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TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2020

TECHNICAL MATHEMATICS: PAPER II

EXAMINATION NUMBER

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

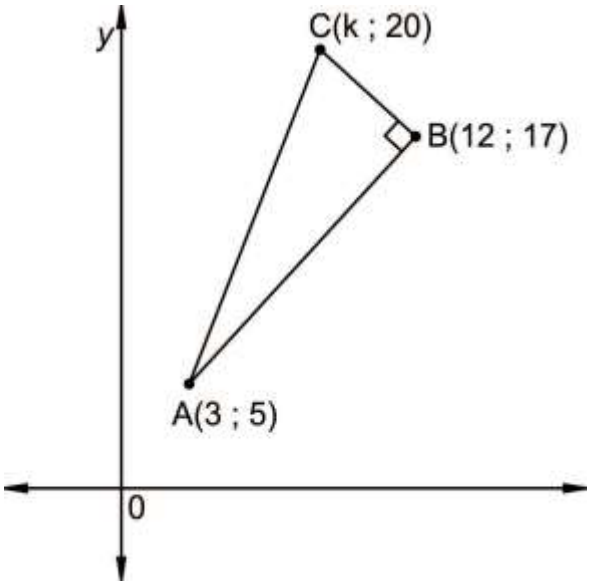
1. This question paper consists of 25 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number in the space provided.**
4. Number your answers exactly as the questions are numbered.
5. Diagrams are not necessarily drawn to scale.
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. Round off your answers to one decimal digit where necessary, unless otherwise stated.
8. All the necessary working details must be clearly shown.
9. It is in your own interest to write legibly and to present your work neatly.
10. One blank page (page 25) is included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
12	**17**	**30**	**15**	**6**	**34**	**21**	**15**	**150**

QUESTION 1

The diagram below, NOT drawn to scale, represents a right-angled triangle. ΔABC. The triangle has vertices A(3 ; 5) , B(12 ; 17) and C(k ; 20).
$CB \perp BA$.



1.1 Calculate the gradient of AB.

(2)

1.2 Determine the numerical value(s) of k.

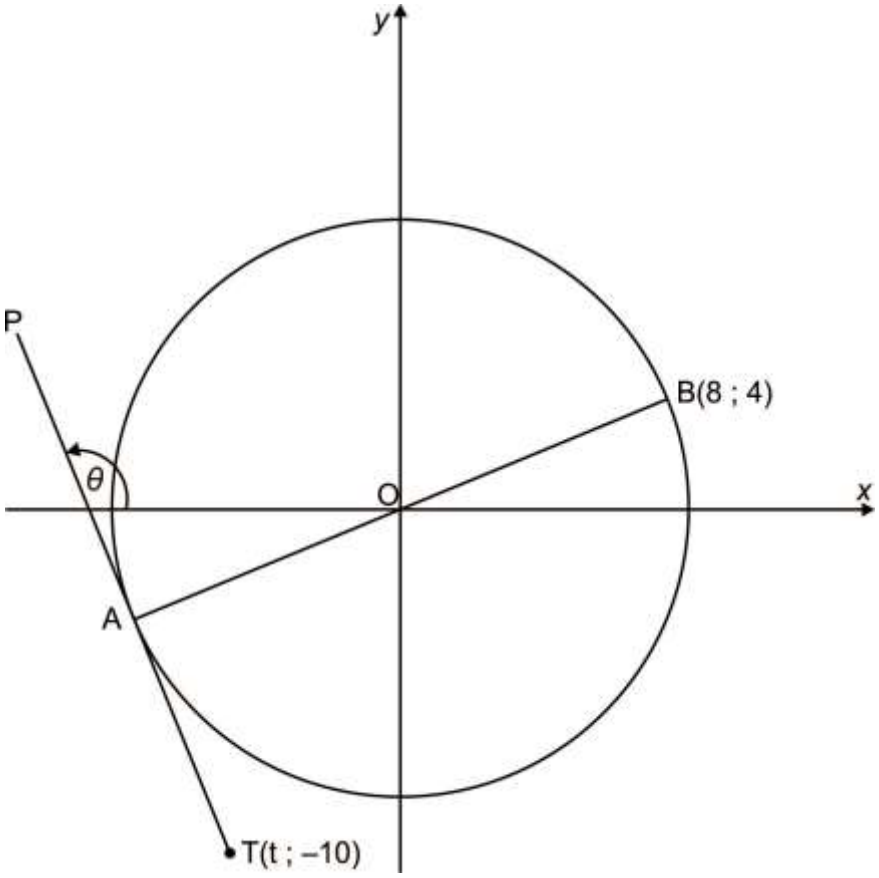
(4)

1.3 If BC = 5 units, calculate the perimeter of ΔABC in simplified surd form.

(6)
[12]

QUESTION 2

2.1 O is the centre of the circle in the diagram below.
B(8 ; 4) is a point on the circle.
AOB is a straight line.
Point T (t ; –10) lies on the tangent to the circle at A.
The inclination angle of line PT is indicated as θ.



