**GEVORDERDEPROGRAM-WISKUNDE: VRAESTEL II**

**NASIENRIGLYNE**

Tyd: 1 uur                                                                                     100 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.
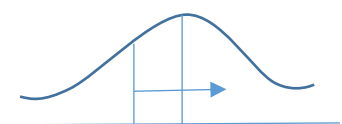
## MODULE 2          STATISTIEK

## VRAAG 1

1.1     (a)     Tweeterm ✓
                ✓✓ ✓ ✓ ✓

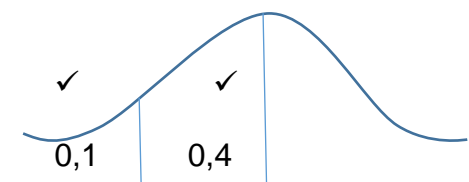Nul of een $\binom{12}{0}(0{,}057)^0(0{,}943)^{12} + \binom{12}{1}(0{,}057)(0{,}943)^{11}$

$= 0{,}8531$ ✓ (7)

     (b)     Die oorblywende 16 het nie AGS nie ✓✓

$(0{,}943)^{16} = 0{,}3910$ ✓✓ (4)

1.2     (a)     Sonder vervanging hipergeometries ✓✓ (2)

     (b)     20 ✓✓ (2)

     (c)     7 ✓ (1)

     (d)     8 ✓✓ (2)

     (e)     2 ✓✓ (2)

     (f)     $7 - k$     $8 - (7 - k) = k + 1$ ✓✓ (2)

**[22]**

## VRAAG 2

2.1     (a)     Laat $X$ die stogastiese veranderlike "gewig van babas" wees.
                $P(X > 2{,}8)$ ✓

$P\left(z > \dfrac{2{,}8 - 3{,}2}{0{,}85}\right)$ ✓

$P(z > -0{,}4706)$ ✓



$0{,}05 + 0{,}1808$ ✓✓

$= 0{,}6808$ ✓

Ongeveer 953 120 babas ✓ (7)

(b)



$z = -1{,}28$ ✓

$$-1{,}28 = \frac{(X-3{,}2)}{0{,}85} \checkmark\checkmark$$

$= 2{,}112$ kg ✓                                                                 (7)

2.2   (a)   61 kg ✓✓                                                            (2)

(b)   $61 + z \times \dfrac{9}{8} = 63$ ✓✓

$z = \dfrac{16}{9} = 1{,}77$ ✓
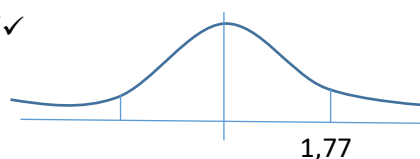


0,461 × 2 ✓
92% vertroue ✓✓                                                              (8)

**[24]**

## VRAAG 3

$H_0 : \mu_x = \mu_y$

$H_1 : \mu_x > \mu_y$ ✓✓

Toetsstatistiek $z = \dfrac{7{,}2 - 8{,}1}{\sqrt{\dfrac{(2{,}85)^2}{35} + \dfrac{4}{38}}} = -1{,}54$ ✓✓



Eenkantige toets ✓✓ ✓
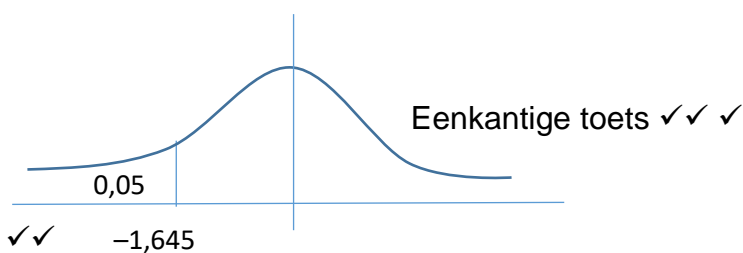
Nie genoeg bewys om die nulhipotese ten gunste van die bewering by die 5%-betekenispeil te verwerp nie.

