TECHNICAL MATHEMATICS: PAPER I

EXAMINATION NUMBER

Time: 3 hours   150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 23 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.

2. Read the questions carefully.

3. **Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number in the space provided.**

4. Diagrams are not necessarily drawn to scale.

5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6. Round off your answers to <u>one decimal digit</u> where necessary, unless otherwise stated.

7. All the necessary working details must be clearly shown.

8. It is in your own interest to write legibly and to present your work neatly.

9. Two blank pages (pages 22 and 23) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	TOTAL
15	16	16	16	15	18	21	14	5	14	150

QUESTION 1

1.1   Solve for x, leaving answers in simplified surd form where necessary:

1.1.1 $2x^2 - x - 6 = 0$

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(3)

1.1.2 $x^2 - 1 = x$

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(4)

1.1.3 $4x^2 - 4x + 1 \leq 0$

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(3)

1.2   Given the equation $3x^2 + 2x + 1 = 0$

Discuss, without solving the equation, the nature of the roots of the equation.

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(3)

1.3   It takes 0,0000333564095 seconds to travel 10 kilometres in a vacuum. Convert this number into scientific notation. No rounding must be done.

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(2)
[15]

QUESTION 2

2.1   Simplify the following without using a calculator:

2.1.1 $\sqrt{9x^4 + 16x^4}$

(2)

2.1.2 $\left(\dfrac{x^{-\frac{1}{3}}}{\sqrt[3]{x^2}}\right)^{-2}$

(3)

2.2   Solve for x:

$\sqrt{5x - 1} - 1 = x$

(5)

2.3   Simplify $\dfrac{2^{2x+3} - 3 \cdot 2^{2x+1}}{2^{x-1}}$

(3)

2.4   If $6 = 3^x$, determine the value of x, correct to one decimal place.

(3)
[16]

QUESTION 3

3.1 $w = a + bi$ and $w^2 = 5 + 2i$.

    3.1.1   Given the complex number $w = a + bi$, determine w^2 in terms of a and b.

        (2)

    3.1.2   Hence, determine all the possible real values of a and b.

        (7)

3.2   Write $z = -2 - 3i$ in polar form and illustrate on an Argand diagram.

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                                                                                          (5)

3.3   Evaluate: $\dfrac{111_2}{35}$

       Show all your calculations.

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                                                                                          (2)
                                                                                        [16]

QUESTION 4

4.1 After just 2 years, an iPhone loses $\dfrac{2}{3}$ of its original value. Assuming that depreciation is calculated using the reducing balance method, determine the annual rate of depreciation.

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(4)

4.2 Bernard runs a small car wash business. He saves to buy equipment by investing in an account paying 7,5% interest per annum compounded quarterly.
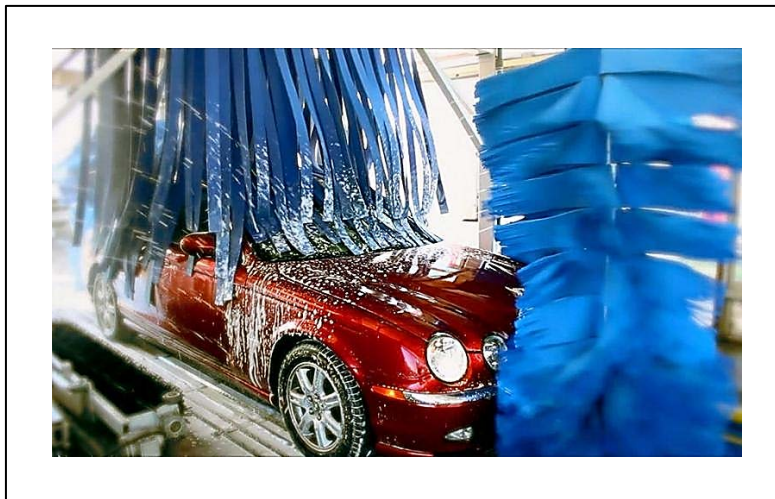
4.2.1 Determine the effective interest rate that would give the same final amount in his account equivalent to the 7,5% per annum compounded quarterly.

