latest 10/12/2018

GRADE 12 EXAMINATION
NOVEMBER 2018

ADVANCED PROGRAMME MATHEMATICS: PAPER I
MODULE 1: CALCULUS AND ALGEBRA

MARKING GUIDELINES

Time: 2 hours

200 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

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7/12/2018

QUESTION 1

1.1 (a) **METHOD 1**

$|x^2+x|=-2x-2$

$\therefore (x^2+x)^2=(-2x-2)^2$ ✓ m – squaring both sides

$\therefore x^4+2x^3+x^2=4x^2+8x+4$

$\therefore x^4+2x^3-3x^2-8x-4=0$ ✓a

by inspection $x=-1$ *is a root* ✓ m

$\therefore (x+1)(x^3+x^2-4x-4)=0$ ✓a

by inspection $x=-1$ *is a root* ✓ m

$\therefore (x+1)(x+1)(x^2-4)=0$ ✓a

$\therefore (x+1)(x+1)(x+2)(x-2)=0$ ✓a

OR

$\therefore x=-1$ *or* ± 2

a check reveals $x=-1$ *or* -2 *only* ✓ m – checking – can be implied ✓a

$(-x)^2-x=-2x-2$
$x^2+x+2=0$
N/S
with other side
correct 4/8

ALTERNATIVE 1

if only 1 case
max 4

$|x^2+x|=-2x-2$

$\therefore x^2+x=-2x-2$ *or* $-x^2-x=-2x-2$ ✓ m – 2 cases ✓ a

$\therefore x^2+3x+2=0$ *or* $x^2-x-2=0$ ✓a✓a

$\therefore x=-1$ *or* -2 *or* $x=-1$ *or* 2 ✓ca ✓ca

but a check reveals $x=-1$ *or* -2 ✓ m – checking – can be implied ✓ a

ALTERNATIVE 2

$|x^2+x|=-2x-2$

if $x^2+x<0$ *then* $-1<x<0$ ✓a *and we have* ✓ m – 2 cases

$-(x^2+x)=-2x-2$ ✓a

$\therefore -x^2-x=-2x-2$

$\therefore x^2-x-2=0$

$\therefore x=2$ *or* -1 (*a contradiction*) ✓ m checking ✓a discarding

if $x^2+x\geq 0$ *then* $x\leq -1$ ✓a *or* $x\geq 0$ *and we have*

$x^2+x=-2x-2$ ✓a

$\therefore x^2+3x+2=0$

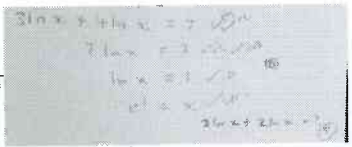
$\therefore (x+2)(x+1)=0$

$\therefore x=-2$ *or* -1 ✓a

Ans only 2/8 x = -2 or x = -1. (8)

(8)

h



answer only $\rightarrow 1$

(b) $\ln x^3 + 2\ln x^2 = 7$

$\therefore \ln x^3 + \ln x^4 = 7$ √m log laws, √a

$\therefore \ln x^7 = 7$ (*can go straight to the answer from here*) ✓a

$\therefore 7\ln x = 7$ ✓a

$\therefore \ln x = 1$ ✓ca

$\therefore x = e$ ✓ca

(overlook $x = 2{,}718$)

if mess up line 1 B/D max 4

√a
√m log law

$3\ln x + 4\ln x = 7$

$\therefore 7\ln x = 7$ √ca √a

$\therefore \ln x = 1$ √ca

$\therefore x = e$ √ca

(6)

1.2 (a) $y = y_0 e^{-kt}$

$\therefore e^{-kt} = \frac{y}{y_0}$ √a

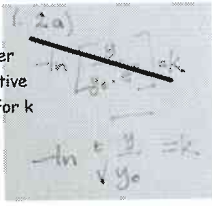
log$_e x$ is okay

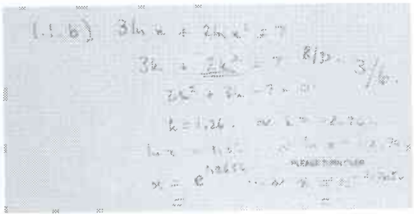
$\therefore -kt = \ln\frac{y}{y_0}$ √m logs √a

$$\boxed{\therefore k = \frac{\ln\frac{y}{y_0}}{-t}} \text{ ✓ ca}$$

OR $\frac{\ln y - \ln y_0}{-t}$ OR $\frac{\ln y_0 - \ln y}{t}$ (4)

1 further alternative forms for k





(b) $k = \frac{\ln\frac{0{,}5y_0}{y_0}}{-5700} \approx 1{,}216 \times 10^{-4}$ ✓m – indicating $\frac{1}{2}$ ✓ca

0,000122 (to 6 dp) not necessary (2)

(c) $0{,}9y_0 = y_0 e^{-kt}$ ✓m – indicating 90% idea/ratio

$\therefore -kt = \ln 0{,}9$ ✓m logs

$\therefore t = \frac{\ln 0{,}9}{-k}$ ✓a

$\therefore t \approx 866$ *years* ✓ca (will get 864 if use rounded value for k) (4)

[24]

accept "exact" answers

QUESTION 2

Alternative:

$x = 3+2i$

$x-3 = 2i$

$x^2-6x+9=-4$

$x^2-6x+13=0$

5 directly

Alternative 2 ↓

2.1 *if* $3+2i$ *is a root then so is* $3-2i$ ✓ m – can be implied

$(x-3+2i)(x+3-2i)$ *so our equation is*:

B/D Max 4

$(x+3)(x-(3+2i))(x-(3-2i))=0$ ✓m ✓a factors

$\therefore (x+3)((x-3)-2i)((x-3)+2i)=0$ ✓m

$\therefore (x+3)((x-3)^2-4i^2)=0$ ✓a

if × out directly 2 - process

$\therefore (x+3)(x^2-6x+13)=0$ ✓ca

$\therefore x^3-3x^2-5x+39=0$ ✓ca✓ca (8)

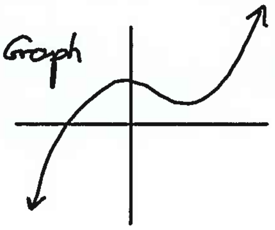
2.2 A cubic equation will have three roots. ✓a✓a Complex roots of polynomials with real coefficients occur in (conjugate) pairs so there must be at least one real root. ✓a✓a (4)

↑NB

not necessary

✓2 pairs
✓1 complex/im.
✓1 hence real

OR Graph



or explanation

2.3 $\dfrac{a+bi}{-b+ai}\times\dfrac{-b-ai}{-b-ai}$ ✓m✓a ×1

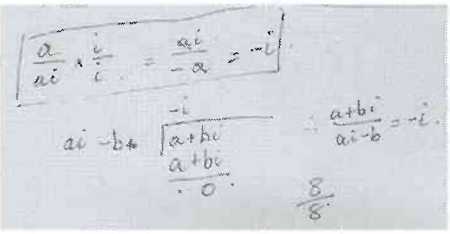
$=\dfrac{-ab-a^2i-b^2i-abi^2}{b^2-a^2i^2}$ ✓ca ✓ca

$=\dfrac{ab-ab-i(a^2+b^2)}{b^2+a^2}$ ✓a ✓a

$=\dfrac{-i(a^2+b^2)}{b^2+a^2}$ ✓a

$=-i$ ✓a (8)

[20]



