Please paste the barcoded label here

TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2021

TECHNICAL MATHEMATICS: PAPER II

EXAMINATION NUMBER

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number in the space provided.**
4. Number your answers exactly as the questions are numbered.
5. Diagrams are not necessarily drawn to scale.
6. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
7. Round off your answers to <u>two decimal digits</u> where necessary, unless otherwise stated.
8. All the necessary working details must be clearly shown.
9. It is in your own interest to write legibly and to present your work neatly.
10. Two blank pages (pages 23 and 24) are included at the end of the paper. If you run out of space for a question, use this page. Clearly indicate the question number of your answer should you use this extra space.

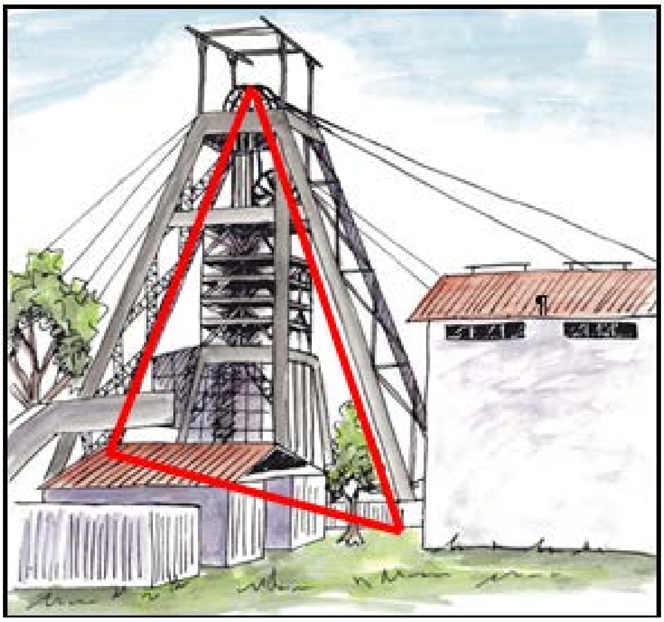
FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

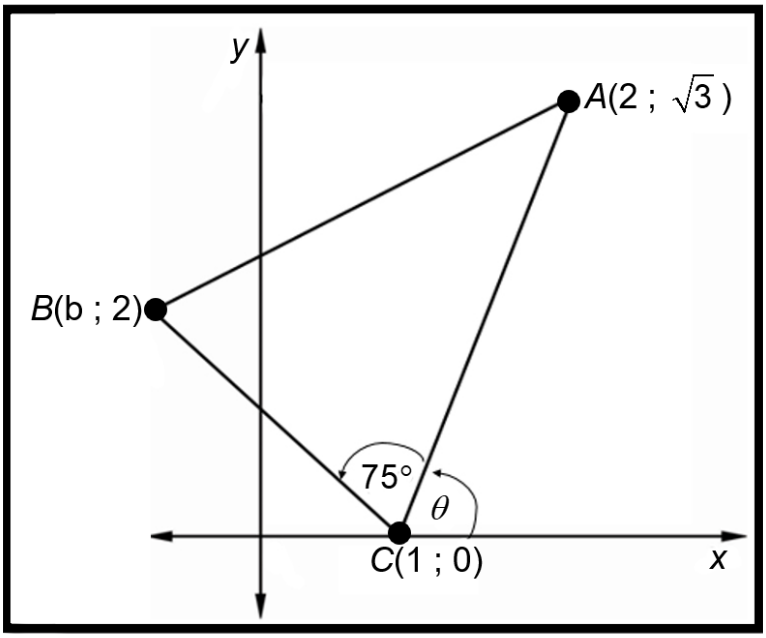
Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	TOTAL
13	**16**	**29**	**10**	**9**	**40**	**9**	**18**	**6**	**150**

QUESTION 1

The picture below shows a mine shaft headgear. The diagram below the picture represents a part of the headgear in the Cartesian plane with origin *O*. Triangle ΔABC with vertices $A(2\ ;\ \sqrt{3})$, $B(b\ ;\ 2)$ and $C(1\ ;\ 0)$ is shown in the diagram.

The acute angle θ is formed by the *x*-axis and line *AC*. Angle $A\hat{C}B = 75°$.





