TECHNICAL MATHEMATICS: PAPER I

EXAMINATION NUMBER

Time: 3 hours   150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 24 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.

2. Read the questions carefully.

3. **Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number in the space provided.**

4. Diagrams are not necessarily drawn to scale.

5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.

6. Round off your answers to <u>one decimal digit</u> where necessary, unless otherwise stated.

7. All the necessary working details must be clearly shown.

8. It is in your own interest to write legibly and to present your work neatly.

9. Two blank pages (pages 23 and 24) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	TOTAL
16	15	14	15	15	10	7	13	33	12	150

QUESTION 1

1.1   Solve for *x* where *x* is an element of {Complex Numbers}, leaving answers in simplified surd form where applicable.

     1.1.1 $2x^2 - 5x = 12$

                                                  (3)

     1.1.2 $2x^2 + 4x + 7 = 0$

                                                  (4)

1.2 An electric motor working for 4 hours and a heater working for 2 hours, use a total of 25 kJ of energy. When the heater works for 3 hours and the electric motor works for 2 hours, they use 18 kJ of energy in total. Determine the energy consumption per hour, of each appliance.

(5)

1.3 Determine the value(s) of k so that $x^2 - 3x + 9k = 0$ will have real and unequal roots.

(4)
[16]

QUESTION 2

2.1   Simplify without using a calculator $\dfrac{3^{2014} + 9^{1007}}{27^{671}}$

_____

_____

_____

_____

_____

(4)

2.2   Solve for x:

2.2.1 $5 - \sqrt{4x+1} = x$

_____

_____

_____

_____

_____

_____

(6)

2.2.2 $2\log x = \log 4 + \log(x-1)$

_____

_____

_____

_____

_____

(5)
[15]

QUESTION 3

3.1   Write $\dfrac{3-2i}{1+5i}$ in the form $a+bi$, without using a calculator.

_____

_____

_____

_____

_____

_____

_____

_____
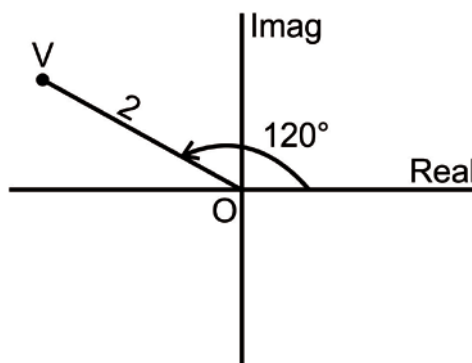
(5)

3.2   The voltage V in an alternating current circuit is represented by the Argand diagram shown below.

3.2.1   Use the diagram to write V in the form $V = r(\cos\theta + i\sin\theta)$.

_____

_____

(2)

3.2.2   Hence, write V in rectangular form.

_____

_____

_____

_____

(4)

3.3   Express 11001_2 in decimal notation. Show all your calculations.

_____

_____

_____

_____

_____

_____

(3)
[14]

QUESTION 4

4.1 A cellular phone has a marked price of R4 800. During a sale, a discount of 13,5% was offered. Calculate the selling price of the phone.

[Source: <www.juzdeals.com>]

(2)

4.2 4.2.1 Determine the nominal interest rate compounded quarterly if the effective interest rate is 9% per annum (correct to two decimal places).

(5)

4.2.2 Dan invests money in a Savings Bond paying simple interest. Determine the interest rate per annum if it takes 15 years for his money to treble in value.

(4)

4.3   Calculate how long it will take for a vehicle to depreciate on a reducing balance at an interest rate of 13% per annum, so that it will be half of its original value. Give your answer correct to the nearest year.