2.1.1 Determine the equation of the circle.

(2)

2.1.2 Write down the coordinates of A.

(2)

2.1.3 Determine the equation of the straight line passing through A and B.

(2)

2.1.4 Hence, determine the inclination angle θ.

(3)

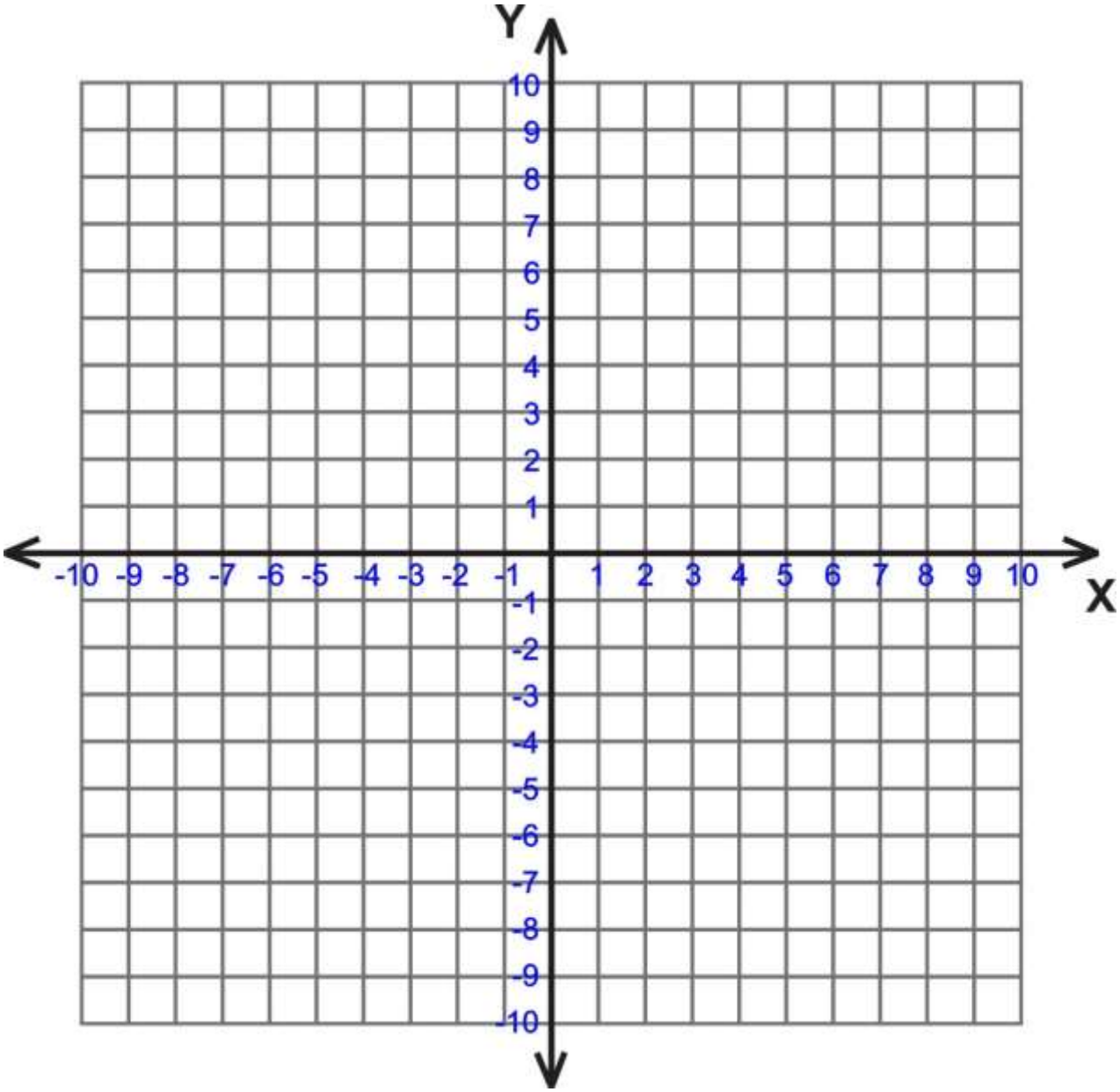
2.1.5 Determine the equation of tangent PT.