$\dfrac{a}{ai}\times\dfrac{i}{i} = \dfrac{ai}{-a} = -i$

$ai-b\overline{)a+bi}$ quotient $-i$; $a+bi$; 0

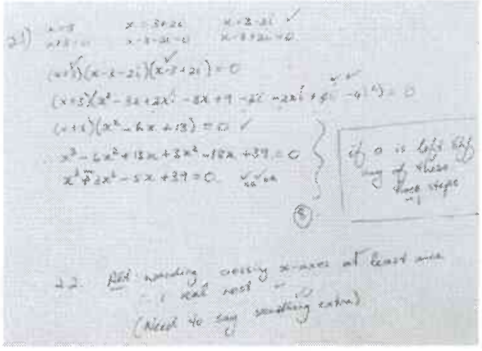
$\therefore \dfrac{a+bi}{ai-b}=-i$

$\dfrac{8}{8}$

Sum/Product $x^2 - \text{sum}\,x + \text{Prod} = 0$

sum $=6$ ✓a Prod $=(3-2i)(3+2i)=9-4i^2=13$ ✓m ✓a

$\therefore x^2-6x+13=0$ ✓ca ✓ca 5 to here



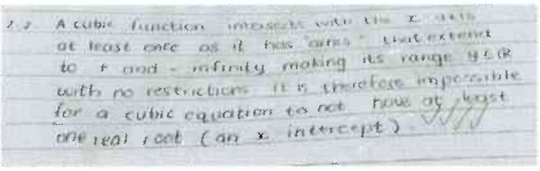
2.1) $(x+3)(x-3-2i)(x-3+2i)=0$

$(x+3)(x^2-6x+13)=0$ ✓

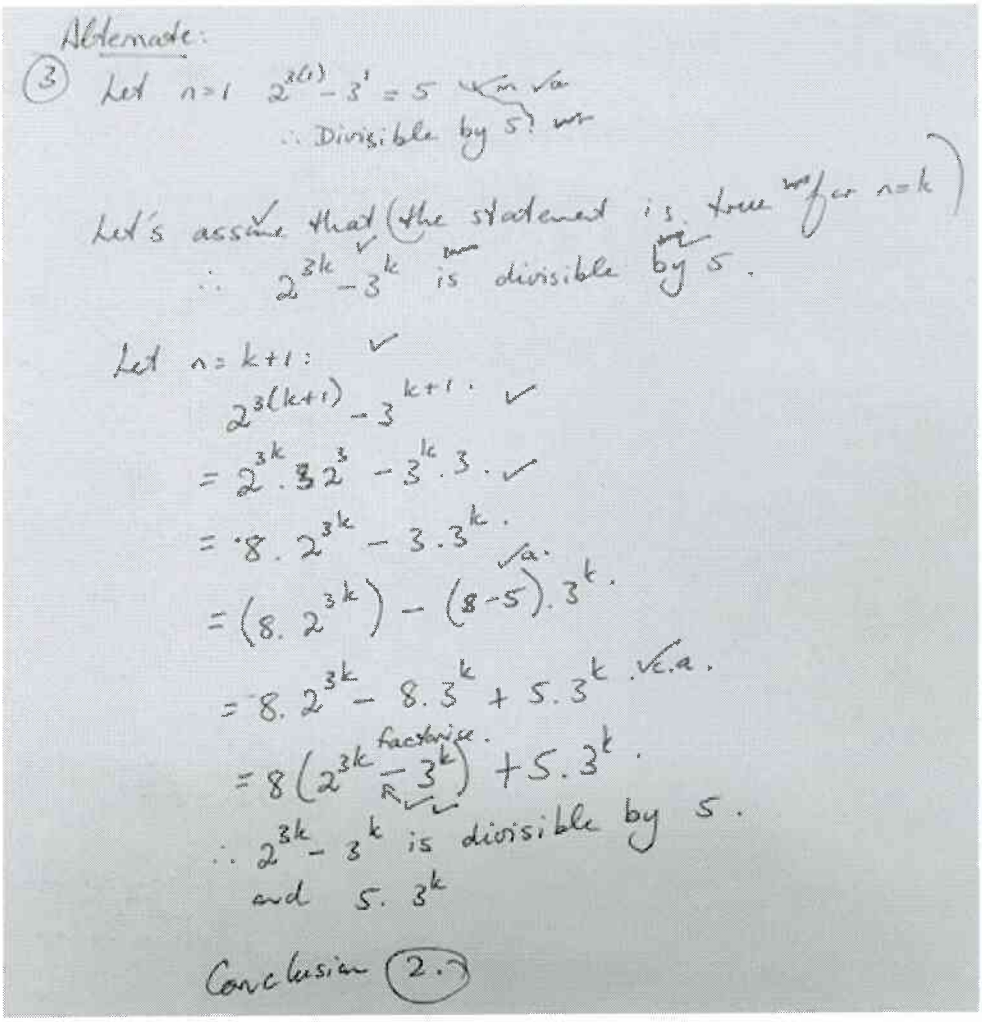
$x^3-3x^2-5x+39=0$ ✓ca✓ca

if 0 is left say any of these three steps -1

2.2 Not wording crossing x-axes at least once -1 real root (Need to say something extra)



2.2 A cubic function intersects with the x axis at least once as it has 'arms' that extend to + and - infinity making its range y∈ℝ with no restrictions. It is therefore impossible for a cubic equation to not have at least one real root (an x intercept). ✓✓✓✓



Alternate:

③ Let n=1 $2^{3(1)} - 3^1 = 5$ ✓m ✓a

∴ Divisible by 5? wr

Let's assume ✓ that (the statement is true for n=k)

∴ $2^{3k} - 3^k$ is divisible by 5.

Let n = k+1: ✓

$2^{3(k+1)} - 3^{k+1}$. ✓

$= 2^{3k}.32^3 - 3^k.3$. ✓

$= 8.2^{3k} - 3.3^k$.

✓a.

$= (8.2^{3k}) - (8-5).3^k$.

$= 8.2^{3k} - 8.3^k + 5.3^k$. ✓c.a.

factorise.

$= 8(2^{3k} - 3^k) + 5.3^k$.

R ✓ ✓

∴ $2^{3k} - 3^k$ is divisible by 5.

and 5.3^k

Conclusion ②

QUESTION 3

Strict – proof

if $n=1$ *we have* $2^3-3=5$ *which is* ~~*clearly*~~ *divisible by 5.* ✓m ✓a

✓ *Assume true for* $n=k$ *viz. that*

$2^{3k}-3^k=5p$ *where* $p\in\mathbb{N}$ (*) ✓m for n=k ✓ for $p \in \mathbb{N}$ or $\mathbb{Z}$.

Now if $n=k+1$ *we have*: ✓m idea of $(k+1)$

$2^{3(k+1)}-3^{k+1}$ ✓a

$=2^{3k+3}-3^{k+1}$

$=2^{3k}\times2^3-3\times3^k$

$=8\times2^{3k}-3\times3^k$ ✓m – splitting ✓m – using * ✓a

from (*) *we have* $2^{3k}=5p+3^k$ *so if* $n=k+1$ *we have*

$=8(5p)+8(3^k)-3\times3^k$ ✓a

$=5(8p)+5(3^k)$ ✓a)

$=5(8p+3^k)$ ✓a) OR

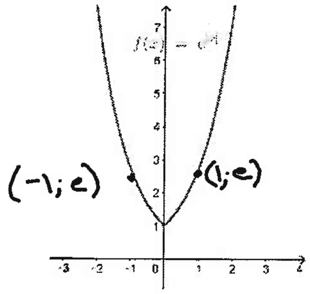
which is ~~*clearly*~~ *divisible by 5*

so, by the Principle of Mathematical Induction we have proved the result for $n\in\mathbb{N}$ ✓a ✓a

OR PMI (or a summary of their argument) [14]

QUESTION 4

4.1 (a)



✓✓ shape

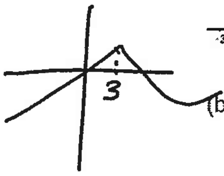
✓✓ symmetry about y-axis

✓✓ range

✓ y-int ✓another pt. if only one branch max4

if a full e^x max 3

(8)



(b) $x=0$ ✓✓ (0; 1) is fine (2)

Consistent with his/her graph if it has a pt. of non-differentiability

4.2 *if f is differentiable at $x=2$*

it must be continuous at $x=2$ ✓m

so $\lim_{x\to2^-} f(x)=\lim_{x\to2^+} f(x)$ OR ←

so $2a-b-1=\overset{\checkmark}{4b-2a+5}$ *or* $\overset{\checkmark a}{4a-5b=6}$ (1) ✓ca

but also, $\lim_{x\to2^-} f'(x)=\lim_{x\to2^+} f'(x)$ ✓m ✓a

$a=4b-a$ *or* $a=2b$ (2) ✓a

(*solving* (1) *and* (2) *simultaneously*) ✓m – solving simultaneously

$8b-5b=6$ ✓a *so* $b=2$ ✓ca *and* $a=4$ ✓ca (10)

Max 8/10 for notation abuse

[20]

QUESTION 5

NB. answer supplied

5.1 *segment = sector – Δ* ✓m – sub. areas

$\therefore 308=\overset{\checkmark m/a}{\frac{1}{2}(18^2)\theta}-\overset{\checkmark m/a}{\frac{1}{2}(18^2)\sin\theta}$ ✓m – equating to 308

$\therefore 162\theta-162\sin\theta-308=0$ (6)

5.2 $f(\theta)=162\theta-162\sin\theta-308$ ✓m – id. f

$\therefore \theta_{n+1}=\theta_n\overset{\checkmark m}{-}\frac{162\theta-162\sin\theta-308\ \checkmark a}{162-162\cos\theta\ \checkmark a}$ ✓m – differentiating

$\theta=2{,}49984$ ✓✓ ✓m subst. 2 – optional (8)

to 5 or more d.p.

