NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2017

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) (1)

$$(x-1)^2 = 2(1-x)$$
$$(x-1)^2 = -2(x-1)$$
$$(x-1)^2 + 2(x-1) = 0$$
$$(x-1)(x-1+2) = 0$$
$$(x-1)(x+1) = 0$$
$$x = 1 \qquad x = -1$$

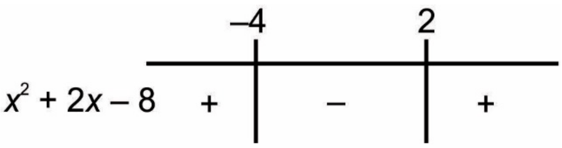
(2)

$$5^{-x}.5^{x-2} = \frac{25^{2x}}{5}$$
$$5^{-x}.5^{x-2} = \frac{5^{4x}}{5^1}$$
$$5^{-x+x-2} = 5^{4x-1}$$
$$-2 = 4x - 1$$
$$x = -\frac{1}{4}$$

(b)

$$(x+1)^2 < 9$$
$$\therefore x^2 + 2x + 1 < 9$$
$$\therefore x^2 + 2x - 8 < 0$$
$$\therefore (x+4)(x-2) < 0$$

Critical Values: –4; 2



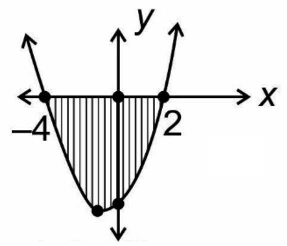
Solution: $\{x : -4 < x < 2\}$

Alternative

$$(x+1)^2 < 9$$
$$x^2 + 2x - 8 < 0$$

Sketch: $y = x^2 + 2x - 8$

$x-\text{int} : x = -4; x = 2$



Solution: $\{x : -4 < x < 2\}$

Alternative

$$(x+1)^2 < 9$$
$$\therefore -3 < x + 1 < 3$$
$$\therefore -4 < x < 2$$

(c)

$$(x-2)(x+4) = 0$$
$$x^2 + 2x - 8 = 0$$
$$\therefore b = 2 \text{ and } c = -8$$

(d) (1) $x-2=\dfrac{-4}{x-2}-4$ let $x-2=y$

$$y=-\frac{4}{y}-4 \text{ LCD: } y$$

$$y^2=-4-4y$$

$$\therefore y^2+4y+4=0$$

(2) $(y+2)^2=0$

$\therefore y=-2$

Roots are real and equal.

Alternative

$y^2+4y+4=0$

$\therefore \Delta=4^2-4(1)(4)$

$\therefore \Delta=0$

$\therefore$ Roots are real and equal.

Alternative

$$x-2=\frac{-4}{x-2}-4$$

$$\therefore (x-2)^2=-4-4(x-2)$$

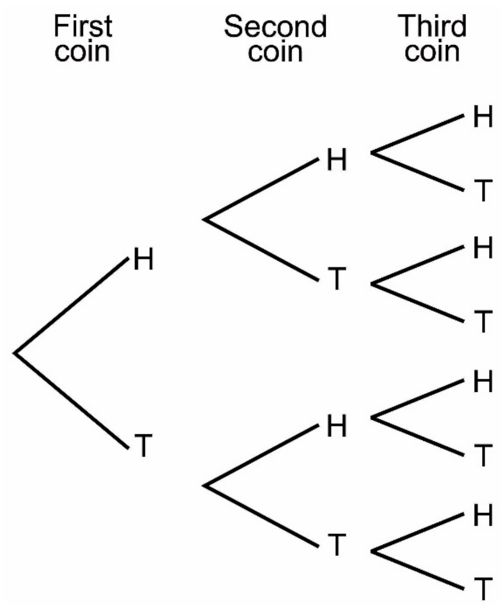
$$\therefore x^2-4x+4=-4-4x+8$$

$$\therefore x^2=0$$

$\therefore$ Roots are real and equal.

QUESTION 2

(a) (1)



(2) $E = \{HTT, THT, TTH\}$

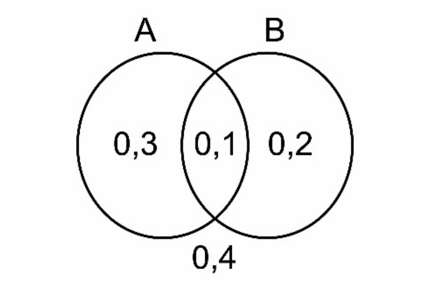
$\therefore$ P(2 tails and 1 head) $= \dfrac{3}{8}$

(b) (1) P $(A \cap B)$ = 0

(2) (i) You cannot pick a R2 and a R5 coin at the same time.

