NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2017

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

QUESTION 1

(a) 0,7337 (The second mark awarded for rounding off correctly)

(b) C

(c) A = 0,6268
B = 0,0264

(d) No, you would be extrapolating (The second mark is for the concept of extrapolation)

QUESTION 2

(a) $m_{OA} = \frac{4-0}{2-0} = 2$

$\tan A\hat{O}B = 2$

$A\hat{O}B = 63,43°$

(b) $m_{\perp} = -\frac{1}{2}$

Midpoint of OA = (1;2)

$\therefore\ y = -\frac{1}{2}x + c$

Subs (1; 2)

$\therefore\ 2 = -\frac{1}{2} + c$

$\therefore\ y = -\frac{1}{2}x + \frac{5}{2}$

(c) $x = 3$

(d) $y = -\frac{1}{2}(3) + \frac{5}{2}$

$y = 1$

$\therefore\ (x-3)^2 + (y-1)^2 = r^2$

Subs (0;0)

$\therefore\ (0-3)^2 + (0-1)^2 = r^2$

$\therefore\ r^2 = 10$

$\therefore\ (x-3)^2 + (y-1)^2 = 10$

QUESTION 3

(a) (1) $\sin(31° + 22°) = k$
$= \sin 53°$
$= k$

(2) $\cos(90° + 53°)$
$= -\sin 53°$
$= -k$

(3) $\cos(75° - 22°)$
$= \cos 53°$
$= \sqrt{1-k^2}$ (There is a method mark for workings.)

(b) $$\frac{\cos\theta}{2\sin\theta\cos\theta} - \frac{\cos^2\theta - \sin^2\theta}{2\sin\theta}$$

$$= \frac{1}{2\sin\theta} - \frac{\cos^2\theta - \sin^2\theta}{2\sin\theta}$$

$$= \frac{1 - \cos^2\theta + \sin^2\theta}{2\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta - \cos^2\theta + \sin^2\theta}{2\sin\theta}$$

$$= \frac{2\sin^2\theta}{2\sin\theta}$$

$$= \sin\theta$$

therefore

LHS = RHS

(c) $3\sin^2\theta - 2\sin\theta = 0$
$\sin\theta(3\sin\theta - 2) = 0$
$\sin\theta = 0$
$\theta = 0° + k180°$
Alternate: $\theta = 0° + k360°$ **OR** $\theta = 180° + k360°$
OR
$\sin\theta = \frac{2}{3}$
$\theta = 41,8° + k360°$ **OR** $\theta = 138,2° + k360°$ $\boldsymbol{K \in Z}$

QUESTION 4

(a) M(3;–1)

(b) $(0-3)^2+(y+1)^2=25$

$y^2+2y-15=0$

$(y+5)(y-3)=0$

$y=-5$ OR $y=3$

C(0;3)

(c) $m_{CM}=\dfrac{3-(-1)}{0-3}=-\dfrac{4}{3}$

$m_{AC}=\dfrac{3}{4}$

$y=\dfrac{3}{4}x+3$

(d) $0=\dfrac{3}{4}x+3$

$x=-4$

A(–4; 0)

$(x-3)^2+(0+1)^2=25$

$(x-3)^2=24$

$x=3\pm\sqrt{24}$

AB = 4 units – 1,9 units

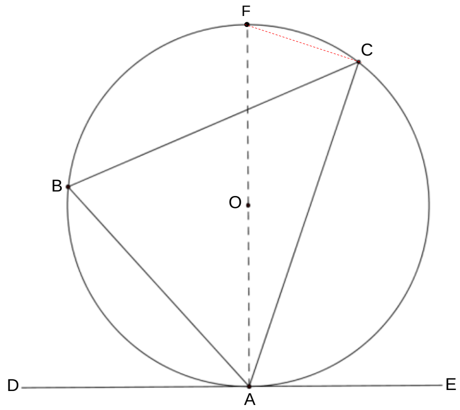
AB = 2,1 units

QUESTION 5

(a) R.T.P: $C\hat{A}E = A\hat{B}C$

Construction: refer to diagram for the construction

Proof:



$O\hat{A}C + C\hat{A}E = 90^\circ$ (Tangent perpendicular to line through centre)

$F\hat{C}A = 90^\circ$ (Angles in semi-circle)