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(3)

4.2.2 Bernard deposits R10 000 immediately into the account. At the end of the first year, the interest rate changes to 7,8% per annum compounded monthly. Six months after he made the deposit, he won R5 000 on Powerball and added his winnings to the total amount to his investment. Determine the final amount that he would have available in the account at the end of the third year.

(5)

4.3 Zelifa invests R20 000 at an interest rate of 4% per annum compounded annually. Determine after how many completed years will her investment be worth at least R25 000.
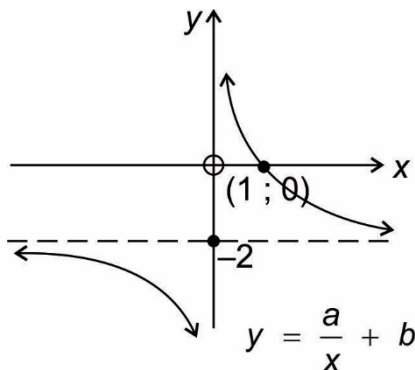
(4)

[16]

QUESTION 5
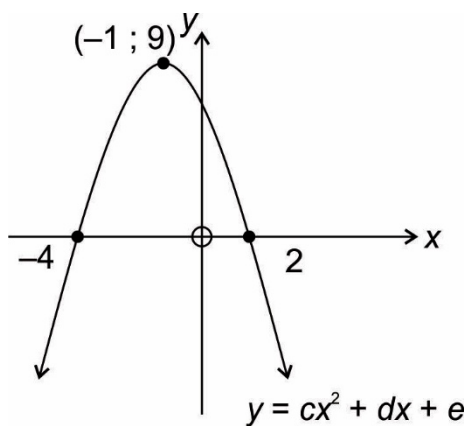
The following four graphs are given with the points on each curve, and the equation underneath each curve as shown.

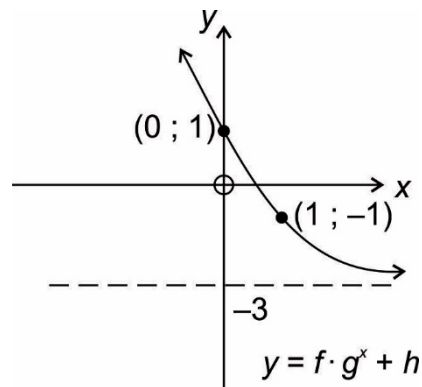5.1   Determine the numerical values of *a* and *b*, showing all calculations where necessary.

$$y = \frac{a}{x} + b$$

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(3)

5.2   The point (–1 ; 9) is the turning point of the graph. Determine the numerical values of *c, d* and *e*, showing all calculations where necessary.
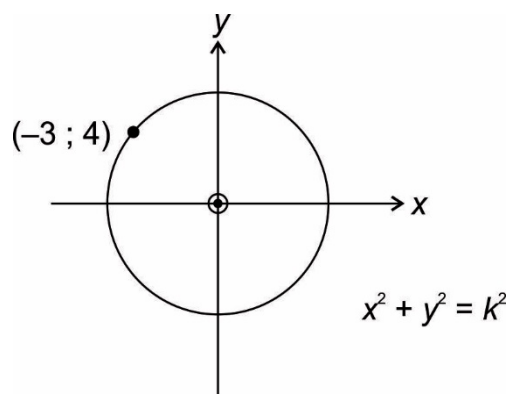

$$y = cx^2 + dx + e$$

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(5)

5.3   Determine the numerical values of *f, g* and *h*, showing all calculations where necessary.

$$y = f \cdot g^x + h$$

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(5)