1.1 Calculate:

1.1.1 The gradient of line *AC*.

(2)

1.1.2 The size of θ.

(2)

1.1.3 The gradient of line *BC*.

(2)

1.2 Show that the numerical value of *b* is equal to –1.

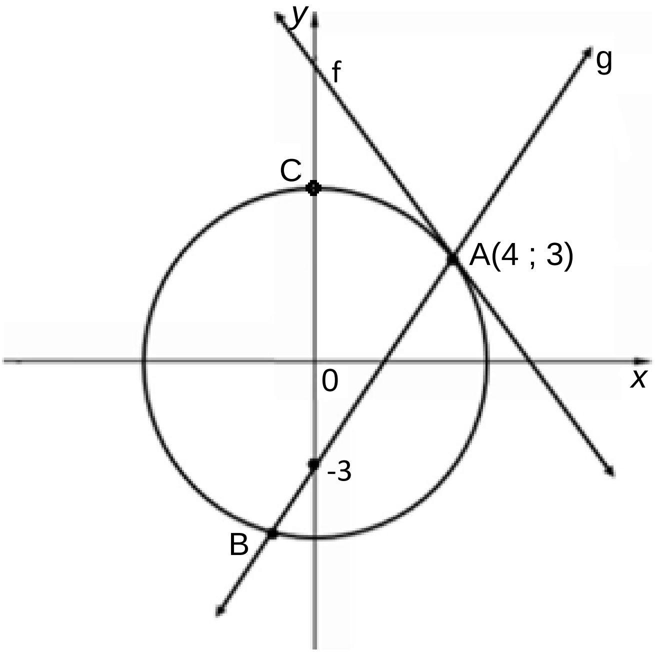
(2)

1.3 Determine the equation of the perpendicular bisector of *BC.*

(5)
[13]

QUESTION 2

2.1 O is the centre of the circle in the diagram below. A(4 ; 3), B and C are three points on the circle. Straight lines f and g intersect at A. Line f is a tangent to the circle at A. Point B $\left(\frac{-16}{13} ; \frac{-63}{13}\right)$ is a point of intersection of g and the circle.
The y-intercept of g is at (0 ; –3).



