



GRADE 12 EXAMINATION  
NOVEMBER 2017

**ADVANCED PROGRAMME MATHEMATICS: PAPER II**  
**MARKING GUIDELINES**

Time: 1 hour

100 marks

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These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

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**MODULE 2            STATISTICS****QUESTION 1**

$$1.1 \quad \frac{\binom{5}{3}\binom{7}{2}}{\binom{12}{5}} = 0.2652$$

$$1.2 \quad P(X = 7) = \binom{10}{7}(0.7)^7(0.3)^3 = 0.2668$$

$$1.3 \quad (a) \quad \frac{5!}{2!2!} + 2 \times \frac{5!}{2!3!} = 50$$

$$(b) \quad 5 \times 2 + 2 = 12 \quad \{(CHLHL) \times 5 + (HLCLH) \times 2 = 12\}$$

**QUESTION 2**

$$2.1 \quad (a) \quad 0,3 \times (0,7) + (0,3)(0,7)^2 + (0,3)(0,7)^3 + (0,3)(0,7)^4 + C = 1$$

$$C = 0,4681$$

$$(b) \quad P(X > 3) = P(X = 4) + P(X = 5)$$

$$= 0,3 (0,7)^4 + 0,4681$$

$$= 0,5401$$

$$2.2 \quad (a) \quad A$$

$$(b) \quad (i) \quad \frac{n}{500} = \frac{0,2278 + 0,2922}{2}$$

$$\therefore n = 130$$

$$(ii) \quad 0,26 + Z\sqrt{\frac{(0,26)(0,74)}{500}} = 0,2922$$

$$Z = 1,64$$

$$\therefore \alpha = 90$$

**QUESTION 3**

3.1  $X \sim N(9, 0,1^2)$

$$\begin{aligned} P(X > 8,9) &= P\left(Z > \frac{8,9-9}{0,1}\right) \\ &= P(Z > -1) \\ &= 0,5 + 0,3413 \\ &= 0,8413 \end{aligned}$$

3.2  $X \sim B(6, 0,8413)$

$$\begin{aligned} P(X \geq 2) &= 1 - \left[ \binom{6}{0}(0,8413)^0(0,1587)^6 + \binom{6}{1}(0,8413)^1(0,1587)^5 \right] \\ &= 0,9995 \end{aligned}$$

3.3  $P(X < a) = 0,04$

$$-1,75 = \frac{a-9}{0,1}$$

$$\therefore a = 8,825 \text{ cm}$$

**QUESTION 4**

4.1 (a)  $1 + m^2 + (m + 1)^2 + 4^2 + 5^2 = 55$

$$2m^2 + 2m - 12 = 0$$

$$m^2 + m - 6 = 0$$

$$(m + 3)(m - 2) = 0$$

$$m \neq -3 \text{ or } m = 2$$

$$\frac{5+t-1+4+3+t}{5} = 3$$

$$2t + 11 = 15$$

$$2t = 4$$

$$t = 2$$

(b)  $r = -0,4$

(c) (i)  $y = 4,2 - 0,4x$

(ii)  $y = 4,2 - 0,4(6)$

$$y = 1,8$$

This is an unreliable estimation as the correlation is weak.

4.2  $H_0 : \mu = 49,5$

$H_1 : \mu < 49,5$

Rejection Region

Reject  $H_0$  if  $Z < -1,48$

Test Statistic:

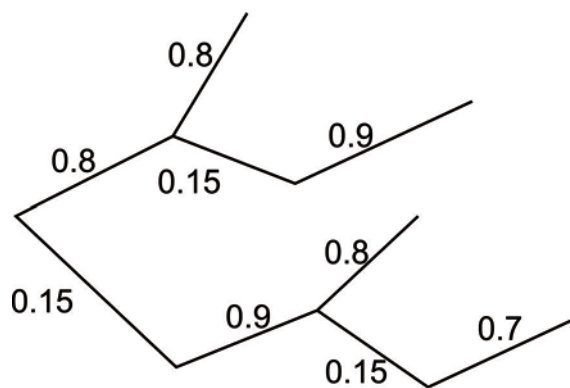
$$Z = \frac{48 - 49,5}{\frac{4,8}{\sqrt{40}}} = -1,98$$

Conclusion: since  $Z < -1,48$ , we reject  $H_0$  at the 7% l.o.s. and suggest sufficient evidence to support the claim that Basi's working hours per week have decreased significantly.