**[9]**

**VRAAG 4**

4.1     $\bar{y} = \dfrac{\sum y}{n}$   $159\dfrac{1}{6} = \dfrac{1910}{n}$

n = 12 ✓✓                                                                    (2)

4.2     $b = \dfrac{12 \times 26270 - 161 \times 1910}{12 \times 2293 - (161)^2} = 4{,}8464$ ✓✓ ✓

$\bar{y} = a + b\bar{x}$

$\dfrac{955}{6} = a + 4{,}8464\left(\dfrac{161}{12}\right)$  $\therefore a = 94{,}1441$ ✓✓

$y = 94{,}1441 + 4{,}8464x$ ✓                                                (6)

4.3     Sterk, positiewe korrelasie ✓✓                                     (2)

4.4     Nee – te ver buite die beeldruimte (ekstrapolasie) ✓✓             (2)

**[12]**

**VRAAG 5**

5.1     $\displaystyle\int_{30}^{60} a(x-30)^2 \; dx = 1$ ✓✓

$\left[\dfrac{a}{3}(x-30)^3\right]_{30}^{60} = 1$ ✓✓

$\dfrac{a}{3}(30)^3 = 1$ ✓✓

$a = \dfrac{3}{(30)^3} = \dfrac{1}{9\,000}$ ✓                                (7)

5.2     $\left[\dfrac{1}{27000}(x-30)^3\right]_{30}^{m} = \dfrac{1}{2}$ ✓✓

$\dfrac{1}{27000}(m-30)^3 = \dfrac{1}{2}$ ✓✓

$(m-30)^3 = 1\,3500$ m ✓✓

$m = 54$ minute ✓                                                           (7)

**[14]**

**VRAAG 6**

6.1     $P(A) = x$  $P(B) = y$  $P(B') = 1 - y$  ✓
        $P(A \cap B) = xy$ ✓
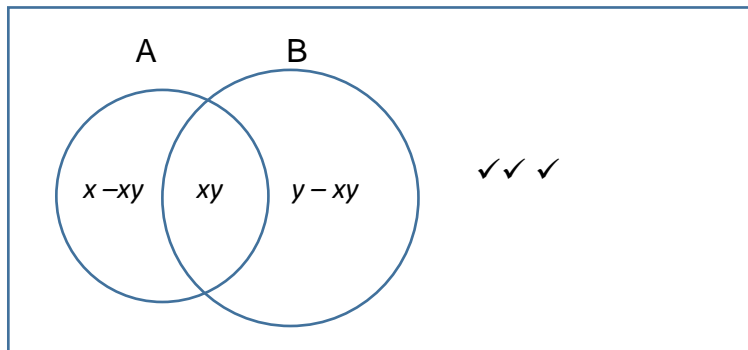


$P(A) \times P(B') = x(1 - y)$ ✓
$P(A \cap B') = x - xy = x(1 - y)$ ✓
Gebeurtenisse A en B' is onafhanklik.                                                    (8)

6.2     $\dbinom{16}{5} - \dbinom{9}{0}\dbinom{7}{5} = 4\,347$                           (4)

6.3     $\dbinom{11}{2}\dbinom{9}{4}\dbinom{5}{5} + \dbinom{11}{1}\dbinom{10}{4}\dbinom{6}{6} + \dbinom{11}{2}\dbinom{9}{3}\dbinom{6}{6} = 13\,860$     (7)

**[19]**

**Totaal vir Module 2: 100 punte**

## MODULE 3     FINANSIES EN MODELLERING

## VRAAG 1

1.1    (a)    $5\,640 \times \dfrac{1}{1,15}$ ✓ $= \mathbf{4\,904,35}$ ✓                          (2) B

       (b)    $\dfrac{1,15 - 1,14}{1,14} \times 100$ ✓ $= 0,00877\ldots$ ✓ $= \mathbf{0,88\%}$ ✓     (3) B