$O\hat{F}C + O\hat{A}C = 90^\circ$ (Angles in triangle)

therefore

$O\hat{F}C = C\hat{A}E$

but

$O\hat{F}C = A\hat{B}C$ (Angles in same segment)

therefore

$C\hat{A}E = A\hat{B}C$

(b) $\hat{B}_3 = 70^\circ$ (Angles in same segment)

$\hat{F}_2 = 52^\circ$

$\hat{G}_1 + \hat{G}_2 = 70^\circ$ (Exterior angle of cyclic quad equal to the interior opposite angle)

$\hat{F}_2 = \hat{G}_2 = 52^\circ$ (tan chord theorem)

$\hat{G}_1 = 18^\circ$

QUESTION 6

(a) A = 50

(b) 400

(c) P = 50 and M = 100

(d) Q_3 = greater than 300 and less than 325 (approximate)
Q_1 = 200
IQR = 110 (ca mark based on values)

(e) Mean = 250

(f) (1) It would stay the same. It is only the upper 25% of data that are affected.
Or
It would decrease. People leave the contract therefore less people.

(2) Standard deviation would decrease. The difference between the new mean and the data would decrease.

(3) It would skew the data to the left. The vales above the median are less spread out. or mean < median

SECTION B

QUESTION 7

(a) $m_{OA} = 3$

Equation of line OA is $y = 3x$

Equation of line EF is $2y + x = 10$

$2(3x) + x = 10$

$7x = 10$

$x = \frac{10}{7}$ (This represents the height of the triangle.)

$y = \frac{30}{7}$

Coordinates of point E

$2y + 0 = 10$

$y = 5$

E(0; 5)

Area of $\Delta EBO = \frac{1}{2} \times 5 \times \frac{10}{7}$

Area of $\Delta EBO = \frac{25}{7}$ units^2

(b) C(4; 0)

$2(0) + x = 10$

$x = 10$

Area of $\Delta DCF = \frac{1}{2} \times 6 \times \frac{18}{5}$

Area of $\Delta DCF = \frac{54}{5}$ units^2

QUESTION 8

(a) (1) $OC = \sqrt{(3-0)^2 + (1-(-2))^2} = \sqrt{18}$

(2) B (6; –2)

(3) $m_{OC} = \frac{3}{3} = 1$

$\therefore\ O\hat{C}B = 45^\circ$

$\therefore\ C\hat{O}B = 90^\circ$ (angles of a Δ)

$\therefore\ C\hat{A}B = 45^\circ$ (angle at centre)

Alternate solution:

$6^2 = 18 + 18 - 2(18)\cos C\hat{O}B$

$C\hat{O}B = 90^\circ$

$\therefore C\hat{A}B = 45^\circ$ (Angle at centre)

(b) Circumference = $2\pi r$

Circumference = $2\pi\sqrt{18}$ units or 26,66 units

$C\hat{O}B = 60^\circ$ (Angle at centre = 2 x angle at circumference)

The size of angle θ after B moves into new position

$\frac{9}{2} = \frac{1}{2}\sqrt{18}\sqrt{18}\sin\theta$ (Area rule)

$\theta = 30^\circ$

$\theta = 180^\circ - 30^\circ = 150^\circ$

B needs to move 90° anti-clockwise

Therefore

Point B needs to move $2\pi\sqrt{18} \times \frac{90^\circ}{360^\circ}$

OR

Point B needs to move 6, 66 units.

QUESTION 9

(a) $\hat{C}$ is a common angle

$\hat{D}_2 = \hat{A}$ (tan chord theorem)

Therefore

$\Delta ADC \,|||\, \Delta DBC$ (A.A.A) **OR** $\hat{B}_2 = A\hat{D}C$ (Angles in a triangle)

(b) $\dfrac{DC}{BC} = \dfrac{AC}{DC}$ ($\Delta ADC \,|||\, \Delta DBC$)

$DC^2 = AC.BC$

but

$AC = AB + BC$

Therefore

$DC^2 = BC\,(AB + BC)$

$DC^2 = AB.BC + BC^2$

$AB.BC = DC^2 - BC^2$

QUESTION 10

(a) $A\hat{D}L = 90°$ (Angle in semi-circle) (one mark for the reason)