2.1.1 Determine the equation of the circle.

____

____ (2)

2.1.2 Determine the length of line AB.

____

____

____

____ (2)

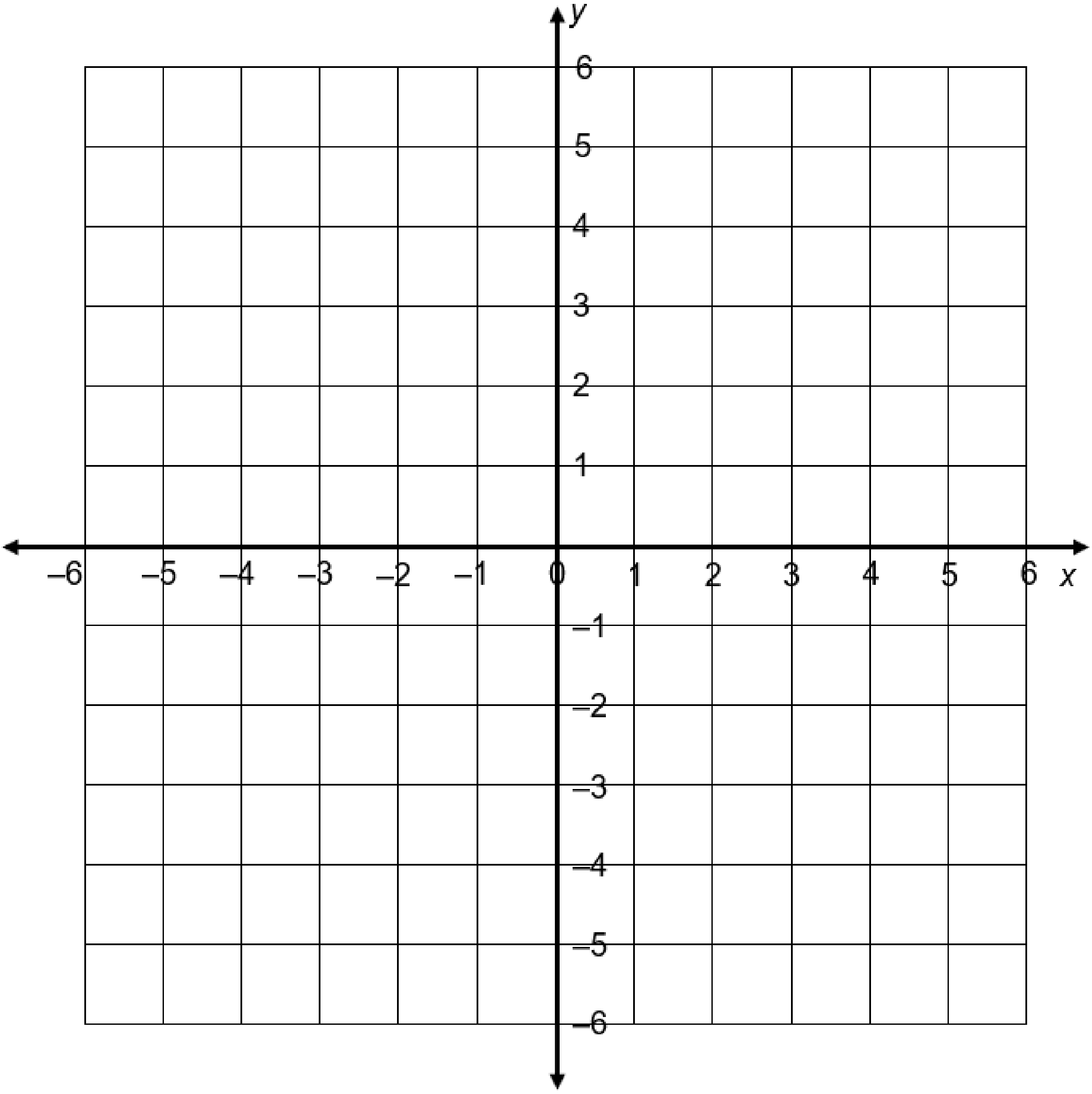
2.1.3 Determine the equation of line **f**, the tangent to the circle at A.

(4)

2.1.4 Determine the x-intercept of a line parallel to line f, through point C.

(4)

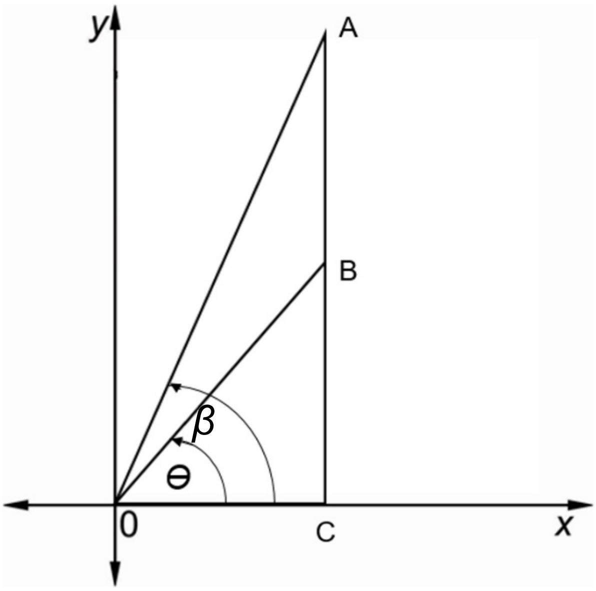
2.2 Sketch the graph defined by $9x^2 + 4y^2 - 36 = 0$ on the set of axes. Clearly show ALL the intercepts with the axes.



(4)
[16]

QUESTION 3

3.1 In the diagram (not drawn to scale) below, ABC is parallel to the y-axis with C on the x-axis. O(0 ; 0) is the origin. OA and OB is drawn with BC = 4 units, $B\hat{O}C = \theta$ and $A\hat{O}C = \beta$ where $\theta = \frac{4\pi}{15}$ and $\beta = 62°$.



Determine the value of each of the following correct to two decimal places.
Show ALL calculations.