_____

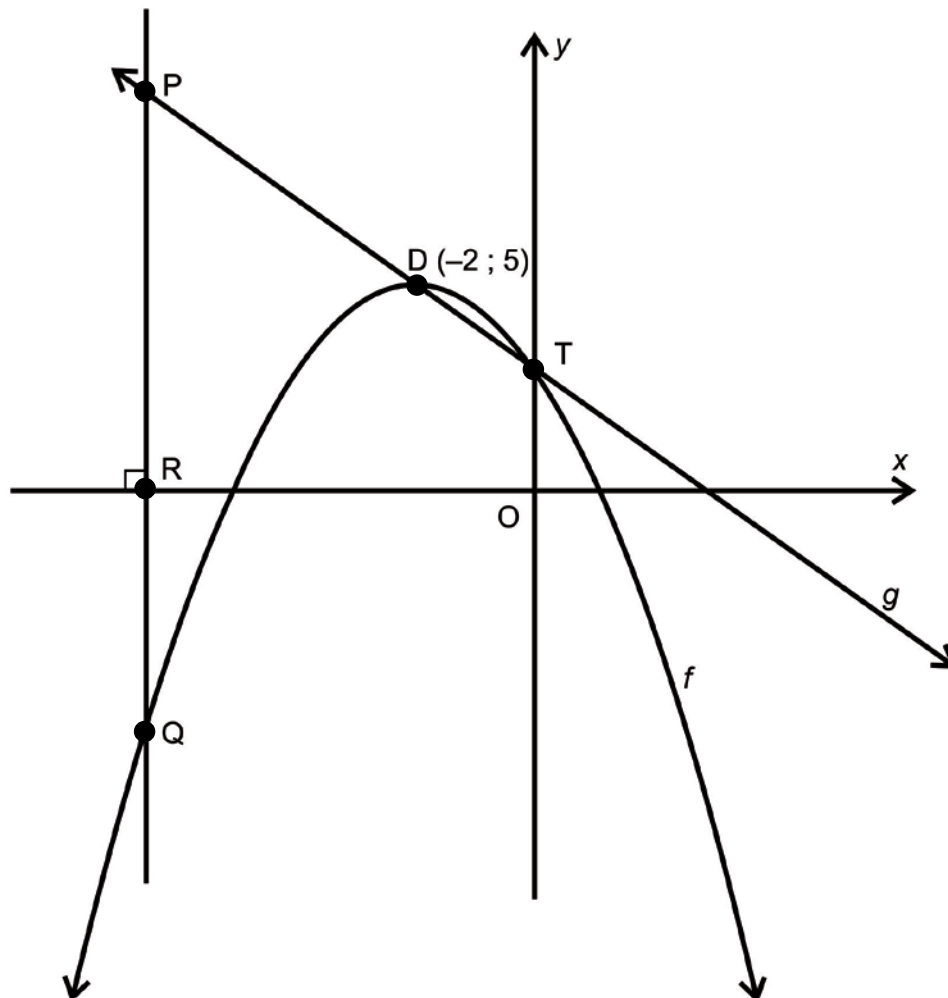_____

_____

_____

_____

_____

(4)
[15]

QUESTION 5

Given below are the graphs of g and f defined by the equations $g(x) = -x + k$ and $f(x) = ax^2 + bx + c$

f and g intersect on the y-axis at point T, and again at $D(-2 ; 5)$, the turning point of f. Line PQ is perpendicular to the x-axis at R, with P on g and Q on f.

5.1   Show that $k = 3$

_____

_____

_____

_____

(2)

5.2   If $a = -\dfrac{1}{2}$, write down the value of c and hence, determine the value of b.

(4)

5.3   P is a point on the straight line defined by $g(x)$ and Q is a point on the parabola defined by $f(x)$, determine a simplified expression in terms of x for the length of PQ.

(2)

5.4   Hence, determine the coordinates of P if $PQ = 12$ units.

(5)

5.5   ***Use your graph*** to determine the values of x for which $g(x) \leq f(x)$

_____

_____

_____

_____
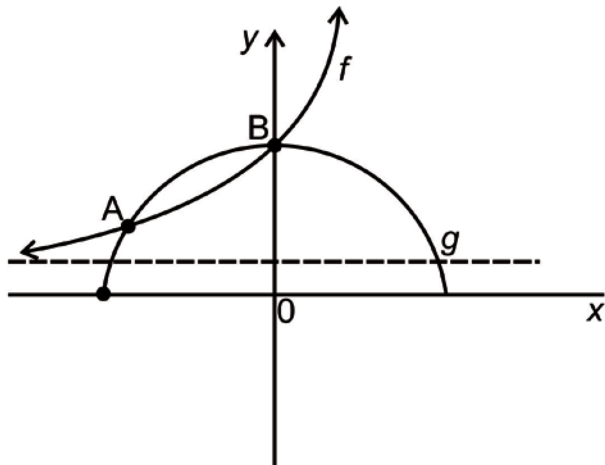
(2)

[15]

QUESTION 6

The diagram shows the graphs of f and g defined by the equations $f(x) = 2^x + 1$ and $g(x) = \sqrt{r^2 - x^2}$ intersecting one another at A and B, where B lies on the y-axis.

