi}} \text{ cm} \quad \textbf{OR} \quad 5\sqrt[3]{\frac{3}{2\pi}} \quad \textbf{OR} \text{ accept } 3{,}9 \text{ or } \frac{5{,}7}{\pi}$$

9.2 9.2.1

$$f(x) = (x+2)(x-3)^2$$
$$= (x+2)(x^2 - 6x + 9)$$
$$= x^3 - 6x^2 + 9x + 2x^2 + 2x + 18$$
$$= x^3 - 4x^2 - 3x + 18$$
$$p = -4 \quad q = 18$$

OR by substitution

$$0 = (-2)^3 + p(-2)^2 - 3(-2) + q$$

and

$$0 = (3)^3 + p(3)^2 - 3(3) + q$$

and solve for p to q

9.2.2

$$m\tan = f'(x) = 3x^2 - 8x - 3$$

At A, $3x^2 - 8x - 3 = 8$

$$3x^2 - 8x - 11 = 0$$
$$(3x - 11)(x + 1) = 0$$
$$x = \frac{11}{3} \text{ or } x_A = -1$$
$$y_A = (-1+2)(-1-3)^2$$
$$= 16$$

OR subs in eqn

$A(-1;16)$

9.2.3 $f(x) = x^3 - 4x^2 - 3x + 18$

$f'(x) = 0$ (t.p.)

$\therefore 3x^2 - 8x - 3 = 0$

$(3x+1)(x-3) = 0$

$\therefore x = -\frac{1}{3}$ or $x = 3$

Turning points $\left(-\frac{1}{3}\,;\,\frac{500}{27}\right)$; (3 ; 0) (5)

9.2.4 At K, $m\tan = f'(0) = -3$

$\therefore m\perp = \frac{1}{3}$

K is $(0\,;18)$

$\therefore h(x) = \frac{1}{3}x + 18$

9.3 9.3.1 $A = -t^2 + 5t + 8$

$A = -(0)^2 + 5(0) + 8$

$= 8 \text{ cm}^2$

9.3.2 $\frac{dA}{dt} = -2t + 5$

at $t = 2$

$= -2(2) + 5$

$= 1 \text{ cm}^2$

9.3.3 $\frac{dA}{dt} = 0$

$-2t + 5 = 0$

$t = \frac{5}{2}$ seconds

QUESTION 10

10.1 (a) $\int 0dx = c$

(b) $\int dx = x + c$

10.2 Determine: $\int\left(3x^2 + \frac{1}{x}\right)dx$

$= \frac{3x}{3} + \ln x + c$

$= x^3 \quad \ln x \quad + c$

10.3 Area $= \int_0^2 x^3 dx$

$= \left[\frac{x^4}{4}\right]_0^2$

$= \frac{16}{4} - 0$

$= 4$

and by symmetry $\int_{-2}^{0} x^3 dx = 4$

$\therefore$ Total area $= 8$ units^2

OR

$\int_{-2}^{0} x^3 dx = \left[\frac{x^4}{0}\right]_{-2}^{0}$

$= -4$

$\therefore$ 8 units^2

Total: 150 marks