NASIONALE SENIOR SERTIFIKAAT-EKSAMEN
NOVEMBER 2021

TEGNIESE WISKUNDE: VRAESTEL I

NASIENRIGLYNE

Tyd: 3 uur 150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulpeksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

VRAAG 1

1.1 1.1.1 $2x^2 - x - 6 = 0$

$(2x + 3)(x - 2) = 0$

$x = -\frac{3}{2}$ of $x = 2$

Alternatief:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-6)}}{2(2)}$$

$$= \frac{1 \pm \sqrt{1 + 48}}{4}$$

$$\therefore x = 2 \text{ of } x = \frac{-3}{2}$$

1.1.2 $x^2 - 1 = x$

$x^2 - x - 1 = 0$

$$x = 1 \pm \frac{\sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{5}}{2}$$

1.1.3 $4x^2 - 4x + 1 \leq 0$

$(2x - 1)(2x - 1) \leq 0$

$(2x - 1)^2 \leq 0$

$$\therefore x = \frac{1}{2}$$

1.2 $3x^2 + 2x + 1 = 0$

$\Delta = 2^2 - 4(3)(1)$

$= 4 - 12 = -8$

Wortels is niereëel

1.3 $3{,}33564095 \times 10^{-5}$

VRAAG 2

2.1 2.1.1 $\sqrt{9x^4 + 16x^4}$

$= \sqrt{25x^4}$

$= 5x^2$

2.1.2 $\left(\dfrac{x^{-\frac{1}{3}}}{\sqrt[3]{x^2}}\right)^{-2}$

$= \left(\dfrac{x^{-\frac{1}{3}}}{x^{\frac{2}{3}}}\right)^{-2}$ OF $(x^{-1})^{-2}$

$= x^2$

$= x^{\frac{2}{3}+\frac{4}{3}}$

$= x^2$

2.2 $\sqrt{5x-1} - 1 = x$

$\sqrt{5x-1} = x + 1$ $x \geq \frac{1}{5};\ x \geq -1$ of kontroleer oplossings

$5x - 1 = x^2 + 2x + 1$

$0 = x^2 - 3x + 2$

$0 = (x - 2)(x - 1)$

$x = 2$ of $x = 1$

Albei geldig

2.3 $\dfrac{2^{2x+3} - 3 \cdot 2^{2x+1}}{2^{x-1}}$

$= \dfrac{2^{2x} \cdot 2^3 - 3 \cdot 2^{2x} \cdot 2^{+1}}{2^x \cdot 2^{-1}}$

$= \dfrac{2^{2x}(8 - 6)}{2^x \cdot \frac{1}{2}}$

$= 4 \cdot 2^x$

Alternatief:

$\dfrac{2^{2x}(2^3 - 3 \cdot 2)}{2^x \cdot 2^{-1}}$

$= \dfrac{2^x(2)}{\cdot 2^{-1}} = 2^x \cdot 4$ **of** 2^{x+2}

2.4 $6 = 3^x$

$\therefore x = \log_3 6$

$= \dfrac{\log 6}{\log 3}$

$\approx 1,6$

Alternatief:

$x \log 3 = \log 6$

$\therefore x = \dfrac{\log 6}{\log 3} \approx 1,6$

VRAAG 3

3.1 3.1.1 $w^2 = (a + bi)^2$

$= a^2 + 2abi + b^2i^2$ OF

$= (a^2 - b^2) + 2abi$

3.1.2 $\therefore\ 2ab = -12$ ① en $a^2 - b^2 = 5$ ②

$$a = -\frac{6}{b}$$

Vervang in ② $\left(-\frac{6}{b}\right)^2 - b^2 = 5$

$$\frac{36}{b^2} - b^2 = 5$$

$$36 - b^4 = 5b^2$$

$$b^4 + 5b^2 - 36$$

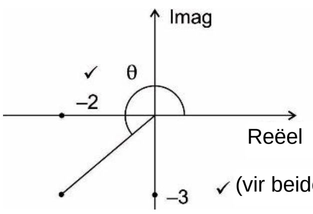
$$0 = (b^2 + 9)(b^2 - 4)$$

$b^2 = -9$ of $b^2 = 4$

ongeldig $b = \pm 2$

$$\left.\begin{array}{l}\therefore\ b = 2,\ a = -3 \\ \text{of } b = -2,\ a = 3\end{array}\right]$$