(3)

2.1.6 Calculate the numerical value of t.

(2)

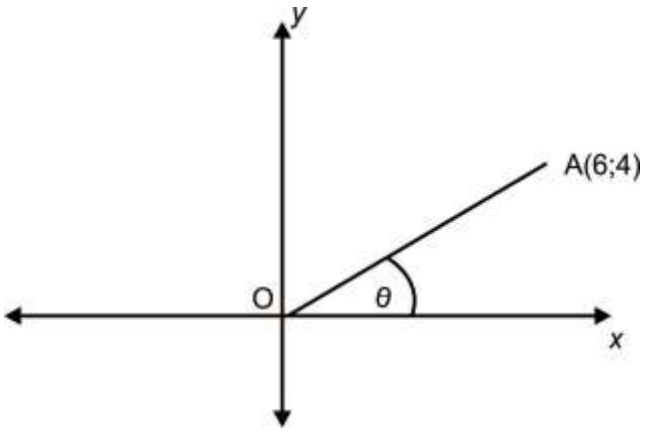
2.2 Sketch, on the system of axes below, the graph defined by $\frac{x^2}{16}+\frac{y^2}{10}=1$
Clearly show ALL the intercepts with the axes.



(3)
[17]

QUESTION 3

3.1 In the diagram below, A(6 ; 4) is a point on the Cartesian plane with origin O(0 ; 0). The acute angle that is formed by OA with the positive *x*-axis, is θ.



Determine, **without the use of a calculator**, the value of each of the following:

3.1.1 $6\sin\theta - 3\tan\theta$

(4)

3.1.2 $\text{cosec}^2\theta$

(2)

3.2 If $\theta = \frac{\pi}{3}$ and $\beta = \frac{\pi}{6}$ determine the value of the following:

3.2.1 $\text{cosec}(\theta - \beta)$

(2)

3.2.2 $2\cot\beta$

(2)

3.3 If $a = 124{,}66°$ and $b = 57{,}46°$

3.3.1 Convert $\boldsymbol{a}$ and $\boldsymbol{b}$ to radians (TWO decimal places).

(2)

3.3.2 Use the values obtained in Question 3.3.1. to calculate: $\sec(a+b)$

(1)

3.4 Simplify the following expression completely:

$$\tan(180°-\alpha).\cos\alpha.\sin(180°+\alpha) + \cos^2(360°+\alpha)$$

(6)

3.5 Without using a calculator, solve for $x \in [0° ; 180°]$ when $\tan(x - 15°) = -1$

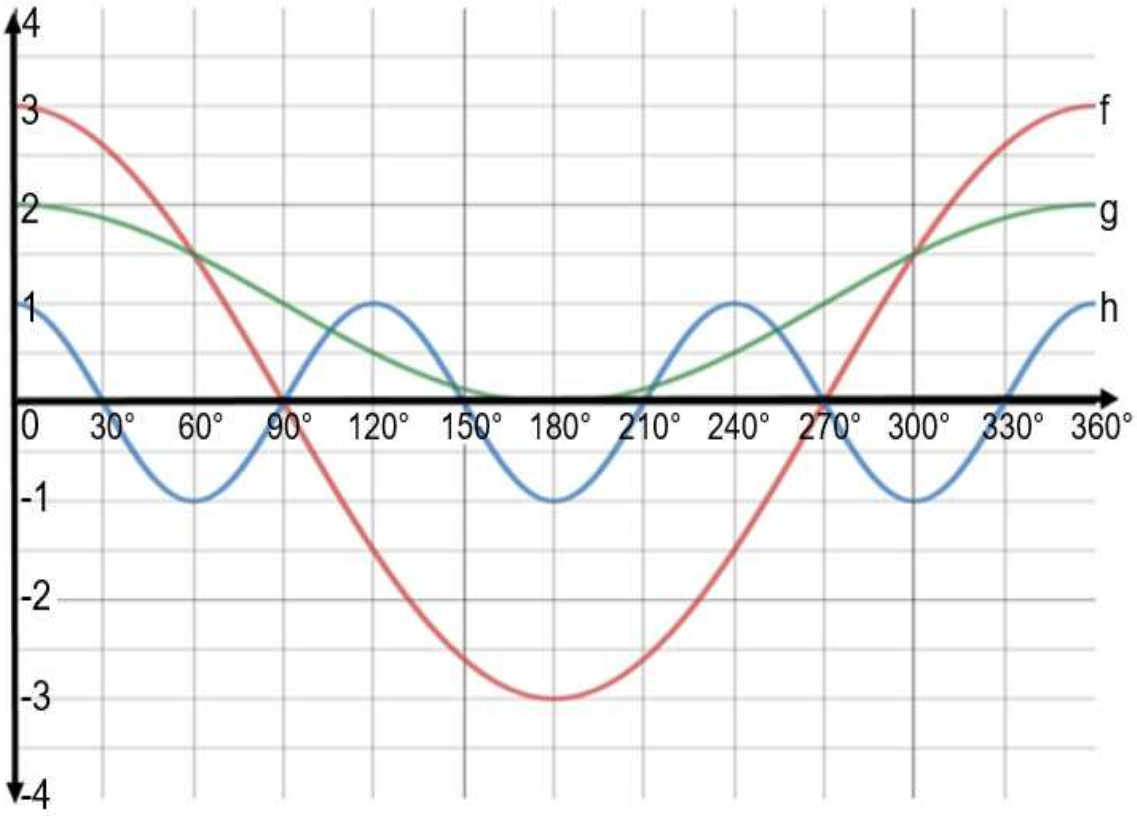
(3)

