GRADE 12 EXAMINATION
NOVEMBER 2018

ADVANCED PROGRAMME MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 2 hours 200 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

MODULE 2 STATISTICS

QUESTION 1

1.1 (a) Binomial ✓

✓✓ ✓ ✓ ✓

Zero or one $\binom{12}{0}(0,057)^0(0,943)^{12} + \binom{12}{1}(0,057)(0,943)^{11}$

= 0,8531. ✓ (7)

(b) The remaining 16 not ADD ✓✓

$(0,943)^{16}$ = 0,3910 ✓✓ (4)

1.2 (a) Without replacement hypergeometric ✓✓ (2)

(b) 20 ✓✓ (2)

(c) 7 ✓ (1)

(d) 8 ✓✓ (2)

(e) 2 ✓✓ (2)

(f) 7 – k 8 – (7 – k) = k + 1 ✓✓ (2)

[22]

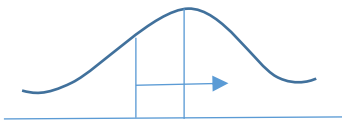
QUESTION 2

2.1 (a) Let X be the random variable "weight of babies"

$P(X > 2,8)$ ✓

$P(z > \frac{2,8 - 3,2}{0,85}$ ✓

$P(z > -0,4706)$ ✓

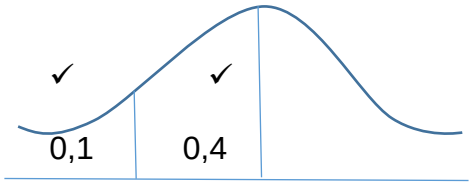


0,05 + 0,1808 ✓✓

= 0,6808 ✓

Approximately 953 120 babies ✓ (7)

(b)



✓ $z = -1{,}28$ ✓

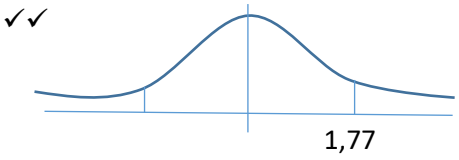
$-1{,}28 = \dfrac{(X - 3{,}2)}{0{,}85}$ ✓✓

= 2,112 kg ✓ (7)

2.2 (a) 61 kg ✓✓ (2)

(b) $61 + z \times \dfrac{9}{8} = 63$ ✓✓

$z = \dfrac{16}{9} = 1{,}77$ ✓



(8)

0,461 × 2 ✓

92% confident ✓✓

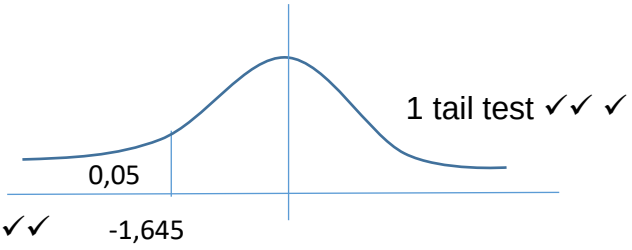
[24]

QUESTION 3

$H_0: \mu_x = \mu_y$

$H_1: \mu_x > \mu_y$ ✓✓

Test statistic $z = \dfrac{7{,}2 - 8{,}1}{\sqrt{\dfrac{(2{,}85)^2}{35} + \dfrac{4}{38}}} = -1{,}54$ ✓✓



Not enough evidence to reject the null hypothesis in favour of the claim at the 5% significance level.

[9]

QUESTION 4

4.1 $\bar{y} = \frac{\sum y}{n} \quad 159\frac{1}{6} = \frac{1910}{n}$

n = 12 ✓✓ (2)

4.2 $b = \frac{12 \times 26270 - 161 \times 1910}{12 \times 2293 - (161)^2} = 4{,}8464$ ✓✓ ✓

$\bar{y} = a + b\bar{x}$

$\frac{955}{6} = a + 4{,}8464\left(\frac{161}{12}\right) \therefore a = 94{,}1441$ ✓✓

$y = 94{,}1441 + 4{,}8464x$ ✓ (6)

4.3 Strong, positive correlation ✓✓ (2)

4.4 No – too far out of the range (extrapolation) ✓✓ (2)

[12]

QUESTION 5

5.1 $\int_{30}^{60} a(x-30)^2 \, dx = 1$ ✓✓

$\left[\frac{a}{3}(x-30)^3\right]_{30}^{60} = 1$ ✓✓

$\frac{a}{3}(30)^3 = 1$ ✓✓

$a = \frac{3}{(30)^3} = \frac{1}{9\,000}$ ✓ (7)