(ii) P(either a R5 or a R2)
$=$ P(A or B)
$=$ P(A) + P(B) mutually exclusive
$= 0{,}36 + 0{,}47$
$= 0{,}83$

(c) (1)



(2) P(exactly one machine is stamping R5 coins)
$= 0{,}3 + 0{,}2$
$= 0{,}5$
$\therefore$ 50%

QUESTION 3

(a) $480\ 163 \div 0{,}502 = \text{R}956\ 500$

(b) $\text{R}956\ 500 \times 5\% = \text{R}47\ 825$

(c) Cost of machinery including import charges $= \text{R}956\ 500 + \text{R}47\ 825$

$$= \text{R}1\ 004\ 325$$

$$\text{A} = \text{P}\ (1+i)^n$$

$$1\,004\ 325 = 225\ 450\left(1+\frac{9{,}5}{100}\right)^n$$

$$\frac{1\,004\ 325}{225\ 450} = \left(\frac{219}{200}\right)^n$$

$$\log_{\left(\frac{219}{200}\right)}\left(\frac{1\,004\ 325}{225\ 450}\right) = n$$

$$n = 16{,}46171594$$

$$\therefore n \approx 16{,}46$$

$\therefore$ approx. 17 years

(d) (1) Loan required: $\text{R}1\ 004\ 325 - \text{R}225\ 450 = \text{R}778\ 875$

$$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$778\ 875 = x\left[\frac{1-\left(1+\frac{12}{1200}\right)^{(-4\times 12)}}{\frac{12}{1200}}\right]$$

$$x = \text{R}20\ 510{,}76607$$

$$\therefore x = \text{R}20\ 510{,}77$$

(2) Outstanding Balance $= A - F$

$$A = 778\ 875\left(1 + \frac{12}{1200}\right)^{24}$$
$$A = 988\ 964{,}5744$$
$$A \approx 988\ 964{,}57$$

$$F = 20\ 510{,}76607\left[\frac{\left(1 + \frac{12}{1200}\right)^{24} - 1}{\frac{12}{1200}}\right]$$
$$F = 553\ 246{,}4277$$
$$F \approx 553\ 246{,}43$$

Outstanding balance $= 988\ 964{,}5744 - 553\ 246{,}4277$
$= \text{R}435\ 718{,}1467 \approx \text{R}435\ 718{,}15$

NB: If A and F are rounded to the nearest cent, consider
Outstanding balance $= 988\ 964{,}57 - 553\ 246{,}43$
$= \text{R}435\ 718{,}14$

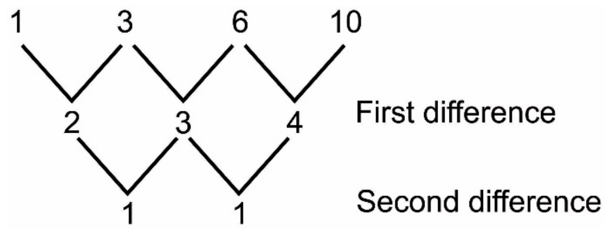
Alternative

Outstanding balance $= 20\ 510{,}76607\left[\frac{1 - \left(1 + \frac{12}{1200}\right)^{-24}}{\frac{12}{1200}}\right]$

$= \text{R}435\ 718{,}1466$
$\approx \text{R}435\ 718{,}15$

QUESTION 4

(a) (1)



Constant second difference

(2) $T_n = an^2 + bn + c$

$T_1 = a + b + c = 1$

$T_2 = 4a + 2b + c = 3$

$T_3 = 9a + 3b + c = 6$

$\therefore 3a + b = 2$ and $5a + b = 3$

Substitute $b = 2 - 3a$ into $5a + b = 3$

$\therefore 5a + (2 - 3a) = 3$

$2a = 1$

$a = \frac{1}{2}$

$b = \frac{1}{2}$ and $c = 0$

$\therefore T_n = \frac{1}{2}n^2 + \frac{1}{2}n$

(b) $T_3 = 52$ cm

$T_7 = 78$ cm

$T_3 = a + 2d = 52$

$T_7 = a + 6d = 78$

$4d = 26 \quad \therefore \quad d = 6\frac{1}{2}$

$\therefore a = 39$ cm

$T_{43} = 39 + 42\left(6\frac{1}{2}\right)$

$T_{43} = 312$ cm

QUESTION 5

(a) (1) $f(x) = x^2 - 6x + 9$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - 6(x+h) + 9 - (x^2 - 6x + 9)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 6x - 6h + 9 - x^2 + 6x - 9}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{2xh + h^2 - 6h}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{h(2x + h - 6)}{h}$$