3.6 Prove the following identity:

$$\sin^2 A + \tan^2 A + \cos^2 A = \sec^2 A$$

(2)

3.7 Three cosine graphs, **f**, **g** and **h**, are represented below:



3.7.1 Write down the values of a, b, c and d if f(x) = a cos bx + c and g(x) = cos x + d

(4)

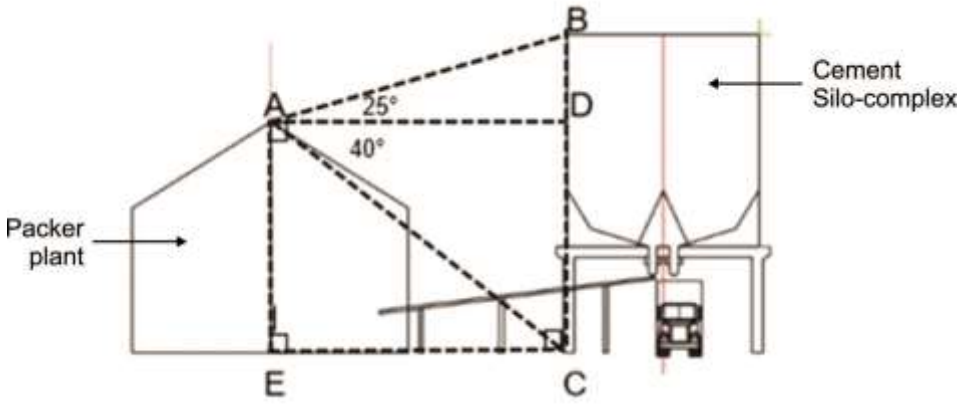
3.7.2 Determine the period of h if h(x) = cos 3x

(2)

[30]

QUESTION 4

4.1 Consider the 6 000 Ton Cement Silo-complex with Packer plant represented below. AE represents the vertical height of the Packer plant and BC the vertical height of the Cement Silo-complex.



From point A at the top of the Packer plant, the angle of elevation of the top of the silo-complex, BC, is 25° and the angle of depression to the base of the silo-complex is 40°. A, B, D, C and E lie in the same vertical plane.

If the height of the silo-complex BC is known to be 30 m, determine the following:

4.1.1 The distance between point A and point B.

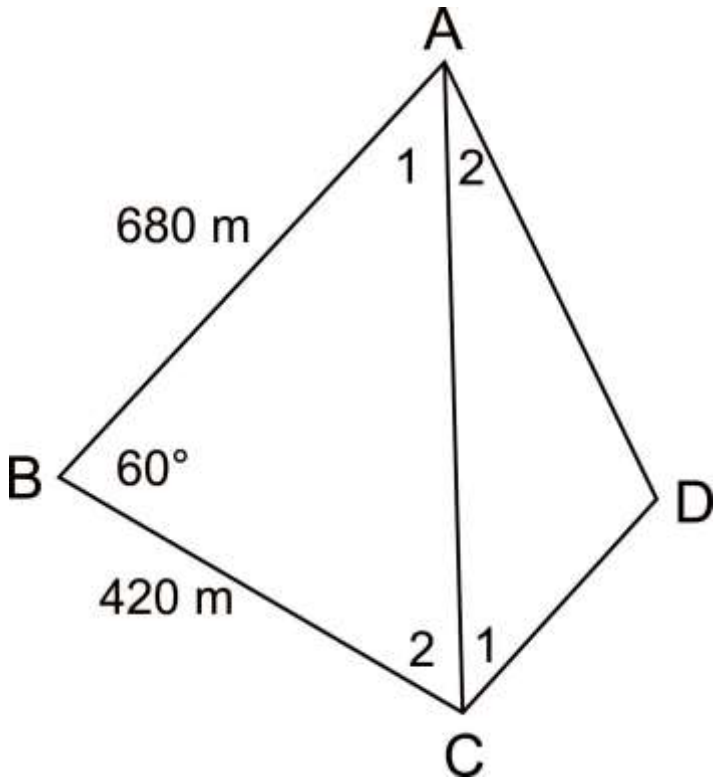
(4)

4.1.2 The distance AD.

(3)

4.2 A plot in the industrial area, in the shape of a cyclic quadrilateral ABCD is given below.

The measurements are given as: AB = 680 m, BC = 420 m, $\hat{C}_1 = 41°$, $\hat{B} = 60°$



Determine:

4.2.1 The length of side AC rounded to the nearest metre.

(4)