5.2 $\left[\frac{1}{27000}(x-30)^3\right]_{30}^{m} = \frac{1}{2}$ ✓✓

$\frac{1}{27000}(m-30)^3 = \frac{1}{2}$ ✓✓

$(m - 30)^3 = 1\,3500$ m ✓✓

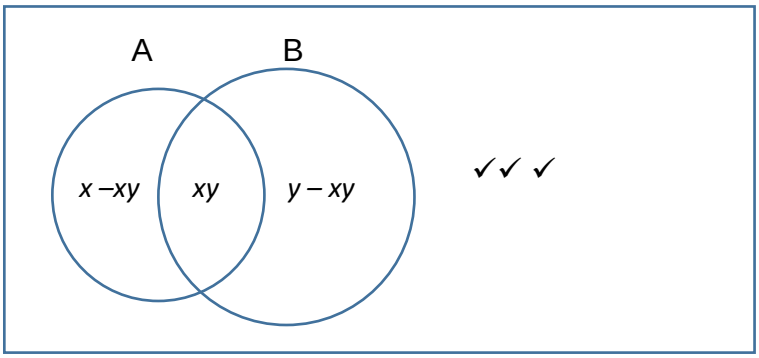
$m = 54$ minutes ✓ (7)

[14]

QUESTION 6

6.1 $P(A)=x\ P(B)=y\ P(B')=1-y$ ✓

$P(A\cap B)=xy$ ✓



✓✓ ✓

$P(A)\times P(B')=x(1-y)$ ✓

$P(A\cap B')=x-xy=x(1-y)$ ✓

Events A and B' are independent (8)

6.2 $\binom{16}{5}-\binom{9}{0}\binom{7}{5}=4\,347$ ✓ ✓ ✓ ✓ (4)

6.3 $\binom{11}{2}\binom{9}{4}\binom{5}{5}+\binom{11}{1}\binom{10}{4}\binom{6}{6}+\binom{11}{2}\binom{9}{3}\binom{6}{6}=13\,860$ ✓✓ ✓✓ ✓✓ ✓ (7)

[19]

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

1.1 (a) $5\,640 \times \frac{1}{1{,}15}$ ✓ = **4 904,35** ✓ (2) B

(b) $\frac{1{,}15 - 1{,}14}{1{,}14} \times 100$ ✓ = 0,00877 … ✓ = **0,88%** ✓ (3) B

1.2 $3x = x(1+i)^{24}$ ✓✓ ∴ i = 0,0468 per month ✓

$2x = x(1+0{,}0468)^{n}$ ✓ ∴ n = 15,1423 ≈ **15 months** ✓ (5) R

1.3 A 3 ✓✓ B 2 ✓✓ C 1 ✓✓ D 5 ✓✓ (8) R
[18]

QUESTION 2

2.1 $500\,000\left(\frac{0{,}088}{12}\right)$ ✓ = 3 666,67 ✓ **interest > payments** ✓ (3) R

2.2 $500\,000$ ✓ $= \dfrac{x\left[1-\left(1+\frac{0{,}088}{12}\right)^{-96}\text{✓}\right]}{\frac{0{,}088}{12}\text{✓}}$ ***x = 7 273,33*** ✓✓ (5) R

2.3 $500\,000$ ✓ $\left(1+\frac{0{,}088}{12}\text{✓}\right)^{95\text{✓}}$ – ✓ $\dfrac{7\,300\text{ ✓}\left[\left(1+\frac{0{,}088}{12}\right)^{95\text{✓}}-1\right]\text{✓}}{\frac{0{,}088}{12}}$ = 3 576,4053 ✓

3 576,4053 $\left(1+\frac{0{,}088}{12}\right)$ ✓✓ = **3 602,63** ✓✓

OR

$500\,000$ ✓ $\left(1+\frac{0{,}088}{12}\text{✓}\right)^{96}$ ✓ – ✓ $\dfrac{7\,300\text{✓}\left(1+\frac{0{,}088}{12}\right)\text{✓✓}\left[\left(1+\frac{0{,}088}{12}\right)^{95\text{✓}}-1\right]\text{✓}}{\frac{0{,}088}{12}}$