[14]

QUESTION 6

6.1 $f(0)=\frac{1}{2}$, *so* y-*int* $\left(0;\frac{1}{2}\right)$ ✓a or $y=\frac{1}{2}$

$\frac{2x^2-3x-2}{x-4}=0$ ✓m letting $y=0$

$\therefore 2x^2-3x-2=0$ ✓m for numerator =1

$\therefore (2x+1)(x-2)=0$ ✓a

$\therefore x$-*ints* $\left(-\frac{1}{2};0\right)$ ✓a *and* $(2;0)$ ✓a Accept answers only (6)

vert. asymptote

y=4 ?

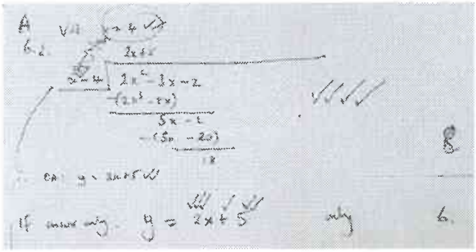
0 →

6.2 *vertical asymptote*: $x=4$ ✓a✓a

$2x^2-3x-2=(x-4)(2x+5)+R$ ✓m✓a✓a✓a – div. by insp / long div / synthetic

so, oblique asymptote is $y=2x+5$ ✓a✓a Accept answers only (8)

(be on the lookout for method involving limit)



A

6.2 VA: x = 4 ✓✓

$$\begin{array}{r} 2x+5 \\ x-4\ \overline{)\ 2x^2-3x-2} \\ -(2x^2-8x) \\ \hline 5x-2 \\ -(5x-20) \\ \hline 18 \end{array}$$

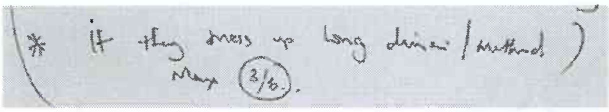
✓✓✓✓

OA: y = 2x+5 ✓✓

8

If answer only: y = 2x + 5 ✓✓✓ ✓ ✓✓ only

6.



(* if they mess up long division / method)

Max (3/8).

Product rule

$f(x) = (2x^2-3x-2)(x-4)^{-1}$

6.3 $f(x)=\dfrac{2x^2-3x-2}{x-4}$

$\therefore f'(x)=\dfrac{(4x-3)\overset{✓a}{(x-4)}-1\overset{✓a}{(2x^2-3x-2)}}{(x-4)^2 ✓a}=0 ✓m$

$=0 \quad \therefore f'(x) = \underset{✓a}{(4x-3)(x-4)^{-1}} - \underset{✓a}{(x-4)^{-2}(2x^2-3x-2)}$

$\therefore 4x^2-19x+12-2x^2+3x+2=0$ ✓a

$\therefore \dfrac{4x-3}{x-4} - \dfrac{2x^2-3x-2}{(x-4)^2} = 0$

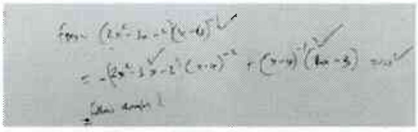
$\therefore 2x^2-16x+14=0$

$\therefore x^2-8x+7=0$

$\therefore (x-1)(x-7)=0$

$\therefore x=1$ or 7 ✓ca

$\therefore (1;1)$ and $(7;25)$ are stationary points
✓ca ✓ca



(8)

6.4 $f''(1)<0$ so $(1;1)$ is a local maximum ✓m✓a

$f''(7)>0$ so $(7;25)$ is a local minimum ✓m✓a

$-\frac{4}{3}$

$\frac{4}{3}$

can't use other methods

(4)

[26]

QUESTION 7

7.1 $x^2+xy+y^2=1$

$\therefore 2x + \overset{a✓}{y} + \overset{✓a}{x\dfrac{dy}{dx}} + \overset{✓a}{2y\dfrac{dy}{dx}} = 0$ ✓a

Be on lookout

$\therefore \dfrac{dy}{dx}(x+2y) = -2x-y$ ✓m factorising

$\therefore \dfrac{dy}{dx} = \dfrac{-2x-y}{x+2y}$

(5)

7.2 At A, $y=0$ $\therefore$ $x=1$ ✓m✓a for $y=0$ $(2;0)$ $x = \pm 1$

so, at A, $\dfrac{dy}{dx}=\dfrac{-2}{1}=-2$ ✓a (for wrong A) max 3 for normal

$\therefore y = -2(x-1)$ ✓m ✓a for str. line

$\therefore y = -2x+2$ (not necessary)

(5)

[10]

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assessment matters

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$\therefore x^4+2x^3-3x^2-8x-4=0$ ✓a

by inspection $x=-1$ *is a root* ✓ m

$\therefore (x+1)(x^3+x^2-4x-4)=0$ ✓a

by inspection $x=-1$ *is a root* ✓ m

$\therefore (x+1)(x+1)(x^2-4)=0$ ✓a

OR

$\therefore (x+1)(x+1)(x+2)(x-2)=0$ ✓a

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a check reveals $x=-1$ *or* -2 *only* ✓ m – checking – can be implied ✓a

$(-x)^2-x=-2x-2$

$x^2+x+2=0$

N/S

with other side correct 4/8

ALTERNATIVE 1

$|x^2+x|=-2x-2$

if only 1 case max 4

$\therefore x^2+x=-2x-2$ *or* $-x^2-x=-2x-2$ ✓ m – 2 cases ✓ a

$\therefore x^2+3x+2=0$ *or* $x^2-x-2=0$ ✓a✓a

$\therefore x=-1$ *or* -2 *or* $x=-1$ *or* 2 ✓ca ✓ca

but a check reveals $x=-1$ *or* -2 ✓ m – checking – can be implied ✓ a

ALTERNATIVE 2

$|x^2+x|=-2x-2$

if $x^2+x<0$ *then* $-1<x<0$ *and we have* ✓a ✓ m – 2 cases

$-(x^2+x)=-2x-2$ ✓a

$\therefore -x^2-x=-2x-2$

$\therefore x^2-x-2=0$

$\therefore x=2$ *or* -1 (*a contradiction*) ✓ m checking ✓a discarding

if $x^2+x\geq 0$ *then* $x\leq -1$ *or* $x\geq 0$ *and we have* ✓a

$x^2+x=-2x-2$ ✓a

$\therefore x^2+3x+2=0$

$\therefore (x+2)(x+1)=0$

$\therefore x=-2$ *or* -1 ✓a

Ans only 2/8 $x=-2$ or $x=-1$

(8)

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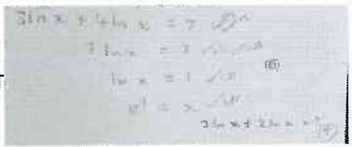

answer only $\rightarrow 1$

if mess up line 1 B/D max 4

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(overlook $x = 2{,}718$)

✓a
✓m log law

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$\therefore x = e$ ✓ ca

(6)

