

**GEVORDERDEPROGRAM-WISKUNDE: VRAESTEL II**

**NASIENRIGLYNE**

Tyd: 1 uur

100 punte

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Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulpeksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

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## MODULE 2 STATISTIEK

### VRAAG 1

1.1 (a)  $X \sim B\left(7; \frac{1}{7}\right)$

$$P(X > 1) = 1 - [P(X = 0) + P(X = 1)]$$

$$= 1 - \left[ \left(\frac{6}{7}\right)^7 + \binom{7}{1} \left(\frac{1}{7}\right) \left(\frac{6}{7}\right)^6 \right]$$

$$= 0,2635$$

(b)  $X \sim B\left(60; \frac{1}{7}\right)$

$np > 5$  en  $nq > 5$

$$X \sim N\left(8,57; \sqrt{7,35}^2\right)$$

$$P(X \geq 13) \rightarrow P(X > 12,5)$$

$$= P\left(Z > \frac{12,5 - 8,57}{\sqrt{7,35}}\right)$$

$$= P(Z > 1,45)$$

$$= 0,5 - 0,4265$$

$$= 0,0735$$

1.2 (a)  $\frac{\binom{5}{2} \binom{2}{1}}{\binom{7}{3}} = \frac{4}{7}$

(b)

$x$	1	2	3
$P(X = x)$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$

(c)  $E[X] = 1\left(\frac{1}{7}\right) + 2\left(\frac{4}{7}\right) + 3\left(\frac{2}{7}\right)$

$$= \frac{15}{7}$$

$$Var(X) = 1^2\left(\frac{1}{7}\right) + 2^2\left(\frac{4}{7}\right) + 3^2\left(\frac{2}{7}\right) - \left(\frac{15}{7}\right)^2$$

$$= \frac{20}{49} \quad (0,408)$$

**VRAAG 2**

2.1 (a)  $X \sim N(7,5 ; 0,75^2)$

$$\begin{aligned} P(X < 8) &= P\left(Z < \frac{8-7,5}{0,75}\right) \\ &= P(Z < 0,67) \\ &= 0,5 + 0,2486 \\ &= 0,7486 \end{aligned}$$

(b)  $P(Q_1 < Z < Q_3) = 0,5$   
 $\therefore P(-0,67 < z < 0,67) = 0,5$   
 $-0,67 = \frac{Q_1 - 7,5}{0,75} \quad \text{en} \quad 0,67 = \frac{Q_3 - 7,5}{0,75}$   
 $\therefore Q_1 = 6,998 \quad Q_3 = 8,003$

(c)  $200(0,7486) = 149,72$   
 $\therefore \approx 149$  volwassenes

2.2 (a)  $\bar{x} = \frac{5,99 + 8,01}{2} = 7$

(b)  $Z\left(\frac{3,5}{\sqrt{50}}\right) = 1,01$   
 $Z = 2,04$   
 $\therefore P(-2,04 < Z < 2,04) = 0,4793 \times 2$   
 $= 0,9586$   
 $\therefore 96\%$

### VRAAG 3

3.1 (a) C

(b) D

(c) C

(d) A

3.2 (a)  $H_0 : \mu = 22$   
 $H_1 : \mu > 22$

(b) Verwerp  $H_0$  indien  $Z > 1,48$

$$\therefore \frac{\bar{X} - 22}{\frac{5}{\sqrt{30}}} > 1,48$$

$$\bar{X} > 23,351$$

### VRAAG 4

(a)  $\int_0^k \frac{2}{k^2} x \, dx$

$$= \left[ \frac{2x^2}{2k^2} \right]_0^k$$

$$= \frac{k^2}{k^2} - 0$$

$$= 1$$

(b)  $\left[ \frac{x^2}{k^2} \right]_0^6 = \frac{1}{4}$

$$\frac{36}{k^2} = \frac{1}{4}$$

$$k^2 = 144$$

$$k = 12$$

### VRAAG 5

$$5.1 \quad (a) \quad \begin{aligned} x^2 + 9x - 2 &= 50 \\ x^2 + 9x - 52 &= 0 \\ (x+13)(x-4) &= 0 \\ x &\neq -13 \cup x = 4 \end{aligned}$$