## QUESTION 5

5.1  $0,8 + 0,15(0,9) = 0,935$

5.2



$$(0,8)(0,8) + (0,8)(0,15)(0,9) + (0,15)(0,9)(0,8) + (0,15)(0,9)(0,15)(0,7) = 0,8702$$

**Total for Module 2: 100 marks**

**MODULE 3 FINANCE AND MODELLING****QUESTION 1**

- 1.1 R1 200 000
- 1.2 50
- 1.3  $\pm$  R1 100 000 method:  $2,3 - 1,2 = 1,1$  million
- 1.4  $\pm$  R900 000
- 1.5 (a) straight line gradient unaffected
- (b) interest has been increased
- OR**
- Withdrawal from loan

**QUESTION 2**

$$2.1 \quad P \left( 1 + \frac{0,0568}{12} \right)^2 = \frac{5154,26 \left[ 1 - \left( 1 + \frac{0,0568}{12} \right)^{-34} \right]}{\frac{0,0568}{12}}$$

$$\mathbf{P = R160\ 000}$$

$$2.2 \quad OB = \frac{5154,26 \left[ 1 - \left( 1 + \frac{0,0568}{12} \right)^{-12} \right]}{\frac{0,0568}{12}}$$

$$\mathbf{OB = R59\ 989,47}$$

- 2.3 Jude paid:  $5\ 154,26 \times 12 = 61\ 851,12$
- Balance decreased:  $116\ 674,09 - 59\ 989,47 = 56\ 684,62$
- Interest paid:  $61\ 851,12 - 56\ 684,62 = \mathbf{R5\ 166,50}$

**QUESTION 3**

$$3.1 \quad 2\,000\,000 = A(1 + 0,068)^8 \quad \mathbf{A = 1\,181\,571,41}$$

$$3.2 \quad 1\,181\,571,41 = 3\,400\,000(1 - i)^8 \quad \mathbf{i = 12,38\%}$$

$$3.3 \quad \left(1 + \frac{0,0764}{12}\right)^{12} = \left(1 + \frac{i}{2}\right)^2 \quad i = 7,7626\%$$

$$5\,500\,000 + 300\,000 \left(1 + \frac{0,077626}{2}\right)^6 = 5\,877\,003,11$$

$$x \frac{\left[\left(1 + \frac{0,077626}{2}\right)^9 - 1\right] \left(1 + \frac{0,077626}{2}\right)^4}{\frac{0,07626}{2}} = 12,264x$$

$$5\,877\,003,611 = 12,264x \quad \mathbf{x = 479\,200,70}$$

**QUESTION 4**

$$4.1 \quad \frac{1\,396 - 1\,300}{1\,300} = \mathbf{7,4\%}$$

$$4.2 \quad Q_{n+1} = 1,05 \cdot Q_n - 50, \quad Q_0 = 6\,500$$

$$4.3 \quad \begin{array}{ll} A = 8\,020 & B = 8\,371 \\ C = 8\,739 & D = 9\,126 \\ E = 9\,532 & F = 9\,959 \end{array}$$

$$4.4 \quad 9\,126 / 4 = 2\,281 < 2\,301 \quad \mathbf{\text{during } 8^{\text{th}} \text{ year}}$$

$$4.5 \quad \frac{2\,655 - 2\,472}{2\,472} = 7,4\% \quad \mathbf{\text{constant exponential growth; thus Malthusian}}$$

**QUESTION 5**

5.1 (a) prey  $\approx 526\,000$  predator  $\approx 4\,500$

(b)  $\pm 4\,180 - 4190$

(c) B

5.2  $S_{n+1} = 4\,000 + 760 - 0,2 \times 4\,000 = \mathbf{3\,960}$

5.3  $2/3 \times 3 \times 8 \times 0,05 = \mathbf{0,8}$

5.4  $533\,300 = 500\,000 + 0,8(500\,000) \left( 1 - \frac{500\,000}{K} \right) - \mathbf{0,4(500\,000)}$   
 $\mathbf{K = 1\,200\,000}$