1.2 (a) $y = y_0 e^{-kt}$

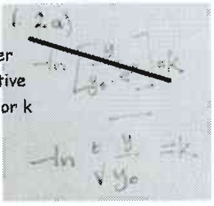
log$_e x$ is okay

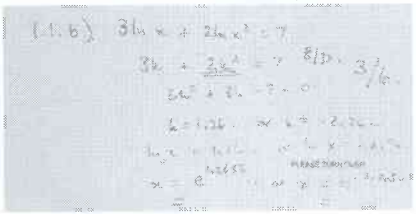
$\therefore e^{-kt} = \dfrac{y}{y_0}$ ✓a

$\therefore -kt = \ln\dfrac{y}{y_0}$ ✓m logs ✓a

$\therefore k = \dfrac{\ln\frac{y}{y_0}}{-t}$ ✓ ca **OR** $\dfrac{\ln y - \ln y_0}{-t}$ **OR** $\dfrac{\ln y_0 - \ln y}{t}$ (4)

1 further alternative forms for k





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$\therefore -kt = \ln 0{,}9$ ✓m logs

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$\therefore t \approx 866$ *years* ✓ca (Will get 864 if use rounded value for k) (4)

[24]

accept "exact" answers

QUESTION 2

2.1 *If $3+2i$ is a root then so is $3-2i$* ✓m – can be implied

$(x-3+2i)(x+3-2i)$

B/D Max 4

so our equation is:

factors

$(x+3)(x-(3+2i))(x-(3-2i))=0$ ✓m ✓a

$\therefore (x+3)((x-3)-2i)((x-3)+2i)=0$ ✓m

$\therefore (x+3)((x-3)^2-4i^2)=0$ ✓a

if x out directly 2 - process

$\therefore (x+3)(x^2-6x+13)=0$ ✓ca

$\therefore x^3-3x^2-5x+39=0$ ✓ca✓ca (8)

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↓

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↑NB

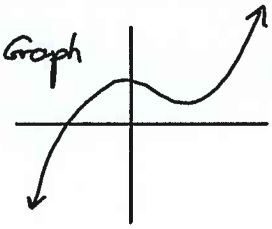
not necessary

2 pairs

✓1 complex/im.

✓1 hence real

OR Graph



or explanation

2.3 $\dfrac{a+bi}{-b+ai}\times\dfrac{-b-ai}{-b-ai}$ ✓m✓a ×1

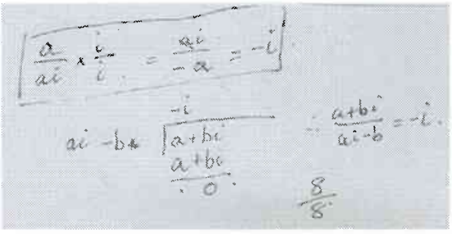
$=\dfrac{-ab-a^2i-b^2i-abi^2}{b^2-a^2i^2}$ ✓ca ✓ca

$=\dfrac{ab-ab-i(a^2+b^2)}{b^2+a^2}$ ✓a ✓a

$=\dfrac{-i(a^2+b^2)}{b^2+a^2}$ ✓a

$=-i$ ✓a (8)

[20]



$\boxed{\dfrac{a}{ai}\times\dfrac{i}{i}=\dfrac{ai}{-a}=-i}$

$ai-b+ \overline{)a+bi}$ with quotient $-i$; $a+bi$; 0

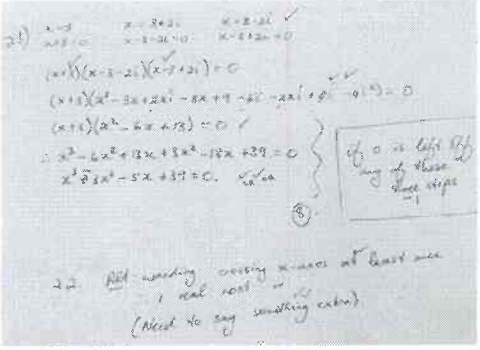
$\therefore \dfrac{a+bi}{ai-b}=-i$

$\dfrac{8}{8}$

Sum/Product $x^2 - \text{sum}\,x + \text{Prod} = 0$

sum $= 6$ ✓a Prod $= (3-2i)(3+2i) = 9-4i^2 = 13$ ✓m ✓a

$\therefore x^2 - 6x + 13 = 0$ ✓ca ✓ca 5 to here



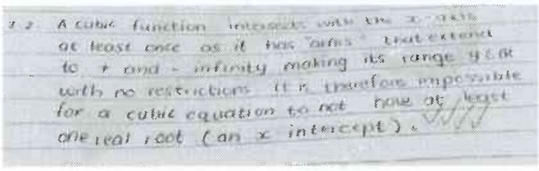
2.1) $(x+3)(x-3-2i)(x-3+2i)=0$

$(x+3)(x^2-6x+13)=0$ ✓

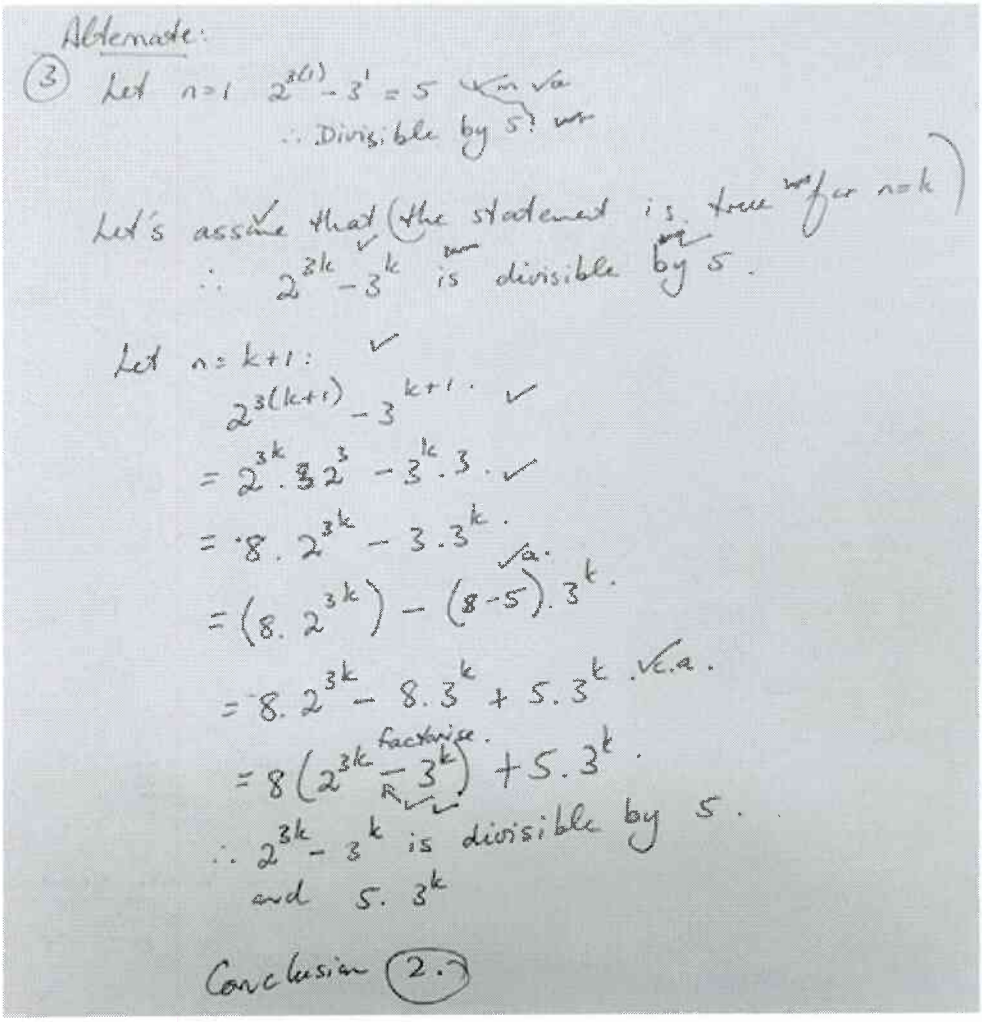
$x^3-3x^2-5x+39=0$ ✓✓

⑧

2.2 Not wording crossing x-axis at least once 1 real root (Need to say something extra)



2.2 A cubic function intersects with the x-axis at least once as it has "arms" that extend to + and - infinity making its range y∈R with no restrictions. It is therefore impossible for a cubic equation to not have at least one real root (an x intercept). ✓✓✓✓



Alternate:

③ Let $n=1$ $2^{3(1)}-3^1=5$ ✓m ✓a

$\therefore$ Divisible by 5! ✓m

Let's assume that (the statement is true for $n=k$)

$\therefore$ $2^{3k}-3^k$ is divisible by 5.