5.4   Determine the numerical value of *k,* showing all calculations where necessary.
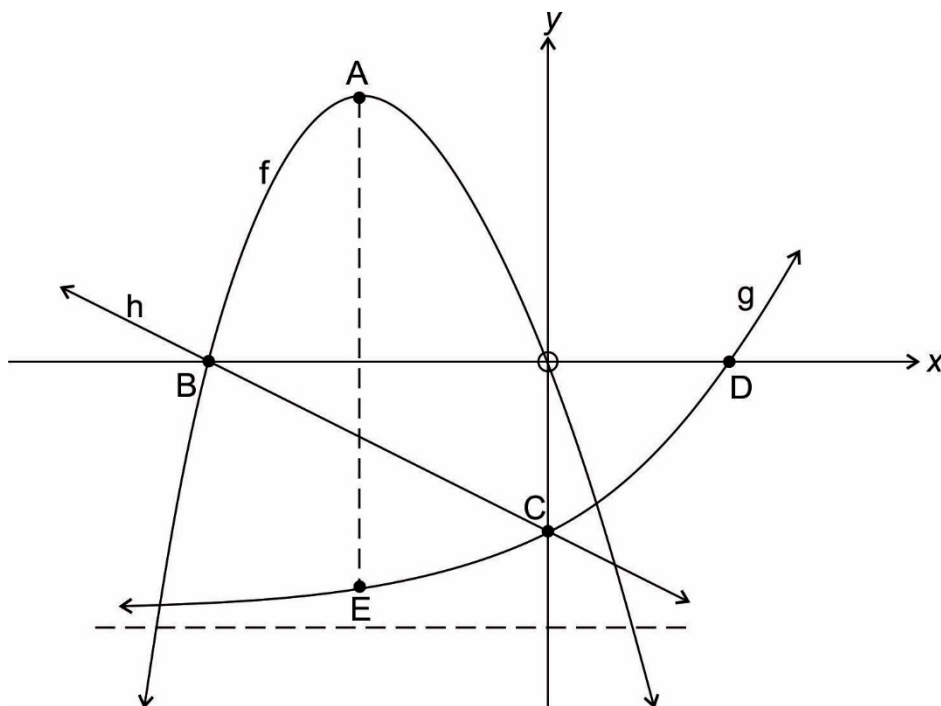

$$x^2 + y^2 = k^2$$

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(2)
[15]

QUESTION 6

The figure below shows functions f and g defined by

$f(x) = -x^2 - 4x$ and $g(x) = 2^x - 8$

A is the turning point of f with points B and D the x-intercepts of f and g respectively. Point C is the y–intercept of g.
E is a point on g such that AE is parallel to the y-axis.

6.1   Determine the co-ordinates of A, B, C and D.

(7)

6.2   Hence, determine the lengths of BD and AE.

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                                                                                        (4)

6.3   Give the range of *f*.

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                                                                                        (1)

6.4   If *h* represents the straight line passing through B and C, determine the defining equation *h*(*x*).

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                                                                                        (2)

6.5   Determine the values of *x* for which $f(x) \cdot h(x) < 0$ and $x < 0$.

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                                                                                        (2)

6.6   Explain (in words) how the graph of *g* should be transformed so that $f(x) = g(x) + k$ has two different real roots that are both negative. (You need not give specific values for *k*).

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                                                                                        (2)
                                                                                       [18]

QUESTION 7

7.1   If $g(x) = \dfrac{x}{3} - 2$, determine $g'(x)$ using first principles.

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(4)

7.2   Determine $\dfrac{dy}{dx}$ if $y = \left(\dfrac{2}{x} - \sqrt{x}\right)^2 - x.$

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(6)

7.3   An open rectangular sensor box used to measure the temperature in a fish tank is shown in the diagram below. It has a square base of side x metres, a height of y metres and a volume of 300 cm³. The cost of material used to make the base is R5 per square metre, and to make the sides is R2 per square metre.

Note: On two sides there are circular openings. These are to be ignored when calculating the surface area.
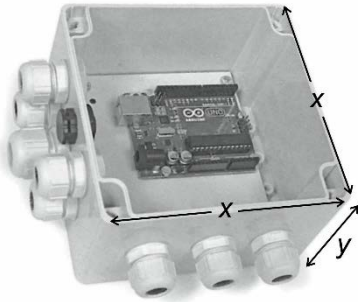
Volume = area of base × height

Surface Area = area of base + circumference of base × height

7.3.1   Express y in terms of x.

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(2)

7.3.2   Express C, the total cost (in rand) of material for the box, in terms of x.

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(4)