$$(b) \quad P(A|B') = \frac{P(A \cap B')}{P(B')}$$

$$= \frac{24}{29}$$

$$(c) \quad P(A) = \frac{32}{50} = \frac{16}{25}$$

$$\therefore P(A|B') \neq P(A)$$

$\therefore$  A en B is nie onafhanklik nie  
OF

$$P(A \cap B) = \frac{4}{25} = 0,16$$

$$\begin{aligned} P(A) \times P(B) &= \frac{16}{25} \times \frac{21}{50} \\ &= 0,2688 \end{aligned}$$

$$P(A \cap B) \neq P(A) \times P(B)$$

$\therefore$  A en B is nie onafhanklik nie

$$5.2 \quad \binom{5}{3} + \binom{5}{2} + \binom{5}{4} + \binom{5}{3} = 35$$

(1E1N 3 ander + 2E1N 2 ander + 1E0N 4 ander + 2E0N 3 ander)

**Totaal vir Module 2: 100 punte**

## MODULE 3 FINANSIES EN MODELLERING

### VRAAG 1

$$1.1 \quad 1\,200\,000(0,85)^8 = R326\,988,63$$

$$1.2 \quad 1\,200\,000(1,055)^8 = R1\,841\,623,82$$

$$1.3 \quad r_{em} = \left(1 + \frac{0,1}{12}\right)^{12} - 1$$

$$= 0,1047$$

$$1.4 \quad 1\,514\,635,19 = \frac{x \left[ \left(1 + \frac{0,1}{12}\right)^{73} - 1 \right] \cdot \left(1 + \frac{0,1}{12}\right)^{24}}{\frac{0,1}{12}}$$

$$- \frac{10\,000 \left[ (1 + 0,1047)^8 - 1 \right]}{0,1047}$$

$$\therefore x = R13373,83$$

## VRAAG 2

2.1 Rente = betalings – vermindering in uitstaande saldo

$$47\,131,31 = 12x - 36\,868,69$$

$$\therefore x = R7\,000$$

2.2 Uitstaande saldo<sub>12</sub> = Lening<sub>12</sub> – Betalings

$$P - 36\,868,69 = P \left( 1 + \frac{0,0925}{12} \right)^{12} - 7\,000 \left[ \frac{\left( 1 + \frac{0,0975}{12} \right)^{12} - 1}{\frac{0,0975}{12}} \right]$$

$$\therefore P = R500\,000$$

$$2.3 \quad 500\,000 = \frac{7\,000 \left[ 1 - \left( 1 + \frac{0,0975}{12} \right)^{-n} \right]}{\frac{0,0975}{12}}$$

$$n = 107,307 \dots$$

$$\therefore 108 \text{ betalings} = 9 \text{ jaar}$$

$$\therefore \text{Finale betaling is 1 Januarie 2030}$$

$$\text{F.P.} = \left[ 500\,000 \left( 1 + \frac{0,0975}{12} \right)^{107} - \frac{7\,000 \left[ \left( 1 + \frac{0,0975}{12} \right)^{107} - 1 \right]}{\frac{0,0975}{12}} \right] \cdot \left( 1 + \frac{0,0975}{12} \right)$$

$$= R2\,159,18 \text{ (12)}$$

### VRAAG 3

$$\begin{aligned}
 3.1 \quad T_1 &= 4 + 5 = -1 = p \\
 T_2 &= 16 - 5(3) = 1 = q \\
 T_3 &= 64 - 5(9) = 19 \\
 19 &= 7(1) + (-1)a \\
 \therefore a &= -12
 \end{aligned}$$

$$3.2 \quad (a) \quad F_{n+1} = 0,8F_n + 1\,000 ; F_0 = 100\,000$$

$$\begin{aligned}
 (b) \quad F_{n+1} &= F_n \\
 \therefore F_n &= 0,8F_n + 1\,000 \\
 F_n &= 5\,000
 \end{aligned}$$

$$\begin{aligned}
 (c) \quad \text{Ewewig} &= 30\,000 \\
 0,2F_n &= 6\,000 \\
 \therefore F_{n+1} &= 0,8F_n + 6\,000 \\
 6\,000 \text{ visse moet elke jaar bygevoeg word}
 \end{aligned}$$