4.2.2 The size of angle D.

(1)

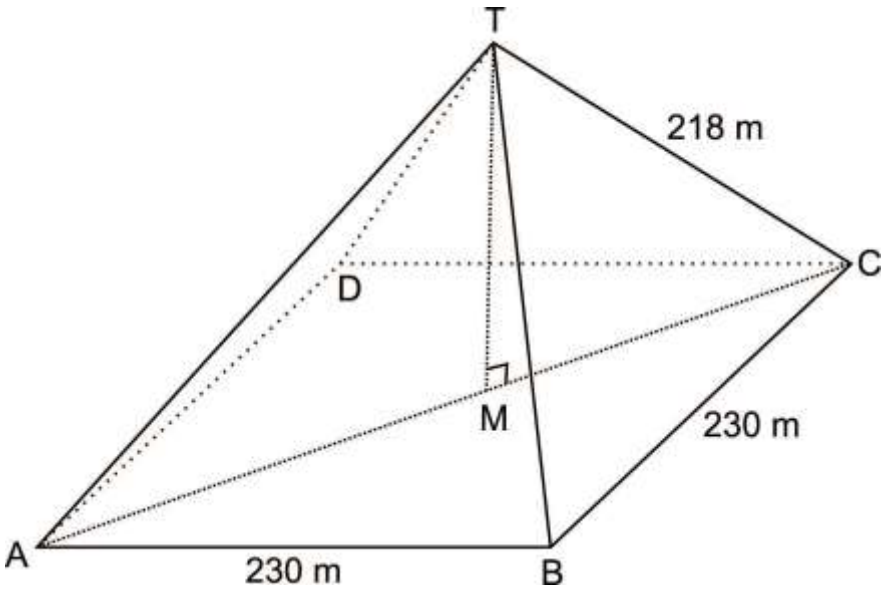
4.2.3 The length of side AD rounded to the nearest metre.

(3)

[15]

QUESTION 5

A right pyramid has a square base ABCD with sides of length 230 metres.
M is the midpoint of AC.
Vertex T is directly above M.
The slant sides of the pyramid have lengths of 218 m each.



5.1 Calculate the length of MC

(3)

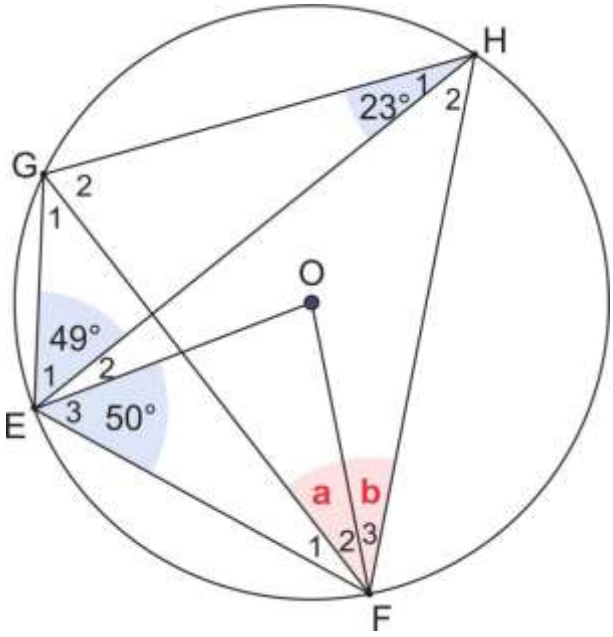
5.2 Calculate the slant angle, $M\hat{C}T$ of the pyramid

(3)
[6]

QUESTION 6

6.1 O is the centre of circle EFHG and radii EO and OF.

$\hat{H}_1 = 23°,\ \hat{E}_1 = 49°,\ \hat{E}_3 = 50°\quad \hat{F}_2 = a\quad \hat{F}_3 = b$



