TEGNIESE WISKUNDE: VRAESTEL II

NASIENRIGLYNE

Tyd: 3 uur   150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulpeksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

VRAAG 1

1.1   1.1.1 $m_{AC} = \dfrac{y_C - y_A}{x_C - x_A} = \dfrac{0 - \sqrt{3}}{1 - 2} = \sqrt{3}$

1.1.2 $\tan\theta = m_{AC}$
$\tan\theta = \sqrt{3}$
$\theta = 60°$

1.1.3 $m_{BC} = \tan(\theta + 75°)$
$m_{BC} = \tan(60° + 75°)$
$= -1$

1.2 $-1 = \dfrac{y_C - y_B}{x_C - x_B} = \dfrac{0 - 2}{1 - b}$
$-1 = \dfrac{-2}{1 - b}$
$-1 + b = -2$
$b = -1$

1.3 $m_{BC} \times m = -1$
$-1 \times m = -1$
$m = 1$
$\text{middelpunt}\left(\dfrac{1 - 1}{2} \; ; \dfrac{0 + 2}{2}\right)$
$\text{middelpunt}(0 \; ; 1)$
$y = mx + c$
$1 = (1)(0) + c$
$1 = c$
$y = x + 1$

VRAAG 2

2.1   2.1.1 $r^2 = x^2 + y^2$

$r^2 = \left(4\right)^2 + \left(3\right)$

$25 = x^2 + y^2$

2.1.2 $\sqrt{\left(x_B - x_A\right)^2 + \left(y_B - y_A\right)^2}$

$= \sqrt{\left(\dfrac{-16}{13} - 4\right)^2 + \left(\dfrac{-63}{13} - 3\right)^2}$

$= \dfrac{34\sqrt{13}}{13} = 9,43$

2.1.3   Gradiënt van radius $= \dfrac{3}{4}$

Gradiënt van raaklyn $= \dfrac{-4}{3}$

Opsie 1

$\therefore y = mx + c$

$\therefore 3 = \dfrac{-4}{3}\left(4\right) + c$

$\therefore \dfrac{25}{3} = c$

$\therefore y = \dfrac{-4}{3}x + \dfrac{25}{3}$

Opsie 2

$y - y_1 = m\left(x - x_1\right)$

$y - 3 = \dfrac{-4}{3}\left(x - 4\right)$

$y - 3 = \dfrac{-4}{3}x + \dfrac{16}{3}$
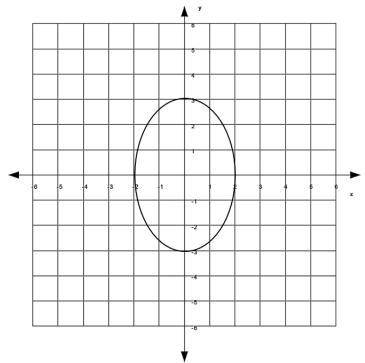
$\therefore y = \dfrac{-4}{3}x + \dfrac{25}{3}$

2.1.4 $y = mx + c$

$\therefore y = \dfrac{-4}{3}x + 5$

$\therefore 0 = \dfrac{-4}{3}x + 5$

$\therefore x = \dfrac{15}{4}$

2.2 $9x^2 + 4y^2 = 36$

$$\frac{9x^2}{36} + \frac{4y^2}{36} = 1$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