**QUESTION 6**

6.1 (a)  $8^{\text{th}}$

(b) A

6.2  $\frac{2a+3b}{a+2b} = \frac{a+2b+a+b}{a+2b} = 1 + \frac{a+b}{a+2b} = 1 + \frac{1}{\frac{a+b+b}{a+b}} = 1 + \frac{1}{1+\frac{b}{a+b}}$   
 $= 1 + \frac{1}{1+\frac{1}{\frac{a+b}{b}}} = 1 + \frac{1}{1+\frac{1}{1+\frac{a}{b}}} = 1 + \frac{1}{1+\frac{1}{1+T_n}}$

**OR**

$$\begin{aligned} \frac{2a+4b-b}{a+2b} &= 2 - \frac{b}{a+2b} = 2 - \frac{1}{\frac{a+2b}{b}} \\ &= 2 - \frac{1}{2\frac{a}{b}} \\ &= 2 - \frac{1}{2+T_n} \end{aligned}$$

**Total for Module 3: 100 marks**

**MODULE 4                  MATRICES AND GRAPH THEORY****QUESTION 1**

1.1    (a)     $1 \times 1$

$$(b) \quad \begin{pmatrix} k+3 & k+2 & 3 \end{pmatrix} \begin{pmatrix} k \\ 3 \\ 1 \end{pmatrix} = (k^2 + 3k) + (3k+6) + 3$$

**OR**

$$\begin{pmatrix} k & 1 & 1 \end{pmatrix} \begin{pmatrix} k+3 \\ 3k+2 \\ 7 \end{pmatrix} = (k^2 + 3k) + (3k+2) + 7$$

$$k^2 + 6k + 9 = 0$$

$$k = -3$$

1.2    (a)    1

(b)    **pr**

(c)    **27p**

(d)    **pr/q**

**QUESTION 2**

$$2.1 \quad \begin{pmatrix} 1 & -3 & 6 & 4 \\ 0 & 3 & -1 & 1 \\ 0 & 2 & 2 & -18 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & -3 & 6 & 4 \\ 0 & 3 & -1 & 1 \\ 0 & 8 & 0 & -16 \end{pmatrix}$$

$$y = -2; \quad z = -7$$

$$x - 3(-2) + 6(-7) = 4 \quad x = 40$$

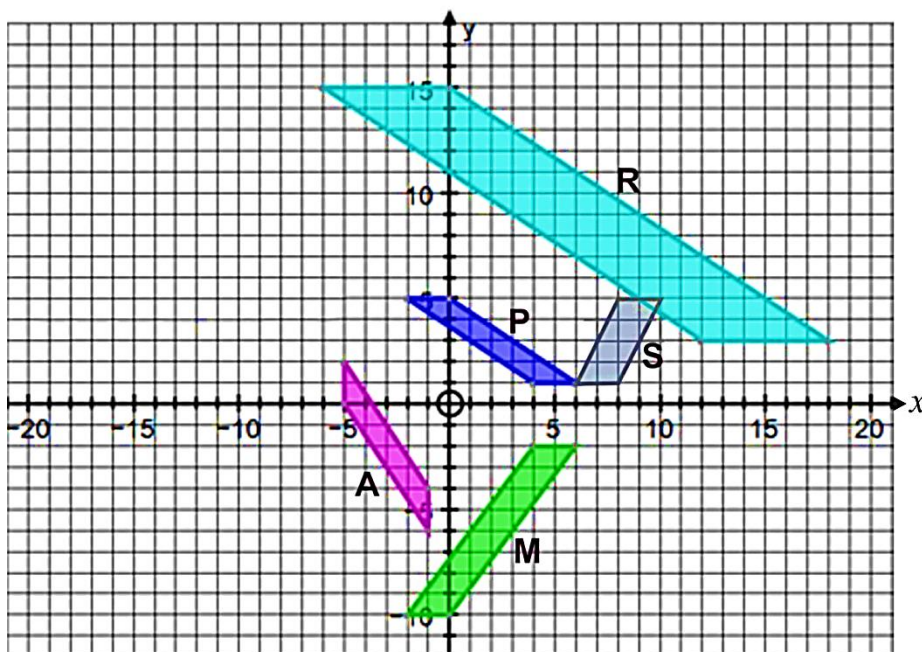
2.2    **L:**             $0x + 0y + 0z \neq 1$  (equation is an inconsistency)