$$f'(x) = 2x - 6$$

(2) $f'(-3) = 2(-3) - 6 = -12$

(b) $y = \pi x^{-1} + 3x^{\frac{1}{3}}$

$$\frac{dy}{dx} = -\pi x^{-2} + x^{-\frac{2}{3}}$$

SECTION B

QUESTION 6

(a) (1) Domain $= x \in \mathbb{R}$; $x \neq 3$

(2) Range $= y \in \mathbb{R}$; $y \neq -3$

(3) (i) 5 units

(ii) 5 units

(b) (1) $y = a.b^x$ substitute $\left(0;\frac{1}{4}\right)$

$\frac{1}{4} = a.b^0$

$a = \frac{1}{4}$

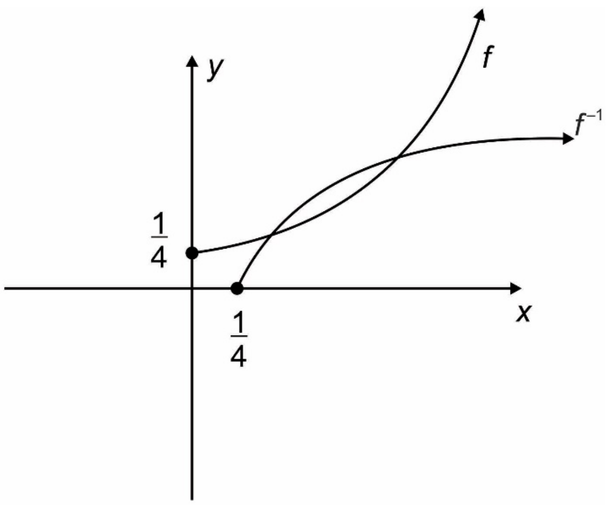
$y = \frac{1}{4}b^x$ substitute $\left(2;\frac{9}{4}\right)$

$\frac{9}{4} = \frac{1}{4}b^2$

$b^2 = 9$

$\therefore b = \pm 3$ but $b > 0$ $\therefore b = 3$

(2)



f: shape
intercept
domain

f^{-1} shape
intercept
range

(3) Range = $\left[\frac{1}{4};\infty\right)$

(4) $f(x) = \frac{1}{4}.3^x$

For f^{-1}: $x = \frac{1}{4}.3^y$; $y \geq 0$

$4x = 3^y$

$y = \log_3(4x)$ for $x \geq \frac{1}{4}$

(5) See graph in Question 6 (b) (2) above.

QUESTION 7

(a) $f(x) = x^2 + 6x + (3)^2 + 5 - (3)^2$

$f(x) = (x+3)^2 - 4$

$\therefore$ T.P.$(-3; -4)$

(b) (1) $x^2 + 6x + 5 = -x - 5$

$x^2 + 7x + 10 = 0$

$x = -2$ or $x = -5$

$A(-5;0)$ and $B(-2;-3)$

(2) Horizontal shift: $\therefore -5 < t < -2$

(c) (1) Length $\text{MN} = (-x-5) - (x^2 + 6x + 5)$

Length $\text{MN} = -x - 5 - x^2 - 6x - 5$

Length MN $= -x^2 - 7x - 10$

For max. length: Let $D_x = 0$

$-2x - 7 = 0$

$x = -\frac{7}{2}$

$\therefore$ Max. length $\text{MN} = -\left(-\frac{7}{2}\right)^2 - 7\left(-\frac{7}{2}\right) - 10$

$\therefore$ Max. length $\text{MN} = \frac{9}{4}$ units i.e. $2{,}25$ units

(2) Vertical shift: $\therefore k > \frac{9}{4}$

QUESTION 8

(a) (1) $\frac{3}{2}\,;-\frac{9}{2}\,;\frac{27}{2}\,;...$

$\therefore r=-3$ and series is geometric

however, series is not convergent since $r<-1$.