3.2 $z = -2 - 3i$



✓ (vir beide komponente en θ)

$$|z|^2 = 4 + 9 = 13$$

$$|z| = \sqrt{13}$$

$$\tan\theta = +\frac{3}{2}$$

$$\theta = 236{,}3^\circ$$

$$z \approx \sqrt{13}\,(\cos 236{,}3^\circ + i\sin 236{,}3^\circ)$$

Alternatief:

$\theta = 4{,}1$ radiale

3.3 $$\frac{111_2}{35} = \frac{1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0}{35}$$

$$= \frac{4 + 2 + 1}{35}$$

$$= \frac{7}{35} = \frac{1}{5}$$

VRAAG 4

4.1 Laat waarde = V

$$\frac{1}{3}\text{V} = \text{V}\,(1 - i)^2$$

$$\sqrt{\frac{1}{3}} = 1 - i$$

$$i = 1 - \sqrt{\frac{1}{3}} \approx 0{,}422649$$

koers ≈ 42,3%

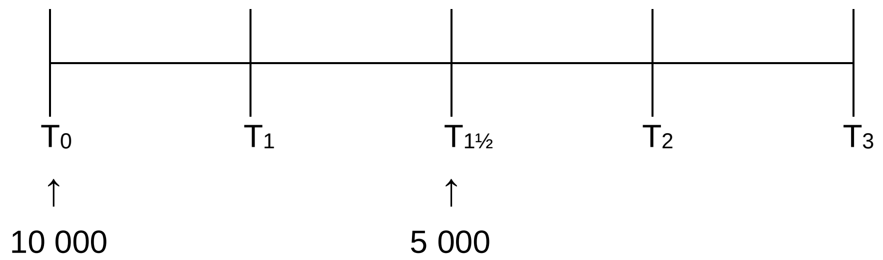
4.2 4.2.1 $1 + i\text{ eff} = \left(1 + \dfrac{0{,}075}{4}\right)^4$

i eff = 0,07718 …

eff koers = 7,7% p.j.

4.2.2

$i = \dfrac{0{,}075}{4}$ $i = \dfrac{0{,}078}{12}$



$$A = 10\,000\left(1 + \frac{0{,}075}{4}\right)^4\left(1 + \frac{0{,}078}{12}\right)^{24} + 5\,000\left(1 + \frac{0{,}078}{12}\right)^{18}$$

≈ R18 201,96

Alternatief

(1) **Tel R5 000 by na 18 maande:**

$$A = \left[10000\left(1 + \frac{0{,}075}{4}\right)^4\left(1 + \frac{0{,}078}{12}\right)^8 + 5000\right] \times \left(1 + \frac{0{,}078}{12}\right)^{1{,}5\times 12} \approx 18201{,}96$$

(2) **STAP-vir-STAP-benadering:**

$$A_1 = 10000\left(1+\frac{0{,}075}{4}\right)^4 \approx 10771{,}35868...$$

$$A_2 = (10771{,}35868...)\left(1+\frac{0{,}078}{12}\right)^6 \approx 11198{,}32744...$$

$$A_2 = 11198{,}32744... + 5000 \approx 16198{,}32744...$$

$$A_3 = (16198{,}32744...)\left(1+\frac{0{,}078}{12}\right)^{18} \approx 18201{,}96$$

4.3 $A = P(1 = i)^n$

$$25\,000 \leq 20\,000\left(1+\frac{4}{100}\right)^n$$

$$\frac{5}{4} \leq (1{,}04)^n$$

$n \geq \log_{1{,}04}\left(\frac{5}{4}\right)$ OF $n \geq \dfrac{\log\left(\frac{5}{4}\right)}{\log 1{,}04}$