Let $n=k+1$: ✓

$2^{3(k+1)}-3^{k+1}$. ✓

$=2^{3k}.32^3-3^k.3$. ✓

$=8.2^{3k}-3.3^k$.

$=(8.2^{3k})-(8-5).3^k$. ✓a.

$=8.2^{3k}-8.3^k+5.3^k$. ✓c.a.

factorise.

$=8(2^{3k}-3^k)+5.3^k$.

R ✓ ✓

$\therefore$ $2^{3k}-3^k$ is divisible by 5.

and 5.3^k

Conclusion ②.

QUESTION 3

Strict – proof

if $n=1$ *we have* $2^3-3=5$ *which is ~~clearly~~ divisible by 5.* ✓m ✓a

✓ *Assume true for* $n=k$ *viz. that*

$2^{3k}-3^k=5p$ *where* $p\in\mathbb{N}$ (*) ✓m for n=k ✓ for $p \in \mathbb{N}$ or $\mathbb{Z}$.

Now if $n=k+1$ *we have:* ✓m idea of $(k+1)$

$2^{3(k+1)}-3^{k+1}$ ✓a

$=2^{3k+3}-3^{k+1}$

$=2^{3k}\times2^3-3\times3^k$

$=8\times2^{3k}-3\times3^k$ ✓m splitting ✓m using * ✓a

from (*) *we have* $2^{3k}=5p+3^k$ *so if* $n=k+1$ *we have*

$=8(5p)+8(3^k)-3\times3^k$ ✓a

$=5(8p)+5(3^k)$ ✓a

$=5(8p+3^k)$ ✓a) OR

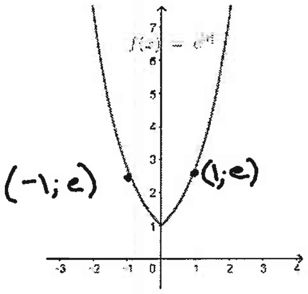
which is ~~clearly~~ divisible by 5

so, by the Principle of Mathematical Induction we have proved the result for $n\in\mathbb{N}$ ✓a ✓a

OR PMI (or a summary of their argument) [14]

QUESTION 4

4.1 (a)

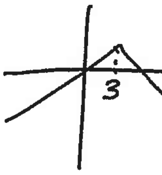


✓✓ shape
✓✓ symmetry about y-axis
✓✓ range
✓ y-int ✓another pt.

if a full e^x max 3

if only one branch max 4

(8)



(b) $x=0$ ✓✓ (0;1) is fine (2)

Consistent with his/her graph if it has a pt. of non-differentiability

4.2 *if f is differentiable at* $x = 2$

it must be continuous at $x = 2$ ✓m

so $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$ OR

so $2a - b - 1 = 4b - 2a + 5$ ✓ *or* $4a - 5b = 6$ ✓a (1) ✓ca

Max 8/10 for notation abuse

but also, $\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^+} f'(x)$ ✓m ✓a

$a = 4b - a$ *or* $a = 2b$ (2) ✓a

(*solving* (1) *and* (2) *simultaneously*) ✓m – solving simultaneously

$8b - 5b = 6$ ✓a *so* $b = 2$ ✓ca *and* $a = 4$ ✓ca (10)

[20]

QUESTION 5

NB. answer supplied

5.1 *segment = sector* – Δ ✓m – sub. areas

$\therefore 308 = \frac{1}{2}(18^2)\theta - \frac{1}{2}(18^2)\sin\theta$ ✓m✓a ✓m✓a ✓m – equating to 308

$\therefore 162\theta - 162\sin\theta - 308 = 0$ (6)

5.2 $f(\theta) = 162\theta - 162\sin\theta - 308$ ✓m – id. f

$\therefore \theta_{n+1} = \theta_n - \frac{162\theta - 162\sin\theta - 308}{162 - 162\cos\theta}$ ✓m ✓a ✓a ✓m – differentiating

$\theta = 2{,}49984$ ✓✓ ✓m subst. 2 – optional (8)

to 5 or more d.p.

[14]

QUESTION 6

6.1 $f(0) = \frac{1}{2}$, *so* *y-int* $\left(0;\frac{1}{2}\right)$ ✓a or $y = \frac{1}{2}$

$\frac{2x^2 - 3x - 2}{x - 4} = 0$ ✓m letting $y = 0$

$\therefore 2x^2 - 3x - 2 = 0$ ✓m for numerator = 1

$\therefore (2x+1)(x-2) = 0$ ✓a

$\therefore$ *x-ints* $\left(-\frac{1}{2};0\right)$ ✓a *and* $(2;0)$ ✓a Accept answers only (6)

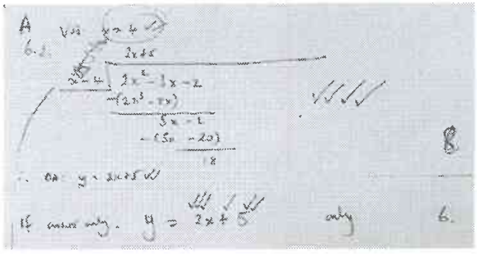
vert. asymptote $y = 4$? 0

6.2 *vertical asymptote* : $x = 4$ ✓a✓a

$2x^2 - 3x - 2 = (x-4)(2x+5) + R$ ✓m✓a✓a✓a div. by insp / long div / synthetic

so, oblique asymptote is $y = 2x + 5$ ✓a✓a Accept answers only (8)

(be on the lookout for method involving limit)



A

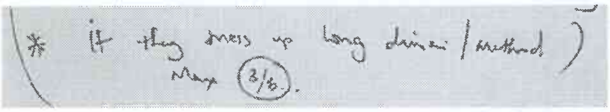
6.2 VA: $x = 4$ ✓✓

$$\begin{array}{r} 2x+5 \\ x-4 \overline{\smash{)}\,2x^2 - 3x - 2} \\ -(2x^2 - 8x) \\ 5x - 2 \\ -(5x - 20) \\ 18 \end{array}$$

✓✓✓✓ 8

∴ OA: $y = 2x+5$ ✓✓

If answer only. $y = 2x + 5$ ✓✓✓ ✓ ✓✓ only 6.



(* if they mess up long division / method)

Max (3/8).

6.3 $f(x)=\dfrac{2x^2-3x-2}{x-4}$

Product rule

$\therefore f'(x)=\dfrac{(4x-3)\overset{\checkmark a}{(x-4)}-1(2x^2-3x-2)^{\checkmark a}}{(x-4)^2\checkmark a}=0\checkmark m$ $=0$

$f(x)=(2x^2-3x-2)(x-4)^{-1}$

$\therefore f'(x)=(4x-3)(x-4)^{-1}-(x-4)^{-2}(2x^2-3x-2)$
✓a ✓a

$\therefore 4x^2-19x+12-2x^2+3x+2=0$ ✓ca ← $\therefore \dfrac{4x-3}{x-4}-\dfrac{2x^2-3x-2}{(x-4)^2}=0$

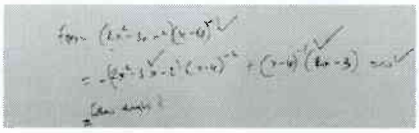
$\therefore 2x^2-16x+14=0$

$\therefore x^2-8x+7=0$

$\therefore (x-1)(x-7)=0$

$\therefore x=1$ *or* 7 ✓ca

$\therefore (1;1)$ *and* $(7;25)$ *are stationary points*
✓ca ✓ca



(8)