#### VRAAG 4

- 4.1 Prooi =  $100 \pm 5$   
Roofdier =  $8 \pm 1$
- 4.2 100 prooi en 50 roofdiere
- 4.3 Die roofdierpopulasie neem op sy vinnigste toe.
- 4.4 Geen effek op prooi nie.  
Toename in  $K \rightarrow$  toename in roofdier

#### VRAAG 5

- 5.1  $a + 1 = 2,6$   
 $\therefore a = 1,6$   
 $1,6 = 0,5 \times \frac{2}{3} \times 6 \times x$   
 $\therefore x = 0,8$  (oorlewingskoers)

$$b = 0,03$$

$$0,9722 = 1 - c$$

$$\therefore c = 0,0278$$

$$\therefore \text{leef tyd 36 siklusse}$$

$$\frac{a}{K} = 0,001778$$

$$\therefore K = 900$$

- 5.2  $F_{n+1} = F_n$   
 $0,0278F_n = 0,00005289R_nF_n$   
 $\therefore R_n = 526$

$$R_{n+1} = R_n$$

$$R_n = 2,6R_n - 0,001778R_n^2 - 0,03R_nF_n$$

$$\therefore 0,03F_n = 1,6 - 0,001778(526)$$

$$\therefore F_n = 22$$

**VRAAG 6**

$$\begin{aligned}
 6.1 \quad P(1+i)^{12} &= P \left(1 + \frac{0,12}{12}\right)^{36} \left(1 + \frac{0,12}{4}\right)^{16} \left(1 + \frac{0,12}{2}\right)^{10} \\
 \therefore i &= \left(1 + \frac{0,12}{12}\right)^3 \left(1 + \frac{0,12}{4}\right)^{\frac{4}{3}} \left(1 + \frac{0,12}{2}\right)^{\frac{5}{6}} - 1 \\
 &= 0,1250 \\
 &= 12,5\% \text{ p.j.}
 \end{aligned}$$

$$\begin{aligned}
 6.2 \quad \text{Toename} &= 1\% \text{ per dag} \\
 &= 365\% \text{ per jaar, daaglik saamgestel} \\
 r_{eNR} &= \left(1 + \frac{3,65}{365}\right)^{365} - 1 \text{ of } (1,01)^{365} - 1 = 36,783434 \text{ uit die groeikoers} \\
 &= 36,783 \dots \\
 &= 3\,678,34\% \text{ p.j.}
 \end{aligned}$$

**Totaal vir Module 3: 100 punte**

## MODULE 4 MATRIKSE EN GRAFIEKTEORIE

### VRAAG 1

1.1  $\tan \vartheta = \sqrt{3}, \vartheta = 60$

$$\begin{pmatrix} \cos 2(60) & \sin 2(60) \\ \sin 2(60) & -\cos 2(60) \end{pmatrix} \begin{pmatrix} 1 & 4 & -1 & 3 \\ 5 & 2 & -1 & -2 \end{pmatrix} \quad (\text{volgorde})$$

$$= \begin{pmatrix} 3.83 & -0.27 & -0.37 & -3.23 \\ 3.37 & 4.46 & -1.37 & 1.6 \end{pmatrix} \quad (\text{tekens, waardes})$$

1.2 (a)  $\begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}$  Teken, dwars matriks, volgorde  $\begin{pmatrix} 1 & 4 & -1 & 3 \\ 5 & 2 & -1 & -2 \end{pmatrix}$

$$\begin{pmatrix} 1 & 4 & -1 & 3 \\ 2 & -10 & 2 & -11 \end{pmatrix}$$

(b) Die oppervlakte van A = Oppervlakte A' faktor 1.