x-afsnitte
y-afsnitte
vorm

VRAAG 3

3.1   3.1.1   **Opsie 1**

$$\theta = \frac{4\pi}{15} \times \frac{180°}{\pi} = 48°$$

$$= \frac{\sec^2(62°) - 1}{\tan(48°)}$$

$$= \frac{\dfrac{1}{\cos^2(62°)} - 1}{\tan(48°)}$$

$$= \frac{\dfrac{1}{\cos^2(62°)} - 1}{\tan(48°)}$$

$$\approx 3,18$$

Opsie 2

$$\theta = \frac{4\pi}{15} \times \frac{180°}{\pi} = 48°$$

$$\frac{\sec^2\beta - 1}{\tan\theta}$$

$$= \frac{\tan^2\beta}{\tan\theta}$$

$$= \frac{\tan^2(62°)}{\tan(48°)}$$

$$\approx 3,18$$

ALTERNATIEF

$$\beta = 62° \times \frac{\pi}{180°} = \frac{31\pi}{90}$$

$$\frac{\sec^2\left(\dfrac{31\pi}{90}\right) - 1}{\tan\left(\dfrac{4\pi}{15}\right)}$$

$$= \frac{\dfrac{1}{\cos^2\left(\dfrac{31\pi}{90}\right)} - 1}{\tan\left(\dfrac{4\pi}{15}\right)}$$

$$\approx 3,18$$

3.1.2 $$\tan 48° = \frac{4}{OC}$$

$$OC = \frac{4}{\tan 48°} \qquad \text{OF} \qquad \frac{4}{\tan\left(\dfrac{4\pi}{15}\right)}$$

$$\therefore OC = 3,6$$

3.1.3 $$\tan 62° = \frac{AC}{OC}$$

$$AC = 3,6 \tan 62°$$

$$\therefore AC \approx 6,77$$

$$\therefore AB = AC - BC$$

$$\therefore AB = 6,77 - 4 = 3,77$$

3.2 $= \dfrac{3\sec^2(180°-30°)\cos180°}{\tan(360°-45°)-\cos^2(180°+60°)}$

$= \dfrac{\dfrac{3}{\cos^2(180°-30°)}(-1)}{-\tan45°-\cos^2 60°}$

$= \dfrac{\dfrac{3}{\cos^2 30°}(-1)}{-\tan45°-\cos^2 60°}$

$= \dfrac{\dfrac{-3}{\left(\dfrac{\sqrt{3}}{2}\right)^2}}{-1-\left(\dfrac{1}{2}\right)^2}$

$= \dfrac{-3\times\dfrac{4}{3}}{-\dfrac{5}{4}} = \dfrac{16}{5}$

3.3 $\sin 2x = 0,473$

Verwysingshoek $= 28,229°$
$\therefore 2x = 28,23°$ of $2x = 180° - 28,23°$
$\therefore x = 14,11°$ of $x \approx 75,89°$

3.4   LK:
$\dfrac{\cos x}{\sin x}+\dfrac{\sin x}{1+\cos x}$

$= \dfrac{\cos x(1+\cos x)+\sin x \,.\sin x}{\sin x(1+\cos x)}$

$= \dfrac{\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)}$

$= \dfrac{\cos x+1}{\sin x(1+\cos x)}$

$= \dfrac{1}{\sin x}$

$\therefore$ LK $=$ RK

3.5 $\dfrac{\sin^2\theta(-\cot\theta)}{-\cos\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta\tan\theta}$

$= \dfrac{\sin^2\theta}{\sin\theta}$

$= \sin\theta$

VRAAG 4

4.1   4 amplitude vir f
1 amplitude vir g

4.2 $A(75,96°\ ;-0,97°)$

4.3   120°

4.4   4.4.1 $\therefore f(x) \geq g(x)$
$\therefore x \in [0° ; 75,96°]$ en $x \in [225,96° ; 360°]$

4.4.2 $x = 120°\ ; 300°$

VRAAG 5

5.1 $\sin 38° = \dfrac{EF}{5\ m}$ OF $\dfrac{EF}{\sin 38°} = \dfrac{5}{\sin 90°}$
$\therefore 5\sin 38° = EF$
$\therefore EF \approx 3,08\ m$

5.2 $\cos 61° = \dfrac{3,08\ m}{AF}$ OF $\dfrac{AF}{\sin 90°} = \dfrac{3,08}{\sin 29°}$
$\therefore AF = \dfrac{3,08}{\cos 61°}$ $AF = \dfrac{3,08}{\sin 29°}$
$\therefore AF \approx 6,35\ m$