3.1.1 $\dfrac{\sec^2\beta - 1}{\tan\theta}$

(4)

3.1.2 Length of OC.

(2)

3.1.3 Length of AB.

(3)

3.2 Calculate, WITHOUT the use of a calculator:

$$\frac{3\sec^2 150^\circ \cdot \cos 180^\circ}{\tan 315^\circ - \cos^2 240^\circ}$$

(8)

3.3 If $\sin 2x = 0{,}473$ for $x \in [0^\circ\,; 180^\circ]$ determine the value(s) of *x.*

(4)

3.4 Prove that $\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{1 + \cos x} = \dfrac{1}{\sin x}$

(4)

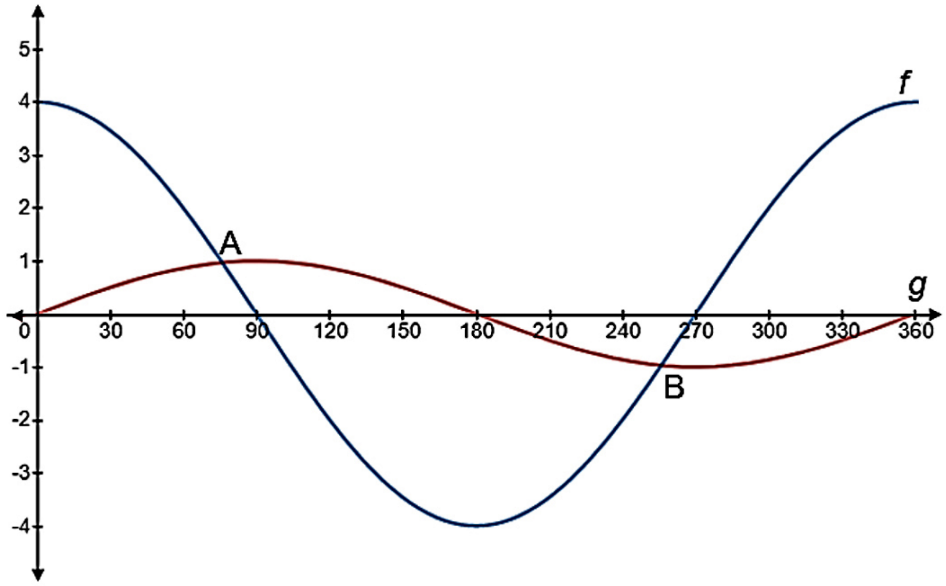
3.5 Simplify $\dfrac{\sin^2(180^\circ + \theta)\cdot\cot(360^\circ - \theta)}{\cos(180^\circ - \theta)}$

(4)

[29]

QUESTION 4

The graphs of *f* and *g* defined by $f(x) = 4\cos x$ and $g(x) = \sin x$ are sketched below for $x \in [0° ; 360°]$.



4.1 Write down the amplitudes of *f* and *g.*

(2)

4.2 If B(256° ; –0,97), write down the coordinates of A.

(2)

4.3 Write down the period of $g(3x)$.

(1)

4.4 Write down the values of *x* for which:

4.4.1 $f(x) - g(x) \geq 0$ $\quad x \in [0° ; 360°]$

(3)

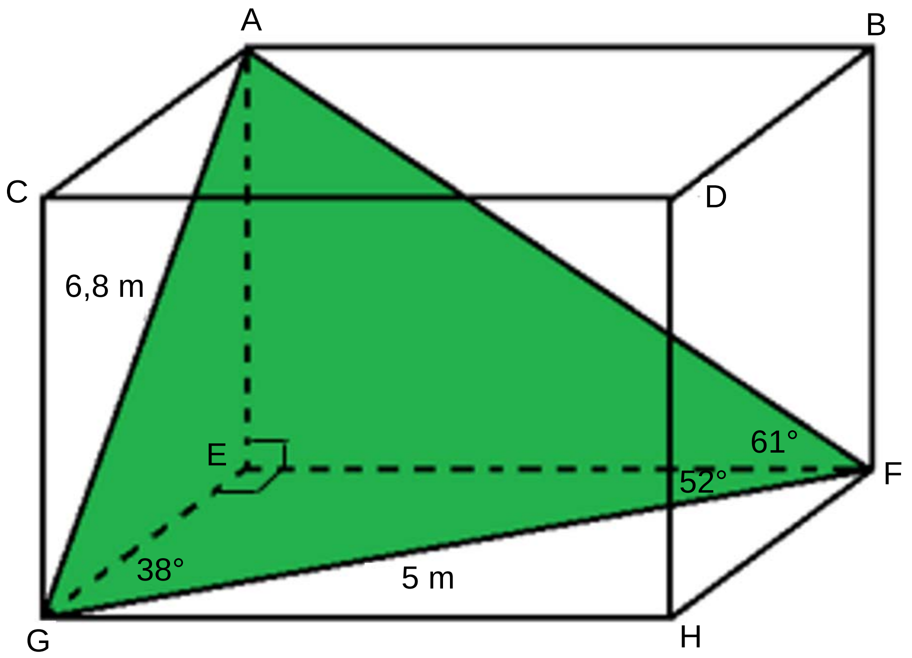
4.4.2 $\dfrac{g(x)}{f(\text{x})}$ will be undefined.