1.2    $3x = x(1 + i)^{24}$ ✓✓                  $\therefore i = 0,0468$ per maand ✓

       $2x = x(1 + 0,0468)^{n}$ ✓                  $\therefore n = 15,1423 \approx \mathbf{15\ maande}$ ✓     (5) R

1.3    A 3 ✓✓        B 2 ✓✓        C 1 ✓✓        D 5 ✓✓                          (8) R

**[18]**

## VRAAG 2

2.1    $500\,000\left(\dfrac{0,088}{12}\right)$ ✓ $= 3\,666,67$ ✓     **rente > betalings** ✓     (3) R

2.2    $500\,000$ ✓ $= \dfrac{x\left[1 - \left(1 + \dfrac{0,088}{12}\right)^{-96}\!\!\text{✓}\right]}{\dfrac{0,088}{12}\text{✓}}$     $\boldsymbol{x = 7\,273,33}$ ✓✓     (5) R

2.3    $500\,000$ ✓ $\left(1 + \dfrac{0,088}{12}\text{✓}\right)^{95\text{✓}} -$ ✓ $\dfrac{7\,300\text{✓}\left[\left(1 + \dfrac{0,088}{12}\right)^{95\text{✓}} - 1\right]\text{✓}}{\dfrac{0,088}{12}} = 3\,576,4053$ ✓

       $3\,576,4053\left(1 + \dfrac{0,088}{12}\right)$ ✓✓ $= \mathbf{3\,602,63}$ ✓✓

       **OF**

       $500\,000$ ✓ $\left(1 + \dfrac{0,088}{12}\text{✓}\right)^{96}$ ✓ $-$ ✓ $\dfrac{7\,300\text{✓}\left(1 + \dfrac{0,088}{12}\right)\text{✓✓}\left[\left(1 + \dfrac{0,088}{12}\right)^{95\text{✓}} - 1\right]\text{✓}}{\dfrac{0,088}{12}}$

       $= 1\,008\,318,445 - 1\,004\,715,812$ ✓ $= \mathbf{3\,602,63}$ ✓✓

**OF**

$$500\,000 \checkmark\checkmark - \checkmark \dfrac{7\,300\checkmark\left[1-\left(1+\dfrac{0,08}{12}\right)^{-95}\checkmark\right]+y\left(1+\dfrac{0,08}{12}\right)^{-96}}{\dfrac{0,08}{12}}\checkmark$$

$$\therefore y = \textbf{3 602,63} \checkmark\checkmark \qquad\qquad (12)\ \text{R}$$

**[20]**

**VRAAG 3**

$$x.\left(1+\dfrac{0,08}{12}\right)^{72}\checkmark.\dfrac{2}{3}.\left(1+\dfrac{0,08}{12}\right)^{24}\checkmark + \checkmark\ x.\checkmark\left(1+\dfrac{0,08}{12}\right)^{72}\checkmark.\dfrac{1}{3}.\left(1+\dfrac{0,1}{4}\right)^{8}\checkmark$$

$$= 20\,702,50 \checkmark\checkmark$$

$$1,9169x = 20\,702,50 \qquad \textbf{\textit{X = 10 800}} \checkmark\checkmark$$

**[12] C**

**VRAAG 4**

4.1   Logistiese Model, $\checkmark$ aanwesigheid van dravermoë $\checkmark$ (2) B

4.2   $V = \dfrac{1}{2}$ (50) $\checkmark$ = **25** $\checkmark$ (2) R

4.3   Die model het regressievergelyking $\dfrac{\Delta P}{P} = -0,0025P + r$.

$r = -Km \checkmark = -50\checkmark.(-0,0025)\checkmark = \textbf{0,125}\checkmark$ (4) R

4.4   $T_{n+1} = \checkmark T_n + 0,13 \checkmark.T_n(1 - T_n/50), \checkmark T_O = 10 \checkmark$