6.4 $f''(1)<0$ *so* $(1;1)$ *is a local maximum* ✓m✓a

$-\frac{4}{3}$

$f''(7)>0$ *so* $(7;25)$ *is a local minimum* ✓m✓a

$\frac{4}{3}$

can't use other methods

(4)

[26]

QUESTION 7

7.1 $x^2+xy+y^2=1$

$\therefore \overset{a\checkmark}{2x}+y+\overset{\checkmark a}{x\dfrac{dy}{dx}}+\overset{\checkmark a}{2y\dfrac{dy}{dx}}=0$ ✓ a

Be on lookout

$\therefore \dfrac{dy}{dx}(x+2y)=-2x-y$ ✓m factorising

$\therefore \dfrac{dy}{dx}=\dfrac{-2x-y}{x+2y}$

(5)

7.2 *At* A, $y=0$ $\therefore$ $x=1$ ✓m✓a / for $y=0$ $(2;0)$ $x=\pm 1$

so, at A, $\dfrac{dy}{dx}=\dfrac{-2}{1}=-2$ ✓a (for wrong A) $\left\{\boxed{\text{max 3 for normal}}\right.$

$\therefore y=\overset{\checkmark m}{-2}(x-1)$ ✓a for str. line

$\therefore y=-2x+2$ (not necessary)

(5)

[10]

QUESTION 8

8.1 $\cos\theta = \frac{FC}{CD}$ ✓m – using trig.

$\therefore FC = 0{,}4\cos\theta$ ✓a

$\therefore A = \Delta CDF + \Delta BEG + BCFG$ ✓m – resolving into areas/trap.

$\therefore A = 2\left(\frac{1}{2}\times 0{,}4\times 0{,}4\overset{✓a}{\cos\theta}\underset{✓ca}{\overset{✓a}{\sin\theta}}\right) + 0{,}4\times 0{,}4\overset{✓a}{\cos\theta}$ $(\Delta CDF = \Delta BEG)$

$\therefore A = 0{,}16\sin\theta\cos\theta + 0{,}16\cos\theta$

$\therefore A = 0{,}08\sin 2\theta + 0{,}16\cos\theta$ ✓ca

$\therefore V = 20(0{,}08\sin 2\theta + 0{,}16\cos\theta)$ ✓m – (for × by 20)

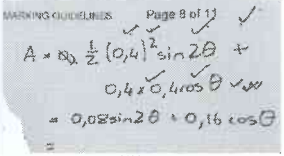
$\therefore V = 1{,}6\sin 2\theta + 3{,}2\cos\theta$

N.B. Answer Given

$FC = 0{,}4\cos\theta$ ✓✓

$GE = 0{,}4\sin\theta$ ✓

putting 2 triangles together



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$A = 2\cdot\frac{1}{2}(0{,}4)^2\sin 2\theta +$

$0{,}4\times 0{,}4\cos\theta$ ✓✓

$= 0{,}08\sin 2\theta + 0{,}16\cos\theta$

(8)

8.2 $V = 1{,}6\sin 2\theta + 3{,}2\cos\theta$

a✓ $\therefore \frac{dV}{d\theta} = 3{,}2\overset{✓a}{\cos 2\theta} - 3{,}2\overset{✓a}{\sin\theta} = 0$ ✓m – equating to zero

$\therefore \cos 2\theta = \sin\theta$ ✓a

$\therefore \cos 2\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ ✓m✓a

$\therefore 2\theta = \frac{\pi}{2} - \theta$

$\therefore 3\theta = \frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{6}$ ✓a

Alternative:

$1 - 2\sin^2 = \sin\theta$ ✓m

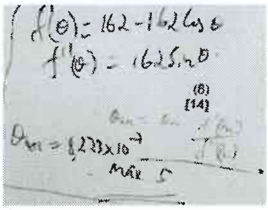
$2\sin^2\theta + \sin\theta - 1 = 0$ ✓a

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\therefore \sin\theta = \frac{1}{2}$ or -1

$\therefore \theta = \frac{\pi}{6}$ (7/8 if keep $-\frac{\pi}{2}$)

if solve for "turning point"



$f(\theta) = 162 - 162\cos\theta$

$f'(\theta) = 162\sin\theta$

(8)

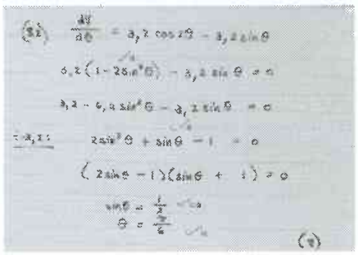
[14]

$\theta_{M} = 1{,}233\times 10^{-7}$

MAX 5

(8)

[16]



(8.2) $\frac{dV}{d\theta} = 3{,}2\cos 2\theta - 3{,}2\sin\theta$

$3{,}2(1 - 2\sin^2\theta) - 3{,}2\sin\theta = 0$

$3{,}2 - 6{,}4\sin^2\theta - 3{,}2\sin\theta = 0$

$\div 3{,}2$: $2\sin^2\theta + \sin\theta - 1 = 0$

$(2\sin\theta - 1)(\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

(8)

QUESTION 9

9.1 (a) $\sin^3\theta = \sin\theta\times\sin^2\theta$ ✓a

$= \sin\theta(1 - \cos^2\theta)$ ✓a✓a

$= \sin\theta - \sin\theta\cos^2\theta$ *as required* ✓a

May go "backwards"

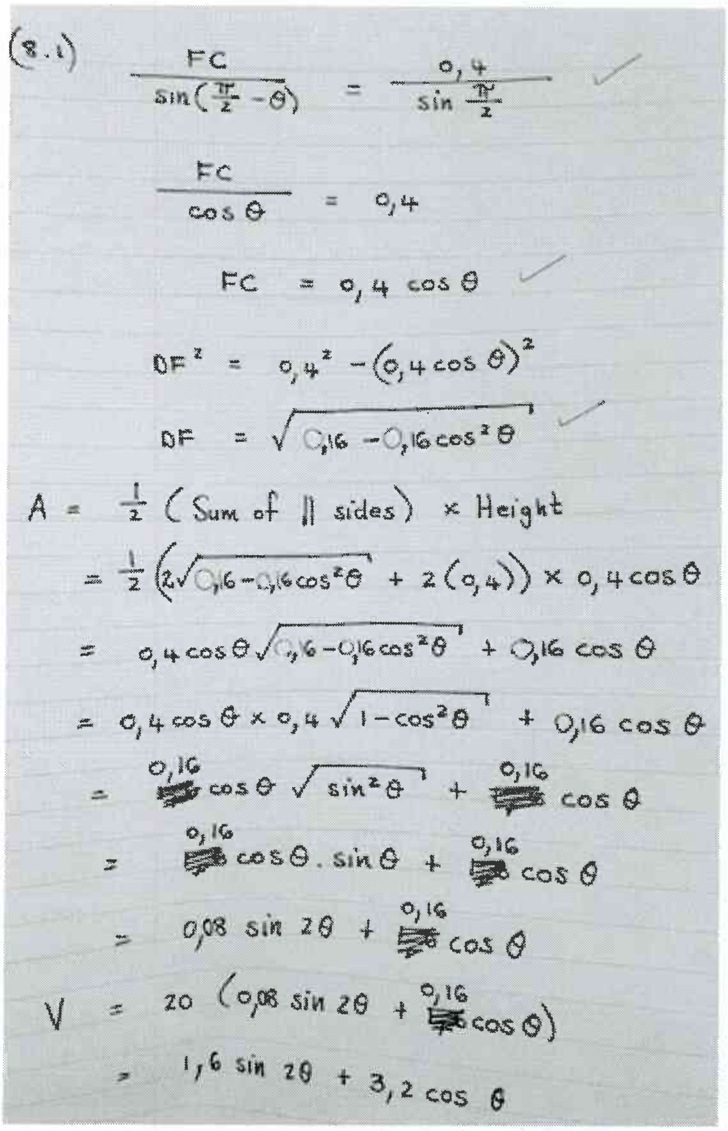
(4)

(b) $\int \sin^3\theta\, d\theta = \int \overset{✓a}{\sin\theta}\, d\theta - \int \sin\overset{✓a}{\theta\cos^2}\theta\, d\theta$

