# Aprendizaje Automático

## **Review of Linear Algebra**



Máster en Bioinformática y Biología Computacional

#### **General Notation**



 $\square$  Vector – We note  $x \in \mathbb{R}^n$  a vector with n entries, where  $x_i \in \mathbb{R}$  is the  $i^{th}$  entry:

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n \qquad \text{Another notation: } \mathbf{x}$$

 $\square$  Matrix – We note  $A \in \mathbb{R}^{m \times n}$  a matrix with n rows and m, where  $A_{i,j} \in \mathbb{R}$  is the entry located in the  $i^{th}$  row and  $j^{th}$  column:

$$A = \begin{pmatrix} A_{1,1} & \cdots & A_{1,n} \\ \vdots & & \vdots \\ A_{m,1} & \cdots & A_{m,n} \end{pmatrix} \in \mathbb{R}^{m \times n}$$
 Another notation: **A**

Remark: the vector x defined above can be viewed as a  $n \times 1$  matrix and is more particularly called a column-vector.

#### **General Notation**

□ **Identity matrix** – The identity matrix  $I \in \mathbb{R}^{n \times n}$  is a square matrix with ones in its diagonal and zero everywhere else:

$$I = \left(\begin{array}{cccc} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \end{array}\right)$$

Remark: for all matrices  $A \in \mathbb{R}^{n \times n}$ , we have  $A \times I = I \times A = A$ .

## **Matrix Operations**

- □ Vector-vector multiplication There are two types of vector-vector products:
  - inner product: for  $x,y \in \mathbb{R}^n$ , we have:

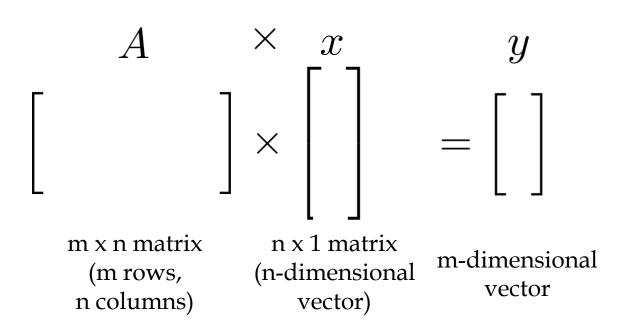
$$x^T y = \sum_{i=1}^n x_i y_i \in \mathbb{R}$$

• outer product: for  $x \in \mathbb{R}^m$ ,  $y \in \mathbb{R}^n$ , we have:

$$xy^T = \begin{pmatrix} x_1y_1 & \cdots & x_1y_n \\ \vdots & & \vdots \\ x_my_1 & \cdots & x_my_n \end{pmatrix} \in \mathbb{R}^{m \times n}$$

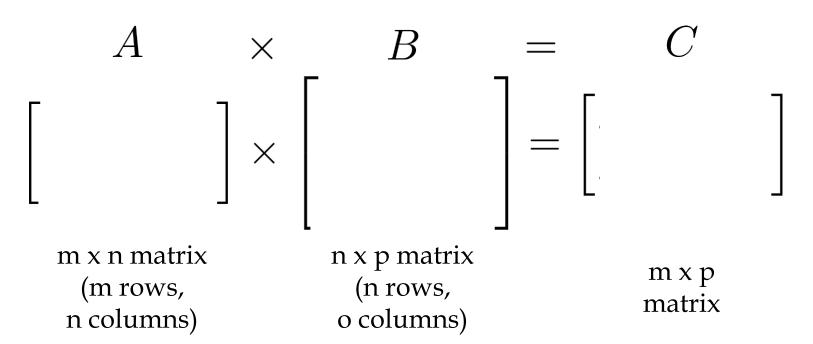
### **Matrix Operations**

□ Matrix-vector multiplication – The product of matrix  $A \in \mathbb{R}^{m \times n}$  and vector  $x \in \mathbb{R}^n$  is a vector of size  $\mathbb{R}^m$ , such that:



## **Matrix Operations**

□ Matrix-matrix multiplication – The product of matrices  $A \in \mathbb{R}^{m \times n}$  and  $B \in \mathbb{R}^{n \times p}$  is a matrix of size  $\mathbb{R}^{m \times p}$ , such that:



## **Matrix Multiplication Properties**

Let A and B be matrices. Then in general,  $A \times B \neq B \times A$ . (not commutative.)

E.g. 
$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

## **Matrix Multiplication Properties**

□ Identity matrix – The identity matrix  $I \in \mathbb{R}^{n \times n}$  is a square matrix with ones in its diagonal and zero everywhere else:

$$I = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 0 \end{pmatrix}$$

Remark: for all matrices  $A \in \mathbb{R}^{n \times n}$ , we have  $A \times I = I \times A = A$ .

#### **Matrix Multiplication Properties**

#### **Matrix inverse:**

If A is an m x m matrix, and if it has an inverse, then:

$$AA^{-1} = A^{-1}A = I.$$

## **Matrix Properties**

#### **Matrix Transpose**

Let A be an m x n matrix, and let  $B = A^T$ .

Then *B* is an n x m matrix, and  $B_{ij} = A_{ji}$ .

Example:

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$$

#### **Exercise**

Given 3 samples of information:  $x^{(1)}$ ,  $x^{(2)}$ ,  $x^{(3)} \in \mathbb{R}^5$ , create matrix X:

$$x^{(1)} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \\ 5 \end{bmatrix}, \quad x^{(2)} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 6 \\ 5 \end{bmatrix}, \quad x^{(3)} = \begin{bmatrix} 5 \\ 4 \\ 2 \\ 3 \\ 1 \end{bmatrix} \qquad \qquad X = \begin{bmatrix} -x^{(1)T} - \\ -x^{(2)T} - \\ -x^{(3)T} - \end{bmatrix} \in \mathbb{R}^{3x5}$$

Using Python, compute the mean and standard deviation of *X*, using the equations:

$$\mu_j = \frac{1}{m} \sum_{i=1}^m x_j^{(i)}$$
 $\sigma_j = \sqrt{\frac{1}{(m)} \sum_{i=1}^m (x_i^{(i)} - \mu_j)^2}$