$n \geq 5{,}7$ jaar

$\therefore$ na 6 jaar

VRAAG 5

5.1 $y = \frac{a}{x} + b$

$b = -2$

$y = \frac{a}{x} - 2$

Vervang (1; 0) : $0 = \frac{a}{1} - 2$

$2 = a$

5.2 $y = c\,(x + 4)(x - 2)$

Vervang (–1; 9) : $9 = c(3)(-3)$

$-1 = c$

$y = -1(x + 4)(x - 2)$

$y = -x^2 - 2x + 8$

$c = -1$

$d = -2$

$e = 8$

OF

$y = c(x + 1)^2 + 9$

vervang óf (–4 ; 0) óf (2 ; 0)

$\therefore\ 0 = c(2 + 1)^2 + 9$

$\therefore\ c = 1$

$\therefore\ y = -1(x + 1)^2 + 9 = -x^2 - 2x - 1 + 9$

$= -x^2 - 2x + 8$

$\therefore\ d = -2$ OF $e = 8$

5.3 $y = f \cdot g^x + h$

Vervang (0; 1) : $1 = f{\cdot}g^\circ + h$

$1 = f + h$

$h = -3$

$\therefore\ 1 = f - 3$

$4 = f$

$y = 4{\cdot}g^x - 3$

Vervang (1; –1) : $-1 = 4{\cdot}g^1 - 3$

$2 = 4g$

$\frac{1}{2} = g$

5.4 $x^2 + y^2 = k^2$

Vervang (–3; 4) : $9 + 16 = k^2$

$k = 5$

VRAAG 6

6.1 Stel $y = 0$: $0 = -x^2 - 4x$

$0 = -x(x + 4)$

$x = 0$ of $x_B = -4$ B is $(-4; 0)$

$x_A = -2$ (volgens simmetrie)

Vervang in f: $y = -(-2)^2 - 4(-2)$

$= 4$ A is $(-2; 4)$

By C, $y = 2^0 - 8 = -7$ C is $(0; -7)$

By D, $y = 0$

$0 = 2^x - 8$

$8 = 2^x$

$2^3 = 2^x$

$x = 3$ D is $(3; 0)$

Alternatief:

$f'(x) = 2x - 4 = 0$

$\therefore x = 2$

6.2 $BD = x_D - x_B$

$= 3 - (-4) = 7$ eenhede

AE : by E, $x = -2$

$y = 2^{-2} - 8 = \frac{1}{4} - 8$

$= -\frac{31}{4}$

$AE = y_A - y_E$

$= 4 - \left(-\frac{31}{4}\right)$

$= \frac{47}{4}$ eenhede (of 11,5 eenhede)

6.3 Terrein van f: $y \in (-\infty; 4]$ OF $y \leq 4$

6.4 $m_{BC} = \dfrac{0+7}{-4-0} = -\dfrac{7}{4}$

d.w.s. $y = -\dfrac{7}{4}x - 7$

6.5 $x \in (-\infty\,; 0)$

6.6 Skuif *g* meer as 7 eenhede vertikaal op (verby oorsprong) maar onder A.

VRAAG 7

7.1 $g(x) = \frac{x}{3} - 2$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h}{3} - 2 - \left(\frac{x}{3} - 2\right)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h-x}{3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{3} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{1}{3}$$

7.2 $y + x = \left(\frac{2}{x} - \sqrt{x}\right)^2 - x$

$$y = \frac{4}{x^2} - \frac{4\sqrt{x}}{x} + x - x$$

$$= 4x^{-2} - 4x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = -8x^{-3} + 2x^{-\frac{3}{2}}$$