$= -\cos\theta + \frac{\cos^3\theta}{3} + c$ ✓a

✓ ✓ ✓ ✓ ✓

(8)



(8.1)

$$\frac{FC}{\sin\left(\frac{\pi}{2}-\theta\right)} = \frac{0{,}4}{\sin\frac{\pi}{2}}$$ ✓

$$\frac{FC}{\cos\theta} = 0{,}4$$

$$FC = 0{,}4\cos\theta$$ ✓

$$DF^2 = 0{,}4^2 - (0{,}4\cos\theta)^2$$

$$DF = \sqrt{0{,}16 - 0{,}16\cos^2\theta}$$ ✓

$$A = \frac{1}{2}(\text{Sum of } \parallel \text{ sides}) \times \text{Height}$$

$$= \frac{1}{2}\left(2\sqrt{0{,}16-0{,}16\cos^2\theta} + 2(0{,}4)\right) \times 0{,}4\cos\theta$$

$$= 0{,}4\cos\theta\sqrt{0{,}16-0{,}16\cos^2\theta} + 0{,}16\cos\theta$$

$$= 0{,}4\cos\theta \times 0{,}4\sqrt{1-\cos^2\theta} + 0{,}16\cos\theta$$

$$= 0{,}16\cos\theta\sqrt{\sin^2\theta} + 0{,}16\cos\theta$$

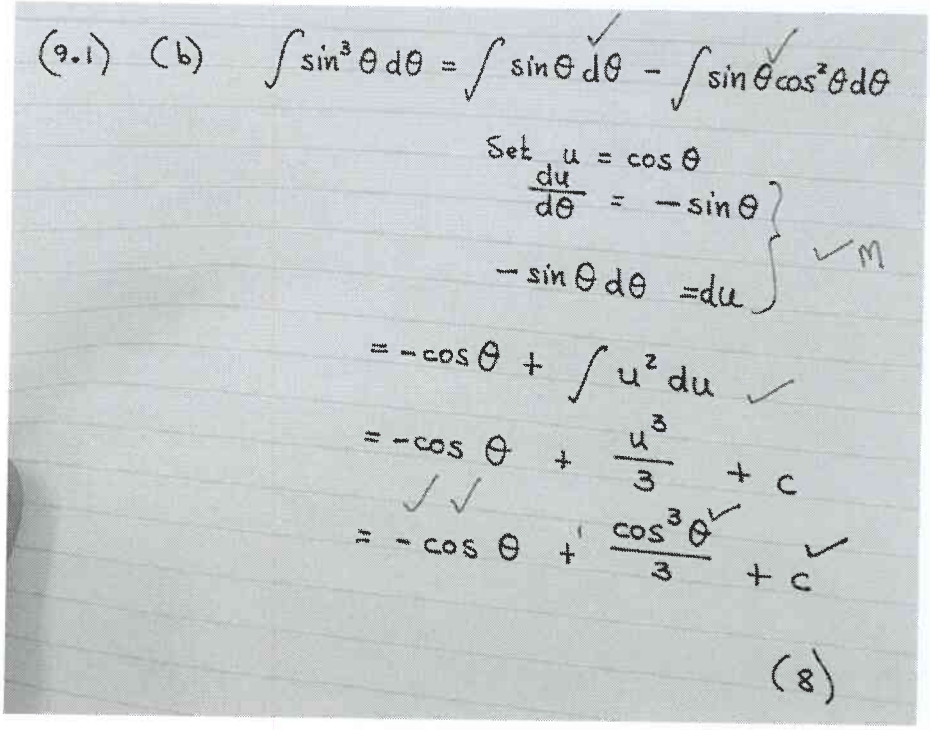
$$= 0{,}16\cos\theta\,.\,\sin\theta + 0{,}16\cos\theta$$

$$= 0{,}08\sin 2\theta + 0{,}16\cos\theta$$

$$V = 20\,(0{,}08\sin 2\theta + 0{,}16\cos\theta)$$

$$= 1{,}6\sin 2\theta + 3{,}2\cos\theta$$

10



(9.1) (b) $\int \sin^3\theta \, d\theta = \int \sin\theta \, d\theta - \int \sin\theta \cos^2\theta \, d\theta$ ✓ ✓

Set $u = \cos\theta$

$\frac{du}{d\theta} = -\sin\theta$

$-\sin\theta \, d\theta = du$ ✓M

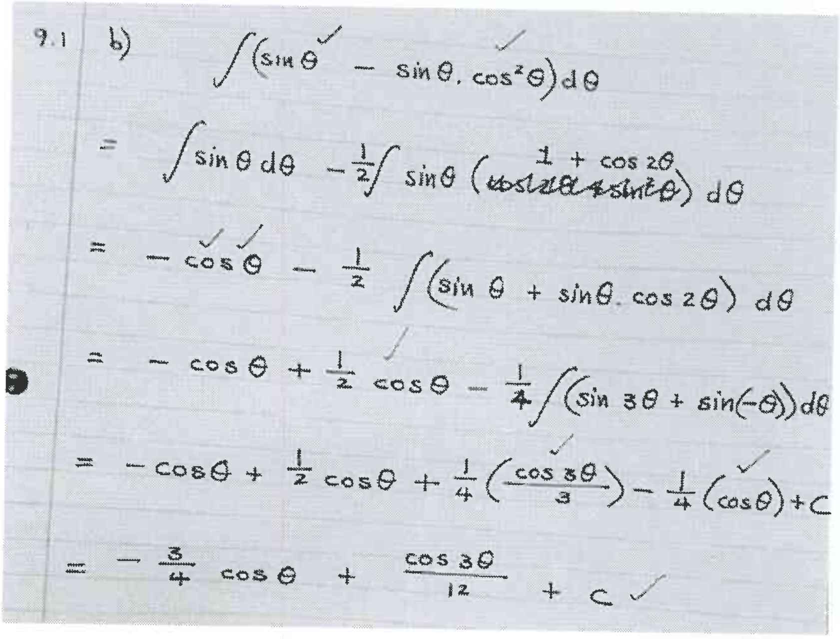
$= -\cos\theta + \int u^2 \, du$ ✓

$= -\cos\theta + \frac{u^3}{3} + c$

✓✓

$= -\cos\theta + \frac{\cos^3\theta}{3} + c$ ✓ ✓

(8)



9.1 b)

$$\int\left(\sin\theta^{\checkmark} - \overset{\checkmark}{\sin\theta.\cos^2\theta}\right)d\theta$$

$$= \int \sin\theta\, d\theta - \frac{1}{2}\int \sin\theta \left(\overset{1 + \cos 2\theta}{\cancel{\cos(2\theta) + \sin^2\theta}}\right) d\theta$$

$$= -\overset{\checkmark\,\checkmark}{\cos\theta} - \frac{1}{2}\int\left(\sin\theta + \sin\theta.\cos 2\theta\right) d\theta$$

$$= -\cos\theta + \overset{\checkmark}{\frac{1}{2}\cos\theta} - \frac{1}{4}\int\left(\sin 3\theta + \sin(-\theta)\right)d\theta$$

$$= -\cos\theta + \frac{1}{2}\cos\theta + \frac{1}{4}\left(\frac{\overset{\checkmark}{\cos 3\theta}}{3}\right) - \frac{1}{4}\left(\overset{\checkmark}{\cos\theta}\right) + C$$

$$= -\frac{3}{4}\cos\theta + \frac{\cos 3\theta}{12} + C \checkmark$$

9.2 **METHOD 1**

$$\int \frac{x}{\sqrt{2+x}}\,dx$$

let $u = 2+x$ ✓m substituting *then* $\overset{\checkmark a}{x = u-2}$ *and* $\overset{\checkmark a}{du = dx}$

$$\therefore \int \frac{u-2}{u^{\frac{1}{2}}}\,du \quad \checkmark a$$

$$= \int u^{\frac{1}{2}} - 2u^{-\frac{1}{2}}\,du \quad \checkmark m \text{ splitting}$$

$$= \frac{2}{3}\overset{\checkmark ca}{u^{\frac{3}{2}}} - \overset{\checkmark ca}{4u^{\frac{1}{2}}} + c$$

$$= \frac{2}{3}(2+x)^{\frac{3}{2}} - 4(2+x)^{\frac{1}{2}} + \underline{c} \quad \checkmark m \text{ subst back}$$

don't penalise c here.