(2)

[10]

QUESTION 5

The picture below shows a triangular tarpaulin tied at points A, G and F with AE a vertical pole. AF, AG and FG are straight lines. FG = 5 m. AG = 6,8 m $A\hat{E}F = 90°$ and $G\hat{E}F = 90°$. The angle of elevation of point A from F is $61°$.
$E\hat{G}F = 38°$ and $E\hat{F}G = 52°$.
Points E, G and F lie in the same horizontal plane.



5.1 Calculate the distance between points E and F.

(2)

5.2 Calculate the side length AF of the tarpaulin.

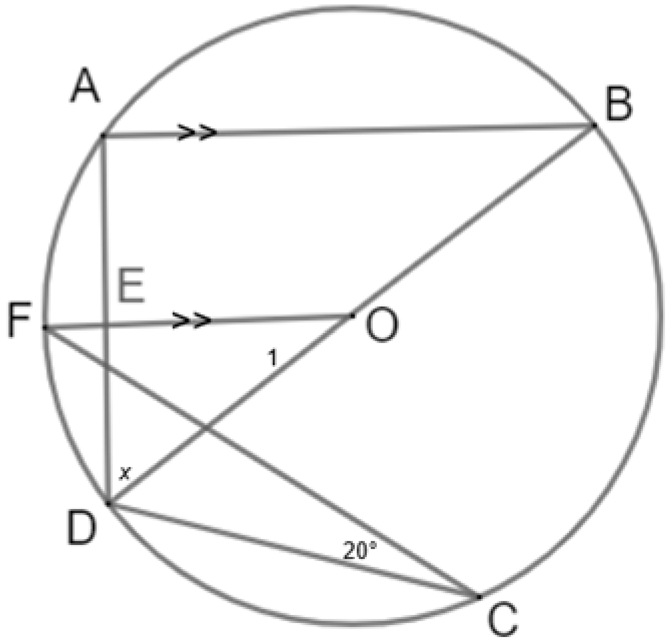
(2)

5.3 Calculate the surface area of the tarpaulin ΔAGF.

(5)
[9]

QUESTION 6

6.1 In the figure below, O is the centre of circle ABCDF. $AB \parallel FO$.
DOB is a diameter. AD and FO intersect at E.
$\hat{C} = 20°$ and $E\hat{D}O = x$.



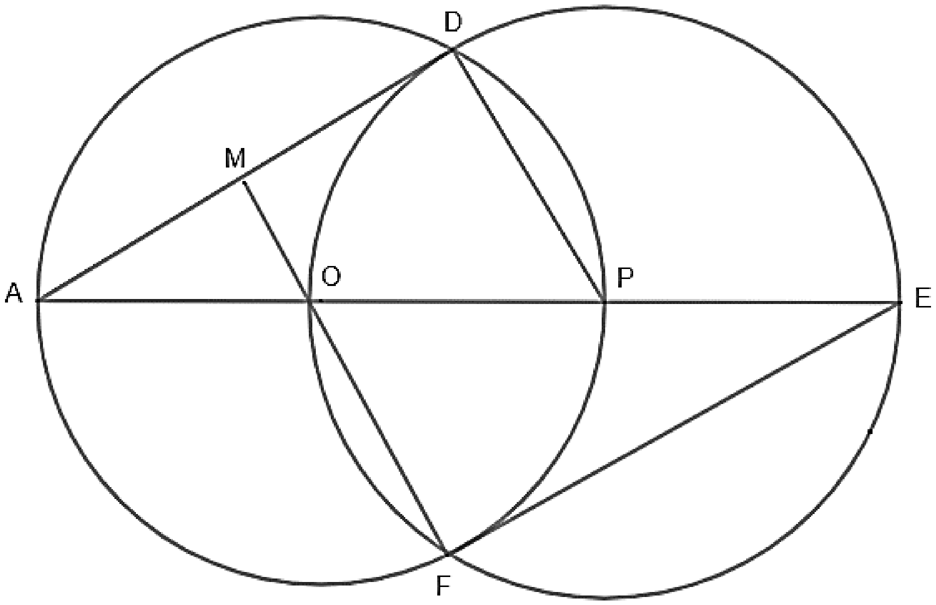
6.1.1 Calculate the size of x, stating reasons.