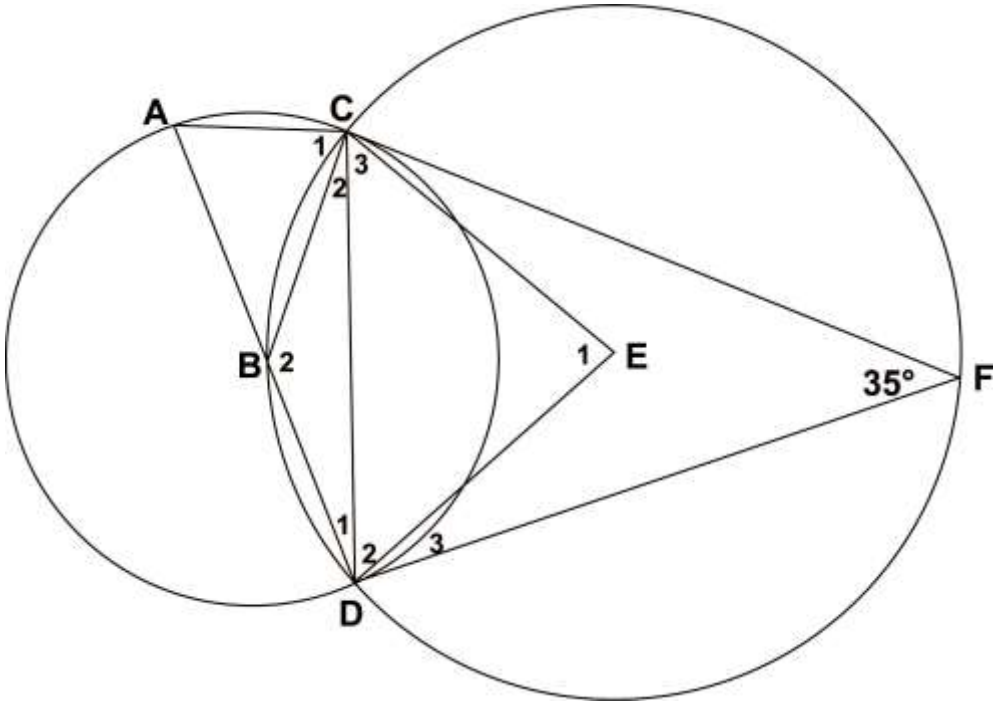
6.1.1 Calculate the size of **a**.

(4)

6.1.2 Calculate the size of **b**.

(2)

6.2 In the diagram below, two circles ACD and CFDB intersect at C and D.
B is the centre of circle ACD
E is the centre of circle CFDB
ABD is a straight line
$\hat{F} = 35°$



Giving reasons:

6.2.1 calculate the numerical value of $\hat{B}_2$.

(2)

6.2.2 calculate the numerical value of $\hat{E}_1$.

(2)

6.2.3 calculate the numerical value of $\hat{C}_3$.

(3)

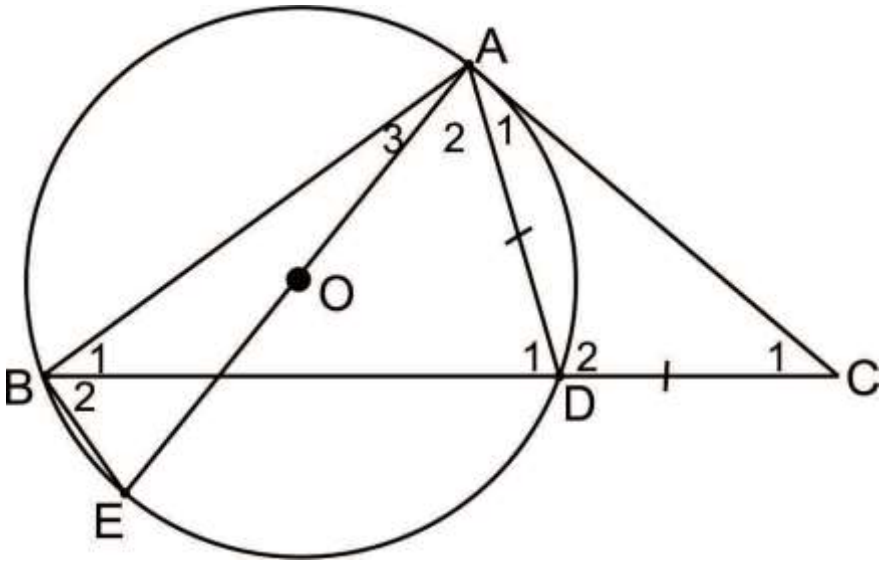
6.2.4 calculate the numerical value of $\hat{A}$.

(2)

6.2.5 calculate the numerical value of $\hat{C}_2$.

(3)

6.3 O is the centre of circle ABED.
The radius of the circle is 6,5 cm. AC is a tangent to the circle at A.
AE is a diameter.
BD is produced to C such that AD = DC. $\hat{C}_1 = 37{,}2°$



6.3.1 Write down the value of $\hat{A}_1$. Give a reason for your answer.

(2)

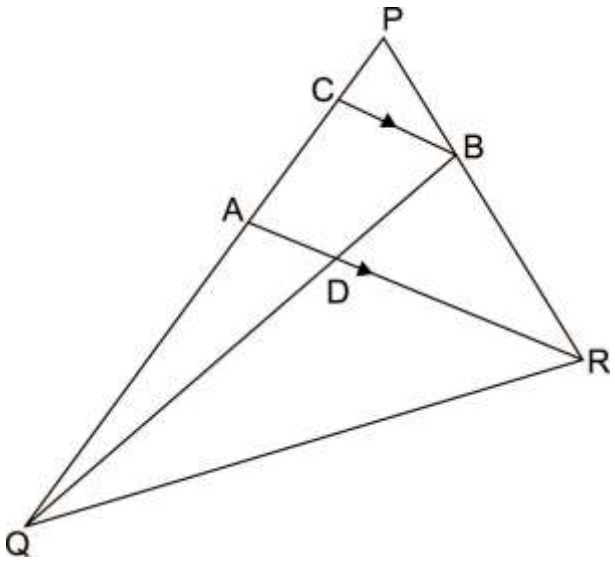
6.3.2 Calculate the size of $\hat{B}_2$.