$\therefore \boldsymbol{x}\neq-\frac{\mathbf{3}}{\mathbf{2}}$

(2) $\frac{x-3}{x+3}=\frac{12-x}{x-3}$

$(x-3)^2=(12-x)(x+3)$

$x^2-6x+9=12x+36-x^2-3x$

$2x^2-15x-27=0$

$x=9$ or $x\neq-\frac{3}{2}$

(b) $S_4=7\frac{1}{2}\,;S_5=15\frac{1}{2}$ and $S_6=31\frac{1}{2}$

$T_5=S_5-S_4$

$T_5=8$

$T_6=S_6-S_5$

$T_6=16$

$T_5=ar^4=8$

$T_6=ar^5=16$

$\frac{T_6}{T_5}=r=2$

$\therefore a=\frac{1}{2}$

$S_n=\frac{\frac{1}{2}(2^n-1)}{2-1}$

$=2^{n-1}-\frac{1}{2}$

QUESTION 9

(a)
$$f(x) = -x^3 + bx^2 + cx - 3$$
$$f(1) = -(1)^3 + b(1)^2 + c(1) - 3 = 4$$
$$b + c = 8$$

$$f'(x) = -3x^2 + 2bx + c$$
$$f''(x) = -6x + 2b$$
$$f''\left(\frac{1}{2}\right) = -6\left(\frac{1}{2}\right) + 2b = 1$$
$$b = 2$$
$$\therefore c = 6$$

(b) For concave up: $f''(x) > 0$
$$-6x + 4 > 0$$
$$x < \frac{2}{3}$$

QUESTION 10

$$\frac{340}{x} - \frac{340}{x+2} = 3 \quad \text{LCD: } x(x+2)$$

$$340(x+2) - 340x = 3x(x+2)$$

$$3x^2 + 6x - 680 = 0$$

$$x = 14{,}09 \quad \text{or} \quad x \neq -16{,}09$$

Therefore Time = $\frac{340}{14{,}09} \approx$ **24,13 seconds**

Alternative

Let original time taken be represented by y.

$$\therefore xy = 340 \ \text{... eq. 1}$$

$$(x+2)(y-3) = 340 \ \text{... eq. 2}$$

From eq. 1 $\quad y = \frac{340}{x}$

$$\therefore (x+2)\left(\frac{340}{x} - 3\right) = 340$$

$$\therefore 3x^2 + 6x - 680 = 0$$

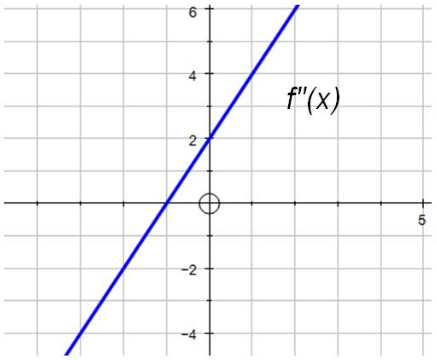
$$\therefore x = 14{,}09 \text{ or } x \neq -16{,}09$$

Therefore Time = $\frac{340}{14{,}09} \approx$ **24,13 seconds**

QUESTION 11

(a) (1) When $x = -2$ and $x = 0$

(2)



(b) $y = \frac{1}{5}x^3 + \frac{3}{4}x + 3$

$\frac{dy}{dx} = \frac{3}{15}x^2 + \frac{3}{4}$ substitute $x = 0$

$\frac{dy}{dx} = \frac{3}{4}$

Equation of tangent: $y = \frac{3}{4}x + c$ where $c = 3$

Equation of tangent: $y = \frac{3}{4}x + 3$

For point of intersection between tangent

and line BC, substitute $x = 2$ into $y = \frac{3}{4}x + 3$

$\therefore y = 4\frac{1}{2} \therefore \text{Pt}\left(2; 4\frac{1}{2}\right)$

Area of Busi's region $= \frac{1}{2}\left(5 + 3\frac{1}{2}\right) \times 2$

$= 8\frac{1}{2}$ units^2

Area of Khanya's region $= \frac{1}{2}\left(3 + 4\frac{1}{2}\right) \times 2$

$= 7\frac{1}{2}$ units^2

Therefore Busi's region is larger.

Total: 150 marks