6.1   Write down the equation of the asymptote of f.

_____

_____

_____

(1)

6.2   Determine the coordinates of point B and hence determine the equation defining g.

_____

_____

_____

_____

(3)

6.3   Write down the domain and range of g.

_____

_____

_____

(2)

6.4   Determine k, correct to 2 decimal places, if A is the point $\left(-1,466 \; ; \; k\right)$ and the average gradient between A and B is given as 0,44.
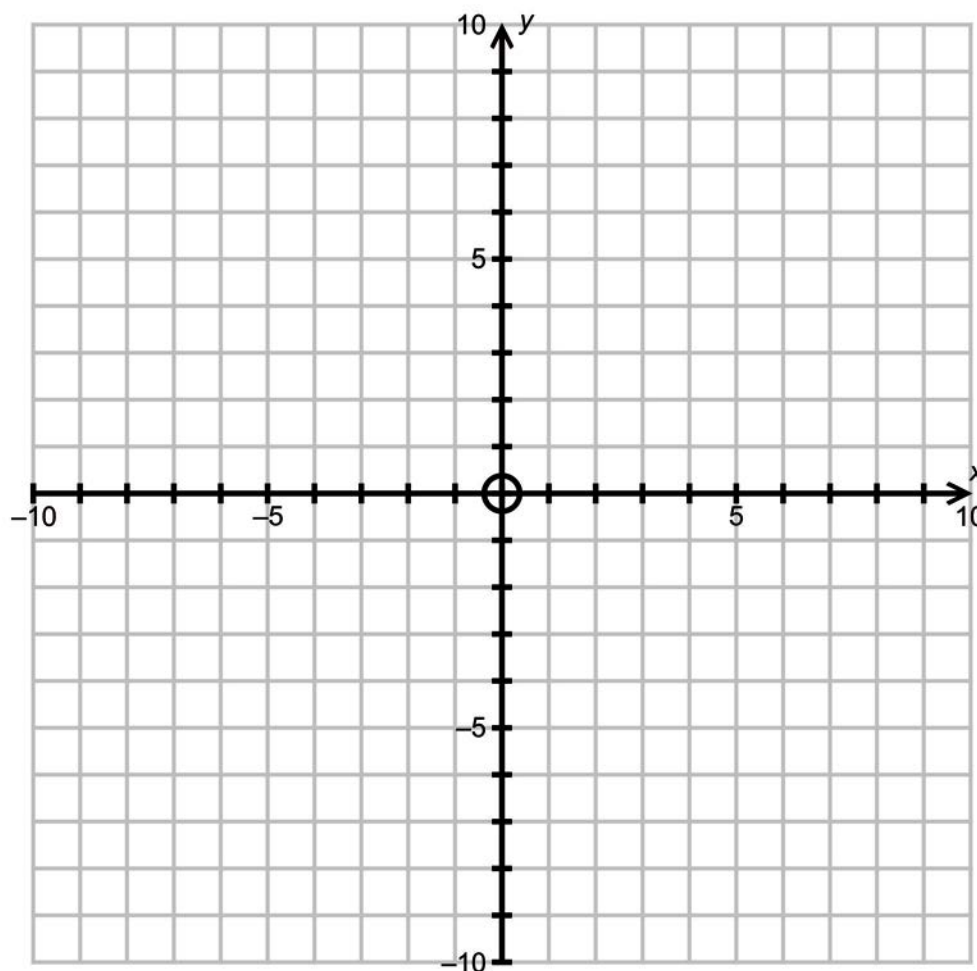
_____

_____

_____

_____

_____

_____

(4)
[10]

QUESTION 7

7.1   Sketch the graphs of *f* and *g* on the system of axes below. Function *f* is defined by
$f(x) = \dfrac{-4}{x} + 3$. Function *g* is a straight line intersecting *f* on the *x*-axis, and
intersecting the asymptote of *f* on the *y*-axis. Show clearly all asymptotes, points
of intersection and intercepts with axes.

(4)

7.2   Function *h* is obtained by translating *g* vertically up 1 unit. Write down the equation defining *h*.

_____

_____

_____

_____

_____

(3)
[7]

QUESTION 8

8.1   If $f(x) = 5 - 2x$, determine $f^{/}(x)$ from first principles.

_____

_____

_____

_____

_____

_____

(5)

8.2   Determine $\dfrac{dy}{dx}$ if $y = \dfrac{x^2 - 4x + 3}{x}$.

_____

_____

_____

_____

_____

(4)

8.3   If $f(x) = 2\sqrt{x} + \dfrac{1}{x^3} - \sqrt{2}x$, determine $f^{/}(x)$.

_____

_____

_____

_____

_____

(4)
[13]

QUESTION 9

9.1   A soft drink company manufactures aluminium cans with a right cylindrical shape. Each can must hold 375 ml of liquid.