(4)

6.3.3 If BE = 3,5 cm, calculate the length of AB.

(3)

6.4 In ΔPQR below, B lies on PR such that 2PB = BR
A lies on PQ such that PA : PQ = 3 : 8
AR and QB intersect at D
BC is drawn parallel to AR



6.4.1 Calculate the numerical value of $\frac{BD}{BQ}$. Show all working.

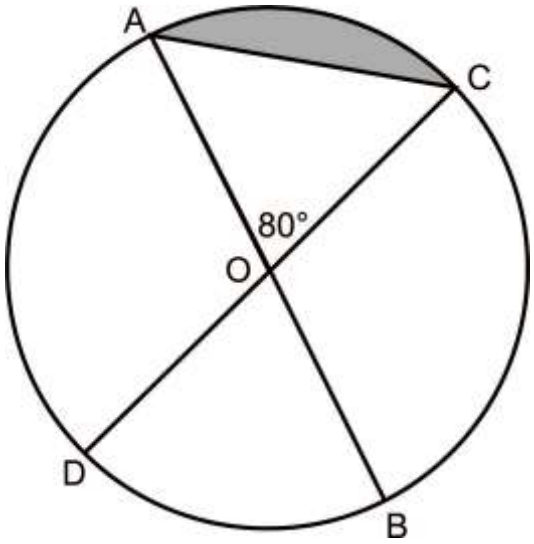
(5)

6.4.2 Determine the numerical value of $\frac{\text{Area of } \Delta PRA}{\text{Area of } \Delta QRA}$

(2)
[34]

QUESTION 7

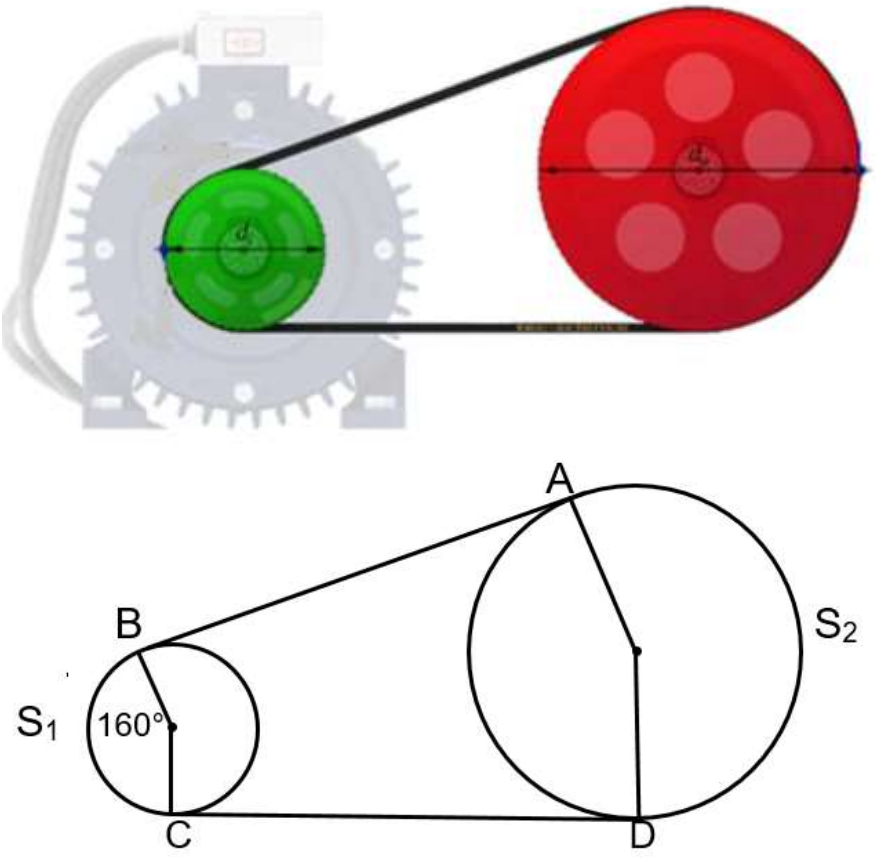
7.1 Circle ACBD with centre O is given with angle $A\hat{O}C = 80°$.
Diameters AB = CD = 18 cm and intersect at point O.



Calculate the size of the shaded area of the circle.