$$\text{OF} = -\frac{8}{x^3} + \frac{2}{x^{\frac{3}{2}}}$$

7.3 7.3.1 Volume = oppervlakte van basis × hoogte

$300 = x^2y$

$\frac{300}{x^2} = y$

7.3.2 Koste in rand = Buiteoppervlakte × Koste/m^2

$$= 5(x^2) + 2(4xy)$$

$$= 5x^2 + 8xy$$

$$C = 5x^2 + \frac{8x \cdot 300}{x^2}$$

$$= 5x^2 + \frac{2\,400}{x}$$

7.3.3 $C(x) = 5x^2 + 2\,450x^{-1}$

$$C'(x) = 10x - 2\,400x^{-2}$$

$$= 10x - \frac{2\,400}{x^2}$$

By min: $10x - \frac{2\,400}{x^2} = 0$

$$10x^3 = 2\,400 \qquad x \neq 0$$

$$x^3 = 240$$

$$x = \sqrt[3]{240}$$

$$\text{Min koste} = 5\left(\sqrt[3]{240}\right)^2 + \frac{2\,400}{\left(\sqrt[3]{240}\right)}$$

$$\approx \text{R}579{,}29$$

VRAAG 8

8.1 $f(x) = -x^3 + 10x^2 - 17x - 28$

$f'(x) = -3x^2 + 20x - 17$

by stasionêre punte: $-3x^2 + 20x - 17 = 0$

$3x^2 - 20x + 17 = 0$

$(3x - 17)(x - 1) = 0$

$x = \frac{17}{3}$ of $x = 1$

$y_E = -1 + 10 - 17 - 28$

$= -36$

E is $(1 ; -36)$

8.2 $x \in \left(1 ; \frac{17}{3}\right)$ OF $1 < x < \frac{17}{3}$

8.3 By F; $m_{\text{raaklyn}} = f'(5) = -3(25) + 20(5) - 17$

$= -75 + 100 - 17$

$= 8$

Vergelyking is $y - 12 = 8(x - 5)$ OF $y = 8x - 28$

8.4 $8x - 28 = -x^3 + 10x^2 - 17x - 28$

$x^3 - 10x^2 + 25x = 0$

$x(x^2 - 10x + 25) = 0$

$x(x - 5)^2 = 0$

$x_G = 0$ d.w.s. y-eenheid van f

G is $(0 ; -28)$

VRAAG 9

$H = 15 + 3t^2 - \frac{2}{3}t^3$

Tempo van verandering = $\frac{dH}{dt} = +6t - 2t^2$

$\therefore\ +6t - 2t^2 = -\frac{1}{2}$

$+12t - 4t^2 = -1$

$0 = 4t^2 - 12t + 1$

$t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(1)}}{2(4)}$

t = 2,99 uur of 0,1 uur

VRAAG 10

10.1 (a) $\int d\theta = \theta + C$

(b) $\int\left(\frac{8}{x} - \frac{5}{x^2} + 6x^3\right)dx = \int\left(\frac{8}{x} - 5x^{-2} + 6x^3\right)dx$

$= 8\ell n(x) - \frac{5x^{-1}}{-1} + \frac{6x^4}{4} = 8\ell n(x) + \frac{5}{x} + \frac{3x^4}{2} + c$

10.2 $\int_0^5 g(x)\,dx = -3$

Indien $g(x) = g(-x)$

$\int_{-5}^{0} g(-x)$ = –3 volgens simmetrie

$\therefore\ \int_{-5}^{5} g(x)$ = –3 + (–3) = –6

10.3 $A=\int_a^b f(x)\,dx=\int_2^4 \left(x^2-4\right)dx$

Ken 1 punt toe vir a = 2
Ken 'n punt toe vir oppervlaktetoepassing
Ken 1 punt toe vir integrasie
Ken 1 + 1 punte toe vir vervanging
Ken 1 punt toe vir vereenvoudiging

Los op:

$$\int_2^4 \left(x^2-4\right)dx=\left(\frac{1}{3}x^3-4x\right)\Bigg|_2^4=\left(\frac{1}{3}\cdot(4)^3-4\cdot 4\right)-\left(\frac{1}{3}\cdot(2)^3-4\cdot 2\right)$$

$$=\left(\frac{64}{3}-16\right)-\left(\frac{8}{3}-8\right)=\frac{32}{3}$$ vierkante eenhede

Totaal: 150 punte