(4)

6.1.2 Prove that $\dfrac{OD \cdot AB}{BD} = EO$

(3)

6.2 In the figure below, O and P are the centres of two identical circles ADPF and DEFO intersecting at D and F. AOPE and MOF are straight lines.



6.2.1 Prove that $\Delta ADP \equiv \Delta EFO$

(5)

6.2.2 Prove AM = MD

(4)

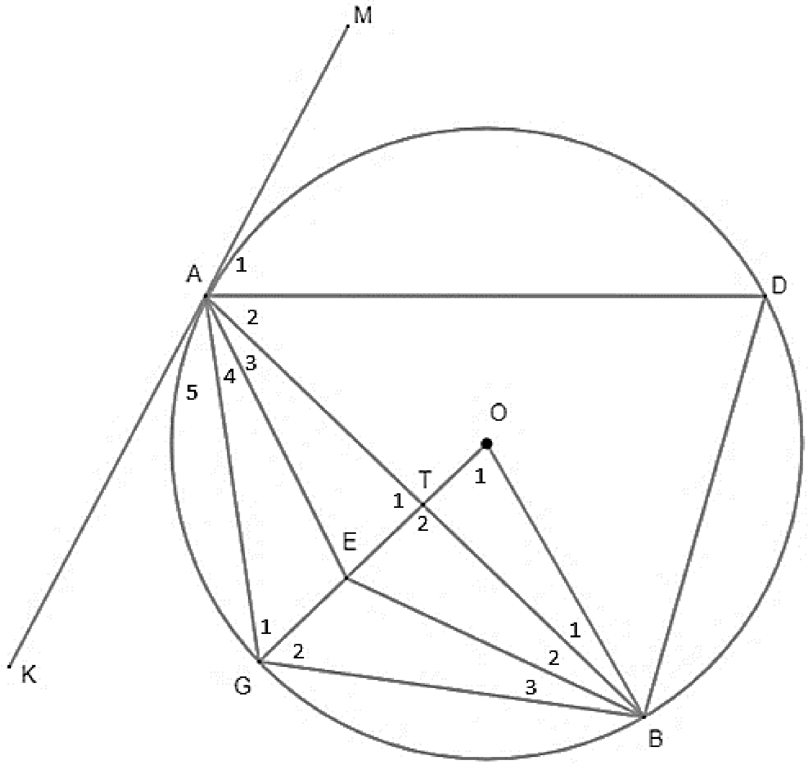
6.2.3 Name two other triangles that are similar to ΔAMO.

(2)

6.2.4 If OE = 4 units, calculate, stating reasons, the length of EF in simplified surd form.

(3)

6.3 In the figure below, O is the centre of circle ADBG.
MAK is a tangent to the circle at A. OG and AB intersect at T with E a point on OG.
AT = TB. AE bisects $T\hat{A}G$ with $\hat{A}_3 = 17°$.