$\textbf{\textit{t = 11}} \checkmark\checkmark$ (6) C

**[14]**

**VRAAG 5**

5.1     (a)     Getal arende wat per jaar gebore word ✓✓                                        (2) B

B       (b)     Doeltreffendheidskoers ✓ waarteen arende prooi in nageslag verander ✓ (2)

        (c)     $f.b.D_n.E_n = 15$
                $f(6\ 000) = 15$ ✓✓               **$f = 0,0025$** ✓                          (3) R

5.2     $a = 0,5 \times 1,5$ ✓ $\times 3$ ✓ $\times 0,67$ ✓      **$a = 1,51$** ✓✓           (5) R

5.3     $6\ 000$ ✓ $= b.(12\ 000)(30)$ ✓✓          **$b = 0,016\ 667$** ✓✓
        vir dassie-ewewig, $E_{n+1} = E_n$ ✓
        $0,1$ ✓ $= 0,003 \times 0,016\ 667 \times D$ ✓    **$D = 1\ 999,96$ ✓ $\approx$ 2 000** ✓

        **OF**

        $6\ 000$ ✓ $= b.(12\ 000)(30)$ ✓✓          **$b = 0,016\ 667$** ✓✓
        $D = \dfrac{c}{fb}$ ✓ $= \dfrac{0,1}{0,003 \times 0,016\ 667}$ ✓✓   **$D = 1\ 999,96$ ✓ $\approx$ 2 000** ✓      (10) C

                                                                                              **[22]**

**VRAAG 6**

6.1     (a)     $T_4 = 75,77$      $T_5 = 84,55$      $T_6 = 91,122$ ✓✓✓✓                       (4) R

        (b)     $64\sqrt{3} =$ **110,8 vierkante eenhede** ✓✓                                  (2) R

6.2     $195 = p.114 + q.60$ ✓✓          en          $114 = p.60 + q.24$ ✓✓
        $p = 2,5$ ✓                      en          $q = -1,5$ ✓
        **$T_n = 5/2.T_{n-1} - 3/2.T_{n-2}$** ✓    **$T_1 = 24,\ T_2 = 60$** ✓               (8) P
                                                                                              **[14]**

                                                        **Totaal vir Module 3: 100 punte**

## MODULE 4          MATRIKSE EN GRAFIEKTEORIE

## VRAAG 1

1.1          $PQ = \begin{pmatrix} 3 & 6 & 2 & -2 \\ 0 & -1 & 4 & 6 \end{pmatrix} \cdot \begin{pmatrix} 3 & 6 \\ -2 & -1 \\ 0 & 5 \\ -7 & 0 \end{pmatrix} = \begin{pmatrix} 11 & 22 \\ -40 & 21 \end{pmatrix}$ ✓✓✓✓          (4) R

1.2     $3x + 2y = 11$          $x - 2z = 0$          $6y + 4z = 5$

| 3 | 2 | 0 | 11 | |
|---|---|---|---|---|
| 1 | 0 | −2 | 0 ✓ | |
| 0 | 6 | 4 | 5 | |
| 3 | 2 | 0 | 11 | |
| 0 | 2 | 6 | 11 | R1 − 3.R2 ✓✓ |
| 0 | 6 | 4 | 5 | |
| 3 | 2 | 0 | 11 | |
| 0 | 2 | 6 | 11 | |
| 0 | 0 | 14 | 28 | 3.R2 − R3 ✓✓ |