(6)

7.2 Two circular pulleys are part of a machine as shown in the picture below.
A, B, C and D are points of contact of the tight driving belt with the pulleys.
S_1 is the arc length of the small pulley and S_2 is the arc length of the larger pulley.
The centres of the pulleys are 52,5 cm apart.
The diameter of the small pulley is 15cm and the diameter of the large pulley is 30 cm.
The small pulley rotates at 4,2 revolutions per second.



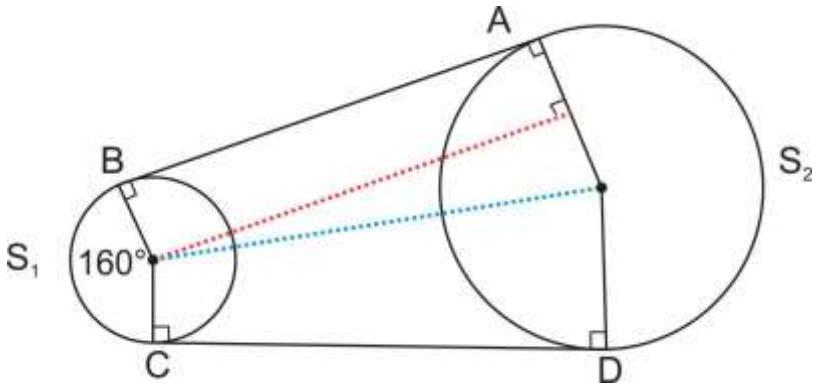
7.2.1 Calculate the circumferential velocity of the small pulley in meters per second.

(4)

7.2.2 Calculate the angular velocity of the smaller pulley in radians per second.

(3)

7.2.3



(a) Calculate S_1 the arc length of the small pulley.

(2)

(b) Calculate S_2 the arc length of the larger pulley.

(2)

(c) Calculate the total length of the driving belt.

(4)

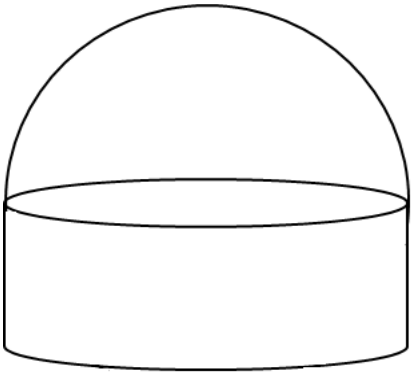
[21]

QUESTION 8

8.1 The tanker in the photo, is equipped with **4** identical tanks to carry fuel.

Each tank is made up of a right cylinder base and a hemisphere top.

The measurements and simplified sketch of one of the tanks are given below.



Hemisphere radius = 20 m

Cylinder radius = 20 m
Cylinder height = 8 m

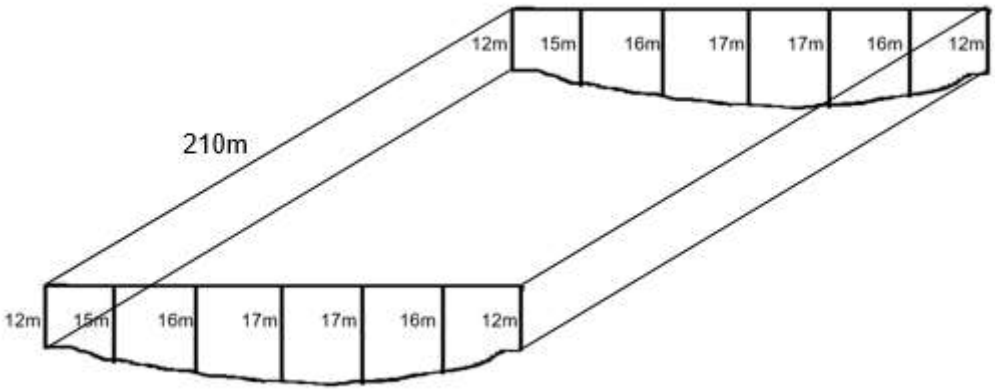
The following formulas may be used:

$\text{Volume} = \frac{4}{3}\pi r^3$	$\text{Volume} = \pi r^2 \times h$

Calculate the **total volume** of fuel that the tanker can carry in these tanks.

(6)

8.2 The following is a cross section of a lock chamber part of the Panama Canal. The indicated section is 210 m long and 120 m wide as represented in the diagram (NOT drawn to scale). The depth at 20 m intervals are given.



8.2.1 Calculate the area of the cross section, using the mid-ordinate rule.

(4)

8.2.2 Hence, calculate the volume (m^3) of the water in this 210 m section of the canal.

(2)

8.2.3 How much water (litres) should be pumped out of the 210 m section of the lock chamber, if the water level must be dropped by 1 m?

(3)
[15]

Total: 150 marks

ADDITIONAL SPACE TO ANSWER QUESTIONS. REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE ALL ANSWERS ARE MARKED.