= 1 008 318,445 – 1 004 715,812 ✓ = **3 602,63** ✓✓

OR

$$500\,000 \checkmark\checkmark - \checkmark \frac{7\,300 \checkmark \left[1-\left(1+\frac{0{,}08}{12}\right)^{-95}\checkmark\right]+y\left(1+\frac{0{,}08}{12}\right)^{-96}}{\frac{0{,}08}{12}} \checkmark$$

$\therefore y =$ **3 602,63** ✓✓ (12) R

[20]

QUESTION 3

$$x.\left(1+\frac{0{,}08}{12}\right)^{72} \checkmark \bullet \frac{2}{3} \bullet \left(1+\frac{0{,}08}{12}\right)^{24} \checkmark + \checkmark\ x. \checkmark \left(1+\frac{0{,}08}{12}\right)^{72} \checkmark \bullet \frac{1}{3} \bullet \left(1+\frac{0{,}1}{4}\right)^{8} \checkmark$$

$= 20\,702{,}50$ ✓✓

$1{,}9169x = 20\,702{,}50$ ***X = 10 800*** ✓✓

[12] C

QUESTION 4

4.1 Logistic Model, ✓ presence of carrying capacity ✓ (2) B

4.2 $V = \frac{1}{2}$ *(50)* ✓ = ***25*** ✓ (2) R

4.3 The model has regression equation $\frac{\Delta P}{P} = -0{,}0025P + r$.

r = – Km ✓ *= – 50* ✓*.(– 0,0025)* ✓ = ***0,125*** ✓ (4) R

4.4 T_{n+1} = ✓ T_n + 0,13 ✓.$T_n(1 - T_n/50)$, ✓ $T_O = 10$ ✓
t = 11 ✓✓ (6) C

[14]

QUESTION 5

5.1 (a) number of eagles born per annum ✓✓ (2) B

(b) efficacy rate ✓ at which eagles turn prey into offspring ✓ (2) B

(c) $f.b.D_n.E_n = 15$
$f\ (6\ 000) = 15$ ✓✓ **$f = 0{,}0025$** ✓ (3) R

5.2 $a = 0{,}5 \times 1{,}5$ ✓ $\times 3$ ✓ $\times 0{,}67$ ✓ **$a = 1{,}51$** ✓✓ (5) R

5.3 $6\ 000$ ✓ $= b.(12\ 000)(30)$ ✓✓ **$b = 0{,}016\ 667$** ✓✓
for dassie equilibrium, $E_{n+1} = E_n$ ✓
$0{,}1$ ✓ $= 0{,}003 \times 0{,}016\ 667 \times D$ ✓ **$D = 1\ 999{,}96$** ✓ **$\approx 2\ 000$** ✓

OR

$6\ 000$ ✓ $= b.(12\ 000)(30)$ ✓✓ **$b = 0{,}016\ 667$** ✓✓
$D = \frac{c}{fb}$ ✓ $= \frac{0{,}1}{0{,}003 \times 0{,}016\ 667}$ ✓✓ **$D = 1\ 999{,}96$** ✓ **$\approx 2\ 000$** ✓ (10) C

[22]

QUESTION 6

6.1 (a) $T_4 = 75{,}77$ $T_5 = 84{,}55$ $T_6 = 91{,}122$ ✓✓✓✓ (4) R

(b) $64\sqrt{3} =$ **110,8 sq units** ✓✓ (2) R

6.2 $195 = p.114 + q.60$ ✓✓ and $114 = p.60 + q.24$ ✓✓
$p = 2{,}5$ ✓ and $q = -1{,}5$ ✓
$T_n = 5/2.T_{n-1} - 3/2.T_{n-2}$ ✓ **$T_1 = 24,\ T_2 = 60$** ✓ (8) P

[14]

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 $PQ = \begin{pmatrix} 3 & 6 & 2 & -2 \\ 0 & -1 & 4 & 6 \end{pmatrix} \cdot \begin{pmatrix} 3 & 6 \\ -2 & -1 \\ 0 & 5 \\ -7 & 0 \end{pmatrix} = \begin{pmatrix} 11 & 22 \\ -40 & 21 \end{pmatrix}$ ✓✓✓✓ (4) R