     **$z = 2,$** ✓   **$y = -\dfrac{1}{2},$** ✓   **$x = 4$** ✓          (8) R

1.3     (a)     3 ✓✓          (2) B

        (b)     0 ✓✓          (2) R

        (c)     $t$ ✓✓          (2) P

          **[18]**

**VRAAG 2**

2.1    refleksie ✓ in $y = x$ ✓ (2) B

2.2    $k = 3$ ✓✓ (2) R

2.3    $C = \dfrac{1}{4}R$ ✓✓ en $R = S$ dus is faktor $\dfrac{1}{4}$ ✓✓ (4) R

2.4    $\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}$ ✓✓✓ $\begin{pmatrix} -0,5 & 0 \\ 0 & -0,5 \end{pmatrix}$ ✓ $= \begin{pmatrix} 1,5 & 0 \\ 0 & -0,5 \end{pmatrix}$ ✓✓ (6) R

OF

$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0,5 & 0 \\ 0 & 0,5 \end{pmatrix} = \begin{pmatrix} 1,5 & 0 \\ 0 & -0,5 \end{pmatrix}$

2.5    $\begin{pmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{pmatrix}$ ✓ $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ ✓ $= \begin{pmatrix} 4,025 \\ 3,578 \end{pmatrix}$ ✓

$5\cos A + 2.\sin A = 4,025$ ✓✓    en    $-2.\cos A + 5\sin A = 3,578$ ✓✓
$\cos A = 0,4472$    en    $\sin A = 0,8944$ ✓
**A = 63,44°**    **A = 63,44°** ✓✓  (10) C
**[24]**

**VRAAG 3**

3.1    det = 25 ✓✓ (2) B

3.2    $\begin{pmatrix} 25 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & 25 \end{pmatrix}$ ✓ ✓ $\begin{pmatrix} -1 & -4 & -10 \\ -2 & -8 & -10 \\ 4 & -9 & 10 \end{pmatrix}$ ✓✓ ✓✓

(6) C

3.3    $\begin{pmatrix} 25 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 25 \end{pmatrix}$ ✓ $\begin{pmatrix} -1 & -4 & 10 \\ 5 & 20 & -25 \\ -4 & 9 & -10 \end{pmatrix}$ ✓

Inverse $= \dfrac{1}{25}$ ✓ $\begin{pmatrix} -1 & -4 & 10 \\ 5 & 20 & -25 \\ -4 & 9 & -10 \end{pmatrix}$ ✓ (4) C
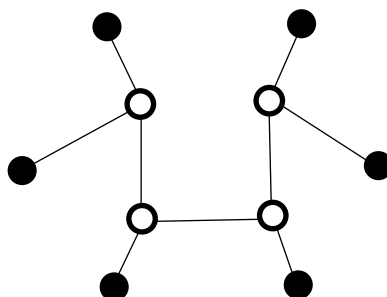
**[12]**

**VRAAG 4**

4.1    Een nodus het 'n onewe graad. ✓✓    (2) B

4.2    Ja; ✓ daar is een paar onewe nodusse. ✓    (2) B

4.3    8 skakels ✓✓    (2) B

4.4    19 x 2 ✓ = 4 x 6 + 2 x 4 + 1 x 1 + e ✓
       **e = 5 ✓✓**    (4) R
       **[10]**


**VRAAG 5**

5.1    TR    3        TV    3 ✓    TS    4 ✓    SU    3 ✓
       RQ    5 ✓      RW    5 ✓    QP    6 ✓    lengte = 29 ✓✓    (8) R

5.2    RT    3        TV    3      VW    7 ✓
       WRQ   10 ✓✓    QP    6 ✓    PS    7 ✓
       SU    3 ✓      UTR   9 ✓✓   **U/B = 48 ✓✓**    (10) R

5.3    37 is die grootste ondergrens ✓ en 41 is die kleinste bogrens ✓    (2) C

5.4    R    Q ✓    P    U ✓    S ✓    T ✓    V ✓    W    R ✓ = 41✓✓    (8) C
       **[28]**


**VRAAG 6**

6.1    3 ✓✓    (2) R

6.2    e = 2n – 3 ✓✓    (2) R

6.3    4 Steiner-nodusse ✓
       9 skakels ✓✓
       Samehangendheid ✓    (4) P



**[8]**

**Totaal vir Module 4: 100 punte**