

# Evolutionary Constrained Multiobjective Optimization: Scalable High-Dimensional Constraint Benchmarks and Algorithm

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## I. THE PROPOSED ALGORITHM

In MTCMO, one main task and one auxiliary task are created, and they use different CHTs to push the population to approach CPF. The main population uses the CDP method to maintain feasibility, and the auxiliary population uses the improved epsilon method to maintain diversity in the infeasible regions. Specially, the improved epsilon method regards the  $CV$  as an additional objective, and uses the multi-objective-based CHT to rank the population. By gradually reducing the constraint boundary, the auxiliary population can search in better infeasible regions and be distributed in the infeasible regions with good diversity. Through the two populations with different search characteristics, MTCMO can balance the constraints and objectives. However, its search ability is limited due to weak evolutionary operators. Thus, we propose a new search algorithm for MTCMO. Next, we will first give the framework and then introduce the new search algorithm.

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### Algorithm 1: The procedure of IMTCMO

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**Input:**  $NP$ : population size;  $MaxFES$ : maximal number of fitness evaluations.

**Output:** The feasible Pareto optimal solutions

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1  $P_1 \leftarrow$  Initialize  $NP$  individuals;
2  $P_2 \leftarrow$  Initialize  $NP$  individuals;
3 Evaluate  $P_1$  and  $P_2$ ;
4  $FES = 2 * NP$ ;
5 while  $FES_j = MaxFES$  do
6    $Off_1 \leftarrow$  Generate  $NP$  offspring using the new
     search algorithm  $\rightarrow$  Algorithm 2;
7    $Off_2 \leftarrow$  Generate  $NP$  offspring using the new
     search algorithm  $\rightarrow$  Algorithm 2;
8   Evaluate  $Off_1$  and  $Off_2$ ;
9    $FES = FES + 2 * NP$ ;
10   $P_1 \leftarrow P_1 \cup Off_1 \cup Off_2$ ;
11   $P_2 \leftarrow P_2 \cup Off_2 \cup Off_1$ ;
12   $P_1 \leftarrow$  Use the CDP method to select  $NP$ 
     individuals from  $P_1$ ;
13   $P_2 \leftarrow$  Use the improved epsilon method to select
      $NP$  individuals from  $P_2$ ;
14 end
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### A. Framework

Algorithm 1 describes the procedure of IMTCMO. First, two populations  $P_1$  and  $P_2$  are initialized with  $NP$  individuals

in the search space, respectively, and they will be evaluated on the CMOP. Then, each parent population uses the search algorithm to generate the offspring population, as shown in Algorithm 2. Next, the environmental selection operator is implemented to obtain the new parent population from the combination of the original parent population and two offspring populations. If computing resources are consumed up, the main loop will be stopped, and the found feasible Pareto optimal solutions will be output.

### B. Search algorithm

In the proposed search algorithm, exploration and exploitation are considered to balance global search ability and local search ability. Algorithm 2 provides the procedure of search algorithm. For local search, we consider the angle-based niching parent selection method [23] to maintain diversity. First, the objective values of all individuals are shifted to the new coordinate axis (Line 2), and the angle values among individuals are calculated (Line 3). Since the local search operator only generates half offspring individuals, population  $P$  is sorted randomly and the sorted population is represented by  $LP$ . For each of the former half individuals in  $LP$ , two neighbors  $ne_{i,1}$  and  $ne_{i,2}$  are first randomly selected from the  $Nr$  neighbors with the smallest angle values.  $Nr$  is set to 10. The scale factor  $F$  is randomly selected from three different values. Finally, the DE/rand/1 strategy and polynomial crossover are employed to generate offspring individuals. The main purpose of local search is to maintain local diversity, thus the niching method and DE/rand/1 strategy are used. Specially, the main population  $P_1$  can use the local search to locate multiple discrete feasible regions. The auxiliary population  $P_2$  can use the local search to fully explore the local infeasible regions.

For global search, we consider the DE/current-to-pbest/1 strategy to increase the feasibility and convergence rate. First,  $P_{best}$  individuals are identified based on their fitness values, in which parameter  $pa$  is set to 0.1. The fitness values of two populations are respectively obtained by the CDP method and the improved epsilon method. Then, the population  $P$  is sorted randomly, and the sorted population is represented by  $GP$ . For each of the former half individuals in  $GP$ , two random individuals are selected from  $GP$ , the learning example  $GP_{best}$  is randomly selected from  $P_{best}$ , and  $F$  is randomly selected from  $F_{set}$ . Finally, the DE/current-to-pbest/1 and polynomial crossover are implemented to obtain

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**Algorithm 2: Search algorithm**


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**Input:**  $NP$ : population size;  $P$ : the population;  $Z_{min}$ : the minimum objective values for  $P$ ;  $N_r$ : neighbor number;  $pa$ : a parameter

**Output:** The offspring population  $Off$

- 1  $obj \leftarrow$  Obtain the objective values of  $P$ ;
- 2  $obj \leftarrow obj - Z_{min}$  ;
- 3  $angle \leftarrow$  Calculate the angle value of each two individuals based on the  $obj$ ;
- 4  $LP \leftarrow$  Randomly sort the  $P$  ;
- 5 **for**  $i = 1 : NP/2$  **do**
- 6      $ne_{i,1}$  and  $ne_{i,2} \leftarrow$  Randomly select two neighbors from the  $N_r$  neighbors with the smallest  $angle$  values;
- 7      $F_i \leftarrow$  Randomly select one value from the set  $F_{set} = [0.6, 0.8, 1.0]$ ;
- 8      $off_{l,i} \leftarrow$  Generate one offspring using the DE/rand/1 mutation operator  $LP_i + F_i * (ne_{i,1} - ne_{i,2})$ ;
- 9      $off_{l,i} \leftarrow$  Implement the polynomial crossover;
- 10 **end**
- 11  $P_{best} \leftarrow$  Determine the former  $(NP * pa)$  individuals with the smaller fitness values;
- 12  $GP \leftarrow$  Randomly sort the  $GP$ ;
- 13 **for**  $i = 1 : NP/2$  **do**
- 14      $DP_{i,1}$  and  $DP_{i,2} \leftarrow$  Randomly select two different individuals from  $P$ ;
- 15      $GP_{best,i} \leftarrow$  Randomly select one individual from  $P_{best}$ ;
- 16      $F_i \leftarrow$  Randomly select one value from the set  $F_{set} = [0.6, 0.8, 1.0]$ ;
- 17      $off_{g,i} \leftarrow$  Generate one offspring using the DE/current-to-pbest/1 mutation operator  $GP_i + F_i * (GP_{best,i} - GP_i + DP_{i,1} - DP_{i,2})$ ;
- 18      $off_{g,i} \leftarrow$  Implement the polynomial crossover;
- 19 **end**
- 20  $off \leftarrow off_{l,i} \cup off_{g,i}$ ;

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the offspring individuals. The main purpose of global search is to enhance the feasibility and convergence rate.  $P_1$  can use the global search to make individuals close to the excellent individuals, so as to improve the population distribution.  $P_2$  can use the global search to speed up the flow of information, so as to find more promising regions.

**The link to published journal paper:**  
<https://ieeexplore.ieee.org/abstract/document/10139843/>

**This algorithm has already participated in CEC 2024 competition[1].**

#### REFERENCES

- [1] X. B. P. C. K. V. P. P. N. S. J. L. G. W. K. Qiao, X. Wen and C. Yue, "Evaluation criteria for cec 2024 competition and special session on numerical optimization considering accuracy and speed, tech. rep." 2024.