7.3.3   Hence, determine the minimum cost of the material for the box.

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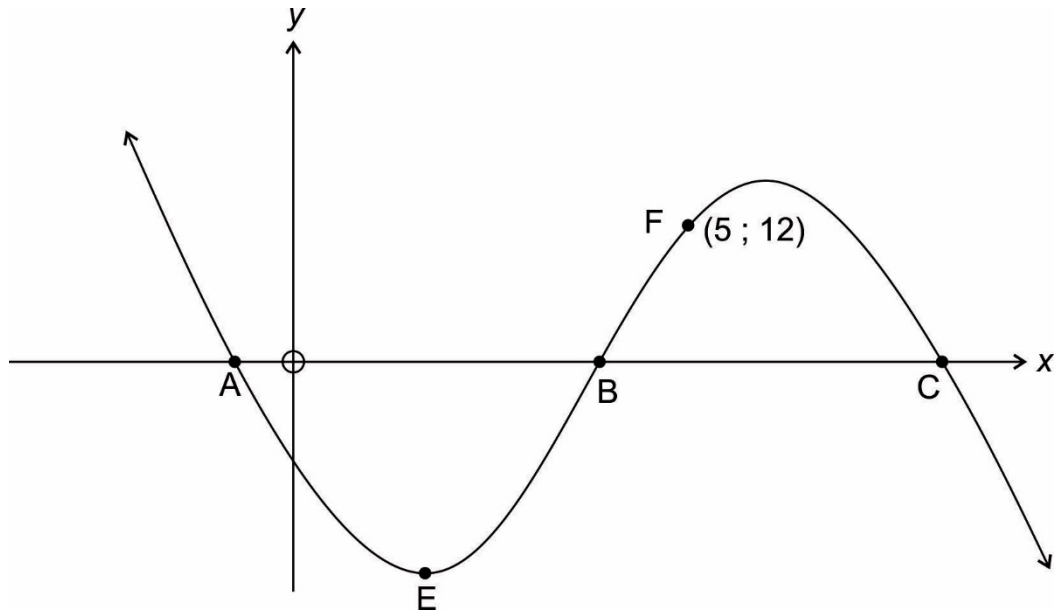
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(5)
[21]

QUESTION 8

The sketch below represents the curve of f defined by

$f(x) = -x^3 + 10x^2 - 17x - 28$

Points A, B and C are the x-intercepts of the curve, with E and F(5 ; 12) two points on the curve.

8.1   Determine the co-ordinates of E, the local minimum point of f.

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                                                                                                    (5)

8.2   For which values of x is f an increasing function?

(2)

8.3   F (5; 12) is a point on f as shown. Show that the equation of the tangent to f at this point is $y = 8x - 28$

(3)

8.4   This tangent at point F intersects curve f at the point G. Find, without using a calculator, the co-ordinates of G.

(4)
[14]

QUESTION 9

The picture above shows water being pumped into a reservoir using a number of pumps. The depth (in meters) of water H in the reservoir t hours after the pumps are opened is given by $H = 15 - 3t^2 - \dfrac{2}{3}t^3$ metres.

Determine how long it would take for the depth of water in the reservoir to be decreasing at a rate of 0,5 m/hour.

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[5]

QUESTION 10

10.1   Find the following integrals:

(a) $\int d\theta$

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(2)

(b) $\int \left(\dfrac{8}{x} - \dfrac{5}{x^2} + 6x^3 \right) dx$

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(4)

10.2   Given $\int_0^5 g(x)\, dx = -3$, determine the value of $\int_{-5}^5 g(x)\, dx$ if $g(x) = g(-x)$

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(2)

10.3 Find the area of the region bounded by the following curves defined by:
$y = x^2 - 4$, $y = 0$ and $x = 4$ as shown in the diagram below.
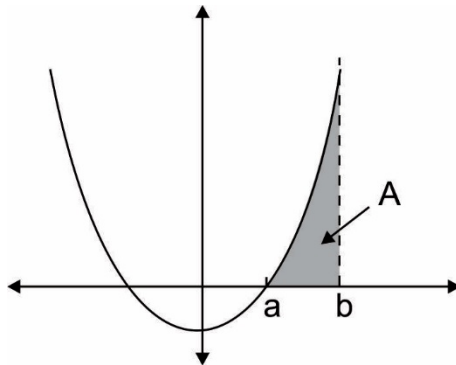
(6)
[14]

Total: 150 marks

ADDITIONAL SPACE (ALL questions)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.