Calculate, stating reasons, the size of:

6.3.1 $\hat{O}_1$

(4)

6.3.2 $\hat{G}_2$

(3)

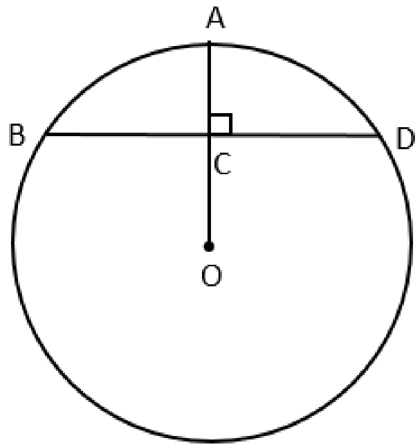
6.3.3 $\hat{A}_5$

(5)

6.3.4 $A\hat{D}B$

(3)

6.4 In the figure below, O is the centre of circle ADB.
$AC \perp BD,\ BD = 4$ units, $AC = 1$ unit.

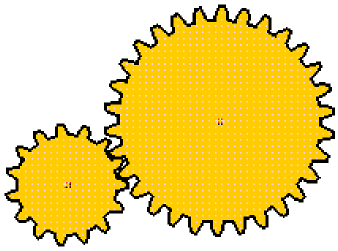


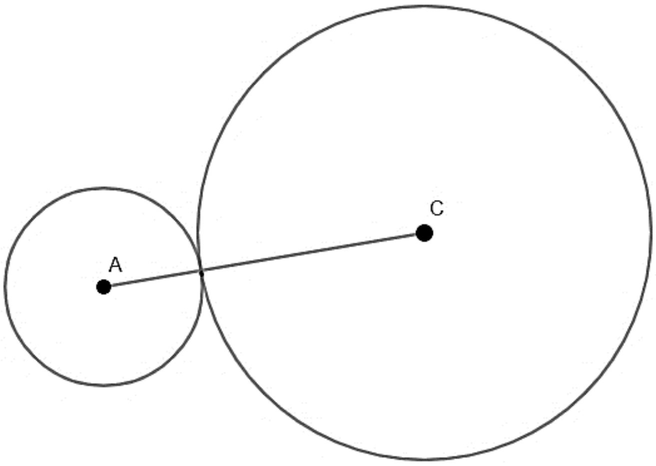
Calculate the radius of circle ADB.

(4)
[40]

QUESTION 7

Two circular gears of different sizes are part of a machine. The larger gear has centre C and the smaller gear has centre A as represented in the given diagram below. The radius of the smaller gear is 12 cm and the radius of the larger gear is 24 cm. The smaller gear completes 5,31 revolutions per second.





Calculate the following:

7.1 The circumferential velocity of the smaller gear in m/s.

(3)

7.2 The number of revolutions the larger gear will complete in one second.

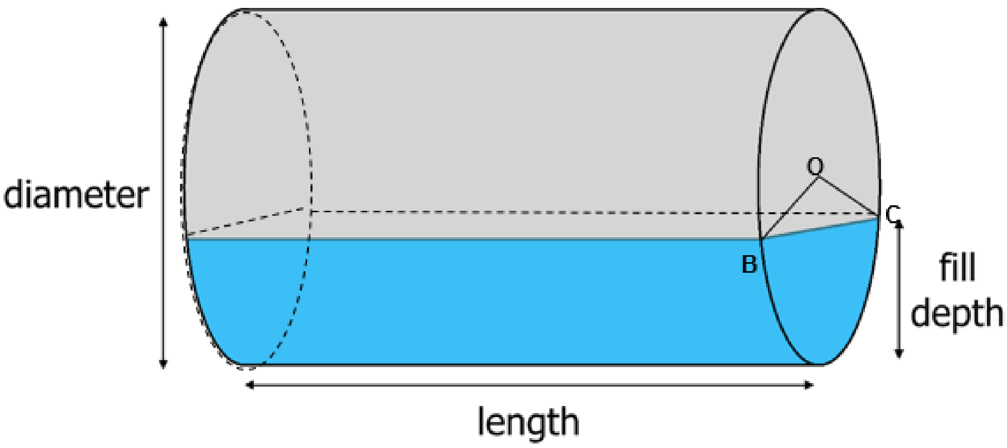
(3)

7.3 The angular velocity of the larger gear in radians per second.

(3)
[9]

QUESTION 8

A right cylindrical tank is filled with diesel as indicated in the diagram below. The diameter of the tank is 3,5 m and the length is 6,25 m. O represents the centre of the circular base with radii OB and OC as shown.