1.2 $3x + 2y = 11$ $x - 2z = 0$ $6y + 4z = 5$

3	2	0	11	
1	0	–2	0 ✓	
0	6	4	5	
3	2	0	11	
0	2	6	11	R1 – 3.R2 ✓✓
0	6	4	5	
3	2	0	11	
0	2	6	11	
0	0	14	28	3.R2 – R3 ✓✓

$z = 2$, ✓ $y = -\frac{1}{2}$, ✓ $x = 4$ ✓ (8) R

1.3 (a) 3 ✓✓ (2) B

(b) 0 ✓✓ (2) R

(c) t ✓✓ (2) P

[18]

QUESTION 2

2.1 reflection ✓ across $y = x$ ✓ (2) B

2.2 $k = 3$ ✓✓ (2) R

2.3 $C = \frac{1}{4}R$ ✓✓ and $R = S$ so factor is $\frac{1}{4}$ ✓✓ (4) R

2.4 $\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}$ ✓✓✓ $\begin{pmatrix} -0,5 & 0 \\ 0 & -0,5 \end{pmatrix}$ ✓ $= \begin{pmatrix} 1,5 & 0 \\ 0 & -0,5 \end{pmatrix}$ ✓✓ (6) R

OR

$$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0,5 & 0 \\ 0 & 0,5 \end{pmatrix} = \begin{pmatrix} 1,5 & 0 \\ 0 & -0,5 \end{pmatrix}$$

2.5 $\begin{pmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{pmatrix}$ ✓ $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ ✓ $= \begin{pmatrix} 4,025 \\ 3,578 \end{pmatrix}$ ✓

$5\cos A + 2.\sin A = 4,025$ ✓✓ and $-2.\cos A + 5\sin A = 3,578$ ✓✓

$\cos A = 0,4472$ and $\sin A = 0,8944$ ✓

A = 63,44° **A = 63,44°** ✓✓ (10) C

[24]

QUESTION 3

3.1 det = 25 ✓✓ (2) B

3.2 $\begin{pmatrix} 25 & 0 & 0 \checmark & -1 & -4 & -10 \checmark\checmark \\ 0 & -10 & 0 & -2 & -8 & -10 \\ 0 & 0 & 25 \checmark & 4 & -9 & 10 \checmark\checmark \end{pmatrix}$

(6) C

3.3 $\begin{pmatrix} 25 & 0 & 0 & -1 & -4 & 10 \\ 0 & 25 & 0 \checmark & 5 & 20 & -25\checmark \\ 0 & 0 & 25 & -4 & 9 & -10 \end{pmatrix}$

$Inverse = \frac{1}{25}$ ✓ $\begin{pmatrix} -1 & -4 & 10 \\ 5 & 20 & -25 \\ -4 & 9 & -10 \end{pmatrix}$ ✓ (4) C

[12]

QUESTION 4

4.1 one vertex has an odd degree ✓✓ (2) B

4.2 yes; ✓ there is one pair of odd vertices ✓ (2) B

4.3 8 edges ✓✓ (2) B

4.4 19×2 ✓ $= 4 \times 6 + 2 \times 4 + 1 \times 1 + e$ ✓
e = 5 ✓✓ (4) R
[10]

QUESTION 5

5.1

TR	3	TV	3 ✓	TS	4 ✓	SU	3 ✓
RQ	5 ✓	RW	5 ✓	QP	6 ✓	length = 29 ✓✓	

(8) R

5.2

RT	3	TV	3	VW	7 ✓
WRQ	10 ✓✓	QP	6 ✓	PS	7 ✓
SU	3 ✓	UTR	9 ✓✓	**U/B = 48** ✓✓	

(10) R

5.3 37 is the largest Lower Bound ✓ and 41 is the smallest Upper Bound ✓ (2) C

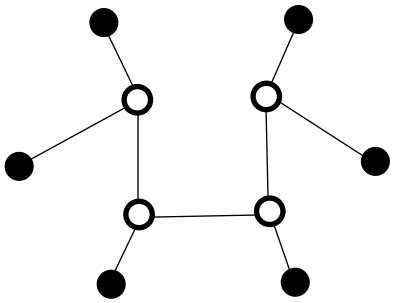
5.4 R Q ✓ P U ✓ S ✓ T ✓ V ✓ W R ✓ = 41✓✓ (8) C
[28]

QUESTION 6

6.1 3 ✓✓ (2) R

6.2 $e = 2n - 3$ ✓✓ (2) R

6.3 4 Steiner Vertices ✓
9 edges ✓✓
Connectivity ✓ (4) P



[8]

Total for Module 4: 100 marks