5.3 $GF^2 = AG^2 + AF^2 - 2(AG)(AF)\cos(G\hat{A}F)$
$(5)^2 = (6,8)^2 + (6,35)^2 - 2(6,8)(6,35)\cos A$
$G\hat{A}F \approx 44,53°$

Oppervlakte $= \dfrac{1}{2}AG\,.\,AF.\,\sin(G\hat{A}F)$
Oppervlakte $= \dfrac{1}{2}(6,8)(6,35)\sin(44,53°)$
Oppervlakte $\approx 15,14\ m^2$

VRAAG 6

6.1   6.1.1 $\hat{A} = 90°$ (hoek in 'n halfsirkel)

$\hat{O}_1 = 40°$ (hoek by middelpunt = 2 × hoek by omtrek)

$\therefore \hat{B}_1 = 40°$ (ooreenkomstige hoeke AB//FO)

$90° + 40° + x = 180°$
$\therefore x = 50°$

6.1.2 $\triangle ADB /// \triangle EDO$ (lyn deur middelpunt || aan 2de sy)

$\therefore \dfrac{OD}{BD} = \dfrac{EO}{AB}$ (eweredigheidstelling)

$\therefore \dfrac{OD \cdot AB}{BD} = EO$

ALTERNATIEWE PUNTETOEKENNING

$\therefore \dfrac{OD}{BD} = \dfrac{EO}{AB}$ (eweredigheidstelling)

$\therefore \dfrac{OD \cdot AB}{BD} = EO$

6.2   6.2.1   In $\triangle ADP \equiv \triangle EFO$

$\hat{D} = \hat{F} = 90°$ (hoeke in halfsirkels)
$DP = FO$ (radii gegee gelyk)
$AP = EO$ (middellyne gegee gelyk)
$\therefore \triangle ADP \equiv \triangle EFO$ (RSS)

6.2.2 $D\hat{P}A = F\hat{O}E$ ($\triangle ADP \equiv \triangle EFO$)
$\therefore DP \parallel MF$ (verwisselende hoeke gelyk)
en $AO = OP$ (radii)
$\therefore AM = MD$ (middelpuntstelling)

6.2.3 $\triangle ADP$ en $\triangle EFO$

6.2.4 $OE = 4$ eenhede   (gegee)
$OF = 2$ eenhede   (radius)
$OE^2 = OF^2 + EF^2$ (Pythagoras)
$4^2 = 2^2 + EF^2$
$12 = EF^2$
$\therefore EF = 2\sqrt{3}$

6.3   6.3.1 $T\hat{A}G = 2T\hat{A}E = 34°$ (gegee)
$\therefore \hat{O}_1 = 2T\hat{A}G$ (hoek by middelpunt = 2 × hoek by omtrek)
$\therefore \hat{O}_1 = 2(34°) = 68°$

6.3.2 $\hat{G}_2 = O\hat{B}G$ (hoeke by gelyke sye; radii)

$\hat{G}_2 + O\hat{B}G + 68° = 180°$ (binnehoeke van driehoek)

$\therefore \hat{G}_2 = O\hat{B}G = 56°$

6.3.3 $AT = TB$ (gegee)

$\therefore \hat{T}_2 = 90°$ (lyn van middelpunt van sirkel na middelpunt van koord)

$\hat{T}_2 + \hat{G}_2 + T\hat{B}G = 180°$ (binnehoeke van driehoek)

$90° + 56° + T\hat{B}G = 180°$

$T\hat{B}G = 34°$

$T\hat{B}G = \hat{A}_5 = 34°$ (raaklyn-koord-stelling)

6.3.4 $K\hat{A}B = K\hat{A}G + T\hat{A}G$

$\therefore K\hat{A}B = 34° + 34° = 68°$ en $K\hat{A}B = A\hat{D}B$ (raaklyn-koord-stelling)