2.3    **D:**             $0x + 0y + 0z = 0$



### QUESTION 3

3.1



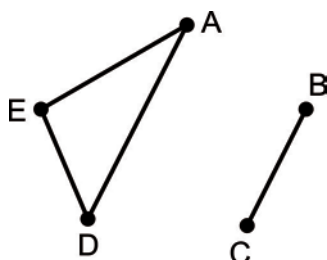
- (a) Reflection; direction  $y = -x$ ; coordinates
- (b) Enlargement; scale  $k = 3$ ; coordinates
- (c) Stretch; invariant line  $y = 0$ ; scale  $k = 2$ ; coordinates
- (d) Shear, invariant line  $y = 0$ ; scale  $k = 2$ , coordinates

3.2 
$$\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 6 \\ 1 \end{pmatrix} = \begin{pmatrix} -5,55 + 10 \\ 5,15 - 1 \end{pmatrix} = \begin{pmatrix} 4,45 \\ 4,15 \end{pmatrix}$$

$6 \cos \theta - \sin \theta = 4,45$       AND       $\cos \theta + 6 \sin \theta = 4,15$   
 $\cos \theta = 0,83 \ 378 \ 378 \dots$       OR       $\sin \theta = 0,55 \ 270 \ 270 \dots$   
 $\theta = 33,5^\circ$        $\theta = 33,6^\circ$

### QUESTION 4

- 4.1 6 edges
- 4.2 not symmetrical
- 4.3 all vertices loop to themselves
- 4.4

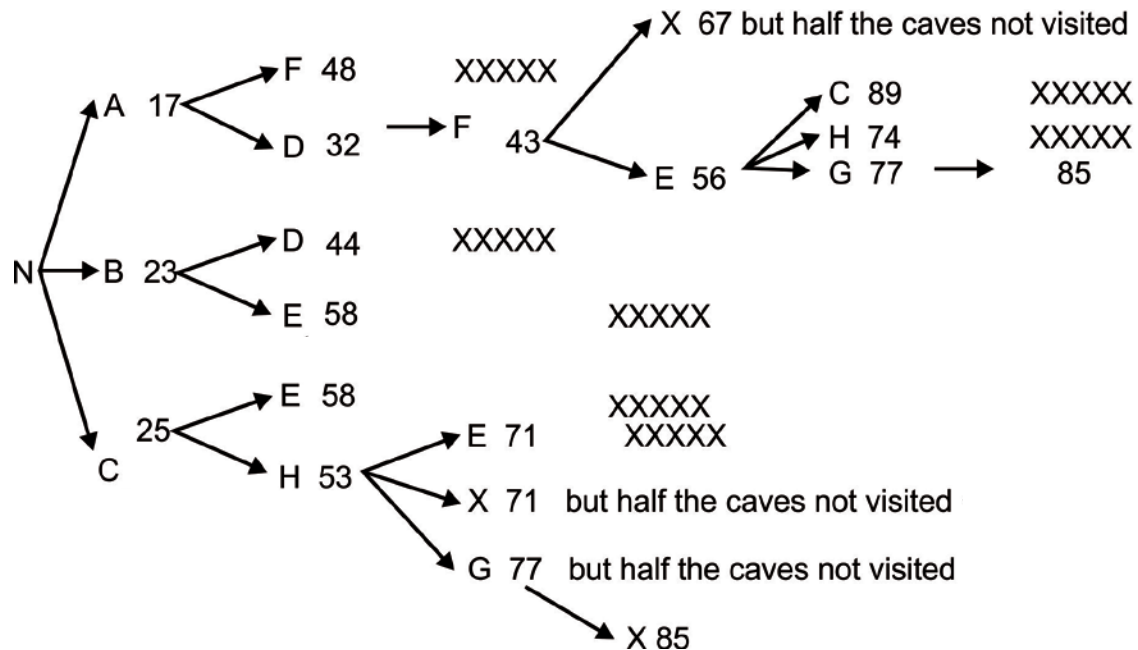


five vertices, four edges  
correct edges, disconnect

### QUESTION 5

|     |    |              |
|-----|----|--------------|
| 5.1 | NA | 17           |
|     | AD | 15           |
|     | DF | 11           |
|     | DB | 21           |
|     | FX | 24           |
|     | XG | 8            |
|     | XH | 18           |
|     | NC | 25           |
|     | EF | 13           |
|     | EH | 18 = 170 min |