The following formulae may be used:
Area of circle $= \pi r^2$.
Volume of a right cylinder $= \pi r^2 \times$ height

8.1 Calculate the total capacity of the tank to the nearest m^3.

(3)

8.2 The size of angle $B\hat{O}C$ was measured as 120° and the area of triangle OBC as 1,326 m^2.

Calculate:

8.2.1 The length of minor arc BC.

(4)

8.2.2 The area of minor sector OBC.

(3)

8.2.3 The area of the shaded segment below chord BC.

(2)

8.2.4 The percentage of the tank that is filled with diesel as indicated in the diagram.

(3)

8.2.5 The surface area of diesel inside the tank exposed to air, when the fill depth of the diesel is reduced to 50 cm.

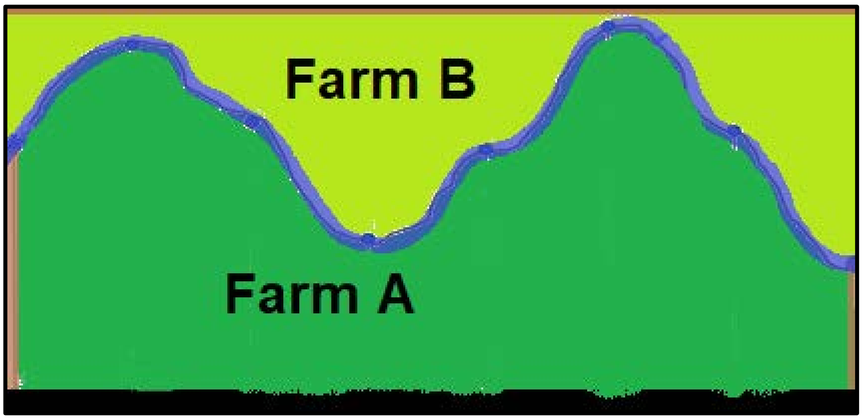
(3)
[18]

QUESTION 9

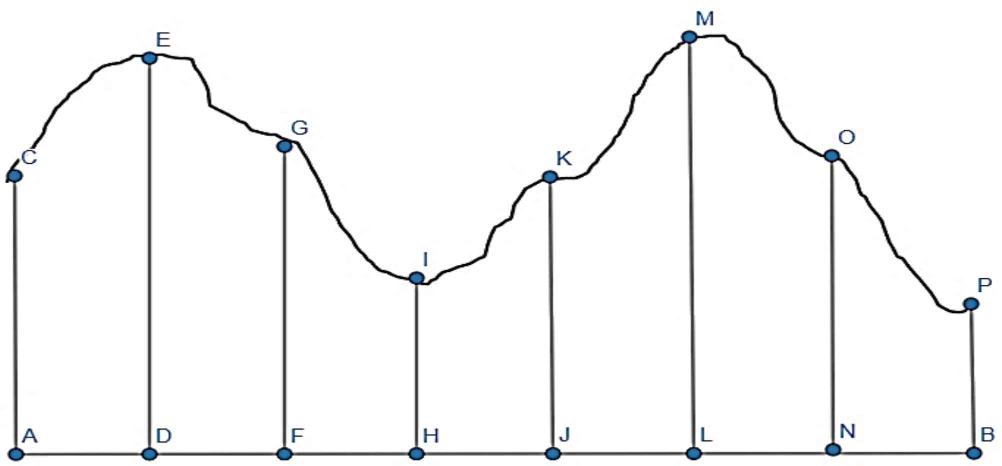
The picture below shows a map of two adjacent Lucerne farms divided by a river.
The diagram below represents Farm A.
The total length of Farm A is represented by AB = 14 km.
It is divided into 7 equal parts.

The following vertical distances were measured:

AC = 0,45 km	DE = 0,62 km	FG = 0,48 km
HI = 0,32 km	JK = 0,46 km	LM = 0,64 km
NO = 0,47 km	BP = 0,21 km	



MAP



DIAGRAM

9.1 Calculate the total area of lucerne for Farm A by using the mid-ordinate rule.

(3)

9.2 The farmer can make 350 bales from 1 hectare (0,01 km^2) and receives R40,00 per bale.

Determine the minimum area in km^2 to be planted with lucerne to receive a minimum income of R525 000 after baling the lucerne on Farm A.

(3)
[6]

Total: 150 marks

ADDITIONAL SPACE TO ANSWER QUESTIONS. REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE ALL ANSWERS ARE MARKED.