### VRAAG 2

2.1  $\text{Tr}(M) = \text{Tr}(N)$   
 $9 = x$

2.2  $\begin{pmatrix} 7-x & 3 & 1 \\ 0 & 2+x & 3 \\ 2 & 8 & -x \end{pmatrix}$

2.3  $(7-x)[(2+x)(-x)-24]+2[9-(2+x)]=0$  teken determinant = 0  
 $(7-x)(-x^2-2x-22)=0$   
 $x=7$  en niereële wortels

### VRAAG 3

3.1  $360/60 = 6^\circ$

3.2 (a)  $6 \times 12 = 72^\circ$ ,

volgorde matriks teken  $\begin{pmatrix} \cos(-72) & -\sin(-72) \\ \sin(-72) & \cos(-72) \end{pmatrix} \begin{pmatrix} 0 \\ 6 \end{pmatrix} = \begin{pmatrix} 5,71 \\ 1,85 \end{pmatrix}$

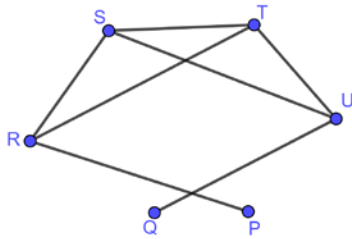
(b)  $360/12 \div 5 = 6^\circ$  – dit is vir 'n volle uur, ons wil 1/5 van 'n uur hê

$$\begin{pmatrix} \cos(-6) & -\sin(-6) \\ \sin(-6) & \cos(-6) \end{pmatrix} \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 0,42 \\ 3,98 \end{pmatrix}$$

### VRAAG 4

4.1 R; S; T; P

4.2



(korrek vanaf elke nodus)

4.3 Elke nodus moet 'n ewe graad hê.

4.4 Q R U P T Q S P Q (laaste regmerkie vir terugkeer na Q)

### VRAAG 5

5.1 Minimum spanboom

5.2 Kruskal

5.3 AB = 1  
AD = 4  
DE = 2  
EF = 2  
FM = 3  
MJ = 1  
MH = 2  
HG = 1

AC = 5  
JK of MK = 6  
KL = 3  
GI = 8 of HI = 8 totaal: 38

5.4 GI /HI JK/MK (kandidaat identifiseer uniekheid spruit uit unieke skakels)

### VRAAG 6

$$\begin{aligned}
 6.1 \quad & EH - 1 \\
 & CE - 2 \\
 & GH - 4 \\
 & BD - 4 \\
 & CD - 4 \\
 & AB - 8 \\
 & + FG - 1 \\
 & + CF - 2 \\
 & = 26
 \end{aligned}$$

$$6.2 \quad A-F-G-H-E-C-D-B = 22 + 8(AB) = 30 \quad (\text{terugkeerpad na A})$$

$$6.3 \quad \text{Enige pad tussen } 26 \leq x \leq 30 \quad (\text{begin en eindig by A, gewig is kleiner as of } = 30, \text{ groter as of } = 26)$$

### VRAAG 7

$$7.1 \quad P^{-1} = \left[ \frac{1}{-11} \begin{pmatrix} -1 & -1 \\ -1 & 10 \end{pmatrix} \right] \quad (1/\det) \quad (\text{matriks})$$

$$\begin{aligned}
 7.2 \quad & A = PDP^{-1} \\
 & = -\frac{1}{11} \begin{pmatrix} 10 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 8 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -1 & 10 \end{pmatrix} \quad \text{Volgorde} \\
 & = -\frac{1}{11} \begin{bmatrix} -77 & -110 \\ -11 & 22 \end{bmatrix} \quad \text{Vermenigvuldiging} \\
 & = A \therefore LK = RK \quad \text{Vereenvoudiging}
 \end{aligned}$$

$$\begin{aligned}
 7.3 \quad & A^5 = A \cdot A \cdot A \cdot A \cdot A \\
 & = (PDP^{-1})(PDP^{-1})(PDP^{-1})(PDP^{-1})(PDP^{-1}) \\
 & = PD^5P^{-1} \quad (\text{identifiseer die patroon}) \\
 & = \frac{-1}{11} \begin{pmatrix} 10 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 8^5 & 0 \\ 0 & -3^5 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -1 & 10 \end{pmatrix} \\
 & = \frac{-1}{11} \begin{pmatrix} 327680 & -243 \\ 32768 & 243 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -1 & 10 \end{pmatrix} \quad \text{of} \quad \frac{-1}{11} \begin{pmatrix} 10 \cdot 8^5 & -3^5 \\ 8^5 & 3^5 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -1 & 10 \end{pmatrix} \\
 & = \begin{pmatrix} 29767 & 30010 \\ 3001 & 2758 \end{pmatrix}
 \end{aligned}$$

**Totaal vir Module 4: 100 punte**