5.2



$\therefore N A D F E G X = 85 \text{ min}$

OR

|                | A                     | B                     | C                     | D                     | E                     | F                     | G                     | H                     | X                     |                            |
|----------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|----------------------------|
| N              | <b>N<sub>17</sub></b> | <b>N<sub>23</sub></b> | <b>N<sub>25</sub></b> |                       |                       |                       |                       |                       |                       |                            |
| A              | N <sub>17</sub>       | N <sub>23</sub>       | N <sub>25</sub>       | <b>A<sub>32</sub></b> | •                     | <b>A<sub>48</sub></b> |                       |                       |                       |                            |
| B              | N <sub>17</sub>       | N <sub>23</sub>       | N <sub>25</sub>       | <b>B<sub>44</sub></b> | <b>B<sub>58</sub></b> | A <sub>48</sub>       |                       |                       |                       |                            |
| C              | N <sub>17</sub>       | N <sub>23</sub>       | N <sub>25</sub>       | A <sub>32</sub>       | <b>C<sub>58</sub></b> | A <sub>48</sub>       | •                     | <b>C<sub>53</sub></b> | •                     |                            |
| D              | N <sub>17</sub>       | N <sub>23</sub>       | N <sub>25</sub>       | A <sub>32</sub>       | B/C <sub>58</sub>     | <b>D<sub>43</sub></b> | •                     | C <sub>53</sub>       | •                     |                            |
| F              | N <sub>17</sub>       | N <sub>23</sub>       | N <sub>25</sub>       | A <sub>32</sub>       | <b>F<sub>56</sub></b> | D <sub>43</sub>       | •                     | C <sub>53</sub>       | <b>F<sub>67</sub></b> | But half caves not visited |
| H              | N <sub>17</sub>       | N <sub>23</sub>       | N <sub>25</sub>       | A <sub>32</sub>       | <b>H<sub>71</sub></b> | D <sub>43</sub>       | <b>H<sub>77</sub></b> | C <sub>53</sub>       | <b>H<sub>71</sub></b> | But half caves not visited |
| E              | N <sub>17</sub>       | N <sub>23</sub>       | N <sub>25</sub>       | A <sub>32</sub>       | F <sub>56</sub>       | D <sub>43</sub>       | <b>E<sub>77</sub></b> | C <sub>53</sub>       | H <sub>71</sub>       |                            |
| G              | N <sub>17</sub>       | N <sub>23</sub>       | N <sub>25</sub>       | A <sub>32</sub>       | F <sub>56</sub>       | D <sub>43</sub>       | H/E <sub>77</sub>     | C <sub>53</sub>       | <b>G<sub>85</sub></b> |                            |
| G              | N <sub>17</sub>       | N <sub>23</sub>       | N <sub>25</sub>       | A <sub>32</sub>       | F <sub>56</sub>       | D <sub>43</sub>       | H <sub>77</sub>       | C <sub>53</sub>       | G <sub>85</sub>       | But half caves not visited |
| $\therefore N$ | A                     | D                     | F                     | E                     | G                     | X                     |                       |                       |                       | $x = 85 \text{ min}$       |

**QUESTION 6**

6.1 (a)  $n - 1$

(b)  $n(n - 1)$

6.2 (a)  $\begin{array}{cc} A B D C A & A C D B A \\ A C B D A & A D B C A \end{array} \left. \vphantom{\begin{array}{cc} A B D C A & A C D B A \\ A C B D A & A D B C A \end{array}} \right\}$

(b)  $(n - 1)!$

**Total for Module 4: 100 marks****Total: 100 marks**