ALTERNATIVE 1

$$\int \frac{x}{\sqrt{2+x}}\,dx$$

$$= \int x(2+x)^{-\frac{1}{2}}\,dx$$

by parts $\overset{\checkmark a}{f = x}$ *and* $\overset{\checkmark ca}{g' = (2+x)^{-\frac{1}{2}}}$

so $f' = 1$ ✓a *and* $\overset{\checkmark ca}{g = 2(2+x)^{\frac{1}{2}}}$ ✓a

$$= 2x(2+x)^{\frac{1}{2}} - \int 2(2+x)^{\frac{1}{2}}\,dx \quad \checkmark ca$$

$$= 2x(2+x)^{\frac{1}{2}} - \frac{4(2+x)^{\frac{3}{2}}\ \checkmark ca}{\underset{\checkmark ca}{3}} + c$$

(don't penalise c again)

(8)

[20]

QUESTION 10

10.1 $Area = \frac{10}{3} + \frac{3}{2(4)} + \frac{1}{6(4^2)}$ ✓m substituting 4 for n ✓ a

$= \frac{119}{32}$ ✓ca or 3,719 (3)

10.2 It will be an over-approximation. ✓a As n gets larger the answer decreases.
✓ca ✓ca under - only → 0 (3)

→ over. approx

gets 2/3 or explain

10.3 $Area = \lim_{n \to \infty} \left(\frac{10}{3} + \frac{3}{2n} + \frac{1}{6n^2} \right)$ ✓m for n tending to infinity

$= \frac{10}{3}$ units^2 ✓a ✓a (3)

10.4 $\frac{10}{3}$✓ca units^2 since this is simply a reflection of the shaded area in the y-axis. ✓a ✓a (3)

[12]

QUESTION 11

✓a limits

$$Area = \int_1^7 g(x) - f(x)\,dx$$ ✓m sub. areas

$$= \int_1^7 f(x) + kx + 1 - f(x)\,dx$$ ✓a

$$= \int_1^7 kx + 1\,dx$$ ✓a

✓ca ✓ca ✓a-notation

$$= \left[\frac{kx^2}{2} + x\right]_1^7$$

✓ca ca✓

$$\text{so,}\quad \frac{49k}{2} + 7 - \frac{k}{2} - 1 = 54$$ ✓

$$\text{so,}\quad 49k + 14 - k - 2 = 108$$ ✓ca

$$\text{so,}\quad 48k = 96$$

$$\text{so}\quad k = 2$$ ✓ca

If he/she does: $\frac{[f(x)]^2}{2}$ Penalise 2

[12]

QUESTION 12

$$vol = \pi\int_a^b [f(x)]^2\,dx$$

for volume formula ✓ h✓ 0✓

$$\therefore 175 = \pi\int_0^h -x^2 + 6x + 4\,dx$$ ✓a

✓a ca✓ ✓ca ✓ca

$$\therefore 175 = \pi\left[-\frac{x^3}{3} + 3x^2 + 4x\right]_0^h$$

$$\therefore 175 = \pi\left(-\frac{h^3}{3} + 3h^2 + 4h\right)$$ ✓ca ✓m substitution

$$\therefore -h^3 + 9h^2 + 12h - \frac{525}{\pi} = 0$$ ✓ca

$$\therefore h = 5{,}28\ cm$$ ✓ca

If double up Max 8

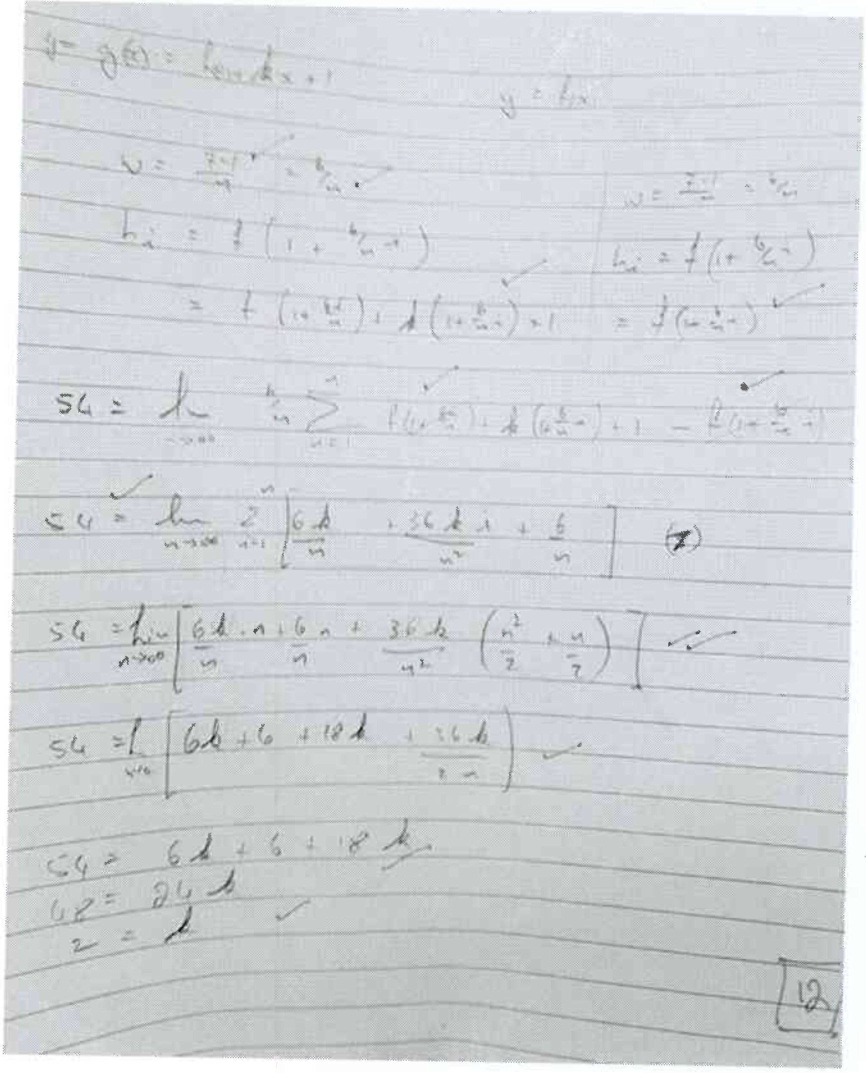
B/D

Max of 9 for (c) use

[12]

Total: 200 marks

alternative for Q11



$y = g(x) = f(x) + kx + 1$

$y = f(x)$

$w = \frac{7-1}{n} = \frac{6}{n}$

$h_i = f\left(1 + \frac{6}{n}i\right)$

$= f\left(1+\frac{6i}{n}\right) + k\left(1+\frac{6}{n}i\right) + 1$

$w = \frac{7-1}{n} = \frac{6}{n}$

$h_i = f\left(1+\frac{6}{n}i\right)$

$= f\left(1+\frac{6}{n}i\right)$

$54 = \lim_{n\to\infty} \frac{6}{n}\sum_{i=1}^{n} f\left(1+\frac{6i}{n}\right) + k\left(1+\frac{6}{n}i\right) + 1 - f\left(1+\frac{6}{n}i\right)$

$54 = \lim_{n\to\infty} \sum_{i=1}^{n}\left[\frac{6k}{n} + \frac{36ki}{n^2} + \frac{6}{n}\right]$

$54 = \lim_{n\to\infty}\left[\frac{6k}{n}\cdot n + \frac{6}{n}n + \frac{36k}{n^2}\left(\frac{n^2}{2} + \frac{n}{2}\right)\right]$

$54 = \lim_{n\to0}\left[6k + 6 + 18k + \frac{36k}{2n}\right)$

$54 = 6k + 6 + 18k$

$48 = 24k$

$2 = k$

$\boxed{12}$