$A\hat{C}B = 90°$

Therefore

DL||CB (Converse: corresponding angles are equal)

(b) LC = LA (radii of the large circle)

SD = SL = SA (radii of small circle)

but

LA = SA + SL

therefore

LC = 2SD

Alternative solution:

AD = DC (converse: midpoint DL||BC)

In ΔACL

DS||CL (midpoint theorem)

$\therefore$ LC = 2SD

(c) AS = SL and AL = LB; radii

$\therefore \dfrac{SL}{AB} = \dfrac{1}{4}$

(d) LB = 15 units (radius)

$\dfrac{9}{16} = \dfrac{LM}{15}$ (prop theorem)

LM = 8,44 units

QUESTION 11

(a) (1) BE = 2OA – ED (radii)

(2) AE = EC (Line from centre is perpendicular to chord)

$BE^2 = BC^2 - EC^2$

$(2OA - ED)^2 = BC^2 - AE^2$

(b) Construction DB

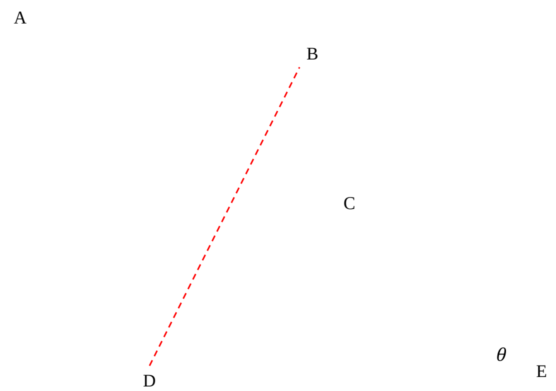
$D\hat{B}E = B\hat{D}E$ (Tangents drawn from common chord)

$$D\hat{B}E = B\hat{D}E = \frac{180^\circ - \theta}{2}$$

$\hat{A} = \frac{180^\circ - \theta}{2}$ (tan chord theorem)

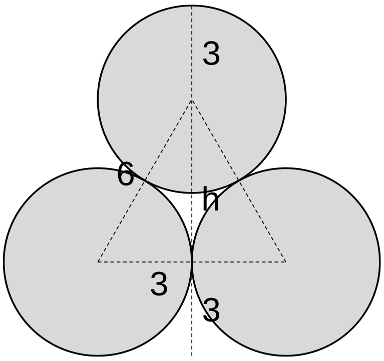
$\hat{C} = 180^\circ - \frac{180^\circ - \theta}{2}$ (Opp angles of cyclic quad)

$$\hat{C} = \frac{180^\circ + \theta}{2} = 90^\circ + \frac{\theta}{2}$$



QUESTION 12

(a) Front view of the circles



$h^2 = 6^2 - 3^2$ (Pythagoras)

$\therefore\ h = 3\sqrt{3}$

$\therefore$ Height of B

$3\sqrt{3} + 6$

(b) (1) $\sin 50° = \dfrac{(11,2)}{\text{AB}}$

$\text{AB} = \dfrac{(11,2)}{\sin 50°}$

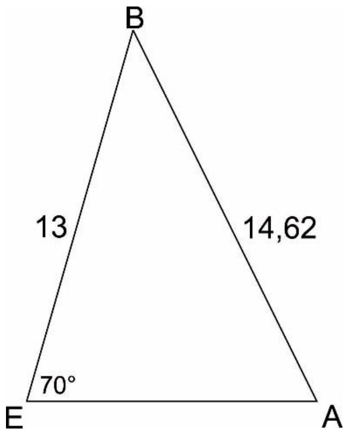
AB = 14,62 metres

(2) A view of the triangle made by the two pieces of rope and the horizontal plane

$\dfrac{\sin A}{13} = \dfrac{\sin 70°}{14,62}$

$\hat{\text{A}} = 56,68°$

$\hat{\text{B}} = 53,32°$



OPTION 1

$\dfrac{\text{EA}}{\sin 53,32°} = \dfrac{14,62}{\sin 70°}$

EA = 12,48 metres

OPTION 2

$\text{EA}^2 = 13^2 + 14,62^2 - 2(13)(14,62)\cos 53,32°$

EA = 12,48 metres

Total: 150 marks