      9.1.1   If the radius of the cylinder is *r*:

          (a)   determine *h* in terms of *r*, where $\text{Volume} = \pi r^2 h$

              (2)

          (b)   hence, show that the total surface area of the cylinder (S) is given by $S = \dfrac{750}{r} + 2\pi r^2$, where $SA = 2\pi r^2 + 2\pi rh$

              (2)

      9.1.2   Determine *r* (in terms of π) if the total surface area of aluminium sheeting needs to be minimised.
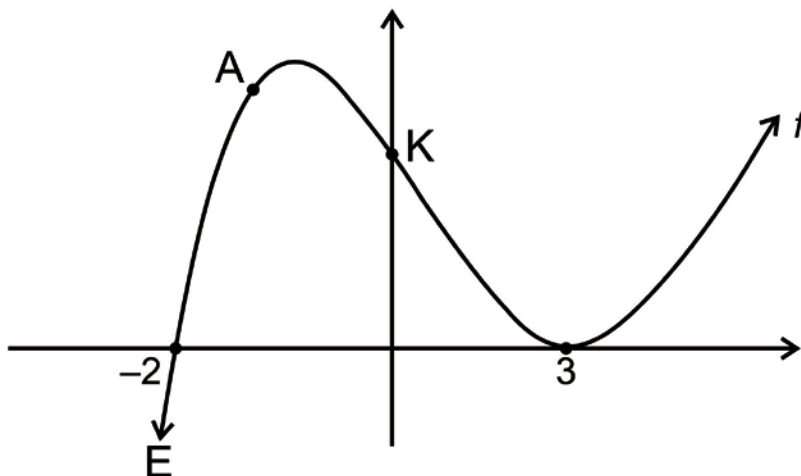
              (5)

9.2   The graph of f is defined by $f(x) = x^3 + px^2 - 3x + q$. Curve f cuts the x-axis at -2 and touches the x-axis at 3. Points A and K lie on f.

9.2.1   Show that the numerical values of p and q are -4 and 18 respectively.

_____

_____

_____

_____

_____

(4)

9.2.2   If $p = -4$ and $q = 18$ and the gradient of curve f at the point A is 8, find the coordinates of A.

_____

_____

_____

_____

_____

_____

(5)

9.2.3   Determine the co-ordinates of the turning points of *f.*

_____

_____

_____

_____

_____

_____

_____

(5)

9.2.4   Determine the equation of a straight line defined by $h(x)$ which is perpendicular to the tangent to *f* at K, the point where *f* meets the *y*-axis.

_____

_____

_____

_____

(3)

9.3   The image below shows an oil leak from a car. The area of the expanding oil leak (in cm^3), t seconds after it has been spilled, is given by the formula $A = -t^2 + 5t + 8$

[Source: < https://oards.com/oil-pan-gasket-leak-symptoms-and-cost/>]

9.3.1   Determine the initial area of the oil leak.

_____

_____

_____

_____
                                                                    (2)

9.3.2   Determine the rate of increase in area at $t = 2$ seconds.

_____

_____

_____

_____
                                                                    (3)

9.3.3   Determine the time at which the oil leak stops spreading.

_____

_____

_____
                                                                    (2)
                                                                  [33]

QUESTION 10

10.1   Simplify:

(a) $\int 0\, dx$

        (1)

(b) $\int dx$
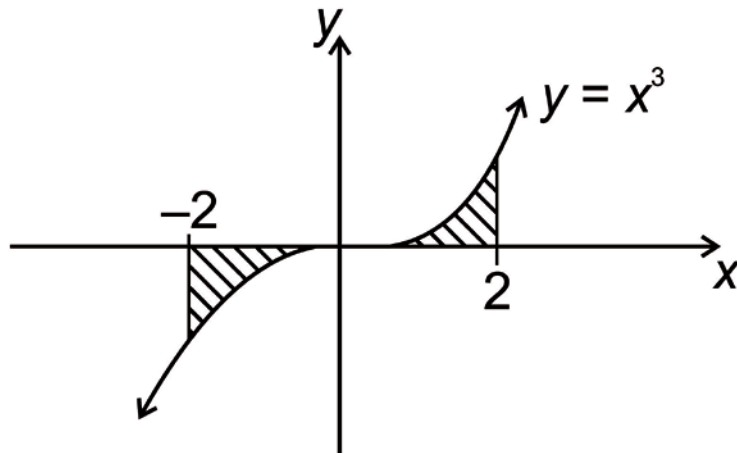
        (1)

10.2   Determine: $\int \left(3x^2 + x^{-1} \right) dx$

        (3)

10.3   Find the total area between the curve $y = x^3$ and the x-axis between $x = -2$ and $x = 2$.

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

(7)
[12]

Total: 150 marks

ADDITIONAL SPACE (ALL questions)

REMEMBER TO CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.