$\therefore A\hat{D}B = 68°$

6.4 $BC = CD = 2$ eenhede   (radius $\perp$ koord)

$OD^2 = CD^2 + OC^2$ (Pythagoras)

$r^2 = 2^2 + (r-1)^2$

$r^2 = 4 + r^2 - 2r + 1$

$2r = 5$

$r = 2,5$ eenhede

VRAAG 7

7.1 $v = \pi D n$

$v = \pi(0,24 \text{ m})(5,31 \text{ rev/s})$

$v = 4 \text{ m/s}$

7.2 $v = \pi D n$

$\dfrac{4 \ m}{s} = \pi(0,48 \ m)n$

$n = 2,65 \text{ rev/s}$

7.3 $\omega = 2\pi n$

$\omega = 2\pi(2,65)$

$\omega = 16,65 \text{ rad/s}$

VRAAG 8

8.1   Volume $= \pi r^2 \times h$

$= \pi (1,75 \text{ m})^2 \times (6,25 \text{ m})$

$= 60,132 \text{ m}^3$

$\approx 60 \text{ m}^3$

8.2   8.2.1 $s = r\theta$

$s = (1,75 \text{ m})(120° \times \dfrac{\pi}{180°})$

$s = 3,67 \text{ m}$

8.2.2   Oppervlakte $= \dfrac{rs}{2}$ **OF** Oppervlakte $= \dfrac{r^2 \theta}{2}$

$= \dfrac{(1,75 \text{ m})(3,67)}{2}$ $= \dfrac{(1,75 \text{ m})^2 \left(120° \times \dfrac{\pi}{180°}\right)}{2}$

$= 3,21 \text{ m}^2$ $= 3,21 \text{ m}^2$

8.2.3   Oppervlakte = Sektoroppervlakte – Driehoekoppervlakte

$= 3,21 \text{ m}^2 - 1,326 \text{ m}^2$

$= 1,88 \text{ m}^2$

8.2.4   Volume van diesel $= 1,88 \text{ m}^2 \times 6,25 \text{ m}$

$= 11,75 \text{ m}^3$

Persentasie gevul $= \dfrac{\text{Gevulde volume}}{\text{Totale volume}} \times 100\%$

$= \dfrac{11,75}{60} \times 100\%$

$\approx 19,58\%$

8.2.5 $4h^2 - 4dh + x^2 = 0$

$4(0,5)^2 - 4(3,5)(0,5) + x^2 = 0$

$\therefore x^2 = 6$

$\therefore x = \sqrt{6} \approx 2,45 \text{ m}$

Oppervlakte = l × b

$= 2,45 \times 6,25$

$\approx 15,31 \text{ m}^2$

VRAAG 9

9.1   **Opsie 1**

$$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + \ldots + o_{n-1}\right)$$

$$A_T = 2\left(\frac{0,45 + 0,21}{2} + 0,62 + 0,48 + 0,32 + 0,46 + 0,64 + 0,47\right)$$

$$A_T = 2(3,32)$$

$$A_T = 6,64 \ \text{km}^2$$

Opsie 2

$$A_T = a\left(m_1 + m_2 + m_3 + \ldots + m_n\right)$$

$$A_T = 2(\frac{0,45 + 0,62}{2} + \frac{0,62 + 0,48}{2} + \frac{0,48 + 0,32}{2} + \frac{0,32 + 0,46}{2} + \frac{0,46 + 0,64}{2} +$$

$$\frac{0,64 + 0,47}{2} + \frac{0,47 + 0,21}{2})$$

$$A_T = 2(3,32)$$

$$A_T = 6,64 \ km^2$$

9.2   R5 250 000 inkomste ÷ R40 per baal = 131 250 bale
Hektaar benodig = 131 250 ÷ 350 = 375

Totale oppervlakte = 375 × 0,01 km^2 = 3,75 km^2

Totaal: 150 punte